Homework for Chapter 2

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1. 试计算氢原子的第一玻尔轨道上电子绕核转动的频率、线速度和加速度.

$$
\begin{aligned}
r_0 &= \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} \\
V_1 = \alpha c &= \frac{c}{137} = 2.19 \times 10^6 \text{ m/s} \\
\nu &= \frac{2\pi r_0}{V_1} = 6.58 \times 10^{15} \text{ Hz} \\
a &= \frac{V_1^2}{r_0} = 9.046 \times 10^{22} \text{ m/s}^2
\end{aligned}
$$

2. 试由氢原子的里德伯常数计算基态氢原子的电离电势和第一激发电势.

The energy of atom at different state are

$$
\begin{aligned}
E_1 &= -hcR_H = -13.6 \text{ eV} \\
E_2 &= -hcR_H \frac{1}{2^2} = -3.4 \text{ eV} \\
E_2 - E_1 &= 10.2 \text{ eV}
\end{aligned}
$$

So that the Ionization potential is 13.6 eV, the First excitation potential is 10.2 eV

3. 用能量为 12.5 电子伏特的电子去激发基态氢原子. 问受激发的氢原子向低能级跃迁时，会出现哪些波长的光谱线?

$$
\begin{aligned}
E_2 &= hcR_H \left(1 - \frac{1}{2^2}\right) = 10.2 \text{ eV} \\
E_3 &= hcR_H \left(1 - \frac{1}{3^2}\right) = 12.1 \text{ eV}
\end{aligned}
$$

$$E_4 = hcR_H\left(1-\frac{1}{4^2}\right) = 12.8\ \text{eV}$$

So that the maximum state the atom may reach is 3, possible wavelength of emitted light during transition are:

$$\lambda_1 = \frac{1}{R_H\left(\frac{1}{1^2}-\frac{1}{3^2}\right)} = 1.025\times10^{-7}\ \text{m}$$
$$\lambda_2 = \frac{1}{R_H\left(\frac{1}{1^2}-\frac{1}{2^2}\right)} = 1.215\times10^{-7}\ \text{m}$$
$$\lambda_3 = \frac{1}{R_H\left(\frac{1}{2^2}-\frac{1}{3^2}\right)} = 6.565\times10^{-7}\ \text{m}$$

4. 试估算一次电离的氦离子 He^+、二次电离的锂离子 Li^{++} 的第一玻尔轨道半径、电离电势、第一激发电势和赖曼系第一条谱线波长分别与氢原子的上述物理量之比值.

$$r_n = a_0\cdot\frac{n^2}{Z}$$
$$E_n = -\frac{m_e e^4}{2\left(4\pi\varepsilon_0\hbar\right)^2}\cdot\frac{Z^2}{n^2}$$
$$\nu = Z^2R\left(\frac{1}{n_1}-\frac{1}{n_2}\right)$$

So that

$$\frac{r_{He}}{r_H} = \frac{1}{2}$$
$$\frac{r_{Li}}{r_H} = \frac{1}{3}$$
$$\frac{E_{He1}}{E_{H1}} = \frac{4}{1}$$
$$\frac{E_{Li1}}{E_{H1}} = \frac{9}{1}$$
$$\frac{E_{He2}-E_{He1}}{E_{H2}-E_{H1}} = 4$$
$$\frac{E_{Li2}-E_{Li1}}{E_{H2}-E_{H1}} = 9$$
$$\frac{\nu_{He}}{\nu_H} = 4$$
$$\frac{\nu_{Li}}{\nu_H} = 9$$

5. 试问二次电离的锂离子 Li^{++} 从其第一激发态向基态跃迁时发出的光子，是否有可能使处于基态的一次电离的氦离子 He^+ 的电子电离掉？

$$E_{Li2} - E_{Li1} = \frac{9}{2}R$$

$$E_{He1} = 4R \leq \frac{9}{2}R$$

The photon emitted by Li^{2+} can ionize He^{+}.

8. 试证明氢原子中的电子从 $n+1$ 轨道跃迁到 n 轨道，发射光子的频率 $\tilde{\nu}_n$. 当 $n \gg 1$ 时光子频率即为电子绕第 n 玻尔轨道转动的频率.

$$\begin{aligned}
\nu &= R_H \left[\frac{1}{n} - \frac{1}{n+1}\right] \\
&= \frac{2\pi^2 e^4 m}{(4\pi\varepsilon_0)^2 h^3 c}\left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right] \\
&= \frac{2\pi^2 e^4 m}{(4\pi\varepsilon_0)^2 h^3 c} \cdot \left[\frac{\mathrm{d}}{\mathrm{d}n}\left(\frac{1}{n^2}\right)\right][n-(n+1)] \\
&= \frac{4\pi^2 e^4 m}{(4\pi\varepsilon_0)^2 h^3 c} \cdot \frac{1}{n^3}
\end{aligned}$$

$$\frac{1}{T} = \frac{V}{2\pi r} = \frac{\alpha c}{2\pi a_0} \cdot \frac{1}{n^3} = \frac{c}{2\pi} \cdot \frac{me^4}{4\pi\varepsilon_0 \hbar c \cdot 4\pi\varepsilon_0 \hbar^2} \cdot \frac{1}{n^3} = \nu$$

9. Li 原子序数 $Z=3$，其光谱的主线系可用下式表示：$\tilde{\nu} = \frac{R}{(1+0.5951)^2} - \frac{R}{(n-0.0401)^2}$. 已知 Li 原子电离成 Li^{+++} 离子需要 203.44 电子伏特的功. 问如要把 Li^{+} 离子电离为 Li^{++} 离子，需要多少电子伏特的功？

Assume that the energy needed to transfer Li to Li^{+} is E_1, the energy to transfer Li^{+} to Li^{2+} is E_2, and the energy to transfer Li^{2+} to Li^{3+} is E_3.

$$E_1 + E_2 + E_3 = 203.44 \text{ eV}$$

$$E_1 = \lim_{n\to\infty}\left[\frac{R}{(1+0.5901)^2} - \frac{R}{(1-0.0401)^2}\right] \cdot hc = 5.35 \text{ eV}$$

$$E_3 = 9Rhc = 122.4 \text{ eV}$$

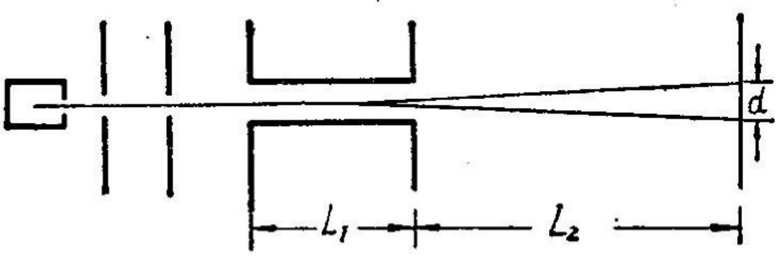
$$E_2 = 203.44 - E_1 - E_2 = 75.7 \text{ eV}$$

10. 具有磁矩的原子，在横向均匀磁场和横向非均匀磁场中运动时有什么不同？

In an uniformed magnet field, where $\frac{\mathrm{d}B}{\mathrm{d}z} = 0$, no force will be applied on the atom. And the electron will move around the nucleus in an qumntumized, eclipse shaped orbit. Also, precession of the orbit will take place if relativity is concerned.

In a varied magnet field, where $\frac{\mathrm{d}B}{\mathrm{d}z} \neq 0$, besides the motion described above, magnetic force will be applied to the atom.

11. 在史特恩-盖拉赫实验中，处于基态的窄银原子束通过不均匀横向磁场，磁场梯度为$\frac{\partial B}{\partial z} = 10^3$特斯拉/米，磁极纵向范围 $L_1 = 0.04$ 米（习题图 2-1），从磁极到屏距离 $L_2 = 0.10$ 米，原子的速度 $v = 5\times10^2$ 米/秒．在屏上两束分开的距离 $d = 0.002$ 米．试确定原子磁矩在磁场方向上投影 μ_z 的大小(设磁场边缘的影响可忽略不计)。



习题图 2·1

$$f = \mu_z \frac{\mathrm{d}B}{\mathrm{d}z}$$

$$a = \frac{f}{m} = \frac{1}{m} \cdot \frac{\mathrm{d}B}{\mathrm{d}z} \cdot \mu_z$$

$$t_1 = \frac{L_1}{v}$$

$$t_2 = \frac{L_2}{v}$$

$$S_1 = \frac{1}{2} a t_1^2 = \frac{1}{2m} \cdot \left(\frac{L_1}{v}\right) \cdot \frac{\mathrm{d}B}{\mathrm{d}z} \cdot \mu_z$$

$$S_2 = a t_1 t_2 = \frac{1}{m} \cdot \frac{\mathrm{d}B}{\mathrm{d}z} \cdot \frac{L_1 L_2}{v^2} \cdot \mu_z$$

$$S_1 + S_2 = \frac{d}{2}$$

$$\frac{1}{2m} \cdot \left(\frac{L_1}{v}\right) \cdot \frac{\mathrm{d}B}{\mathrm{d}z} \cdot \mu_z + \frac{1}{m} \cdot \frac{\mathrm{d}B}{\mathrm{d}z} \cdot \frac{L_1 L_2}{v^2} \cdot \mu_z = \frac{d}{2}$$

$$\frac{1}{2m} \cdot \frac{L_1 (L_1 + 2L_2)}{v^2} \cdot \frac{\mathrm{d}B}{\mathrm{d}z} \cdot \mu_z = \frac{d}{2}$$

$$\mu_z = 9.3 \times 10^{-24}\ \mathrm{J} \cdot \mathrm{T}^{-1}$$