Week 12 Further Topics in Moral Hazard

Wenhao Wu
wuwh2@shanghaitech.edu.cn

SEM, ShanghaiTech U

2024 Spring

Outline

Outline

Risk

The meaning of "risk"

If you take an action that results in a certain consequence, there is no risk. By contrast, if your action results in probabilities of reaching different outcomes, you are facing a risk.

The definition

The outcome of a **risky** project is represented by a random variable Z, whose possible realizations are $z \in \{z_1, \ldots, z_n\}$. The probability of z_i is $p(z_i)$, such that $p(z_i) \geq 0$ and $\sum_{i=1}^{n} p(z_i) = 1$.

Risk Averse vs. Risk Neutral

Risk averse and **risk neutral** are descriptions of preferences over risky projects.
Given two projects. (A) return monetary payoff $\tilde{m}$ with 100%; (B) a risky project with $\mathbb{E}[Z] = \tilde{m}$.

- Risk averse: always prefer (A)
- Risk neutral: indifferent between (A) and (B)

Utility Functions of Two Preferences

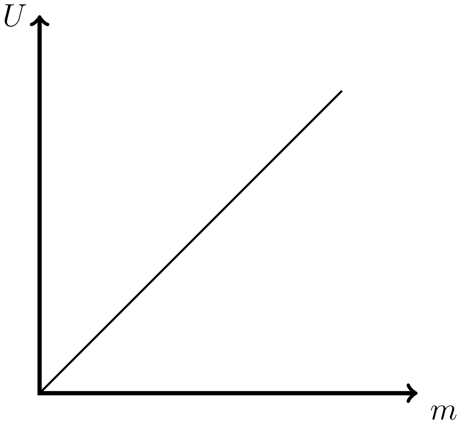


Figure: Risk Neutral

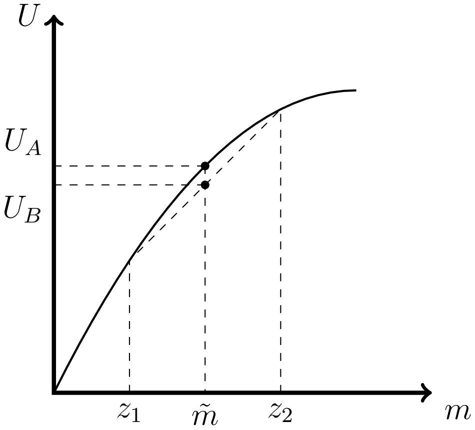


Figure: Risk Averse

上海科技大学 ShanghaiTech University

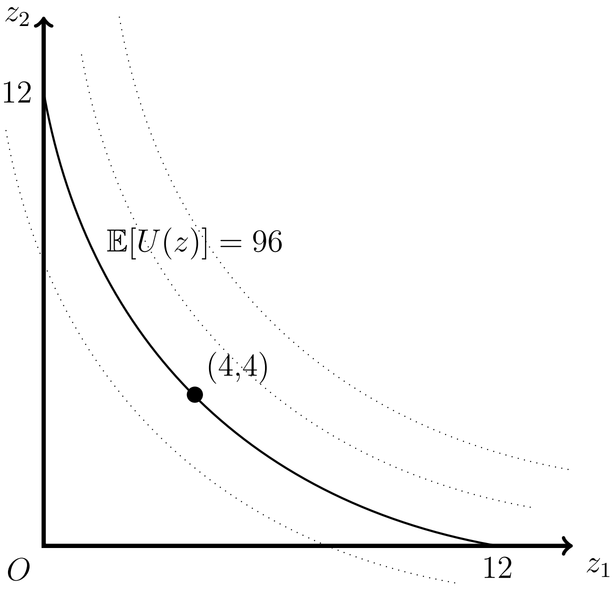
Indifference Curves

Definition

A set of bundles that contribute to the same utility level.

Suppose $U(z) = z(28 - z)$, where $z \in [0, 14]$.
There are two states z_1 and z_2 with probabilities 0.5 and 0.5.
What is the indifference curve for the *expected* utility equal to 96?
$\mathbb{E}[U(z)] = 0.5U(z_1) + 0.5U(z_2) = 96$

There are <u>infinite</u> indifference curves in the state space corresponding to different utility levels.



z_2
12
$\mathbb{E}[U(z)] = 96$
(4,4)
O
12
z_1

Slope of Indifference Curve

The slope of an indifference curve $\mathbb{E}[U(x,y)] = \bar{U}$ can be calculated by the Implicit Function Theorem.

$$\frac{dy}{dx} = -\frac{\partial U/\partial x}{\partial U/\partial y}$$

For $\mathbb{E}[U(z)] = 96$, at (4,4):

$$\frac{dz_2}{dz_1} = -\frac{0.5U_x(4)}{0.5U_y(4)} = -1$$

Outline

A Prevalent Phenomenon

Most *durables* have some type of warranty promising payment from the producer, conditional on performance of the product.

Two observations

1. They provide less than full insurance against unsatisfactory performance.
2. They are supplied by the seller of the product rather than by independent insurance companies.

Double Moral Hazard

Incentive problems on both sides

- Will buyers take care of the product after the purchase?
- Will sellers build in high quality during the production?

Bilateral agreement by buyer and seller

We seek for a warranty contract that disciplines *unobservable* behavior with regard to levels of quality and customer care.

Setup

- One buyer purchases one unit of product from one seller.
- The seller sets up the price p and the warranty ratio $s \in [0, 1]$.
- Product quality q
- Customer care e
- The probability that the product works well is $\Pi(e, q) = \alpha e + \beta q$.
- Seller's cost $C(q) = \frac{1}{2}q^2$ is a convex function.
- Buyer's payoff loss $g(e) = \frac{1}{2}e^2$ is also a convex function.

Two-Stage Game

First Stage

Buyer and Seller forms an agreement with respect to (p, s), where s specifies the ratio of the compensation that the seller should return to the buyer if the product does not work within a limited amount of time.

Second Stage

Given (p, s), the buyers and seller chooses their effort level e and quality level q, simultaneously.

Motivations

Buyer's utility

$$U(e, q, p, s) = \Pi z + (1 - \Pi)sz - p - \frac{1}{2}e^2$$

where z is the strength of the product's utility effect.

Seller's expected payoff

$$V(e, q, p, s) = p - (1 - \Pi)sz - \frac{1}{2}q^2$$

First-Best Solution

When buyer and seller are in a cooperative relationship

They choose e and q so as to maximize the joint profit $\max_{e,q} U + V$.

$$\begin{aligned} \hat{e} &= \alpha z \\ \hat{q} &= \beta z \end{aligned} \tag{1}$$

Second-Best Solution

Backward Induction - Stage II

In the second stage, the buyer and seller optimize under the warranty contract (p, s)。
The buyer $\max_e U(e, q, p, s)$, giving rise to

$$e^* = \alpha(1 - s)z \tag{2}$$

The seller $\max_q V(e, q, p, s)$, giving rise to

$$q^* = \beta s z \tag{3}$$

A notable fact is that $e^* < \hat{e}$ and $q^* < \hat{q}$.

上海科技大学 ShanghaiTech University

Second-Best Solution

Backward Induction - Stage I

The seller chooses the optimal s^* by maximizing the joint profit $\max_s U + V = \Pi z - \frac{1}{2}e^2 - \frac{1}{2}q^2$.
FOC gives that

$$s^* = \frac{\beta^2}{\alpha^2 + \beta^2}$$

Price p is determined by the Participation Constraint

p will be set so that the buyer's expected profit equals her reservation utility, and we skip the details.

Outline

A Key Concept in Economics

Moral hazard is an important topic because it studies **incentives**, one of the central concepts of economics. To solve the problem, we need to find a paradigm of providing the right incentives for effort by satisfying a participation constraint and an incentive compatibility constraint.

The term "moral hazard" comes from the insurance industry, which is the focus of next example.

Insurance Game

Players

Smith and an insurance company. Smith has a car with market value 12.

The Order of Play

1. The insurance company offers a contract of form (x, y), under which Smith pays premium x and receives compensation y if there is a theft.
2. Smith decides whether to accept or reject it.
3. Smith chooses either $Careful$ or $Careless$.
4. Nature chooses whether there is a theft, with probability 0.5 if Smith is $Careful$ or 0.75 if Smith is $Careless$.

Insurance Game

Payoffs

Smith is risk averse and the insurance company is risk neutral.

If Smith chooses $Careful$,

$$\begin{aligned} \bar{\pi}_S &= 0.5U(12 - x) + 0.5U(y - x) \\ \bar{\pi}_I &= 0.5x + 0.5(x - y) \end{aligned}$$

If Smith chooses $Careless$,

$$\begin{aligned} \underline{\pi}_S &= 0.25U(12 - x) + 0.75U(y - x) \\ \underline{\pi}_I &= 0.25x + 0.75(x - y) \end{aligned}$$

Suppose when Smith is *indifferent*, he will choose $Careful$.

State-space Diagram

We will use a geometric approach to solve the problem, instead of algebraic equations. This approach relies on a **state-space diagram**, a diagram whose axes measure the values of one variable in two different states of the world.

Two states in this example:

- Smith's payment when $Safe$, π_S^{safe}
- Smith's payment when $Theft$, π_S^{theft}

Mapping From $(\pi_S^{safe}, \pi_S^{theft})$ to (x,y)

Each combination $(\pi_S^{safe}, \pi_S^{theft})$ represents a certain contract such that

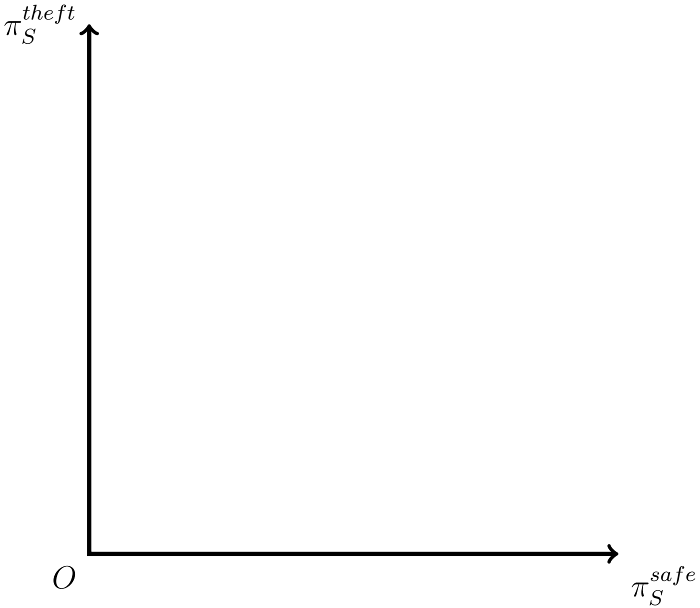
- $x = 12 - \pi_S^{safe}$
- $y = \pi_S^{theft} + x = 12 + \pi_S^{theft} - \pi_S^{safe}$

Translation of Utility Functions

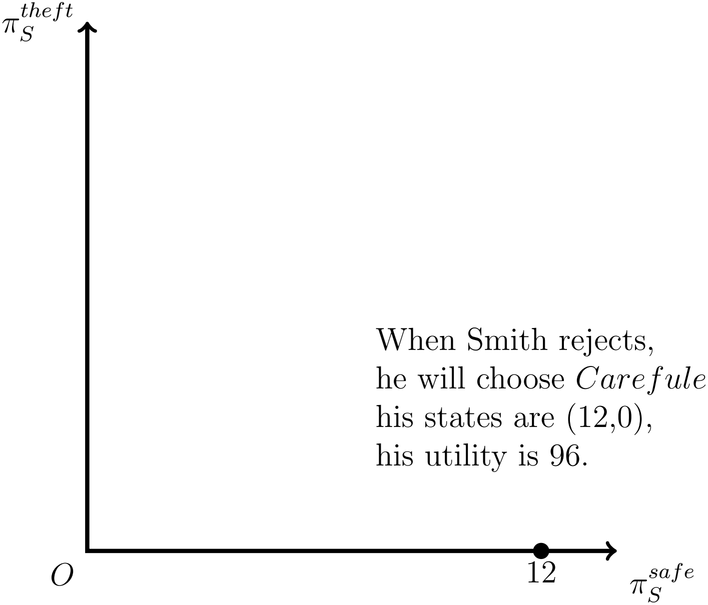
$$
\begin{aligned}
\bar{\pi}_S &= 0.5U(\pi_S^{safe}) + 0.5U(\pi_S^{theft}) \\
\bar{\pi}_I &= 6 - 0.5\pi_S^{safe} - 0.5\pi_S^{theft} \\
\underline{\pi}_S &= 0.25U(\pi_S^{safe}) + 0.75U(\pi_S^{theft}) \\
\underline{\pi}_I &= 3 - 0.25\pi_S^{safe} - 0.75\pi_S^{theft}
\end{aligned}
$$

上海科技大学 ShanghaiTech University

Smith's Reservation Utility

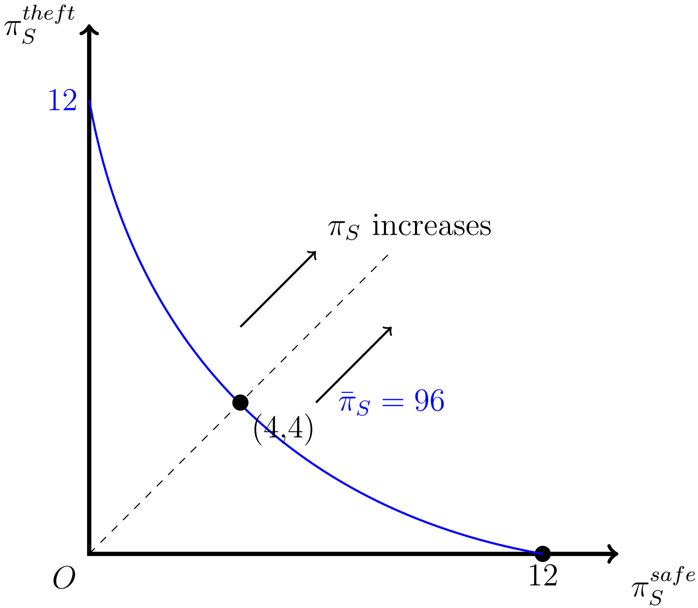


Smith's Reservation Utility

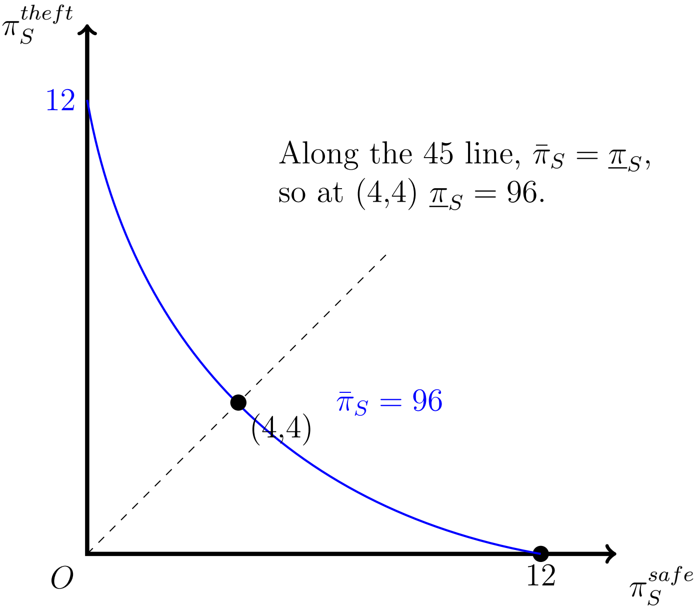


When Smith rejects,
he will choose $Carefule$
his states are (12,0),
his utility is 96.

Smith's Reservation Utility

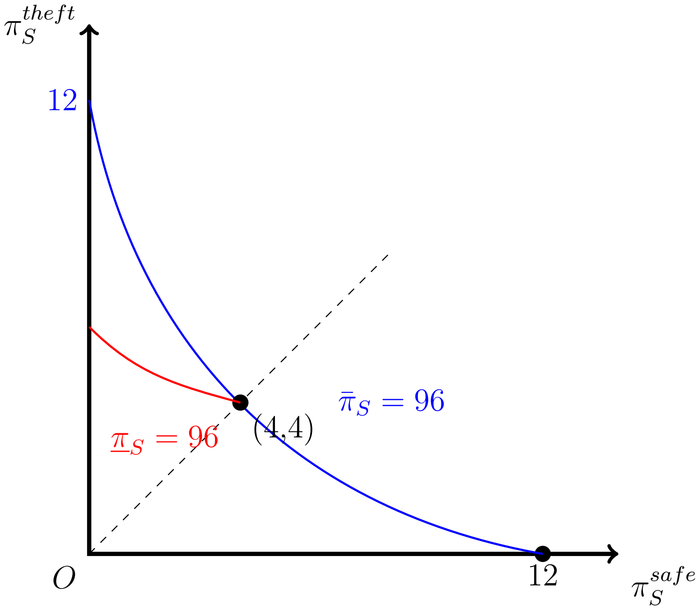


Smith's Reservation Utility

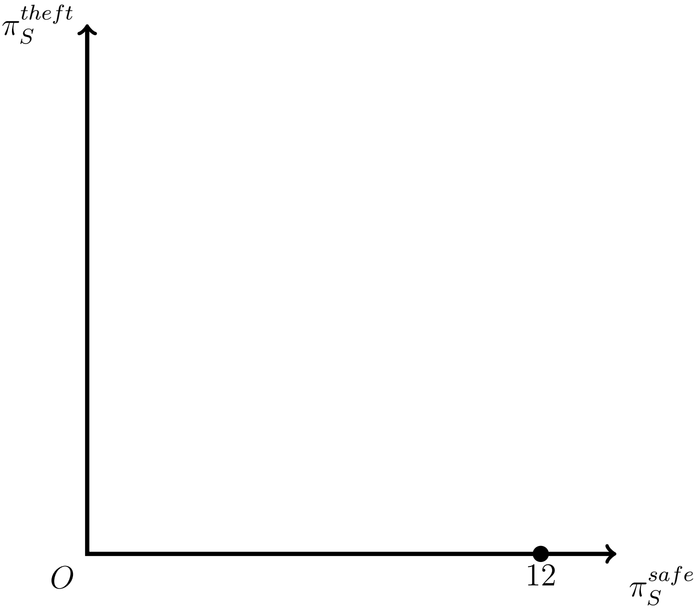


Along the 45 line, $\bar{\pi}_S = \underline{\pi}_S$,
so at (4,4) $\underline{\pi}_S = 96$.

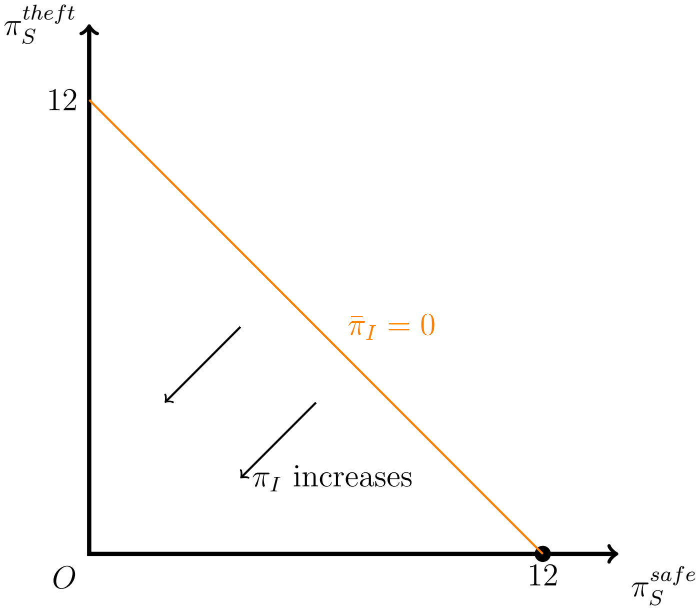
Smith's Reservation Utility



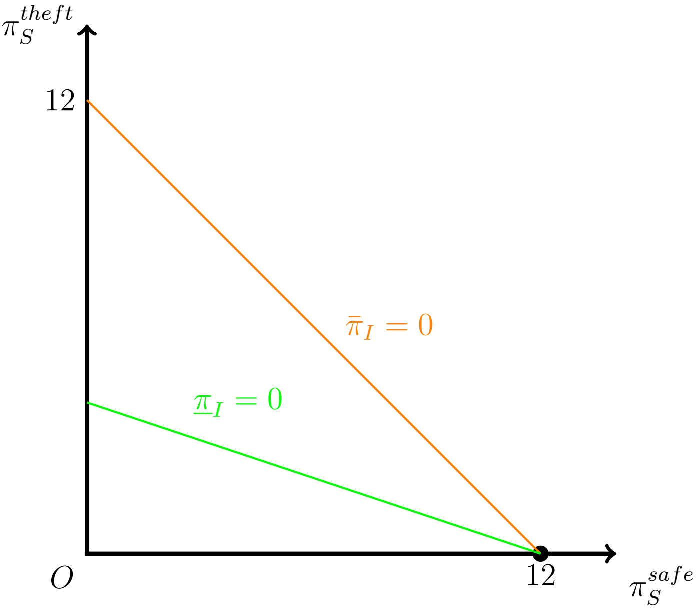
Insurance Company's Zero-Profit Curves



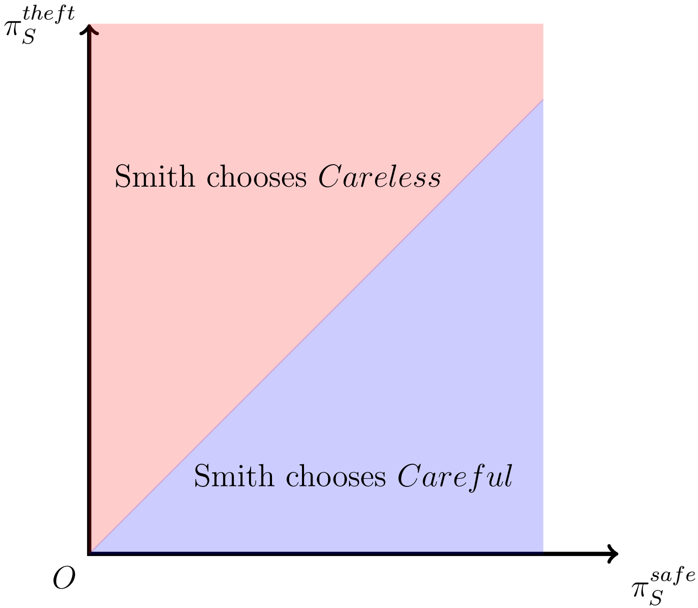
Insurance Company's Zero-Profit Curves



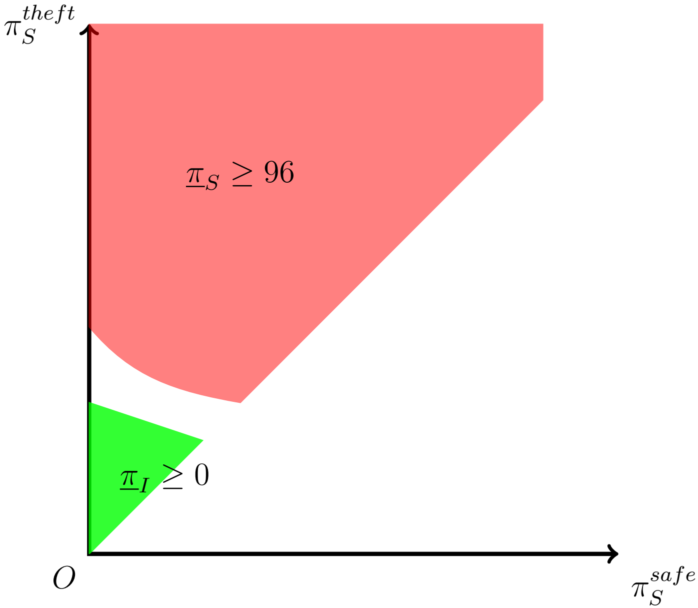
Insurance Company's Zero-Profit Curves



Smith's Choice

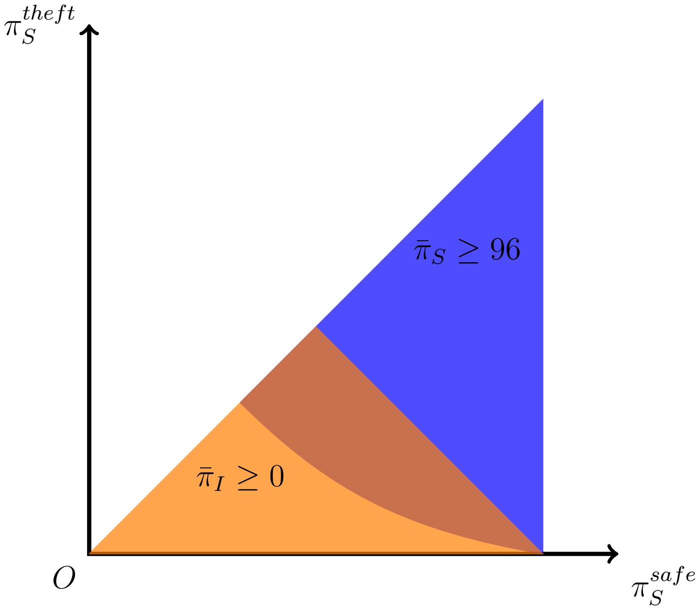


If Smith Chooses *Careless*

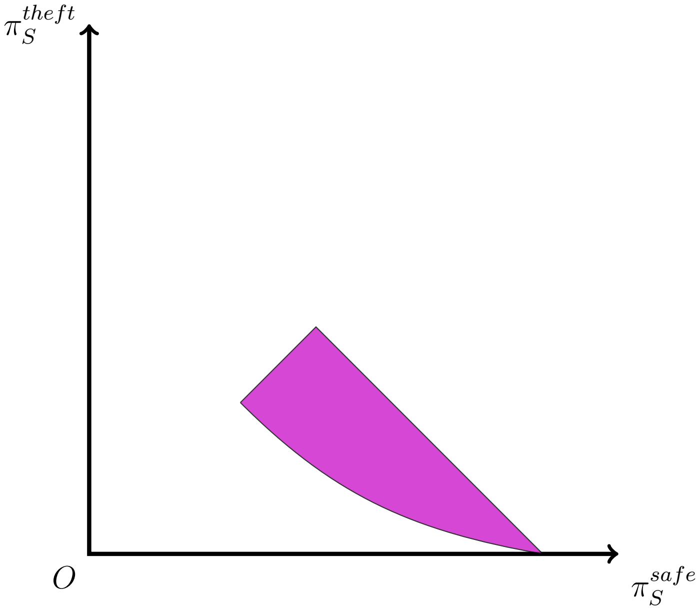


上海科技大学 ShanghaiTech University

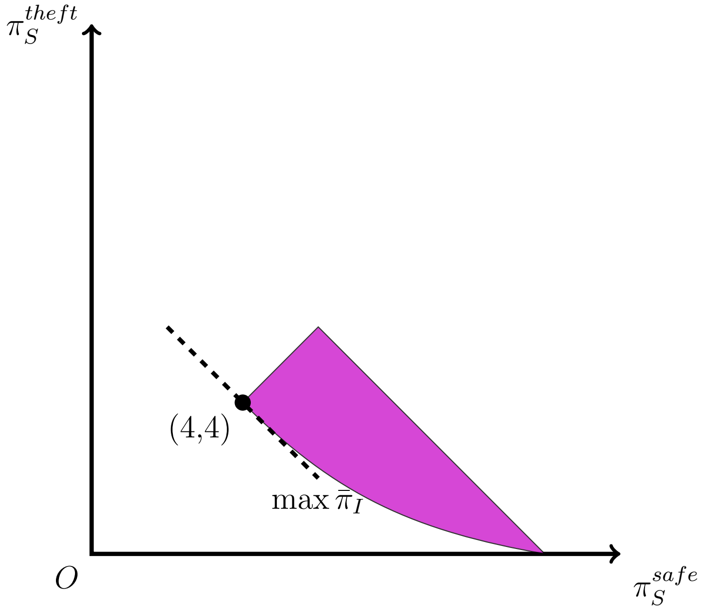
If Smith Chooses $Careful$



Contracts Both Can Agree to



Contracts Both Can Agree to



上海科技大学 ShanghaiTech University

Optimal Contract

It can be checked that the slope of the indifference curve for $\bar{\pi}_I$ equals the derivatives of the indifference curve for $\bar{\pi}_S$ at (4,4). So that the optimal contract for the insurance company is represented by (4,4) in the state-space diagram.

Under the optimal contract, Smith pays the premium at the price of 8, while the insurance company pays Smith 12 if his car is stolen. In such a case, when Smith gets equal payoffs in both states, we say Smith is **fully insured**.

What is the insurance company's profit under the optimal contract?

Outline

Allocation of Effort

Holmstrom & Milgrom (1991) point out one place that has been ignored by the standard principal-agent problems — sometimes P wants A to split his time onto different tasks.

If some task is NOT *observable*, and therefore NOT **contractible**, then A has no incentive to spend any time on it.

Examples

- Faculty: research, teaching, service
- Salesman: sales, attitude to customers, customary network

Multi-Task Game

Players

Principal and Agent.

The Order of Play

1. P wants A to carry out Task 1 and Task 2. The outputs are q_1 and q_2. Only q_1 is observable.
2. P provides a contract $w(q_1)$ as a function of q_1.
3. A decides whether to accept the contract or reject it.
4. If accept, A chooses his effort levels for two projects e_1 and e_2, where $e_1 + e_2 = 1$.
5. Outputs are $q_1(e_1)$ and $q_2(e_2)$, where $q_1(e) = q_2(e) = 2e - e^2$.

上海科技大学 ShanghaiTech University

Multi-Task Game

Payoffs

If A rejects, both P and A have payoff 0.

If A accepts, then

$$\begin{aligned}\pi_P &= q_1 + \beta q_2 - w \\ \pi_A &= w - e_1^2 - e_2^2\end{aligned}$$

$\beta \in (0,1)$ measures the importance of Task 2.

Solve for the optimal linear contract $w(q_1) = a + bq_1$.

Incentive Compatibility

Effort induced by linear contract

$\max a + bq_1(e_1) - e_1^2 - e_2^2$

FOC given that $e_2 = 1 - e_1$

$$\begin{aligned} e_1^* &= 1 - \tfrac{1}{2+b} \\ e_2^* &= \tfrac{1}{2+b} \end{aligned} \tag{4}$$

- e_1^* is increasing in b.
- How much effort A puts in is only related with b (marginal return) but not a (fixed wage).

Optimal b

FOC to $\max \pi_P$

$$2 - 2e_1 + \beta(2 - 2e_2)(-1) - b(2 - 2e_1) = 0$$

Solve for e_1^*

$$e_1^* = 1 - \frac{\beta}{1 + \beta - b} \tag{5}$$

Combining Eq. (1)(2)

$$b = \frac{1 - \beta}{1 + \beta}$$

Note that b is decreasing in β. It means that the more important the unobservable task is, the less incentive the principal should provide for the observable task.

Participation

After we solve for b, e_1^*, e_2^*, P picks a so that A's utility equals 0. That is

$$a + bq(e_1^*) - (e_1^*)^2 - (e_2^*)^2 = 0 \tag{6}$$

a^* is the solution to Eq.(3).

Vocabulary

incentive	动机	state-space diagram	状态空间图
insurance game	保险博弈	fully insured	全保险
premium	保费	multi-task game	多任务博弈