Weeks 3, 4 Information

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Outline

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Simultaneous vs. Sequential Moves

Simultaneous-move game

Action = Strategy

Sequential-move game

The second player has *information*.

How to describe information?

- Information set in a game tree
- Harsanyi transformation (prior belief)
- Bayes rule (posterior belief)

The Extensive Form and the Game Tree

- A **node** is a point in the game at which some player or Nature takes an action, or the game ends.
- A **successor** to node X is a node that may occur later in the game if X has been reached.
- A **predecessor** to node X is a node that must be reached before X can be reached.
- A **starting node** is a node with no predecessors.
- An **end node** or **end point** is a node with no successors.
- A **branch** is one action in a player's action set at a particular node.
- A **path** is a sequence of nodes and branches leading from the starting node to an end node.

The Extensive Form and the Game Tree

The **extensive form** is a description of a game consisting of
(1) A configuration of nodes and branches running without any closed loops from a single starting node to its end nodes.
(2) An indication of which node belongs to which player.
(3) The probabilities that *Nature* uses to choose different branches at its nodes.
(4) The information sets into which each player's nodes are divided.
(5) The payoffs for each player at each end node.

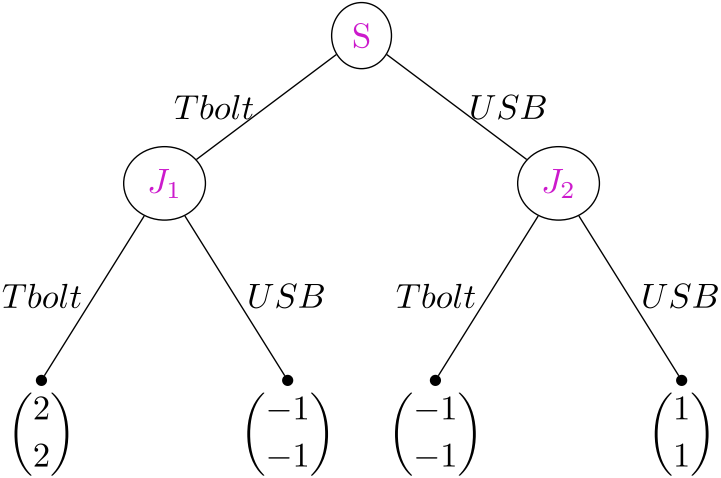
The **game tree** is the same as the extensive form except that (5) is replaced with
(5') The outcomes at each end node.

Follow-the-Leader

Example

Smith and Jones are computer manufacturers. They decide whether to use Thunderbolt or USB port. Both players will sell more if their interfaces are compatible. Smith makes his choice first. After that, Jones sees Smith's choice and then make his choice.

Follow-the-Leader



Strategic Form

The **strategic form** (or **normal form**) consists of
(1) All possible *strategy profiles* $\{s\}$;
(2) Payoff function π_i for each player i.

The Strategies for Jones

$$\left\{\begin{array}{l} \textit{Tbolt} \text{ if Smith chooses } \textit{Tbolt},\ \textit{USB} \text{ if Smith chooses } \textit{USB}. \\ \textit{USB} \text{ if Smith chooses } \textit{Tbolt},\ \textit{Tbolt} \text{ if Smith chooses } \textit{USB}. \\ \textit{Tbolt} \text{ No Matter What.} \\ \textit{USB} \text{ No Matter What.} \end{array}\right\}$$

which we will abbreviate as

$$\left\{\begin{array}{c} (T|T, U|U) \\ (U|T, T|U) \\ (T|T, T|U) \\ (U|T, U|U) \end{array}\right\}$$

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Strategic Form of Follow-the-Leader

	T|T, U|U	U|T, T|U	T|T, T|U	U|T, U|U
T	$\boxed{2}$, $\boxed{2}$	-1, -1	$\boxed{2}$, $\boxed{2}$	-1, -1
U	1, 1	-1, -1	-1, -1	$\boxed{1}$, $\boxed{1}$

Table: Strategic Form

Equilibrium	Strategies	Outcome
E_1	$\{T, T\|T, U\|U\}$	Both pick Tbolt
E_2	$\{T, T\|T, T\|U\}$	Both pick Tbolt
E_3	$\{U, U\|T, U\|U\}$	Both pick USB

Table: Equilibria, and equilibium outcomes

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Strategic Form of Follow-the-Leader

	T|T, U|U	U|T, T|U	T|T, T|U	U|T, U|U
T	$\boxed{2}$, $\boxed{2}$	-1, -1	$\boxed{2}$, $\boxed{2}$	-1, -1
U	1, 1	-1, -1	-1, -1	$\boxed{1}$, $\boxed{1}$

Table: Strategic Form

	Equilibrium	**Strategies**	**Outcome**
Sensible	E_1	$\{T, T\|T, U\|U\}$	Both pick Tbolt
Not Sensible	E_2	$\{T, T\|T, T\|U\}$	Both pick Tbolt
Not Sensible	E_3	$\{U, U\|T, U\|U\}$	Both pick USB

Table: Equilibria, and equilibium outcomes

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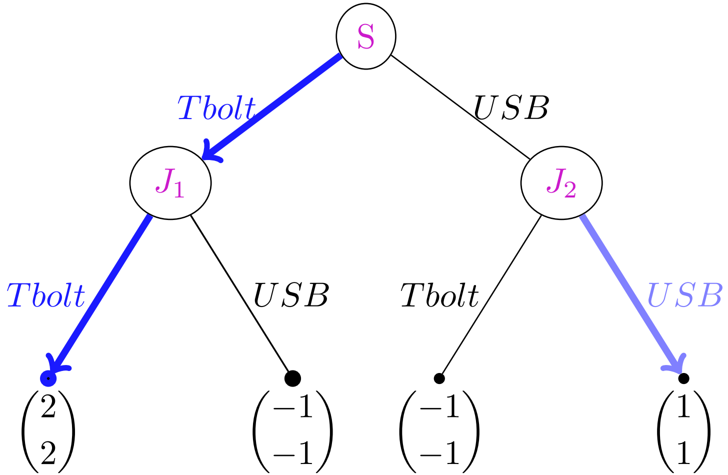
Representation of One Equilibrium



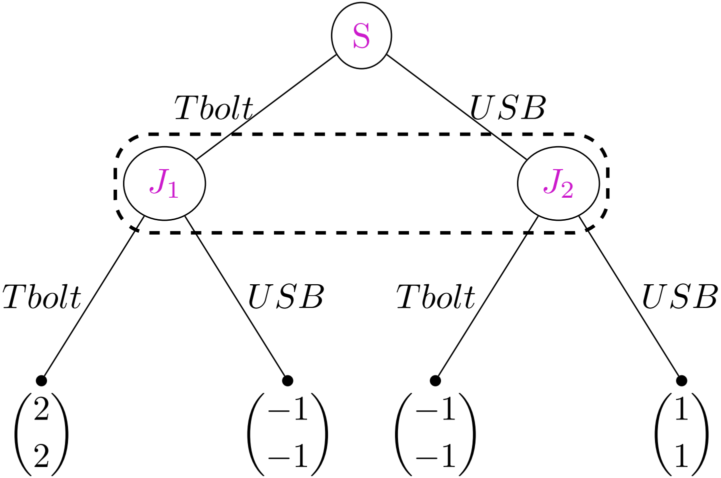
Ranked Coordination

Example

If Smith and Jones move simultaneously instead of sequentially, then we have another completely different game, called *Ranked Coordination*. In what follows we present its strategic and extensive forms.

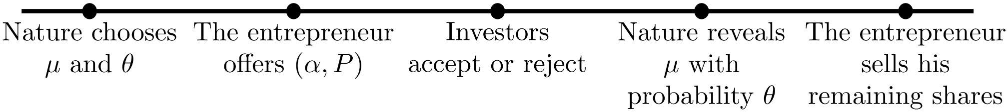
$$
\begin{array}{ccc|cc}
 & & & \multicolumn{2}{c}{\text{Jones}} \\
 & & & \textit{Thunderbolt} & \textit{USB} \\
\text{Smith} & \textit{Thunderbolt} & & 2,\ 2 \ \longleftarrow & -1,\ -1 \\
 & & & \uparrow & \downarrow \\
 & \textit{USB} & & -1,\ -1 \ \longrightarrow & 1,\ 1
\end{array}
;
$$

Extensive Form for *Ranked Coordination*



Time Line

The **time line** shows the order of events.
It is particularly useful for games with continuous strategies, exogenous arrival of information, and multiple periods games.

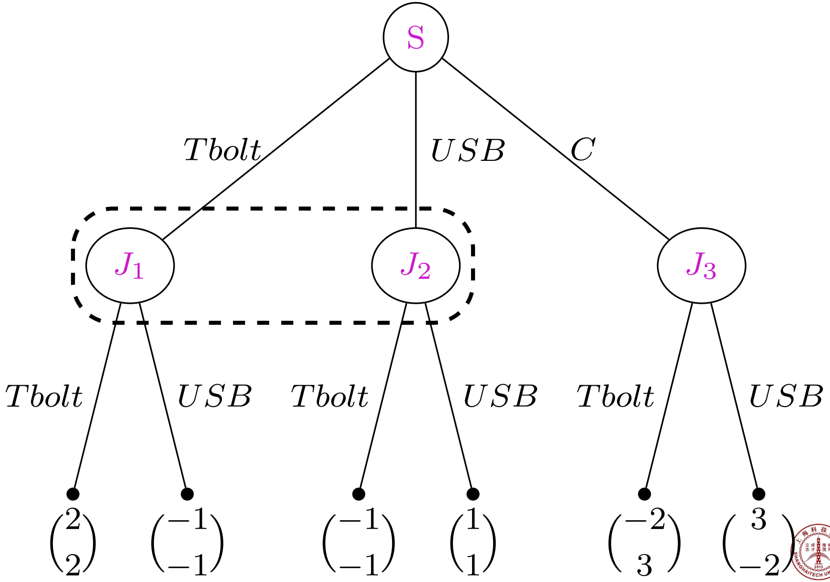


Outline

Who Knows What, and When?

Player i's **information set** ω_i at any particular point of the game is the set of different nodes in the game tree that he knows might be the actual node, but between which he cannot distinguish by direct observation.

Follow-the-Leader II



Features of Information Sets

The idea behind a "cloud"

- Nodes in an information set belong to *one* player, but on *different* paths.
- The player knows it is his turn.
- The player does not know the *exact* location the game has reached.

Restrictions on information sets

- One node cannot belong to two different information sets.
- The action sets must be the same at all nodes within one "cloud."

The Set of Information Sets

Player i's **information partition** is a collection of his information sets such that:
(1) Each path is represented by one node in a single information set in the partition, and
(2) (**Perfect Recall**) Players know everything they have known before, e.g. the actions taken publicly or the states of the world they knew before.

An information set that contains only one node is called a **singleton**.

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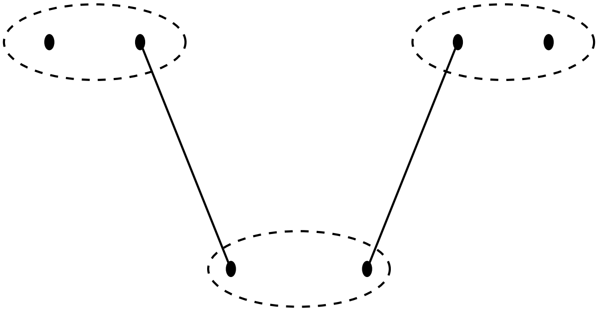
An Example that Violates Perfect Recall

If these three information sets belong to the **same** player, then the following situation does not satisfy the assumption of perfect recall.



Common Knowledge

Information is **common knowledge** if it is known to all the players, if each player knows that all the players know it, if each player knows that all the players know that all the players know it, and so forth *ad infinitum*.

The rules of the game (or game tree) is assume to be common knowledge among players.

Belief Hierarchy

Belief Hierarchy

“ I know you think you understand what you thought I said but I'm not sure you realize that what you heard is not what I meant”

– Alan Greenspan

Information Categories

We categorize the information structure of a game in 3 different ways.

Category	**Meaning**
Perfect	Each information set is a singleton.
Symmetric	No player has information different from other players when he moves, or at the end nodes.
Asymmetric	Someone has **private information** .
Complete	Nature does not move first, or her initial move is observed by every player.

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Poker Examples of Information Classification

1. All cards are dealt face up.
2. All cards are dealt face down and a player cannot look even at his own cards before he bets.
3. All cards are dealt face down, and a player can look at his own cards.
4. All cards are dealt face up, but each player then scoops up his hand and secretly discards one card.
5. All cards are dealt face up, the players bet, and then each player receives one more card face up.
6. All cards are dealt face down, but then each player scoops up his cards without looking at them and holds them against his forehead so all the *other* players can see them (Indian poker).

Poker Examples of Information Classification

1. Perfect, Symmetric, Complete
2. Imperfect, Symmetric, Incomplete
3. Imperfect, Asymmetric, Incomplete
4. Imperfect, Asymmetric, Complete
5. Perfect, Symmetric, Complete
6. Imperfect, Asymmetric, Incomplete

Outline

A Game With Unclear Rules

Think of a game where players might not know the "rules" of the game. That is, they are uncertain about:

- payoff functions
- the strategies available to various players
- the information other players have about the game

Until 1967, game theorists thought that this kind of game cannot be analyzed.

Harsanyi (1967) pointed out that any game with *unclear* rules could be remodelled as a game with *clear* rules without changing its essentials.

Harsanyi Transformation

Harsanyi suggests that to deal with a game with unclear rules, we can **transform** the game by adding an initial move in which Nature chooses between different sets of rules.

In the transformed game, all players know the new meta-rules, including the fact that Nature has made an initial move *unobserved* by them.

A **state of the world** is a move by Nature.

Follow-the-Leader III

- Smith *knows* the structure of the game.
- Jones *does not know* the structure of the game.
- Jones believes that the states of the world is A with 70%, B with 10%, and C with 20%.
- Smith knows Johns' beliefs.

There are Three States of the World

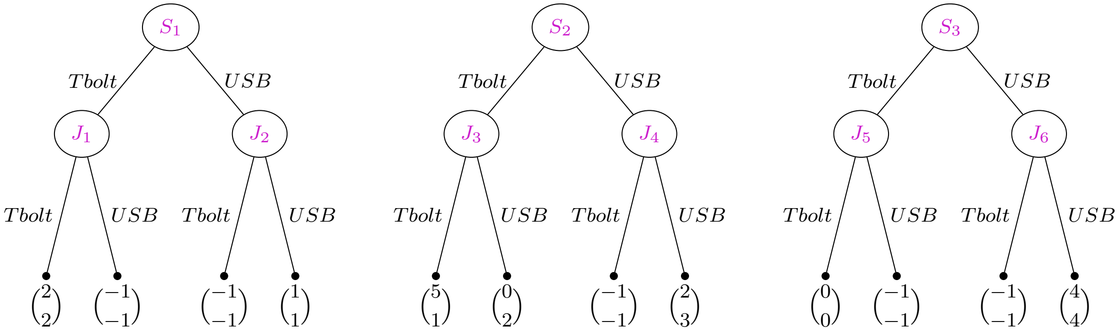
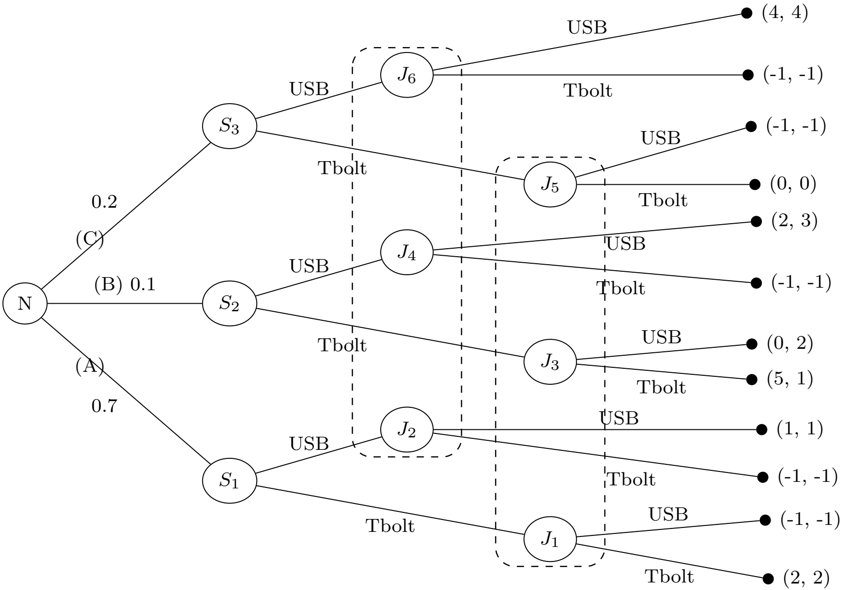


Figure: (A) 70%

Figure: (B) 10%

Figure: (C) 20%

Harsanyi Transformation



Harsanyi Doctrine

A player's **type** is the strategy set, information partition, and payoff function which nature chooses for him at the start of a game of incomplete information.

People's Opinions

- Jones has opinions about Smith's possible type.
- Smith knows what Jones' possible opinions are.
- Jones knows his opinions are just opinions. . .

Harsanyi Doctrine

- Different beliefs is modelled as the effect of observing different moves by Nature.
- Players start with the *same* probabilities of Nature moves.

Updating Beliefs with Bayes Rule

Players start with **prior beliefs** (or **priors**) concerning the types of other players. Then they update their beliefs to **posterior beliefs** in the course of the game, under the assumption that *players are following equilibrium behavior.*

Bayesian Equilibrium is a Nash equilibrium in which players update their beliefs according to Bayes' rule.

Checking Bayesian Equilibrium

1. Propose a strategy profile.
2. See what beliefs the strategy profile generates when players update their beliefs in response to each others' moves.
3. Check that, given those beliefs together with the strategies of the other players, each player is choosing a best response for himself.

Follow-the-Leader III

1. Strategy profile: $\{T|A, T|B, U|C; T|T, U|U\}$.
2. At information set (J_1, J_3, J_5), the posterior beliefs about (A, B, C) are (0.875, 0.125, 0). At information set (J_2, J_4, J_6), the posterior beliefs are (0, 0, 1).
3. Optimization
   - At information set (J_1, J_3, J_5), it is optimal for Jones to choose T.
     $0.875 \times 2 + 0.125 \times 1 = 1.875 > -0.625 = 0.875 \times (-1) + 0.125 \times 2$
   - At information set (J_2, J_4, J_6), it is optimal for Jones to choose U.
     $4 > -1$
   - Given that Jones will imitate his action, Smith does best by following his equilibrium strategy.

Outline

Cascade Model

Example

- A sequence of players 1, . . . , N.
- There is one project. The quality of project could be *good* (50%), or bad (50%).
- One by one, players decide to *accept* or *reject* the project.
  - If he accepts, cost = 0.5, benefit = 1 (good) or 0 (bad).
  - If he rejects, cost = 0, benefit = 0.
- Suppose each of them maximizes his *expected* payoff.

Private Signals

Before making decisions, each player is able to observe previous actions of other players and one **private signal** about the true quality of the project.

Each signal may take on two values *high* and *low*.
$Pr(high|good) = p > 0.5$ and $Pr(high|bad) = 1 - p$.

Herding Effect

One Bayesian Equilibrium looks like this:

- Player 1: If sees signal *high*, accept; if sees signal *low*, reject.
- Player 2: imitates player 1's action, regardless of the value of his signal.
- Player 3: imitates 1 and 2's action, regardless of the value of his signal.
- So on and so forth.

The Png Settlement Game

Example

The **plaintiff** alleges that the **defendant** was negligent in providing safety equipment at a chemical plant, a charge which is true with probability q. The plaintiff files **suit**, but the case is not decided immediately. In the meantime, the defendant and the plaintiff can **settle** out of court.

The game is made up of two games:
the one in which the defendant is **liable** for damages, and the one in which he is **blameless**.
We use *Harsanyi transformation* to start the game with a move by Nature.

The Order of Play

(0a) Nature chooses the defendant to be Liable with probability $q = 0.13$ and Blameless otherwise.

(0b) The defendant observes this but the plaintiff does not.

(1) The plaintiff decides to Sue or just to Grumble.

(2) The defendant Offers a settlement amount of S=0.15 to the plaintiff, or Resist and goes to Trial.

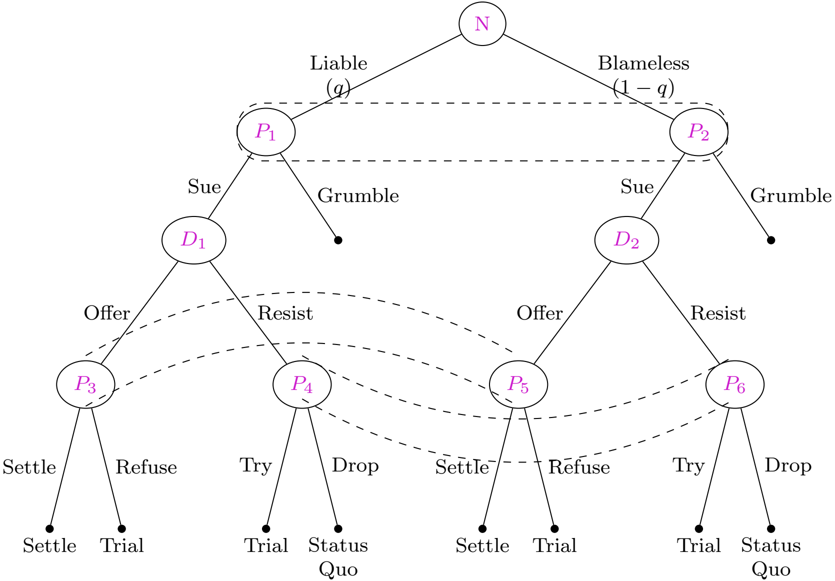
(3a) If the defendant offered S=0.15, the plaintiff agrees to Settle or he Refuses and goes to trial.

(3b) If the defendant offered S=0, the plaintiff Drops the case, for legal cost of P=0 and D=0 for himself and the defendant, or chooses to Try it, creating legal costs of P=0.1 and D=0.2.

(4) If the case goes to trial, the plaintiff wins damages of W=1 if the defendant is Liable and W=0 if the defendant is Blameless. If the case is dropped, W=0.

The plaintiff's payoff is S+W-P. The defendant's payoff is -S-W-D.

Png Settlement Game



Two Nash Equilibria

Rule out the plaintiff's strategy *Grumble*, which is weakly dominated by (Sue, Settle, Drop).

One Nash Equilibrium

{(Sue, Settle, Try), (Offer, Offer)}

Another Nash Equilibrium

{(Sue, Refuse, Try), (Resist, Resist)}

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Final Outcomes

	S	W	P	D	S+W-P	-S-W-D
Settle	0.15	0	0	0	0.15	-0.15
Trial	0	x	0.1	0.2	x - 0.1	-x-0.2
Status Quo	0	0	0	0	0	0

Remark: x is the posterior belief that the defendant is *guilty*.

Checking {(Sue, Settle, Try), (Offer, Offer)}

Optimization of Defendant

Plaintiff would go to trial if he resists in any case, so it is better for him to settle.

Optimization of Plaintiff

- Since Defendant would settle in any case, Plaintiff cannot differentiate between whether he is liable or blameless after Defendant chooses to settle.
- In this case, $Prob(Liable) = 0.13$, i.e., $x = 0.13$.
- At the last stage, it is better for Plaintiff to settle rather than refuse.

Checking {(Sue, Refuse, Try), (Resist, Resist)}

Optimization of Defendant

Plaintiff would go to trial no matter what choice Defendant makes, so he is indifferent between Offer and Resist.

Optimization of Plaintiff

- Since Defendant would resist in any case, Plaintiff cannot differentiate between whether he is liable or blameless after Defendant chooses to resist.
- In this case, $Prob(Liable) = 0.13$, i.e., $x = 0.13$.
- At the last stage, it is better for Plaintiff to go to trial rather than stay in status quo.

Food for Thought

Is there an equilibrium where Defendant plays (Offer, Resist)?

Vocabulary

strategic form	策略式	extensive form	扩展式
outcome matrix	结果矩阵	ranked coordination	分级协调
follow-the-leader	跟随领头羊	node	结
successor	后续结	predecessor	前续结
time line	时间线	information set	信息集
common knowledge	共同知识	private information	私人信息
perfect	完美	symmetric	对称
incomplete information	不完全信息	type	类型
state of the world	世界状态	Bayesian equilibrium	贝叶斯均衡
Harsanyi transformation	海萨尼转换	cascade model	瀑布模型
Png Settlement Game	潘格赔偿博弈	status quo	最初状态