Solutions for Chapter 1

Exercise 1.1

(a) $R = 5\text{k} + 10\text{k} = \boxed{\mathbf{15}\text{k}\boldsymbol{\Omega}}$

(b) $R = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{5\text{k} \cdot 10\text{k}}{5\text{k} + 10\text{k}} = \boxed{\mathbf{3.33}\text{k}\boldsymbol{\Omega}}$

Exercise 1.2

$P = IV = \left(\dfrac{V}{R}\right) V = \dfrac{(12\text{V})^2}{1\Omega} = \boxed{\mathbf{144}\text{W}}$

Exercise 1.3

TODO: Solve this problem

Exercise 1.4

TODO: Solve this problem

Exercise 1.5

Given that $P = \dfrac{V^2}{R}$, we know that the maximum voltage we can achieve is 15V and the smallest resistance we can have across the resistor in question is 1kΩ. Therefore, the maximum amount of power dissipated can be given by

$$P = \frac{V^2}{R} = \frac{(15\text{V})^2}{1\text{k}\Omega} = \boxed{\mathbf{0.225}\text{W}}$$

This is less than the 1/4W power rating.

Exercise 1.10

(a) With two equal-value resistors, the output voltage is half the input voltage.

$$V_{out} = \frac{1}{2} V_{in} = \frac{30\text{V}}{2} = \boxed{\mathbf{15}\text{V}}$$

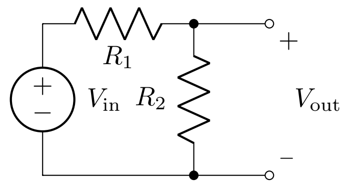
(b) To treat R_2 and R_{load} as a single resistor, combine the two resistors which are in parallel to find that the combined (equivalent) resistance is 5kΩ. Now, we have a simple voltage divider with a 10kΩ resistor in series with the 5kΩ equivalent resistor. The output voltage is across this equivalent resistance. The output voltage is given by

$$V_{out} = V_{in} \frac{5\text{k}\Omega}{10\text{k}\Omega + 5\text{k}\Omega} = \frac{30\text{V}}{3} = \boxed{\mathbf{10}\text{V}}$$

TODO: Add a diagram to make this clearer

(c) We can redraw the voltage divider circuit to make the "port" clearer.

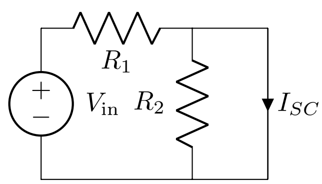
Figure 1: Voltage divider with port shown.



We can find V_{Th} by leaving the ports open (open circuit) and measuring V_{out}, the voltage across R_2. This comes out to be half the input voltage when $R_1 = R_2$, so $V_{\text{out}} = 15\text{V}$. Thus $V_{\text{Th}} = \boxed{\mathbf{15}\text{V}}$.

To find the Thévinen resistance, we need to find the short circuit current, I_{SC}. We short circuit the port and measure the current flowing through it.

Figure 2: Voltage divider with short circuit on the output.

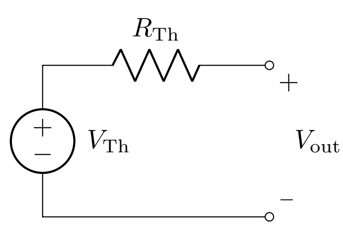


In this circuit, no current flows through R_2, flowing through the short instead. Thus we have $I_{SC} = \frac{V_{\text{in}}}{R_1}$. From this, we can find R_{Th} from $R_{\text{Th}} = \frac{V_{\text{Th}}}{I_{SC}}$. This gives us

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_{SC}} = \frac{V_{\text{Th}}}{V_{\text{in}}/R_1} = \frac{15\text{V}}{30\text{V}/10\text{k}\Omega} = \boxed{\mathbf{5k\Omega}}$$

The Thévenin equivalent circuit takes the form shown below.

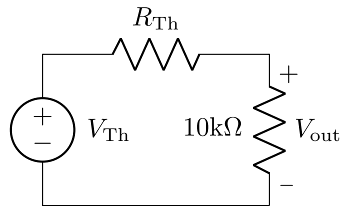
Figure 3: Thévenin equivalent circuit.



In terms of behavior at the ports, this circuit is equivalent to the circuit in Figure 1.

(d) We connect the 10kΩ load to the port of the Thévenin equivalent circuit in Figure 3 to get the following circuit.

Figure 4: Thévenin equivalent circuit with 10kΩ load.



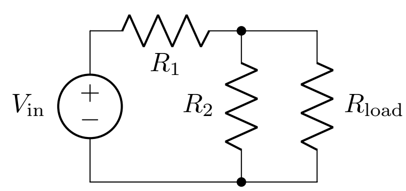
From here, we can find V_{out}, treating this circuit as a voltage divider.

$$V_{\text{out}} = \frac{10\text{k}\Omega}{R_{\text{Th}} + 10\text{k}\Omega} V_{\text{Th}} = \frac{10\text{k}\Omega}{5\text{k}\Omega + 10\text{k}\Omega} \cdot 15\text{V} = \boxed{\mathbf{10}\text{V}}$$

This is the same answer we got in part (b).

(e) To find the power dissipated in each resistor, we return to the original three-resistor circuit.

Figure 5: Original voltage divider with 10kΩ load attached.



From part (d), we know that the output voltage is 10V and that this is the voltage across the load resistor. Since $P = IV = \frac{V^2}{R}$, we find that the power through R_{load} is

$$P_{\text{load}} = \frac{V^2}{R_{\text{load}}} = \frac{(10\text{V})^2}{10\text{k}\Omega} = \boxed{\mathbf{10}\text{mW}}$$

Similarly, we know that the power across R_2 is the same since the voltage across R_2 is the same as the voltage across R_{load}. Thus we have

$$P_2 = \boxed{\mathbf{10}\text{mW}}$$

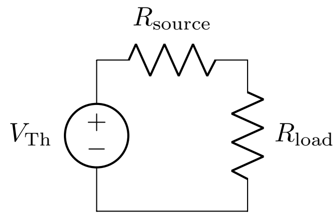
To find the power dissipated in R_1, we first have to find the voltage across it. From Kirchoff's loop rule, we know that the voltage around any closed loop in the circuit must be zero. We can choose the loop going through the voltage source, R_1, and R_2. The voltage supplied by the source is 30V. The voltage dropped across R_2 is 10V as discussed before. Thus the voltage dropped across R_1 must be $30\text{V} - 10\text{V} - 20\text{V}$. Now we know the voltage across and the resistance of R_1. We use the same formula as before to find the power dissipated.

$$P_1 = \frac{V^2}{R_1} = \frac{(20\text{V})^2}{10\text{k}\Omega} = \boxed{\mathbf{40}\text{mW}}$$

Exercise 1.11

Consider the following Thévenin circuit where R_{source} is just another name for the Thévenin resistance, R_{Th}.

Figure 6: Standard Thévenin circuit with attached load.



We will first calculate the power dissipated in the load and then maximize it with calculus. We can find the power through a resistor using current and resistence since $P = IV = I(IR) = I^2R$. To find the total current flowing through the resistors, we find the equivalent resistance which is $R_{\text{source}} + R_{\text{load}}$. Thus the total current flowing is $I = \frac{V_{\text{Th}}}{R_{\text{source}}+R_{\text{load}}}$. The power dissipated in R_{load} is thus

$$P_{\text{load}} = I^2 R_{\text{load}} = \frac{V_{\text{Th}}^2 R_{\text{load}}}{(R_{\text{source}} + R_{\text{load}})^2}$$

To maximize this function, we take the derivative and set it equal to 0.

$$\begin{aligned}\frac{dP_{\text{load}}}{dR_{\text{load}}} &= V_{\text{Th}} \frac{(R_{\text{source}} + R_{\text{load}})^2 - 2R_{\text{load}}(R_{\text{source}} + R_{\text{load}})}{(R_{\text{source}} + R_{\text{load}})^4} = 0 \\ &\implies R_{\text{source}} + R_{\text{load}} = 2R_{\text{load}} \\ &\implies R_{\text{source}} = R_{\text{load}}\end{aligned}$$