

# HW.6

$$1. \quad \left[ \begin{array}{cccc|c} 1.19 & 2.11 & -100 & 1 & 1.12 \\ 14.2 & -0.112 & 12.2 & -1 & 3.44 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 15.3 & 0.110 & -13.1 & -1 & 4.16 \end{array} \right]$$

$m_{21} = \frac{14.2}{15.3}$   
 $m_{41} = \frac{1.19}{15.3}$

$$\Rightarrow \left[ \begin{array}{cccc|c} 15.3 & 0.110 & -13.1 & -1 & 4.16 \\ 0 & -0.214 & 24.358 & -0.072 & -0.421 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 0 & 2.101 & -98.98 & 1.078 & 0.796 \end{array} \right]$$

$m_{32} = \frac{-0.214}{100}$   
 $m_{42} = \frac{2.101}{100}$

$$\Rightarrow \left[ \begin{array}{cccc|c} 15.3 & 0.110 & -13.1 & -1 & 4.16 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 0 & 0 & 24.144 - 0.070 & -0.416 & \\ 0 & 0 & -96.88 & 1.057 & 0.751 \end{array} \right]$$

$m_{43} = \frac{24.144}{-96.88}$

$$\Rightarrow \left[ \begin{array}{cccc|c} 15.3 & 0.110 & -13.1 & -1 & 4.16 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 0 & 0 & -96.88 & 1.057 & 0.751 \\ 0 & 0 & 0 & 0.193 & -0.229 \end{array} \right]$$

$$\left\{ \begin{array}{l} 15.3x_1 + 0.110x_2 - 13.1x_3 - x_4 = 4.16 \\ 100x_2 - 99.9x_3 + x_4 = 2.15 \\ -96.88x_3 + 1.057x_4 = 0.751 \\ 0.193x_4 = -0.229 \end{array} \right.$$

2

$$A = \begin{bmatrix} 4 & 1 & -1 & 0 \\ 1 & 3 & -1 & 0 \\ -1 & -1 & 6 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix}$$

1

$$\left| \begin{array}{cccc|cccc} 4 & 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 3 & -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 6 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 5 & 0 & 0 & 0 & 1 \end{array} \right|$$

$\xrightarrow{x_1 \times 1}$   
 $\xrightarrow{x_2 - x_1}$   
 $\xrightarrow{x_3 + x_1}$

2.

$$\left| \begin{array}{cccc|cccc} 1 & 0.25 & -0.25 & 0 & 0.25 & 0 & 0 & 0 \\ 0 & 2.75 & -0.75 & 0 & -0.25 & 1 & 0 & 0 \\ 0 & -0.75 & 5.75 & 2 & 0.25 & 0 & 1 & 0 \\ 0 & 0 & 2 & 5 & 0 & 0 & 0 & 1 \end{array} \right|$$

$\xrightarrow{\frac{1}{2.75}}$   
 $\xrightarrow{x_3 \times 0.75}$

3.

$$\left| \begin{array}{cccc|cccc} 1 & 0.25 & -0.25 & 0 & 0.25 & 0 & 0 & 0 \\ 0 & 1 & -0.273 & 0 & -0.091 & 0.364 & 0 & 0 \\ 0 & 0 & 5.545 & 2 & 0.181 & 0.273 & 1 & 0 \\ 0 & 0 & 2 & 5 & 0 & 0 & 0 & 1 \end{array} \right|$$

$\xrightarrow{\frac{1}{5.545}}$   
 $\xrightarrow{x_3 - 2x_1}$

4

$$\left| \begin{array}{cccc|cccc} 1 & 0.25 & -0.25 & 0 & 0.25 & 0 & 0 & 0 \\ 0 & 1 & -0.273 & 0 & -0.091 & 0.364 & 0 & 0 \\ 0 & 0 & 1 & 0.361 & 0.033 & 0.049 & 0.18 & 0 \\ 0 & 0 & 0 & 4.178 & -0.066 & -0.098 & -0.36 & 1 \end{array} \right|$$

5.

$$\left| \begin{array}{cccc|cccc} 1 & 0.25 & -0.25 & 0 & 0.25 & 0 & 0 & 0 \\ 0 & 1 & -0.273 & 0 & -0.091 & 0.364 & 0 & 0 \\ 0 & 0 & 1 & 0.361 & 0.033 & 0.049 & 0.18 & 0 \\ 0 & 0 & 0 & 4.278 & -0.066 & -0.098 & -0.36 & 1 \end{array} \right| \times \frac{1}{4.278} \quad \begin{matrix} \swarrow \times 0.25 \\ \swarrow \times 0.273 \end{matrix}$$

6

$$\left| \begin{array}{cccc|cccc} 1 & 0.25 & 0 & 0 & 0.26 & 0.014 & 0.053 & -0.021 \\ 0 & 1 & 0 & 0 & -0.08 & 0.379 & 0.057 & -0.023 \\ 0 & 0 & 1 & 0 & 0.038 & 0.057 & 0.211 & -0.084 \\ 0 & 0 & 0 & 1 & -0.015 & -0.023 & -0.084 & 0.234 \end{array} \right| \quad \begin{matrix} \swarrow \times -0.25 \end{matrix}$$

7

$$\left| \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0.28 & -0.08 & 0.038 & -0.015 \\ 0 & 1 & 0 & 0 & -0.08 & 0.379 & 0.057 & -0.023 \\ 0 & 0 & 1 & 0 & 0.038 & 0.057 & 0.211 & -0.084 \\ 0 & 0 & 0 & 1 & -0.015 & -0.023 & -0.084 & 0.234 \end{array} \right|$$

$$A^{-1} = \begin{bmatrix} 0.28 & -0.08 & 0.038 & -0.015 \\ -0.08 & 0.379 & 0.057 & -0.023 \\ 0.038 & 0.057 & 0.211 & -0.084 \\ -0.015 & -0.023 & -0.084 & 0.234 \end{bmatrix}$$

$$3. \quad A = \begin{bmatrix} 3 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix} \quad Ax = b$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

$$A = LU = \begin{bmatrix} 3 & 0 & 0 & 0 \\ -1 & \frac{8}{3} & 0 & 0 \\ 0 & -1 & \frac{4}{3} & 0 \\ 0 & 0 & -1 & \frac{55}{27} \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{3} & 0 & 0 \\ 0 & 1 & -\frac{3}{8} & 0 \\ 0 & 0 & 1 & -\frac{8}{27} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$LUx = b \quad \Leftrightarrow \quad UX = y$$

$$Ly = b \quad y = \begin{bmatrix} \frac{2}{3} \\ \frac{11}{8} \\ \frac{43}{27} \\ \frac{64}{55} \end{bmatrix}$$

$$UX = y \quad X = \begin{bmatrix} 1.436 \\ 2.309 \\ 2.491 \\ 1.164 \end{bmatrix}$$