1.

a. Use the composite trapezoidal rule

$$\int_a^b f(x)dx = \frac{h}{2}[f(a) + 2\sum_{i=1}^{2n-1} f(x_i) + f(b)]$$

b. Use the composite Simpson's rule

$$\int_a^b f(x)dx = \frac{h}{3}[f(x_0) + 2\sum_{i=2}^{n} f(x_{2i-2}) + 4\sum_{i=1}^{n} f(x_{2i-1}) + f(x_{2n})]$$

c. Use the midpoint rule

$$\int_a^b f(x)dx = 2h[f(x_1) + f(x_3) + f(x_5) + \cdots f(x_{2n-1})]$$

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a.Composite Trapezoidal Rule approximation: 0.396147592214901
b.Composite Simpson's Method approximation: 0.38566359602374467
c.Composite Midpoint Rule approximation: 0.3808047983772932
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2、 $\int_1^{1.5} x^2 \ln x \, dx = \frac{1}{2} \int_{-1}^{1} f\left[\frac{2.5}{2} + \frac{1}{2}\eta\right] d\eta$

$$= \frac{1}{2} \sum_{i=1}^{n} c_i f\left[\frac{a+b}{2} + \frac{b-a}{2}\eta_i\right]$$

find $n = 3$, $n = 4$

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n = 3 answer:0.1922593772568790З
n = 4 answer:0.192259357804863２
exact value:0.19225935773279604
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3、

由 1、2、題公式做雙重積分

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Simpson's Rule : 0.5118230071056833
Gauss-Legendre Quadrature : 0.5118655399452959
Exact Integral : 0.5118446353109126
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4 the composite Simpson's rule

$$\int_a^b f(x)dx = \frac{h}{3}\left[f(x_0)+2\sum_{i=1}^{\frac{n}{2}} f(x_{2i-2}) + 4\sum_{i=1}^{\frac{n}{2}} f(x_{2i-1}) + f(x_n)\right]$$

a. 可以直接計算

b. $\int_1^\infty x^{-4}\sin x\,dx$ improper integral transform $t = x^{-1}$ $dx = -\frac{1}{t^2}dt$

$x=\infty$ $t=0$

$x=1$ $t=1$

$$\int_1^\infty x^{-4}\sin x\,dx = \int_1^0 t^4 \sin\frac{1}{t}\left(-\frac{1}{t^2}dt\right) = \int_0^1 t^2 \sin\left(\frac{1}{t}\right)dt$$

a.Composite Simpson's Rule for ∫ x^(-1/4) sin(x) dx: 0.5259312819330653

b.Composite Simpson's Rule for ∫ x^(-4) sin(x) dx (Transformed): 0.2744816127051007