Optics

Xiping Hu

https://hxp.plus/

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History of Optics

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Electromagnetic Theory and photons

2.1 Maxwell's Equation

Faraday's Induction Law

$$\oint_c \vec{E} \cdot \mathrm{d}\vec{l} = -\frac{\mathrm{d}}{\mathrm{d}t} \iint \vec{B} \cdot \mathrm{d}\vec{S}$$

Gauss's Law

$$\oiint_A \vec{E} \cdot \mathrm{d}\vec{S} = \frac{1}{\varepsilon_0} \iiint_v \rho \,\mathrm{d}V$$

$$\oiint_A \vec{B} \cdot \mathrm{d}\vec{S} = 0$$

Ampere's Circuital Law

$$\oint_C \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 \iint_A \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \cdot \mathrm{d}\vec{S}$$

We can now take the derivatives of the 4 equations

$$\begin{cases} \nabla \times \vec{E} = -\dfrac{\partial B}{\partial t} \\ \nabla \times \vec{B} = \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} \\ \nabla \cdot \vec{E} = \dfrac{\rho}{\varepsilon_0} \\ \nabla \cdot \vec{B} = 0 \end{cases}$$

The Following Equation can be derived from Maxwell Equation above

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$
$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

Coincidentally

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Which indicates the speed of electromagnetic wave is the speed of light.

Furthermore, it can be seen that the electric field and magnetic field are transverse. They are perpendicular to each other. We assume the electric field is parallel to the y-axis.

$$E_y(x,t) = E_0 \cos\left[\omega\left(t - x/c\right) + \varepsilon\right]$$

According to Faraday's Law

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_Z}{\partial t}$$

We can calculate B_z

$$B_z = \frac{1}{c} E_0 \cos\left[\omega\left(t - x/c\right) + \varepsilon\right]$$

So that

$$E_y = cB_z$$

When not in vacuum, similarly

$$E_y = vB_z$$
$$v = \frac{1}{\varepsilon\mu}$$

2.2 Energy

$$\begin{aligned}
u_E &= \frac{\varepsilon_0}{2} E^2 \\
u_B &= \frac{1}{2\mu_0} B^2 \\
u_E &= u_B \\
u &= u_E + u_B = \varepsilon_0 E^2 = \frac{1}{\mu_0} B^2 \\
S &= uc \\
\vec{S} &= \frac{1}{\mu} \vec{E} \times \vec{B} \quad \text{(PoyntingVector)} \\
I &= \frac{1}{2} \varepsilon v E_0^2
\end{aligned}$$

2.3 Radiation Pressure

$$P(t) = \frac{S(t)}{c} = u = u_E + u_B$$

$$\langle P(t) \rangle_T = \frac{I}{c}$$

$$p_V = \frac{S}{c^2}$$

2.4 Light in Bulk Matter

2.4.1 Speed of light and Dielectric Constant

$$v = \frac{1}{\sqrt{\varepsilon\mu}}$$

$$n = \frac{c}{v} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \sqrt{\frac{\varepsilon}{\varepsilon_0}}$$

2.4.2 Dispersion

For gas and solid

$$m_e\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+\gamma m_e\frac{\mathrm{d}x}{\mathrm{d}t}+m_e\omega_0^2x=-eE\left(t\right)$$
$$E\left(t\right)=E_0\exp\left(-i\omega t\right)$$

Assume

$$x=x_0\exp\left(-i\omega t\right)$$

We got a solution

$$x_0\left(\omega_0^2-\omega^2-i\gamma\omega\right)=-\frac{eE_0}{m_e}$$
$$x_0=-\frac{eE_0}{m_e\left(\omega_0^2-\omega^2-i\gamma\omega\right)}$$
$$x\left(t\right)=-\frac{eE\left(t\right)}{m_e\left(\omega_0^2-\omega^2-i\gamma\omega\right)}$$
$$P\left(t\right)=-Nex\left(t\right)=\frac{Ne^2E\left(t\right)}{m_e\left(\omega_0^2-\omega^2-i\gamma\omega\right)}$$
$$\varepsilon_r=\frac{\varepsilon}{\varepsilon_0}=n^2=1+\frac{P}{\varepsilon_0E}=1+\frac{Ne^2}{\varepsilon_0m_e\left(\omega_0^2-\omega^2-i\gamma\omega\right)}$$
$$\begin{cases}\mathrm{Re}\left(\varepsilon_r\right)=1+\dfrac{Ne^2\left(\omega_0^2-\omega^2\right)}{\varepsilon_0m_e\left[\left(\omega_0^2-\omega^2\right)^2+\gamma^2\omega^2\right]}\\ \mathrm{Im}\left(\varepsilon_r\right)=\dfrac{Ne^2\gamma\omega}{\varepsilon_0m_e\left[\left(\omega_0^2-\omega^2\right)^2+\gamma^2\omega^2\right]}\end{cases}$$

When $\gamma=0$

$$\varepsilon_r=n^2=1+\frac{Ne^2}{\varepsilon_0m\left(\omega_0^2-\omega^2\right)}$$

For metal

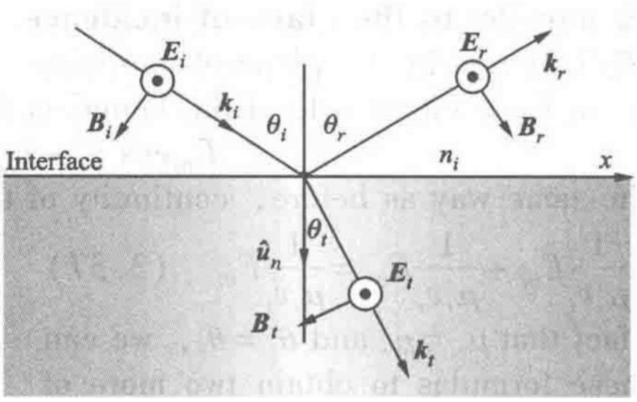
$$m_e\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+\gamma m_e\frac{\mathrm{d}x}{\mathrm{d}t}=-eE\left(t\right)$$
$$\varepsilon_r=1-\frac{Ne^2}{\varepsilon_0m_e\left(\omega^2+i\gamma\omega\right)}=1-\frac{\omega_p^2}{\omega\left(\omega+i\gamma\right)}$$

Chapter 3

The Propagation of Light

3.1 The Fresnel Equations

3.1.1 Electric Field Perpendicular to Plane of Incidence



$$
\begin{cases}
E_i + E_r = E_t \\
B_i \cos\theta_i = B_r \cos\theta_r + B_t \cos\theta_t
\end{cases}
$$

$$
\begin{cases}
E = vB \\
v = \dfrac{c}{n}
\end{cases}
$$

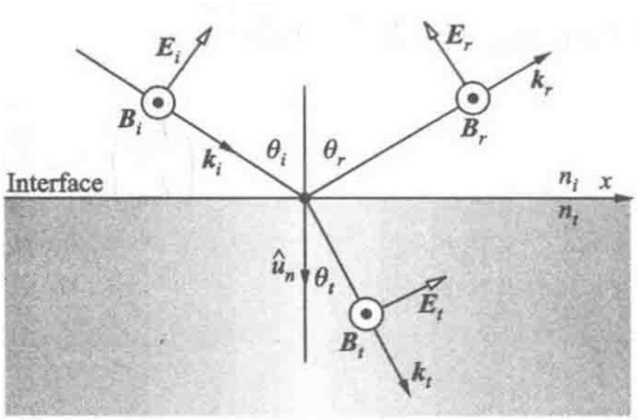
We define the amplitude reflection coefficient r, the amplitude transmission coefficient t

$$
r = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}
$$
$$
t = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}
$$

Considered that $n_i \sin\theta_i = n_t \sin\theta_t$

$$
r_\perp = \frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}
$$
$$
t_\perp = \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)}
$$

3.1.2 Electric Field Parallel to Plane of Incidence



$$\begin{cases} B_i + B_r = B_t \\ E_i \cos\theta_i = E_r \cos\theta_r + E_t \cos\theta_t \end{cases}$$

$$\begin{cases} E = vB \\ v = \dfrac{c}{n} \end{cases}$$

We define the amplitude reflection coefficient r, the amplitude transmission coefficient t

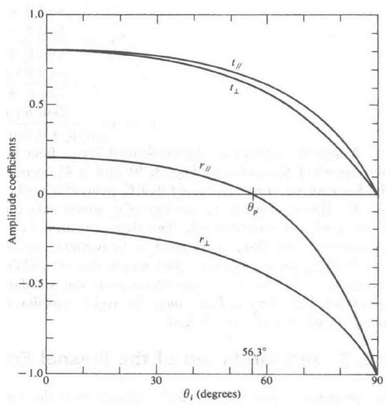
$$r = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$
$$t = \frac{2n_i \cos\theta_i}{n_t \cos\theta_i + n_i \cos\theta_t}$$

Considered that $n_i \sin\theta_i = n_t \sin\theta_t$

$$r_\parallel = \frac{\sin(2\theta_i) - \sin(2\theta_t)}{\sin(2\theta_i) + \sin(2\theta_t)} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$
$$t_\parallel = \frac{2\sin\theta_t\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

3.2 Polarization Angle

$$\theta_p = \arctan\left(\frac{\theta_p}{\theta_i}\right)$$



3.3 Critical Angle

$$\theta_C = \arcsin\left(\frac{n_t}{n_i}\right)$$