

$$1. \cos(0.698) = 0.7661, \cos(0.733) = 0.7432 \quad \cos(0.768) = 0.7193, \cos(0.803) = 0.6946$$

$$\text{approximate} \quad \cos(0.750) = 0.7317$$

degree one, $x_0 = 0.698 \quad x_1 = 0.733$

$$y(x) = f(x_0) \frac{x - x_1}{x_0 - x_1} + f(x_1) \frac{x - x_0}{x_1 - x_0}$$

$$= 0.7661 \times \frac{x - 0.733}{0.698 - 0.733} + 0.7432 \times \frac{x - 0.698}{0.733 - 0.698}$$

$$x = 0.750, \quad y(0.750) \approx 0.732077$$

error bound

$$E_1(x) = \frac{(x - x_0)(x - x_1)}{2!} f''(\varepsilon)$$

$$f''(\varepsilon) \leq 1$$

$$E_1(0.750) \leq \frac{|(0.750 - 0.698)(0.75 - 0.733)|}{2!}$$

$$\leq 4.42 \times 10^{-4}$$

degree two

$$x_0 = 0.698, x_1 = 0.733, x_2 = 0.768$$

$$f(x) = 0.7661 \times \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + 0.7432 \times \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + 0.7193 \times \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$x=0.75 \text{ 代入 } f(0.75) = 0.731716$$

error bound

$$E_2(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{3!} f'''(\varepsilon)$$

$$|f'''(\varepsilon)| \leq |$$

$$E_2(0.75) \leq \left| \frac{(0.75-0.698)(0.75-0.733)(0.75-0.768)}{3!} \right|$$

$$\leq 2.652 \times 10^{-6}$$

degree three

$$\cos(0.698) = 0.7661, \cos(0.733) = 0.7432 \quad \cos(0.768) = 0.7193, \cos(0.803) = 0.6946$$

$$f(x) = 0.7661 \times \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + 0.7432 \times \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \\ + 0.7193 \times \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} + 0.6946 \times \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$x = 0.75 \quad f(0.75) = 0.731704$$

error bound

$$E_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{4!} f^4(\varepsilon)$$

$$|f^4(\varepsilon)| \leq 1 \quad E(0.75) \leq \frac{(0.75-0.698)(0.75-0.733)(0.75-0.768)(0.75-0.803)}{24}$$

$$\leq 3.5139 \times 10^{-8}$$

degree four

找不到 degree four

因為 Lagrange n=4 插值 需要 n+1 個點

$$2. \quad X \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6$$

$$e^X \quad 0.740818 \quad 0.67032 \quad 0.606531 \quad 0.548812$$

$$f(X) = X - e^X \quad -0.440818 \quad -0.270320 \quad -0.106531 \quad 0.05188$$

$$f(X) = -0.440818 \times \frac{(X-X_1)(X-X_2)(X-X_3)}{(X_0-X_1)(X_0-X_2)(X_0-X_3)} - 0.270320 \times \frac{(X-X_0)(X-X_2)(X-X_3)}{(X_1-X_0)(X_1-X_2)(X_1-X_3)}$$

$$-0.106531 \times \frac{(X-X_0)(X-X_1)(X-X_3)}{(X_2-X_0)(X_2-X_1)(X_2-X_3)} + 0.05188 \times \frac{(X-X_0)(X-X_1)(X-X_2)}{(X_3-X_0)(X_3-X_1)(X_3-X_2)}$$

$$f(X) = 0, \text{ 求 } X$$

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from sympy import symbols, Eq, solve
x = symbols('x')
y = 0

equation = Eq([
    -0.440818 * ((x-0.4)*(x-0.5)*(x-0.6)) / ((0.3-0.4)*(0.3-0.5)*(0.3-0.6)) +
    -0.270320 * ((x-0.3)*(x-0.5)*(x-0.6)) / ((0.4-0.3)*(0.4-0.5)*(0.4-0.6)) +
    -0.106531 * ((x-0.4)*(x-0.3)*(x-0.6)) / ((0.5-0.4)*(0.5-0.3)*(0.5-0.6)) +
    0.05188 * ((x-0.4)*(x-0.5)*(x-0.3)) / ((0.6-0.4)*(0.6-0.5)*(0.6-0.3)),
    y # 等式右邊是 y
])

# 解方程式
solutions = solve(equation, x)
print(solutions)

```

複數不符

[0.566900850448465, 1.62894150561146 - 3.53445762087103*I, 1.62894150561146 + 3.53445762087103*I]

$$X \approx 0.566901$$

3.

step 1. 用 python 繪會製 hermite polynomial

step 2 找出 $t=10$ 的距離及速度 -- (a)

step 3. 繪會黑點找出 (b) 及 (c) 的答案

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numerical_hw3_problem3
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a.
Distance at time 10 seconds: 768.96 feet
Velocity at time 10 seconds: 74.64 feet/second

b.
The car first exceeds 80.69 ft/s at time 3.14 seconds
c.
The predicted maximum speed is 92.04 feet/second