Homework for Chapter 4

Xiping Hu

https://hxp.plus/

April 6, 2020

1. 已知 Li 原子光谱主线系最长波长 $\lambda=6707\text{Å}$，辅线系系限波长 $\lambda_\infty=3519\text{Å}$. 求 Li 原子第一激发电势和电离电势.

$$V_1=\frac{hc}{\lambda e}=1.850\ \text{eV}$$

$$V_\infty=\frac{hc}{\lambda e}+\frac{hc}{\lambda_\infty e}=5.375\ \text{eV}$$

2. Na 原子的基态为 3S. 已知其共振线波长为 5893Å，漫线系第一条的波长为 8193Å，基线系第一条的波长为 18459Å，主线系的系限波长为 2413Å. 试求 3S、3P、3D、4F 各谱项的项值.

$$T_{3S}=\frac{1}{\lambda_{p_\infty}}=4.144\times10^6\ \text{m}^{-1}$$

$$T_{3P}=\frac{1}{\lambda_{p_\infty}}-\frac{1}{\lambda_{p_{max}}}=2.447\times10^6\ \text{m}^{-1}$$

$$T_{3D}=T_{3P}-\frac{1}{\lambda_{d_{max}}}=1.227\times10^6\ \text{m}^{-1}$$

$$T_{4F}=T_{3P}-\frac{1}{\lambda_{f_{max}}}=0.685\times10^6\ \text{m}^{-1}$$

3. K 原子共振线波长为 7665Å, 主线系系限波长为 2858Å. 已知 K 原子基态为 4S. 试求 4S、4P 谱项的量子数修正项 Δ_S、Δ_P 值各为多少？

$$\begin{cases} T_{4S} = \dfrac{1}{\lambda_{p_\infty}} \\ T_{4S} = \dfrac{R}{(4-\Delta_s)^2} \end{cases}$$

$$\begin{cases} T_{4P} = \dfrac{1}{\lambda_{p_\infty}} - \dfrac{1}{\lambda_{p_{max}}} \\ T_{4P} = \dfrac{R}{(4-\Delta_p)^2} \end{cases}$$

$$\begin{cases} \Delta_s = 2.229 \\ \Delta_p = 1.764 \end{cases}$$

5. 为什么谱项 S 项的精细结构总是单层结构？试直接从碱金属光谱双线的规律性和从电子自旋与轨道相互作用的物理概念两方面分别说明之。

Experiments showed that the principal series yellow sodium line 3s-3p is not a simple line but is a doublet. And this phenomenon exists in other alkalis metal atoms. Hence the levels 3p, 4p, 5p, etc, must be double while 3s should be single.

What's more, the angular momentum of the whole electron, including it's self-spin angular momentum and it's orbital momentum, is

$$p_j = \sqrt{j(j+1)} \cdot \frac{h}{2\pi}$$

Where

$$j = l \pm \frac{1}{2}$$

When the electron is in s orbital, $l = 0$, since p_j must be real, j cannot be $-\dfrac{1}{2}$. This will cause p_j to be a single value. Which means that the electron can only has one possible direction to self-spin. Which, finally cause the energy level of s orbit to be a single level.

8. 原子在热平衡条件下处在各种不同能量激发态的原子的数目是按玻耳兹曼分布的，即能量为 E 的激发态原子数目 $N=N_0\dfrac{g}{g_0}e^{-\left(\frac{E-E_0}{kT}\right)}$. 其中 N_0 是能量为 E_0 的状态的原子数，g 和 g_0 是相应能量状态的统计权重. k 是玻耳兹曼常数. 从高温铯原子气体光谱中测出其共振光谱双线 $\lambda_1=8943.5$Å、$\lambda_2=8521.1$Å 的强度比 $I_1:I_2=2:3$. 试估算此气体的温度. 已知相应能级的统计权重 $g_1=2, g_2=4$.

$$\begin{cases} E_1 = \dfrac{hc}{\lambda_1} \\ E_2 = \dfrac{hc}{\lambda_2} \end{cases}$$

$$\frac{I_1}{I_2} = \frac{N_1}{N_2} = \frac{g_1}{g_2} \frac{\exp\left[-\left(E_1 - E_0\right) / \left(kT\right)\right]}{\exp\left[-\left(E_2 - E_0\right) / \left(kT\right)\right]} = \frac{g_1}{g_2} \exp\left[-\frac{E_1 - E_2}{kT}\right]$$

$$\frac{2}{3} = \frac{2}{4} \cdot \exp\left[\frac{E_2 - E_1}{kT}\right]$$

$$T = \frac{E_2 - E_1}{k \ln\left[\dfrac{4}{3}\right]} = 2773 \text{ K}$$