

Homework for Chapter 3

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1. 波长为 1\AA 的 X 光光子的动量和能量各为多少?

$$E = \frac{hc}{\lambda} = 1.98645 \times 10^{-15} \text{ J} \cdot \text{m}$$

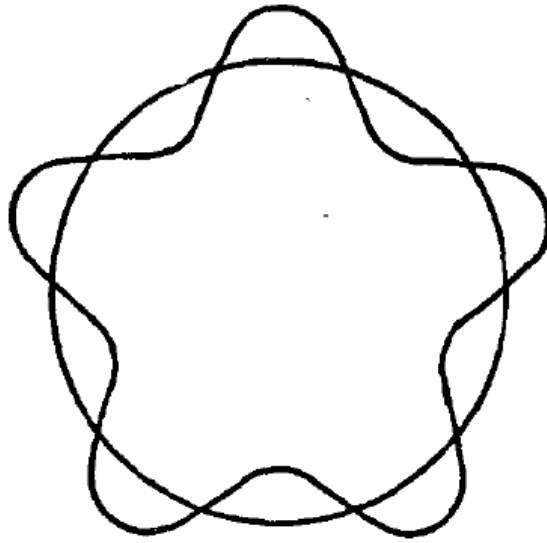
$$p = \frac{h}{\lambda} = 6.62607 \times 10^{-24} \text{ J} \cdot \text{s}$$

2. 经过 10000 伏特电势差加速的电子束的德布罗意波波长 $\lambda = ?$ 用上述电压加速的质子束, 其德布罗意波波长是多少 \AA ?

$$v = \sqrt{\frac{2E_k}{m_0}} = 5.93097 \times 10^7 \text{ m/s}$$

$$\lambda = \frac{h}{m_0 v} = 1.22643 \times 10^{-11} \text{ m}$$

4. 试证明氢原子稳定轨道上正好能容纳下整数个电子的德布罗意波长(习题图 3.1)。上述结果不但适用于圆轨道, 同样适用于椭圆轨道, 试证



习题图 3.1

明之.

In round orbits

$$\oint p_r dq_r = n_r h$$

$$\oint p_\phi dq_\phi = n_\phi h$$

Since $p_r = 0$

$$\oint m r^2 \dot{\phi} d\phi = n_\phi h$$

$$2\pi m v r = n_\phi h$$

$$2\pi r = n_\phi \frac{h}{mv}$$

The wave length of the electron is

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

So that

$$2\pi r = n_\phi \lambda$$

When the orbit is shaped in eclipse

$$\oint p_r dq_r = n_r h$$

$$\oint p_\phi dq_\phi = n_\phi h$$

So that

$$\oint (p_r dr + p_\phi d\phi) = nh$$

$$\oint (m\dot{r} dr + mr^2 \dot{\phi} d\phi) = nh$$

$$\oint (m\dot{r}^2 dt + mr^2 \dot{\phi}^2 dt) = nh$$

$$\oint [m(\dot{r}^2 + r^2 \dot{\phi}^2)] dt = nh$$

$$\oint [mv^2] dt = nh$$

$$\oint mv ds = nh$$

$$\oint \frac{mv}{h} ds = n$$

The wavelength of the electron is

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

insert it into

$$\oint \frac{ds}{\lambda} = n$$

$$\oint ds = n\lambda$$

5. 带电粒子在威耳孙云室（一种径迹探测器）中的轨迹是一串小雾滴，雾滴的线度约为 1 微米。当观察能量为 1000 电子伏的电子径迹时其动量与经典力学动量的相对偏差不小于多少？

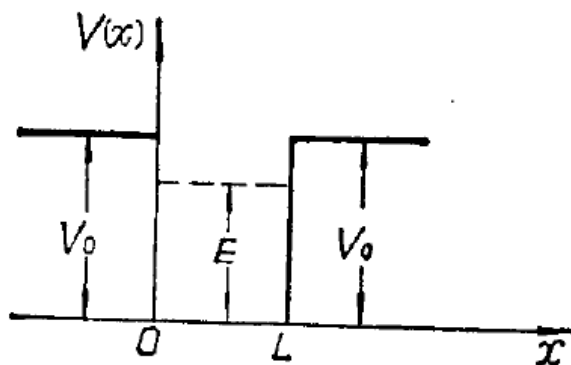
$$\Delta p \Delta x \geq \frac{\hbar}{2}$$

$$p = \sqrt{2mE_k}$$

$$\frac{\Delta p}{p} = \frac{\hbar}{2\Delta x \sqrt{2mE_k}} = 3.046 \times 10^{-5}$$

7. 粒子位于一维对称势场中, 势场形式如下图, 即

$$\begin{cases} 0 < x < L, & V = 0, \\ x < 0, x > L, & V = V_0. \end{cases}$$



习题图 3.2

(1) 试推导粒子在 $E < V_0$ 情况下其总能量 E 满足的关系式.

(2) 试利用上述关系式, 以图解法证明, 粒子的能量只能是一些不连续的值.

Solution for problem (1)

$$\frac{d^2\Psi}{dx^2} + \frac{2m(E - V)}{\hbar^2}\Psi = 0$$

For $x < 0$

$$\frac{d^2\Psi}{dx^2} - \frac{2m(V_0 - E)}{\hbar^2}\Psi = 0$$

$$\Psi_1(x) = A \exp \left[\sqrt{\frac{2m(V_0 - E)}{\hbar^2}} x \right]$$

For $0 < x < L$

$$\frac{d^2\Psi}{dx^2} + \frac{2mE}{\hbar^2}\Psi = 0$$

$$\Psi_2(x) = B \sin \left[\sqrt{\frac{2mE}{\hbar^2}} x + \phi \right]$$

For $x > L$

$$\frac{d^2\Psi}{dx^2} - \frac{2m(V_0 - E)}{\hbar^2}\Psi = 0$$

$$\Psi_3(x) = C \exp \left[-\sqrt{\frac{2m(V_0 - E)}{\hbar^2}}x \right]$$

In conclusion

$$\begin{cases} \Psi_1(x) = A \exp \left[\sqrt{\frac{2m(V_0 - E)}{\hbar^2}}x \right] = A \exp [\alpha x] \\ \Psi_2(x) = B \sin \left[\sqrt{\frac{2mE}{\hbar^2}}x + \phi \right] = B \sin [\beta x + \phi] \\ \Psi_3(x) = C \exp \left[-\sqrt{\frac{2m(V_0 - E)}{\hbar^2}}x \right] = C \exp [-\alpha x] \end{cases}$$

By taking logarithms

$$\begin{cases} \log [\Psi_1(x)] = \log A + \alpha x \\ \log [\Psi_2(x)] = \log B + \log [\sin (\beta x + \phi)] \\ \log [\Psi_3(x)] = \log C - \alpha x \end{cases}$$

By taking derivatives

$$\begin{cases} \frac{1}{\Psi_1(x)} \frac{d\Psi_1(x)}{dx} = \alpha \\ \frac{1}{\Psi_2(x)} \frac{d\Psi_2(x)}{dx} = \beta \cot [\beta x + \phi] \\ \frac{1}{\Psi_3(x)} \frac{d\Psi_3(x)}{dx} = -\alpha \end{cases}$$

Due to the continuity of $\Psi(x)$

$$\begin{cases} \alpha = \beta \cot \phi \\ \beta \cot [\beta L + \phi] = -\alpha \end{cases}$$

So that

$$\begin{cases} \tan \phi = \frac{\beta}{\alpha} \\ \tan [\beta L + \phi] = -\frac{\beta}{\alpha} \end{cases}$$

Finally

$$\beta L = -2\phi + n\pi = -2 \arctan \frac{\beta}{\alpha} + n\pi$$

Whereas

$$\alpha = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

$$\beta = \sqrt{\frac{2mE}{\hbar^2}}$$

Solution for Problem (2)

$$\beta L = -2 \arctan \frac{\beta}{\alpha} + n\pi$$

$$\frac{\beta}{\alpha} = \tan \left(\frac{n\pi}{2} - \frac{\beta L}{2} \right)$$

When $n = 1, 3, 5 \dots$

$$\frac{\beta}{\alpha} = \cot \left(\frac{\beta L}{2} \right)$$

$$\sqrt{\frac{E}{V_0 - E}} = \cot \left(\frac{mEL^2}{2\hbar^2} \right)$$

When $n = 0, 2, 4 \dots$

$$\frac{\beta}{\alpha} = -\tan \left(\frac{\beta L}{2} \right)$$

$$\sqrt{\frac{E}{V_0 - E}} = -\tan \left(\frac{mEL^2}{2\hbar^2} \right)$$

So that E is discontinued.

8. 有一粒子, 其质量为 m , 在一个三维势箱中运动. 势箱的长、宽、高分别为 a, b, c . 在势箱外, 势能 $V = \infty$; 在势箱内, $V = 0$. 试算出粒子可能具有的能量.