Solutions for Chapter 1

Exercise 1.1

(a) $R = 5\text{k} + 10\text{k} = \boxed{\mathbf{15}\text{k}\boldsymbol{\Omega}}$

(b) $R = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{5\text{k} \cdot 10\text{k}}{5\text{k} + 10\text{k}} = \boxed{\mathbf{3.33}\text{k}\boldsymbol{\Omega}}$

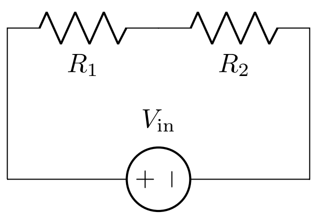
Exercise 1.2

$$P = IV = \left(\frac{V}{R}\right) V = \frac{(12\text{V})^2}{1\Omega} = \boxed{\mathbf{144}\text{W}}$$

Exercise 1.3

Consider a simple series resistor circuit.

Figure 1: A basic series circuit.



By **TODO: Solve this problem**

Exercise 1.4

TODO: Solve this problem

Exercise 1.5

Given that $P = \dfrac{V^2}{R}$, we know that the maximum voltage we can achieve is 15V and the smallest resistance we can have across the resistor in question is 1kΩ. Therefore, the maximum amount of power dissipated can be given by

$$P = \frac{V^2}{R} = \frac{(15\text{V})^2}{1\text{k}\Omega} = \boxed{\mathbf{0.225}\text{W}}$$

This is less than the 1/4W power rating.

Exercise 1.6

(a) The total current required by New York City that will flow through the cable is $I = \frac{P}{V} = \frac{10^{10}\text{ W}}{115\text{ V}} = 869.6$ MA. Therefore, the total power lost per foot of cable can be calculated by:

$$P = I^2 R = (869.6 \times 10^6\text{ A})^2 \times \left(5 \times 10^{-8}\ \frac{\Omega}{\text{ft}}\right) = \boxed{\mathbf{3.78 \times 10^8}\frac{\text{W}}{\text{ft}}}$$

(b) The length of cable over which all 10^{10} W will be lost is:

$$L = \frac{10^{10}\text{ W}}{3.78 \times 10^8\ \frac{\text{W}}{\text{ft}}} = \boxed{\mathbf{26.45\ ft}}$$

(c) To calculate the heat dissipated by the cable, we can use the Stefan-Boltzmann equation $T = \sqrt[4]{\frac{P}{A\sigma}}$, with A corresponding to the cylindrical surface area of the 26.45 foot long section of 1 foot diameter cable. Note that σ is given in cm^2, so we will need to use consistent units.

$$A = \pi D L = \pi \times 30.48\text{ cm } \times 806.196\text{ cm} = 7.72 \times 10^4\text{ cm}^2$$

Therefore,

$$T = \sqrt[4]{\frac{P}{A\sigma}} = \sqrt[4]{\frac{10^{10}\text{ W}}{7.72 \times 10^4\text{ cm}^2 \times 6 \times 10^{-12}\ \frac{\text{W}}{\text{K}^4\text{cm}^2}}} = \boxed{\mathbf{12,121\ K}}$$

This is indeed a preposterous temperature, more than twice that at the surface of the Sun! The "solution" to this problem is to look at the melting point of copper, which is ~1358 K at standard pressure. The copper cable will melt long before such a temperature is reached.

Exercise 1.10

(a) With two equal-value resistors, the output voltage is half the input voltage.

$$V_{out} = \frac{1}{2}V_{in} = \frac{30\text{V}}{2} = \boxed{\mathbf{15}\text{V}}$$

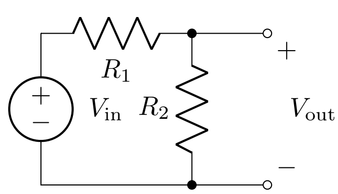
(b) To treat R_2 and R_{load} as a single resistor, combine the two resistors which are in parallel to find that the combined (equivalent) resistance is 5kΩ. Now, we have a simple voltage divider with a 10kΩ resistor in series with the 5kΩ equivalent resistor. The output voltage is across this equivalent resistance. The output voltage is given by

$$V_{out} = V_{in}\frac{5\text{k}\Omega}{10\text{k}\Omega + 5\text{k}\Omega} = \frac{30\text{V}}{3} = \boxed{\mathbf{10}\text{V}}$$

TODO: Add a diagram to make this clearer

(c) We can redraw the voltage divider circuit to make the "port" clearer.

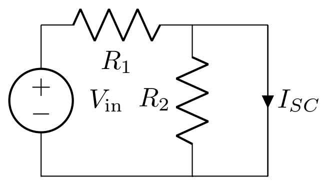
Figure 2: Voltage divider with port shown.



We can find V_{Th} by leaving the ports open (open circuit) and measuring V_{out}, the voltage across R_2. This comes out to be half the input voltage when $R_1 = R_2$, so $V_{\text{out}} = 15\text{V}$. Thus $V_{\text{Th}} = \boxed{\mathbf{15}\text{V}}$.

To find the Thévinen resistance, we need to find the short circuit current, I_{SC}. We short circuit the port and measure the current flowing through it.

Figure 3: Voltage divider with short circuit on the output.

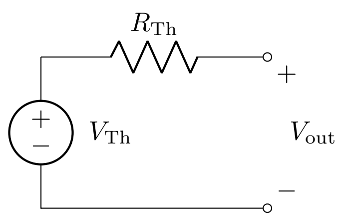


In this circuit, no current flows through R_2, flowing through the short instead. Thus we have $I_{SC} = \dfrac{V_{\text{in}}}{R_1}$. From this, we can find R_{Th} from $R_{\text{Th}} = \dfrac{V_{\text{Th}}}{I_{SC}}$. This gives us

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_{SC}} = \frac{V_{\text{Th}}}{V_{\text{in}}/R_1} = \frac{15\text{V}}{30\text{V}/10\text{k}\Omega} = \boxed{\mathbf{5k\Omega}}$$

The Thévenin equivalent circuit takes the form shown below.

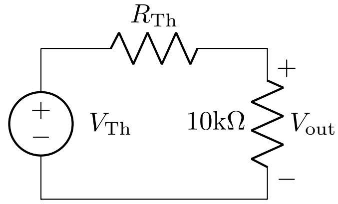
Figure 4: Thévenin equivalent circuit.



In terms of behavior at the ports, this circuit is equivalent to the circuit in Figure 2.

(d) We connect the 10kΩ load to the port of the Thévenin equivalent circuit in Figure 4 to get the following circuit.

Figure 5: Thévenin equivalent circuit with 10kΩ load.



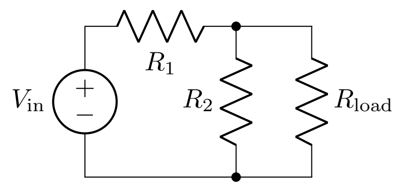
From here, we can find V_{out}, treating this circuit as a voltage divider.

$$V_{\text{out}} = \frac{10\text{k}\Omega}{R_{\text{Th}} + 10\text{k}\Omega} V_{\text{Th}} = \frac{10\text{k}\Omega}{5\text{k}\Omega + 10\text{k}\Omega} \cdot 15\text{V} = \boxed{\mathbf{10}\text{V}}$$

This is the same answer we got in part (b).

(e) To find the power dissipated in each resistor, we return to the original three-resistor circuit.

Figure 6: Original voltage divider with 10kΩ load attached.



From part (d), we know that the output voltage is 10V and that this is the voltage across the load resistor. Since $P = IV = \frac{V^2}{R}$, we find that the power through R_{load} is

$$P_{\text{load}} = \frac{V^2}{R_{\text{load}}} = \frac{(10\text{V})^2}{10\text{k}\Omega} = \boxed{\mathbf{10}\text{mW}}$$

Similarly, we know that the power across R_2 is the same since the voltage across R_2 is the same as the voltage across R_{load}. Thus we have

$$P_2 = \boxed{\mathbf{10}\text{mW}}$$

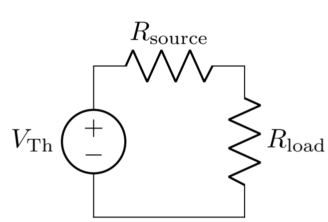
To find the power dissipated in R_1, we first have to find the voltage across it. From Kirchoff's loop rule, we know that the voltage around any closed loop in the circuit must be zero. We can choose the loop going through the voltage source, R_1, and R_2. The voltage supplied by the source is 30V. The voltage dropped across R_2 is 10V as discussed before. Thus the voltage dropped across R_1 must be $30\text{V} - 10\text{V} - 20\text{V}$. Now we know the voltage across and the resistance of R_1. We use the same formula as before to find the power dissipated.

$$P_1 = \frac{V^2}{R_1} = \frac{(20\text{V})^2}{10\text{k}\Omega} = \boxed{\mathbf{40}\text{mW}}$$

Exercise 1.11

Consider the following Thévenin circuit where R_{source} is just another name for the Thévenin resistance, R_{Th}.

Figure 7: Standard Thévenin circuit with attached load.



We will first calculate the power dissipated in the load and then maximize it with calculus. We can find the power through a resistor using current and resistence since $P = IV = I(IR) = I^2 R$. To find the total current flowing through the resistors, we find the equivalent resistance which is $R_{\text{source}} + R_{\text{load}}$. Thus the total current flowing is $I = \frac{V_{\text{Th}}}{R_{\text{source}}+R_{\text{load}}}$. The power dissipated in R_{load} is thus

$$P_{\text{load}} = I^2 R_{\text{load}} = \frac{V_{\text{Th}}^2 R_{\text{load}}}{(R_{\text{source}} + R_{\text{load}})^2}$$

To maximize this function, we take the derivative and set it equal to 0.

$$\begin{aligned}
\frac{dP_{\text{load}}}{dR_{\text{load}}} &= V_{\text{Th}} \frac{(R_{\text{source}} + R_{\text{load}})^2 - 2R_{\text{load}}(R_{\text{source}} + R_{\text{load}})}{(R_{\text{source}} + R_{\text{load}})^4} = 0 \\
&\implies R_{\text{source}} + R_{\text{load}} = 2R_{\text{load}} \\
&\implies R_{\text{source}} = R_{\text{load}}
\end{aligned}$$

Exercise 1.12

(a) Voltage ratio: $\frac{V_2}{V_1} = 10^{db/20} = 10^{3/20} = \boxed{\mathbf{1.413}}$

Power ratio: $\frac{P_2}{P_1} = 10^{db/10} = 10^{3/10} = \boxed{\mathbf{1.995}}$

(b) Voltage ratio: $\frac{V_2}{V_1} = 10^{db/20} = 10^{6/20} = \boxed{\mathbf{1.995}}$

Power ratio: $\frac{P_2}{P_1} = 10^{db/10} = 10^{6/10} = \boxed{\mathbf{3.981}}$

(c) Voltage ratio: $\frac{V_2}{V_1} = 10^{db/20} = 10^{10/20} = \boxed{\mathbf{3.162}}$

Power ratio: $\frac{P_2}{P_1} = 10^{db/10} = 10^{10/10} = \boxed{\mathbf{10}}$

(d) Voltage ratio: $\frac{V_2}{V_1} = 10^{db/20} = 10^{20/20} = \boxed{\mathbf{10}}$

Power ratio: $\frac{P_2}{P_1} = 10^{db/10} = 10^{20/10} = \boxed{\mathbf{100}}$

Exercise 1.13

There are two important facts to notice from Exericse 1.12:

1. An increase of 3dB corresponds to doubling the power
2. An increase of 10dB corresponds to 10 times the power.

Using these two facts, we can fill in the table. Start from 10dB. Fill in 7dB, 4dB, and 1dB using fact 1. Then fill in 11dB using fact 2. Then fill in 8dB, 5dB, and 2dB using fact 1 and approximating 3.125 as π.

dB	ratio(P/P_0)
0	1
1	$\boxed{\mathbf{1.25}}$
2	$\boxed{\pi/\mathbf{2}}$
3	2
4	$\boxed{\mathbf{2.5}}$
5	$\boxed{\mathbf{3.125} \approx \pi}$
6	4
7	$\boxed{\mathbf{5}}$
8	$\boxed{\mathbf{6.25}}$
9	8
10	10
11	$\boxed{\mathbf{12.5}}$

Exercise 1.14

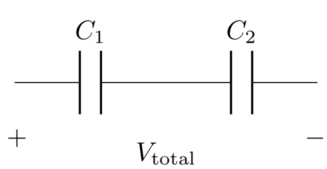
Recall the relationship between I, V, and C: $I = C\frac{dV}{dt}$. Now, we perform the integration:

$$\begin{aligned}
\int dU &= \int_{t_0}^{t_1} VI dt \\
U &= \int_{t_0}^{t_1} CV\frac{dV}{dt} dt \\
&= C\int_{0}^{V_f} V dV \\
U &= \frac{1}{2} C V_f^2
\end{aligned}$$

Exercise 1.15

Consider the following two capacitors in series.

Figure 8: Two capacitors in series.



To prove the capacitance formula, we need to express the total capacitance of both of these capacitors in terms of the individual capacitances. From the definition of capacitance, we have

$$C_{\text{total}} = \frac{Q_{\text{total}}}{V_{\text{total}}}$$

Notice that V_{total} is the sum of the voltages across C_1 and C_2. We can get each of these voltages using the definition of capacitance.

$$V_{\text{total}} = V_1 + V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

The key observation now is that because the right plate of C_1 is connected to the left plate of C_2, the charge stored on both plates must be of equal magnitude.[1]Therefore, we have $Q_1 = Q_2$. Let us call this charge stored Q (i.e. $Q = Q_1 = Q_2$). Now, we know that the total charge stored is also Q.[2]Therefore, we know that $Q_{\text{total}} = Q$. Now, we have

$$C_{\text{total}} = \frac{Q_{\text{total}}}{V_{\text{total}}} = \frac{Q}{Q_1/C_1 + Q_2/C_2} = \frac{Q}{Q/C_1 + Q/C_2} = \frac{1}{1/C_1 + 1/C_2}$$

Exercise 1.16

Equation 1.21 gives us the relationship between the time and the voltage (V_{out}) across the capacitor while charging. To find the rise time, subtract the time it takes to reach 10% of the final value from the time it takes to reach 90% of the final value.

$$\begin{aligned} V_{\text{out}} &= 0.1V_f = V_f(1 - e^{-t_1/RC}) \\ 0.1 &= 1 - e^{-t_1/RC} \\ t_1 &= -RC\ln(0.9) \end{aligned}$$

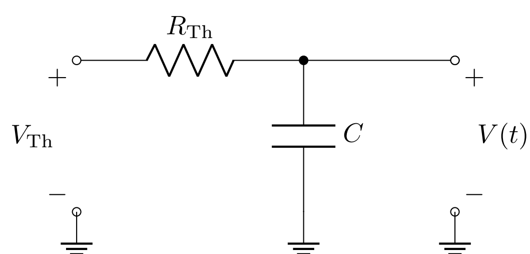
Similarly, we find that $t_2 = -RC\ln(0.1)$. Subtracting these two gives us

$$t_2 - t_1 = -RC(\ln(0.1) - \ln(0.9)) = 2.2RC$$

Exercise 1.17

The voltage divider on the left side of the circuit can be replaced with the Thévenin equivalent circuit found Exercise 1.10 (c). Recall that $V_{\text{Th}} = \frac{1}{2}V_{\text{in}}$ and $R_{\text{Th}} = 5\text{k}\Omega$. This gives us the following circuit.

Figure 9: Thévenin equivalent circuit to Figure 1.36 from the textbook.



Now we have a simple RC circuit which we can apply Equation 1.21 to. The voltage across the capacitor is given by

$$V(t) = V_{\text{final}}(1 - e^{-t/RC}) = V_{\text{Th}}(1 - e^{-t/R_{\text{Th}}C} = \boxed{\mathbf{\frac{1}{2}V_{\text{in}}(1 - e^{-t/5\times10^{-4}})}}$$

TODO: Add graph

[2]If this were not true, then there would be a net charge on these two plates and the wire between them. Because we assume that the capacitors started out with no net charge and there is no way for charge to leave the middle wire or the two plates it connects, this is impossible.

[2]If you are having trouble seeing this, suppose we apply a positive voltage to the left plate of C_1 relative to the right plate of C_2. Suppose this causes the left plate of C_1 to charge to some charge q. We now must have a charge of $-q$ on the right plate of C_1 because q units of charge are now pushed onto the left plate of C_2. Now the left of C_2 has q units of charge which causes a corresponding $-q$ charge on the right side of C_2. Thus the overall total charge separated across these two capacitors is q.

Exercise 1.18

From the capacitor equation in the previous paragraph, we have

$$V(t) = (I/C)t = (1\text{mA}/1\mu\text{F})t = 10\text{V}$$

This gives us

$$\boxed{\mathbf{t = 0.01}\text{s}}$$

Exercise 1.19

The magnetic flux produced within the coil is proportional to the number of turns. Now, because the inductance of the inductor is proportional to the amount of magnetic flux that passes through all the coils, it is proportional to the product of the magnetic flux and the number of coils. Thus the inductance is propotional to the square of the number of turns. **TODO: Check/clarify this answer**

Exercise 1.20

We can use the formula for the full-wave rectifier ripple voltage to find the capacitance.

$$\frac{I_{\text{load}}}{2fC} = \Delta V \leq 0.1\text{V}_{\text{p-p}}$$

The maximum load current is 10mA and assuming a standard wall outlet frequency of 60 Hz, we have

$$C \geq \frac{10\text{mA}}{2 \times 60\text{Hz} \times 0.1\text{V}} = \boxed{\mathbf{833}\mu\text{F}}$$

Now we need to find the AC input voltage. The peak voltage after rectification must be 10V (per the requirements). Since each phase of the AC signal must pass through 2 diode drops, we have to add this to find out what our AC peak-to-peak voltage must be. Thus we have

$$V_{\text{in,p-p}} = 10\text{V} + 2(0.6\text{V}) = \boxed{\mathbf{11.2}\text{V}}$$

Exercise 1.30

V_{out} is simply the voltage at the output of an impedance voltage divider. We know that $Z_R = R$ and $Z_C = \frac{1}{j\omega C}$. Thus we have

$$V_{\text{out}} = \frac{Z_C}{Z_R + Z_C} V_{\text{in}} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} V_{\text{in}} = \frac{1}{1 + j\omega RC} V_{\text{in}}$$

The magnitude of this expression can be found by multiplying by the complex conjugate and taking the square root.

$$sqrt V_{\text{out}} V_{\text{out}}^* = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} V_{\text{in}}$$