

Atomic Physics

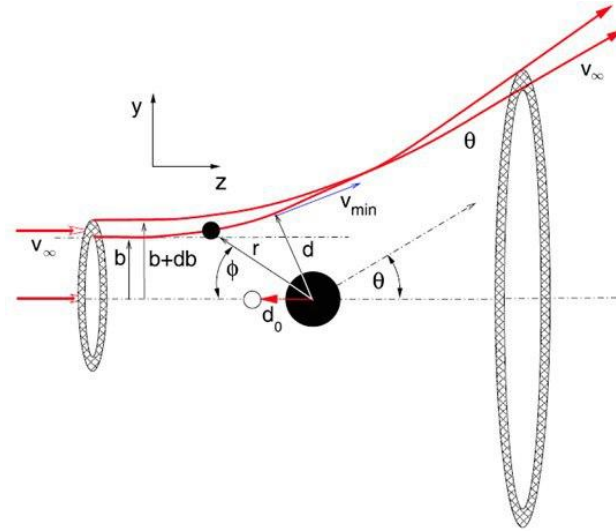
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Chapter 1

Rutherford's Alpha Particle Scattering Experiment



According to Coulomb's Law:

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{Z_1 Z_2 e^2}{r^2} = \frac{C}{r^2}$$

$$C = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}$$

$$F_y = F \sin \phi = \frac{C}{r^2} \sin \phi$$

Law of momentum and angular momentum

$$mv_y = \int F_y dt$$

$$mr^2 \dot{\phi} = mv_\infty b$$

Then we integrate

$$\begin{aligned} v_y &= \frac{1}{m} \int \frac{C}{r^2} \sin \phi dt = \frac{1}{m} \int \frac{C}{r^2} \sin \phi \frac{dt}{d\phi} d\phi = \frac{1}{m} \int \frac{C}{r^2} \sin \phi \frac{r^2}{v_\infty b} d\phi = \frac{C}{mv_\infty b} \int_0^{\pi-\theta} \sin \phi d\phi \\ &= \frac{C}{mv_\infty b} (1 + \cos \theta) \end{aligned}$$

Now we need to relate θ with b , Since

$$v_y = v_\infty \sin \theta$$

We have

$$\frac{C}{mv_\infty b} (1 + \cos \theta) = v_\infty \sin \theta$$

So that

$$\cot \frac{\theta}{2} = \frac{1 + \cos \theta}{\sin \theta} = \frac{mv_\infty^2 b}{C} = \frac{2E_0 b}{C}$$

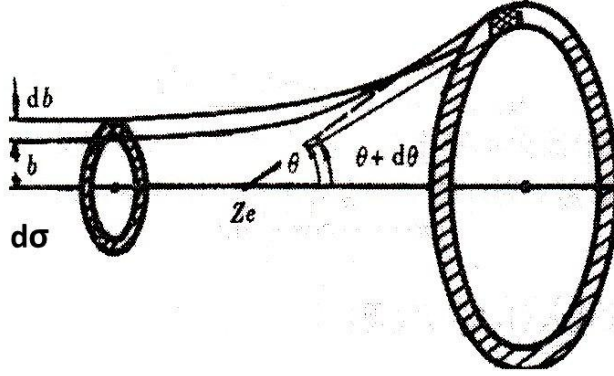
Note that this trigonometry transform is used

$$\frac{1 + \cos \theta}{\sin \theta} = \frac{2 \cos^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} = \cot \frac{\theta}{2}$$

Finally

$$b = \frac{C}{2E_0} \cdot \cot \frac{\theta}{2} = \frac{1}{4\pi\epsilon_0} \frac{Z_1 Z_2 e^2}{2E_0} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Z_1 Z_2 e^2}{mv_\infty^2} \cdot \cot \frac{\theta}{2}$$

Now we begin to find the relation between db and $d\Omega$



$$d\sigma = 2\pi b db = \pi \left(\frac{1}{4\pi\epsilon_0} \right)^2 \left(\frac{Z_1 Z_2 e^2}{mv_\infty^2} \right)^2 \frac{\cos \frac{\theta}{2}}{\sin^3 \frac{\theta}{2}} d\theta$$

$$\Omega = 2\pi (1 - \cos \theta)$$

$$d\Omega = 2\pi \sin \theta d\theta = 4\pi \sin \frac{\theta}{2} \cos \frac{\theta}{2} d\theta$$

Then we found $\frac{d\sigma}{d\Omega}$ is only related with θ

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{4\pi\epsilon_0} \right)^2 \left(\frac{Z_1 Z_2 e^2}{2mv_\infty^2} \right)^2 \sin^{-4} \frac{\theta}{2}$$

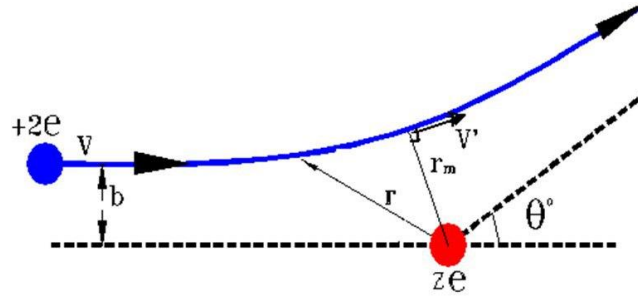
As for a thin gold leaf, we assume there's only one layer of atoms, the density of atoms is N , the area and thickness of gold leaf are A and t

When n particles passed through the gold leaf, dn of them ended up in $d\Omega$

$$\frac{dn}{n} = \frac{NAt d\sigma}{A} = Nt d\sigma = Nt \left(\frac{1}{4\pi\epsilon_0} \right)^2 \left(\frac{Z_1 Z_2 e^2}{2mv_\infty^2} \right)^2 \sin^{-4} \frac{\theta}{2} d\Omega$$

$$\frac{dn}{d\Omega} \sin^4 \frac{\theta}{2} = nNt \left(\frac{1}{4\pi\epsilon_0} \right)^2 \left(\frac{Z_1 Z_2 e^2}{2mv_\infty^2} \right)^2 = \text{const}$$

For alpha particles, $Z_1 = 2$. We can also take the closest distance between the 2 particles as the radius of a particle.



What we have already known are:

$$\frac{1}{2}MV^2 = \frac{1}{2}MV'^2 + \frac{2Ze^2}{4\pi\epsilon_0 r_m}$$

$$MVb = MV'r_m$$

$$b = \frac{1}{4\pi\epsilon_0} \cdot \frac{2Ze^2}{MV} \cdot \cot \frac{\theta}{2}$$

We solve r_m with the equation we known.

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{2Ze^2}{M} \cdot \cot \frac{\theta}{2} = V'r_m$$

$$\frac{1}{2}MV^2 = \frac{1}{2}M \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{2Ze^2}{Mr_m} \cdot \cot \frac{\theta}{2} \right)^2 + \frac{2Ze^2}{4\pi\epsilon_0 r_m}$$

$$\frac{1}{2}MV^2 r_m^2 - \frac{2Ze^2}{4\pi\epsilon_0} r_m - \frac{1}{2}M \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{2Ze^2}{M} \cdot \cot \frac{\theta}{2} \right)^2 = 0$$

Finally

$$r_m = \frac{1}{4\pi\epsilon_0} \frac{2ze^2}{MV^2} \left(1 + \frac{1}{\sin \frac{\theta}{2}} \right)$$