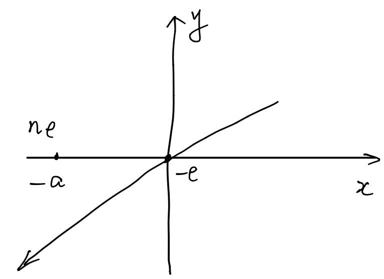
一、 $\vec{r}=(x,y,z)$



$$U=\frac{1}{4\pi\varepsilon_0}\left(\frac{ne}{r'}-\frac{e}{r}\right)$$

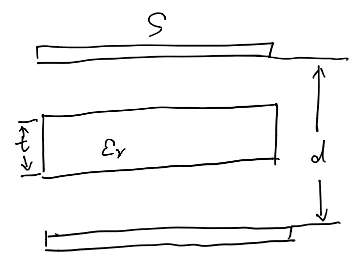
$$U=0 \Rightarrow \quad n^2=\frac{(r')^2}{r^2}=\frac{(x+a)^2+y^2+z^2}{x^2+y^2+z^2}$$

$$(n^2-1)(x^2+y^2+z^2)=2ax+a^2$$

$$\left(x-\frac{a}{n^2-1}\right)^2+y^2+z^2=\frac{a^2n^2}{(n^2-1)^2}$$

电势为0等势面是一个球面，球心在 $-e$ 延长线的外边

二、



(1) $$U=\frac{D}{\varepsilon_0}(d-t)+\frac{D}{\varepsilon_0\varepsilon_r}t$$

$$D=\frac{\varepsilon_0\varepsilon_r U}{\varepsilon_r d+(1-\varepsilon_r)t}$$

电介质中 $$E=\frac{D}{\varepsilon_0\varepsilon_r}=\frac{U}{\varepsilon_r d+(1-\varepsilon_r)t}$$

$$P=D-\varepsilon_0 E=\frac{\varepsilon_0(\varepsilon_r-1)U}{\varepsilon_r d+(1-\varepsilon_r)t}$$

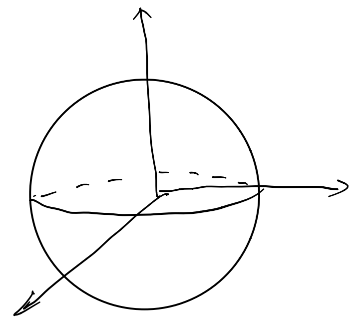
(2) 真空中 $$E_0=\frac{D}{\varepsilon_0}=\frac{\varepsilon_r U}{\varepsilon_r d+(1-\varepsilon_r)t}$$

$$Q=S\sigma_0=SD=\frac{\varepsilon_0\varepsilon_r US}{\varepsilon_r d+(1-\varepsilon_r)t}$$

(3) 间隙场强 $$E_0=\frac{\varepsilon_r U}{\varepsilon_r d+(1-\varepsilon_r)t}$$

(4) $$C=\frac{Q}{U}=\frac{\varepsilon_0\varepsilon_r S}{\varepsilon_r d+(1-\varepsilon_r)t}$$

三、 球外场强 $\vec{E} = \dfrac{Q}{4\pi\varepsilon_0 r^2}\vec{e}_r$



$$\sigma = \frac{Q}{4\pi R^2}$$

$$dq = \sigma R^2 \sin\theta\, d\theta\, d\varphi$$

$$F = \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} d\theta \ \frac{E}{2}\cos\theta\ \sigma R^2 \sin\theta$$

$$= 2\pi \int_0^{\frac{\pi}{2}} \frac{Q}{8\pi\varepsilon_0 R^2}\frac{Q}{4\pi}\sin\theta\cos\theta\, d\theta$$

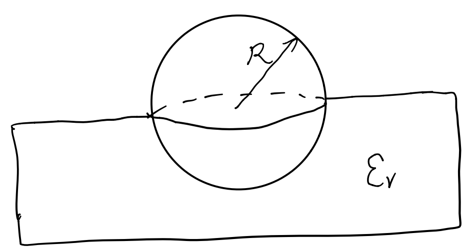
$$E = \begin{cases} 0 & , r<R \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & , r>R \end{cases}$$

$$= \frac{Q^2}{16\pi\varepsilon_0 R^2}\frac{1}{2}$$

$$= \frac{Q^2}{32\pi\varepsilon_0 R^2}$$

四、

$C_1 = 2\pi\varepsilon_0 R$, $C_2 = 2\pi\varepsilon_0\varepsilon_r R$



$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{1}{\varepsilon_r}$$

$$Q_1 + Q_2 = Q \Rightarrow \begin{cases} Q_1 = \dfrac{Q}{1+\varepsilon_r} \\ Q_2 = \dfrac{\varepsilon_r Q}{1+\varepsilon_r} \end{cases}$$

$$\begin{cases} D_1 = \dfrac{Q}{2(1+\varepsilon_r)\pi r^2} \\ D_2 = \dfrac{Q\varepsilon_r}{2(1+\varepsilon_r)\pi r^2} \end{cases} \qquad E = \frac{Q}{2(1+\varepsilon_r)\varepsilon_0\pi r^2}$$

$$P = D_2 - \varepsilon_0 E = \frac{(\varepsilon_r - 1)Q}{2(1+\varepsilon_r)\pi r^2} \qquad \sigma = -P = -\frac{(\varepsilon_r-1)Q}{2(1+\varepsilon_r)\pi R^2}$$

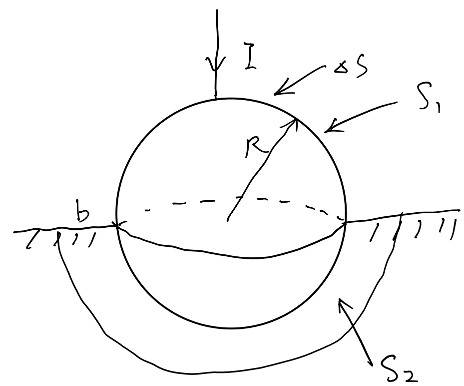
$$\rho = -\vec{\nabla}\cdot\vec{P} = 0$$

五、

(1) $R' = \int_0^b \rho \frac{dx}{S}$

$= \int_0^b \rho \frac{1}{2\pi (R+x)^2} dx$

$= \frac{\rho}{2\pi} \left(\frac{1}{R} - \frac{1}{R+b}\right)$



$U = IR'$

$E = -\frac{\partial U}{\partial b} = -\frac{\rho}{\pi} \frac{I}{(R+b)^2}$

$D = \varepsilon_0 \varepsilon_r E(0) = \frac{\rho \varepsilon_0 \varepsilon_r}{2\pi} \frac{I}{R^2}$

自由电荷 $Q = DS = 2\rho \varepsilon_0 \varepsilon_r I$

总电荷 $Q' = \varepsilon_0 E S = 2\rho \varepsilon_0 I$

(2) 全部埋入大地，半球变成整个球，

$Q = \rho \varepsilon_0 \varepsilon_r I$