Optics

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Contents

Chapter 1

Electromagnetic Theory and photons

1.1 Longitudinal and Transverse

- Longitudinal: medium is in the direction of motion of wave.
- Transverse: medium is perpendicular to the motion of wave.

1.2 Wave Equation

$$\psi\left(x,t\right)=f\left(x+vt\right)$$

$$\begin{cases}\dfrac{\partial}{\partial x}=\dfrac{\partial}{\partial\left(x+vt\right)}\cdot\dfrac{\partial\left(x+vt\right)}{\partial x}=\dfrac{\partial}{\partial\left(x+vt\right)}\\\dfrac{\partial}{\partial t}=\dfrac{\partial}{\partial\left(x+vt\right)}\cdot\dfrac{\partial\left(x+vt\right)}{t}=v\cdot\dfrac{\partial}{\partial\left(x+vt\right)}\end{cases}\Rightarrow\begin{cases}\dfrac{\partial^2}{\partial x^2}=\dfrac{\partial^2}{\partial\left(x+vt\right)^2}\\\dfrac{\partial^2}{\partial t^2}=v^2\cdot\dfrac{\partial^2}{\partial\left(x+vt\right)^2}\end{cases}\Rightarrow\begin{cases}\dfrac{\partial^2\psi}{\partial x^2}=\dfrac{\partial^2\psi}{\partial\left(x+vt\right)^2}\\\dfrac{\partial^2\psi}{\partial t^2}=v^2\cdot\dfrac{\partial^2\psi}{\partial\left(x+vt\right)^2}\end{cases}$$

$$\Rightarrow\frac{\partial^2\psi}{\partial x^2}=\frac{1}{v^2}\cdot\frac{\partial^2\psi}{\partial t^2}\quad\Rightarrow\quad\nabla^2\psi\left(x,y,z\right)=\frac{1}{v^2}\cdot\frac{\partial^2\psi\left(x,y,z\right)}{\partial t^2}$$

1.3 Maxwell's Equation

Faraday's Induction Law

$$\oint_c\vec{E}\cdot\mathrm{d}\vec{l}=-\frac{\mathrm{d}}{\mathrm{d}t}\iint\vec{B}\cdot\mathrm{d}\vec{S}\quad\Rightarrow\quad\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

Gauss's Law

$$\oiint_A\vec{E}\cdot\mathrm{d}\vec{S}=\frac{1}{\varepsilon_0}\iiint_v\rho\,\mathrm{d}V\quad\Rightarrow\quad\nabla\cdot\vec{E}=\frac{\rho}{\varepsilon_0}$$

$$\oiint_A\vec{B}\cdot\mathrm{d}\vec{S}=0\quad\Rightarrow\quad\nabla\cdot\vec{B}=0$$

Ampere's Circuital Law

$$\oint_C \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 \iint_A \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \cdot \mathrm{d}\vec{S} \quad \Rightarrow \quad \nabla \times \vec{B} = \mu_0 \varepsilon_0 \cdot \frac{\partial \vec{E}}{\partial t}$$

We can now take the derivatives of the 4 equations

$$\begin{cases} \nabla \times \vec{E} = -\dfrac{\partial B}{\partial t} \\ \nabla \times \vec{B} = \mu_0 \varepsilon_0 \cdot \dfrac{\partial \vec{E}}{\partial t} \\ \nabla \cdot \vec{E} = \dfrac{\rho}{\varepsilon_0} \\ \nabla \cdot \vec{B} = 0 \end{cases} \quad \Rightarrow \quad \begin{cases} \nabla^2 \vec{E} = \mu_0 \varepsilon_0 \cdot \dfrac{\partial^2 \vec{E}}{\partial t^2} = \dfrac{1}{c^2} \cdot \dfrac{\partial^2 \vec{E}}{\partial t^2} \\ \nabla^2 \vec{B} = \mu_0 \varepsilon_0 \cdot \dfrac{\partial^2 \vec{B}}{\partial t^2} = \dfrac{1}{c^2} \cdot \dfrac{\partial^2 \vec{B}}{\partial t^2} \end{cases}$$

Which indicates the speed of electromagnetic wave is exactly the speed of light.

Furthermore, it can be seen that the electric field and magnetic field are transverse. They are perpendicular to each other. We assume the electric field is parallel to the y-axis.

$$E_y(x,t) = E_0 \cos\left[\omega\left(t - x/c\right) + \varepsilon\right]$$

According to Faraday's Law

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

We can calculate B_z

$$B_z = \frac{1}{c} \cdot E_0 \cos\left[\omega\left(t - x/c\right) + \varepsilon\right]$$

So that

$$E_y = vB_z = \begin{cases} \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} \cdot B_z & \text{in vacuum} \\ \dfrac{1}{\sqrt{\mu \varepsilon}} \cdot B_z & \text{not in vacuum} \end{cases}$$

1.4 Energy

$$u_E = \frac{1}{2} \cdot \frac{\varepsilon_0}{1} \cdot E^2 \qquad u_B = \frac{1}{2} \cdot \frac{1}{\mu_0} \cdot B^2 \qquad u_E = u_B \qquad u = u_E + u_B = \varepsilon_0 E^2 = \frac{1}{\mu_0} B^2$$

$$S = uc = \varepsilon_0 c E^2 \qquad \text{(Power: Transport of Energy per unit time across a unit area)}$$

$$\vec{S} = \frac{1}{\mu} \cdot \vec{E} \times \vec{B} = c^2 \varepsilon \cdot \vec{E} \times \vec{B} \qquad \text{(Poynting Vector)} \qquad I = \frac{S}{2} = \frac{\varepsilon_0 c}{2} E_0^2 \qquad \text{(Irradiance)}$$

1.5 Radiation Pressure

$$P(t) = u = u_E + u_B = \frac{S}{c} \qquad \text{Radiation Pressure equals energy density of the EM wave}$$

$$\langle P(t)\rangle_T = \frac{1}{2} \cdot \frac{S}{c} = \frac{I}{c} \qquad \text{Average Radiation Pressure}$$

$$AP = \frac{\Delta p}{\Delta t} \quad \Rightarrow \quad Ac\Delta t P = c\Delta p \quad \Rightarrow \quad p_V = \frac{P}{c} = \frac{S}{c^2} \qquad \text{Momentum per Volume}$$

1.6 Light in Bulk Matter

1.6.1 Speed of light and Dielectric Constant

$$v = \frac{1}{\sqrt{\varepsilon\mu}} \qquad n = \frac{c}{v} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \sqrt{\frac{\varepsilon}{\varepsilon_0}}$$

1.6.2 Dispersion

For gas and solid

$$m_e \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \gamma m_e \frac{\mathrm{d}x}{\mathrm{d}t} + m_e \omega_0^2 x = -eE(t)$$
$$E(t) = E_0 \exp(-i\omega t)$$

Assume

$$x = x_0 \exp(-i\omega t)$$

We got a solution

$$x_0 \left(\omega_0^2 - \omega^2 - i\gamma\omega\right) = -\frac{eE_0}{m_e}$$
$$x_0 = -\frac{eE_0}{m_e\left(\omega_0^2 - \omega^2 - i\gamma\omega\right)}$$
$$x(t) = -\frac{eE(t)}{m_e\left(\omega_0^2 - \omega^2 - i\gamma\omega\right)}$$
$$P(t) = -Nex(t) = \frac{Ne^2 E(t)}{m_e\left(\omega_0^2 - \omega^2 - i\gamma\omega\right)}$$
$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = n^2 = 1 + \frac{P}{\varepsilon_0 E} = 1 + \frac{Ne^2}{\varepsilon_0 m_e\left(\omega_0^2 - \omega^2 - i\gamma\omega\right)}$$
$$\begin{cases} \mathrm{Re}(\varepsilon_r) = 1 + \dfrac{Ne^2\left(\omega_0^2 - \omega^2\right)}{\varepsilon_0 m_e\left[\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\omega^2\right]} \\ \mathrm{Im}(\varepsilon_r) = \dfrac{Ne^2\gamma\omega}{\varepsilon_0 m_e\left[\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\omega^2\right]} \end{cases}$$

When $\gamma = 0$

$$\varepsilon_r = n^2 = 1 + \frac{Ne^2}{\varepsilon_0 m\left(\omega_0^2 - \omega^2\right)}$$

For metal

$$m_e \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \gamma m_e \frac{\mathrm{d}x}{\mathrm{d}t} = -eE(t)$$

$$\varepsilon_r = 1 - \frac{Ne^2}{\varepsilon_0 m_e\left(\omega^2 + i\gamma\omega\right)} = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$$

Chapter 2

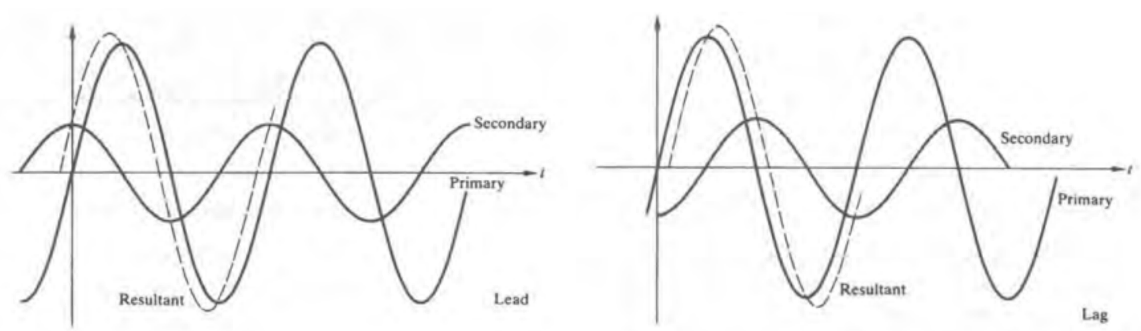
The Propagation of Light

2.1 Scattering and Interference

Destructive interference of the scattering light

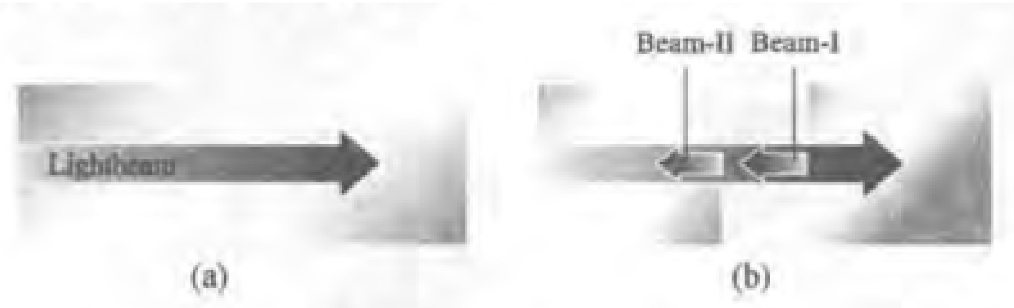
- The denser the substance through which light advances, the less the lateral scattering.
- The longer the wavelength, the less the lateral scattering.
- On an overcast day, sky looks white because of large water droplets scatters all lights. On sunny day, sky only scatters blue light. And if there were no atmosphere, sky would be black as it is on moon.
- All molecules have electronic resonances in UV, the closer driving frequency is to a resonance, the more vigorously the oscillator responds. Blue and violet response more than red, sky is blue.

2.2 Speed of Light in Medium



If the phase of light in dielectric lags behind vacuum one, the resultant lags, and vice versa.

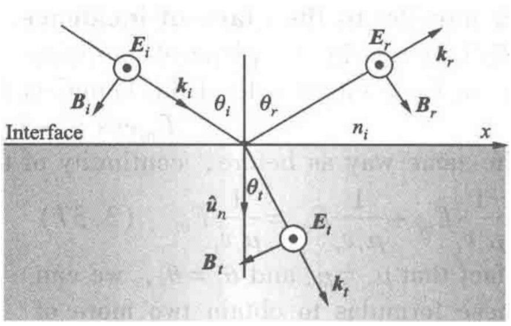
2.3 Internal and External Reflection



Beam I (internal reflection) and Beam II (external reflection) has 180° phase shift, when the gap between right part and left part of the glass in picture b becomes zero, two beams diminishes. This case is the same as picture a where the glass has not been cut.

2.4 The Fresnel Equations

2.4.1 Electric Field Perpendicular to Plane of Incidence

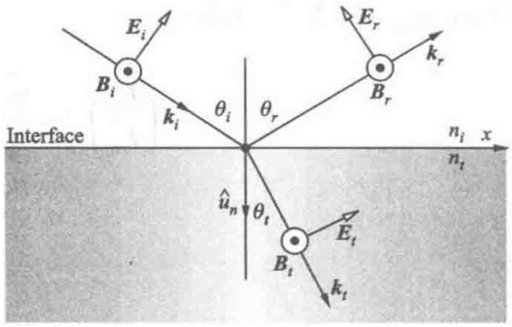


$$\begin{cases} E_i + E_r = E_t \\ B_i \cos\theta_i = B_r \cos\theta_r + B_t \cos\theta_t \end{cases} \Rightarrow \begin{cases} E = vB \\ v = \dfrac{c}{n} \end{cases}$$

We define the amplitude reflection coefficient r, the amplitude transmission coefficient t

$$\begin{cases} r = \dfrac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \\ t = \dfrac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} \end{cases} + \quad n_i \sin\theta_i = n_t \sin\theta_t \quad \Rightarrow \begin{cases} r_\perp = \dfrac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \\ t_\perp = \dfrac{2 \sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)} \end{cases}$$

2.4.2 Electric Field Parallel to Plane of Incidence



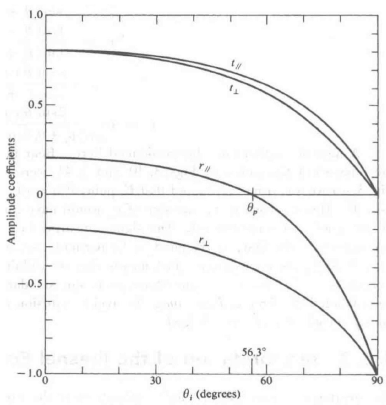
$$\begin{cases} B_i + B_r = B_t \\ E_i \cos\theta_i = E_r \cos\theta_r + E_t \cos\theta_t \end{cases} \quad \Rightarrow \quad \begin{cases} E = vB \\ v = \dfrac{c}{n} \end{cases}$$

We define the amplitude reflection coefficient r, the amplitude transmission coefficient t

$$\begin{cases} r = \dfrac{n_t\cos\theta_i - n_i\cos\theta_t}{n_t\cos\theta_i + n_i\cos\theta_t} \\ t = \dfrac{2n_i\cos\theta_i}{n_t\cos\theta_i + n_i\cos\theta_t} \end{cases} \quad + \quad n_i\sin\theta_i = n_t\sin\theta_t \quad \Rightarrow \quad \begin{cases} r_\parallel = \dfrac{\sin(2\theta_i) - \sin(2\theta_t)}{\sin(2\theta_i) + \sin(2\theta_t)} = \dfrac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \\ t_\parallel = \dfrac{2\sin\theta_t\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)} \end{cases}$$

2.5 Polarization Angle

$$\theta_p = \arctan\left(\frac{\theta_p}{\theta_i}\right)$$



2.6 Critical Angle

$$\theta_c = \arcsin\left(\frac{n_t}{n_i}\right)$$

2.7 Phase Shift

When $\theta_i = 0$

$$\begin{aligned} r_\perp = -r_\parallel &= \frac{n_i - n_t}{n_i + n_t} \\ t_\parallel = \quad t_\perp &= \frac{2n_i}{n_i + n_t} \end{aligned}$$

While $n_i > n_t$ (Inner reflection)

$$\begin{aligned} r_\parallel &< 0 \\ r_\perp &> 0 \end{aligned}$$

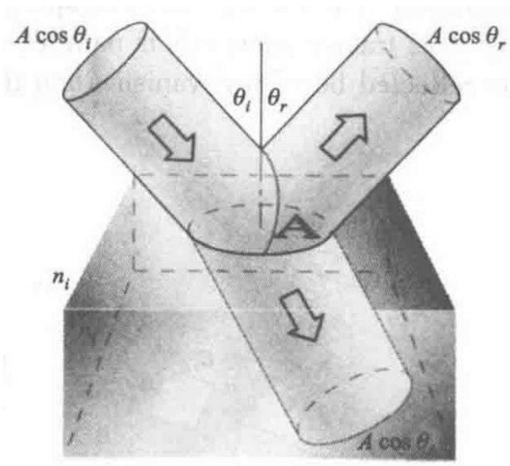
No phase shift.

While $n_i < n_t$ (Outer reflection)

$$\begin{aligned} r_\parallel &> 0 \\ r_\perp &< 0 \end{aligned}$$

Phase shifted by π.

2.8 Reflectance and Transmittance



$$\begin{cases} R = \dfrac{I_r A\cos\theta_r}{I_i A\cos\theta_i} = \dfrac{I_r}{I_i} \\ T = \dfrac{I_t A\cos\theta_t}{I_i A\cos\theta_i} = \dfrac{I_t\cos\theta_t}{I_i\cos\theta_i} \end{cases}$$

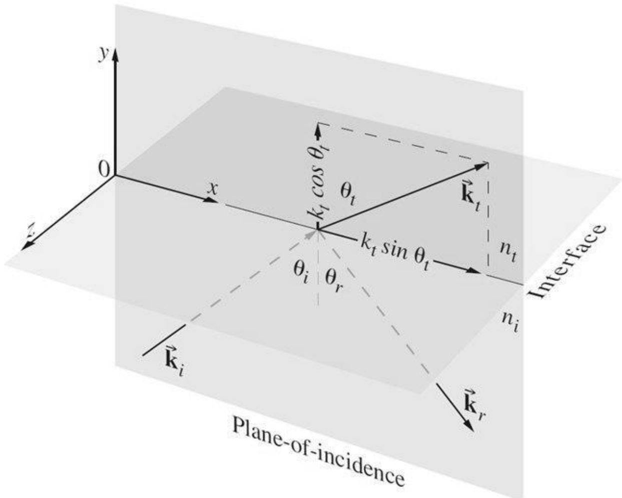
$$\vec{S} = c^2\varepsilon_0 \vec{E}\times\vec{B} \quad \Rightarrow \quad I = \frac{1}{2}\varepsilon v E_0^2 = \frac{1}{2}\varepsilon_0\varepsilon_r v E_0^2 = \frac{1}{2}\varepsilon_0 n^2 v E_0^2 = \frac{1}{2}\varepsilon_0 n c E_0^2$$

$$\begin{cases} R = \dfrac{I_r}{I_i} = \left(\dfrac{E_{0r}}{E_{0i}}\right)^2 = r^2 \\ T = \dfrac{I_t\cos\theta_t}{I_i\cos\theta_i} = \left(\dfrac{n_t\cos\theta_t}{n_i\cos\theta_i}\right)\left(\dfrac{E_{0t}}{E_{0i}}\right)^2 = \left(\dfrac{n_t\cos\theta_t}{n_i\cos\theta_i}\right)t^2 \end{cases} \Rightarrow \begin{cases} R_\perp = r_\perp^2 \\ R_\parallel = r_\parallel^2 \\ T_\perp = \left(\dfrac{n_t\cos\theta_t}{n_i\cos\theta_i}\right)t_\perp^2 \\ T_\parallel = \left(\dfrac{n_t\cos\theta_t}{n_i\cos\theta_i}\right)t_\parallel^2 \end{cases} \Rightarrow \begin{cases} R_\perp + T_\perp = 1 \\ R_\parallel + T_\parallel = 1 \\ R + T = 1 \end{cases}$$

When $\theta_i = 0$, any distinction between the parallel and perpendicular components of R and T vanishes. Thus

$$\begin{cases} R = R_{\parallel} = R_{\perp} = \left(\dfrac{n_t - n_i}{n_t + n_i}\right)^2 \\ T = T_{\parallel} = T_{\perp} = \dfrac{4n_t n_i}{(n_i + n_t)^2} \end{cases}$$

2.9 The Evanescent Wave



$$\vec{E}_t = \vec{E}_{0t} \exp\left[i\left(\vec{k}_t \cdot \vec{r} - \omega t\right)\right]$$

$$\vec{k}_t \cdot \vec{r} = k_{tx}x + k_{ty}y$$

$$k_{tx} = k_t \sin\theta_t = \left(\frac{n_i}{n_t}\right) k_t \sin\theta_i = n_i k_0 \sin\theta_i$$

$$k_{ty} = k_t \cos\theta_t = ik_t\sqrt{\frac{n_i^2 \sin^2\theta_i}{n_t^2} - 1} = i\beta$$

$$\vec{E}_t = \vec{E}_{0t} \exp\left(-\beta y\right) \exp\left[i\left(n_i k_0 x \sin\theta_i - \omega t\right)\right]$$

2.10 Optical Properties of Metals

The index of refraction of metal is complex

$$\tilde{n} = n_R - in_I$$

$$\nabla \times \vec{H} = \varepsilon_0\varepsilon_r \frac{\partial \vec{E}}{\partial t} + \sigma\vec{E} = -i\omega\varepsilon_0\varepsilon_r\vec{E} + \sigma\vec{E} = -i\omega\varepsilon_0\tilde{\varepsilon}_r\vec{E}$$

Whereas

$$\tilde{\varepsilon}_r = \varepsilon_r + i\frac{\sigma}{\omega\varepsilon_0}$$

$$\tilde{n}^2 = \tilde{\varepsilon}_r = \varepsilon_r + i\frac{\sigma}{\omega\varepsilon_0} = (n_R + in_I)^2$$

Since $\dfrac{\sigma}{\omega\varepsilon_0\varepsilon_r} \gg 1$

$$n_I \approx n_R = \sqrt{\frac{\sigma}{2\omega\varepsilon_0}}$$

Skin depth

$$\delta = \sqrt{\frac{1}{2\omega\mu_0\sigma}}$$

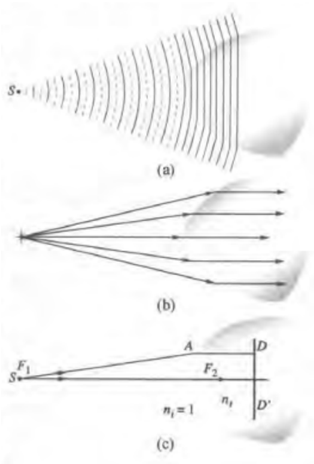
Reflectance

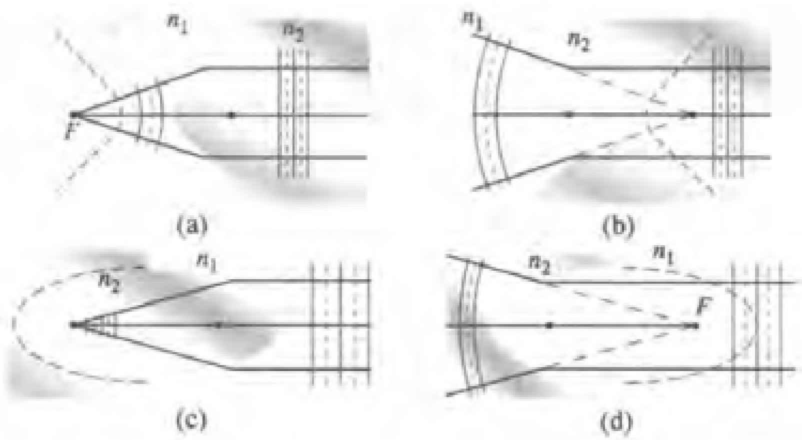
$$R = \left|\frac{n_i - n_t}{n_i + n_t}\right|^2 = \left(\frac{\tilde{n} - 1}{\tilde{n} + 1}\right)\left(\frac{\tilde{n} - 1}{\tilde{n} + 1}\right)^* = \frac{(n_R - 1)^2 + n_I^2}{(n_R + 1)^2 + n_I^2}$$

Chapter 3

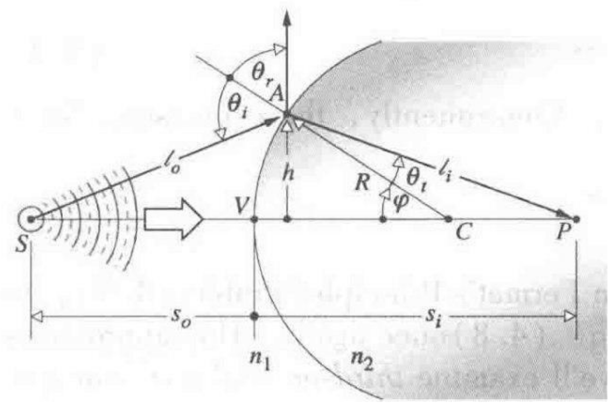
Geometrical Optics

3.1 Aspherical Surface

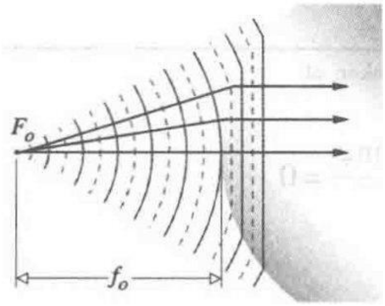


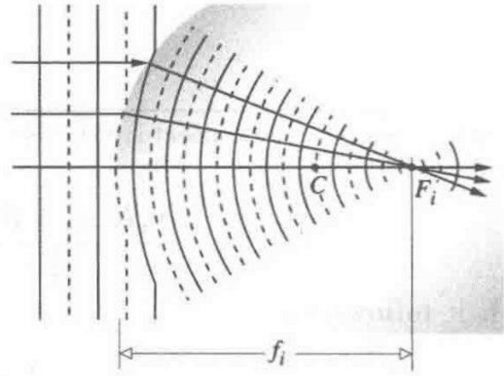


3.2 Refraction at a Spherical Interface

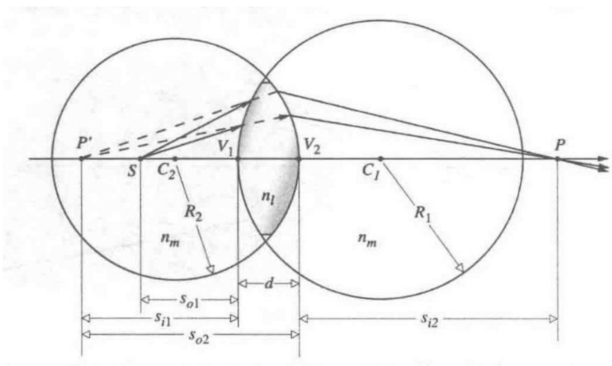


$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} = \Phi \quad \Rightarrow \quad \begin{cases} f_0 = \dfrac{n_1}{n_2 - n_1} R & (s_i = \infty) \\ f_i = \dfrac{n_2}{n_2 - n_1} R & (s_o = \infty) \end{cases}$$





3.3 Lenses



$$\frac{n_m}{s_{o1}} + \frac{n_m}{s_{i2}} = (n_l - n_m)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{n_l d}{(s_{i1} - d)\, s_{i1}}$$

For lenses in the air, where $n_m = 1$

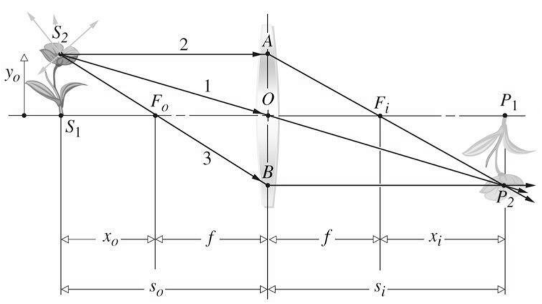
$$\frac{1}{s_{o1}} + \frac{1}{s_{i2}} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{n_l d}{(s_{i1} - d)\, s_{i1}}$$

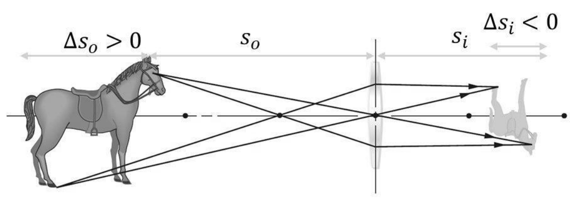
For thin lenses, $d \approx 0$

$$\frac{1}{s_{o1}} + \frac{1}{s_{i2}} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$$

Which is the Lensmaker's Formula.

3.4 Magnification





$$\begin{cases} \dfrac{y_o}{|y_i|} = \dfrac{f}{x_i} \\ \dfrac{|y_i|}{y_o} = \dfrac{f}{x_o} \end{cases} \Rightarrow \quad x_o x_i = f^2 \qquad \text{(Newton's formula)}$$

Transverse Magnification

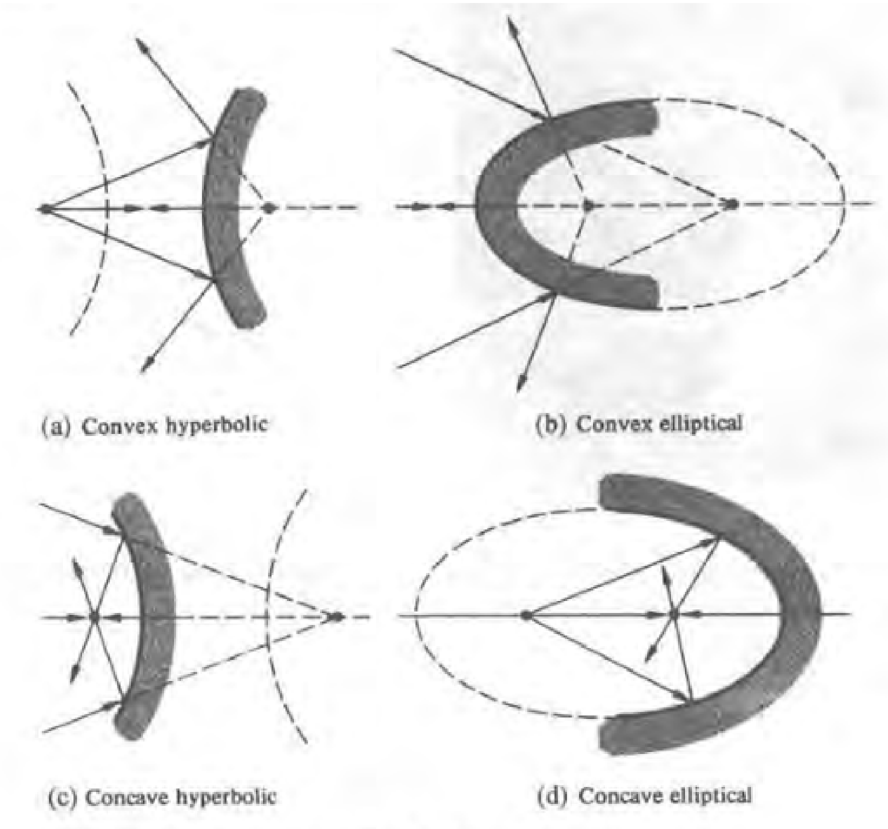
$$M_T = \frac{y_i}{|y_o|} = -\frac{s_o}{s_i} = -\frac{f}{x_o} = -\frac{x_i}{f}$$

Longitudinal Magnification

$$M_L = \frac{\mathrm{d}x_i}{\mathrm{d}x_o} = \frac{\mathrm{d}}{\mathrm{d}x_o}\left(\frac{f^2}{x_o}\right) = -\frac{f^2}{x_o^2} = -M_T^2$$

3.5 Mirrors

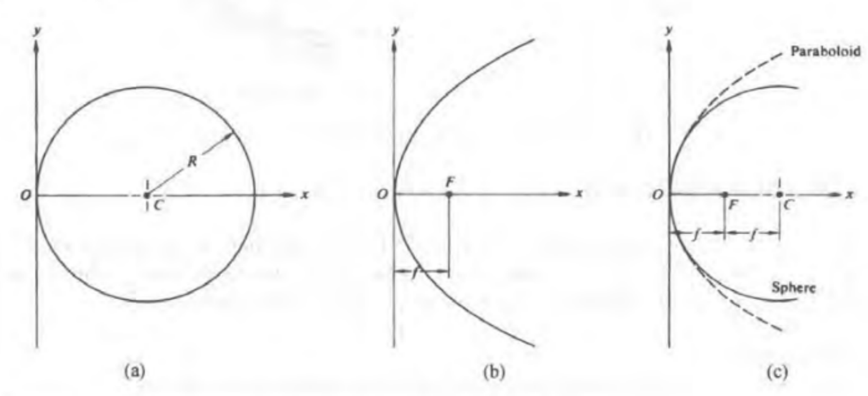
3.5.1 Aspherical Mirrors



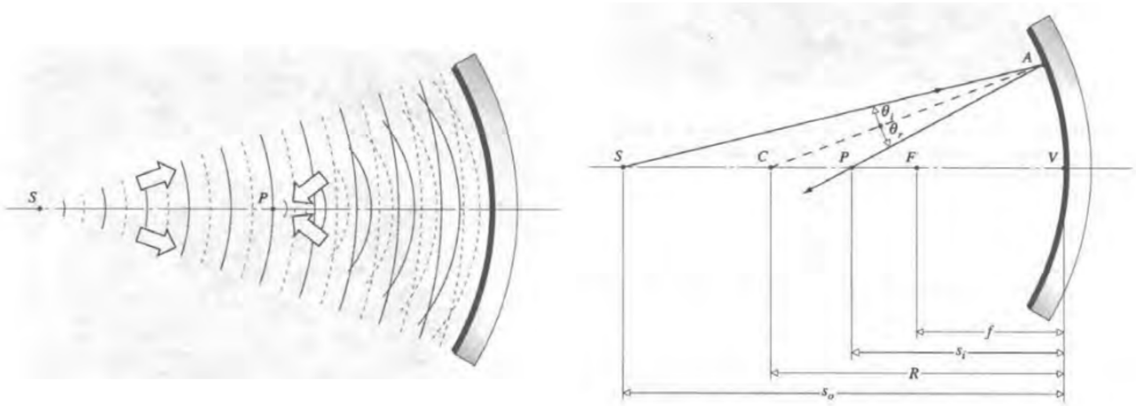
Precise aspheric surface are difficult and expensive to fabricate.

3.5.2 Spherical Mirrors

The difference between spherical and paraboloidal mirror will be appreciable only if y is large.



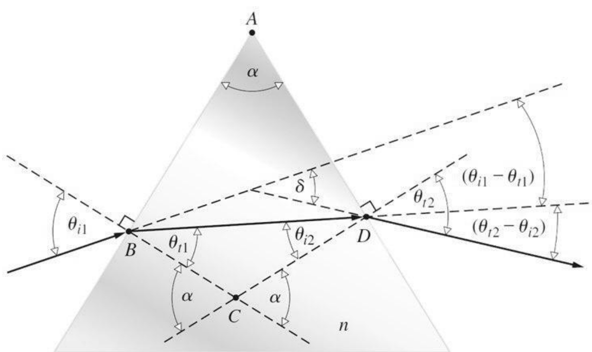
Mirror Formula



$$\frac{\overline{SC}}{\overline{SA}} = \frac{\overline{CP}}{\overline{PA}} \quad + \quad \begin{cases} \overline{SC} = \quad s_o - |R| = \quad s_o + R \\ \overline{CP} = -s_i + |R| = -(s_i + R) \\ \overline{SA} = s_o \\ \overline{PA} = s_i \end{cases} \quad \Rightarrow \quad \frac{s_o + R}{s_o} = -\frac{s_i + R}{s_i} \quad \Rightarrow \quad \frac{1}{s_o} + \frac{1}{s_i} = -\frac{2}{R}$$

$$\begin{cases} s_o = \infty \quad \Rightarrow \quad s_i = f = -R/2 \\ s_i = \infty \quad \Rightarrow \quad s_o = f = -R/2 \end{cases} \quad \Rightarrow \quad \begin{cases} s_o = 2f \quad \Rightarrow \quad s_i = 2f \\ s_o = \ \ f \quad \Rightarrow \quad s_i = \infty \\ s_i = \ \ f \quad \Rightarrow \quad s_o = \infty \end{cases}$$

3.6 Prism



$$\begin{cases} \delta = (\theta_{i1} - \theta_{t1}) + (\theta_{t2} - \theta_{i2}) \\ \alpha = \theta_{t1} + \theta_{i2} \end{cases} \quad \Rightarrow \quad \delta = \theta_{i1} + \theta_{t2} - \alpha$$

$$\begin{aligned} \theta_{t2} &= \sin^{-1}(n \sin\theta_{i2}) = \sin^{-1}[n\sin(\alpha - \theta_{t1})] = \sin^{-1}[n(\sin\alpha\cos\theta_{t1} - \cos\alpha\sin\theta_{t1})] \\ &= \sin^{-1}\left[n\left(\sin\alpha\sqrt{1-\sin^2\theta_{t1}} - \cos\alpha\sin\theta_{t1}\right)\right] = \sin^{-1}\left[\sin\alpha\sqrt{n^2-\sin^2\theta_{i1}} - \cos\alpha\sin\theta_{i1}\right] \end{aligned}$$

$$\delta = \theta_{i1} + \sin^{-1}\left[\sin\alpha\sqrt{n^2-\sin^2\theta_{i1}} - \cos\alpha\sin\theta_{i1}\right] - \alpha$$

Minimum deviation:

$$\begin{cases} \dfrac{\mathrm{d}\delta}{\mathrm{d}\theta_{i1}} = 0 \quad \Rightarrow \quad 1 + \dfrac{\mathrm{d}\theta_{t2}}{\mathrm{d}\theta_{i1}} = 0 \quad \Rightarrow \quad \mathrm{d}\theta_{i1} = -\,\mathrm{d}\theta_{t2} \\ \mathrm{d}\alpha = 0 \quad \Rightarrow \quad \mathrm{d}(\theta_{t1} + \theta_{i2}) = 0 \quad \Rightarrow \quad \mathrm{d}\theta_{t1} = -\,\mathrm{d}\theta_{i2} \\ \sin\theta_{i1} = n\sin\theta_{t1} \quad \Rightarrow \quad \cos\theta_{i1}\,\mathrm{d}\theta_{i1} = n\cos\theta_{t1}\,\mathrm{d}\theta_{t1} \\ \sin\theta_{t2} = n\sin\theta_{i2} \quad \Rightarrow \quad \cos\theta_{t2}\,\mathrm{d}\theta_{t2} = n\cos\theta_{i2}\,\mathrm{d}\theta_{i2} \end{cases} \quad \Rightarrow \quad \frac{\cos\theta_{i1}}{\cos\theta_{t2}} = \frac{\cos\theta_{t1}}{\cos\theta_{i2}}$$

$\Rightarrow$

$$\frac{1-\sin^2\theta_{i1}}{1-\sin^2\theta_{t2}} = \frac{1-n^2\sin^2\theta_{i1}}{1-n^2\sin^2\theta_{t2}} \quad \Rightarrow \quad \theta_{i1} = \theta_{t2}$$

This means that the ray for which the deviation is a minimum traverses the prism symmetrically.

$$\theta_{t1} = \theta_{i2} = \alpha/2 \qquad \theta_{i1} = \theta_{t2} = \frac{\delta_m + \alpha}{2} \qquad n = \frac{\sin[(\delta_m + \alpha)/2]}{\sin\alpha/2}$$

Chapter 4

The Superstition of Waves

4.1 The Addition of Waves

4.1.1 The Algebraic Method

Wave function

$$E(x,t) = E_0 \sin[\omega t - (kx + \varepsilon)]$$

let

$$\alpha(x,\varepsilon) = -(kx+\varepsilon)$$

Then

$$E(x,t) = E_0 \sin[\omega t + \alpha(x,\varepsilon)]$$

Waves for which ϵ are same are **coherent**.

Two waves of the same frequency

$$\begin{cases} E_1 = E_{01}\sin(\omega t + \alpha_1) \\ E_2 = E_{02}\sin(\omega t + \alpha_2) \end{cases}$$

$$\begin{aligned} E = E_1 + E_2 &= E_{01}(\sin\omega t\cos\alpha_1 + \cos\omega t\sin\alpha_1) + E_{02}(\sin\omega t\cos\alpha_2 + \cos\omega t\sin\alpha_2) \\ &= (E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2)\sin\omega t + (E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2)\cos\omega t \\ &= E_0\cos\alpha\sin\omega t + E_0\sin\alpha\cos\omega t \\ &= E_0\sin(\omega t + \alpha) \end{aligned}$$

$$\begin{cases} E_0\cos\alpha = E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2 \\ E_0\sin\alpha = E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2 \end{cases} \quad\Rightarrow\quad \begin{cases} E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_2 - \alpha_1) \\ \tan\alpha = \dfrac{E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2}{E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2} \end{cases}$$

The phase difference

$$\alpha_2 - \alpha_1 = \delta = (kx_1 + \varepsilon_1) - (kx_2 + \varepsilon_2) = \frac{2\pi}{\lambda}(x_1 - x_2) + (\varepsilon_1 - \varepsilon_2)$$

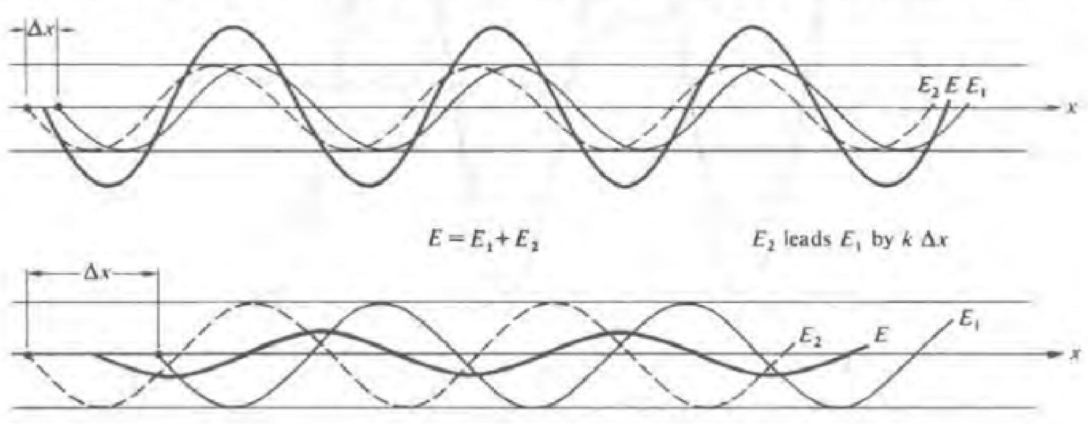
When $E_{01} = E_{02}$ and $\alpha_2 - \alpha_1 = k\Delta x$, two beams of light are coherent and have the same amplification.

$$\begin{cases} E_1 = E_{01} \sin\left[\omega t - k\left(x + \Delta x\right)\right] \\ E_2 = E_{02} \sin\left[\omega t - kx\right] \end{cases}$$

E_2 leads E_1 by Δx

$$E = 2E_{01} \cos\left(\frac{k\Delta x}{2}\right) \sin\left[\omega t - k\left(x + \frac{\Delta x}{2}\right)\right]$$

The add resultant E lags behind E_1 but leads E_2.



$$E_0^2 = 2E_{01}^2 + 2E_{01}^2 \cos\left(k\Delta x\right) = 2E_{01}^2 \left[1 + \cos\left(k\Delta x\right)\right]$$

$$\cos 2x = 2\cos^2 x - 1 \Rightarrow \cos\left(k\Delta x\right) = 2\cos^2\left(\frac{k\Delta x}{2}\right) - 1$$

$\Rightarrow$

$$E_0^2 = 2E_{01}^2 \cos^2\left(\frac{k\Delta x}{2}\right)$$

$$k\Delta x = \frac{2\pi}{\lambda}\left(x_1 - x_2\right) = \frac{2\pi}{\lambda_0} n\left(x_1 - x_2\right) = \Lambda \frac{2\pi}{\lambda_0} \qquad \Lambda = n\left(x_1 - x_2\right)$$

$$\begin{aligned} \Lambda &= n\lambda_0 && \Rightarrow \quad \text{constructive} \\ \Lambda &= \left(n + 0.5\right)\lambda_0 && \Rightarrow \quad \text{destructive} \end{aligned}$$

Period of the amplitude of addition

$$\frac{k\Delta x}{2} = \frac{\pi}{2} \Rightarrow k\left(\alpha_2 - \alpha_1\right) = \pi \Rightarrow \Delta x = \alpha_2 - \alpha_1 = \frac{\lambda}{2}$$

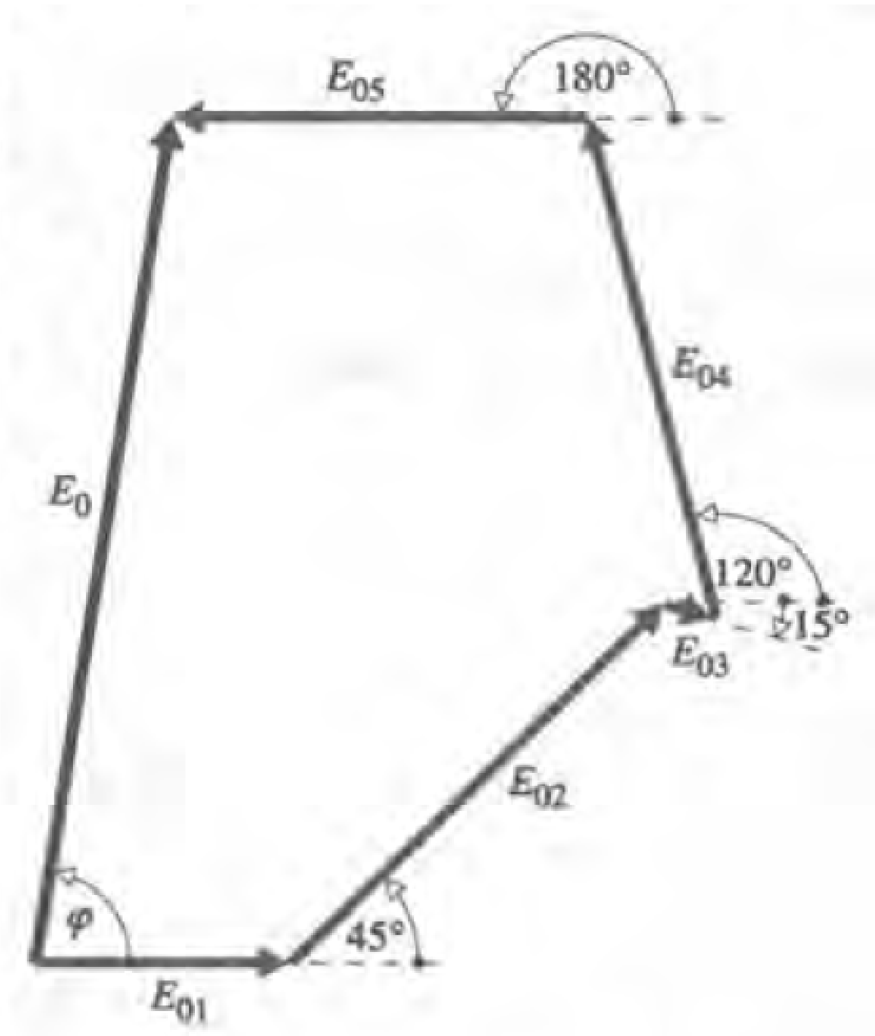
4.1.2 The Complex Method

$$E_1 = E_{01}\cos(kx \pm \omega t) \Rightarrow \tilde{E}_1 = E_{01}\exp[i(kx \pm \omega t)]$$

$$\begin{cases} E_1 = E_{01}\exp[i\alpha_1] \\ E_2 = E_{02}\exp[i\alpha_2] \\ E_0 = E_1 + E_2 \end{cases}$$

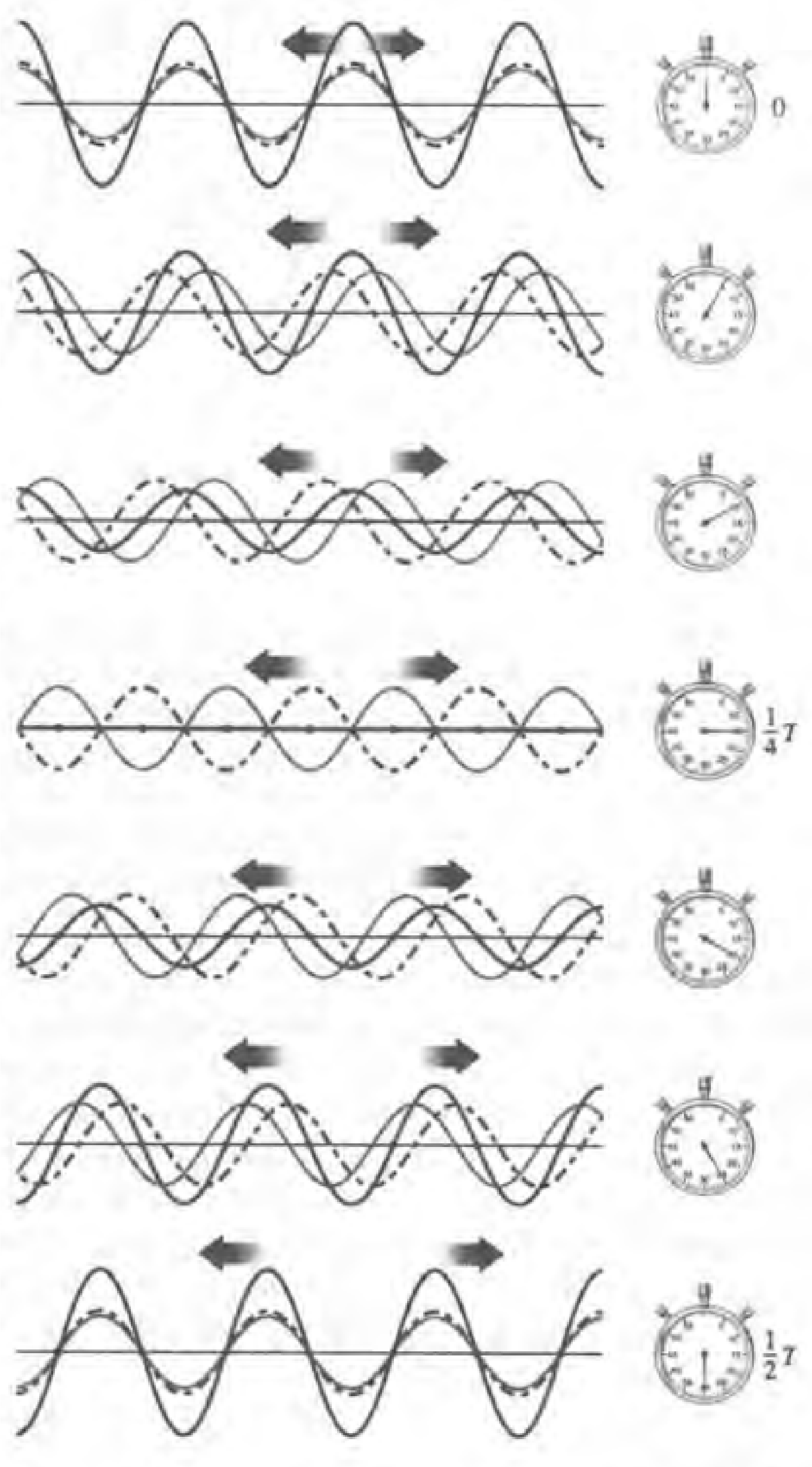
$$\begin{aligned} E_0^2 &= (E_{01}\exp[i\alpha_1] + E_{02}\exp[i\alpha_2]) \cdot (E_{01}\exp[-i\alpha_1] + E_{02}\exp[-i\alpha_2]) \\ &= E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_1 - \alpha_2) \end{aligned}$$

4.1.3 Phasor Addition Method



4.2 Standing Waves

Incoming wave E_I strikes a mirror and reflected. The reflected wave have a phase shift π.



$$E_I = E_{0t} \sin (kx + \omega t)$$
$$E_R = E_{0t} \sin (kx - \omega t) \quad + \quad \sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \quad \Rightarrow \quad E = 2E_{0t}\sin kx \cos\omega t$$
$$E = E_I + E_R$$

4.3 Addition of Waves of Different Frequency

$$\begin{cases} E_1 = E_{01}\cos\left(k_1 x - \omega_1 t\right) \\ E_2 = E_{02}\cos\left(k_2 x - \omega_2 t\right) \\ E = E_1 + E_2 \end{cases} \quad + \quad \cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$\Rightarrow$

$$\begin{aligned} E &= E_{01}\left[\cos\left(k_1 x - \omega t\right) + \cos\left(k_2 x - \omega_2 t\right)\right] \\ &= 2E_{01}\cos\frac{1}{2}\left[\left(k_1 + k_2\right)x - \left(\omega_1 + \omega_2\right)t\right] \times \cos\frac{1}{2}\left[\left(k_1 - k_2\right)x - \left(\omega_1 - \omega_2\right)t\right] \end{aligned}$$

Define

$$\begin{aligned} \bar{\omega} &= \frac{1}{2}\left(\omega_1 + \omega_2\right) & \omega_m &= \frac{1}{2}\left(\omega_1 - \omega_2\right) \\ \bar{k} &= \frac{1}{2}\left(k_1 + k_2\right) & k_m &= \frac{1}{2}\left(k_1 - k_2\right) \end{aligned}$$

Then

$$E = 2E_{01}\cos\left(k_m x - \omega_m t\right)\cos\left(\bar{k}x - \bar{\omega}t\right)$$

Define

$$E_0\left(x,t\right) = 2E_{01}\cos\left(k_m x - \omega_m t\right)$$

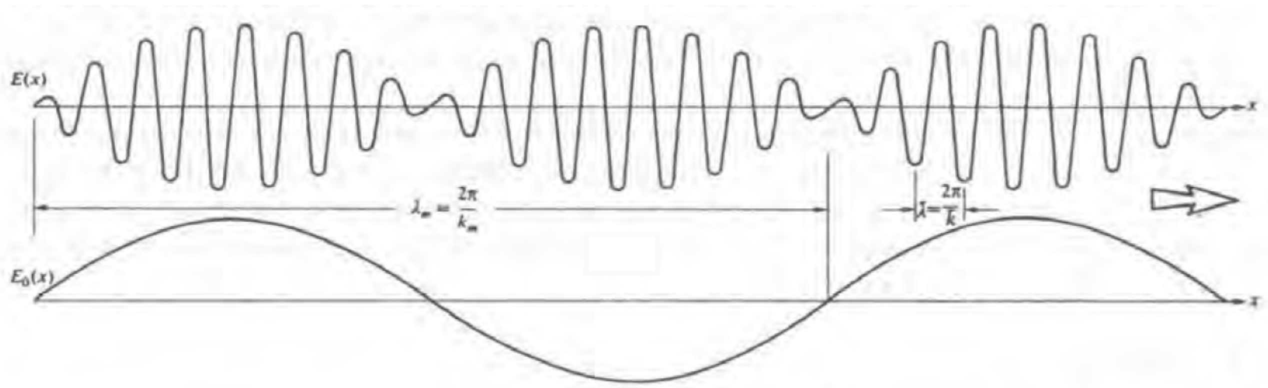
Then

$$E = E_0\left(x,t\right)\cos\left(\bar{k}x - \bar{\omega}t\right)$$

Noted that

$$\begin{aligned} \bar{\omega} &= \frac{1}{2}\left(\omega_1 + \omega_2\right) \\ \bar{k} &= \frac{1}{2}\left(k_1 + k_2\right) \end{aligned} \quad \gg \quad \begin{aligned} \omega_m &= \frac{1}{2}\left(\omega_1 - \omega_2\right) \\ k_m &= \frac{1}{2}\left(k_1 - k_2\right) \end{aligned}$$

$E_0\left(x,t\right) = 2E_{01}\cos\left(k_m x - \omega_m t\right)$ varies far less frequently than $\cos\left(\bar{k}x - \bar{\omega}t\right)$



The picture above shows what a standing wave looks like. So we define

Beat Frequency (Time): $2\omega_m$

Beat Frequency (Space): $2k_m$

Group Frequency: $v_g = \omega_m/k_m$

Phase Velocity: $v_p = \bar{\omega}/\bar{k}$

4.4 Light in Dispersible Media

Average Phase Velocity: $\bar{v}_p = \dfrac{c}{\bar{n}}$

Group Velocity: $v_g = \dfrac{c}{\bar{n}}\left(1 + \dfrac{\bar{\lambda}}{\bar{n}}\dfrac{\Delta n}{\Delta \lambda}\right)$

Normal Dispersion Media: $\bar{v}_p > v_g$

Anomalous Dispersion Media: $\bar{v}_p < v_g$

Chapter 5

Polarization

5.1 Circular Polarization

Phase shift: $\varepsilon = -\pi/2 + 2m\pi$

$$\begin{cases} \vec{E}_x(z,t) = & \vec{\imath} E_0 \cos(kx - \omega t) \\ \vec{E}_y(z,t) = & \vec{\jmath} E_0 \sin(kx - \omega t) \end{cases} \Rightarrow \vec{E} = E_0 [\vec{\imath} \cos(kx - \omega t) + \vec{\jmath} \sin(kx - \omega t)] \qquad \text{Right-circularly polarized}$$

Phase shift: $\varepsilon = +\pi/2 + 2m\pi$

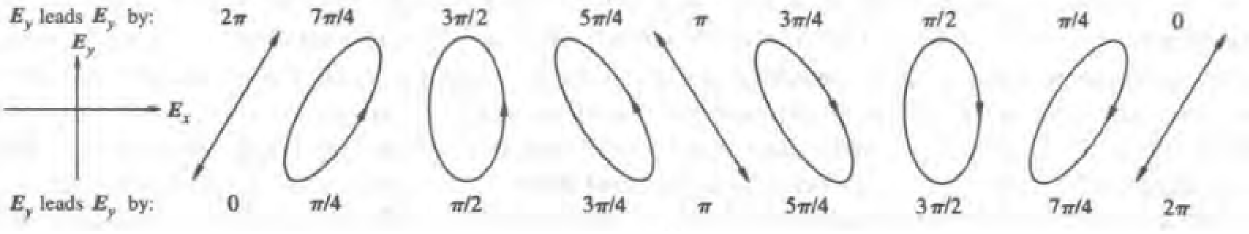
$$\begin{cases} \vec{E}_x(z,t) = & \vec{\imath} E_0 \cos(kz - \omega t) \\ \vec{E}_y(z,t) = & -\vec{\jmath} E_0 \sin(kz - \omega t) \end{cases} \Rightarrow \vec{E} = E_0 [\vec{\imath} \cos(kx - \omega t) - \vec{\jmath} \sin(kx - \omega t)] \qquad \text{Left-circularly polarized}$$

When a Right-circularly polarized light reflected by a mirror, $\vec{E}_x$ and $\vec{E}_y$ all gain a phase shift by π, the reflected light will be also Right-circularly polarized, but only in the same coordinates as before. In the new coordinates, light will be Left-circularly polarized.

5.2 Elliptical Polarization

When the phase shift of the two perpendicular light is not $m\pi$, elliptical-polarized light will form.

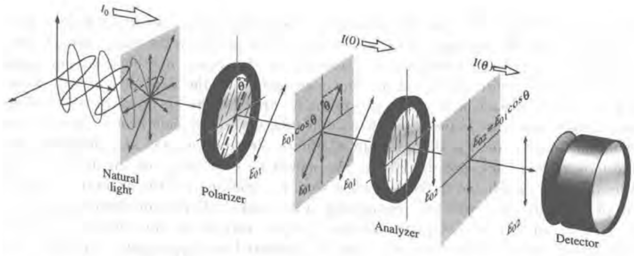
$$\begin{cases} \vec{E}_x = E_{0x} \cos(kx - \omega t) \\ \vec{E}_y = E_{0y} \cos(kz - \omega t + \epsilon) \end{cases} \qquad \text{Elliptical Polarization}$$



5.3 Angular Momentum

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \omega M = \omega \frac{\mathrm{d}L}{\mathrm{d}t} \Rightarrow L = \frac{E}{\omega} = \frac{h\nu}{\omega} = \pm\hbar = \begin{cases} -\hbar & \text{Right-circularly polarized} \\ +\hbar & \text{Left-circularly polarized} \end{cases}$$

5.4 Malus's Law

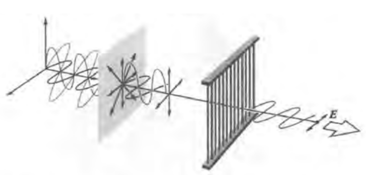
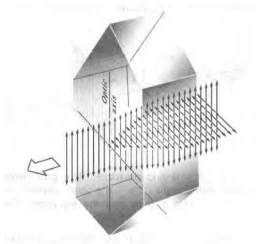
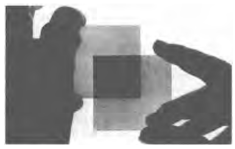


$$E_{02} = E_{01}\cos\theta \qquad\qquad I(\theta) = \frac{c\varepsilon_0}{2}E_{01}^2\cos^2\theta = I(0)\cos^2\theta$$

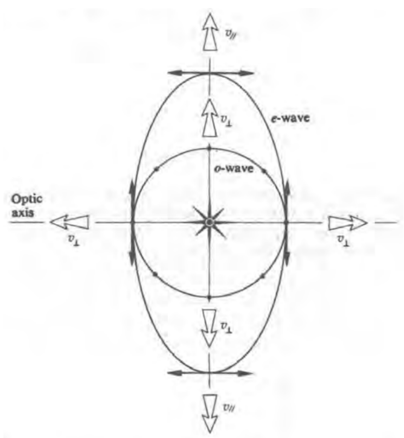
5.5 Dichroism

5.5.1 The Wire-Grid Polarizer and Dichroic Crystals, Polaroid

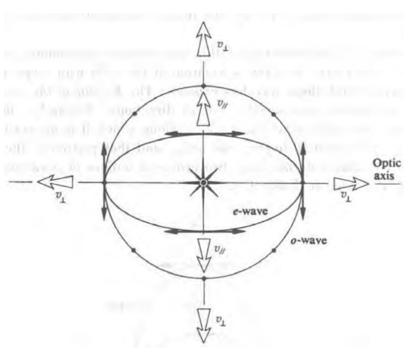
- Extraordinary wave: parallel to optic axis
- Ordinary wave: perpendicular to optic axis

5.6 Birefringent Crystals



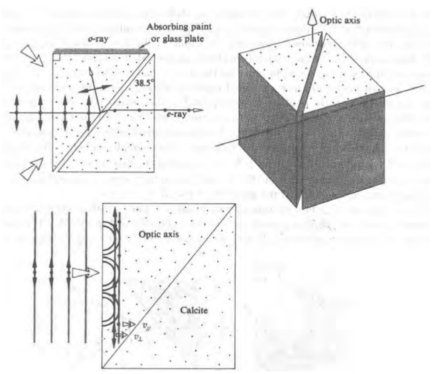
(a) $v_\perp < v_\parallel \Rightarrow n_o > n_e \Rightarrow$ negative uniaxial



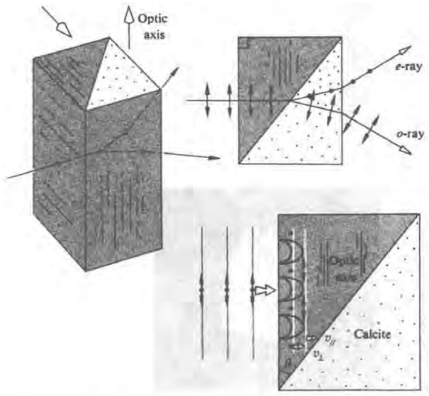
(b) $v_\perp > v_\parallel \Rightarrow n_o > n_e \Rightarrow$ positive uniaxial

Figure 5.2: negative and positive uniaxial

5.7 Polarizers



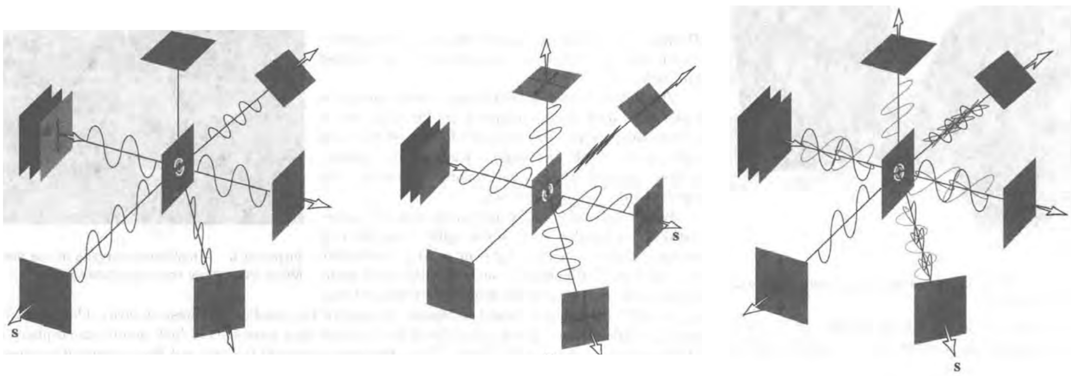
(a) The Glan-Foucault Prism



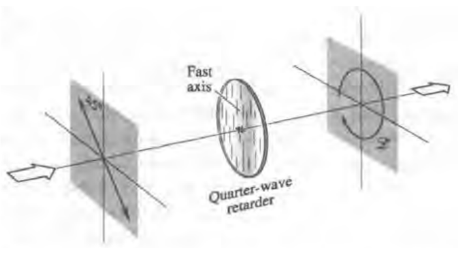
(b) The Wollaston Prism

Figure 5.3: Tow Birefringent Polarizers

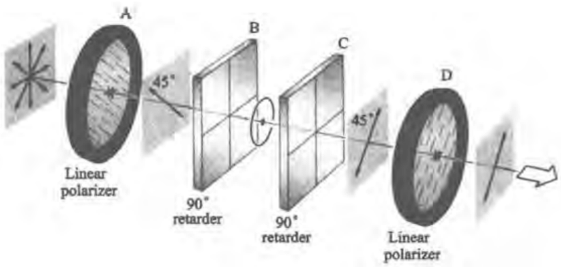
5.8 Scattering and Polarization



5.9 Retarders



(a) Quarter-wave Retarder



(b) Two Linear Polarizers and Two Quarter-wave Retarders

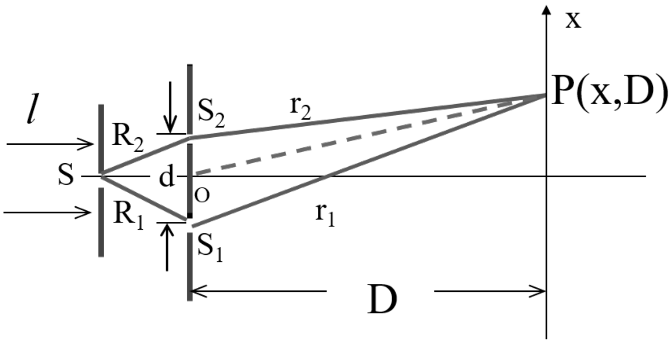
Figure 5.5: Quarter-wave Retarder and its Application

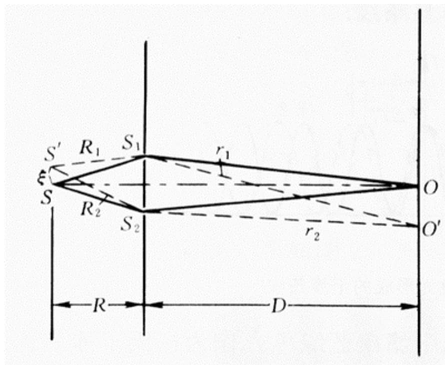
$$d\left(n_o - n_e\right) = \frac{4m+1}{4}\lambda_0$$

Chapter 6

Interference

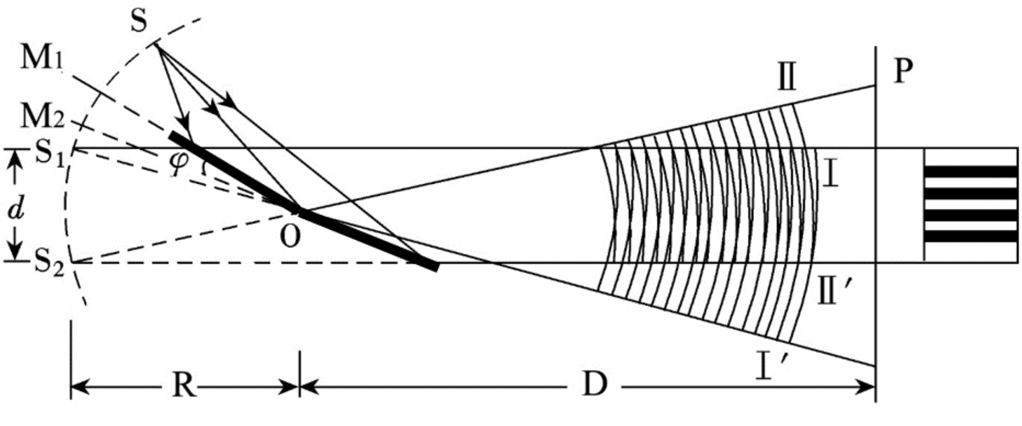
6.1 Young's Experiment





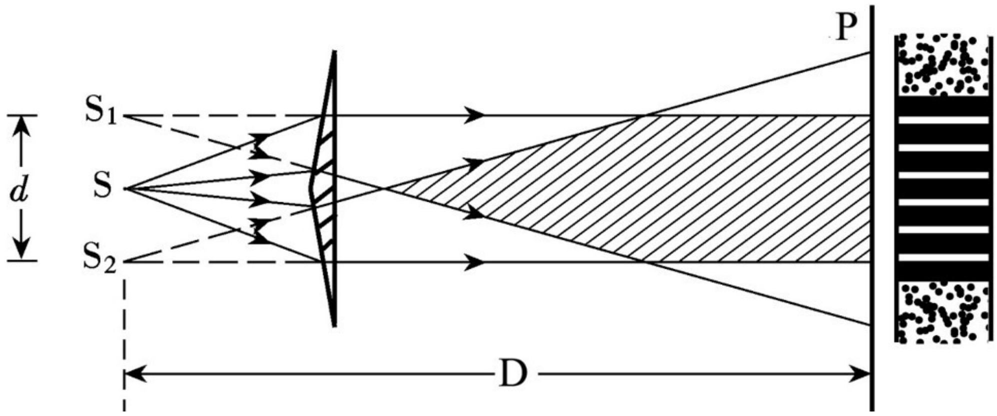
$$\Delta x = \frac{D}{d}\lambda \qquad b \leq \lambda R \frac{1}{d} \qquad I = I_0 \cos^2\left(\frac{d\pi}{D\lambda}x\right) \qquad x_0 = -\frac{D}{R}\xi$$

6.2 Fresnel's Double Mirror



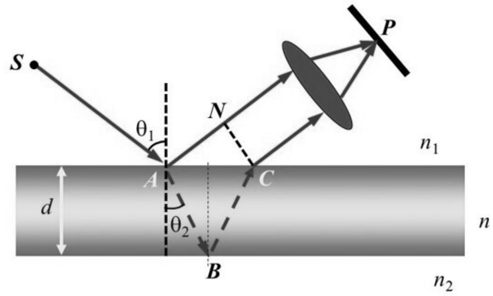
$$x_{white} = k\lambda\frac{D}{d} \qquad x_{black} = \frac{2k+1}{2}\lambda\frac{D}{d}$$

6.3 Fresnel's Double Prism



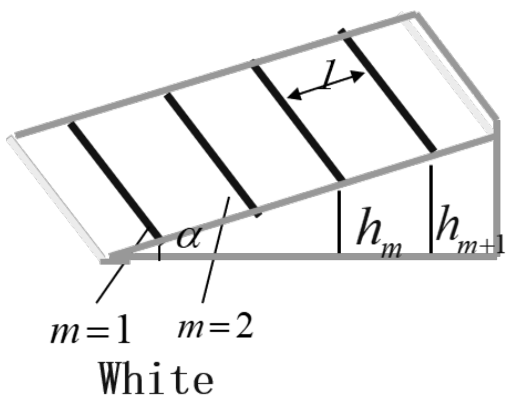
$$x_{white} = k\lambda \frac{D}{d} \qquad x_{black} = \frac{2k+1}{2}\lambda \frac{D}{d}$$

6.4 Equal Inclination Interference



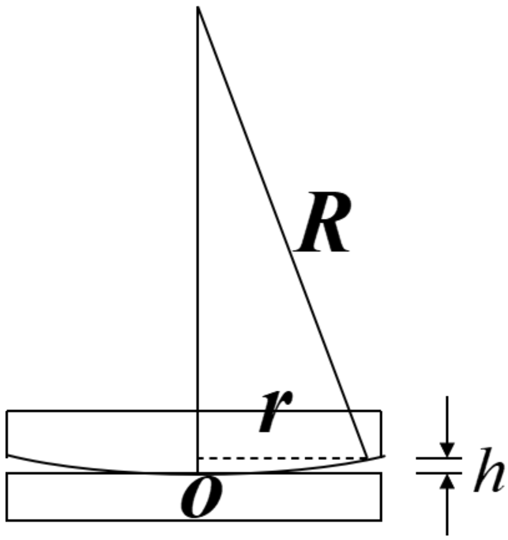
$$\Lambda = \begin{cases} 2nk_0 d\cos\theta_2 \pm \pi & n_1 > n_2 < n_3 \text{ OR } n_1 < n_2 > n_3 \\ 2nk_0 d\cos\theta_2 & n_1 < n_2 < n_3 \text{ OR } n_1 > n_2 > n_3 \end{cases}$$

6.5 Equal Thickness Interference



$$e = \Delta h = \frac{\lambda}{2n} \qquad l = \frac{e}{\sin\alpha} = \frac{\lambda}{2n\alpha} \approx \frac{\lambda}{2n\alpha}$$

6.6 Newton's Rings



$$\Delta = 2nh + \frac{\lambda}{2} = \begin{cases} k\lambda & \text{White} \\ \left(k + \frac{1}{2}\right)\lambda & \text{Black} \end{cases}$$

$$h = R - \sqrt{R^2 - r^2} = R\left[1 - \sqrt{1 - \left(\frac{r}{R}\right)^2}\right] \approx \frac{r^2}{2R}$$

$$r^2 = \begin{cases} \left(k - \frac{1}{2}\right)\frac{R\lambda}{n} & \text{White} \\ \frac{kR\lambda}{n} & \text{Black} \end{cases}$$

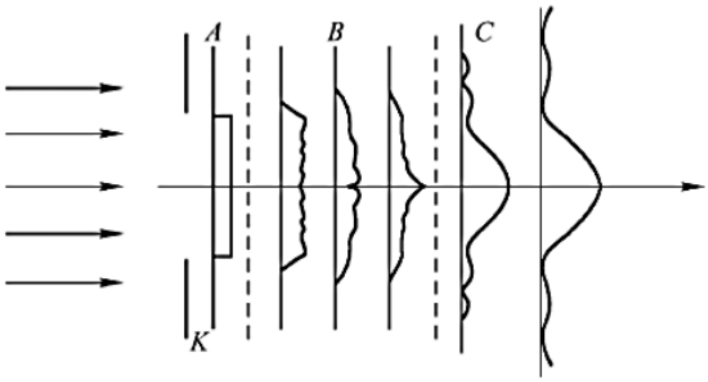
Chapter 7

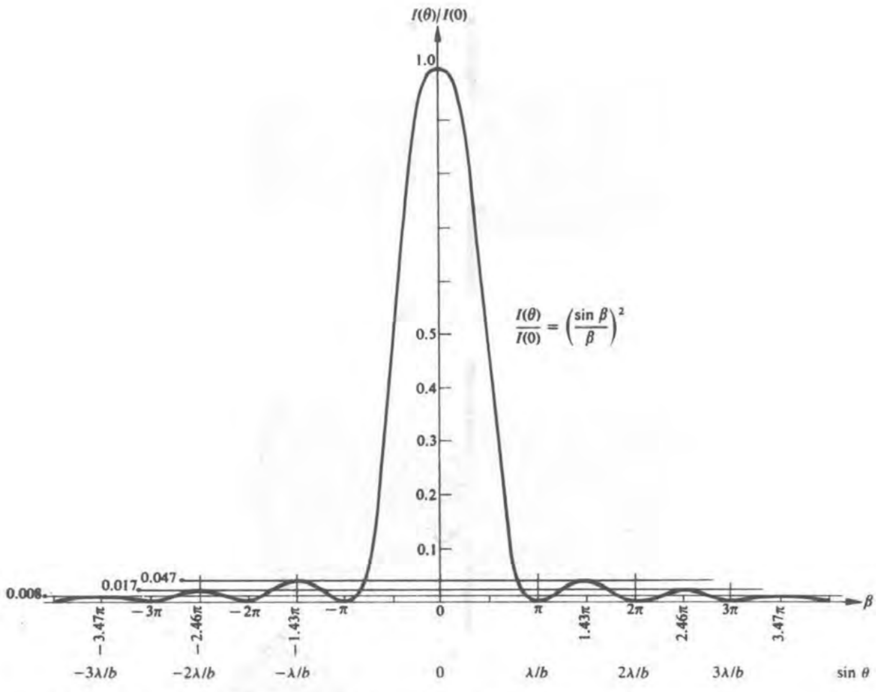
Diffraction

Chapter 8

Fraunhofer and Fresnel Diffraction

8.1 Fraunhofer Diffraction





White fringes:

$$\begin{cases} b\sin\theta = 0 & \text{Central Fringe} \\ \sin\theta = \pm(2m+1)\cdot\dfrac{\lambda}{2b} & m = 1, 2, 3, \ldots \end{cases} \qquad \begin{cases} \Delta\theta_0 = 2\cdot\dfrac{\lambda}{b} \\ \Delta\theta = \dfrac{\lambda}{b} \end{cases}$$

Dark fringes:

$$\sin\theta = \pm m\cdot\frac{\lambda}{b} \qquad m = 1, 2, 3, \ldots$$