Optics

Xiping Hu

https://hxp.plus/

July 10, 2020

Contents

1 History of Optics **5**

2 Electromagnetic Theory and photons **7**
2.1 Longitudinal and Transverse 7
2.2 Wave Equation 7
2.3 Maxwell's Equation 7
2.4 Energy 9
2.5 Radiation Pressure 9
2.6 Light in Bulk Matter 9
2.6.1 Speed of light and Dielectric Constant 9
2.6.2 Dispersion 9

3 The Propagation of Light **11**
3.1 The Fresnel Equations 11
3.1.1 Electric Field Perpendicular to Plane of Incidence 11
3.1.2 Electric Field Parallel to Plane of Incidence 12
3.2 Polarization Angle 12
3.3 Critical Angle 13
3.4 Phase Shift 13
3.5 Reflectance and Transmittance 13
3.6 The Evanescent Wave 14
3.7 Optical Properties of Metals 14

4 Geometrical Optics **17**
4.1 Refraction at a Spherical Interface 17
4.2 Lenses 18
4.3 Magnification 18
4.4 Prism 19

5 The Superstition of Waves **21**
5.1 The Addition of Waves 21
5.1.1 The Algebraic Method 21
5.1.2 The Complex Method 22
5.1.3 Phasor Addition Method 22
5.2 Standing Waves 22
5.3 Addition of Waves of Different Frequency 23
5.4 Light in Dispersible Media 24

6 Polarization **25**
6.1 Circular Polarization 25
6.2 Elliptical Polarization 25
6.3 Angular Momentum 25
6.4 Malus's Law 25
6.5 Dichroism 26
6.5.1 The Wire-Grid Polarizer and Dichroic Crystals, Polaroid 26

6.6 Birefringent Crystals . . . 26
6.7 Polarizers . . . 26
6.8 Scattering and Polarization . . . 27
6.9 Retarders . . . 27

7 Interference **29**
7.1 Young's Experiment . . . 29
7.2 Fresnel's Double Mirror . . . 29
7.3 Fresnel's Double Prism . . . 30
7.4 Equal Inclination Interference . . . 30
7.5 Equal Thickness Interference . . . 30
7.6 Newton's Rings . . . 31

8 Diffraction **33**

9 Fraunhofer and Fresnel Diffraction **35**
9.1 Fraunhofer Diffraction . . . 35

Chapter 1

History of Optics

Chapter 2

Electromagnetic Theory and photons

2.1 Longitudinal and Transverse

- Longitudinal: medium is in the direction of motion of wave.
- Transverse: medium is perpendicular to the motion of wave.

2.2 Wave Equation

$$\psi\left(x,t\right)=f\left(x+vt\right)$$

$$\begin{cases}\dfrac{\partial}{\partial x}=\dfrac{\partial}{\partial\left(x+vt\right)}\cdot\dfrac{\partial\left(x+vt\right)}{\partial x}=\dfrac{\partial}{\partial\left(x+vt\right)}\\ \dfrac{\partial}{\partial t}=\dfrac{\partial}{\partial\left(x+vt\right)}\cdot\dfrac{\partial\left(x+vt\right)}{t}=v\cdot\dfrac{\partial}{\partial\left(x+vt\right)}\end{cases}\Rightarrow\begin{cases}\dfrac{\partial^2}{\partial x^2}=\dfrac{\partial^2}{\partial\left(x+vt\right)^2}\\ \dfrac{\partial^2}{\partial t^2}=v^2\cdot\dfrac{\partial^2}{\partial\left(x+vt\right)^2}\end{cases}\Rightarrow\begin{cases}\dfrac{\partial^2\psi}{\partial x^2}=\dfrac{\partial^2\psi}{\partial\left(x+vt\right)^2}\\ \dfrac{\partial^2\psi}{\partial t^2}=v^2\cdot\dfrac{\partial^2\psi}{\partial\left(x+vt\right)^2}\end{cases}$$

$$\Rightarrow\frac{\partial^2\psi}{\partial x^2}=\frac{1}{v^2}\cdot\frac{\partial^2\psi}{\partial t^2}\quad\Rightarrow\quad\nabla^2\psi\left(x,y,z\right)=\frac{1}{v^2}\cdot\frac{\partial^2\psi\left(x,y,z\right)}{\partial t^2}$$

2.3 Maxwell's Equation

Faraday's Induction Law

$$\oint_c\vec{E}\cdot\mathrm{d}\vec{l}=-\frac{\mathrm{d}}{\mathrm{d}t}\iint\vec{B}\cdot\mathrm{d}\vec{S}$$

Gauss's Law

$$\oiint_A\vec{E}\cdot\mathrm{d}\vec{S}=\frac{1}{\varepsilon_0}\iiint_v\rho\,\mathrm{d}V$$

$$\oiint_A\vec{B}\cdot\mathrm{d}\vec{S}=0$$

Ampere's Circuital Law

$$\oint_C\vec{B}\cdot\mathrm{d}\vec{l}=\mu_0\iint_A\left(\vec{J}+\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)\cdot\mathrm{d}\vec{S}$$

We can now take the derivatives of the 4 equations

$$\begin{cases} \nabla \times \vec{E} = -\dfrac{\partial B}{\partial t} \\ \nabla \times \vec{B} = \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} \\ \nabla \cdot \vec{E} = \dfrac{\rho}{\varepsilon_0} \\ \nabla \cdot \vec{B} = 0 \end{cases}$$

The Following Equation can be derived from Maxwell Equation above

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$
$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

Coincidentally

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Which indicates the speed of electromagnetic wave is the speed of light.

Furthermore, it can be seen that the electric field and magnetic field are transverse. They are perpendicular to each other. We assume the electric field is parallel to the y-axis.

$$E_y(x,t) = E_0 \cos\left[\omega\left(t - x/c\right) + \varepsilon\right]$$

According to Faraday's Law

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_Z}{\partial t}$$

We can calculate B_z

$$B_z = \frac{1}{c} E_0 \cos\left[\omega\left(t - x/c\right) + \varepsilon\right]$$

So that

$$E_y = cB_z$$

When not in vacuum, similarly

$$E_y = vB_z$$
$$v = \frac{1}{\varepsilon\mu}$$

2.4 Energy

$$
\begin{aligned}
u_E &= \frac{\varepsilon_0}{2} E^2 \\
u_B &= \frac{1}{2\mu_0} B^2 \\
u_E &= u_B \\
u &= u_E + u_B = \varepsilon_0 E^2 = \frac{1}{\mu_0} B^2 \\
S &= uc \\
\vec{S} &= \frac{1}{\mu} \vec{E} \times \vec{B} \quad \text{(PoyntingVector)} \\
I &= \frac{1}{2} \varepsilon v E_0^2
\end{aligned}
$$

2.5 Radiation Pressure

$$
P(t) = \frac{S(t)}{c} = u = u_E + u_B
$$

$$
\langle P(t) \rangle_T = \frac{I}{c}
$$

$$
p_V = \frac{S}{c^2}
$$

2.6 Light in Bulk Matter

2.6.1 Speed of light and Dielectric Constant

$$
v = \frac{1}{\sqrt{\varepsilon\mu}}
$$

$$
n = \frac{c}{v} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \sqrt{\frac{\varepsilon}{\varepsilon_0}}
$$

2.6.2 Dispersion

For gas and solid

$$
m_e \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \gamma m_e \frac{\mathrm{d}x}{\mathrm{d}t} + m_e \omega_0^2 x = -eE\left(t\right)
$$

$$
E\left(t\right) = E_0 \exp\left(-i\omega t\right)
$$

Assume

$$
x = x_0 \exp\left(-i\omega t\right)
$$

We got a solution

$$
x_0 \left(\omega_0^2 - \omega^2 - i\gamma\omega\right) = -\frac{eE_0}{m_e}
$$

$$
x_0 = -\frac{eE_0}{m_e\left(\omega_0^2 - \omega^2 - i\gamma\omega\right)}
$$

$$
x\left(t\right) = -\frac{eE\left(t\right)}{m_e\left(\omega_0^2 - \omega^2 - i\gamma\omega\right)}
$$

$$
P\left(t\right) = -Nex\left(t\right) = \frac{Ne^2 E\left(t\right)}{m_e\left(\omega_0^2 - \omega^2 - i\gamma\omega\right)}
$$

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = n^2 = 1 + \frac{P}{\varepsilon_0 E} = 1 + \frac{Ne^2}{\varepsilon_0 m_e \left(\omega_0^2 - \omega^2 - i\gamma\omega\right)}$$

$$\begin{cases} \mathrm{Re}\left(\varepsilon_r\right) = 1 + \dfrac{Ne^2\left(\omega_0^2 - \omega^2\right)}{\varepsilon_0 m_e \left[\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\omega^2\right]} \\ \mathrm{Im}\left(\varepsilon_r\right) = \dfrac{Ne^2\gamma\omega}{\varepsilon_0 m_e \left[\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\omega^2\right]} \end{cases}$$

When $\gamma = 0$

$$\varepsilon_r = n^2 = 1 + \frac{Ne^2}{\varepsilon_0 m \left(\omega_0^2 - \omega^2\right)}$$

For metal

$$m_e \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \gamma m_e \frac{\mathrm{d}x}{\mathrm{d}t} = -eE\left(t\right)$$

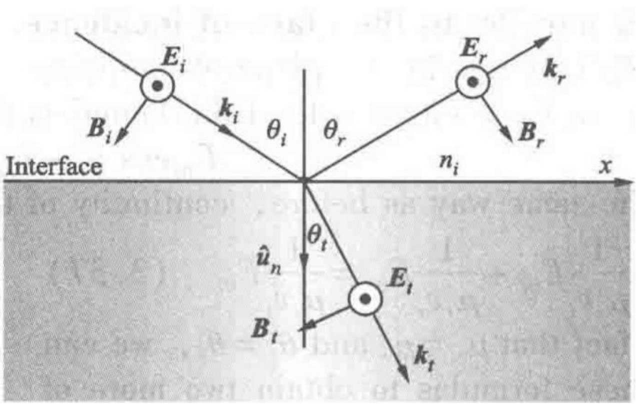
$$\varepsilon_r = 1 - \frac{Ne^2}{\varepsilon_0 m_e \left(\omega^2 + i\gamma\omega\right)} = 1 - \frac{\omega_p^2}{\omega\left(\omega + i\gamma\right)}$$

Chapter 3

The Propagation of Light

3.1 The Fresnel Equations

3.1.1 Electric Field Perpendicular to Plane of Incidence



$$\begin{cases} E_i + E_r = E_t \\ B_i \cos\theta_i = B_r \cos\theta_r + B_t \cos\theta_t \end{cases}$$

$$\begin{cases} E = vB \\ v = \dfrac{c}{n} \end{cases}$$

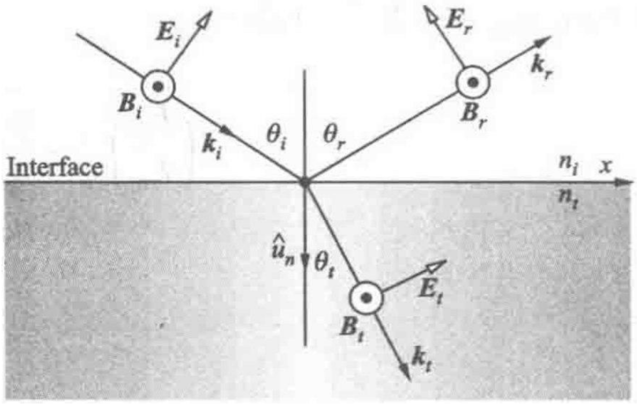
We define the amplitude reflection coefficient r, the amplitude transmission coefficient t

$$r = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$
$$t = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

Considered that $n_i \sin\theta_i = n_t \sin\theta_t$

$$r_\perp = \frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$
$$t_\perp = \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)}$$

3.1.2 Electric Field Parallel to Plane of Incidence



$$\begin{cases} B_i + B_r = B_t \\ E_i \cos\theta_i = E_r \cos\theta_r + E_t \cos\theta_t \end{cases}$$

$$\begin{cases} E = vB \\ v = \dfrac{c}{n} \end{cases}$$

We define the amplitude reflection coefficient r, the amplitude transmission coefficient t

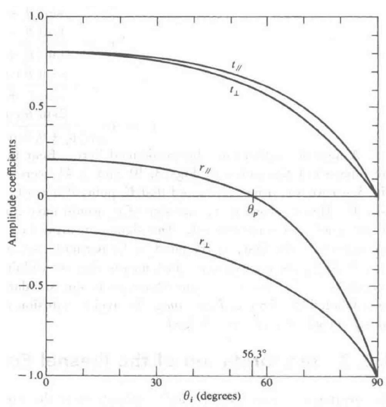
$$r = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$
$$t = \frac{2n_i \cos\theta_i}{n_t \cos\theta_i + n_i \cos\theta_t}$$

Considered that $n_i \sin\theta_i = n_t \sin\theta_t$

$$r_{\|} = \frac{\sin(2\theta_i) - \sin(2\theta_t)}{\sin(2\theta_i) + \sin(2\theta_t)} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$
$$t_{\|} = \frac{2\sin\theta_t\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

3.2 Polarization Angle

$$\theta_p = \arctan\left(\frac{\theta_p}{\theta_i}\right)$$



3.3 Critical Angle

$$\theta_c = \arcsin\left(\frac{n_t}{n_i}\right)$$

3.4 Phase Shift

When $\theta_i = 0$

$$\begin{aligned} r_\perp = -r_\parallel &= \frac{n_i - n_t}{n_i + n_t} \\ t_\parallel = \quad t_\perp &= \frac{2n_i}{n_i + n_t} \end{aligned}$$

While $n_i > n_t$ (Inner reflection)

$$\begin{aligned} r_\parallel &< 0 \\ r_\perp &> 0 \end{aligned}$$

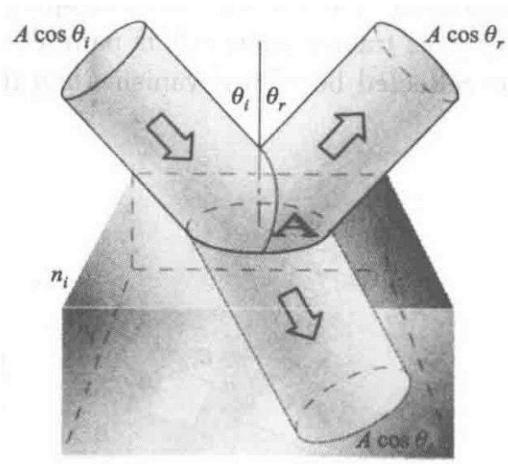
No phase shift.

While $n_i < n_t$ (Outer reflection)

$$\begin{aligned} r_\parallel &> 0 \\ r_\perp &< 0 \end{aligned}$$

Phase shifted by π.

3.5 Reflectance and Transmittance

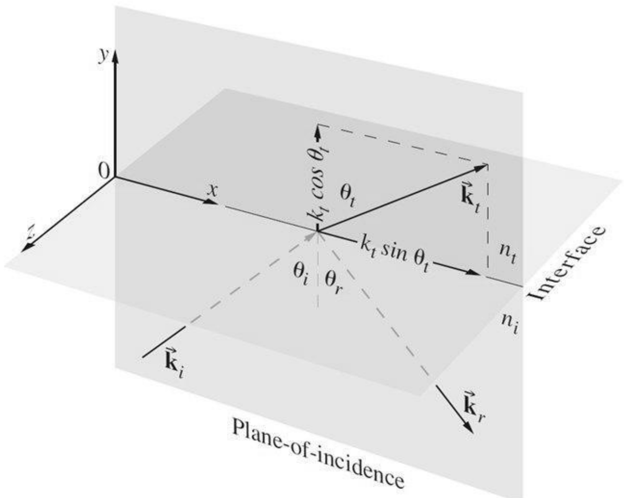


$$\begin{cases} R = \dfrac{I_t A \cos\theta_r}{I_i A \cos\theta_i} = \dfrac{I_t}{I_i} \\ T = \dfrac{I_t A \cos\theta_t}{I_i A \cos\theta_i} = \dfrac{I_t \cos\theta_t}{I_i \cos\theta_i} \end{cases}$$

$$I = \frac{1}{2}\varepsilon v E_0^2 = \frac{1}{2}\varepsilon_0\varepsilon_r v E_0^2 = \frac{1}{2}\varepsilon_0 n^2 v E_0^2 = \frac{1}{2}\varepsilon_0 n c E_0^2$$

$$\begin{cases} R = \dfrac{I_t}{I_i} = \left(\dfrac{E_{0t}}{E_{0i}}\right)^2 = r^2 \\ T = \dfrac{I_t \cos\theta_t}{I_i \cos\theta_i} = \left(\dfrac{n_t \cos\theta_t}{n_i \cos\theta_i}\right)\left(\dfrac{E_{0t}}{E_{0i}}\right)^2 = \left(\dfrac{n_t \cos\theta_t}{n_i \cos\theta_i}\right) t^2 \end{cases}$$

3.6 The Evanescent Wave



$$\vec{E}_t = \vec{E}_{0t} \exp\left[i\left(\vec{k}_t \cdot \vec{r} - \omega t\right)\right]$$

$$\vec{k}_t \cdot \vec{r} = k_{tx} x + k_{ty} y$$

$$\begin{aligned} k_{tx} &= k_t \sin\theta_t = \left(\frac{n_i}{n_t}\right) k_t \sin\theta_i = n_i k_0 \sin\theta_i \\ k_{ty} &= k_t \cos\theta_t = i k_t \sqrt{\frac{n_i^2 \sin^2\theta_i}{n_t^2} - 1} = i\beta \end{aligned}$$

$$\vec{E}_t = \vec{E}_{0t} \exp(-\beta y) \exp\left[i\left(n_i k_0 x \sin\theta_i - \omega t\right)\right]$$

3.7 Optical Properties of Metals

The index of refraction of metal is complex

$$\tilde{n} = n_R - i n_I$$

$$\nabla \times \vec{H} = \varepsilon_0 \varepsilon_r \frac{\partial \vec{E}}{\partial t} + \sigma \vec{E} = -i\omega\varepsilon_0\varepsilon_r \vec{E} + \sigma\vec{E} = -i\omega\varepsilon_0\tilde{\varepsilon}_r \vec{E}$$

Whereas

$$\tilde{\varepsilon}_r = \varepsilon_r + i\frac{\sigma}{\omega\varepsilon_0}$$

$$\tilde{n}^2 = \tilde{\varepsilon}_r = \varepsilon_r + i\frac{\sigma}{\omega\varepsilon_0} = (n_R + in_I)^2$$

Since $\dfrac{\sigma}{\omega\varepsilon_0\varepsilon_r} \gg 1$

$$n_I \approx n_R = \sqrt{\frac{\sigma}{2\omega\varepsilon_0}}$$

Skin depth

$$\delta = \sqrt{\frac{1}{2\omega\mu_0\sigma}}$$

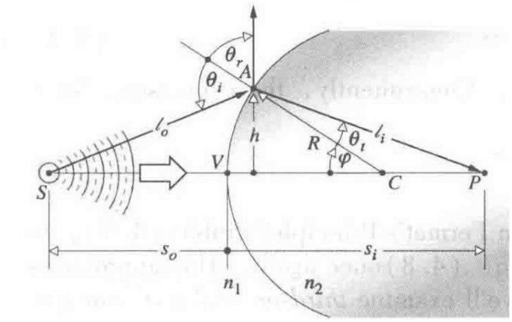
Reflectance

$$R = \left|\frac{n_i - n_t}{n_i + n_t}\right|^2 = \left(\frac{\tilde{n}-1}{\tilde{n}+1}\right)\left(\frac{\tilde{n}-1}{\tilde{n}+1}\right)^* = \frac{(n_R-1)^2 + n_I^2}{(n_R+1)^2 + n_I^2}$$

Chapter 4

Geometrical Optics

4.1 Refraction at a Spherical Interface



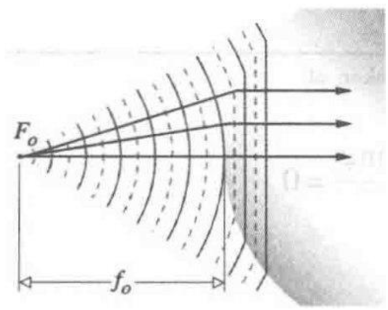
$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

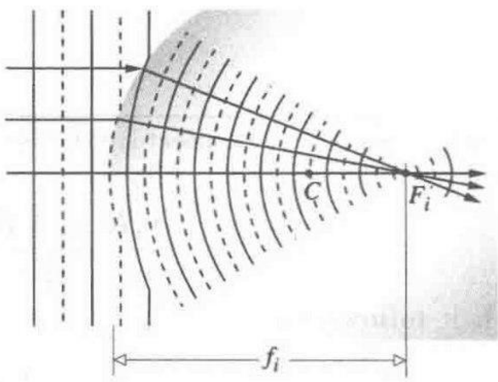
Let $s_i = \infty$, the object focus

$$f_0 = \frac{n_1}{n_2 - n_1} R$$

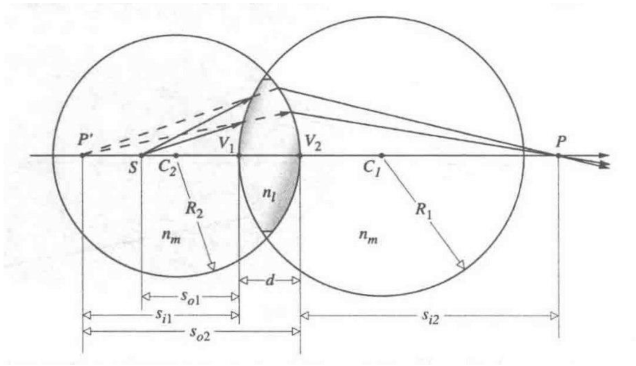
Let $s_o = \infty$, the image focus

$$f_i = \frac{n_2}{n_2 - n_1} R$$





4.2 Lenses



$$\frac{n_m}{s_{o1}} + \frac{n_m}{s_{i2}} = (n_l - n_m)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{n_l d}{(s_{i1} - d)\, s_{i1}}$$

For lenses in the air, where $n_m = 1$

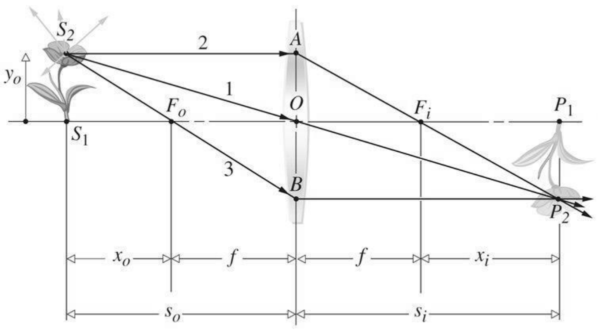
$$\frac{1}{s_{o1}} + \frac{1}{s_{i2}} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{n_l d}{(s_{i1} - d)\, s_{i1}}$$

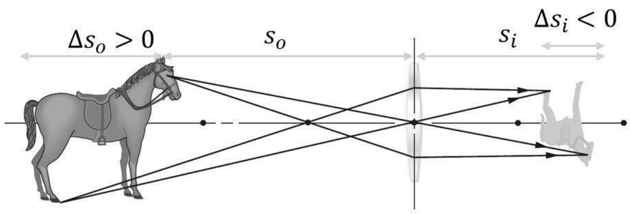
For thin lenses, $d \approx 0$

$$\frac{1}{s_{o1}} + \frac{1}{s_{i2}} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$$

Which is the Gaussian Lens Formula.

4.3 Magnification





$$\begin{cases} \dfrac{y_o}{|y_i|} = \dfrac{f}{x_i} \\ \dfrac{|y_i|}{y_o} = \dfrac{f}{x_o} \end{cases}$$

Newton's formula

$$x_o x_i = f^2$$

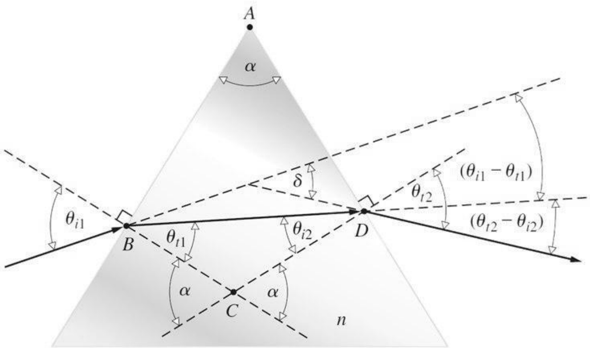
Transverse Magnification

$$M_T = \frac{y_i}{|y_o|} = -\frac{s_o}{s_i} = -\frac{f}{x_o} = -\frac{x_i}{f}$$

Longitudinal Magnification

$$M_L = \frac{\mathrm{d}x_i}{\mathrm{d}x_o} = \frac{\mathrm{d}}{\mathrm{d}x_o}\left(\frac{f^2}{x_o}\right) = -\frac{f^2}{x_o^2} = -M_T^2$$

4.4 Prism



$$\begin{cases} \delta = (\theta_{i1} - \theta_{t1}) + (\theta_{t2} - \theta_{i2}) \\ \alpha = \theta_{t1} + \theta_{i2} \end{cases}$$

$$\delta = \theta_{i1} + \theta_{t2} - \alpha$$

$$\begin{aligned} \theta_{t2} &= \arcsin\left(n \sin\theta_{i2}\right) = \arcsin\left[n \sin\left(\alpha - \theta_{t1}\right)\right] = \arcsin\left[n\left(\sin\alpha\cos\theta_{t1} - \cos\alpha\sin\theta_{t1}\right)\right] \\ &= \arcsin\left[n\left(\sin\alpha\sqrt{1-\sin^2\theta_{t1}} - \cos\alpha\sin\theta_{t1}\right)\right] \\ &= \arcsin\left[n\left(\sin\alpha\sqrt{1-n^2\sin^2\theta_{i1}} - \cos\alpha\sin\theta_{t1}\right)\right] \\ \delta &= \theta_{i1} + \arcsin\left[n\left(\sin\alpha\sqrt{1-n^2\sin^2\theta_{i1}} - \cos\alpha\sin\theta_{t1}\right)\right] - \alpha \end{aligned}$$

Chapter 5

The Superstition of Waves

5.1 The Addition of Waves

5.1.1 The Algebraic Method

$$E\left(x,t\right)=E_0\sin\left[\omega t-\left(kx+\varepsilon\right)\right]$$

let

$$\alpha\left(x,\varepsilon\right)=-\left(kx+\varepsilon\right)$$

Then

$$E\left(x,t\right)=E_0\sin\left[\omega t+\alpha\left(x,\varepsilon\right)\right]$$

Two waves of the same frequency

$$\begin{cases}E_1=E_{01}\sin\left(\omega t+\alpha_1\right)\\E_2=E_{02}\sin\left(\omega t+\alpha_2\right)\end{cases}$$

$$\begin{aligned}E=E_1+E_2&=E_{01}\left(\sin\omega t\cos\alpha_1+\cos\omega t\sin\alpha_1\right)+E_{02}\left(\sin\omega t\cos\alpha_2+\cos\omega t\sin\alpha_2\right)\\&=\left(E_{01}\cos\alpha_1+E_{02}\cos\alpha_2\right)\sin\omega t+\left(E_{01}\sin\alpha_1+E_{02}\sin\alpha_2\right)\cos\omega t\\&=E_0\cos\alpha\sin\omega t+E_0\sin\alpha\cos\omega t\\&=E_0\sin\left(\omega t+\alpha\right)\end{aligned}$$

$$\begin{cases}E_0\cos\alpha=E_{01}\cos\alpha_1+E_{02}\cos\alpha_2\\E_0\sin\alpha=E_{01}\sin\alpha_1+E_{02}\sin\alpha_2\end{cases}\Rightarrow\begin{cases}E_0^2=E_{01}^2+E_{02}^2+2E_{01}E_{02}\cos\left(\alpha_2-\alpha_1\right)\\\tan\alpha=\dfrac{E_{01}\sin\alpha_1+E_{02}\sin\alpha_2}{E_{01}\cos\alpha_1+E_{02}\cos\alpha_2}\end{cases}$$

The phase difference

$$\delta=\left(kx_1+\varepsilon_1\right)-\left(kx_2+\varepsilon_2\right)=\frac{2\pi}{\lambda}\left(x_1-x_2\right)+\left(\varepsilon_1-\varepsilon_2\right)$$

When $E_{01}=E_{02}$ and $\alpha_2-\alpha_1=\Delta x$

$$E_0^2=2E_{01}^2+2E_{01}^2\cos\left(k\Delta x\right)=2E_{01}^2\left[1+\cos\left(k\Delta x\right)\right]$$

$$\cos 2x = 2\cos^2 x - 1 \Rightarrow \cos(k\Delta x) = 2\cos^2\left(\frac{k\Delta x}{2}\right) - 1$$

$\Rightarrow$

$$E_0^2 = 2E_{01}^2 \cos^2\left(\frac{k\Delta x}{2}\right)$$

Period of the amplitude of addition

$$\frac{k\Delta x}{2} = \frac{\pi}{2} \Rightarrow k(\alpha_2 - \alpha_1) = \pi \Rightarrow \Delta x = \alpha_2 - \alpha_1 = \frac{\lambda}{2}$$

5.1.2 The Complex Method

$$E_1 = E_{01}\cos(kx \pm \omega t) \Rightarrow \tilde{E}_1 = E_{01}\exp[i(kx \pm \omega t)]$$

$$\begin{cases} E_1 = E_{01}\exp[i\alpha_1] \\ E_2 = E_{02}\exp[i\alpha_2] \\ E_0 = E_1 + E_2 \end{cases}$$

$$\begin{aligned} E_0^2 &= (E_{01}\exp[i\alpha_1] + E_{02}\exp[i\alpha_2]) \cdot (E_{01}\exp[-i\alpha_1] + E_{02}\exp[-i\alpha_2]) \\ &= E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_1 - \alpha_2) \end{aligned}$$

5.1.3 Phasor Addition Method

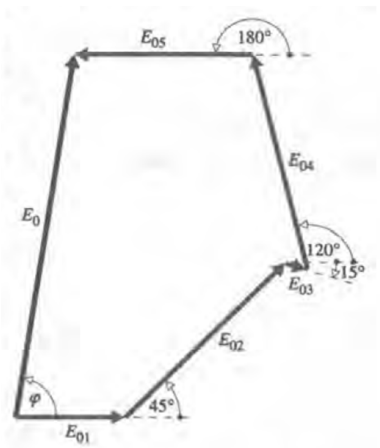


Figure 5.1: Phasor Addition Method

5.2 Standing Waves

$$\begin{aligned} &E_L = E_{0t}\sin(kx - \omega t) \\ &E_R = E_{0t}\sin(kx - \omega t) \quad E = E_L + E_R \end{aligned}$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$\Rightarrow$

$$E = 2E_{0t}\sin kx \cos\omega t$$

5.3 Addition of Waves of Different Frequency

$$\begin{cases} E_1 = E_{01}\cos(k_1 x - \omega_1 t) \\ E_2 = E_{02}\cos(k_2 x - \omega_2 t) \\ E = E_1 + E_2 \end{cases}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$\Rightarrow$

$$\begin{aligned} E &= E_{01}\left[\cos(k_1 x - \omega t) + \cos(k_2 x - \omega_2 t)\right] \\ &= 2E_{01}\cos\frac{1}{2}\left[(k_1+k_2)x - (\omega_1+\omega_2)t\right] \times \cos\frac{1}{2}\left[(k_1-k_2)x - (\omega_1-\omega_2)t\right] \end{aligned}$$

Define

$$\begin{aligned} \bar{\omega} &= \frac{1}{2}(\omega_1+\omega_2) & \omega_m &= \frac{1}{2}(\omega_1-\omega_2) \\ \bar{k} &= \frac{1}{2}(k_1+k_2) & k_m &= \frac{1}{2}(k_1-k_2) \end{aligned}$$

Then

$$E = 2E_{01}\cos(k_m x - \omega_m t)\cos\left(\bar{k}x - \bar{\omega}t\right) = E_0(x,t)\cos\left(\bar{k}x - \bar{\omega}t\right)$$

Noted that

$$\begin{aligned} \bar{\omega} &= \frac{1}{2}(\omega_1+\omega_2) & & \omega_m = \frac{1}{2}(\omega_1-\omega_2) \\ & & \gg & \\ \bar{k} &= \frac{1}{2}(k_1+k_2) & & k_m = \frac{1}{2}(k_1-k_2) \end{aligned}$$

$E_0 = 2E_{01}\cos(k_m x - \omega_m t)$ varies far less frequently than $\cos\left(\bar{k}x - \bar{\omega}t\right)$

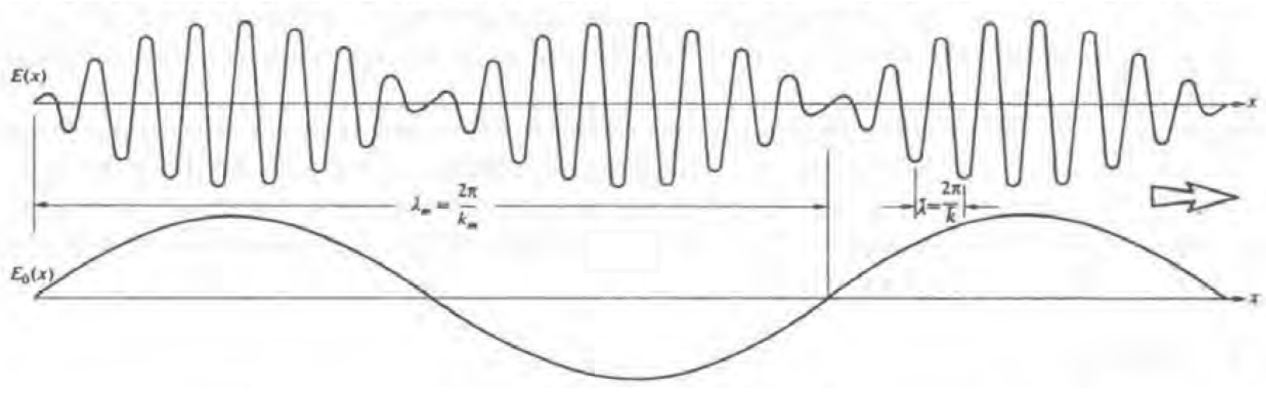


Figure 5.2: Standing Wave

Beat Frequency (Time)	$2\omega_m$
Beat Frequency (Space)	$2k_m$

Group Frequency	$v_g = \omega_m/k_m$
Phase Velocity	$v_p = \bar{\omega}/\bar{k}$

5.4 Light in Dispersible Media

Average Phase Velocity	$\bar{v}_p = \frac{c}{\bar{n}}$
Group Velocity	$v_g = \frac{c}{\bar{n}}\left(1 + \frac{\bar{\lambda}}{\bar{n}}\frac{\Delta n}{\Delta \lambda}\right)$

Normal Dispersion Media	$\bar{v}_p > v_g$
Anomalous Dispersion Media	$\bar{v}_p < v_g$

Chapter 6

Polarization

6.1 Circular Polarization

$$\begin{cases} \vec{E}_x(z,t) = \vec{\imath} E_0 \cos(kx - \omega t) \\ \vec{E}_y(z,t) = \vec{\jmath} E_0 \sin(kx - \omega t) \end{cases} \Rightarrow \vec{E} = E_0 \left[\vec{\imath}\cos(kx - \omega t) + \vec{\jmath}\sin(kx - \omega t)\right] \qquad \text{Right-circularly polarized}$$

$$\begin{cases} \vec{E}_x(z,t) = \vec{\imath} E_0 \cos(kz - \omega t) \\ \vec{E}_y(z,t) = -\vec{\jmath} E_0 \sin(kz - \omega t) \end{cases} \Rightarrow \vec{E} = E_0 \left[\vec{\imath}\cos(kx - \omega t) - \vec{\jmath}\sin(kx - \omega t)\right] \qquad \text{Left-circularly polarized}$$

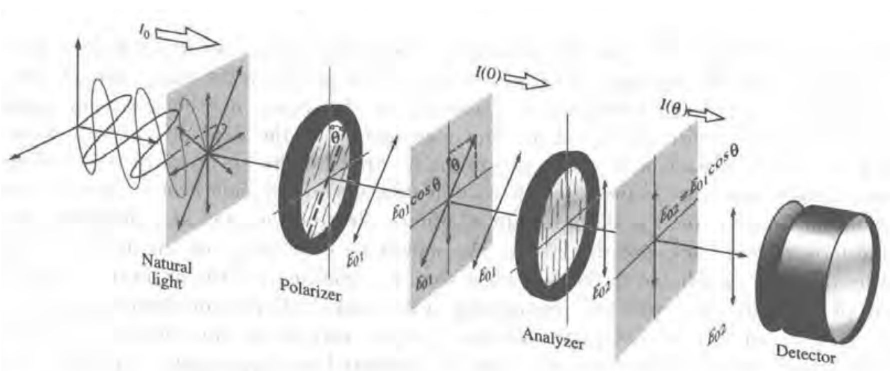
6.2 Elliptical Polarization

$$\begin{cases} \vec{E}_x = E_{0x} \cos(kx - \omega t) \\ \vec{E}_y = E_{0y} \cos(kz - \omega t + \epsilon) \end{cases} \qquad \text{Elliptical Polarization}$$

6.3 Angular Momentum

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \omega M = \omega \frac{\mathrm{d}L}{\mathrm{d}t} \Rightarrow L = \frac{E}{\omega} = \frac{h\nu}{\omega} = \pm\hbar = \begin{cases} -\hbar & \text{Right-circularly polarized} \\ +\hbar & \text{Left-circularly polarized} \end{cases}$$

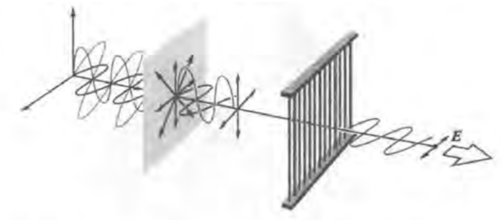
6.4 Malus's Law

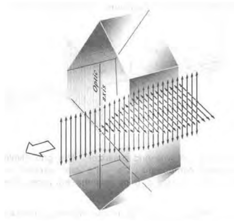


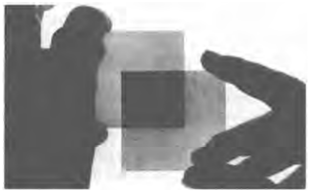
$$\begin{cases} E_{02} = E_{01} \cos\theta \\ I(\theta) = \dfrac{c\varepsilon_0}{2} E_{01}^2 \cos^2\theta = I(0)\cos^2\theta \end{cases}$$

6.5 Dichroism

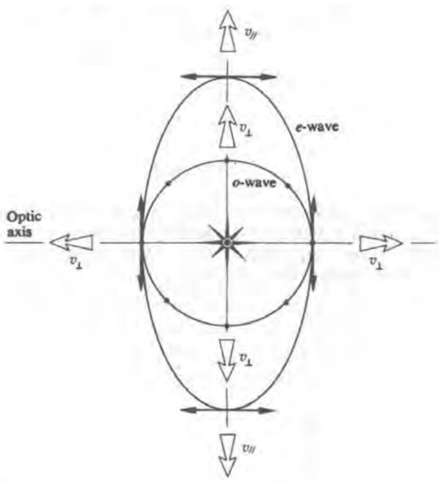
6.5.1 The Wire-Grid Polarizer and Dichroic Crystals, Polaroid



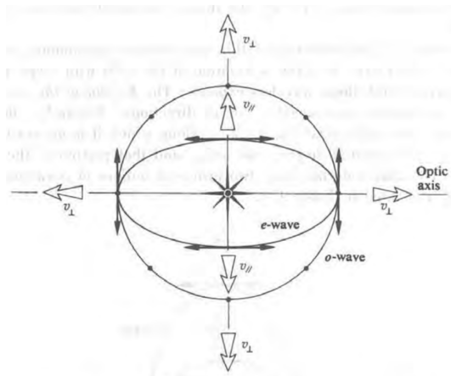




6.6 Birefringent Crystals



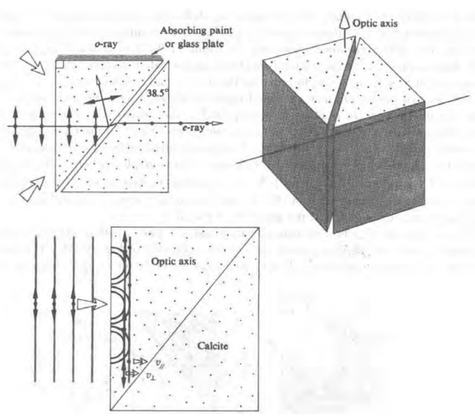
(a) $v_\perp < v_\parallel \Rightarrow n_o > n_e \Rightarrow$ negative uniaxial



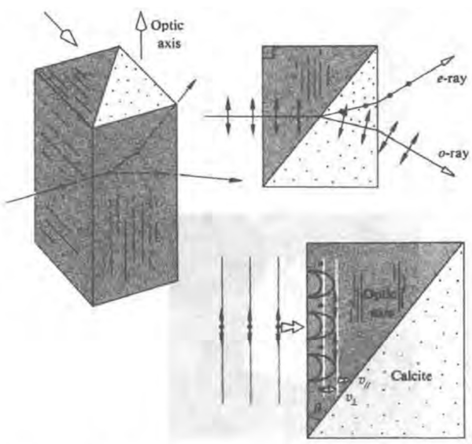
(b) $v_\perp > v_\parallel \Rightarrow n_o > n_e \Rightarrow$ positive uniaxial

Figure 6.2: negative and positive uniaxial

6.7 Polarizers



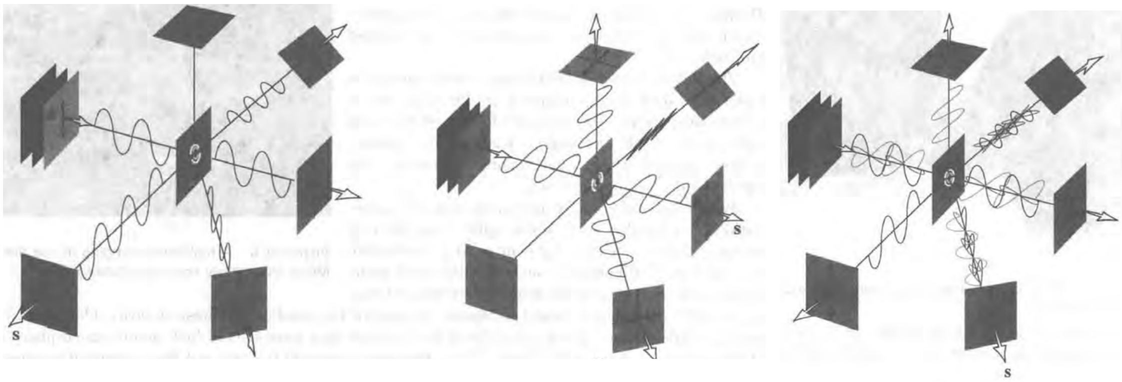
(a) The Glan-Foucault Prism



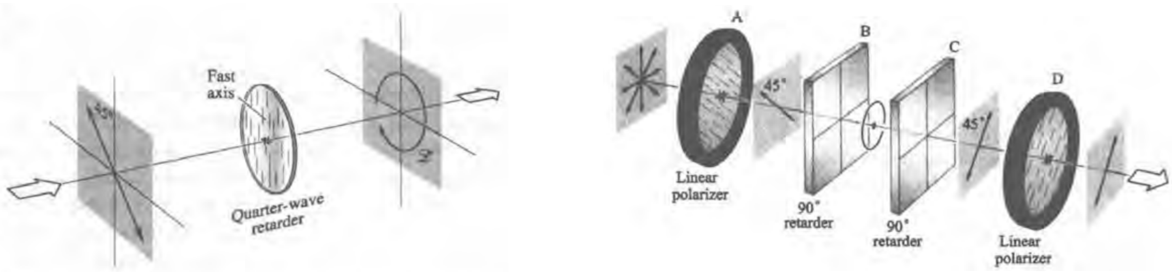
(b) The Wollaston Prism

Figure 6.3: Tow Birefringent Polarizers

6.8 Scattering and Polarization



6.9 Retarders



(a) Quarter-wave Retarder (b) Two Linear Polarizers and Two Quarter-wave Retarders

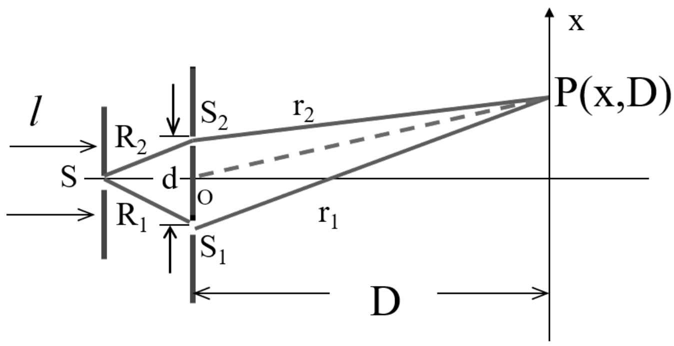
Figure 6.5: Quarter-wave Retarder and its Application

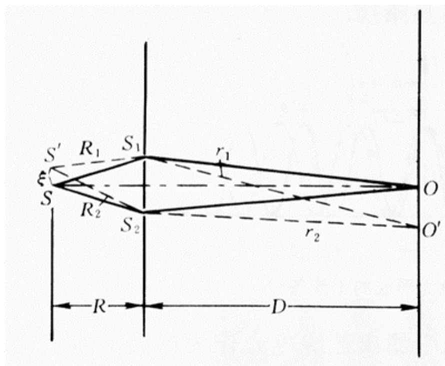
$$d\,(n_o - n_e) = \frac{4m+1}{4}\lambda_0$$

Chapter 7

Interference

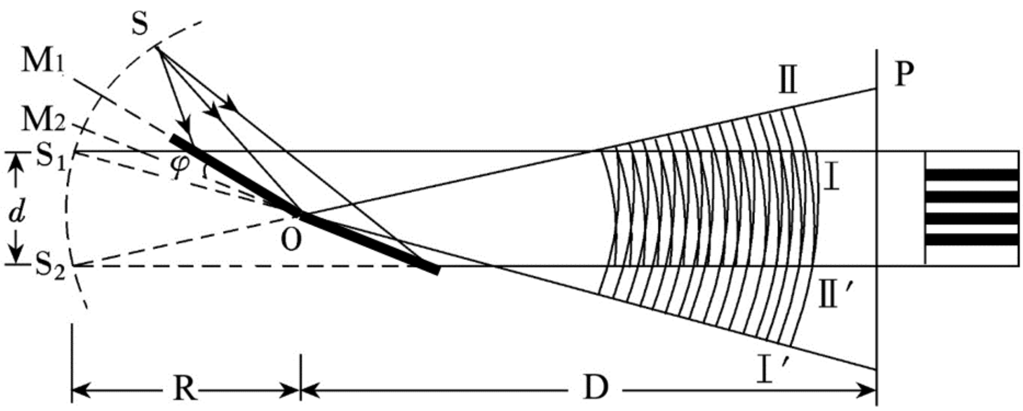
7.1 Young's Experiment





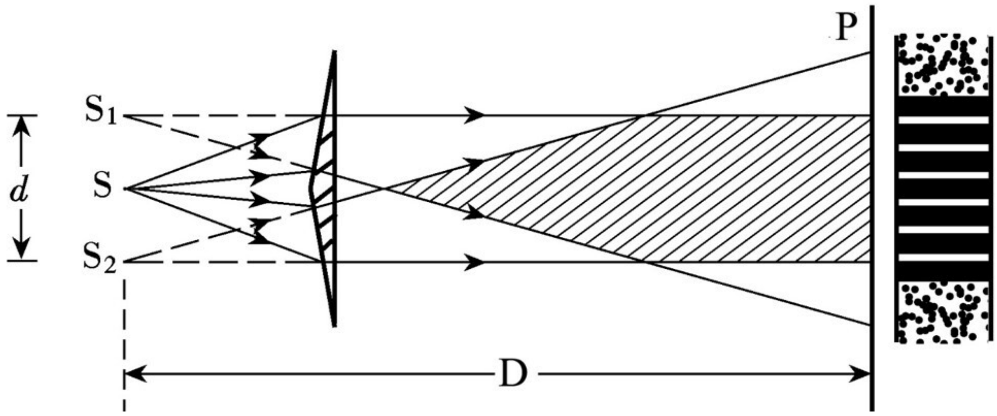
$$\Delta x = \frac{D}{d}\lambda \qquad b \leq \lambda R \frac{1}{d} \qquad I = I_0 \cos^2\left(\frac{d\pi}{D\lambda}x\right) \qquad x_0 = -\frac{D}{R}\xi$$

7.2 Fresnel's Double Mirror



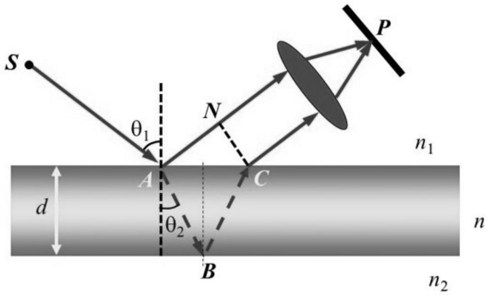
$$x_{white} = k\lambda\frac{D}{d} \qquad x_{black} = \frac{2k+1}{2}\lambda\frac{D}{d}$$

7.3 Fresnel's Double Prism



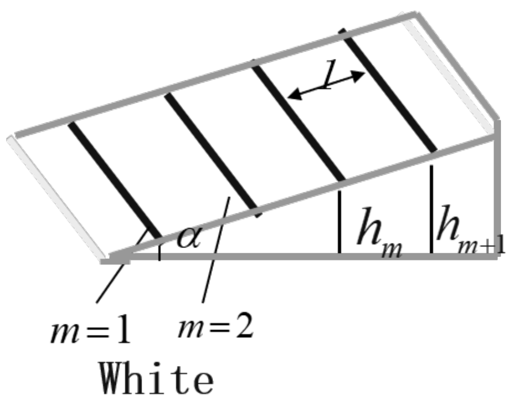
$$x_{white} = k\lambda\frac{D}{d} \qquad x_{black} = \frac{2k+1}{2}\lambda\frac{D}{d}$$

7.4 Equal Inclination Interference



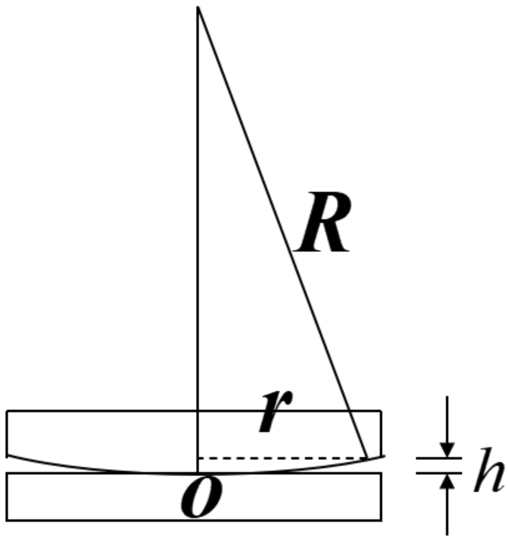
$$\Lambda = \begin{cases} 2nk_0 d\cos\theta_2 \pm \pi & n_1 > n_2 < n_3 \text{ OR } n_1 < n_2 > n_3 \\ 2nk_0 d\cos\theta_2 & n_1 < n_2 < n_3 \text{ OR } n_1 > n_2 > n_3 \end{cases}$$

7.5 Equal Thickness Interference



$$e = \Delta h = \frac{\lambda}{2n} \qquad l = \frac{e}{\sin\alpha} = \frac{\lambda}{2n\alpha} \approx \frac{\lambda}{2n\alpha}$$

7.6 Newton's Rings



$$\Delta = 2nh + \frac{\lambda}{2} = \begin{cases} k\lambda & \text{White} \\ \left(k + \frac{1}{2}\right)\lambda & \text{Black} \end{cases}$$

$$h = R - \sqrt{R^2 - r^2} = R\left[1 - \sqrt{1 - \left(\frac{r}{R}\right)^2}\right] \approx \frac{r^2}{2R}$$

$$r^2 = \begin{cases} \left(k - \frac{1}{2}\right)\frac{R\lambda}{n} & \text{White} \\ \frac{kR\lambda}{n} & \text{Black} \end{cases}$$

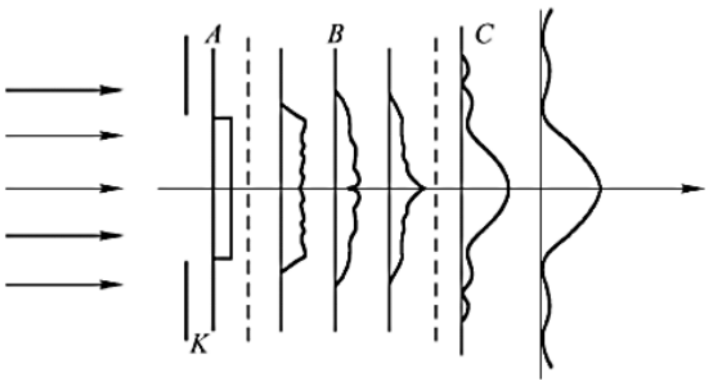
Chapter 8

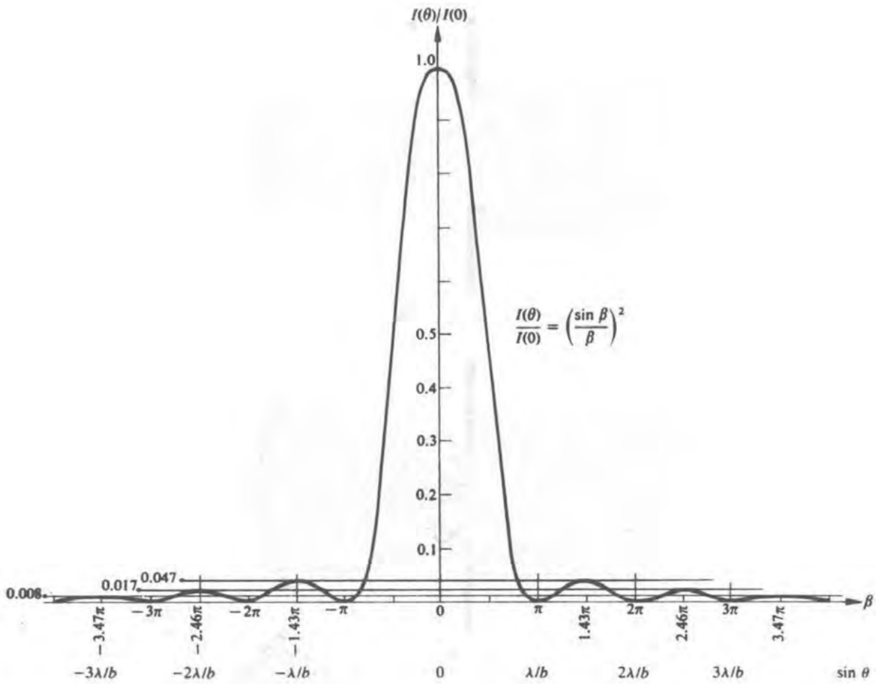
Diffraction

Chapter 9

Fraunhofer and Fresnel Diffraction

9.1 Fraunhofer Diffraction





White fringes:

$$\begin{cases} b\sin\theta = 0 & \text{Central Fringe} \\ \sin\theta = \pm(2m+1)\cdot\dfrac{\lambda}{2b} & m = 1, 2, 3, \ldots \end{cases} \qquad \begin{cases} \Delta\theta_0 = 2\cdot\dfrac{\lambda}{b} \\ \Delta\theta = \dfrac{\lambda}{b} \end{cases}$$

Dark fringes:

$$\sin\theta = \pm m\cdot\frac{\lambda}{b} \qquad m = 1, 2, 3, \ldots$$