Ecuaciones Diferenciales de Segundo Orden que se reducen en primer orden.

1) $xy''+y'=4x \to y'=p \to y''=p' \to p'=\frac{dp}{dx}$

$xp'+p=4x \to x\frac{dp}{dx}+p=4x \to \frac{dp}{dx}+\frac{p}{x}=\frac{4x}{x}$

$\frac{dp}{dx}+\frac{p}{x}=4 \to e^{\int\frac{dx}{x}} \to e^{\ln x} \to FI=x$

$x\frac{dp}{dx}+x\left(\frac{p}{x}\right)=4x \to x\frac{dp}{dx}+p=4x \to \int\frac{d}{dx}[xp]=\int 4x+c$

$xp=4\int x\,dx+k_1 \to xp=2\frac{x^2}{2}+C_1 \to p=2\frac{x^2}{x}+\frac{c_1}{x}$

$p=2x+\frac{C_1}{x} \to y'=2x+\frac{C}{x} \to \frac{dy}{dx}=2x+\frac{C_1}{x}$

$\int dy=\int\left(2x+\frac{c_1}{x}\right)dx \to y=\int 2x\,dx+c_1\int\frac{dx}{x}$

$y=x^2+c_1\ln|x|+C_2 \to \boxed{y=x^{2}+C_1\ln x+C_2}$

2) $y''\cos x=y' \to y'=p \to y''=p' \to p'=\frac{dp}{dx}$

$p'\cos x=p \to \frac{dp}{dx}\cos x=p \to \int\frac{dp}{p}=\int\sec x\,dx$

$\ln p=\ln(\sec x+\tan x)+C_1 \to e^{\ln p}=e^{\ln(\sec x+\tan x)}+e^{C_1}$

$p=\sec x+\tan x+C_1 \to \frac{dy}{dx}=\sec x+\tan x+C_1$

$\int dy=\int(\sec x+\tan x+C_1)dx \to y=\int\sec x\,dx+\int\tan x\,dx+\int C_1\,dx$

$y=\ln|\sec x+\tan x|+\ln|\sec x|+C_1x+C \to$ Para $x>0$

$\boxed{y=\ln(\sec x+\tan x)+\ln(\sec x)+C_1x+C}$

3) $y''=2\,\text{sen}\,x\cos^2x-\text{sen}^3x \to y'=p \to y''=p' \to p'=\frac{dp}{dx}$

$\int y''=2\int\text{sen}\,x\cos^2x\,dx-\int\text{sen}\,x\,\text{sen}^2x\,dx$ $\quad u=\cos x - du=\text{sen}\,x\,dx$

$\int y'=-2\int u^2du-\int(1-\cos^2x)\,\text{sen}\,x\,dx \to \int y'=-\frac{2}{3}u^3-\int\text{sen}\,x\,dx+\int\text{sen}\,x\cos^2x\,dx$

$\int y'=-\frac{2}{3}\cos^3x+\cos x-\int u^2du+C_1 \to \int y'=-\frac{2}{3}\cos^3x+\cos x-\frac{1}{3}\cos^3x+C_1$

$\int y'=-\frac{2}{3}\int\cos^3x+\int\cos x\,dx-\frac{1}{3}\int\cos^3x\,dx+\int C_1\,dx+C_2$

$y=-\int\cos^3x\,dx+\int\cos x\,dx+\int C_1\,dx+C_2$

$y=-\int\cos^2x\cos x\,dx+\text{sen}\,x+C_1x+C_2$

$y=-\int(1-\text{sen}^2x)\cos x\,dx+\text{sen}\,x+C_1x+C_2$ $\quad v=\text{sen}\,x$, $du=\cos x\,dx$

$y=-\int\cos x\,dx+\int\text{sen}^2x\cos x\,dx+\text{sen}\,x+C_1x+C_2$

$y=\cancel{-\text{sen}\,x}+\int u^2du+\cancel{\text{sen}\,x}+C_1x+C_2$

$y=\frac{1}{3}u^3+C_1x+C_2$

$\boxed{y=\frac{1}{3}\text{sen}^3x+C_1x+C_2}$

4) $y'''=x+\cos x \to y'=p \to y''=p' \to p'=\frac{dp}{dx}$

$\int y'''=\int x\,dx+\int\cos x\,dx \to y''=\frac{1}{2}x^2+\text{sen}\,x+C_1$

$\int y''=\frac{1}{2}\int x^2dx+\int\text{sen}\,x\,dx+C_1\int dx \to y'=\frac{1}{2}\frac{x^3}{3}-\cos x+C_1x+C_2$

$y'=\frac{1}{6}x^3-\cos x+C_1x+C_2 \to \int y'=\frac{1}{6}\int x^3dx-$

$\int y'=\frac{1}{6}\int x^3dx-\int\cos x\,dx+C_1\int x\,dx+C_2\int dx$

$y=\frac{1}{6}\frac{x^4}{4}-\text{sen}\,x+C_1\frac{x^2}{2}+C_2x+C_3$

$\boxed{y=\frac{1}{24}x^4-\text{sen}\,x-\frac{1}{2}C_1x^2+C_2x+C_3}$

*5) $x^2y''+(y')^2=0 \to y'=p \to y''=p' \to \frac{dp}{dx}=y''$

$x^2p'+p^2=0 \to x^2\frac{dp}{dx}+p^2=0 \to x^2\frac{dp}{dx}=-p^2$

$\frac{x^2dp}{p^2}=dx \to \int\frac{dp}{p^2}=-\int\frac{dx}{x^2} \to \int p^{-2}dp=-\int x^{-2}dx$

$-p^{-1}=x^{-1}+C \to -\frac{1}{p}=\frac{1}{x}+C \to -\frac{1}{p}=\frac{1+C_1x}{x}$

$-x=(1+C_1x)p \to p=\frac{-x}{1+C_1x} \to \frac{dy}{dx}=\frac{-x}{1+C_1x}$

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6) $y''=2x(y')^2 \to y'=p \to y''=p' \to \frac{dp}{dx}=y''$

$p'=2x\,p^2 \to \frac{dp}{dx}=2xp^2 \to \int\frac{dp}{p^2}=\int 2x\,dx$

$-\frac{1}{p}=x^2+C_1 \to -1=(x^2+C_1)p \to \frac{-1}{x^2+C_1}=p$

$\frac{dy}{dx}=\frac{-1}{x^2+C_1} \to \int dy=\int\frac{-1}{x^2+C_1}$

$\sqrt{a^2}=\sqrt{C_1}$ $\quad u^2=x^2 \to \int dy=-\int\frac{du}{u^2+a^2}$

$a=C_1$ $\quad u=x$

$du=dx$

$\boxed{y=-\frac{1}{C_1}\tan^{-1}\frac{x}{C_1}+C_2}$

7) $y''+y=0 \to p\frac{dy}{dx} \to y''=p' \to p'=p\frac{dp}{dx}$

$p\frac{dp}{dy}+y=0 \to p\frac{dp}{dy}=-y \to \int p\,dp=-\int y\,dy$

$\frac{p^2}{2}=-\frac{1}{2}y^2+C_1 \to p^2=\left(-\frac{1}{2}\right)(2)y^2+2C_1 \to p=\sqrt{2C_1-y^2}$

$\frac{dy}{dx}=\sqrt{2C_1-y^2} \to \int\frac{dy}{\sqrt{2C_1-y^2}}=\int dx \to \sqrt{a^2}=\sqrt{2C_1}$ $\quad u^2=y^2$

$a=\sqrt{2C_1}$ $\quad u=y$ $\quad du=dy$

$\int\frac{du}{\sqrt{a^2-u^2}}=x \to \boxed{x=\text{sen}^{-1}\frac{y}{\sqrt{2C_1}}+C_2}$

7) $y''=2y(y')^3 \rightarrow y'=p \rightarrow y''=p' \rightarrow y''=p\frac{dp}{dy}$

$p\frac{dp}{dy}=2yp^3 \rightarrow \frac{dp}{dy}=\frac{2yp^{\cancel{3}}}{\cancel{p}} \rightarrow \int\frac{dp}{p^2}=\int 2y\,dy$

$-\frac{1}{p}=y^2+C_1 \rightarrow -1=(y^2+C_1)p \rightarrow \frac{-1}{y^2+C_1}=p$

$\frac{dy}{dx}=\frac{-1}{y^2+C_1} \rightarrow dy(y^2+C_1)= \quad dx$

$-\int(y^2+C_1)dy=\int dx \rightarrow \boxed{x=-\frac{1}{3}y^3-C_1x+C_2}$

8) $yy''+(y')^2=yy' \rightarrow y'=p \rightarrow y''=p \rightarrow y''=p\frac{dp}{dy}$

$yp\frac{dp}{dy}+p^2=yp \rightarrow p\frac{dp}{dy}+\frac{p^2}{y}=\frac{\cancel{y}p}{\cancel{y}}$

$p\frac{dp}{dy}+\frac{p^2}{y}=p \rightarrow \frac{dp}{dy}+\frac{p^{\cancel{2}}}{\cancel{p}y}=\frac{\cancel{p}}{\cancel{p}}$

$\frac{dp}{dy}+\frac{p}{y}=1 \rightarrow e^{\int\frac{dy}{y}} \rightarrow e^{\ln y} \rightarrow F.I=y$

$y\frac{dp}{dy}+\cancel{y}\left(\frac{p}{\cancel{y}}\right)=y(1) \rightarrow y\frac{dp}{dy}+p=y$

$\int\frac{d}{dy}[yp]=\int y\,dy+C_1 \rightarrow yp=\frac{1}{2}y^2+C_1$

$p=\frac{1}{2}\frac{y^{\cancel{2}}}{\cancel{y}}+\frac{C_1}{y} \rightarrow \frac{dy}{dx}=\frac{y}{2}+\frac{C_1}{y}$

$\frac{dy}{dx}=\frac{y^2+C_1}{2y} \rightarrow \int\frac{2y\,dy}{y^2+C_1}=\int dx$ $\quad u=y^2+C_1$, $du=2y\,dy$, $\frac{du}{2}=y\,dy$

$x=\left(\frac{1}{2}\right)(2)\ln u+C_2 \rightarrow \boxed{x=\ln(y^2+C_1)+C_2}$

9) $y''(2y+3)=2(y')^2 \rightarrow y'=p \rightarrow y''=p' \rightarrow y''=p\frac{dp}{dy}$

$p\frac{dp}{dy}(2y+3)=2p^2 \rightarrow \frac{dp}{dy}(2y+3)=\frac{2p^{\cancel{2}}}{\cancel{p}}$

$\frac{dp}{dy}(2y+3)=2p \rightarrow \frac{dp(2y+3)}{dy\;2p}=1$

$\int\frac{dp}{2p}=\int\frac{dy}{2y+3} \rightarrow \frac{\cancel{1}}{\cancel{2}}\ln p=\frac{\cancel{1}}{\cancel{2}}\ln(2y+3)+C_1$

$e^{\ln p}=e^{\ln(2y+3)}+e^{C_1} \rightarrow p=2y+3+C_1$

$\frac{dy}{dx}=2y+3+C_1 \rightarrow \int\frac{dy}{2y+3+C_1}=\int dx$

$\boxed{x=\frac{1}{2}\ln(2y+3+C_1)+C_2}$

10) $yy''+(y')^2+1=0 \rightarrow y'=p \rightarrow y''=p' \rightarrow y''=p\frac{dp}{dy}$

$yp\frac{dp}{dy}+(p^2+1)=0 \rightarrow yp\frac{dp}{dy}=-(p^2+1) \rightarrow \int\frac{p\,dp}{p^2+1}=\int-\frac{dy}{y}$

$\frac{1}{2}\ln|p^2+1|=-\ln y+C_1 \rightarrow \ln(p^2+1)^{1/2}=-\ln y+C_1$

$\frac{1}{2}\ln(p^2+1)+\ln y+\ln C_1=0 \rightarrow \frac{1}{2}\ln(p^2+1)=-\ln(C_1y)$

$\ln(p^2+1)=-2\ln(C_1y) \rightarrow e^{\ln(p^2+1)}=e^{-2\ln(C_1y)}$

$e^{\ln(p^2+1)}=e^{\ln(C_1y)^{-2}} \rightarrow p^2+1=(C_1y)^{-2} \rightarrow p^2=(C_1\cdot y)^{-2}-1$

$p=\sqrt{(C_1y)^{-2}-1} \rightarrow p=\sqrt{\frac{C_1}{y^2}-1} \rightarrow p=\sqrt{\frac{C_1-y^2}{y^2}}$

$\frac{dy}{dx}=\frac{1}{y}\sqrt{C_1-y^2} \rightarrow \int\frac{y\,dy}{\sqrt{C_1-y^2}}=\int dx \rightarrow u=C_1-y^2$, $du=-2y\,dy$, $-\frac{du}{2}=y\,dy$

$-\frac{1}{2}u^{-1/2}=x \rightarrow -\frac{1}{2}\frac{u^{1/2}}{1/2}+C_2=x$

$x=\left(-\frac{1}{\cancel{2}}\right)\left(\frac{\cancel{2}}{1}\right)u^{1/2}+C_2 \rightarrow \boxed{x=C_2+\sqrt{C_1-y^2}}$

11) $xy''=y'\ln\left(\frac{y'}{x}\right) \rightarrow y'=p \rightarrow y''=p' \rightarrow y''=\frac{dp}{dx}$

$xp'=p\ln\left(\frac{p}{x}\right) \rightarrow x\frac{dp}{dx}=p\ln\left(\frac{p}{x}\right) \rightarrow x\,dp=p\ln\left(\frac{p}{x}\right)dx$

$p=ux \rightarrow u=p/x$
$dp=u\,dx+x\,du$ $\quad x[u\,dx+x\,du]=ux\ln\left(\frac{ux}{x}\right)dx$

$ux\,dx+x^2du=ux\ln(u)\,dx \rightarrow x^2du=(ux\ln u-ux)dx$

$x^{\cancel{2}}du=\cancel{x}(u\ln u-u)dx \rightarrow x\,du=(u\ln u-u)dx$

$\int\frac{du}{u\ln u-u}=\int\frac{dx}{x} \rightarrow \int\frac{du}{u(\ln u-1)}=\int\frac{dx}{x}$ $\quad w=\ln u-1$, $dw=\frac{du}{u}$

$\int\frac{dw}{w}=\int\frac{dx}{x} \rightarrow \ln|\ln u-1|=\ln|x|+C_1 \rightarrow \ln(\ln\frac{p}{x}-1)=\ln x+C_1$

$e^{\ln(\ln(\frac{p}{x})-1)}=e^{\ln x}+e^{C_1} \rightarrow \ln\left(\frac{p}{x}\right)-1=x+C_1$

$\ln\left(\frac{p}{x}\right)=x-1+C_1 \rightarrow e^{\ln(\frac{p}{x})}=e^x-e^{-1}+e^{C_1}$

$\frac{p}{x}=e^x-e^{-1}+e^{C_1} \rightarrow p=\frac{e^x}{x}-\frac{e^{-1}}{x}+\frac{e^{C_1}}{x}$

$\frac{dy}{dx}=x^{-1}e^x-\frac{e^{-1}}{x}+\frac{e^{C_1}}{x} \rightarrow \int dy=\int x^{-1}e^x\,dx-e^{-1}\int\frac{dx}{x}+e^{C_1}\int\frac{dx}{x}$

$u=x^{-1}$ $\quad dv=e^x$
$du=-\frac{1}{x^2}$ $\quad v=e^x$ $\quad y=\frac{e^x}{x}-\int e^x$