Atomic Physics

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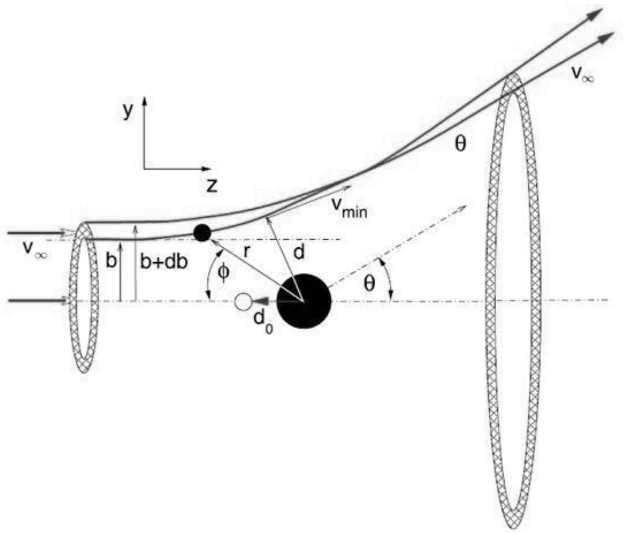
April 16, 2020

Chapter 1

Rutherford's Alpha Particle Scattering Experiment



According to Columb's Law:

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Z_1 Z_2 e^2}{r^2} = \frac{C}{r^2}$$

$$C = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}$$

$$F_y = F \sin\phi = \frac{C}{r^2} \sin\phi$$

Law of momentum and angular momentum

$$mv_y = \int F_y \, \mathrm{d}t$$

$$mr^2 \dot{\phi} = mv_\infty b$$

Then we integrate

$$v_y = \frac{1}{m}\int \frac{C}{r^2}\sin\phi\,\mathrm{d}t = \frac{1}{m}\int \frac{C}{r^2}\sin\phi\frac{\mathrm{d}t}{\mathrm{d}\phi}\,\mathrm{d}\phi = \frac{1}{m}\int \frac{C}{r^2}\sin\phi\frac{r^2}{v_\infty b}\,\mathrm{d}\phi = \frac{C}{mv_\infty b}\int_0^{\pi-\theta}\sin\phi\,\mathrm{d}\phi$$
$$= \frac{C}{mv_\infty b}(1+\cos\theta)$$

Now we need to relate θ with b, Since

$$v_y = v_\infty \sin\theta$$

We have

$$\frac{C}{mv_\infty b}(1+\cos\theta) = v_\infty\sin\theta$$

So that

$$\cot\frac{\theta}{2} = \frac{1+\cos\theta}{\sin\theta} = \frac{mv_\infty^2 b}{C} = \frac{2E_0 b}{C}$$

Note that this trigonometry transform is used

$$\frac{1+\cos\theta}{\sin\theta} = \frac{2\cos^2\dfrac{\theta}{2}}{2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}} = \cot\frac{\theta}{2}$$

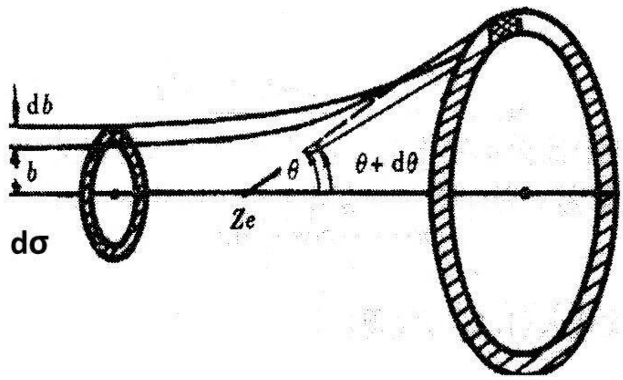
Finally

$$b = \frac{C}{2E_0}\cdot\cot\frac{\theta}{2} = \frac{1}{4\pi\varepsilon_0}\frac{Z_1Z_2e^2}{2E_0} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Z_1Z_2e^2}{mv_\infty^2}\cdot\cot\frac{\theta}{2}$$

$\Rightarrow$

$$\cot\frac{\theta}{2} = 4\pi\varepsilon_0\frac{mv_\infty^2}{Z_1Z_2e^2}b \tag{1.1}$$

Now we begin to find the relation between $\mathrm{d}b$ and $\mathrm{d}\Omega$



$$\mathrm{d}\sigma = 2\pi b\,\mathrm{d}b = \pi\left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{Z_1Z_2e^2}{mv_\infty^2}\right)^2\frac{\cos\dfrac{\theta}{2}}{\sin^3\dfrac{\theta}{2}}\,\mathrm{d}\theta$$

$$\Omega = 2\pi\,(1-\cos\theta)$$
$$\mathrm{d}\Omega = 2\pi\sin\theta\,\mathrm{d}\theta = 4\pi\sin\frac{\theta}{2}\cos\frac{\theta}{2}\,\mathrm{d}\theta$$

Then we found $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$ is only related with θ

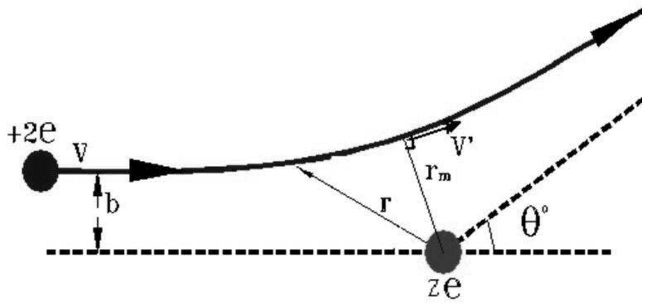
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{Z_1Z_2e^2}{2mv_\infty^2}\right)^2\sin^{-4}\frac{\theta}{2} \tag{1.2}$$

As for a thin gold leaf, we assume there's only one layer of atoms, the density of atoms is N, the area and thickness of gold leaf are A and t

When n particles passed through the gold leaf, $\mathrm{d}n$ of them ended up in $\mathrm{d}\Omega$

$$\frac{\mathrm{d}n}{n} = \frac{NAt\,\mathrm{d}\sigma}{A} = Nt\,\mathrm{d}\sigma = Nt\left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{Z_1Z_2e^2}{2mv_\infty^2}\right)^2\sin^{-4}\frac{\theta}{2}\,\mathrm{d}\Omega$$
$$\frac{\mathrm{d}n}{\mathrm{d}\Omega}\sin^4\frac{\theta}{2} = nNt\left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{Z_1Z_2e^2}{2mv_\infty^2}\right)^2 = \mathrm{const} \tag{1.3}$$

For alpha particles, $Z_1 = 2$. We can also take the closest distance between the 2 particles as the radius of a particle.



What we have already known are:

$$\frac{1}{2}MV^2 = \frac{1}{2}MV'^2 + \frac{2Ze^2}{4\pi\varepsilon_0 r_m}$$
$$MVb = MV'r_m$$
$$b = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{MV}\cdot\cot\frac{\theta}{2}$$

We solve r_m with the equation we known.

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{M}\cdot\cot\frac{\theta}{2} = V'r_m$$

$$\frac{1}{2}MV^2 = \frac{1}{2}M\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{Mr_m}\cdot\cot\frac{\theta}{2}\right)^2 + \frac{2Ze^2}{4\pi\varepsilon_0 r_m}$$

$$\frac{1}{2}MV^2r_m^2 - \frac{2Ze^2}{4\pi\varepsilon_0}r_m - \frac{1}{2}M\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{M}\cdot\cot\frac{\theta}{2}\right)^2 = 0$$

Finally

$$r_m = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{MV^2}\left(1 + \frac{1}{\sin\dfrac{\theta}{2}}\right) \tag{1.4}$$

Chapter 2

The Energy and Radiation of Atoms

2.1 Rydberg Constant and Wavelength of Radiation

Wave number of Hydrogen Atoms:

$$\tilde{\nu} = \frac{1}{\lambda} = Z^2 R \left[\frac{1}{m^2} - \frac{1}{n^2}\right] \tag{2.1}$$

Where $Z = 1$

Lyman Series	$\tilde{\nu} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$	$n = 2, 3, 4, \ldots$
Balmer Series	$\tilde{\nu} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$	$n = 3, 4, 5, \ldots$
Paschen Series	$\tilde{\nu} = R\left[\frac{1}{3^2} - \frac{1}{n^2}\right]$	$n = 4, 5, 6, \ldots$
Brackett Series	$\tilde{\nu} = R\left[\frac{1}{4^2} - \frac{1}{n^2}\right]$	$n = 5, 6, 7, \ldots$
Pfund Series	$\tilde{\nu} = R\left[\frac{1}{5^2} - \frac{1}{n^2}\right]$	$n = 6, 7, 8, \ldots$

Spectroscopic term

$$T(m) = \frac{Z^2 R}{m^2} \quad T(n) = \frac{Z^2 R}{n^2}$$

Then

$$\tilde{\nu} = T\,(m) - T\,(n) \tag{2.2}$$

Energy of emitted light

$$E = \frac{hc}{\lambda} = hc\tilde{v} = hc\,[T(m) - T(n)] = \frac{Rhc}{m^2} - \frac{Rhc}{n^2}$$

2.2 Bohr's Theory of Hydrogen Atoms

Energy of steady states:

$$E_n = -\frac{1}{2}\frac{Ze^2}{4\pi\varepsilon_0 r_n}$$

Transition

$$h\nu = E_n - E_m$$

Angular Momentum

$$L = n \cdot \frac{h}{2\pi} = n\hbar = m_e v_n r_n$$
$$m_e \frac{v_n^2}{r_n} = \frac{Ze^2}{4\pi\varepsilon_0 r_n^2}$$

So that

$$\frac{n^2\hbar^2}{m_e r_n^3} = \frac{Ze^2}{4\pi\varepsilon_0 r_n^2}$$

From which we can indicate that the radius is quantized.

$$r_n = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} \cdot \frac{n^2}{Z} = a_0 \cdot \frac{n^2}{Z} \qquad \left(a_0 = \frac{4\pi\varepsilon_0\hbar}{m_e e^2}\right) \tag{2.3}$$

For hydrogen atoms, $Z = 1$. The energy is also quantized.

$$E_n = -\frac{1}{2}\frac{Ze^2}{4\pi\varepsilon_0}\frac{m_e e^2}{4\pi\varepsilon_0\hbar^2} \cdot \frac{Z}{n^2} = -\frac{m_e e^4}{2\left(4\pi\varepsilon_0\hbar\right)^2} \cdot \frac{Z^2}{n^2}$$

The velocity is also qumntumized

$$v_n = \frac{n\hbar}{m_e r_n} = \frac{n\hbar}{m_e}\frac{m_e e^2}{4\pi\varepsilon_0\hbar^2} \cdot \frac{Z}{n^2} = \frac{e^2}{4\pi\varepsilon_0\hbar} \cdot \frac{Z}{n} = \frac{Z\alpha c}{n}$$
$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{1}{137}$$

Calculate the value of Rydberg Constant

$$E = -\frac{m_e e^4}{2\left(4\pi\varepsilon_0\hbar\right)^2} \cdot \frac{Z^2}{n^2} = -\frac{2\pi^2 m_e e^4}{\left(4\pi\varepsilon_0 h\right)^2} \cdot \frac{Z^2}{n^2}$$
$$\frac{hc}{\lambda} = E_2 - E_1 = \frac{2\pi^2 m_e e^4 Z^2}{\left(4\pi\varepsilon_0 h\right)^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$
$$\tilde{\nu} = \frac{1}{\lambda} = \frac{2\pi^2 m_e e^4 Z^2}{\left(4\pi\varepsilon_0\right)^2 h^3 c}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$\Rightarrow$

$$R = \frac{2\pi^2 m_e e^4}{\left(4\pi\varepsilon_0\right)^2 h^3 c}$$

2.3 Rydburg Constant of Different Atoms

In previous sections, we assumed that the nucleus is fixed at a point, with the electron surrounded. But in fact, the nucleus's mass is not infinity, and it moves as well. So that for different atoms, the Rydburg Constants varies. In a two-body system, we define the Reduced Mass of a system as

$$\mu = \frac{Mm}{M+m}$$

And we replace the Reduced Mass with m_e

$$\begin{aligned} R_\infty &= \frac{2\pi^2 me^4}{(4\pi\varepsilon_0)^2 h^3 c} \\ R_A &= R_\infty \cdot \left[\frac{1}{1+\frac{m}{M}}\right] \end{aligned} \tag{2.4}$$

2.4 Sommerfeld's Quantize Condition

For any coordinate q and its momentum p

$$\oint p \, \mathrm{d}q = nh$$

holds

2.5 Quantized Magneton

$$\begin{aligned} \mu &= iA \\ i &= \frac{e}{\tau} \\ A &= \int_0^{2\pi} \frac{1}{2} r \cdot r \, \mathrm{d}\phi \\ &= \frac{1}{2}\int_0^{\tau} r^2 \omega \, \mathrm{d}t \\ &= \frac{1}{2m}\int_o^{\tau} mr^2 \omega \, \mathrm{d}t \\ &= \frac{p_\phi}{2m}\tau \end{aligned}$$

Consider that

$$\int_0^{2\pi} p_\phi \, \mathrm{d}\phi = 2\pi p_\phi = nh$$

We have

$$\mu = \frac{e}{2m} p_\phi = \frac{eh}{4\pi m} \cdot n$$

Let

$$\mu_B = \frac{eh}{4\pi m} \tag{2.5}$$

Then μ_B is the minimum unit of magneton.

Chapter 3

Basics of Quantum Mechanics

3.1 De Broglie Wave

$$\lambda = \frac{h}{p}$$

3.2 Uncertianty Principle

$$\Delta p \Delta q \geq \frac{\hbar}{2}$$

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

3.3 Wave Function

$$\psi = \psi_0 \exp\left[\frac{i}{\hbar}\left(\vec{p}\cdot\vec{r} - Et\right)\right]$$

$$\frac{\partial \psi}{\partial x} = -\frac{i}{\hbar} p_x \psi$$
$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar} E \psi$$

Three properties

- limited
- Normalized
- Continuous

3.4 Schrodinger Equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial \psi}{\partial t}$$