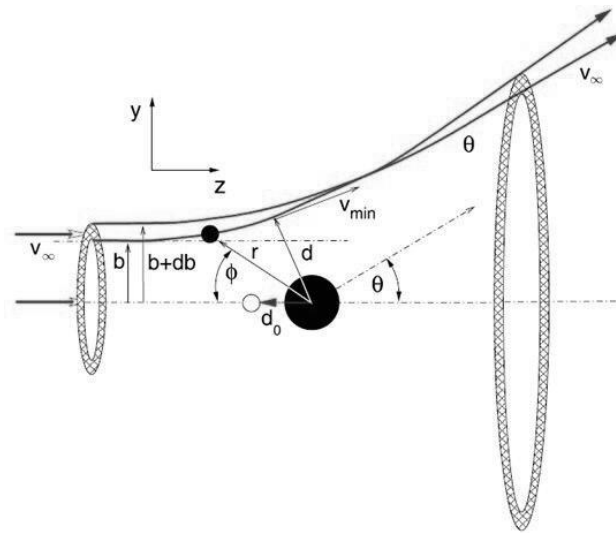# Atomic Physics

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# Chapter 1

# Rutherford's Alpha Particle Scattering Experiment



According to Columb's Law:

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Z_1 Z_2 e^2}{r^2} = \frac{C}{r^2}$$

$$C = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}$$

$$F_y = F \sin\phi = \frac{C}{r^2} \sin\phi$$

Law of momentum and angular momentum

$$m v_y = \int F_y \, \mathrm{d}t$$

$$m r^2 \dot{\phi} = m v_\infty b$$

Then we integrate

$$v_y = \frac{1}{m} \int \frac{C}{r^2} \sin\phi \, dt = \frac{1}{m} \int \frac{C}{r^2} \sin\phi \frac{dt}{d\phi} \, d\phi = \frac{1}{m} \int \frac{C}{r^2} \sin\phi \frac{r^2}{v_\infty b} \, d\phi = \frac{C}{mv_\infty b} \int_0^{\pi-\theta} \sin\phi \, d\phi$$

$$= \frac{C}{mv_\infty b} (1 + \cos\theta)$$

Now we need to relate $\theta$ with $b$, Since

$$v_y = v_\infty \sin\theta$$

We have

$$\frac{C}{mv_\infty b} (1 + \cos\theta) = v_\infty \sin\theta$$

So that

$$\cot\frac{\theta}{2} = \frac{1 + \cos\theta}{\sin\theta} = \frac{mv_\infty^2 b}{C} = \frac{2E_0 b}{C}$$

Note that this trigonometry transform is used

$$\frac{1 + \cos\theta}{\sin\theta} = \frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \cot\frac{\theta}{2}$$

Finally
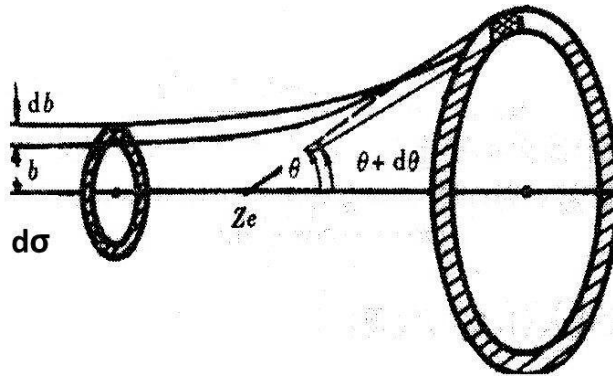
$$b = \frac{C}{2E_0} \cdot \cot\frac{\theta}{2} = \frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2 e^2}{2E_0} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Z_1 Z_2 e^2}{mv_\infty^2} \cdot \cot\frac{\theta}{2}$$

$\Rightarrow$

$$\cot\frac{\theta}{2} = 4\pi\varepsilon_0 \frac{mv_\infty^2}{Z_1 Z_2 e^2} b \tag{1.1}$$

Now we begin to find the relation between $db$ and $d\Omega$



$$d\sigma = 2\pi b \, db = \pi \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \left(\frac{Z_1 Z_2 e^2}{mv_\infty^2}\right)^2 \frac{\cos\frac{\theta}{2}}{\sin^3\frac{\theta}{2}} \, d\theta$$

$$\Omega = 2\pi \left(1 - \cos\theta\right)$$

$$d\Omega = 2\pi \sin\theta \, d\theta = 4\pi \sin\frac{\theta}{2}\cos\frac{\theta}{2} \, d\theta$$

Then we found $\dfrac{d\sigma}{d\Omega}$ is only related with $\theta$

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \left(\frac{Z_1 Z_2 e^2}{2mv_\infty^2}\right)^2 \sin^{-4}\frac{\theta}{2} \tag{1.2}$$

As for a thin gold leaf, we assume there's only one layer of atoms, the density of atoms is $N$, the area and thickness of gold leaf are $A$ and $t$

When $n$ particles passed through the gold leaf, $dn$ of them ended up in $d\Omega$

$$\frac{dn}{n} = \frac{NAt \, d\sigma}{A} = Nt \, d\sigma = Nt \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \left(\frac{Z_1 Z_2 e^2}{2mv_\infty^2}\right)^2 \sin^{-4}\frac{\theta}{2} \, d\Omega$$

$$\frac{dn}{d\Omega}\sin^4\frac{\theta}{2} = nNt \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \left(\frac{Z_1 Z_2 e^2}{2mv_\infty^2}\right)^2 = \text{const} \tag{1.3}$$

For alpha particles, $Z_1 = 2$. We can also take the closest distance between the 2 particles as the radius of a particle.
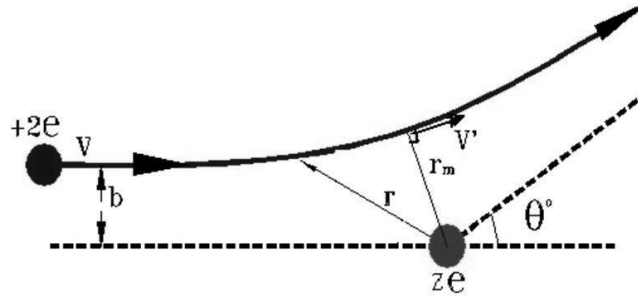


What we have already known are:

$$\frac{1}{2}MV^2 = \frac{1}{2}MV'^2 + \frac{2Ze^2}{4\pi\varepsilon_0 r_m}$$

$$MVb = MV'r_m$$

$$b = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{MV} \cdot \cot\frac{\theta}{2}$$

We solve $r_m$ with the equation we known.

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{M} \cdot \cot\frac{\theta}{2} = V'r_m$$

$$\frac{1}{2}MV^2 = \frac{1}{2}M \left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{Mr_m} \cdot \cot\frac{\theta}{2}\right)^2 + \frac{2Ze^2}{4\pi\varepsilon_0 r_m}$$

$$\frac{1}{2}MV^2 r_m^2 - \frac{2Ze^2}{4\pi\varepsilon_0}r_m - \frac{1}{2}M \left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{M} \cdot \cot\frac{\theta}{2}\right)^2 = 0$$

Finally

$$r_m = \frac{1}{4\pi\varepsilon_0} \frac{2Ze^2}{MV^2} \left( 1 + \frac{1}{\sin\dfrac{\theta}{2}} \right) \tag{1.4}$$

# Chapter 2

# The Energy and Radiation of Atoms

## 2.1  Rydberg Constant and Wavelength of Radiation

Wave number of Hydrogen Atoms:

$$\tilde{\nu} = \frac{1}{\lambda} = Z^2 R \left[ \frac{1}{m^2} - \frac{1}{n^2} \right] \tag{2.1}$$

Where $Z = 1$

| Lyman Series | $\tilde{\nu} = R \left[ \dfrac{1}{1^2} - \dfrac{1}{n^2} \right]$ | $n = 2, 3, 4, \dots$ |
|---|---|---|
| Balmer Series | $\tilde{\nu} = R \left[ \dfrac{1}{2^2} - \dfrac{1}{n^2} \right]$ | $n = 3, 4, 5, \dots$ |
| Paschen Series | $\tilde{\nu} = R \left[ \dfrac{1}{3^2} - \dfrac{1}{n^2} \right]$ | $n = 4, 5, 6, \dots$ |
| Brackett Series | $\tilde{\nu} = R \left[ \dfrac{1}{4^2} - \dfrac{1}{n^2} \right]$ | $n = 5, 6, 7, \dots$ |
| Pfund Series | $\tilde{\nu} = R \left[ \dfrac{1}{5^2} - \dfrac{1}{n^2} \right]$ | $n = 6, 7, 8, \dots$ |

Spectroscopic term

$$T(m) = \frac{Z^2 R}{m^2} \quad T(n) = \frac{Z^2 R}{n^2}$$

Then

$$\tilde{\nu} = T(m) - T(n) \tag{2.2}$$

Energy of emitted light

$$E = \frac{hc}{\lambda} = hc\tilde{v} = hc \left[ T(m) - T(n) \right] = \frac{Rhc}{m^2} - \frac{Rhc}{n^2}$$

## 2.2   Bohr's Theory of Hydrogen Atoms

Energy of steady states:

$$E_n = -\frac{1}{2}\frac{Ze^2}{4\pi\varepsilon_0 r_n}$$

Transition

$$h\nu = E_n - E_m$$

Angular Momentum

$$L = n \cdot \frac{h}{2\pi} = n\hbar = m_e v_n r_n$$

$$m_e \frac{v_n^2}{r_n} = \frac{Ze^2}{4\pi\varepsilon_0 r_n^2}$$

So that

$$\frac{n^2\hbar^2}{m_e r_n^3} = \frac{Ze^2}{4\pi\varepsilon_0 r_n^2}$$

From which we can indicate that the radius is quantized.

$$r_n = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} \cdot \frac{n^2}{Z} = a_0 \cdot \frac{n^2}{Z} \quad \left(a_0 = \frac{4\pi\varepsilon_0\hbar}{m_e e^2}\right) \tag{2.3}$$

For hydrogen atoms, $Z = 1$. The energy is also quantized.

$$E_n = -\frac{1}{2}\frac{Ze^2}{4\pi\varepsilon_0}\frac{m_e e^2}{4\pi\varepsilon_0\hbar^2} \cdot \frac{Z}{n^2} = -\frac{m_e e^4}{2\left(4\pi\varepsilon_0\hbar\right)^2} \cdot \frac{Z^2}{n^2}$$

The velocity is also qumntumized

$$v_n = \frac{n\hbar}{m_e r_n} = \frac{n\hbar}{m_e}\frac{m_e e^2}{4\pi\varepsilon_0\hbar^2} \cdot \frac{Z}{n^2} = \frac{e^2}{4\pi\varepsilon_0\hbar} \cdot \frac{Z}{n} = \frac{Z\alpha c}{n}$$

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{1}{137}$$

Calculate the value of Rydberg Constant

$$E = -\frac{m_e e^4}{2\left(4\pi\varepsilon_0\hbar\right)^2} \cdot \frac{Z^2}{n^2} = -\frac{2\pi^2 m_e e^4}{\left(4\pi\varepsilon_0 h\right)^2} \cdot \frac{Z^2}{n^2}$$

$$\frac{hc}{\lambda} = E_2 - E_1 = \frac{2\pi^2 m_e e^4 Z^2}{\left(4\pi\varepsilon_0 h\right)^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\tilde{\nu} = \frac{1}{\lambda} = \frac{2\pi^2 m_e e^4 Z^2}{\left(4\pi\varepsilon_0\right)^2 h^3 c}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$\Rightarrow$

$$R = \frac{2\pi^2 m_e e^4}{\left(4\pi\varepsilon_0\right)^2 h^3 c}$$

## 2.3  Rydburg Constant of Different Atoms

In previous sections, we assumed that the nucleus is fixed at a point, with the electron surrounded. But in fact, the nucleus's mass is not infinity, and it moves as well. So that for different atoms, the Rydburg Constants varies. In a two-body system, we define the Reduced Mass of a system as

$$\mu = \frac{Mm}{M + m}$$

And we replace the Reduced Mass with $m_e$

$$R_\infty = \frac{2\pi^2 m e^4}{\left(4\pi\varepsilon_0\right)^2 h^3 c}$$

$$R_A = R_\infty \cdot \left[\frac{1}{1 + \dfrac{m}{M}}\right]$$

(2.4)

## 2.4  Sommerfeld's Quantize Condition

For any coordinate $q$ and its momentum $p$

$$\oint p \, \mathrm{d}q = nh$$

holds

## 2.5  Quantized Magneton

$$\mu = iA$$
$$i = \frac{e}{\tau}$$
$$A = \int_0^{2\pi} \frac{1}{2} r \cdot r \, \mathrm{d}\phi$$
$$= \frac{1}{2} \int_0^{\tau} r^2 \omega \, \mathrm{d}t$$
$$= \frac{1}{2m} \int_o^{\tau} m r^2 \omega \, \mathrm{d}t$$
$$= \frac{p_\phi}{2m} \tau$$

Consider that

$$\int_0^{2\pi} p_\phi \, \mathrm{d}\phi = 2\pi p_\phi = nh$$

We have

$$\mu = \frac{e}{2m} p_\phi = \frac{eh}{4\pi m} \cdot n$$

Let

$$\mu_B = \frac{eh}{4\pi m} = \frac{e}{2m} \hbar$$

(2.5)

Then $\mu_B$ is the minimum unit of magneton.

# Chapter 3

# Basics of Quantum Mechanics

## 3.1 De Broglie Wave

$$\lambda = \frac{h}{p} = \frac{h}{mv} \tag{3.1}$$

## 3.2 Uncertianty Principle

$$\Delta p \Delta q \geq \frac{\hbar}{2} \qquad \Delta E \Delta t \geq \frac{\hbar}{2} \tag{3.2}$$

## 3.3 Wave Function

$$\psi = \psi_0 \exp\left[\frac{i}{\hbar}\left(\vec{p}\cdot\vec{r} - Et\right)\right] \tag{3.3}$$

$$\begin{cases} \nabla\psi = -i\dfrac{\vec{p}}{\hbar}\psi \\ \nabla^2\psi = -\dfrac{p^2}{\hbar^2}\psi \\ \dfrac{\partial\psi}{\partial t} = -\dfrac{i}{\hbar}E\psi \end{cases} \tag{3.4}$$

Three properties

- limited
- Normalized
- Continuous

## 3.4 Schrodinger Equation

$$\frac{p^2}{2m} + V = E \quad \Rightarrow \quad \frac{p^2}{2m}\psi + V\psi = E\psi$$

$\Rightarrow$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t} \tag{3.5}$$

Simplified form for $E$ is constant:

$$Hu = Eu \qquad\qquad\qquad H = \left[ -\frac{\hbar^2}{2m}\nabla^2 + V \right]$$

$$Hu = Eu \qquad\qquad\qquad H = \left[ -\frac{\hbar^2}{2m}\nabla^2 + V \right]$$

# Chapter 4

# Alkalis and Self-spin of Electrons

## 4.1 Four Series of Spectrum

| Principle Series | $_p\tilde{v}_n = \dfrac{R}{(2-\Delta_s)^2} - \dfrac{R}{(n-\Delta_p)^2}$ | $n = 2, 3, 4, \ldots$ | $P \to S$ |
|---|---|---|---|
| Sharp (Second Subordinate) Series | $_s\tilde{v}_n = \dfrac{R}{(2-\Delta_p)^2} - \dfrac{R}{(n-\Delta_s)^2}$ | $n = 3, 4, 5, \ldots$ | $S \to P$ |
| Diffuse (First Subordinate) Series | $_d\tilde{v}_n = \dfrac{R}{(2-\Delta_p)^2} - \dfrac{R}{(n-\Delta_d)^2}$ | $n = 3, 4, 5, \ldots$ | $D \to P$ |
| Fundamental (Bergmann) Series | $_f\tilde{v}_n = \dfrac{R}{(3-\Delta_d)^2} - \dfrac{R}{(n-\Delta_f)^2}$ | $n = 4, 5, 6, \ldots$ | $F \to D$ |

The Spectroscopic term of alkalis

$$T = \frac{R}{n^{*2}} = \frac{R}{(n-\Delta)^2} \tag{4.1}$$
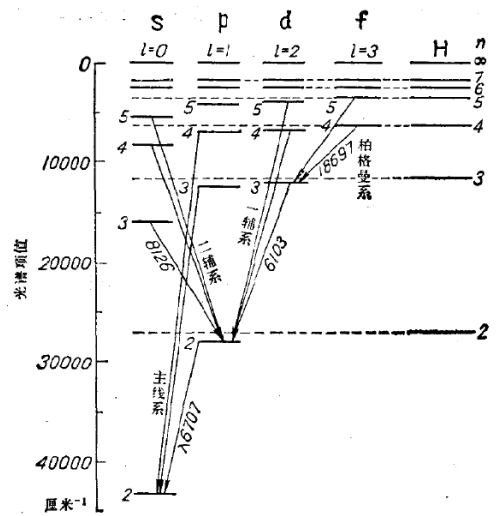


Figure 4.1: Energy Level of Lithium Atom

The wavenumbers of spectroscopic lines

$$\tilde{\nu}_n = \tilde{\nu}_\infty - \frac{R}{n^{*2}}$$

(4.2)

## 4.2   Self-spin and Orbital Angular Momentum

$$s = \frac{1}{2} \quad l = 1, 2, 3, \dots \quad j = l \pm s$$

| | | |
|---|---|---|
| Self-spin | $p_s = \sqrt{s(s+1)}\hbar$ | $\mu_s = \dfrac{e}{m}p_s$ |
| Orbital | $p_l = \sqrt{l(l+1)}\hbar$ | $\mu_s = \dfrac{e}{2m}p_l$ |
| Total | $p_j = \sqrt{j(j+1)}\hbar$ | $\mu_j = g\dfrac{e}{2m}p_j$ |

$$\vec{p}_s + \vec{p}_l = \vec{p}_j$$

## 4.3   Selection Rule in Transition

$$\begin{cases} \Delta l = \pm 1 \\ \Delta j = 0, \pm 1 \end{cases}$$

# Chapter 5

# Atoms with Two Electrons

## 5.1   LS Coupling

| Compound Self-spin Angular Momentum | $S = 1, 0$ | $P_S = \sqrt{S(S+1)}\hbar$ |
| --- | --- | --- |
| Compound Orbital Angular Momentum | $L = |l_1 + l_2|, |l_1 + l_2| - 1, \ldots, |l_1 - l_2|$ | $P_S = \sqrt{L(L+1)}\hbar$ |
| Compound Total Angular Momentum | $J = |L + S|, |L + S| - 1, \ldots, |L - S|$ | $P_S = \sqrt{S(S+1)}\hbar$ |

### 5.1.1   Hund's Law

- For a given electron configuration, the term with maximum multiplicity has the lowest energy. The multiplicity is equal to $2S + 1$ , where $S$ is the total spin angular momentum for all electrons.

- For a given multiplicity, the term with the largest value of the total orbital angular momentum quantum number $L$ , has the lowest energy.

- For a given term, in an atom with outermost subshell half-filled or less, the level with the lowest value of the total angular momentum quantum number $J$ , (for the operator $J = L + S$) lies lowest in energy. If the outermost shell is more than half-filled, the level with the highest value of $J$, is lowest in energy.

### 5.1.2   Rules of Transition

| $\Delta S = 0$ | $\Delta L = 0, \pm 1$ | $\Delta J = 0, \pm 1 \quad (0 \to 0\, forbidden)$ |
| --- | --- | --- |

## 5.2   jj Coupling

| Self-spin Angular Momentum | $s_1 = \dfrac{1}{2}$ | $s_2 = \dfrac{1}{2}$ |
| --- | --- | --- |
| Orbital Angular Momentum | $l_1$ | $l_2$ |
| Total Angular Momentum | $j_1 = l_1 \pm s_1$ | $j_2 = l_2 \pm s_2$ |
| Compound Total Angular Momentum | $J = |j_1 + j_2|, |j_1 + j_2| - 1, \ldots, |j_1 - j_2|$ | $P_J = \sqrt{J(J+1)}\hbar$ |

## 5.3   Pauli exclusion principle

For state $nsns$ or $ns^2$ (for instance, $1s1s$ of $2s2s$), $^3S_1$ can not exist.

# Chapter 6

# Atoms in Magnetic Field

## 6.1  Magnetic Moment of Atoms

$$\mu_L = -\frac{e}{2m}p_l = -\sqrt{l\,(l+1)}\mu_B \qquad\qquad \mu_B = \frac{eh}{2\pi m} = \frac{e\hbar}{2m} \quad \text{(Bohr magneton)}$$

## 6.2  Lande Factor

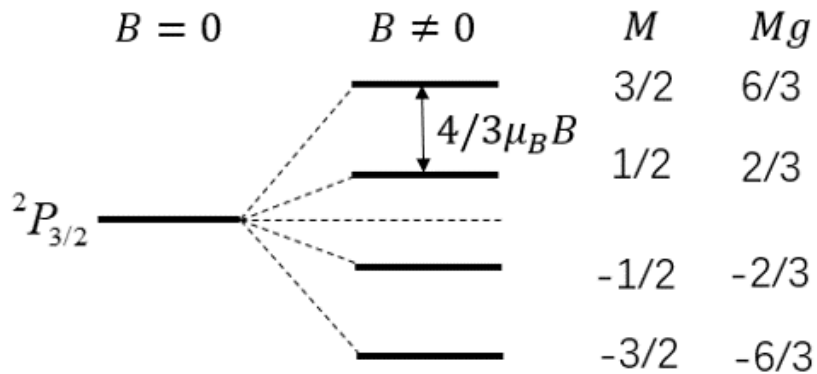$$g = 1 + \frac{j\,(j+1) - l\,(l+1) + s\,(s+1)}{2j\,(j+1)}$$

For single electron atom:

$$\mu_j = g\frac{e}{2m}p_j$$

For multi electron atom:

$$\mu_J = -g\frac{e}{2m}P_J$$

## 6.3  Energy of Atom in Magnetic Field



$$\Delta E = (M_2 g_2 - M_1 g_1)\,\mu_B B \qquad\qquad M = J, J-1, J-2, \ldots, -J$$

$$\Delta\tilde{\nu} = \frac{1}{\lambda'} - \frac{1}{\lambda} = [M_2g_2 - M_1g_1]\frac{Be}{4\pi mc} = [M_2g_2 - M_1g_1]L \qquad\qquad L = \frac{Be}{4\pi mc}$$

$^2\mathrm{P}_{3/2} \longrightarrow {}^2\mathrm{S}_{1/2}$

| M | 3/2 | 1/2 | -1/2 | -3/2 |
|---|---|---|---|---|
| $M_2g_2$ | 6/3 | 2/3 | -2/3 | -6/3 |

$\searrow \;\mid\; \times \;\mid\; \diagup$

| $M_1g_1$ | 1 | -1 |
|---|---|---|

-5/3  -3/3   -1/3      1/3      3/3  5/3

$$\Delta\tilde{\nu} = \Delta(\frac{1}{\lambda}) = (-\frac{5}{3}, -\frac{3}{3}, -\frac{1}{3}, \frac{1}{3}, \frac{3}{3}, \frac{5}{3})L$$

## 6.4   Selection Rule

$$\Delta M = 0 \quad \Rightarrow \quad \pi$$
$$\Delta M = \pm 1 \quad \Rightarrow \quad \sigma$$

$\pi$ beam cannot be observed in the perspective which is parallel to $B$, both beam can be observed in the perspective perpendicular to $B$.

## 6.5   Abnormal Zeeman effect

$$\Delta\tilde{\nu} = [M_2g_2 - M_1g_1]L \neq \pm L \quad \Rightarrow \quad M_2g_2 - M_1g_1 \neq \pm 1$$