Solution for Test 1

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1 Problem 1

$$\Psi\left(z,t\right)=A\exp\left[-\left(a^2z^2+b^2t^2+2abzt\right)\right]=A\exp\left[-\left(az+bt\right)^2\right]$$

波函数

$$\frac{\partial^2\Psi}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2\Psi}{\partial t^2}$$

求导

$$\begin{aligned}
\frac{\partial\Psi}{\partial z}&=A\exp\left[-\left(az+bt\right)^2\right]\cdot\left(-2\left(az+bt\right)\right)\cdot 2a=-4A\exp\left[-\left(az+bt\right)^2\right]\cdot\left(a^2z+abt\right)\\
\frac{\partial^2\Psi}{\partial z^2}&=-4A\left\{\exp\left[-\left(az+bt\right)^2\right]\cdot a^2-4\left(a^2z+abt\right)^2\exp\left[-\left(az+bt\right)\right]\right\}\\
\frac{\partial\Psi}{\partial t}&=A\exp\left[-\left(az+bt\right)^2\right]\cdot\left(-2\left(az+bt\right)\right)\cdot 2b=-4A\exp\left[-\left(az+bt\right)^2\right]\cdot\left(abz+b^2t\right)\\
\frac{\partial^2\Psi}{\partial t^2}&=-4A\left\{\exp\left[-\left(az+bt\right)^2\right]\cdot b^2-4\left(abz+b^2t\right)^2\exp\left[-\left(az+bt\right)\right]\right\}
\end{aligned}$$

因此

$$\begin{cases}\dfrac{\partial^2\Psi}{\partial x^2}=\dfrac{1}{v^2}\dfrac{\partial^2\Psi}{\partial t^2}\\ v=\dfrac{b}{a}\end{cases}$$

波沿 z 轴正方向传播

2 Problem 2

$$\vec{E}=\left(-6\hat{\imath}+3\sqrt{5}\hat{\jmath}\right)\left(10^4V/m\right)\exp\left\{i\left[\frac{1}{3}\left(\sqrt{5}x+2y\right)\pi\times10^7-9.42\times10^{15}t\right]\right\}$$

显然波是在 xy 平面传播的，将上面的方程化成如下形式

$$\vec{E}(\vec{r},t)=\vec{A}\exp\left[i\left(\vec{k}\cdot\vec{r}-\omega t\right)\right]$$

$$\begin{cases}\vec{A}=\left(-6,3\sqrt{5},0\right)\left(10^{4}V/m\right)\\ \vec{k}=\dfrac{\pi}{3}\times10^{7}\times\left(\sqrt{5},2,0\right)\\ \omega=9.42\times10^{15}\end{cases}$$

上述各量的大小为

$$\begin{cases}A=9\times10^{4}\ \mathrm{V/m}\\ k=\pi\times10^{7}\ \mathrm{m}^{-1}\\ \omega=9.42\times10^{15}\ \mathrm{s}^{-1}\end{cases}$$

由此求得

$$\begin{cases}\lambda=\dfrac{2\pi}{k}=2\times10^{-7}\ \mathrm{m}\\ \kappa=\dfrac{1}{\lambda}=5\times10^{6}\ \mathrm{m}^{-1}\\ f=\dfrac{\omega}{2\pi}=1.50\times10^{15}\ \mathrm{s}^{-1}\\ \dfrac{\vec{A}}{|A|}=\left(-\dfrac{2}{3},\dfrac{\sqrt{5}}{3},0\right)\\ \dfrac{\vec{k}}{|k|}=\left(\dfrac{\sqrt{5}}{3},\dfrac{2}{3},0\right)\end{cases}$$

3 Problem 3

$$\vec{E}=E_0\hat{\jmath}\cos\left(\frac{\pi z}{z_0}\right)\cos\left(kx-\omega t\right)$$

从电场的表示形式来看，应该是沿着 x 方向传播的驻波，驻波的振幅随著 z 的变化而变化。积化和差得到

$$\vec{E}=\frac{E_0}{2}\hat{\jmath}\left[\cos\left(\frac{\pi z}{z_0}+kx-\omega t\right)+\cos\left(\frac{\pi z}{z_0}-kx+\omega t\right)\right]$$

说明这个光波是由两个沿 x 方向，传播方向相反的光波叠加形成的。

已知 k，相速度为

$$v_p = \frac{\omega}{k}$$

4 Problem 4

$$m_e\ddot{x} + m_e\gamma\dot{x} + m_e\omega_0^2 x = q_e E(t)$$

等式左边第一项是电子受到的总共的力，第二项是受到的阻力，第三项是线性的回复力。等式右边是电场给电子的力。将

$$E = E_0 \exp(i\omega t)$$
$$x = x_0 \exp[i(\omega t - \alpha)]$$

代入

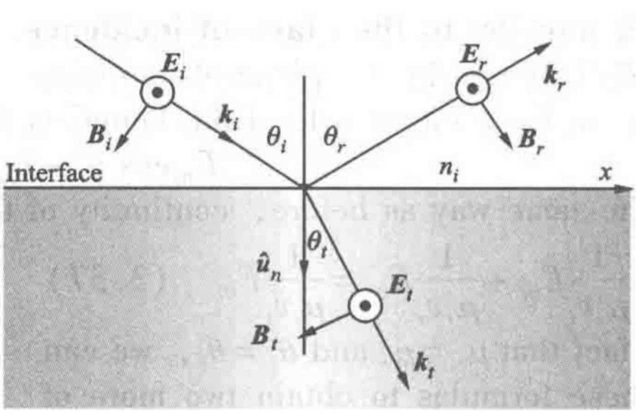
$$-\omega^2 x + i\omega\gamma x + \omega_0^2 x = \frac{q_e E_0}{m_e} \exp(i\omega t)$$

$$\left[\left(\omega_0^2 - \omega^2\right) + i\omega\gamma\right] x_0 \exp[i(\omega t - \alpha)] = \frac{q_e E_0}{m_e} \exp(i\omega t)$$

$$\left[\left(\omega_0^2 - \omega^2\right)^2 + \omega^2\gamma^2\right] x_0 \exp[i(\omega t - \alpha)] = \frac{q_e E_0}{m_e} \exp(i\omega t) \left[\left(\omega_0^2 - \omega^2\right) - i\omega\gamma\right]$$

$$x_0 = \frac{q_e E_0}{m_e} \frac{1}{\left[\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\omega^2\right]} \exp(i\alpha) \left[\left(\omega_0^2 - \omega^2\right) - i\omega\gamma\right]$$

5 Problem 5



$$
\begin{cases}
E_i + E_r = E_t \\
B_i \cos\theta_i = B_r \cos\theta_r + B_t \cos\theta_t
\end{cases}
$$

$$
\begin{cases}
E = vB \\
v = \dfrac{c}{n}
\end{cases}
$$

$$
\begin{cases}
E_i + E_r = E_t \\
\dfrac{E_i}{v_i} \cos\theta_i = \dfrac{E_r}{v_r} \cos\theta_r + \dfrac{E_t}{v_t} \cos\theta_t
\end{cases}
$$

$$
\begin{cases}
E_i + E_r = E_t \\
n_i E_i \cos\theta_i = n_r E_r \cos\theta_r + n_t E_t \cos\theta_t
\end{cases}
$$

定义

$$
r = \frac{E_r}{E_i}
$$
$$
t = \frac{E_t}{E_i}
$$

则有

$$
\begin{cases}
1 + r = t \\
n_i \cos\theta_i = n_r r \cos\theta_r + n_t t \cos\theta_t
\end{cases}
$$

解得菲涅耳公式

$$
\begin{cases}
r_\perp = \dfrac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \\
t_\perp = \dfrac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}
\end{cases}
$$

又因为 $n_i \sin\theta_i = n_t \sin\theta_t$

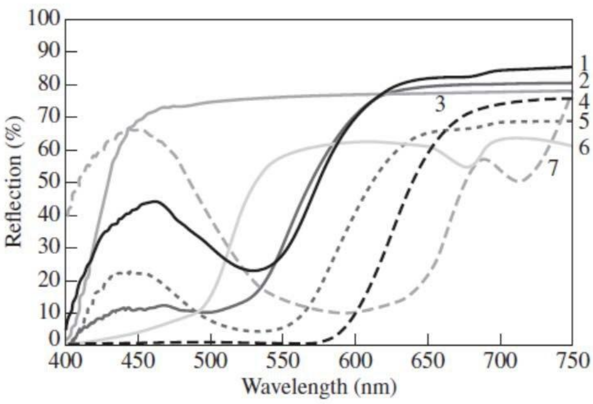
$$
\begin{cases}
r_\perp = \dfrac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} = \dfrac{\cos\theta_i - \dfrac{n_t}{n_i}\cos\theta_t}{\cos\theta_i + \dfrac{n_t}{n_i}\cos\theta_t} = \dfrac{\cos\theta_i - \dfrac{\sin\theta_i}{\sin\theta_t}\cos\theta_t}{\cos\theta_i + \dfrac{\sin\theta_i}{\sin\theta_t}\cos\theta_t} = \dfrac{\sin\theta_t \cos\theta_i - \sin\theta_i \cos\theta_t}{\sin\theta_t \cos\theta_i + \sin\theta_i \cos\theta_t} \\
t_\perp = \dfrac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} = \dfrac{2\cos\theta_i}{\cos\theta_i + \dfrac{n_i}{n_t}\cos\theta_t} = \dfrac{2\cos\theta_i}{\cos\theta_i + \dfrac{\sin\theta_i}{\sin\theta_t}\cos\theta_t} = \dfrac{2\sin\theta_t \cos\theta_i}{\sin\theta_t \cos\theta_i + \sin\theta_i \cos\theta_t}
\end{cases}
$$

即

$$
\begin{cases}
r_\perp = \dfrac{\sin(\theta_t - \theta_i)}{\sin(\theta_i + \theta_t)} \\
t_\perp = \dfrac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i + \theta_t)}
\end{cases}
$$

$$
t_\perp - r_\perp = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i + \theta_t)} - \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_i + \theta_t)} = \frac{2\sin\theta_t\cos\theta_i - \sin\theta_t\cos\theta_i + \sin\theta_i\cos\theta_t}{\sin\theta_i\cos\theta_t + \sin\theta_t\cos\theta_i} = 1
$$

6 Problem 6



Vacuum Wavelength Ranges for the Various Colors

Color	λ_0 (nm)	ν (THz)*
Red	780–622	384–482
Orange	622–597	482–503
Yellow	597–577	503–520
Green	577–492	520–610
Blue	492–455	610–659
Violet	455–390	659–769

*1 terahertz (THz) = 10^{12} Hz, 1 nanometer (nm) = 10^{-9} m.

白色的是 3，反射光里面各种波长都有。

黄色的是 6，粉色由红色和蓝色混合而成，应该是 1，蓝色是 7，橘红色是 5，红色是 4。