Composite pulses for robust control of spin qubits

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References

- **Single-qubit gates:**
  "Composite pulses for robust universal control of singlet-triplet qubits"
  XW, L.S. Bishop, J.P. Kestner, E. Barnes, K. Sun, and S. Das Sarma
  Nature Commun. **3**, 997 (2012)
- **Optimized Single-qubit gates and two-qubit gates for exchange-coupled qubits:**
  "Noise-resistant control for a spin qubit array"
  J.P. Kestner, XW, L.S. Bishop, E. Barnes, and S. Das Sarma
  Phys. Rev. Lett. **110**, 140502 (2013)
- ***Two-qubit gates for capacitively coupled qubits:***
  "Improving the gate fidelity of capacitively coupled spin qubits"
  XW, E. Barnes, and S. Das Sarma
  npj Quantum Information **1**, 15003 (2015)

References

- ***Benchmarking:***
  "Noise filtering of composite pulses for singlet-triplet qubits"
  X.-C. Yang and XW
  Sci. Rep. **6**, 28996 (2016)
- **Smooth pulses:**
  "Robust quantum control using smooth pulses and topological winding"
  E. Barnes, XW, and S. Das Sarma
  Sci. Rep. **5**, 12685 (2015)

Outline

- Background
- Noise-compensating single qubit-gates
- Two-qubit gates
- Benchmarking
- Robust control using smooth pulses

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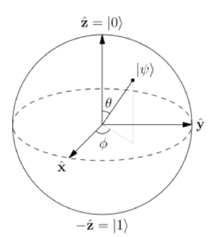
Essential ingredients for computation

- Classical computation:
  - Bits: **0** and **1**
  - Gates: *NOT*: **0** <-> **1**
    AND & *OR*
    } Universal computation
- Quantum computation:
  - **Qu**bits: $|0\rangle$ and $|1\rangle$

    $$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$
  - Quantum gates:
    Arbitrary single-qubit rotation, and one "entangling" two-qubit gate
    → Universal quantum computation



Courtesy of Wikipedia

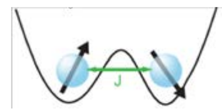
Bloch, Phys. Rev. **70**, 460 (1946)
Nielsen and Chuang, *Quantum Computation and Quantum Information* (Cambridge, 2000)

Spin qubit (a single electron)

- Simplest proposal: a single electron spin (S=1/2) serves as a qubit (Loss—DiVincenzo qubit)

$$|0\rangle = |\uparrow\rangle$$
$$|1\rangle = |\downarrow\rangle$$

- Two-qubit gates are achieved by Heisenberg exchange interaction between the two spins

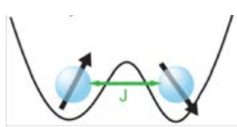


- Single-qubit gates are done using e.g. Electron Spin Resonance (ESR).
- However, experimentally it is difficult to address a single electron without affecting its neighbors.

Loss and DiVincenzo, Phys. Rev. A **57**, 120 (1998)

Singlet-triplet (S-T) qubit

- A qubit can alternatively be encoded in the $S_z = 0$ singlet and triplet states of two electrons, confined in a double quantum dot system.



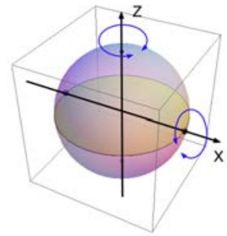
$$|0\rangle = |T_0\rangle = \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)/\sqrt{2}$$
$$|1\rangle = |S\rangle = \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

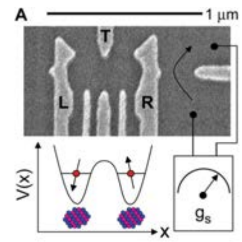
- Other triplet states, $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$, can be split away by a homogeneous magnetic field (B-field).
(minimizing leakage; robust against homogenous fluctuations in the B-field)
- Focus of this talk

Petta *et al.*, *Science* **309**, 2180 (2005)

Controlling a S-T qubit

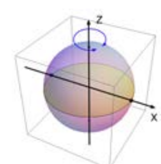
- Full single-qubit control requires ability to rotate a Bloch vector around two independent axes of the Bloch sphere (e.g. x and z axes)
- In the gate-defined quantum dot system, the control is achieved by applying "gate voltages".





Petta *et al.*, *Science* **309**, 2180 (2005)

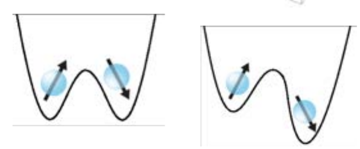
Z-rotation



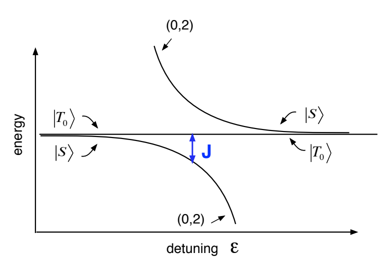
- Gate voltages changes the "detuning", which as a consequence changes the exchange interaction J -- the energy splitting between the singlet and triplet states.
- The control over J suffices for Z-rotation.
- J is constrained:

$$0 \le J \le J_{\max}$$

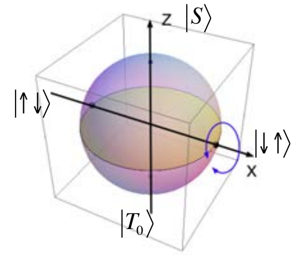
  - One may only rotate along one direction.
  - One cannot do "delta" pulse.



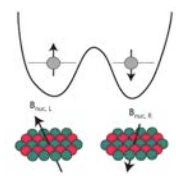
Detuning ε small　　ε large



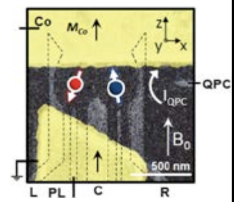
X-rotation



- X-rotation is achieved by an **in**homogeneous Zeeman field
  $h = g\mu_B \Delta B$
  where ΔB is the magnetic field gradient.
- h is generated either by Overhauser nuclear field or a micromagnet.
- h cannot be directly controlled.



Foletti et al, Nat. Phys. 2009



Brunner et al, PRL 2011

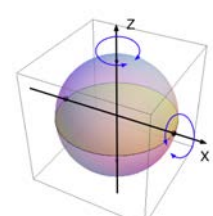
Control Hamiltonian

$$H = \frac{J(t)}{2}\sigma_z + \frac{h}{2}\sigma_x$$

$$0 \le J \le J_{\max}$$

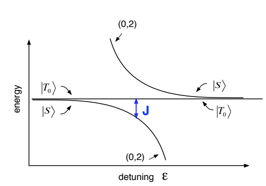
Hamiltonian of an S-T qubit

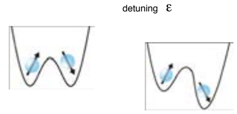
$$H = \frac{J(t)}{2}\sigma_z + \frac{h}{2}\sigma_x$$



z-rotation: exchange interaction J tuned by detuning (gate voltage)

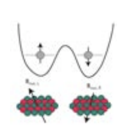
$$0 \le J \le J_{\max}$$



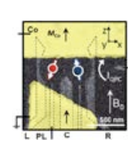


x-rotation: magnetic field gradient (two dots feel different B-field).

cannot vary "dynamically"



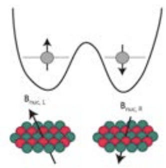
Foletti et al, Nat. Phys. 2009



Brunner et al, PRL 2011

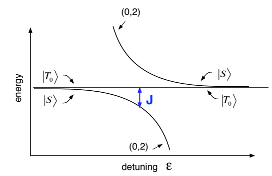
Noises

- **Nuclear (Overhauser) noise**: Fluctuations in the nuclear magnetic field surrounding the quantum dots



$h \to h + \delta h$

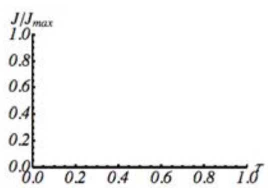
- **Charge noise**: Shift of energy levels induced by charged impurities

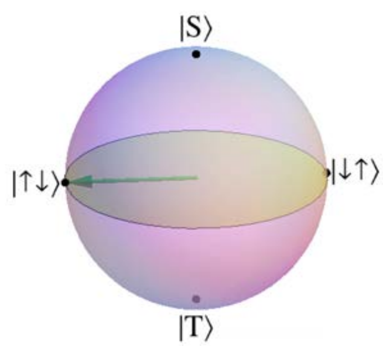


$J(t) \to J(t) + g(J)\delta\varepsilon$

- **Assumption**: Noises are assumed to be "static" for gate control purpose, because they vary on a time scale ($\sim$ ms) much longer than the gate operation time ($\sim$ ns).

Z-rotation affected by noise



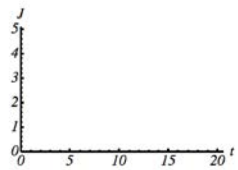


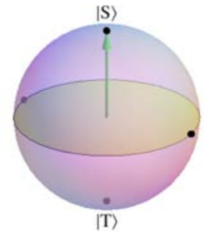
Z-rotation of π (fix $h=0$) affected by a nonzero (but constant) δh

XW *et al.*, *Nature Commun.* **3**, 997 (2012)

Combating noise: dynamical decoupling

"Hahn Spin echo"





- Dynamical decoupling: a powerful technique to preserve the qubit state against noise
- Basic idea: to make the noise compensate itself by applying a π-pulse
- The "static" noise approximation plays a central role.

Question: can we make the noise compensate itself in performing a gate operation (dynamically corrected gate)?

Hahn, Phys. Rev. **80**, 580 (1950)

Why is this problem challenging?

- Existing dynamical decoupling techniques only work for preserving a quantum state ("quantum memory") rather than a quantum gate.
- Dynamically corrected gates developed in NMR literature, such as *CORPSE* and *SCROFULOUS*, requires the control field $J(t)$ to change its sign.
- Naïve search of a pulse sequences involves a high dimensional optimization problem, and is nonlinear.
- The use of optimization algorithms, such as GRAPE, is limited due to constraint $0 \le J \le J_{\max}$.
- **Goal: search in a reduced-dimension space, while respecting the constraint $0 \le J \le J_{\max}$.**

Bando, Ichikawa and Kondo, J. Phys. Soc. Jpn. **82**, 014004 (2013)

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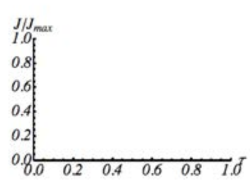
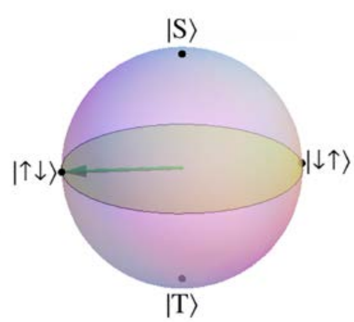
A simple dynamically corrected gate





- We wish to do a π rotation around the z axis
- We rotate a total of 3π around the Bloch sphere, but carefully design the rotating rate and time so that the error cancels itself

XW *et al.*, *Nature Commun.* **3**, 997 (2012)

Why it works

- The corresponding evolution operator is

$$U(T_f,0) = -\exp\left(-i\frac{\pi}{2}\sigma_z\right) + \frac{\delta h^2}{2J_{\max}^2}3\pi I + \mathcal{O}\Big[\Big(\frac{\delta h}{J_{\max}}\Big)^3\Big]$$

- The first order error due to δh vanished
- The minus sign at the front is due to the fact that we are doing 3π rotation instead of a π rotation

- **Idea: Cancel the leading order error due to noise, by a "composite" pulse sequence, with time/strength of each pulse carefully tailored.**

XW *et al.*, *Nature Commun.* **3**, 997 (2012)

Simple case: *h=0*

- We consider a simple case with *h=0*, but having the Overhauser noise δh.
- In this case, the evolution operator can be expanded in powers of δh:

$$U(T,0) = \mathcal{T}\left\{\exp\left[-i\int_0^T dt\left(\frac{\delta h}{2}\sigma_x + \frac{J(t)}{2}\sigma_z\right)\right]\right\}$$

$$U(T,0) = \sum_{n=0}^{\infty}\delta h^n \Pi_n$$

$$\Pi_0 = \cos\left[f(T)\right]I - i\sin\left[f(T)\right]\sigma_z$$

$$n>0:\ \Pi_n = \left(-\frac{i}{2}\right)^n\left(\prod_{m=n}^{1}\int_0^{t'_{m+1}}dt'_m\right)\left\{\cos\left[\frac{f(\tau)}{2} - \sum_{k=1}^{n}(-1)^{k-n}f(t'_k)\right]A_n + \sin\left[\frac{f(\tau)}{2} - \sum_{k=1}^{n}(-1)^{k-n}f(t'_k)\right]B_n\right\}$$

$$f(T) = \int_0^T dt J(t), \qquad A_n \equiv \begin{cases} I & n \text{ even} \\ \sigma_x & n \text{ odd}\end{cases}, \qquad B_n \equiv \begin{cases} -i\sigma_z & n \text{ even} \\ \sigma_y & n \text{ odd}\end{cases}$$

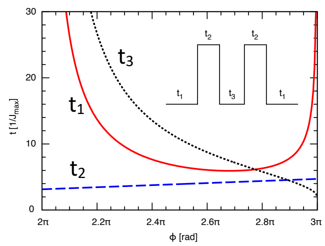
- For the n^{th} order error term to vanish, one must satisfy

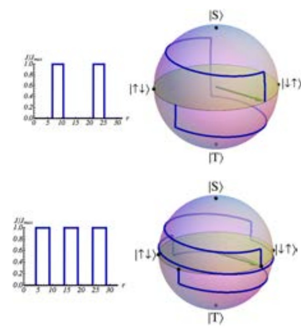
$$\left(\prod_{m=n}^{1}\int_0^{t'_{m+1}}dt'_m\right)\exp\left[i\sum_{k=1}^{n}(-1)^k f(t'_k)\right] = 0$$

XW *et al.*, *Nature Commun.* **3**, 997 (2012)

Results for *h=0*

- We have designed 5-piece, 7-piece, and 9-piece pulses, which cancels error in the evolution operator due to δh to 2nd, 2nd and 3rd order respectively.
- Only "symmetric" pulses, i.e. $J(t) = J(T_f - t)$ are considered. (One can show that in this case the σ_y component always vanishes)
- Our method reduces the dimensionality of the search space, and automatically respects the constraint $0 \le J \le J_{\max}$



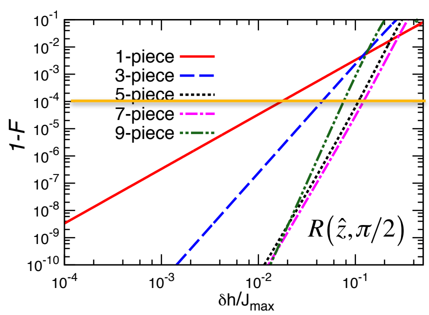


XW *et al.*, *Nature Commun.* **3**, 997 (2012)

Gate Error

- We calculate the average error per gate as a function of δh.
- The error is substantially reduced for small nuclear noise.
- The pulse sequences extend the range of δh that one can tolerate below an error threshold of $\sim 10^{-4}$.
- The nine-piece pulse, although having a higher-order scaling, does not offer much advantage.

$$1 - F = 1 - \overline{|\langle \psi_i | V^\dagger U(T_f) | \psi_i \rangle|^2}$$



XW *et al.*, *Nature Commun.* **3**, 997 (2012)

General single-qubit gates

- For *h=0*, only z-axis rotation may be achieved.
- Since it is very slow to change the value of h during the computation, one loses access to arbitrary single-qubit rotation.
- We therefore need to consider the case with a nonzero h. Once set, its value remains constant for the duration of the computation
- While a single piece of rotation $R(J\hat{z}+h\hat{x},\phi)$ only covers rotation axes in a part of the first quadrant, other arbitrary rotations can be made by combining these rotations
- For example, a z-axis rotation can be done by the following pulse sequence:

$$R(\hat{z},\phi) = -R(\hat{x}+\hat{z},\pi)R(\hat{x},\phi)R(\hat{x}+\hat{z},\pi)$$

Hanson et al., *PRL* **98**, 050502 (2007)
Ramon, *PRB* **84**, 155329 (2011)

"One-piece" rotations

- Expansion of *R(J,ϕ)* to first order in δJ and δh:

$$R(J,\phi) \equiv \exp\left[-i\left(\frac{h+\delta h}{2}\sigma_x + \frac{J+\delta J}{2}\sigma_z\right)\frac{\phi}{\sqrt{h^2+J^2}}\right] = \exp\left[-i\left(\frac{h}{2}\sigma_x + \frac{J}{2}\sigma_z\right)\frac{\phi}{\sqrt{h^2+J^2}}\right]$$
$$\times\left\{I + i\frac{\delta h}{2(h^2+J^2)^{3/2}}\left[(-h^2\phi - J^2\sin\phi)\sigma_x + 2J\sqrt{h^2+J^2}\sin^2\frac{\phi}{2}\sigma_y + hJ(\sin\phi-\phi)\sigma_z\right] + \mathcal{O}(\delta h^2)\right.$$
$$\left. + i\frac{\delta J}{2(h^2+J^2)^{3/2}}\left[hJ(\sin\phi-\phi)\sigma_x + 2h\sqrt{h^2+J^2}\sin^2\frac{\phi}{2}\sigma_y + (-J^2\phi - h^2\sin\phi)\sigma_z\right] + \mathcal{O}(\delta J^2)\right\}$$

- **Goal**: Design an imperfect identity operation (being exact identity in absence of fluctuations)

$$\widetilde{I} = I + (a_1\sigma_x + a_2\sigma_y + a_3\sigma_z)\delta h + (b_1\sigma_x + b_2\sigma_y + b_3\sigma_z)\delta J$$

such that when R and $\widetilde{I}$ are applied back-to-back, the errors (at the first order) exactly cancel.

Kestner *et al.*, *PRL* **110**, 140502 (2013)

Corrected rotation for *R(J,ϕ)*

- One can perform a corrected *R(J,ϕ)* rotation as:

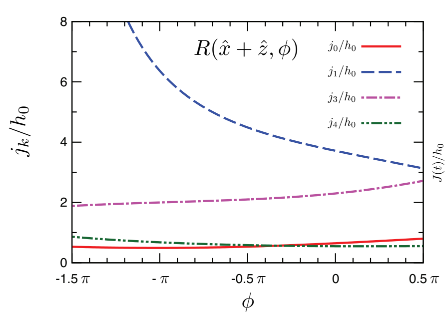
$$R(J,\phi)\times$$
$$\boxed{\begin{aligned}&R(J,\pi-\frac{\phi}{2})R(j_4,\pi)R(j_3,\pi)R(j_2,\pi)R(j_1,\pi)R(j_0,4\pi)\\ &\qquad\times R(j_1,\pi)R(j_2,\pi)R(j_3,\pi)R(j_4,\pi)R(J,\pi+\frac{\phi}{2})\end{aligned}}$$

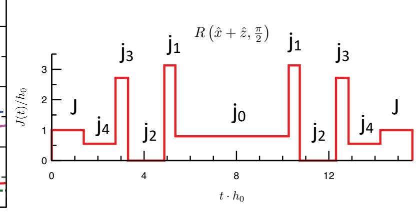
- Here the identity is formed by "interrupted 2π rotations" around certain axis
- Symmetry considerations can reduce the number of unknowns (6 $\rightarrow$4), making the numerical search easier

Kestner *et al.*, *PRL* **110**, 140502 (2013)

Corrected rotation for *R(J,ϕ)*

- Results for *J=h*:

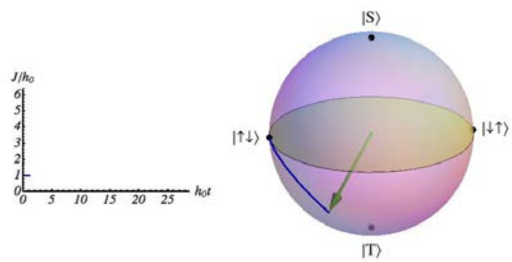




- The pulse sequence cancels both δJ and δh errors.
- Total duration: $\sim$ 14-16π

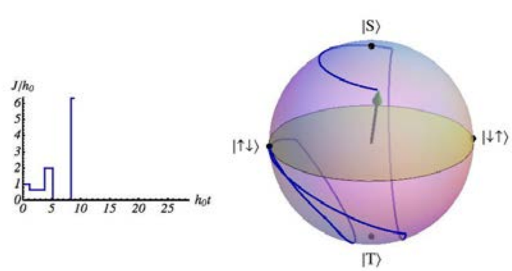
Kestner *et al.*, *PRL* **110**, 140502 (2013)

R(x+z,π) (Hadamard gate)



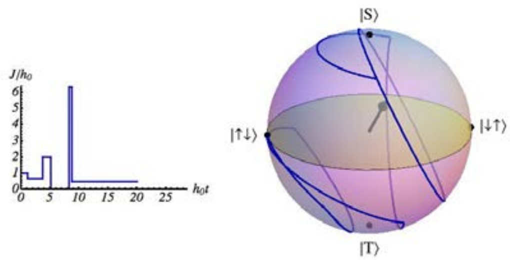
$$\boxed{R(J=h_0,\frac{\pi}{2})}R(j_4,\pi)R(j_3,\pi)R(j_2,\pi)R(j_1,\pi)R(j_0,4\pi)$$
$$\times R(j_1,\pi)R(j_2,\pi)R(j_3,\pi)R(j_4,\pi)R(J=h_0,\frac{\pi}{2})$$

R(x+z,π) (Hadamard gate)



$$\boxed{R(J=h_0,\frac{\pi}{2})R(j_4,\pi)R(j_3,\pi)R(j_2,\pi)R(j_1,\pi)}R(j_0,4\pi)$$
$$\times R(j_1,\pi)R(j_2,\pi)R(j_3,\pi)R(j_4,\pi)R(J=h_0,\frac{\pi}{2})$$

R(x+z,π) (Hadamard gate)



$$R(J=h_0,\frac{\pi}{2})R(j_4,\pi)R(j_3,\pi)R(j_2,\pi)R(j_1,\pi)R(j_0,4\pi)$$
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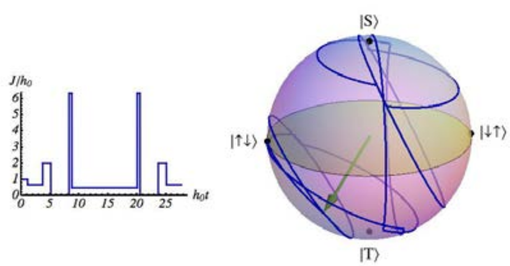
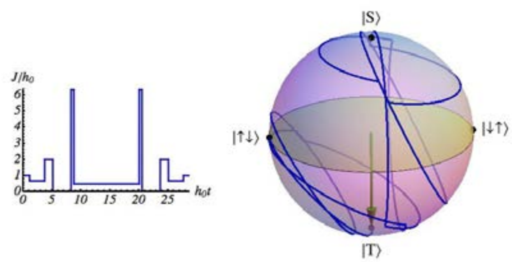
R(x+z,π) (Hadamard gate)



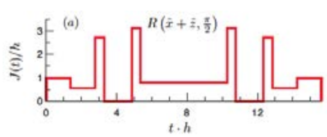
$$R(J=h_0,\frac{\pi}{2})R(j_4,\pi)R(j_3,\pi)R(j_2,\pi)R(j_1,\pi)R(j_0,4\pi)$$
$$\times R(j_1,\pi)R(j_2,\pi)R(j_3,\pi)R(j_4,\pi)R(J=h_0,\frac{\pi}{2})$$

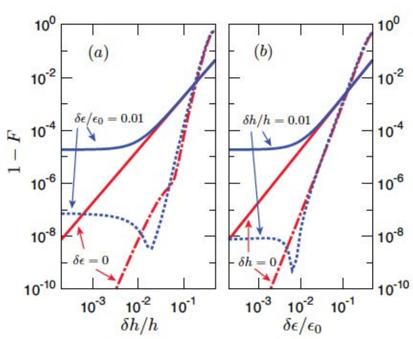
R(x+z,π) (Hadamard gate)



$$R(J = h_0, \frac{\pi}{2})R(j_4, \pi)R(j_3, \pi)R(j_2, \pi)R(j_1, \pi)R(j_0, 4\pi)$$
$$\times R(j_1, \pi)R(j_2, \pi)R(j_3, \pi)R(j_4, \pi)R(J = h_0, \frac{\pi}{2})$$

Performance of a corrected single-qubit gate

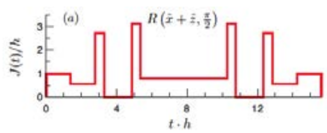


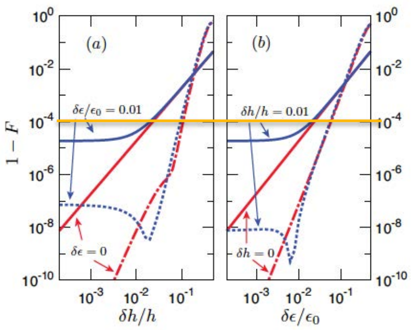


Gate error is substantially reduced for small nuclear noise and charge noise

Kestner *et al.*, *PRL* **110**, 140502 (2013)

Performance of a corrected single-qubit gate





The range of noises that can be tolerated with infidelity under 10^{-4} is extended

Kestner *et al.*, *PRL* **110**, 140502 (2013)

DCGs in lab

PRL 112, 050503 (2014) PHYSICAL REVIEW LETTERS week ending 7 FEBRUARY 2014

Implementation of Dynamically Corrected Gates on a Single Electron Spin in Diamond

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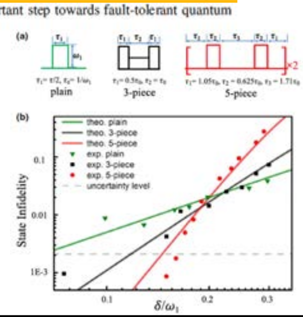
Precise control of an open quantum system is critical to quantum information processing but is challenging due to inevitable interactions between the quantum system and the environment. We demonstrated experimentally a type of dynamically corrected gates using only bounded-strength pulses on the nitrogen-vacancy centers in diamond. The infidelity of quantum gates caused by a nuclear-spin bath is reduced from being the second order to the sixth order of the noise-to-control-field ratio, which offers greater efficiency in reducing infidelity. The quantum gates have been protected to the limit essentially set by the spin-lattice relaxation time T_1. Our work marks an important step towards fault-tolerant quantum computation in realistic systems.

TABLE I. Infidelity of the SUPCODE π gate as a function of δ/ω_1. The words "plain" and "n piece" stand for a normal rectangle pulse and an n-piece SUPCODE pulse, respectively (see Ref. [22]).

Sequence	Infidelity Δ
Plain	$0.5(\delta/\omega_1)^2 + O(\delta/\omega_1)^4$
Three piece	$11.1(\delta/\omega_1)^4 + O(\delta/\omega_1)^6$
Five piece	$64.1(\delta/\omega_1)^6 + O(\delta/\omega_1)^8$
Nine piece	$317\,237(\delta/\omega_1)^8 + O(\delta/\omega_1)^{10}$



Summary for single-qubit gates

- *Arbitrary* single-qubit gates can be made robust against both nuclear noise and charge noise simultaneously, by using composite pulse sequences.
- The Dynamically Corrected Gates (DCGs) reduce the gate error at the cost of extending the sequence ($\sim$14-30π)

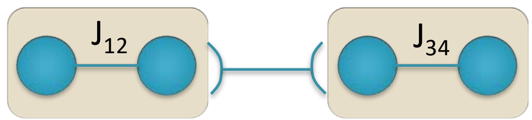
XW *et al.*, *Nature Commun.* **3**, 997 (2012)
Kestner *et al.*, *PRL* **110**, 140502 (2013)

Outline

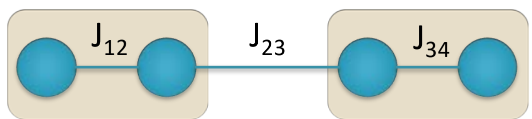
- Background
- Noise-compensating single qubit-gates
- Two-qubit gates
- Benchmarking
- Robust control using smooth pulses

Coupling spin qubits

"Capacitively coupled "

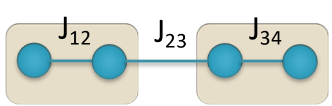


"Exchange-coupled "



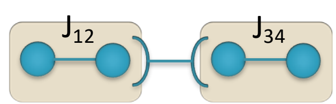
Two ways to couple spin qubits

"Exchange-coupled"

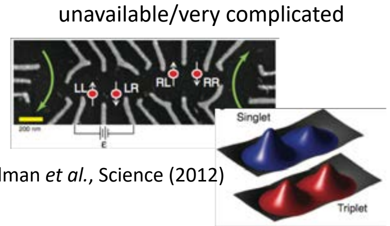


- Theoretically elegant (and simpler) but experimentally less feasible
  - Breaking down the Hilbert space into 2*2 subspaces (treating two spins each time)
  - Analytical results available
  - "Gate cross-talk"

"Capacitively coupled"

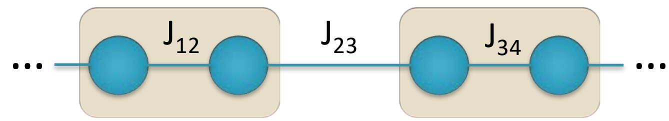


- Experimentally demonstrated but theoretically difficult
  - Full 4*4 Hilbert space
  - Analytical results unavailable/very complicated

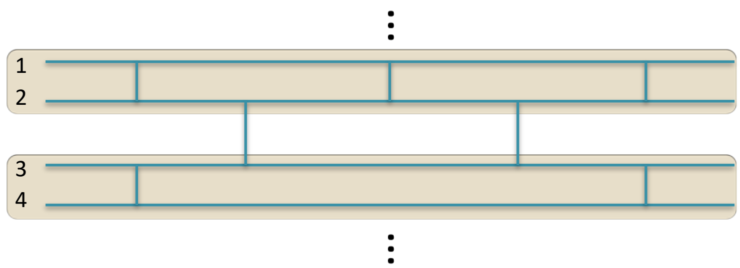


Shulman *et al.*, Science (2012)

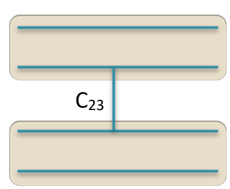
Exchange-coupled S-T qubit array



- Inter-qubit exchange links entangle qubits



"Zeroth" step to a robust gate

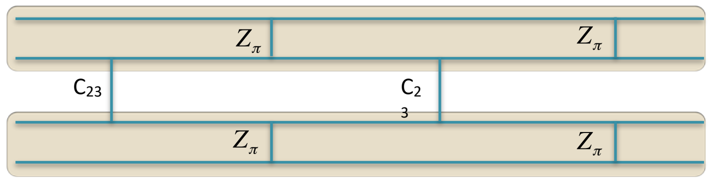


- To achieve any entangling two qubit gate, one must invoke the inter-qubit rotation, C_{23}.
- However, (1) a naïve implementation of C_{23} possibly causes leakage:

$$|\uparrow\downarrow\uparrow\downarrow\rangle \rightarrow |\uparrow\uparrow\downarrow\downarrow\rangle$$

(2) During the operation of C_{23}, the two qubits acquire an undesired phase dependent on the local magnetic field

"Zeroth" step to a robust gate



Klinovaja et al., Phys. Rev. B 86, 085423 (2012)

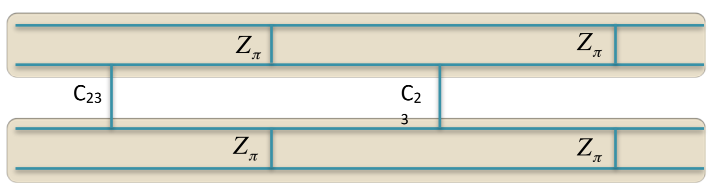
C_{23} needs to be a $2N\pi$ rotation (around whatever axis) in the subspace of the 2nd and 3rd spins to eliminate leakage, i.e.

$$\left|\uparrow_2\downarrow_3\right\rangle \rightarrow e^{i\phi}\left|\uparrow_2\downarrow_3\right\rangle$$

It also acquires a phase proportional to $J_{23}T/2$ (what we want)

During time T, the two-qubit state also acquires a phase dependent on local magnetic field, but this is cancelled by swapping all spins (Z_π) after a C_{23} gate, then repeat once.

Zeroth-order Ising gate



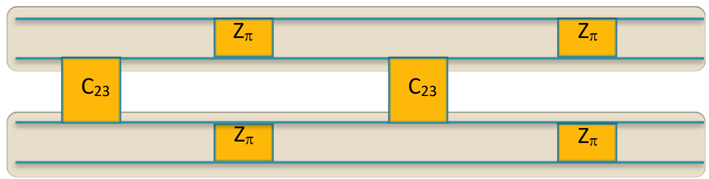
Klinovaja et al., Phys. Rev. B 86, 085423 (2012)

$$U_{xx}(\alpha) \equiv \left(Z_\pi^{12} \otimes Z_\pi^{34}\right) C_{23}\left(\frac{\alpha}{2}\right)\left(Z_\pi^{12} \otimes Z_\pi^{34}\right) C_{23}\left(\frac{\alpha}{2}\right)$$

$$= e^{i\sigma_x \otimes \sigma_x \frac{\alpha}{2}}$$

$$\alpha = 2\int J_{23}(t)\,dt$$

First step to a robust gate



- Replace all gates by our corrected single-qubit gates
- Corrects leakage in C_{23} due to both nuclear noise and charge noise
- Corrects the Z_π gates
- Net effect: a partially corrected Ising gate
- Angle (pulse area) still sensitive to charge noise

Kestner *et al.*, *PRL* **110**, 140502 (2013)

Effect of charge noise

$$U_{xx}(\alpha) = e^{i\sigma_x \otimes \sigma_x \frac{\alpha}{2}}$$

$$\alpha = 2\int J_{23}(t)\,dt$$

If $J_{23} \to J_{23} + dJ$

Then α contains error.
Worse, the error in α will rapidly accumulate as our rotations are $\sim$10 times longer than a naïve rotation

Kestner *et al.*, *PRL* **110**, 140502 (2013)

Second step to a robust gate

- We must correct error in α due to charge noise.
- Intuition is gained from NMR: sequences available to correct over-rotation error.
- "BB1" (Broad-Band):

$$X(\varepsilon, \theta) = \exp\left[-i\sigma_x \frac{(1+\varepsilon)\theta}{2}\right]$$

$$X'(\varepsilon, \theta, \phi) = R(\hat{z}, -\phi) X(\varepsilon, \theta) R(\hat{z}, \phi)$$

$$X'(\varepsilon, \pi, \phi_1) X'(\varepsilon, 2\pi, 3\phi_1) X'(\varepsilon, \pi, \phi_1) X(\varepsilon, \theta)$$
$$= R(\hat{x}, \theta)\left[I - \frac{i}{2}(\theta + 4\pi\cos\phi_1)\varepsilon\right] + \mathcal{O}(\varepsilon^2)$$

1st order noise is cancelled when $\phi_1 = \pm\arccos\left(-\frac{\theta}{4\pi}\right)$

Wimperis, *J. Magn. Reson. B* **109**, 221 (1994)
Jones, *PRA* **67**, 012317 (2003)

Generalizing BB1

$$X'(\varepsilon, \pi, \phi_1) X'(\varepsilon, 2\pi, 3\phi_1) X'(\varepsilon, \pi, \phi_1) X(\varepsilon, \theta)$$
$$= R(\hat{x}, \theta)\left[I - \frac{i}{2}(\theta + 4\pi\cos\phi_1)\varepsilon\right] + \mathcal{O}(\varepsilon^2)$$

$\phi_1 = \pm\arccos\left(-\frac{\theta}{4\pi}\right)$ Good when $-4\pi \leq \theta \leq 4\pi$

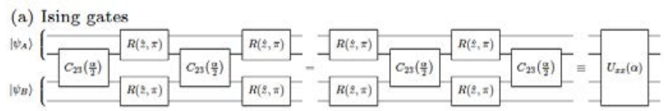
We can generalize it to arbitrarily large angles as

$$X'\left[\varepsilon, (4N \pm 1)\pi, \phi_1\right] X'\left[\varepsilon, (8N \pm 2)\pi, 3\phi_1\right]$$
$$\cdot X'\left[\varepsilon, (4N \pm 1)\pi, \phi_1\right] X(\varepsilon, 4N\pi + \theta)$$
$$= R(\hat{x}, \theta)\left\{I - \frac{i}{2}\left\{\theta + 4\pi\left[N + (4N \pm 1)\cos\phi_1\right]\right\}\varepsilon\right\}$$
$$+ \mathcal{O}(\varepsilon^2),$$

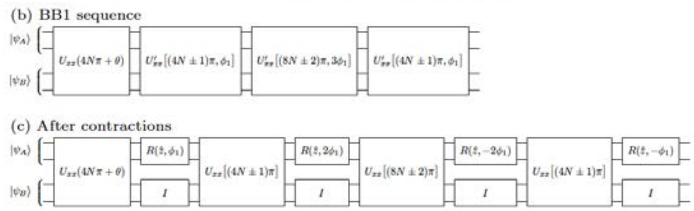
$$\phi_1 = \pm\arccos\left[-\frac{4N\pi + \theta}{(16N \pm 4)\pi}\right]$$

These discussions are for single-qubit rotations. According to Jones *[PRA 67, 012317 (2003)]*, the same sequence would apply to two-qubit gates

Fully corrected gate



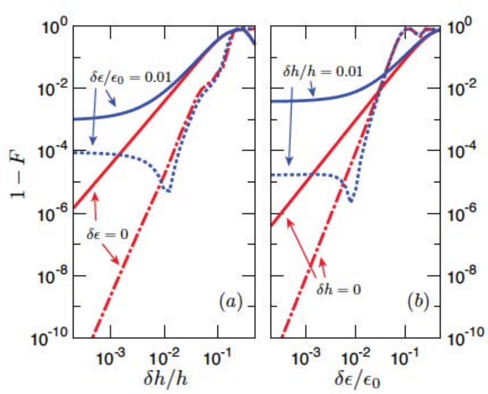
"Tilted" Ising gate: $$U'_{xx}(\alpha,\phi) \equiv R^{(A)}(\hat{z},-\phi)\,U_{xx}(\alpha)\,R^{(A)}(\hat{z},\phi) = [R(\hat{z},-\phi)\otimes I]U_{xx}(\alpha)[R(\hat{z},\phi)\otimes I]$$



- First concrete example of dynamically corrected two-qubit gate in spin qubit literature (locally equivalent to CNOT)
- First order nuclear and charge noise corrected
- 20 composite pulses, of $\sim 360\pi$ rotation

Kestner *et al.*, *PRL* **110**, 140502 (2013)

Performance of the corrected Ising gate



- Substantial error reduction for small fluctuations in the magnetic field or charge noise
- Further optimization possible

Kestner *et al.*, *PRL* **110**, 140502 (2013)

Capacitively coupled S-T qubits

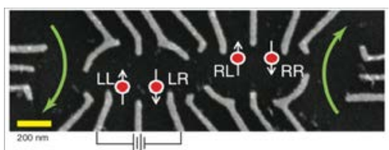
$$H(\{J^A, h^A\}, \{J^B, h^B\}, J^{AB}) =$$
$$\boxed{(J^A\sigma_z + h^A\sigma_x) \otimes I + I \otimes (J^B\sigma_z + h^B\sigma_x)}$$
$$+ \boxed{J^{AB}\sigma_z \otimes \sigma_z}$$

Two-qubit part

Single-qubit part

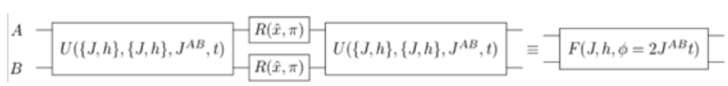
It has the usual constraint for S-T qubit: $0 \leq J^A, J^B \leq J_{\max}$

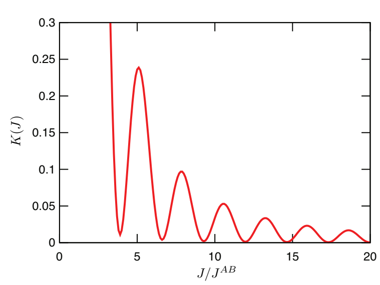
In principle $J^{AB} \leq J^A, J^B$ while in practice $J^{AB} \ll J^A, J^B$

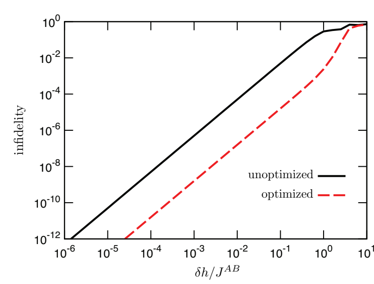


Shulman *et al.*, Science (2012)

Nuclear noise reduction for h=0 case



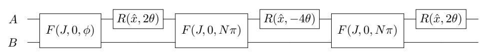


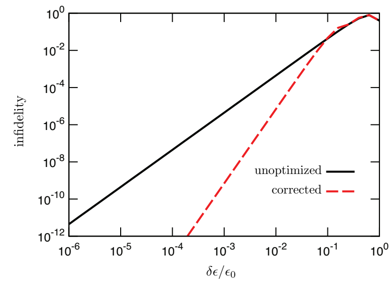


Parameters: Unoptimized pulse: $J = 3J^{AB}$, $\phi = 3\pi/4$ [$t = 3\pi/(8J^{AB})$];
Optimized pulse: $J = 9.2901J^{AB}$, $\phi = 3\pi/4$ [$t = 3\pi/(8J^{AB})$].

XW et al., npj Quantum Information **1**, 15003 (2015)

Charge noise reduction for h=0 case

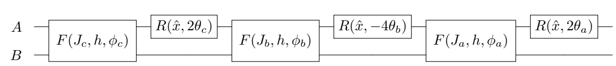


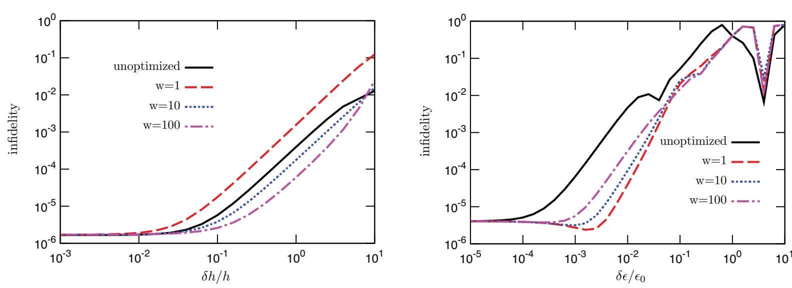


Parameters: Unoptimized pulse, $J = 3J^{AB}$, $\phi = 3\pi/4$,
Corrected pulse: $J = 3J^{AB}$, $\phi = 3\pi/4$, $N = 4$, $\theta = \frac{1}{2}\arccos\left(-\frac{\phi}{2N\pi}\right)$

XW et al., npj Quantum Information **1**, 15003 (2015)

Noise reduction for the general case





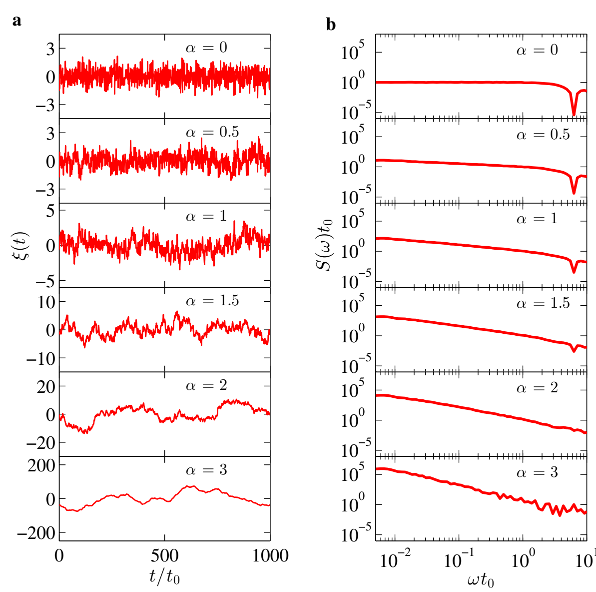
Parameters: Unoptimized pulse: $J = 64.128J^{AB}$, $\phi = 2.3596$.
Optimized pulses:
$w = 1$: $J_a = 151.56J^{AB}$, $J_b = 58.609J^{AB}$, $J_c = 33.712J^{AB}$, $\phi_a = 0.21612$, $\phi_b = 0.35106$, $\phi_c = 0.22303$, $\theta_a = 14.200$, $\theta_b = 1.5999$ and $\theta_c = 3.0629$.
$w = 10$: $J_a = 89.590J^{AB}$, $J_b = 83.522J^{AB}$, $J_c = 38.413J^{AB}$, $\phi_a = 0.33514$, $\phi_b = 0.27385$, $\phi_c = 0.17992$, $\theta_a = 22.690$, $\theta_b = 1.5253$ and $\theta_c = 3.0535$.
$w = 100$: $J_a = 115.02J^{AB}$, $J_b = 64.424J^{AB}$, $J_c = 36.711J^{AB}$, $\phi_a = 0.24015$, $\phi_b = 0.36612$, $\phi_c = 0.18231$, $\theta_a = 12.940$, $\theta_b = 1.5308$ and $\theta_c = 3.1414$.

XW et al., npj Quantum Information **1**, 15003 (2015)

Outline

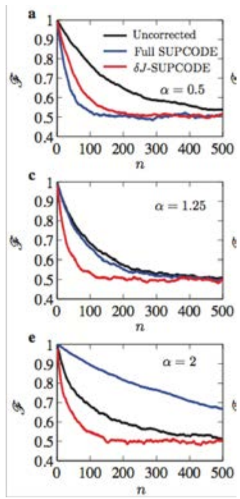
- Background
- Noise-compensating single qubit-gates
- Two-qubit gates
- Benchmarking
- Robust control using smooth pulses

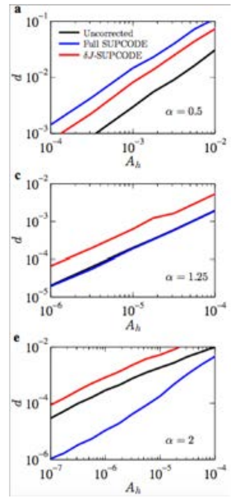
1/f Noises and spectra



X.-C. Yang and XW, *Sci. Rep.* **6**, 28996 (2016)

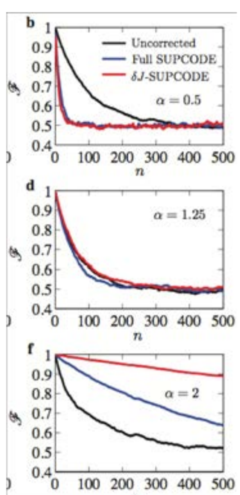
Randomized benchmarking: Results for the nuclear noise

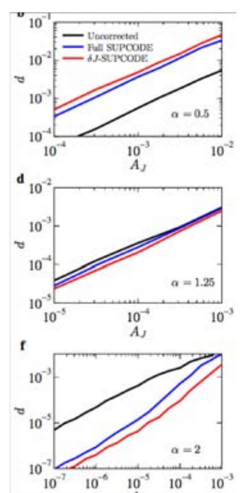




X.-C. Yang and XW, *Sci. Rep.* **6**, 28996 (2016)

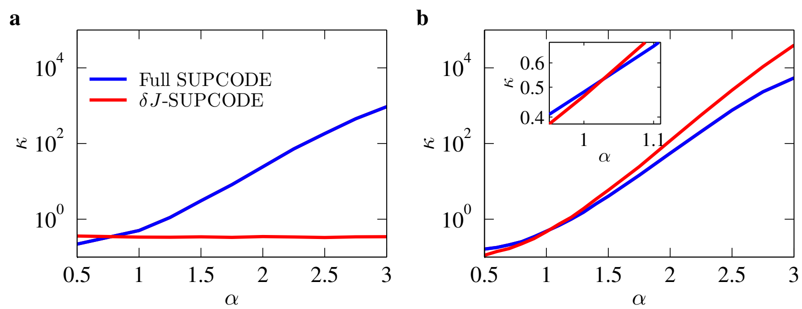
Randomized benchmarking: Results for the charge noise





X.-C. Yang and XW, *Sci. Rep.* **6**, 28996 (2016)

Improvement ratio v.s. noise exponent



nuclear noise

charge noise

X.-C. Yang and XW, *Sci. Rep.* **6**, 28996 (2016)

Conclusions

- Dynamically corrected gates: useful technique to perform noise-resistent operations
- DCGs are proposed for single- and two-qubit operations for singlet-triplet qubits
- 1/f noise environment: DCGs will work for α>1, but not otherwise

Outline

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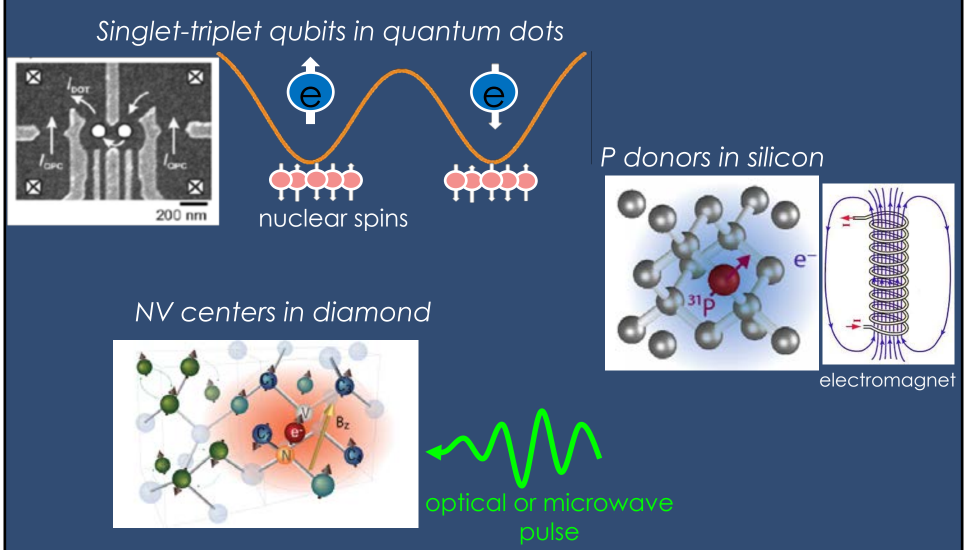
Quantum systems in noisy environments



Controlling quantum systems in noisy environments

$$H = \frac{J(t)}{2}\sigma_z + \frac{h}{2}\sigma_x$$

$J(t) =$ e.g., tunable energy splitting, exchange coupling, time-dependent external electric/magnetic field

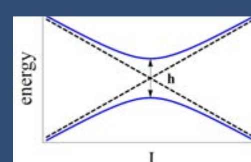
$h =$ e.g., fixed energy splitting, laser/microwave detuning, constant external magnetic field

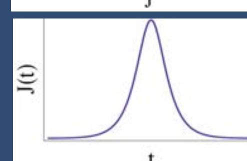
The environment causes fluctuations δh and δJ in the Hamiltonian, altering the system evolution.

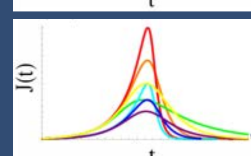
Analytical solutions: 1932-2012

$$H = \frac{J(t)}{2}\sigma_z + \frac{h}{2}\sigma_x$$

- Square pulse
  $J(t) = J_0$
- Landau-Majorana-Stueckelberg-Zener (1932)
  $J(t) = vt$
- Rosen-Zener sech pulse (1932)
  $J(t) = A\mathrm{sech}(\sigma t)$
- Generalizations of the sech (1980s)
- Solutions based on Heun functions (2000s)
- Solutions based on elliptic functions (2010s)







Traditional approach to finding new solutions

Single-axis driving: $H = \frac{1}{2}\begin{pmatrix} J(t) & h \\ h & -J(t) \end{pmatrix}$

Evolution operator: $U = \begin{pmatrix} u_{11} & -u_{21}^* \\ u_{21} & u_{11}^* \end{pmatrix}$

Evolution in rotating frame: $D_\pm = \frac{1}{\sqrt{2}} e^{\pm iht/2}(u_{11} \pm u_{21})$

Schrödinger equation: $\ddot{D}_+ + (-ih - \dot{J}/J)\dot{D}_+ + (J^2/4)D_+ = 0$

Old strategy: Pick $J(t)$ to obtain a familiar equation
e.g. $J(t) = A\mathrm{sech}(\sigma t)$ gives hypergeometric equation

Can we find new solutions more systematically?

Our approach: partial reverse-engineering

Schrödinger equation: $\ddot{D}_+ + (-ih - \dot{J}/J)\dot{D}_+ + (J^2/4)D_+ = 0$

Instead of guessing $J(t)$ and solving for D_+, think of this as an equation for $J(t)$.

Surprisingly, we found that this non-linear equation can be solved exactly:

$$J(t) = \pm \frac{\dot{D}_+ e^{-iht}}{\sqrt{c - \frac{1}{4}D_+^2 e^{-2iht} - \frac{ih}{2}\int_0^t dt' e^{-2iht'} D_+^2(t')}}$$

- First choose the evolution D_+
- Use the formula to find the $J(t)$ that generates that evolution
- Enforce unitarity

Barnes and Das Sarma, PRL **109**, 060401 (2012)

Reverse engineering a two-level Hamiltonian

$$H = \frac{J(t)}{2}\sigma_z + \frac{h}{2}\sigma_x$$

Both Hamiltonian and evolution determined from auxiliary "latitude" function $\chi(t)$

$$U = \begin{pmatrix} \cos\chi e^{i\psi_+} & -i\sin\chi e^{-i\psi_-} \\ -i\sin\chi e^{i\psi_-} & \cos\chi e^{-i\psi_+} \end{pmatrix}$$

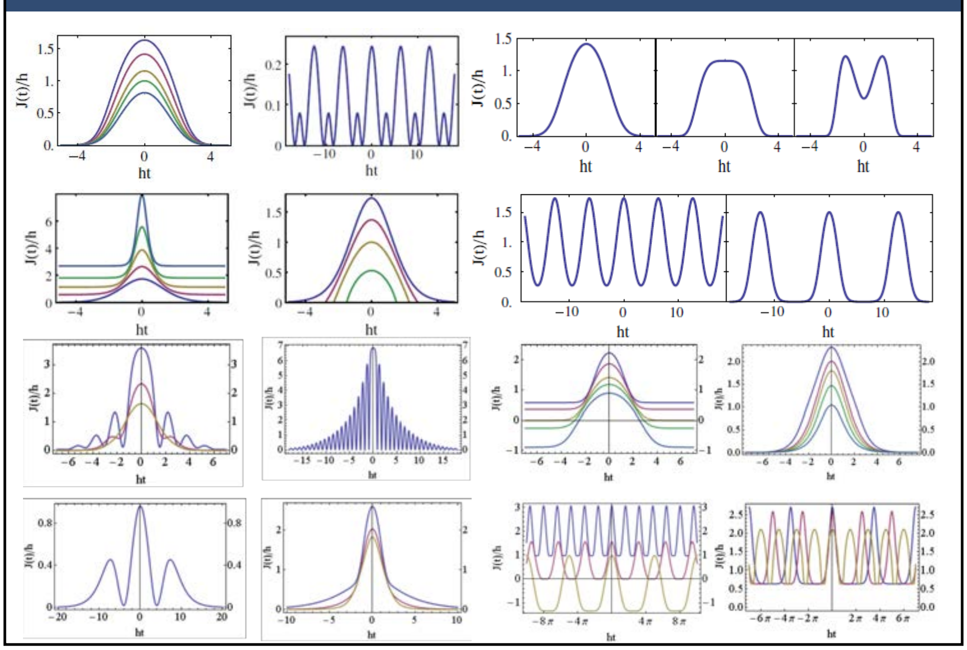
$$\psi_\pm = \pm\frac{1}{2}\arccos(2\dot{\chi}/h) + \int_0^t dt'\sqrt{h^2/4 - \dot{\chi}^2}\,\csc(2\chi)$$

$$J(t) = \frac{\ddot{\chi}}{\sqrt{h^2/4 - \dot{\chi}^2}} - 2\sqrt{h^2/4 - \dot{\chi}^2}\cot(2\chi)$$

$\chi(t)$ must obey the inequality: $|\dot{\chi}| \leq |h/2|$

Barnes, PRA **88**, 013818 (2013)

Some new analytically soluble examples



How can we include fluctuations?

- Fluctuations in the Hamiltonian

$$H = \frac{1}{2}\begin{pmatrix} J(t) + g(t)\delta\epsilon & h_0 + \delta h \\ h_0 + \delta h & -J(t) - g(t)\delta\epsilon \end{pmatrix}$$

can be thought of as fluctuations coming from $\chi(t)$:

$$\chi(t) = \chi_0(t) + \underbrace{\delta_h \chi(t) + \delta_\epsilon \chi(t)}_{\text{encode response of system to noise \textit{and} driving}}$$

- The response functions depend on the driving field.
- If we know the response functions, we can use the result to compute δU and find driving fields that cancel errors.

Computing response functions

$$\chi(t) = \chi_0(t) + \delta_h \chi(t) + \delta_\epsilon \chi(t)$$

$$J(t) + g(t)\delta\epsilon = \frac{\ddot{\chi}}{\sqrt{h^2/4 - \dot{\chi}^2}} - 2\sqrt{h^2/4 - \dot{\chi}^2}\cot(2\chi)$$

- Computing $\delta_h \chi[\chi_0(t)]$ and $\delta_\epsilon \chi[\chi_0(t)]$ is hard because they are determined by solving complicated differential equations.

e.g.,

$$(h^2 - 4\dot{\chi}_0^2)\delta_h \ddot{\chi} + 2\dot{\chi}_0\left[2\ddot{\chi}_0 + (h^2 - 4\dot{\chi}_0^2)\cot(2\chi_0)\right]\delta_h \dot{\chi} + \csc^2(2\chi_0)(h^2 - 4\dot{\chi}_0^2)^2 \delta_h \chi = 0$$

- We need a general solution to these equations to make progress.

Exact expressions for response functions

- There exists a reparameterization that yields the exact solution:

$$w \equiv \log\tan\chi + i\arcsin(2\dot{\chi}/h)$$

$$J(t) = i\dot{w} - h\sinh w$$

$$\delta_h\chi(t) = \delta h\frac{2}{h_0}\left\{\frac{1}{8}\sin[4\chi_0(t)] + \mathrm{Re}\left[e^{-2i\psi_0(t)}\int_0^t dt'\dot{\chi}_0(t')\sin^2[2\chi_0(t')]e^{2i\psi_0(t')}\right]\right\}$$

$$\delta_\epsilon\chi(t) = \delta\epsilon\frac{1}{2}\mathrm{Im}\left[e^{-2i\psi_0(t)}\int_0^t dt'\sin[2\chi_0(t')]g(t')e^{2i\psi_0(t')}\right]$$

Barnes et al., Sci. Rep. **5**, 12685 (2015)

$$\psi_0(t) = \int_0^t dt'\sqrt{h_0^2/4 - \dot{\chi}_0^2}\csc(2\chi_0(t'))$$

- Adjust parameters in $\chi_0(t)$ to cancel these variations ($\delta U \sim \delta\chi$)

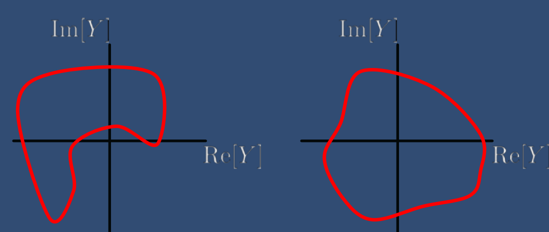
Difficulty: how to fix the target evolution?

- In adjusting the driving (χ_0) to cancel errors, we also alter a phase in the target evolution:

$$\Phi = \int_0^t dt'\sqrt{h_0^2/4 - \dot{\chi}_0^2}\csc(2\chi_0(t'))$$

- We can hold the phase fixed if we view it as a topological winding number:

$$\Phi = \lambda * \arg Y(t)$$



$\Phi(t_f) = 0$ $\Phi(t_f) = 2\pi\lambda$

Robust driving from the topological phase

- Φ determines the driving field
- Boundary conditions on Φ determine target evolution
- $\delta U = 0$ if $\Phi(\chi)$ obeys the following constraints

Canceling noise in h

$$\sin(4\chi_f) + 8e^{-2i\Phi(\chi_f)} \int_0^{\chi_f} d\chi \sin^2(2\chi) e^{2i\Phi(\chi)} = 0$$

$$\int_0^{\chi_f} d\chi \sin^2(2\chi)\Phi'(\chi) = 0$$

Canceling noise in $J(t)$

$$\int_0^{\chi_f} d\chi \sin(2\chi)\tilde{g}(\chi)\sqrt{1+[\Phi'(\chi)]^2\sin^2(2\chi)}\, e^{2i\Phi(\chi)} = 0$$

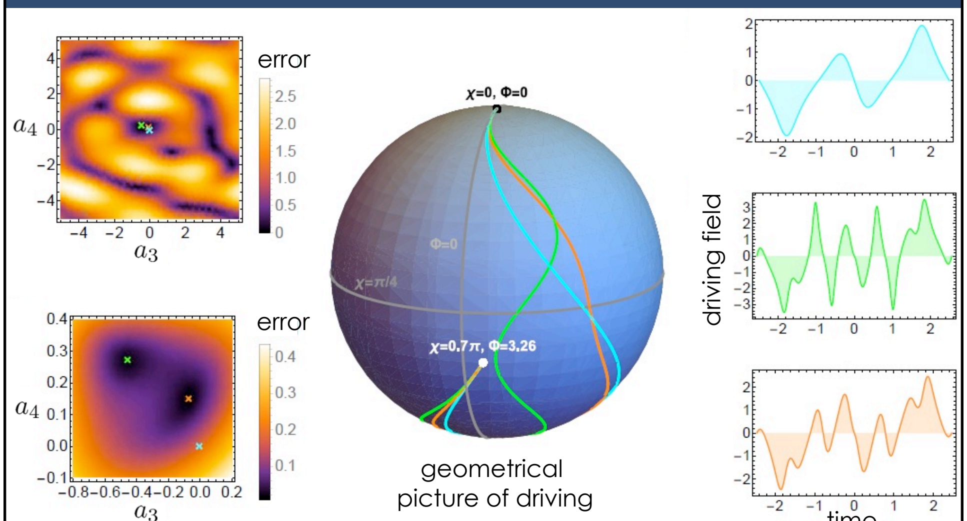
$$\int_0^{\chi_f} d\chi \cos(2\chi)\tilde{g}(\chi)\sqrt{1+[\Phi'(\chi)]^2\sin^2(2\chi)} = 0$$

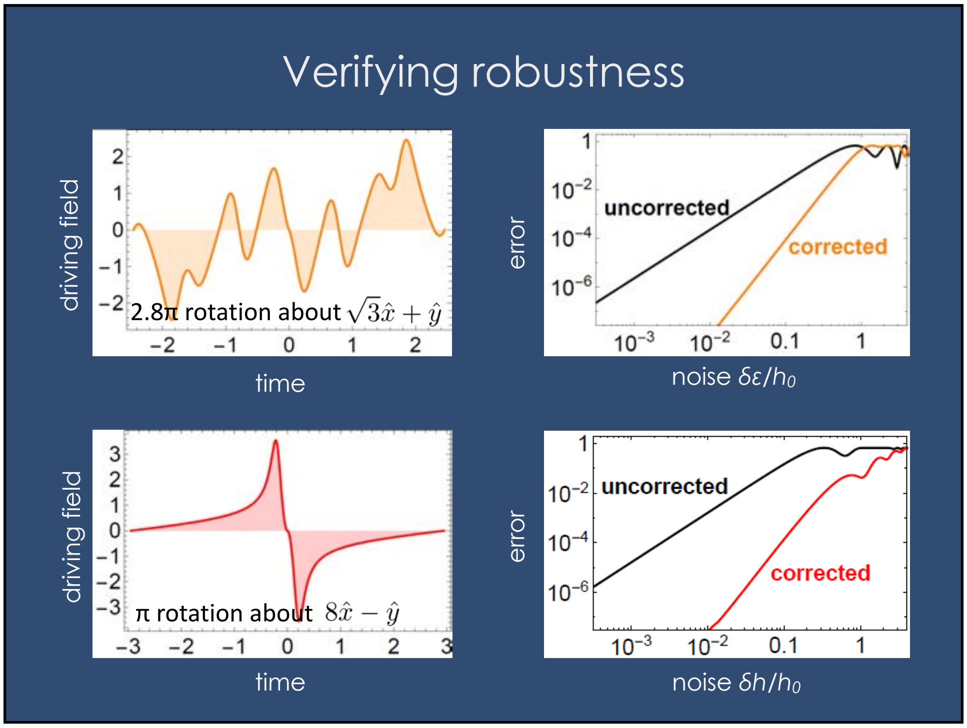
- All robust driving fields MUST satisfy these constraints:
  General solution to robust quantum control!

Barnes et al., Sci. Rep. **5**, 12685 (2015)

Example: canceling noise in the driving field

Ansatz: $\Phi(\chi) = a_1\chi^2 + a_2\chi^3 + a_3\sin^3(4\pi\chi/\phi) + a_4\sin^3(8\pi\chi/\phi)$





Verifying robustness
driving field
2.8π rotation about $\sqrt{3}\hat{x} + \hat{y}$
time
error
uncorrected
corrected
noise $\delta\varepsilon/h_0$
driving field
π rotation about $8\hat{x} - \hat{y}$
time
error
uncorrected
corrected
noise $\delta h/h_0$