SI114H-Computational Science and Engineering, 2025 Spring

Homework Set #2

Requirements:

1) Deadline: **11pm, 12 May 2025**.
2) About your codes:
   a) Make sure that your codes can run and are consistent with your results.
   b) Attach a Readme.txt file to clearly identify the function of each file.
3) You need to compress three files **code** (Only accept MATLAB language), **readme** (Add supplementary explanations to the code), and **PDF** (Show your results) into one file, name this file as student ID + your name and send it to the blackboard system.

Problem 1. (100 points)

Consider the following problem

$$\begin{cases} -\frac{d}{dx}[C(x)\frac{du}{dx}] = 1 \ , \ 0 < x < 1, \\ u(0) = u(1) = 0, \end{cases} \tag{1}$$

where

$$C(x) = \begin{cases} 1 \ , \ 0 < x < \frac{1}{2}, \\ \frac{1}{2} \ , \ \frac{1}{2} \leq x < 1. \end{cases} \tag{2}$$

Program the finite element method (FEM) to solve the problem (1). Denote the number of elements as n. Exhibit the corresponding solutions with $n = 4, 8, 1000$ in your report.

1) (30 points) Give the stiffness matrix $\boldsymbol{A}$, vector $\mathbf{f}$ and solution $\mathbf{u}$ for $n = 4$.
2) (60 points) Give the value of $u(\frac{1}{4})$ and $u(\frac{3}{4})$ for $n = 4, 8, 1000$.
3) (10 points) Plot the solutions in one figure for $n = 4, 8, 1000$.

Solution:

1) The stiffness matrix $\boldsymbol{A}$, vector $\mathbf{f}$ and solution $\mathbf{u}$ for $n = 4$ are as follows:

$$\boldsymbol{A} = \begin{bmatrix} 4 & -4 & 0 & 0 & 0 \\ -4 & 8 & -4 & 0 & 0 \\ 0 & -4 & 6 & -2 & 0 \\ 0 & 0 & -2 & 4 & -2 \\ 0 & 0 & 0 & -2 & 2 \end{bmatrix}, \mathbf{f} = \begin{bmatrix} 0.1250 \\ 0.2500 \\ 0.2500 \\ 0.2500 \\ 0.1250 \end{bmatrix}, \mathbf{u} = \begin{bmatrix} 0 \\ 0.1146 \\ 0.1667 \\ 0.1458 \\ 0 \end{bmatrix}. \tag{3}$$

2) The values of $u(\frac{1}{4})$ and $u(\frac{3}{4})$ for $n = 4, 8, 1000$ are as follows:

Results for u(1/4) and u(3/4):

n = 4: u(1/4) = 0.114583, u(3/4) = 0.145833

n = 8: u(1/4) = 0.114583, u(3/4) = 0.145833

n = 1000: u(1/4) = 0.114583, u(3/4) = 0.145833

This is a known property: for a 1D problem $-u^{''} = f$ with f constant, linear elements give exact nodal values. If f is piecewise constant and discontinuities are nodes, it's also exact.

3) The plot of the solutions for $n = 4, 8, 1000$ is shown below:

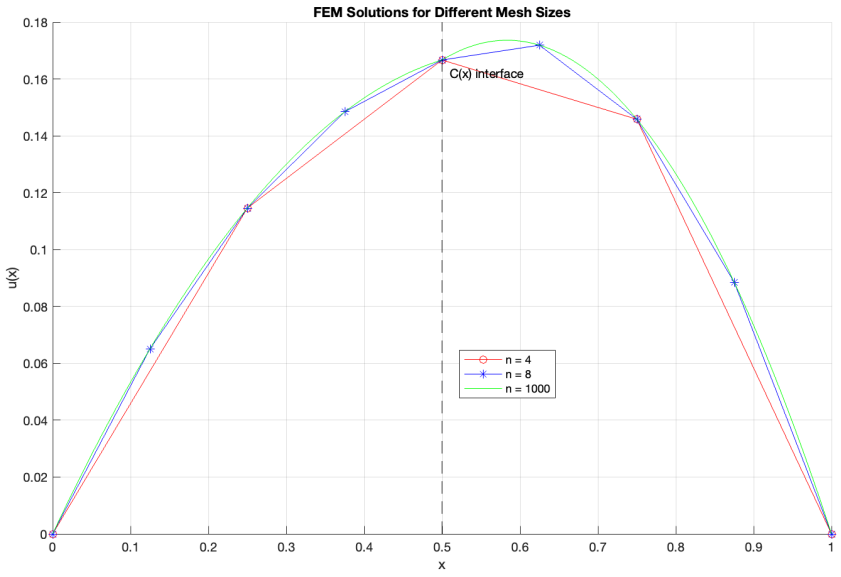

Figure 1. Plot of the solutions for $n = 4, 8, 1000$