Weeks 10-11 Dynamic Games with Incomplete Information

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Outline

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Incorporate Incomplete Information

Incomplete information is particularly important in dynamic games

When the players have several moves in sequence, their earlier moves may convey private information that is relevant to the decisions of players moving later on.

Revealing vs. concealing information

- Basis of strategic behavior
- Useful explanation of actions

Subgame Perfectness is Not Enough

Idea of "perfectness"

In *dynamic games with symmetric information*, **subgame perfectness** captures the idea that each player optimizes at EVERY point of his moves, after reconsidering the situation as games proceed.

Singleton vs. non-trivial information set

- Each subgame starts with a singleton.
- Under incomplete information, many information sets are non-trivial.
- Optimization at each information set is not reflected in *subgame perfectness*.

Refer to the example of Entry Deterrence II.

A New Refinement

Recall

Nash equilibrium is a strategy profile consisting of mutual best responses.

- Each player maximizes his payoff, given others' strategies.

Here

The equilibrium notion is a combination of *strategies and beliefs*.

- Each player make the optimal choice, given others' strategies and his belief about states.
- Beliefs are updated based on what has occurred previously.

Perfect Bayesian Equilibrium

A **perfect Bayesian equilibrium (PBE)** is a strategy profile s and a set of beliefs μ such that at each information set of the game:

(1) Sequential Rationality The strategies for the remainder of the game are Nash given the beliefs and strategies of the other players.

(2) Belief Consistency The beliefs at each information set are rational given the evidence appearing thus far in the game (meaning that there are based, if possible, on priors updated by Bayes' Rule, given the observed actions of the other players under the hypothesis that they are in equilibrium).

Remarks

Strategy

Each strategy $s_i \in s$ is a contingent plan that calls for actions for each information set of player i.

Beliefs

The beliefs μ assign a probability distribution over each information set.

Belief updating

The beliefs μ are updated

- By Bayes rule, *on* the equilibrium path.
- Arbitrarily, *off* the equilibrium path. (Bayes rule does not apply.)

Outline

Entry Deterrence II

Players

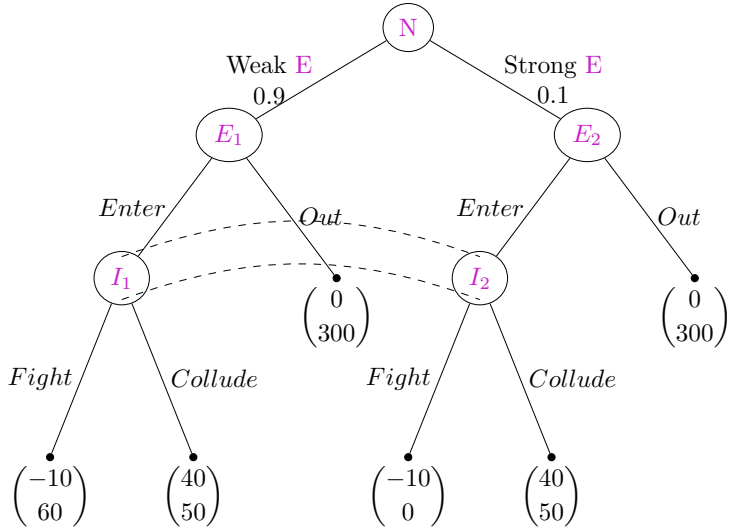
Two firms, Entrant and Incumbent.

The Order of Play

1. Nature chooses the type of Entrant, *Weak* or *Strong*.
2. Entrant is informed, but not Incumbent.
3. Entrant decides whether to *Enter* or *Stay Out*.
4. If Entrant enters, Incumbent can *Collude* with him, or *Fight* by cutting the price drastically.

Payoffs

- Monopoly: profits are 300
- Duopoly: profits are 100, split evenly.
- Entry cost: 10.
- To *Fight* a *Strong* Entrant is more costly than a *Weak* one.



Payoff: (Entrant, Incumbent)

Figure: Entry Deterrence II

Two Categories of PBE

The PBE can be divided into two distinct categories, depending on the outcome.

- **separating equilibrium**, the weak E stays out and the strong E enters.
- **pooling equilibrium**, both types enter or stay out.

Information in 3 Equilibria

What is the belief over the information set $\{I_1, I_2\}$?

- separating equilibrium:
- pooling equilibrium (enter):
- pooling equilibrium (out):

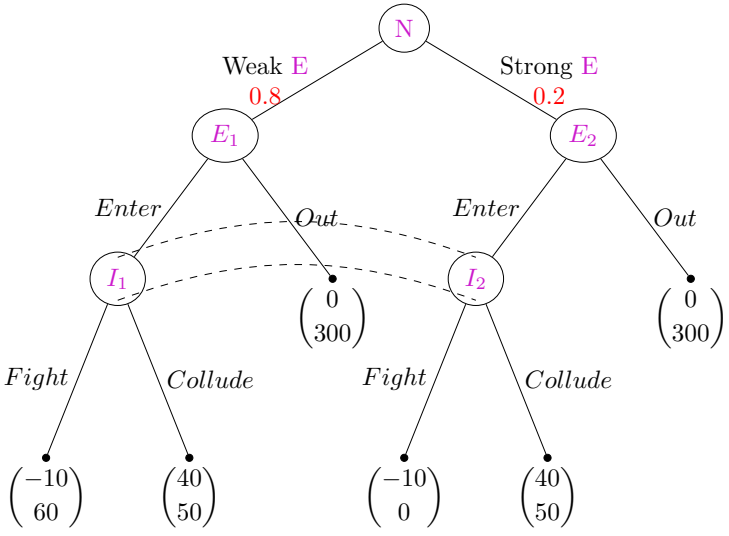
PBE in This Example

Is there a separating equilibrium?

Is there a pooling equilibrium where both types enter?

Is there a pooling equilibrium where both types stay out?

What if We Increase $Pr(Strong)$



Payoff: (Entrant, Incumbent)

Change in the PBE

Is there a separating equilibrium?

Is there a pooling equilibrium where both types enter?

Is there a pooling equilibrium where both types stay out?

No Separating Equilibrium

Irrespective of the prior probability, there is NO separating equilibrium.

- If the $Weak$ enters and the $Strong$ stays out, Incumbent would $Fight$. So the $Weak$ must stay out.
- If the $Strong$ enters and the $Weak$ stays out, Incumbent would $Collude$. But then the $Weak$ must enter.

Out-of-Equilibrium Beliefs

When Bayes rule does not apply

In the case when both types stay out, the posterior belief following *Enter* cannot be calculated by Bayes rule as below:

$$Prob(Weak|Enter) = \frac{Prob(Enter|Weak) \cdot Prob(Weak)}{Prob(Enter)}$$

Requirement of PBE

PBE does not put any restriction on out-of-equilibrium beliefs.

- Any probability is allowed, as long as *sequential rationality* is satisfied.

A Restriction on Out-of-Equilibrium Beliefs

Definition

A convenient way to form beliefs is retaining the prior after observing out-of-equilibrium actions, which is called **passive conjecture**.

When both types *Stay Out*

- $Prior(Weak) = 0.9 \rightarrow Prob(Weak|Enter) = 0.9$
- $Prior(Weak) = 0.8 \rightarrow Prob(Weak|Enter) = 0.8$

Question

Under the passive conjecture, is it still an equilibrium where both types choose *Stay Out*?

Outline

PhD Admission

Players

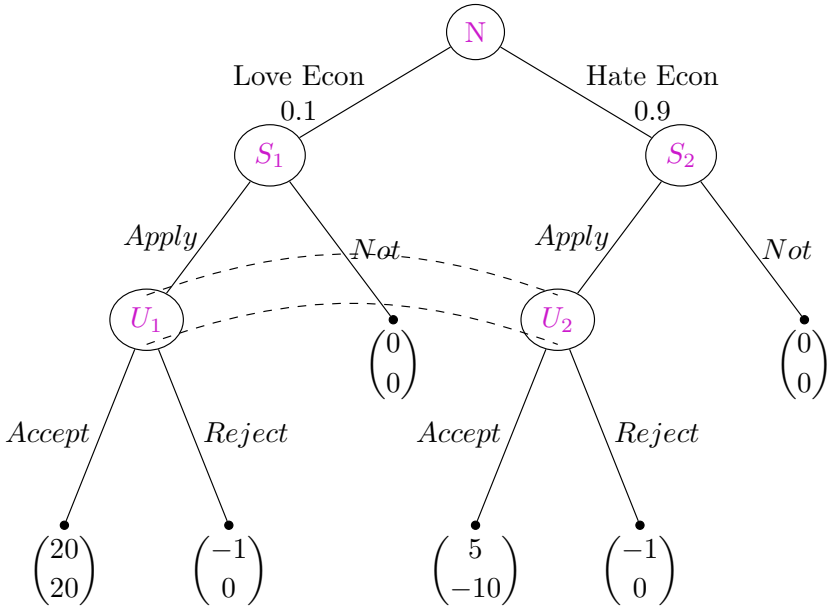
University and Student.

The Order of Play

1. Nature chooses the type of Student, *Love* or *Hate* Economics.
2. Student is informed, but not University.
3. Student decides whether to *Apply* to University.
4. If Student applies, University can *Accept* or *Reject* the application.

Payoffs

- Student wants to get into the PhD program, while the benefit for a student who is truly interested in Economics is larger.
- Application fee =1
- University only wants to recruit students with high motivations.



Payoff: (Student, University)

Is There Pooling Equilibrium?

A More Interesting Equilibrium

Please solve for a PBE where S_1 applies, S_2 probably applies, and U only accepts a proportion of applicants.

Vocabulary

perfect Bayesian equilibrium	完美贝叶斯均衡	passive conjecture	消极推测
separating equilibrium	分离均衡	pooling equilibrium	混同均衡
PhD admission	博士录取	out of equilibrium	非均衡路径