$$\begin{tikzcd}
X \arrow[r, mapsto] \arrow[d, dashed] & Y_i \arrow[d, dashed] \\
(x_i, & y_i)
\end{tikzcd}$$

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

$$\mathbb{E}\,\varepsilon_i = 0$$

<u>LSM</u>: $$\min_{(\beta_0, \beta_1)} \underbrace{\sum_{i=1}^{n} \left[y_i - \underbrace{(\beta_0 + \beta_1 x_i)}_{} \right]^2}_{RSS}$$

$$\log\left(\frac{p(x_i)}{1 - p(x_i)} \right) = \underbrace{\beta_0 + \beta_1 x_i}_{}$$

$$\boxed{?}$$

- $y_i = \underbrace{\beta_0 + \beta_1 x_i}$ + ε_i

  $\varepsilon_i \sim \text{Normal}(0, \sigma^2)$

- $y_i \sim \text{Normal}(\beta_0 + \beta_1 x_i, \sigma^2)$

- independent observations, i.e. $\{y_i\}_{i=1}^n$ are mutually indp.

Joint distribution $\{y_i\}_{i=1}^n$

$$\boxed{\prod_{i=1}^n \frac{1}{\sqrt{2\pi}\,\sigma} \prod_{i=1}^n \exp\left\{ \underbrace{-\frac{(y_i - (\beta_0 + \beta_1 x))^2}{2\sigma^2}} \right\}}$$

the density for y_i

likelihood function for $\{y_i\}_{i=1}^n$

$L(\beta_0, \beta_1)$ ← we want to $\max L(\beta_0, \beta_1)$

$$\exp\left\{-\frac{\sum_{i=1}^{n}\left(y_i - (\beta_0 + \beta_1 x_i)\right)^2}{2\sigma^2}\right\}$$

$$\exp\{-a\} \cdot \exp\{-b\} = \exp\{-(a+b)\}$$

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maximum likelihood

$$p_i = \frac{e^{\beta_0+\beta_1 x_i}}{1+e^{\beta_0+\beta_1 x_i}}$$

$$\log\left(\frac{p(x_i)}{1-p(x_i)}\right) = \beta_0 + \beta_1 x_i$$

$$p_i = p(x_i) = \Pr(y_i = 1 \mid x_i) = E(y_i \mid x_i)$$

(x_i, y_i)

PMF for y_i

$$\Pr(y_i = 1) = p_i \qquad \Pr(y_i = 0) = 1 - p_i$$

$$L(\beta_0, \beta_1) = \prod_{i=1}^{n} p_i^{y_i} (1-p_i)^{1-y_i}$$

$$\max L(\beta_0, \beta_1)$$

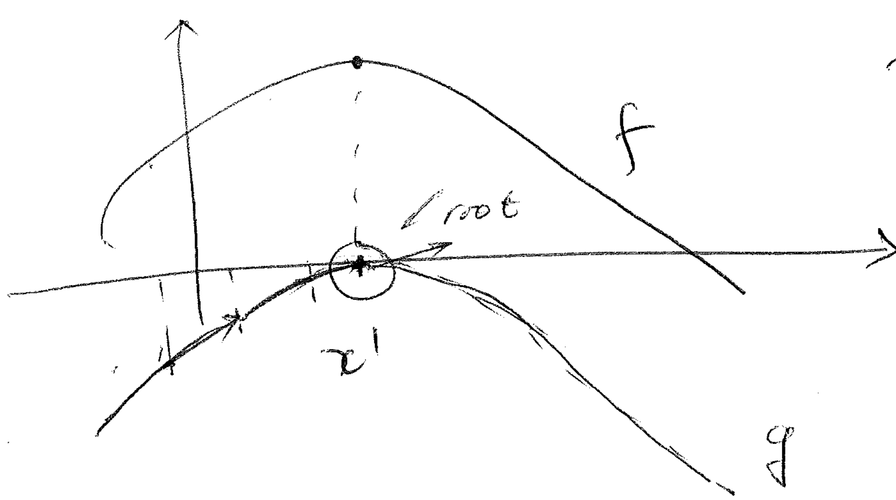
$$L(\beta_0, \beta_1) = \prod_{i=1}^{n} p_i^{y_i} (1-p_i)^{1-y_i}$$

$$p_i = \frac{e^{\beta_0 + \beta_1 x_i}}{1 + e^{\beta_0 + \beta_1 x_i}}$$

$$\log L(\beta_0, \beta_1) = \sum_{i=1}^{n} \log \left[p_i^{y_i} (1-p_i)^{1-y_i} \right]$$

$$= \sum_{i=1}^{n} \left[y_i \log p_i + (1-y_i) \log (1-p_i) \right]$$

$$(\hat{\beta}_0, \hat{\beta}_1) \in \underset{(\beta_0, \beta_1)}{\operatorname{argmax}} \log L(\beta_0, \beta_1)$$



x' f

g $\underline{x'}$