

**Input:**

Training data:  $(X_1, Y_1), (X_2, Y_2), \dots, (X_n, Y_n)$

Current weights:  $W_k^2, W_k^3, \dots, W_k^L$

Current biases:  $b_k^2, b_k^3, \dots, b_k^L$

Learning rate:  $\eta \in \mathbb{R}$

**Output:**

Updated weights:  $W_{k+1}^2, W_{k+1}^3, \dots, W_{k+1}^L$

Updated biases:  $b_{k+1}^2, b_{k+1}^3, \dots, b_{k+1}^L$

**for**  $i \leftarrow 1$  **to**  $n$  **do**

    Forward pass:

**begin**

$$a^1 = X_i$$

**for**  $\ell \leftarrow 2$  **to**  $L$  **do**

$$z^\ell = W_k^\ell a^{\ell-1} + b^\ell$$

$$a^\ell = \tilde{\sigma}(z^\ell)$$

**end**

**end**

    Compute errors (backpropagation):

**begin**

$$\delta^L = [a^L(X_i) - Y_i] \odot \tilde{\sigma}'(z^L)$$

$$\frac{\partial C_i}{\partial W_k^L} = \delta^L (a^{L-1})^T$$

$$\frac{\partial C_i}{\partial b_k^L} = \delta^L$$

**for**  $\ell \leftarrow L - 1, L - 2, \dots$  **to**  $2$  **do**

$$\delta^\ell = \left[ (W^{\ell+1})^T \delta^{\ell+1} \right] \odot \tilde{\sigma}'(z^\ell)$$

$$\frac{\partial C_i}{\partial W_k^\ell} = \delta^\ell (a^{\ell-1})^T$$

$$\frac{\partial C_i}{\partial b_k^\ell} = \delta^\ell$$

**end**

**end**

**end**

Compute gradients:

**begin**

**for**  $\ell \leftarrow 2$  **to**  $L$  **do**

$$\frac{\partial C}{\partial W_k^\ell} = \frac{1}{n} \sum_{i=1}^n \frac{\partial C_i}{\partial W_k^\ell}$$

$$\frac{\partial C}{\partial b_k^\ell} = \frac{1}{n} \sum_{i=1}^n \frac{\partial C_i}{\partial b_k^\ell}$$

**end**

**end**

Update weights and biases (gradient descent):

**begin**

**for**  $\ell \leftarrow 2$  **to**  $L$  **do**

$$W_{k+1}^\ell = W_k^\ell - \eta \frac{\partial C}{\partial W_k^\ell}$$

$$b_{k+1}^\ell = b_k^\ell - \eta \frac{\partial C}{\partial b_k^\ell}$$

**end**

**end**