Homework for Chapter 6

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1. 已知钒原子的基态是 $^4F_{\frac{3}{2}}$. (1)问钒原子束在不均匀横向磁场中将分裂为几束？(2)求基态钒原子的有效磁矩 μ_J.

$$l = 3 \qquad j = \frac{3}{2} \qquad s = \frac{3}{2}$$

$\Rightarrow$

$$g = 1 + \frac{j\left(j+1\right) - l\left(l+1\right) + s\left(s+1\right)}{2j\left(j+1\right)} = 0.4$$

$\Rightarrow$

$$m = j, j-1, \ldots, -j = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$$

Since m has 4 different values, the beam of atoms in magnet field will split into 4 sub-beams.

$$\mu_l = g\sqrt{j\left(j+1\right)}\mu_B = 7.18361 \times 10^{22}\ \mathrm{A} \cdot \mathrm{m}^2$$

3. Li 漫线系的一条($3^2D_{\frac{3}{2}} \to 2^2P_{\frac{1}{2}}$)在磁场中将分裂成多少条光谱线？试作出相应的能级跃迁图.

For $3^2D_{\frac{3}{2}}$

$$l = 2 \qquad j = \frac{3}{2} \qquad s = \frac{1}{2}$$

$\Rightarrow$

$$m = j, j-1, \ldots, -j = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$$

$$g = 1 + \frac{j\left(j+1\right) - l\left(l+1\right) + s\left(s+1\right)}{2j\left(j+1\right)} = \frac{4}{5}$$

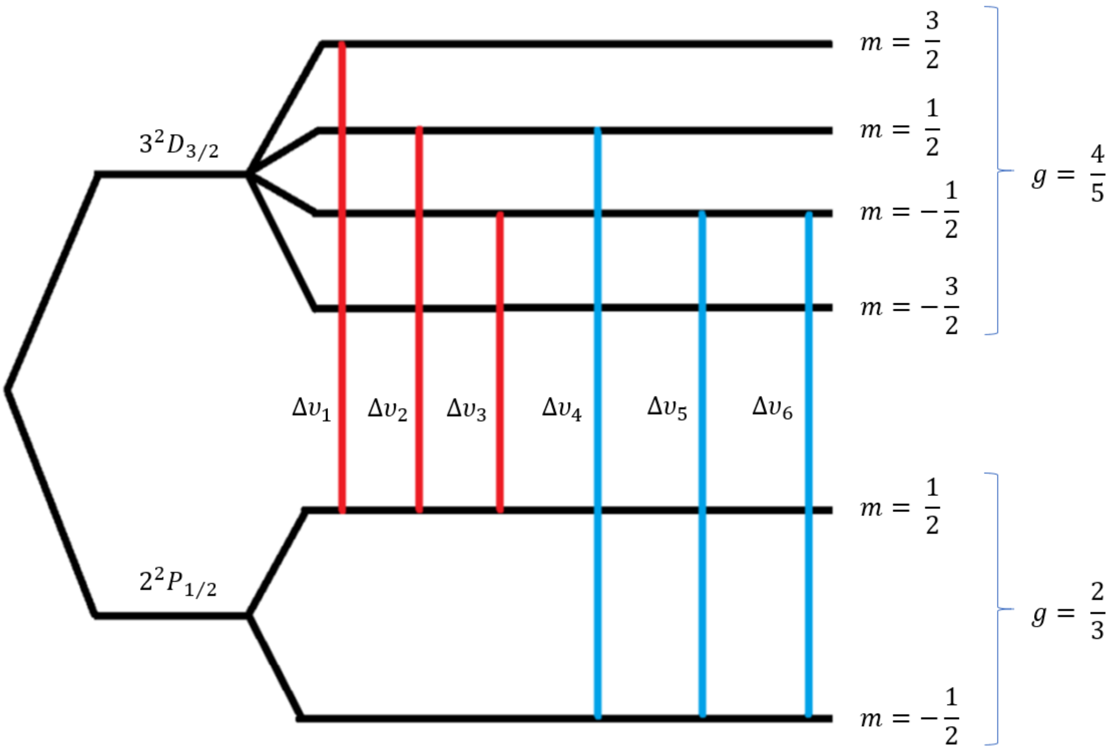
For $2^2P_{\frac{1}{2}}$

$$l = 1 \qquad j = \frac{1}{2} \qquad s = \frac{1}{2}$$

$\Rightarrow$

$$m = j, j-1, \ldots, -j = \frac{1}{2}, -\frac{1}{2}$$

$$g = 1 + \frac{j\left(j+1\right) - l\left(l+1\right) + s\left(s+1\right)}{2j\left(j+1\right)} = \frac{2}{3}$$



$$\nu_1 = \left(\frac{3}{2} \cdot \frac{4}{5} - \frac{1}{2} \cdot \frac{2}{3}\right) L = \frac{13}{15} L$$
$$\nu_2 = \left(\frac{1}{2} \cdot \frac{4}{5} - \frac{1}{2} \cdot \frac{2}{3}\right) L = \frac{1}{15} L$$
$$\nu_3 = \left(-\frac{1}{2} \cdot \frac{4}{5} - \frac{1}{2} \cdot \frac{2}{3}\right) L = -\frac{11}{15} L$$
$$\nu_4 = \left(\frac{1}{2} \cdot \frac{4}{5} + \frac{1}{2} \cdot \frac{2}{3}\right) L = \frac{11}{15} L$$
$$\nu_5 = \left(-\frac{1}{2} \cdot \frac{4}{5} + \frac{1}{2} \cdot \frac{2}{3}\right) L = -\frac{1}{15} L$$
$$\nu_6 = \left(-\frac{3}{2} \cdot \frac{4}{5} + \frac{1}{2} \cdot \frac{2}{3}\right) L = -\frac{13}{15} L$$

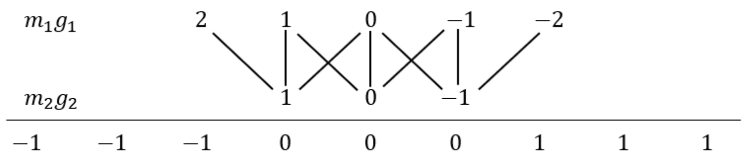
5. 氦原子光谱中波长为 6678.1Å(1s3d$^1D_2$→1s2p$^1P_1$) 及 7065.1Å(1s3s $^3S_1$→1s2p$^3P_0$) 的两条谱线，在磁场中发生塞曼效应时各分裂成几条？分别作出能级跃迁图。

$1s3d^1D_2 \to 1s2p^1P_1$:

$$
\begin{array}{ll}
l_1 = 2 & l_2 = 1 \\
s_1 = 0 & s_2 = 0 \\
j_1 = 2 & j_2 = 1
\end{array}
$$

$$
g = 1 + \frac{j(j+1) - l(l+1) + s(s+1)}{2j(j+1)} \Rightarrow \begin{cases} g_1 = 1 \\ g_2 = 1 \end{cases}
$$

$$
m = j, j-1, \ldots, -j \Rightarrow \begin{cases} m_1 = 2, 1, 0, -1, -2 \\ m_2 = 1, 0, -1 \end{cases}
$$



The spectral line will split into 3 components in Zeeman effect.

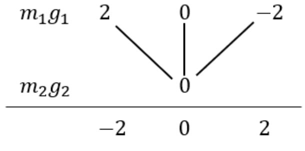
$1s3s^3S_1 \to 1s2p^3P_0$:

$$
\begin{array}{ll}
l_1 = 0 & l_2 = 1 \\
s_1 = 1 & s_2 = 1 \\
j_1 = 1 & j_2 = 0
\end{array}
$$

$\Rightarrow$

$$
\begin{cases} g_1 = 2 \\ g_2 = 0 \end{cases}
$$

$$
m = j, j-1, \ldots, -j \Rightarrow \begin{cases} m_1 = 1, 0, -1 \\ m_2 = 0 \end{cases}
$$



The spectral line will split into 3 components in Zeeman effect.

6. Na 原子从 $3^2P_{\frac{1}{2}} \to 3^2S_{\frac{1}{2}}$ 跃迁的光谱线波长为 5896 Å，在 $B = 2.5$ 韦伯/米^2 的磁场中发生塞曼分裂. 问从垂直于磁场方向观察，其分裂为多少条光谱线？其中波长最长和最短的两条光谱线的波长各多少 Å？

$$
\begin{aligned}
l_1 &= 1 & l_2 &= 0 \\
s_1 &= \frac{1}{2} & s_2 &= \frac{1}{2} \\
j_1 &= \frac{1}{2} & j_2 &= \frac{1}{2}
\end{aligned}
$$

$\Rightarrow$

$$
\begin{cases}
g_1 = \dfrac{2}{3} \\
g_2 = 2
\end{cases}
$$

$$
m = j, j-1, \dots, -j \Rightarrow \begin{cases}
m_1 = \dfrac{1}{2}, -\dfrac{1}{2} \\
m_2 = \dfrac{1}{2}, -\dfrac{1}{2}
\end{cases}
$$

$$
\begin{array}{ccccc}
m_1 g_1 & & \frac{1}{3} & -\frac{1}{3} & \\
m_2 g_2 & & 1 & -1 & \\
\hline
 & -\frac{4}{3} & -\frac{2}{3} & \frac{2}{3} & \frac{4}{3} \\
 & \sigma & \pi & \pi & \sigma
\end{array}
$$

Observing in perpendicular orientation, the spectral line will split into 4 components.

$$
\Delta\left(\frac{1}{\lambda}\right) = -\frac{\Delta\lambda}{\lambda^2} = \frac{4}{3}L \Rightarrow \Delta\lambda_{max} = \frac{8}{3}L\lambda^2 = 1.08 \text{ Å}
$$