Week 11 Moral Hazard: Hidden Actions

Wenhao Wu
wuwh2@shanghaitech.edu.cn

SEM, ShanghaiTech U

2024 Spring

Outline

Outline

上海科技大学 ShanghaiTech University

Economists' answers in response to peculiar behavior that seems to contradict basic theory:

- Incomplete competition
- Market failure
- Price discrimination
- **Information asymmetry**

The Principal-Agent Model

Two representative players

The **principal** (or **uninformed player**) has less information than the **agent** (or **informed player**).

What they do

The principal (P) hires an agent (A) to perform a task, and the agent acquires an informational advantage about his type, his actions, or the outside world at some point of the game. P and A can make a binding **contract** at some point in the game, based on which P pays A an agreed sum if he observes a certain outcome.

Four Categories

- **Moral Hazard**: P and A begin with symmetric information and agree to a contract, but then A takes an action unobserved by P.
- **Adverse Selection**: Nature begins the game by choosing A's type, unobserved by P. P and A then agree to a contract.
- **Signaling**: Nature begins the game by choosing A's type, unobserved by P. To demonstrate his type, A takes actions that P can observe. Then they agree to a contract.
- **Screening**: Nature begins the game by choosing A's type, unobserved by P. Then they agree to a contract. A takes actions that reveal information about his type.

Illustrations

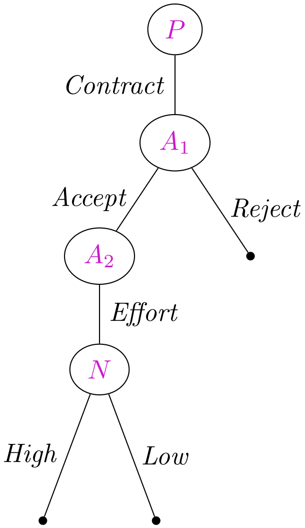


Figure: Moral Hazard

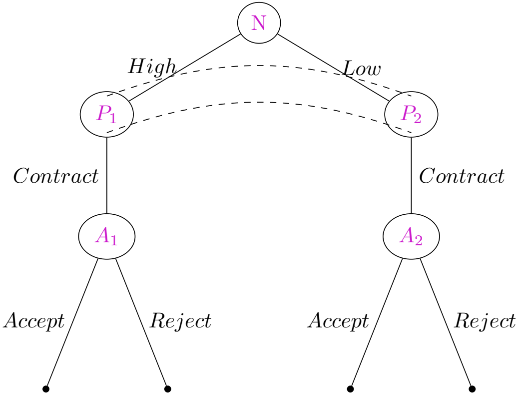


Figure: Adverse Selection

Illustrations

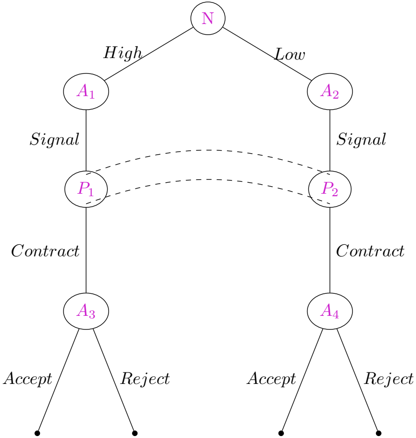


Figure: Signaling

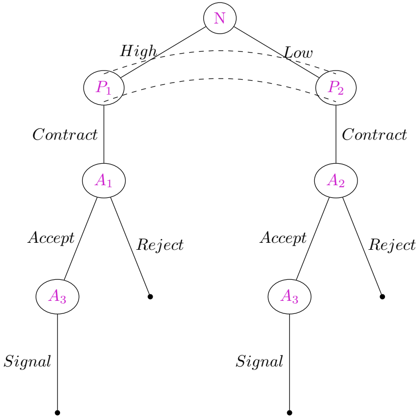


Figure: Screening

上海科技大学
ShanghaiTech University

Applications

	Principal	**Agent**	Type or Signal
Moral hazard	Insurance company	Policyholder	Care to avoid theft
	Insurance company	Policyholder	Drinking or Smoking
	Bondholders	Stockholders	Riskiness of corporate projects
Adverse selection	Insurance company	Policyholder	Infection with HIV
	Employer	Worker	Skill
	Buyer	Seller	Used car quality
Signaling and Screening	Employer	Worker	Education
	Investor	Stock issuer	Stock value and percentage retained

Outline

The Production Game

Players

The Principal and the Agent.

The Order of Play

1. P offers A a wage contract w.
2. A decides whether to accept or reject it.
3. If A accepts, he exerts effort e.
4. Output equals $q(e)$, where $q' > 0$.

Payoffs

- If A rejects, $\pi_A = \bar{U}$ and $\pi_P = 0$, where $\bar{U}$ is a real number.
- If A accepts, then $\pi_A = U(e, w)$ and $\pi_P = V(q(e) - w)$.

上海科技大学 ShanghaiTech University

Interpretations

Function properties

$U(e, w)$ decreases in e and increases in w.
$V(q - w)$ increases in $q - w$.

A's outside option

$\bar{U}$ is A's **reservation utility**, which is the minimum for which he will accept the job.

P moves first

The order of play allows P to make a **take-it-or-leave-it** offer, leaving A little bargaining room as if he had to compete with multiple other A's.

Solving for the Optimal Contract

Problem for P

$$\max_{\tilde{w}(\cdot)} V(q(e^*) - \tilde{w}(e^*)) \tag{1}$$

subject to

$$e^* = \arg\max_{e} U(e, \tilde{w}(e))$$

$$U(e^*, \tilde{w}(e^*)) = \bar{U} \tag{2}$$

上海科技大学 ShanghaiTech University

First Order Condition

Given that $\tilde{w}$ is a continuous function, FOC of Eq. (1) is

$$V'(q(e) - \tilde{w}(e)) \left(\frac{\partial q}{\partial e} - \frac{\partial \tilde{w}}{\partial e} \right) = 0$$

We need that

$$\frac{\partial q}{\partial e} - \frac{\partial \tilde{w}}{\partial e} = 0$$

First Order Condition

From Eq. (2), Implicit Function Theorem implies that

$$\frac{\partial \tilde{w}}{\partial e} = -\left(\frac{\partial U/\partial e}{\partial U/\partial \tilde{w}}\right)$$

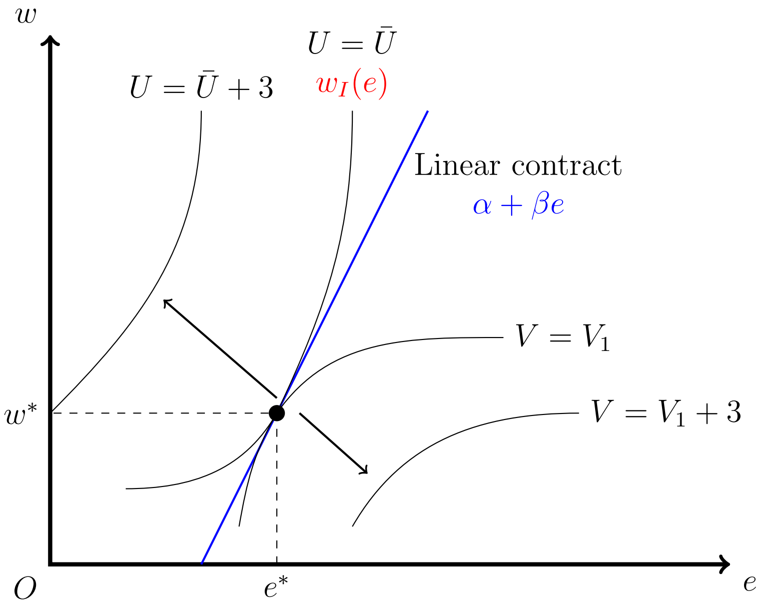
Finally, we have

$$\left(\frac{\partial U}{\partial \tilde{w}}\right)\left(\frac{\partial q}{\partial e}\right) = -\left(\frac{\partial U}{\partial e}\right) \tag{3}$$

Interpretation of Eq.(3)

- LHS: marginal benefit of effort
- RHS: marginal cost of effort

Indifference Curves



Shape of Indifference Curves

Assuming

- $\partial^2 U/\partial w \partial e < 0$ High-income worker dislike working more.
- $\partial^2 U/\partial e^2 < 0$ (increasing marginal cost)
- $q'' < 0$ (diminishing returns to effort)

We can verify by the Implicit Function Theorem.

Optimal Linear Contracts

Slopes of indiff. curves

The picture shows indifference curves of P and A that slope upwards, because the effect of having *higher wage* should be canceled out by putting in *more effort* or equivalently, yielding *higher outcome*.

Exploiting A

Under perfect competition among A's, his profit is 0, i.e., Eq. (2) is satisfied.

Optimal contract: $w = \alpha + \beta e$

Under this contract,

- A maximizes his utility by choosing e^* and P pays w^*.
- V is maximized provided that Eq.(2) is satisfied.

A Parametric Example

Assumptions

- Output: $q(e) = 100log(1+e)$
- Reservation utility: $\bar{U} = 3$
- A's payoff: $U(e,w) = log(w) - e^2$
- P's payoff: $\pi_P = q(e) - w(e)$

Solving the Problem

Indiff. curve $w_I(e)$ for A

Eq.(2) becomes $log(w_I(e)) - e^2 = 3$, which translates to

$$w_I(e) = Exp(3 + e^2) \tag{4}$$

Exp() is the exponential function.

FOC

Eq.(3) translates to

$$\left(\frac{1}{w}\right)\left(\frac{100}{1+e}\right) = 2e \tag{5}$$

Solving the Problem

Plug Eq.(4) into Eq.(5), we obtain that

$$\left(\frac{100}{1+e}\right) - 2e(Exp(3+e^2)) = 0$$

Solution

Solving this equation by computer, we have $e^* = 0.77$, $w^* = 37$, $q^* = 57$, $\pi_A = 3$, $\pi_P = 20$.

Linear contract

At e^*, the derivative of w_I equals 56.

$w = -7 + 56e$

Outline

Constrained optimization problem

In moral hazard problems, P maximizes his utility knowing that

- A can reject the contract entirely
- He must give A an incentive to choose the desired effort

These two constraints are named the **participation constraint** and the **incentive compatibility constraint**.

In Math

Problem for P

$$\max_{\tilde{w}(\cdot)} V(q(e^*) - \tilde{w}(e^*))$$

subject to

$$e^* = \arg\max_{e} U(e, \tilde{w}(e)) \qquad \text{(Incentive Compatibility)}$$

$$U(e^*, \tilde{w}(e^*)) = \bar{U} \qquad \text{(Participation)}$$

Outline

上海科技大学 ShanghaiTech University

The Purposes of Studying this Game

Outcome-based vs. effort-based contracts

In the Production Game, the contract is contingent on A's effort. But there are cases where effort cannot be observed or verified by P. In Broadway Game, the contract is based on the outcome.

Method

Instead of using the First-order approach, we will simply use the incentive compatibility and participation constraints to solve for the optimal contract.

Broadway Game

Players

The Investor and the Producer.

The Order of Play

1. I offer a wage contract $w(q)$ as a function of revenue q.
2. P accepts or rejects the contract.
3. If accept, P chooses to *Embezzle* or *Be Honest*.
4. Nature picks the profit q with probability depending on P's action.

Payoffs

- $$\pi_P = \begin{cases} U(w(q) + 50), & \text{if Embezzle} \\ U(w(q)), & \text{if Honest} \end{cases}$$
- $\pi_I = q - w(q)$.

Assumptions

Utility Function of P

- $U(w) = 100w - 0.1w^2$
- $\bar{U} = U(100)$

q	-100	100	500
Embezzle	70%	20%	10%
Honest	10%	20%	70%

Table: Profit Distribution and Action

上海科技大学 ShanghaiTech University

Two Constraints

Incentive compatibility (be honest)

$$0.1U(w(-100)) + 0.2U(w(100)) + 0.7U(w(500)) \geq$$
$$0.7U(w(-100) + 50) + 0.2U(w(100) + 50) + 0.1U(w(500) + 50)$$

Participation

$$0.1U(w(-100)) + 0.2U(w(100)) + 0.7U(w(500)) \geq U(100)$$

Solution - solving by computer

Optimal contract

$$w(q) = \begin{cases} 29, & \text{q=-100} \\ 102, & \text{q=100} \\ 110, & \text{q=500} \end{cases}$$

Utility

- $\pi_P = 9000 = U(100)$
- $\pi_I = 259$

Vocabulary

moral hazard	道德风险	adverse selection	逆向选择
signaling	信号传递	screening	信息甄别
principal	委托人	agent	代理人
reservation utility	保留效用	linear contract	线性合同
incentive compatibility	激励相容	participation constraint	参与约束
production game	生产博弈	Broadway game	百老汇博弈
embezzle	贪污		