Trayectorias Ortogonales

1) Ecuación de las trayectorias ortogonales de la familia $x^2+y^2=C$

$2x+2yy'=0 \rightarrow 2yy'=-2x \rightarrow y'=-\frac{\cancel{2}x}{\cancel{2}y} \rightarrow y'=-\frac{x}{y}$

$\frac{dy}{dx}=-\frac{1}{-\frac{x}{y}} \rightarrow \frac{dy}{dx}=(-)\left(-\frac{y}{x}\right) \rightarrow \frac{dy}{dx}=\frac{y}{x}$

$\int\frac{dy}{y}=\int\frac{dx}{x} \rightarrow \ln|y|=\ln|x|+\ln|c|$

$e^{\ln|y|}=e^{\ln|x|}+e^{\ln|c|} \rightarrow y=x+C \rightarrow \boxed{y=xC}$

2) Ecuación de las trayectorias ortogonales de la familia $x^2+y^2=cx$

$2x+2yy'=C \rightarrow 2x+2y\frac{dy}{dx}=c \rightarrow 2y\frac{dy}{dx}=c-2x$

$\frac{dy}{dx}=\frac{y^2-x^2}{2xy} \rightarrow \frac{dy}{dx}=\frac{-1}{\frac{y^2-x^2}{2xy}} \rightarrow \frac{dy}{dx}=-\left(\frac{2xy}{y^2-x^2}\right)$

$\frac{dy}{dx}=\frac{2xy}{x^2-y^2} \rightarrow dy(x^2-y^2)=2xy\,dx$ $\quad x=vy \quad v=\frac{x}{y} \quad dx=v\,dy+y\,dv$

$(v^2y^2-y^2)dy=2(vy)y(v\,dy+y\,dv)$

$y^2(v^2-1)dy=2vy^2(v\,dy+y\,dv) \rightarrow y^2(v^2-1)dy=$ Jeje

$y^2(v^2-1)dy=2v^2y^2dy+2vy^3dv \rightarrow y^2(v^2-1)dy-2v^2y^2dy=2vy^3dv$

$y^2(v^2-1-2v^2)dy=2vy^3dv \rightarrow y^2(-v^2-1)dy=2vy^3dv$

$\int\frac{y^2}{y^3}dy=\int\frac{2v}{-v^2-1}dv \rightarrow \int\frac{y^2}{y^3}dy=-\int\frac{2v}{v^2+1}dv$ $\quad u=v^2+1 \quad du=2v\,dv$

$\int\frac{dy}{y}=-\int\frac{u}{du} \rightarrow \ln|y|=-\ln|u|+C \rightarrow \ln|y|+\ln|u|+C=0$

$\ln|y|+\ln|v^2+1|+c=0 \rightarrow \ln|y|+\ln\left|\frac{x^2}{y^2}+1\right|+c=0 \rightarrow e$

$y+\frac{x^2+y^2}{y^2}=C \rightarrow$ Verificar Ecuación

3) Encuentre las trayectorias ortogonales de la familia $y=x+Ce^{-x}$

$\frac{dy}{dx}=1-Ce^{-x} \rightarrow \frac{dy}{dx}=1+x-y \rightarrow \frac{dy}{dx}=\frac{-1}{1+x-y}$

$\frac{dy}{dx}=\frac{1}{1+x-y} \rightarrow (1+x-y)dy=dx \rightarrow dx+(1+x-y)dy=0$

$\frac{\partial M}{\partial y}=0 \quad \frac{\partial N}{\partial x}=1 \quad \frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$ No Exacta $\quad \frac{0-1}{1}=-1\rightarrow e^{\int dy}$

$e^y\rightarrow F.I \rightarrow e^y dx+e^y(1+x-y)dy=0$

$f(x,y)=\int e^y dx+c(y) \rightarrow f(x,y)=xe^y+c(y)$

$\frac{\partial f(x,y)}{\partial y}=xe^y+c'(y) \rightarrow \cancel{xe^y}+c'(y)=e^y+\cancel{xe^y}-ye^y$

$c'(y)=e^y-ye^y \rightarrow \boxed{f(x,y)=xe^y+e^y-ye^y}$

4) Encontrar las trayectorias ortogonales de $y^2=4px$.

$y^2=4px \rightarrow \frac{y^2}{x}=4p \rightarrow \frac{x\frac{dy}{dx}(y^2)-y^2\frac{dy}{dx}(x)}{x^2}=0$

$\frac{x(2y)y'-y^2(1)}{x^2}=0 \rightarrow 2xyy'-y^2=0$

$2xyy'=y^2 \rightarrow y'=\frac{y^2}{2xy} \rightarrow \frac{dy}{dx}=\frac{y}{2x} \rightarrow \frac{dy}{dx}=-\frac{1}{y/2x}$

$\frac{dy}{dx}=(-1)\left(\frac{2x}{y}\right) \rightarrow \frac{dy}{dx}=-\frac{2x}{y} \rightarrow y\,dy=-2x\,dx$

$\int y\,dy=-2\int x\,dx \rightarrow \frac{y^2}{2}=-\cancel{2}\frac{x^2}{\cancel{2}}+c$

$y^2=-2(x^2+c) \rightarrow \boxed{y^2=2C-2x^2}$

5) Determinar las trayectorias ortogonales de $x^2+(y-c)^2=c^2$

$x^2+y^2-2yc+c^2=c^2 \rightarrow x^2+y^2-2yc=\cancel{c^2}-\cancel{c^2}$

$x^2+y^2-2yc=0 \rightarrow x^2+y^2=2yc \rightarrow c=\frac{x^2+y^2}{2y}$ u v

$\frac{2y\frac{dy}{dx}(x^2+y^2)-(x^2+y^2)\frac{dy}{dx}(2y)}{(2y)^2}=0 \rightarrow \frac{2y(2x+2yy')-(x^2+y^2)(2y')}{4y^2}$

$2xy+4yy'-2x^2y-2y^2y'=0 \rightarrow y'(4y-2y^2)=2x^2y-2xy$

$2y'(2y-y^2)=2(x^2y-xy) \rightarrow y'=\frac{\cancel{2}(x^2y-xy)}{\cancel{2}(2y-y^2)}$

$y'=\frac{x^2y-xy}{2y-y^2} \rightarrow y'=\frac{-1}{\frac{x^2y-xy}{2y-y^2}} \rightarrow y'=(-1)\left(\frac{2y-y^2}{x^2y-xy}\right)$

$\frac{dy}{dx}=-\frac{2y-y^2}{x^2y-xy} \rightarrow (x^2y-xy)dy=-(2y-y^2)dx$

$\cancel{y}(x^2-x)dy=-\cancel{y}(2+y)dx \rightarrow (x^2-x)dy=-(2+y)dx$

$\int\frac{dy}{2+y}+\int\frac{dx}{x^2-x}=0 \rightarrow \ln|2+y|+\ln|x-1|-\ln|x|+C=0$

$e^{\ln|2+y|}+e^{\ln|x-1|}-e^{\ln|x|}=e^{\ln|C|} \rightarrow 2+y+\cancel{x}-1-\cancel{x}=C$

$\boxed{y=-C} \rightarrow$ Verificar esta vaina

6) Encontrar las trayectorias ortogonales de $x^2-xy+y^2=c^2$ $\quad y=ux \quad dy=u\,dx+x\,du$

$2x-\left[x\frac{dy}{dx}(y)+y\frac{dy}{dx}(x)\right]+2yy'=0$

$2x-xy'+y+2yy'=0 \rightarrow y'(2y-x)=y-2x$

$y'=\frac{y-2x}{2y-x} \rightarrow y'=\frac{-1}{\frac{y-2x}{2y-x}} \rightarrow \frac{dy}{dx}=-\frac{2y-x}{y-2x}$

$y=ux \quad \frac{dy}{dx}=u\frac{dy}{dx}(x)+x\frac{du}{dx} \rightarrow \frac{dy}{dx}=u+x\frac{du}{dx}$

$u+x\frac{du}{dx}=-$ Verificar

7) Determina las trayectorias ortogonales de ~~[illegible]~~. $y = Ky^2$

$$x^2+y^2-2Kx=0 \rightarrow x^2+y^2 = 2Kx \rightarrow \frac{x^2+y^2}{x} = 2K$$

$y=$

$$\frac{x\frac{d}{dx}(x^2+y^2)-(x^2+y^2)\frac{d}{dx}(x)}{x^2} = 0 \rightarrow x(2x+2yy')-(x^2+y^2)(1) = 0$$

$$2x^2+2xyy'-x^2-y^2=0 \rightarrow x^2-y^2+2xyy'=0$$

$$2xyy' = y^2-x^2 \rightarrow y' = \frac{y^2-x^2}{2xy} \rightarrow \frac{dy}{dx} = \frac{-1}{\frac{y^2-x^2}{2xy}}$$

$$\frac{dy}{dx} = \frac{2xy}{x^2-y^2} \rightarrow (x^2-y^2)dy = (2xy)dx \qquad x = vy$$

$$v = \frac{x}{y} \qquad dx = v\,dy + y\,dv$$

$$(v^2y^2-y^2)dy = (2vy^2)[v\,dy+y\,dv]$$

$$(v^2y^2-y^2)dy = [2v^2y^2dy + 2vy^3dv] \rightarrow (v^2y^2-y^2-2v^2y^2)dy = 2vy^3dv$$

$$-(v^2y^2-y^2)dy = 2vy^3dv \rightarrow -y^2(v^2-1)dy = 2vy^3dv$$

$$\int \frac{-y^2}{y^3}dy = \int \frac{2v\,dv}{v^2-1} \rightarrow -\int \frac{dy}{y} = \int \frac{2v\,dv}{v^2-1} \rightarrow -\ln y = \ln|v^2-1| + C$$

$$\ln|v^2-1| + \ln|y| = C \rightarrow \ln(y)(v^2-1) = C$$

$$e^{\ln(y)(v^2-1)} = e^C \rightarrow (y)(v^2-1) = 0 \rightarrow y\left(\frac{x^2}{y^2}-1\right) = 0$$

$$y\left(\frac{x^2-y^2}{y^2}\right) = 0 \rightarrow \boxed{\frac{x^2-y^2}{y} = 0}$$