Ecuación Diferencial de Bernoulli

1) $$\frac{dy}{dx} + \frac{y}{x} = x^2y^2 \quad n=2,\ P(x)=\frac{1}{x},\ Q(x)=x^2 \rightarrow y^2=(u^{-1})^2$$

$$u = y^{1-2} \rightarrow u = y^{-1} \rightarrow y = u^{-1} \rightarrow \frac{dy}{dx} = -u^{-2}\frac{du}{dx} \quad y^2=u^{-2}$$

$$-u^{-2}\frac{du}{dx} + \frac{u^{-1}}{x} = x^2u^{-2} \rightarrow \frac{du}{dx} + \frac{\frac{u^{-1}}{x}}{-u^{-2}} = \frac{x^2\cancel{u^{-2}}}{-\cancel{u^{-2}}}$$

$$\frac{du}{dx} + \left(\frac{u^{-1}}{x}\right)(-u^2) = -x^2 \rightarrow \frac{du}{dx} + \left(\frac{1}{xu}\right)\cdot -u^2 = -x^2$$

$$\frac{du}{dx} - \frac{u}{x} = -x^2 \rightarrow e^{-\int\frac{dx}{x}} \rightarrow e^{-\ln x} \rightarrow e^{\ln x^{-1}} = F.I = \frac{1}{x}$$

$$\frac{1}{x}\frac{du}{dx} - \left(\frac{1}{x}\right)\left(\frac{u}{x}\right) = \frac{1}{x}(-x^2) \rightarrow \frac{1}{x}\frac{du}{dx} - \frac{u}{x^2} = -x$$

$$\int\frac{d}{dx}\left[\frac{1}{x}u\right] = -\int x\,dx + C \rightarrow \frac{1}{x}u = -\frac{x^2}{2} + C$$

$$u = \frac{-\frac{x^2}{2}}{1/x} + \frac{C}{1/x} \rightarrow u = \left(\frac{-x^2}{2}\right)\left(\frac{x}{1}\right) + C\left(\frac{x}{1}\right)$$

$$u = -\frac{x^3}{2} + Cx \rightarrow \boxed{y = \frac{2}{Cx - x^3}}$$

2) $$\frac{dy}{dx} - y = e^{2x}y^3 \rightarrow n=3,\ P(x)=-1,\ Q(x)=e^{2x}$$

$$\frac{1}{y^3}\frac{dy}{dx} - \frac{\cancel{y}}{y^3} = \frac{e^{2x}\cancel{y^3}}{\cancel{y^3}} \rightarrow y^{-3}\frac{dy}{dx} - y^{-2} = e^{-2x}$$

$$u = y^{-2} \quad \frac{du}{dx} = -2y^{-3}\frac{du}{dx}$$

$$(\cancel{y^3})(-2\cancel{y^{-3}})\frac{du}{dx} - u = e^{-2x} \rightarrow -2\frac{du}{dx} - u = e^{-2x}$$

$$\frac{du}{dx} + 2u = -2e^{-2x} \rightarrow e^{\int 2dx} \rightarrow e^{2x} = FI$$

$$e^{2x}\frac{du}{dx} + e^{2x}2u = -2\cancel{e^{2x}e^{-2x}} \rightarrow e^{2x}\frac{du}{dx} + 2ue^{2x} = -2$$

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$$\frac{d}{dx}\left[ue^{2x}\right] =$$

3) $$\frac{dy}{dx} = \frac{2y}{x} - x^2y^2 \rightarrow \frac{dy}{dx} - \frac{2y}{x} = -x^2y^2 \rightarrow n=2,\ P(x)=\frac{1}{x},\ Q(x)=-x^2$$

$$y^{-2}\frac{dy}{dx} - \left(\frac{2y}{x}\right)\left(\frac{1}{y^2}\right) = \frac{-x^2\cancel{y^2}}{\cancel{y^2}} \rightarrow y^{-2}\frac{dy}{dx} - 2x^{-1}y^{-1} = -x^2$$

$$u = y^{-1} \quad \frac{dy}{dx} = -2y^{-2}\cdot\frac{du}{dx} \rightarrow -2\frac{du}{dx} - 2x^{-1}u = -x^2$$

$$\frac{du}{dx} + \frac{2x^{-1}u}{2} = x^2 \rightarrow \frac{du}{dx} + 2\frac{u}{x} = x^2 \rightarrow e^{2\int\frac{dx}{x}} \rightarrow F.I = x^2$$

$$x^2\frac{du}{dx} + 2x^{\cancel{2}}\left(\frac{u}{\cancel{x}}\right) = (x^2)(x^2) \rightarrow x^2\frac{du}{dx} + 2xu = x^4$$

$$\int\frac{d}{dx}\left[x^2u\right] = \int x^4dx + C_1 \rightarrow x^2u = \frac{x^5}{5} + C_1 \rightarrow u = \frac{x^{5\,3}}{5x^2} + \frac{C_1}{x^2}$$

$$u = \frac{x^3}{5} + \frac{C_1}{x^2} \rightarrow \frac{1}{y} = \frac{x^3}{5} + \frac{C_1}{x^2}$$

$$y = \frac{1}{\frac{x^3}{5}} + \frac{1}{\frac{C_1}{x^2}} \rightarrow \boxed{y = \frac{5}{x^3} + \frac{x^2}{C_1}}$$

4) $$\frac{dy}{dx} + \frac{y}{x-2} = 5(x-2)y^{1/2} \rightarrow \frac{1}{y^{1/2}}\frac{dy}{dx} + \left(\frac{y^{1/2}}{x-2}\right)\left(\frac{1}{y^{1/2}}\right) = 5(x-2)\frac{\cancel{y^{1/2}}}{\cancel{y^{1/2}}}$$

$$y^{-1/2}\frac{dy}{dx} + \frac{y^{1/2}}{x-2} = 5(x-2) \rightarrow u = y^{1/2} \rightarrow \frac{du}{dx} = \frac{1}{2}y^{-1/2}\frac{du}{dx}$$

$$2\frac{du}{dx} = y^{-1/2}\frac{du}{dx} \rightarrow 2\frac{du}{dx} + \frac{u}{x-2} = 5(x-2) \rightarrow \frac{du}{dx} + \frac{u}{2(x-2)} = \frac{5}{2}(x-2)$$

$$e^{\frac{1}{2}\int\frac{dx}{x-2}} \rightarrow e^{\frac{1}{2}\ln(x-2)} \rightarrow e^{\ln(x-2)^{1/2}} \rightarrow e^{\ln\sqrt{x-2}} \rightarrow F.I = \sqrt{x-2}$$

$$\sqrt{x-2}\frac{du}{dx} + \sqrt{x-2}\frac{u}{x-2} = 5\sqrt{x-2}(x-2) \rightarrow \frac{d}{dx}\left[u\sqrt{x-2}\right] =$$

$$\int\frac{d}{dx}\left[u\sqrt{x-2}\right] = 5\int\sqrt{x-2}(x-2)\,dx + C_1$$

$$u\sqrt{x-2} = \frac{2}{5}(x-2)^{5/2} + C_1 \rightarrow u = \frac{2}{5}\frac{(x-2)^{5/2}}{(x-2)^{1/2}} + \frac{C_1}{(x-2)^{1/2}}$$

$$u = \frac{2}{5}(x-2)^2 + C_1\sqrt{x-2} \rightarrow \boxed{y = \left(\frac{2}{5}(x-2)^2 + C_1\sqrt{x-2}\right)^2}$$

5) $$\frac{dx}{dt} + tx^3 + \frac{x}{t} = 0 \rightarrow \frac{dx}{dt} + \frac{x}{t} = -tx^3 \rightarrow \frac{1}{x^3}\frac{dx}{dt} + \left(\frac{1}{x^3}\right)\left(\frac{x}{t}\right) = -\frac{tx^3}{x^3}$$

$$x^{-3}\frac{dx}{dt} + x^{-2}t^{-1} = -t \rightarrow u = x^{-2} \quad \frac{dy}{dx} = -2x^{-3}\frac{du}{dt} \rightarrow -\frac{1}{2}\frac{dy}{dx} = x^{-3}\frac{du}{dt}$$

$$-\frac{1}{2}\frac{du}{dt} + \frac{u}{t} = -t \rightarrow \frac{du}{dt} - 2\frac{u}{t} = +2t \quad e^{-2\int\frac{dt}{t}} \rightarrow e^{\ln t^{-2}} = \frac{1}{t^2}$$

$$-\frac{1}{2}\left(\frac{1}{t^2}\right)\frac{du}{dt} + 2\left(\frac{1}{t^2}\right)\frac{u}{t} = 2t\left(\frac{1}{t^2}\right) \rightarrow -\frac{1}{t^2}\frac{du}{dt} - \frac{2u}{t^3} = -\frac{2}{t}$$

$$\int\frac{d}{dt}\left[\frac{u}{t^2}\right] = +2\int\frac{dt}{t} + C_1 \rightarrow ut^{-2} = +2\ln t + C_1$$

$$u = \frac{+2}{t^{-2}}\ln t + \frac{C_1}{t^{-2}} \rightarrow u = +2t^2\ln t + C_1t^2$$

$$\frac{1}{x^2} = +2t^2\ln t + C_1t^2 \rightarrow \boxed{x^{-2} = 2t^2\ln t + C_1t^2}$$

6) $$\frac{dy}{dx} + y = e^xy^{-2} \rightarrow y^2\frac{dy}{dx} + (y^2)(y) = \cancel{(y^2)}(e^x\cancel{y^{-2}})$$

$$y^2\frac{dy}{dx} + y^3 = e^x \rightarrow u = y^3 \rightarrow \frac{du}{dx} = 3y^2\frac{du}{dx} \rightarrow \frac{1}{3}\frac{du}{dx} = y^2\frac{du}{dx}$$

$$\frac{1}{3}\frac{du}{dx} + u = e^x \rightarrow \frac{du}{dx} + 3u = 3e^x \rightarrow e^{3\int dx} \rightarrow e^{3x} = F.I.$$

$$e^{3x}\frac{du}{dx} + 3ue^{3x} = (3)(e^x)(e^{3x}) \rightarrow e^{3x}\frac{du}{dx} + 3ue^{3x} = 3e^{4x}$$

$$\int\frac{d}{dx}\left[ue^{3x}\right] = 3\int e^{4x}dx + C_1 \rightarrow ue^{3x} = \frac{3}{4}e^{4x} + C_1$$

$$u = \frac{3}{4}\frac{e^{4x}}{e^{3x}} + \frac{C_1}{e^{3x}} \rightarrow u = \frac{3}{4}e^x + C_1e^{-3x}$$

$$y^3 = \frac{3}{4}e^x + C_1e^{-3x} \rightarrow \boxed{y = \sqrt[3]{\frac{3}{4}e^x + C_1e^{-3x}}}$$

7) $$\frac{dr}{d\theta} = \frac{r^2 + 2r\theta}{\theta^2} \rightarrow \frac{dr}{d\theta} = \frac{r^2}{\theta^2} + \frac{2r\theta}{\theta^2} \rightarrow \frac{dr}{d\theta} - \frac{r^2}{\theta^2} = 2r\theta^{-1}$$

$$\frac{dr}{d\theta} - \frac{2r}{\theta} = r^2\theta^{-2} \rightarrow \frac{1}{r^2}\frac{dr}{d\theta} - \frac{1}{r^2}\left(\frac{2r}{\theta}\right) = \frac{1}{\cancel{r^2}}(\cancel{r^2}\theta^{-2})$$

$$\frac{1}{r^2}\frac{dr}{d\theta} - r^{-1}\theta^{-1} = \theta^{-2} \rightarrow r^{-2}\frac{dr}{d\theta} - \frac{r^{-1}}{\theta} = \theta^{-2} \quad u = r^{-1} \quad \frac{du}{d\theta} = -2r^{-2}\frac{dr}{d\theta}$$

$$-\frac{1}{2}\frac{du}{d\theta} = r^{-2}\frac{dr}{d\theta} \rightarrow -\frac{1}{2}\frac{du}{d\theta} - \frac{u}{\theta} = \theta^{-2} \rightarrow \frac{du}{d\theta} + 2\frac{u}{\theta} = -2\theta^{-2}$$

$$e^{2\int\frac{d\theta}{\theta}} \rightarrow e^{2\ln\theta} \rightarrow F.I = \theta^2 \rightarrow \theta^2\frac{dr}{d\theta} + 2\theta^2\left(\frac{u}{\theta}\right) = -2\cancel{\theta^2}(\cancel{\theta^{-2}})$$

$$\theta^2\frac{dr}{d\theta} + 2\theta^{-1}u = -2 \rightarrow \theta^2\frac{dr}{d\theta} + \frac{2}{\theta}u = -1 \rightarrow \frac{d}{d\theta}\left[u\theta^2\right] = -\int d\theta + C_1$$

$u\theta^2 = -\theta + C_1 \to u = -\frac{\theta}{\theta^2} + \frac{C_1}{\theta^2} \to u = -\theta^{-1} + C_1\theta^{-2}$

$\frac{1}{r} = \frac{-\theta + C_1}{\theta^2} \to \boxed{r = \frac{\theta^2}{C_1 - \theta}}$

8) $\frac{dy}{dx} + y^3x + y = 0 \to \frac{dy}{dx} + y = -y^3x$

$\frac{1}{y^3}\frac{dy}{dx} + \frac{1}{y^3}(y) = \frac{-1}{y^3}(y^3x) \to \frac{1}{y^3}\frac{dy}{dx} + y^{-2} = -x$

$u = y^{-2} \to \frac{du}{dx} = -2y^{-3}\frac{dy}{dx} \to -\frac{1}{2}\frac{du}{dx} = y^{-3}\frac{dy}{dx}$

$-\frac{1}{2}\frac{du}{dx} + u = -x \to \frac{du}{dx} - 2u = 2x \to$

$e^{-2\int dx} \to F.I = e^{-2x} \to e^{-2x}\frac{du}{dx} - 2ue^{-2x} = 2xe^{-2x}$

$\int \frac{d}{dx}\left[ue^{-2x}\right] = \int 2xe^{-2x}dx + C_1$

Derivada		Integral
$2x$	$+$	e^{-2x}
2	$-$	$-\frac{1}{2}e^{-2x}$
0		$\frac{1}{4}e^{-2x}$

$ue^{-2x} = -xe^{-2x} - \frac{1}{2}e^{-2x} + C_1$

$u = -x\frac{e^{-2x}}{e^{-2x}} - \frac{1}{2}\frac{e^{-2x}}{e^{-2x}} + \frac{C_1}{e^{-2x}} \to u = -x - \frac{1}{2} + C_1e^{2x}$

$\frac{1}{y^2} = -x - \frac{1}{2} + C_1e^{2x} \to \boxed{y = \frac{1}{\sqrt{-x - \frac{1}{2} + C_1e^{2x}}}}$

9) $\frac{dy}{dx} + \frac{y}{x} = x^2y^2 \quad \frac{1}{y^2}\frac{dy}{dx} + \frac{1}{y^2}\left(\frac{y}{x}\right) = \frac{x^2y^2}{y^2}$

$\frac{1}{y^2}\frac{dy}{dx} + \frac{y^{-1}}{x} = x^2 \to u = y^{-1} \quad -\frac{du}{dx} = \frac{1}{y^2}\frac{dy}{dx}$

$-\frac{du}{dx} + \frac{u}{x} = x^2 \to \frac{du}{dx} - \frac{u}{x} = -x^2$

$e^{-\int\frac{dx}{x}} \to e^{-\ln x} \to e^{\ln x^{-1}} \to F.I = \frac{1}{x}$

$\frac{1}{x}\frac{du}{dx} - \frac{1}{x}\left(\frac{u}{x}\right) = \left(\frac{1}{x}\right)(-x^2) \to \frac{1}{x}\frac{du}{dx} - \frac{u}{x^2} = -x$

$\int\frac{d}{dx}\left[\frac{u}{x}\right] = -\int x\,dx + C_1 \to \frac{u}{x} = -\frac{x^2}{2} + C_1$

$u = \frac{-x^3}{2} + C_1x \to \boxed{y = \frac{2}{Cx - x^3}}$

10) $\frac{dy}{dx} = y(xy^3 - 1) \to \frac{dy}{dx} = xy^4 - y \to \frac{dy}{dx} + y = xy^4$

$\frac{1}{y^4}\frac{dy}{dx} + \frac{1}{y^4}(y) = \frac{xy^4}{y^4} \to y^{-4}\frac{dy}{dx} + y^{-3} = x$

$u = y^{-3} \quad \frac{du}{dx} = -3y^{-4}\frac{dy}{dx} \to -\frac{1}{3}\frac{du}{dx} = y^{-4}\frac{dy}{dx}$

$-\frac{1}{3}\frac{dy}{dx} + u = x \to \frac{du}{dx} - 3u = -3x \to e^{-3\int dx} \to e^{-3x} = F.I.$

$e^{-3x}\frac{du}{dx} - 3ue^{-3x} = -3xe^{-3x} \to \int\frac{d}{dx}\left[ue^{-3x}\right] = \int -3xe^{-3x}dx + C_1$

Derivada		Integral
$-3x$	$+$	e^{-3x}
-3	$-$	$-\frac{1}{3}e^{-3x}$
0		$-\frac{1}{9}e^{-3x}$

$ue^{-3x} = xe^{-3x} - \frac{1}{3}e^{-3x} + C_1$

$u = x - \frac{1}{3} + C_1e^{3x}$

$\frac{1}{y^3} = x + \frac{1}{3} + C_1e^{3x}$

$y^3 = \frac{1}{x} + 3 + \frac{1}{C_1e^{3x}} \to \boxed{y = \sqrt[3]{\frac{1}{x} + 3 + \frac{1}{C_1e^{3x}}}}$

11) $\to x\frac{dy}{dx} + \frac{y}{x} = \frac{1}{y^2} \to \frac{dy}{dx} + \frac{y}{x} = \frac{1}{xy^2}$

$\frac{dy}{dx} + \frac{y}{x} = x^{-1}y^{-2} \to y^2\frac{dy}{dx} + (y^2)\left(\frac{y}{x}\right) = (y^2)(x^{-1}y^{-2})$

$y^2\frac{dy}{dx} + \frac{y^3}{x} = x^{-1} \quad u = y^3 \quad \frac{du}{dx} = 3y^2\frac{dy}{dx} \to \frac{1}{3}\frac{du}{dx} = y^2\frac{dy}{dx}$

$\frac{1}{3}\frac{du}{dx} + \frac{u}{x} = x^{-1} \to \frac{du}{dx} + 3\frac{u}{x} = 3x^{-1} \to e^{3\int\frac{dx}{x}} \to e^{\ln x^3} \to x^3$

$x^3\frac{du}{dx} + (x^3)\left(\frac{u}{x}\right) = 3(x^3)\left(\frac{1}{x}\right) \to x^3\frac{du}{dx} + x^2u = 3x^2$

$\to \int\frac{d}{dx}\left[ux^3\right] = 3\int x^2dx + C_1 \to ux^3 = \frac{3}{3}x^3 + C_1 \to u = \frac{3}{3}\frac{x^3}{x^3} + \frac{C_1}{x^3}$

$u = \frac{3}{3} + \frac{C_1}{x^3} \to y^3 = \frac{3}{3} + \frac{C_1}{x^3} \to \boxed{y = \sqrt[3]{1 + \frac{C_1}{x^3}}}$

12) $\frac{dy}{dx} - y = e^xy^2 \to \frac{1}{y^2}\frac{dy}{dx} - \left(\frac{1}{y^2}\right)(y) = \frac{e^xy^2}{y^2} \to \frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y} = e^x$

$\frac{1}{y^2}\frac{dy}{dx} - y^{-1} = e^x \to u = y^{-1} \quad \frac{du}{dx} = -\frac{1}{y^2}\frac{dy}{dx} \to -\frac{du}{dx} = \frac{1}{y^2}\frac{dy}{dx}$

$-\frac{du}{dx} - u = e^x \to \frac{du}{dx} + u = -e^x \to e^{\int dx} \to e^x = F.I$

$e^x\frac{du}{dx} + e^xu = -(e^x)(e^x) \to e^x\frac{du}{dx} + e^xu = -e^{2x}$

$\int\frac{d}{dx}\left[ue^x\right] = -\int e^{2x}dx + C_1 \to ue^x = -\frac{1}{2}e^{2x} + C_1$

$u = -\frac{1}{2}\frac{e^{2x}}{e^x} + \frac{C_1}{e^{2x}} \to u = -\frac{1}{2}e^x + \frac{C_1}{e^{2x}} \to y^{-1} = -\frac{1}{2}e^x + \frac{C_1}{e^{2x}}$

$\frac{1}{y} = -\frac{1}{2}e^x + \frac{C_1}{e^{2x}} \to \boxed{y = -\frac{2}{e^x} + \frac{e^{2x}}{C_1}}$

13) $x\frac{dy}{dx} - (1+x)y = xy^2 \to \frac{dy}{dx} - \left(\frac{1}{x} + \frac{x}{x}\right)y = \frac{xy^2}{x}$

$\frac{dy}{dx} - \left(\frac{1}{x} + 1\right)y = y^2 \to \frac{1}{y^2}\frac{dy}{dx} - \left(\frac{1}{x} + 1\right)\left(\frac{1}{y^2}\right)(y) = \frac{y^2}{y^2}$

$\frac{1}{y^2}\frac{dy}{dx} - \left(\frac{1}{y}\right)\left(1 + \frac{1}{x}\right) = 1 \to u = y^{-1} \to \frac{du}{dx} = -\frac{1}{y^2}\frac{dy}{dx} \to -\frac{du}{dx} = \frac{1}{y^2}\frac{dy}{dx}$

$-\frac{dy}{dx} - u\left(1 + \frac{1}{x}\right) = 1 \to \frac{du}{dx} + u\left(1 + \frac{1}{x}\right) = -1 \to e^{\int\left(1 + \frac{1}{x}\right)dx}$

$e^{\int dx + \int\frac{dx}{x}} \to e^{x + \ln x} \to e^xe^{\ln x} \to F.I = xe^x$

$xe^x\frac{du}{dx} + (xe^x)u\left(1 + \frac{1}{x}\right) = -xe^x \to \int\frac{d}{dx}\left[uxe^x\right] = -\int xe^xdx + C_1$

Derivada		Integral
$-x$	$+$	e^x
-1	$-$	e^x
0		e^x

$uxe^x = -xe^x + e^x + C_1$

$u = -\frac{xe^x}{xe^x} + \frac{e^x}{xe^x} + \frac{C_1}{xe^x}$

$u = -1 + \frac{1}{x} + \frac{C_1}{xe^x}$

$\frac{1}{y} = -1 + \frac{1}{x} + \frac{C_1}{xe^x} \to \boxed{y = \frac{xe^x}{C_1} + x - 1}$

Ecuación Diferencial de Jacob "El pollo del mes" Bernoulli

14) $t^2\frac{dy}{dt}+y^2=ty \to \frac{dy}{dt}+\frac{y^2}{t^2}=\frac{ty}{t^2} \to \frac{dy}{dt}+\frac{y^2}{t^2}=\frac{y}{t}$

$\frac{dy}{dt}-\frac{y}{t}=-y^2t^{-2} \to \frac{1}{y^2}\frac{dy}{dt}-\left(\frac{1}{\cancel{y^2}}\right)\left(\frac{\cancel{y}}{t}\right)=-\frac{\cancel{y^2}t^{-2}}{\cancel{y^2}}$

$\frac{1}{y^2}\frac{dy}{dt}-\frac{y^{-1}}{t}=-t^{-2} \to u=y^{-1} \quad -\frac{du}{dt}=\frac{1}{y^2}\frac{dy}{dt}$

$-\frac{du}{dt}-\frac{u}{t}=-t^{-2} \to \frac{du}{dt}+\frac{u}{t}=t^{-2} \to e^{\int\frac{dt}{t}} \to F.I.=t$

$t\frac{du}{dt}+\cancel{t}\left(\frac{u}{\cancel{t}}\right)=(\cancel{t})\left(\frac{1}{t^2}\right) \to t\frac{du}{dt}+u=\frac{1}{t}$

$\int\frac{d}{du}[ut]=\int\frac{1}{t}+C_1 \to ut=\ln t+C_1 \to u=\frac{\ln t}{t}+\frac{C_1}{t}$

$\frac{1}{y}=\frac{\ln t}{t}+\frac{C_1}{t} \to \boxed{y=\frac{t}{\ln t}+\frac{t}{C_1}}$

15) $3(1+t^2)\frac{dy}{dt}=2ty(y^3-1)$

$3(1+t^2)\frac{dy}{dt}=2ty^4-2ty \to 3(1+t^2)\frac{dy}{dt}+2ty=2ty^4$

$\frac{3}{y^4}(1+t^2)\frac{dy}{dt}+\frac{2ty}{y^{4}{}_3}=\frac{2t\cancel{y^4}}{\cancel{y^4}} \to \frac{3}{y^4}(1+t^2)\frac{dy}{dt}+2ty^{-3}=2t$

$3y^{-4}(1+t^2)\frac{dy}{dt}+2ty^{-3}=2t \to u=y^{-3} \quad \frac{du}{dt}=-3y^{-4}\frac{dy}{dt}$

$-\frac{du}{dt}=3y^{-4}\frac{dy}{dt} \to -(1+t^2)\frac{du}{dt}+2ut=2t$

$\frac{du}{dt}-\frac{2ut}{1+t^2}=-\frac{2t}{1+t^2} \to e^{-\int\frac{2t}{1+t^2}} \to e^{-\ln(1+t^2)}$

$v=1+t^2$

$dv=2tdt$

$\frac{dv}{2}=tdt$

$F.I.=\frac{1}{1+t^2} \to \frac{1}{1+t^2}\frac{du}{dt}-\frac{2ut}{(1+t^2)^2}=\frac{-2t}{(1+t^2)^2}$

$\int\frac{d}{dt}\left[\frac{u}{1+t^2}\right]=\int\frac{-2t}{(1+t^2)^2}dt+C_1 \to \frac{u}{1+t^2}=-\int\frac{dv}{v^2}+C_1$

$\frac{u}{1+t^2}=+v^{-1}+C_1 \to \frac{u}{1+t^2}=\frac{1}{1+t^2}+C_1$

$u=\frac{\cancel{1+t^2}}{\cancel{1+t^2}}+C_1(1+t^2) \to y^{-3}=1+C_1(1+t^2)$

$\frac{1}{y^3}=1+C_1(1+t^2) \to \boxed{y=\sqrt[3]{1+\frac{1}{C_1(1+t^2)}}}$

16) $x^2\frac{dy}{dx}-2xy=3y^4 \to \frac{dy}{dx}-\frac{2xy}{x^2}=\frac{3y^4}{x^2}$

$\frac{dy}{dx}-\frac{2y}{x}=3x^{-2}y^4 \to \frac{1}{y^4}\frac{dy}{dx}-\frac{1}{y^{4}{}_3}\left(\frac{2y}{x}\right)=\frac{3x^{-2}\cancel{y^4}}{\cancel{y^4}}$

$\frac{1}{y^4}\frac{dy}{dx}-\frac{2y^{-3}}{x}=3x^{-2} \to u=y^{-3} \to \frac{du}{dx}=-3y^{-4}\frac{dy}{dx}$

$-\frac{1}{3}\frac{du}{dx}=3y^{-4}\frac{dy}{dx} \to -\frac{1}{3}\frac{du}{dx}-2\frac{u}{x}=3x^{-2}$

$\to e^{6\int\frac{dx}{x}} \to F.I.=x^6$

$\frac{du}{dx}+6\frac{u}{x}=-9x^{-2}$

$x^6\frac{du}{dx}+6\frac{u}{x}(x^{\cancel{6}\,5})=-9\frac{x^6}{\cancel{x^2}} \to x^6\frac{du}{dx}+6ux^5=-9x^4$

$\int\frac{d}{dx}[ux^6]=-9\int x^4dx+C_1 \to ux^6=\frac{-9}{5}x^5+C_1$

$u=\frac{-9}{5}\frac{x^5}{x^6}+\frac{C_1}{x^6} \to u=-\frac{9}{5}\cdot\frac{1}{x}+\frac{C_1}{x^6}$

$\frac{1}{y^3}=-\frac{9}{5x}+\frac{C_1}{x^6} \to y^3=-\frac{5x}{9}+\frac{x^6}{C_1} \to \boxed{y=\sqrt[3]{\frac{x^6}{C_1}-\frac{5x}{9}}}$

17) $y^{1/2}\frac{dy}{dx}+y^{3/2}=1 \to \frac{dy}{dx}+\frac{y^{3/2}}{y^{1/2}}=\frac{1}{y^{1/2}}$

$\frac{dy}{dx}+y=y^{-1/2} \to y^{1/2}\frac{dy}{dx}+(y^{1/2})(y)=(\cancel{y^{1/2}})(\cancel{y^{-1/2}})\,1$

$y^{1/2}\frac{dy}{dx}+y^{3/2}=1 \to u=y^{3/2} \to \frac{du}{dx}=\frac{3}{2}y^{1/2}\frac{dy}{dx} \to \frac{2}{3}\frac{du}{dx}=y^{1/2}\frac{dy}{dx}$

$\frac{2}{3}\frac{du}{dx}+u=1 \to \frac{du}{dx}+\frac{3}{2}u=\frac{3}{2} \to e^{\int\frac{3}{2}dx} \to e^{3/2x}=F.I.$

$e^{3/2x}\frac{du}{dx}+ue^{3/2x}=\frac{3}{2}e^{3/2x} \to \int\frac{d}{dx}\left[ue^{3/2x}\right]=\frac{3}{2}\int e^{3/2x}dx+C_1$

$v=3/2x$

$dv=3/2dx$

$2/3dv=dx$

$ue^{3/2x}=\left(\frac{2}{3}\right)\left(\frac{3}{2}\right)e^{3/2x}+C_1$

$u=\frac{\cancel{e^{3/2x}}}{\cancel{e^{3/2x}}}+\frac{C_1}{e^{3/2x}} \to \boxed{y^{3/2}=1+C_1e^{-3/2x}}$