概率论与数理统计

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1　随机事件与概率

1.1　条件概率

有事件 A、B，$P(B) \neq 0$，则 A 在 B 发生条件下的概率

$$P(A|B) = \frac{P(AB)}{P(B)}$$

事件 B 是发生的前提条件，也有

$$P(AB) = P(A|B)P(B) = P(B|A)P(A)$$

全概率公式 (p9)

$$P(B) = \sum_{i=1}^{n} P(B|A_i)P(A_i) \tag{1}$$

贝叶斯公式

$$P(A_i|B)P(B) = P(B|A_i)P(A_i)$$

将公式 1代入

$$P(A_i|B)\sum_{i=1}^{n} P(B|A_i)P(A_i) = P(B|A_i)P(A_i)$$

最终得到

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum\limits_{i=1}^{n} P(B|A_i)P(A_i)}$$

1.2　随机变量的分布

1.2.1　离散型

X	x_1	x_2	$\cdots$	x_i	$\cdots$
P	p_1	p_2	$\cdots$	p_i	$\cdots$

一维分布列　其中 $\sum\limits_i p_i = 1$，分布函数 $F(x) = P(X \leq x)$

X \ Y	y_1	y_2	y_3	
x_1	p_{11}	p_{12}	p_{13}	$p_{1\cdot}$
x_2	p_{21}	p_{22}	p_{23}	$p_{2\cdot}$
	$p_{\cdot 1}$	$p_{\cdot 2}$	$p_{\cdot 3}$	

二维分布列　其中 $\sum\limits_{i,j} p_{ij} = 1$

二项分布 $X \sim B(n,p)$　共实验 n 次, 成功的概率是 p，分布列为

$$P(X=k) = C_n^k p^k \left(1-p\right)^{n-k}$$

分布列先增后减, 中心项 $P(X = [(n+1)\,p])$ 最大

泊松分布 $X \sim P(\lambda)$　当二项分布的 n 很大，p 很小时，可近似为泊松分布

$$P(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

先增后减，中心项为 $P(X = [\lambda])$

超几何分布（略）p19

几何分布　分布列为

$$P(X=k) = pq^{k-1}$$

计算分布函数要用到等比数列的求和公式

1.2.2 连续型随机变量

一维随机变量概率密度与分布函数

$$F(x) = \int_{-\infty}^{x} f(t)\mathrm{d}t = P(X<x)$$

$$F(+\infty) = \int_{-\infty}^{+\infty} f(x)\mathrm{d}t = 1$$

二维随机变量概率密度与分布函数

$$F(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f(u,v)\mathrm{d}u\mathrm{d}v$$

$$F(+\infty,+\infty) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(u,v)\mathrm{d}u\mathrm{d}v = 1$$

边缘概率密度

$$f_X(x) = \int_{-\infty}^{+\infty} f\left(x,y\right)\mathrm{d}y$$

$$f_Y(y)=\int_{-\infty}^{+\infty} f(x,y)\,\mathrm{d}x$$

二维随机变量求分布函数需要画图分区域讨论，详见：p39 例 3.4

均匀分布 $X \sim U(a,b)$

$$f(x)=\begin{cases} \dfrac{1}{b-a} & ,a \le x \le b \\ 0 & ,\text{其他} \end{cases}$$

$$F(x)=\begin{cases} 0 & ,x<a \\ \dfrac{x-a}{b-a} & ,a \le x < b \\ 1 & ,x \ge b \end{cases}$$

指数分布 $X \sim E(\lambda)$

$$f(x)=\begin{cases} \lambda e^{-\lambda x} & ,x \ge 0 \\ 0 & ,x<0 \end{cases}$$

$$F(x)=\begin{cases} 0 & ,x<0 \\ 1-e^{-\lambda x} & ,x \ge 0 \end{cases}$$

正态分布 $N \sim (\mu,\sigma^2)$ **一维情形**

$$f(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\dfrac{(x-\mu)^2}{2\sigma^2}}$$

$$F(x)=\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{x} e^{-\dfrac{(x-\mu)^2}{2\sigma^2}}\mathrm{d}x=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\dfrac{\left(\dfrac{x-\mu}{\sigma}\right)^2}{2}}\mathrm{d}\left(\frac{x-\mu}{\sigma}\right) \tag{2}$$

定义标准正态分布 $N \sim (0,1)$ 的分布函数为

$$\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\dfrac{t^2}{2}}\mathrm{d}t$$

则方程 2可化为

$$F(x)=\Phi\left(\frac{x-\mu}{\sigma}\right)=P(X<x)$$

二维情形

相关系数为零，则

$$f(x)=\frac{1}{\sqrt{2\pi}\sigma_1}e^{-\dfrac{(x-\mu_1)^2}{2\sigma_1^2}}\frac{1}{\sqrt{2\pi}\sigma_2}e^{-\dfrac{(x-\mu_2)^2}{2\sigma_2^2}}$$

详见 p41

1.2.3 随机变量函数的分布 p28

对于离散型随机变量，根据分布列求概率就可以。对于连续型，先根据概率求分布函数，再对分布函数求导，得到概率密度。

条件分布　根据公式 1.1，相似的，定义

$$f(y|x) = \frac{f(x,y)}{f_X(x)}$$

为 $X = x$ 时 Y 的条件密度

1.2.4 随机变量的独立性

离散型

$$P(X_1 = x_1, X_2 = x_2) = P(X_1 = x_1)P(X_2 = x_2)$$

连续型

$$f(x,y) = f_X(x)f_Y(y)$$

2 数字特征

2.1 数学期望与方差

2.1.1 离散型

$$EX = \sum_{i=1}^{\infty} x_i p_i$$

$$DX = E(X - EX)^2 = EX^2 - E(X^2)$$

2.1.2 连续型

$$EX = \int_{-\infty}^{+\infty} x f(x) \mathrm{d}x$$

$$DX = \int_{-\infty}^{+\infty} (x - EX)^2 f(x) \mathrm{d}x$$

2.2 不同分布的数学期望与方差

2.2.1 离散型

X	EX	DX
$B(n,p)$	np	$np(1-p)$
$P(\lambda)$	λ	λ

2.2.2 连续型

X	EX	DX
$N(\mu,\sigma^2)$	μ	σ^2
$E(\lambda)$	$\dfrac{1}{\lambda}$	$\dfrac{1}{\lambda^2}$
$U(a,b)$	$\dfrac{b-a}{2}$	$\dfrac{(b-a)^2}{12}$

2.3 切比雪夫不等式

$$P\left(|X-EX|\geq\epsilon\right)\leq\frac{DX}{\epsilon^2}$$

2.4 协方差

$$Cov(X,Y)=E\left[\left(X-EX\right)\left(Y-EY\right)\right]=E\left(XY\right)-EXEY$$

协方差的性质有

$$Cov(aX,bY)=abCov(X,Y)$$

$$Cov(X_1+X_2,Y)=Cov(X_1,Y)+Cov(X_2,Y)$$

特别地

$$DX=Cov(X,X)$$

2.5 相关系数

$$\rho_{xy}=\frac{Cov(X,Y)}{\sqrt{DX}\sqrt{DY}}$$

2.6 矩

k 阶原点矩　$\nu_k=EX^k$

k 阶原点绝对矩　$\alpha_k=E|X|^k$

k 阶中心矩　$\mu_k=E(X-EX)^k$

k 阶原点绝对矩　$\beta_k=E|X-EX|^k$

3 统计量的分布

3.1 统计量

均值 $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

方差 $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$

k **阶原点矩** $A_k = \frac{1}{n}\sum_{i=1}^{n} X_i^k$

k **阶中心距** $B_k = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$

3.2 分布

χ^2 **分布** $X_n \sim N\left(0,1\right)$，$X_1 + \cdots + X_n$ 满足自由度为 n 的 χ^2 分布，该分布具有可加性。

t **分布** $X_n \sim N\left(0,1\right)$，$Y \sim \chi^2(n)$，则 $T = \frac{X}{\sqrt{Y/n}} \sim t(n)$

F **分布** $X \sim \chi^2(n_1)$，$Y \sim \chi^2(n_2)$，则 $F = \frac{X/n_1}{Y/n_2} \sim F\left(n_1, n_2\right)$

3.2.1 正态总体样本均值与方差满足的分布

均值 $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

推论 1 $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

推论 2 $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim t(n-1)$

方差 $\frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}{\sigma^2} = (n-1)\frac{S^2}{\sigma^2} \sim \chi^2(n-1)$

4 参数估计

4.1 矩估计

- 求 EX 与 EX^2
- $A_1 = \frac{1}{n}\sum_{i=1}^{n} X_i = \bar{X}$
- $A_2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2$

- $A_2 - A_1^2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2 - \bar{X}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2 = \tilde{S}^2$
- 用上面的式子解出分布中的参数，消去 A_1 和 A_2

例子详见 p101

4.2 极大似然估计

- 根据概率密度用连乘的形式写出似然方程
- 对似然方程取对数，求导判断参数取什么值的时候，似然函数最大
- 如果参数取多个值的时候似然函数都能最大，则取最大值作为极大似然估计

详见 p103

4.3 无偏估计

$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$ 是 σ^2 的无偏估计，$\tilde{S}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$ 是 σ^2 的有偏估计

4.4 区间估计（置信水平 $1-\alpha$）

4.4.1 均值 μ 的区间估计

已知 σ^2，由章节 3.2.1，推论 1： $\left(\bar{X} - \frac{\sigma}{\sqrt{n}}u_{\alpha/2}, \bar{X} + \frac{\sigma}{\sqrt{n}}u_{\alpha/2}\right)$

未知 σ^2，由章节 3.2.1，推论 2： $\left(\bar{X} - \frac{S}{\sqrt{n}}u_{\alpha/2}, \bar{X} + \frac{S}{\sqrt{n}}u_{\alpha/2}\right)$

4.4.2 方差 σ 的区间估计

未知 μ，由章节 3.2.1，方差满足的分布： $\left(\frac{\sqrt{n-1}S}{\sqrt{\chi^2_{\alpha/2}(n-1)}}, \frac{\sqrt{n-1}S}{\sqrt{\chi^2_{1-\alpha/2}(n-1)}}\right)$

4.4.3 单侧置信区间

单侧置信下限 $\underline{\theta}$ $\quad P(\theta > \underline{\theta}) = 1-\alpha$

单侧置信上限 $\overline{\theta}$ $\quad P(\theta < \overline{\theta}) = 1-\alpha$

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