1 The rules of game

PAPI: Players, Actions, Payoffs, Information Dominant Strategy: A strategy is dominant if it is the best strategy for A regardless of what B does. Dominated Strategy: A strategy is dominated if there is another strategy that always gives a higher payoff. For player i, strategy si is weakly dominated if there exists some other strategy s i' which is possibly better and never worse, yielding a higher payoff in some strategy profile and never yielding a lower payoff.
Nash Equilibrium: The strategy profile s* is a Nash equilibrium if no player has incentive to deviate from his strategy given that the other players do not deviate.
Perfect: Each information set is a singleton. Symmetric: No player has information different from other players when he moves, or at the end nodes. Asymmetric: Someone has private information . Complete: Nature does not move first, or her initial move is observed by every player.
Harsanyi Transformation: Harsanyi suggests that to deal with a game with unclear rules, we can transform the game by adding an initial move in which Nature chooses between different sets of rules. In the transformed game, all players know the new meta-rules, including the fact that Nature has made an initial move unobserved by them. A state of the world is a move by Nature.
Harsanyi Doctrine: Different beliefs is modelled as the effect of observing different moves by Nature. Players start with the same probabilities of Nature moves.
Herding Effect: One Bayesian Equilibrium looks like this: Player 1: If sees signal high, accept; if sees signal low, reject. Player 2: imitates player 1's action, regardless of the value of his signal. Player 3: imitates 1 and 2's action, regardless of the value of his signal.
The welfare game:

The Welfare Game

Example
A government wishes to aid a pauper if he searchers for work but not otherwise, and a pauper who searches for work only if he cannot depend on government aid.

		Pauper	
		Work (γ_w)	*Loaf* ($1-\gamma_w$)
Government	*Aid* (θ_a)	3, 2	-1, 3
	No Aid (1-θ_a)	-1, 1	0, 0

;

Definitions Payoff-Equating Method Symmetric Mixed Strategy Equilibrium Randomizing vs. Mixing The Cournot Game

How Government Responds?

Government's expected payoff

$$\begin{aligned}\pi_G &= \theta_a[3\gamma_w + (-1)(1-\gamma_w)] + [1-\theta_a][-\gamma_w + 0(1-\gamma_w)]\\ &= \theta_a[3\gamma_w - 1 + \gamma_w] - \gamma_w + \theta_a\gamma_w\\ &= \theta_a[5\gamma_w - 1] - \gamma_w\end{aligned}$$

Government chooses θ_a

- If $\gamma_w > 0.2$, $\theta_a = 1$.
- If $\gamma_w = 0.2$, $\theta_a \in [0,1]$.
- If $\gamma_w < 0.2$, $\theta_a = 0$.

Solving for θ_a

$$\pi_P(Work) = 2\theta_a + (1-\theta_a) = 3\theta_a + 0(1-\theta_a) = \pi_P(Loaf)$$

The solution is $\theta_a = 0.5$

Solving for γ_w

$$\pi_G(Aid) = 3\gamma_w - (1-\gamma_w) = -\gamma_w + 0(1-\gamma_w) = \pi_G(No\,Aid)$$

The solution is $\gamma_w = 0.2$

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In the mixed strategy equilibrium, Government must be indifferent between Aid and No Aid and Pauper must be indifferent between Work and Loaf.
War of Attrition:

- The War of Attrition is like *Chicken* stretched out over time.
- Smith and Jones control two firms in an industry which is natural monopoly, with demand strong enough for one firm to operate profitably, but not two.
- There could be infinite periods.
- In each period, the possible actions are to *Exit* or to *Continue*.
  - Both continue, each earns -1. The game continues.
  - Continue and the opponent exits, earns 3. The game ends.
  - Exit, earns 0. The game ends.
- Discount rate r.

Solve for a symmetric equilibrium

Each player exits with probability θ in any period.

V_{stay}: the expected discounted value of Smith's payoffs if he stays.
V_{exit}: the expected discounted value of Smith's payoffs if he exits.

$$V_{stay} = 3\theta + (1-\theta)\left(-1 + \frac{V_{stay}}{1+r}\right) \quad (1)$$

$$V_{stay} = V_{exit} = 0 \quad (2)$$

Combining Eq. (1)(2), we have $\theta = \frac{1}{4}$.

Patent Game: The cumulative distribution function for N-players game is $M(x) = (x/V)^{1/(N-1)}$
The Cournot Game:

Nash Equilibrium

The best response of Company A to q_b

$q_b = 0$, $q_a = 60$.
Generally, given q_b, the problem for Company A is

$$\max_{q_a} 120q_a - q_a^2 - q_aq_b$$

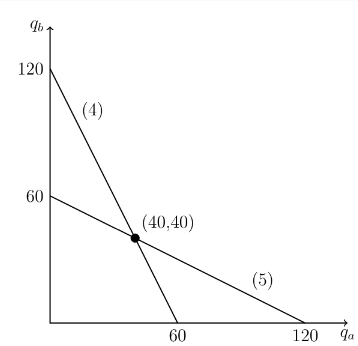
$$F.O.C. \quad q_a = 60 - q_b/2 \quad (4)$$

Eq.(4) is the **best response function for Company A**.

The best response function for Company B

Similarly, we have

$$q_b = 60 - q_a/2 \quad (5)$$



Equilibrium Path: is the path through the game tree that is followed in equilibrium. If a player deviates from the equilibrium strategy, we say it is off the equilibrium path.

A subgame is a game consisting of a node which is a singleton in every player's information partition, that node's successors, and the payoffs at the associated end nodes.
A strategy profile is a subgame perfect equilibrium (SPE) if (a) it is a Nash equilibrium for the entire game; and (b) its relevant action rules are a Nash equilibrium for every subgame. The small probability of a mistake is called a tremble. The trembling hand approach is featured by allowing players to make mistakes with small probabilities and examining whether a Nash equilibrium is robust to this change in the game.
Chainstore Paradox: SPE: Grim Strategy(Start by choosing Deny. If the other player ever chooses Confess, then choose Confess forever after.) Tit-for-Tat(Start by Start by choosing Deny. In period n, choose the action that the other player chose in period (n - 1)) Tit-for-Tat is not an equilibrium strategy
Minimax Strategy and the Folk Theorem(Note that the crossing part is the The set of SPE payoff profiles)

The set of strategies s^*_{-i} is a set of $(n-1)$ **minimax strategies** chosen by all the players except i to keep i's payoff as low as possible, no matter how he responds. s^*_{-i} solves

$$\min_{s_{-i}} \max_{s_i} \pi_i(s_i, s_{-i}) \qquad (1)$$

Player i's **minimax payoff**, **minimax value**, or **security value** is his payoff from the solution of Eq. (1).

Interpretation: Everybody punishes player i and he protects himself as best as he can.

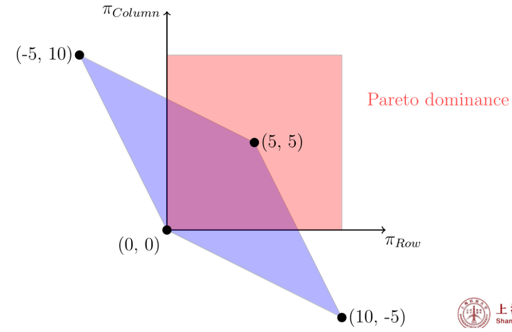
Theorem (The Folk Theorem)

In an infinitely repeated n-person game with finite action sets at each repetition, any combination of actions observed in any finite number of repetitions is the unique outcome of some subgame perfect equilibrium given

Condition 1 δ *is sufficiently large; and*

Condition 2 *The set of payoff profile Pareto dominates the minimax payoff profile in the mixed extension of the stage game.*

- Pareto dominance: all players get better off than certain levels.
- Mixed extension: the convex hull of payoff points.



Perfect Bayesian Equilibrium: Separating Equilibrium(Can update beliefs) and Pooling Equilibrium(No update beliefs). When the equilibrium is out of the path, you need to somehow choose your beliefs, whether it is passive conjecture or others.

- **Moral Hazard**: P and A begin with symmetric information and agree to a contract, but then A takes an action unobserved by P.
- **Adverse Selection**: Nature begins the game by choosing A's type, unobserved by P. P and A then agree to a contract.
- **Signaling**: Nature begins the game by choosing A's type, unobserved by P. To demonstrate his type, A takes actions that P can observe. Then they agree to a contract.
- **Screening**: Nature begins the game by choosing A's type, unobserved by P. Then they agree to a contract. A takes actions that reveal information about his type.

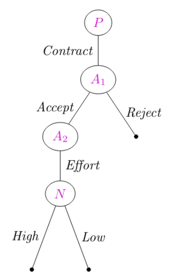


Figure: Moral Hazard

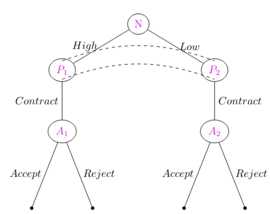


Figure: Adverse Selection

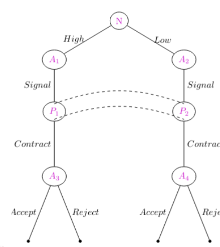


Figure: Signaling

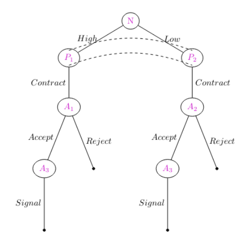


Figure: Screening

Players
The Principal and the Agent.
The Order of Play
1. P offers A a wage contract w.
2. A decides whether to accept or reject it.
3. If A accepts, he exerts effort e.
4. Output equals $q(e)$, where $q' > 0$.

Payoffs
- If A rejects, $\pi_A = \bar{U}$ and $\pi_P = 0$, where $\bar{U}$ is a real number.
- If A accepts, then $\pi_A = U(e, w)$ and $\pi_P = V(q(e) - w)$.

$$\max_{\tilde{w}(\cdot)} V(q(e^*) - \tilde{w}(e^*)) \qquad (1)$$

$$e^* = \arg\max_e U(e, \tilde{w}(e))$$

$$U(e^*, \tilde{w}(e^*)) = \bar{U} \qquad (2)$$

$$\left(\frac{\partial U}{\partial \tilde{w}}\right)\left(\frac{\partial q}{\partial e}\right) = -\left(\frac{\partial U}{\partial e}\right) \qquad (3)$$

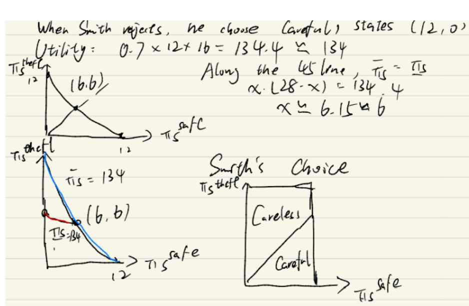
Linear contract

At e^*, the derivative of w_I equals 56.
$w = -7 + 56e$

Production Game: Incentive compatibility: the payoff to be honest is no less than the payoff to be embezzle. Participation: the payoff to be honest is higher than Reservation utility

Translation of Utility Functions

$$\begin{aligned} \bar{\pi}_S &= 0.5U(\pi_S^{safe}) + 0.5U(\pi_S^{theft}) \\ \bar{\pi}_I &= 6 - 0.5\pi_S^{safe} - 0.5\pi_S^{theft} \\ \underline{\pi}_S &= 0.25U(\pi_S^{safe}) + 0.75U(\pi_S^{theft}) \\ \underline{\pi}_I &= 3 - 0.25\pi_S^{safe} - 0.75\pi_S^{theft} \end{aligned}$$



Education III has no pooling equilibrium, because if one E tried to offer the zero profit pooling contract, $w(0) = 4$, the other E would offer $w(1) = 6$ to draw away the High type.

In this case, the Low type sticks with $s = 0$, because
$U_L(s = 0) = 4 \geq 2 = U_L(s = 1)$.
And the High type wants to deviate, since
$U_H(s = 1) = 6 - \frac{4}{3} = 4.66 > 4 = U_H(s = 0)$

Separating Equilibrium

$$\begin{cases} w(0) = 2, \quad w(1) = 6, \\ s(Low) = 0, \quad s(High) = 1. \end{cases}$$

Signaling: Separating Equilibrium and Pooling Equilibrium are all under participation and incentive compatibility.