Atomic Physics

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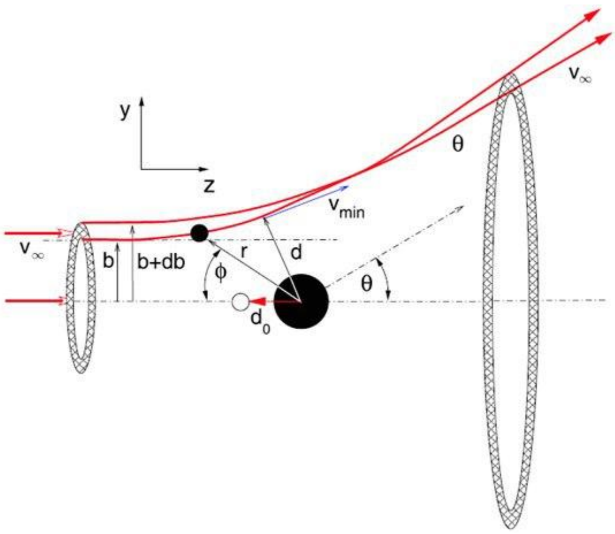
Chapter 1

Rutherford's Alpha Particle Scattering Experiment



According to Columb's Law:

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Z_1 Z_2 e^2}{r^2} = \frac{C}{r^2}$$

$$C = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}$$

$$F_y = F \sin\phi = \frac{C}{r^2} \sin\phi$$

Law of momentum and angular momentum

$$m v_y = \int F_y \, \mathrm{d}t$$

$$m r^2 \dot{\phi} = m v_\infty b$$

Then we integrate

$$v_y = \frac{1}{m} \int \frac{C}{r^2} \sin\phi \, \mathrm{d}t = \frac{1}{m} \int \frac{C}{r^2} \sin\phi \frac{\mathrm{d}t}{\mathrm{d}\phi} \, \mathrm{d}\phi = \frac{1}{m} \int \frac{C}{r^2} \sin\phi \frac{r^2}{v_\infty b} \, \mathrm{d}\phi = \frac{C}{m v_\infty b} \int_0^{\pi-\theta} \sin\phi \, \mathrm{d}\phi$$
$$= \frac{C}{m v_\infty b} (1 + \cos\theta)$$

Now we need to relate θ with b, Since

$$v_y = v_\infty \sin\theta$$

We have

$$\frac{C}{mv_\infty b}(1+\cos\theta) = v_\infty \sin\theta$$

So that

$$\cot\frac{\theta}{2} = \frac{1+\cos\theta}{\sin\theta} = \frac{mv_\infty^2 b}{C} = \frac{2E_0 b}{C}$$

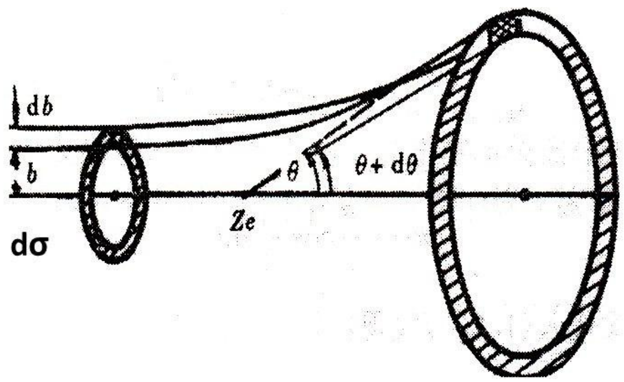
Note that this trigonometry transform is used

$$\frac{1+\cos\theta}{\sin\theta} = \frac{2\cos^2\dfrac{\theta}{2}}{2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}} = \cot\frac{\theta}{2}$$

Finally

$$b = \frac{C}{2E_0}\cdot\cot\frac{\theta}{2} = \frac{1}{4\pi\varepsilon_0}\frac{Z_1Z_2e^2}{2E_0} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Z_1Z_2e^2}{mv_\infty}\cdot\cot\frac{\theta}{2}$$

Now we begin to find the relation between $\mathrm{d}b$ and $\mathrm{d}\Omega$



$$\mathrm{d}\sigma = 2\pi b\,\mathrm{d}b = \pi\left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{Z_1Z_2e^2}{mv_\infty^2}\right)^2\frac{\cos\dfrac{\theta}{2}}{\sin^3\dfrac{\theta}{2}}\,\mathrm{d}\theta$$

$$\Omega = 2\pi(1-\cos\theta)$$

$$\mathrm{d}\Omega = 2\pi\sin\theta\,\mathrm{d}\theta = 4\pi\sin\frac{\theta}{2}\cos\frac{\theta}{2}\,\mathrm{d}\theta$$

Then we found $\dfrac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$ is only related with θ

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{Z_1Z_2e^2}{2mv_\infty^2}\right)^2\sin^{-4}\frac{\theta}{2}$$

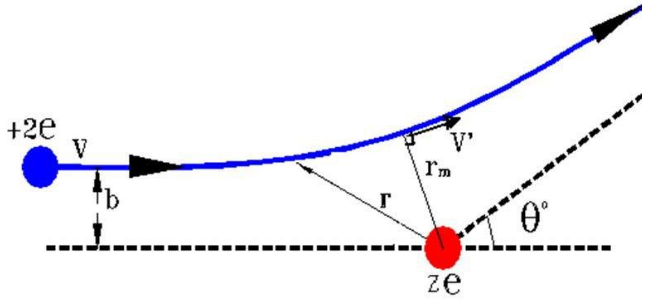
As for a thin gold leaf, we assume there's only one layer of atoms, the density of atoms is N, the area and thickness of gold leaf are A and t

When n particles passed through the gold leaf, $\mathrm{d}n$ of them ended up in $\mathrm{d}\Omega$

$$\frac{\mathrm{d}n}{n} = \frac{NAt\,\mathrm{d}\sigma}{A} = Nt\,\mathrm{d}\sigma = Nt\left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{Z_1Z_2e^2}{2mv_\infty^2}\right)^2\sin^{-4}\frac{\theta}{2}\,\mathrm{d}\Omega$$

$$\frac{\mathrm{d}n}{\mathrm{d}\Omega}\sin^4\frac{\theta}{2} = nNt\left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{Z_1 Z_2 e^2}{2mv_\infty^2}\right)^2 = \text{const}$$

For alpha particles, $Z_1 = 2$. We can also take the closest distance between the 2 particles as the radius of a particle.



What we have already known are:

$$\frac{1}{2}MV^2 = \frac{1}{2}MV'^2 + \frac{2Ze^2}{4\pi\varepsilon_0 r_m}$$

$$MVb = MV'r_m$$

$$b = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{MV}\cdot\cot\frac{\theta}{2}$$

We solve r_m with the equation we known.

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{M}\cdot\cot\frac{\theta}{2} = V'r_m$$

$$\frac{1}{2}MV^2 = \frac{1}{2}M\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{Mr_m}\cdot\cot\frac{\theta}{2}\right)^2 + \frac{2Ze^2}{4\pi\varepsilon_0 r_m}$$

$$\frac{1}{2}MV^2 r_m^2 - \frac{2Ze^2}{4\pi\varepsilon_0}r_m - \frac{1}{2}M\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{M}\cdot\cot\frac{\theta}{2}\right)^2 = 0$$

Finally

$$r_m = \frac{1}{4\pi\varepsilon_0}\frac{2ze^2}{MV^2}\left(1 + \frac{1}{\sin\dfrac{\theta}{2}}\right)$$