Import Required Libraries

Import the necessary libraries, including NumPy and Matplotlib.

In [1]:
```
# Import Required Libraries
import numpy as np
import matplotlib.pyplot as plt
```

Define the Random Variable N

Define the random variable N based on the given condition.

In [2]:
```
# Define the Random Variable N

# Function to generate the random variable N
def generate_N():
    product = 1
    n = 0
    while product >= np.exp(-1):
        product *= np.random.uniform(0, 1)
        n += 1
    return n - 1

# Generate 5000 samples of N
samples = [generate_N() for _ in range(5000)]

# Display the first 10 samples as a sanity check
samples[:10]
```

Out[2]:
```
[1, 0, 0, 0, 0, 0, 2, 0, 2, 1]
```

Estimate E(N)

Estimate the expected value of N by calculating the sample mean of the generated samples.

In [3]:
```
# Estimate E(N)

# Calculate the sample mean of the generated samples
E_N = np.mean(samples)

# Display the estimated expected value of N
E_N
```

Out[3]:
```
0.9914
```

Estimate Var(N)

Estimate the variance of N by calculating the sample variance of the generated samples.

In [4]:
```
# Estimate Var(N)

# Calculate the sample variance of the generated samples
Var_N = np.var(samples)

# Display the estimated variance of N
Var_N
```

Out[4]:
```
1.01492604
```

Estimate P(N=i) for i=0,1,2,3

Estimate the probabilities P(N=i) for i=0,1,2,3 using the generated samples.

In [5]:
```
# Estimate P(N=i) for i=0,1,2,3

# Calculate the probabilities P(N=i) for i=0,1,2,3
probabilities = {i: np.mean(np.array(samples) == i) for i in range(0,4)}

# Display the estimated probabilities
probabilities
```

Out[5]:
```
{0: 0.3744, 1: 0.367, 2: 0.1764, 3: 0.0616}
```