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Survey Paper

Theory and Applications of Adaptive Control—A Survey*

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A survey of adaptive control theory and its applications indicates that when designed with adequate practical precautions it may be used successfully in a variety of applications even though many theoretical problems remain to be solved.

Key Words—Adaptive control; model reference; self-tuning regulators; gain scheduling; stability analysis; stochastic control theory; dual control; auto-tuning.

Abstract—Progress in theory and applications of adaptive control is reviewed. Different approaches are discussed with particular emphasis on model reference adaptive systems and self-tuning regulators. Techniques for analysing adaptive systems are discussed. This includes stability and convergence analysis. It is shown that adaptive control laws can also be obtained from stochastic control theory. Issues of importance for applications are covered. This includes parameterization, tuning, and tracking, as well as different ways of using adaptive control. An overview of applications is given. This includes feasibility studies as well as products based on adaptive techniques.

1. INTRODUCTION

ACCORDING to Webster's dictionary to *adapt* means "to change (oneself) so that one's behavior will conform to new or changed circumstances". The words adaptive control have been used at least from the beginning of the 1950s. There is, for example, a patent on an daptive regulator by Caldwell (1950).

Over the years there have been many attempts to define adaptive control (Truxal, 1964; Saridis, Mendel and Nikolic, 1973). Intuitively an adaptive regulator can change its behaviour in response to changes in the dynamics of the process and the disturbances. Since ordinary feedback was introduced for the same purpose the question of the difference between feedback control and adaptive control immediately arises. A meaningful definition of adaptive control which makes it possible to look at a regulator and decide if it is adaptive or not is still missing. There appears, however, to be a consensus that a constant gain feedback is not an adaptive system. In this paper I will therefore take the pragmatic approach that adaptive control is simply a special type of nonlinear feedback control. Adaptive control often has the characteristic that the states of the process can be separated into two categories, which change at different rates. The slowly changing states are viewed as parameters.

Research on adaptive control was very active in the early 1950s. It was motivated by design of autopilots for high performance aircraft. Such aircraft operate over a wide range of speeds and altitudes. It was found that ordinary constant gain, linear feedback can work well in one operating condition. However, difficulties can be encountered when operating conditions change. A more sophisticated regulator which works well over a wide range of operating conditions is therefore needed. The work on adaptive flight control was characterized by a lot of enthusiasm, bad hardware and nonexisting theory. A presentation of the results is given in Gregory (1959) and Mishkin and Braun (1961). Interest in the area diminished due to lack of insight and a disaster in a flight test (see Taylor and Adkins, 1965).

In the 1960s there were many contributions to control theory, which were important for the development of adaptive control. State space and stability theory were introduced. There were also important results in stochastic control theory. Dynamic programming, introduced by Bellman (1957, 1961) and dual control theory introduced by Feldbaum (1960a, b, 1961a, b, 1965), increased the understanding of adaptive processes. Fundamental contributions were also made by Tsypkin (1971), who showed that many schemes for learning and adaptive control could be described in a common framework as recursive equations of the stochastic approximation type. There were also major developments in system identification and in parameter estimation (Åström and Eykhoff, 1971).

The interest in adaptive control was renewed in the 1970s. The progress in control theory during the previous decade contributed to an improved understanding of adaptive control. The rapid and revolutionary progress in microelectronics has made it possible to implement adaptive regulators simply and cheaply. There is now a vigorous development of the field both at universities and in industry.

There are several surveys on adaptive control. The early work was surveyed by Aseltine, Mancini and Sarture (1958); Stromer (1959) and Jacobs (1961). Surveys of special areas in the field are given by Landau (1974); Wittenmark (1975); Unbehauen and Schmidt (1975); Parks, Schaufelberger and Schmid (1980). The papers by Truxal (1964) and Tsypkin (1973) also given an enlightening perspective. An extensive bibliography which covers more than 700 papers is given by Asher, Andrisani and Dorato (1976). Three books, Narendra and Monopoli (1980); Unbehauen (1980); Harris and Billings (1981), contains representative collections of papers dealing with recent applications.

When selecting the material for this paper I deliberately choose to focus on the simplest types of adaptive regulators. The idea is to describe the principles behind those adaptive schemes which are now finding their way towards applications and products. This means that many interesting adaptive schemes are left out. Self-optimizing controls were recently surveyed by Sternby (1980). Other forms of adaptation that occurs in learning systems and in biological systems are described in Saradis (1977); Mendel and Fu (1970).

2. APPROACHES TO ADAPTIVE CONTROL

Three schemes for parameter adaptive control—gain scheduling, model reference control and self-tuning regulators—are described in a common framework. The starting point is an ordinary feedback control loop with a process and a regulator with adjustable parameters. The key problem is to find a convenient way of changing the regulator parameters in response

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to changes in process and disturbance dynamics. The schemes differ only in the way the parameters of the regulator are adjusted.

Gain scheduling

It is sometimes possible to find auxiliary variables which correlate well with the changes in process dynamics. It is then possible to reduce the effects of parameter variations by changing the parameters of the regulator as functions of the auxiliary variables (Fig. 1). This approach is called *gain scheduling* because the scheme was originally used to accommodate changes in process gain only.

The concept of gain scheduling originated in connection with development of flight control systems. In this application the Mach-number and the dynamic pressure are measured by air data sensors and used as scheduling variables.

The key problem in the design of systems with gain scheduling is to find suitable scheduling variables. This is normally done based on knowledge of the physics of a system. For process control the production rate can often be chosen as a scheduling variable since time constants and time delays are often inversely proportional to production rate.

When scheduling variables have been obtained, the regulator parameters are determined at a number of operating conditions using some suitable design method. Stability and performance of the system are typically evaluated by simulation. Particular attention is given to the transition between different operating conditions. The number of operating conditions are increased if necessary.

It is sometimes possible to obtain gain schedules by introducing normalized dimension-free parameters in such a way that the normalized model does not depend on the operating conditions. The auxiliary measurements are used together with the process measurements to calculate the normalized measurement variables. The normalized control variable is calculated and retransformed before it is applied to the process. An example of this is given in Källström and co-workers (1979). The flow control scheme proposed by Niemi (1981) is also of this type. The regulator obtained can be regarded as composed of two nonlinear static systems with a linear regulator in between. Sometimes the calculation of the normalized variables is based on variables obtained by Kalman filtering. The system then becomes even more complex.

One drawback of gain scheduling is that it is an open-loop compensation. There is no feedback which compensates for an incorrect schedule. Gain scheduling can thus be viewed as feedback control system where the feedback gains are adjusted by feed forward compensation. Another drawback of gain scheduling is that the design is time consuming. The regulator parameters must be determined for many operating conditions. The performance must be checked by extensive simulations. This difficulty is partly avoided if scheduling is based on normalized variables. Gain scheduling has the advantage that the parameters can be changed very quickly in response to process changes. The limiting factors depend on how quickly the auxiliary measurements respond to process changes.

There is a controversy in nomenclature whether gain scheduling should be considered as an adaptive system or not because the parameters are changed in open loop. Irrespective of this discussion, gain scheduling is a very useful technique to reduce the effects of parameter variations. It is in fact the predominant method to handle parameter variations in flight control systems (Stein, 1980). There is a commercial regulator for process control Micro-Scan 1300 made by Taylor Instruments which is based on gain scheduling (Andreiev, 1977).

Model reference adaptive systems MRAS

Another way to adjust the parameters of the regulator is shown in Fig. 2. This scheme was originally developed by Whitaker, Yamron and Kezer (1958) for the servo problem. The specifications are given in terms of a reference model which tells how the process output ideally should respond to the command signal. Notice that the reference model is part of the control system. The regulator can be thought of as consisting of two loops. The inner loop is an ordinary control loop composed of the process and the regulator. The parameters of the regulator are adjusted by the outer loop, in such a way that the error e between the model output y_m and the process output y becomes small. The outer loop is thus also a regulator loop. The key problem is to determine the adjustment mechanism so that a stable system which brings the error to zero is obtained. This problem is nontrivial. It is easy to show that it cannot be solved with a simple linear feedback from the error to the controller parameters.

The following parameter adjustment mechanism, called the 'MIT-rule', was used in the original MRAS

$$\frac{d\theta}{dt} = ke\,\mathrm{grad}_\theta\, e. \tag{1}$$

In this equation e denotes the model error. The components of the vector θ are the adjustable regulator parameters. The components of the vector $\mathrm{grad}_\theta\, e$ are the sensitivity derivatives of the error with respect to the adjustable parameters. The sensitivity derivatives can be generated as outputs of a linear system driven by process inputs and outputs. The number k is a parameter which determines the adaptation rate.

Whitaker and co-workers motivated the rule (1) as follows. Assume that the parameters θ change much slower than the other system variables. To make the square of the error e small it seems reasonable to change the parameters in the direction of the negative gradient of e^2.

If (1) is rewritten as

$$\theta(t) = -k \int^t e(s)\,\mathrm{grad}_\theta\, e(s)\,\mathrm{d}s$$

it is seen that the adjustment mechanism can be thought of as composed of three parts: a linear filter for computing the sensitivity derivatives from process inputs and outputs, a multiplier and an integrator. This configuration is typical for many adaptive systems.

The MIT-rule will perform well if the parameter k is small. The allowable size depends on the magnitude of the reference signal. Consequently it is not possible to give fixed limits which guarantee stability. The MIT-rule can thus give an unstable closed loop system. Modified adjustment rules can be obtained using stability theory. These rules are similar to the MIT-rule. The sensitivity derivatives in (1) will be replaced by other functions. This is discussed further in Section 3.

The MRAS was originally proposed by Whitaker and co-workers (1958). Further work was done by Parks (1966), Hang and Parks (1973), Monopoli (1973), Landau (1974) and Ionescu and Monopoli (1977). There has been a steady interest in the method (Hang and Parks, 1973). Landau's book (Landau, 1979)

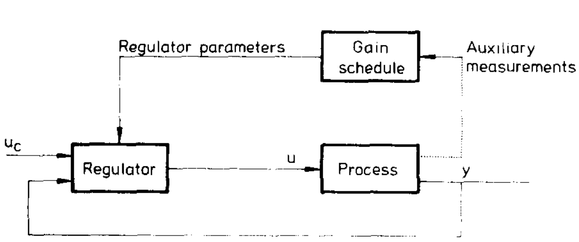

FIG. 1. Block diagram of a system where influences of parameter variations are reduced by gain scheduling.

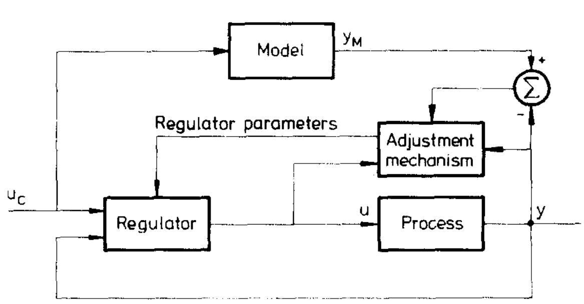

FIG. 2. Block diagram of model reference adaptive system (MRAS).

gives a comprehensive treatment of work up to 1978. It also includes many references. Recent contributions are discussed in Section 4.

The MRAS shown in Fig. 2 is called a *direct* scheme because the regulator parameters are updated directly. There are also other MRAS schemes where the regulator parameters are updated *indirectly* (Narendra and Valavani, 1979).

Self-tuning regulators STR

A third method for adjusting the parameters is to use the self-tuning regulator. Such a system is shown in Fig. 3. The regulator can be thought of as composed of two loops. The inner loop consists of the process and an ordinary linear feedback regulator. The parameters of the regulator are adjusted by the outer loop, which is composed of a recursive parameter estimator and a design calculation. To obtain good estimates it may also be necessary to introduce perturbation signals. This function is not shown in Fig. 3 in order to keep the figure simple.

Notice that the box labelled 'regulator design' in Fig. 3 represents an on-line solution to a design problem for a system with known parameters. This is called the *underlying design problem.* Such a problem can be associated with most adaptive control schemes. However, the problem is often given indirectly. To evaluate adaptive control schemes it is often useful to find the underlying design problem because it will give the characteristics of the system under the ideal conditions when the parameters are known exactly.

The self-tuning regulator was originally developed for the stochastic minimum variance control problem (Åström and Wittenmark, 1973). Since the approach is very flexible with respect to the underlying design method many different extensions have been made. Self-tuners based on phase and amplitude margins are discussed in Åström (1982). Pole-placement self-tuners have been investigated by many authors: Edmunds (1976); Wouters (1977); Wellstead and co-workers (1979); Wellstead, Prager and Zanker; Åström and Wittenmark (1980). Minimum variance self-tuners with different extensions are treated in Peterka (1970, 1982); Åström and Wittenmark (1973, 1974); Clarke and Gawthrop (1975, 1979); Gawthrop (1977). The LQG design method is the basis for the self-tuners presented in Peterka and Åström (1973); Åström (1974); Åström and Zhao-ying (1982); Menga and Mosca (1980).

The self-tuner also contains a recursive parameter estimator. Many different estimation schemes have been used, for example stochastic approximation, least squares, extended and generalized least squares, instrumental variables, extended Kalman filtering and the maximum likelihood method.

The self-tuning regulator was originally proposed by Kalman (1958), who built a special-purpose computer to implement the regulator. Several experimental investigations were carried out as digital computers became available. The self-tuning regulator has recently received considerable attention because it is flexible, easy to understand and easy to implement with microprocessors (Åström, 1980a; Kurz, Isermann and Schumann 1980; Isermann, 1980a).

The self-tuner shown in Fig. 3 is called an *explicit* STR or an STR based on estimation of an explicit process model. It is sometimes possible to re-parameterize the process so that it can be expressed in terms of the regulator parameters. This gives a significant simplification of the algorithm because the design calculations are eliminated. Such a self-tuner is called an *implicit* STR because it is based on estimation of an implicit process model.

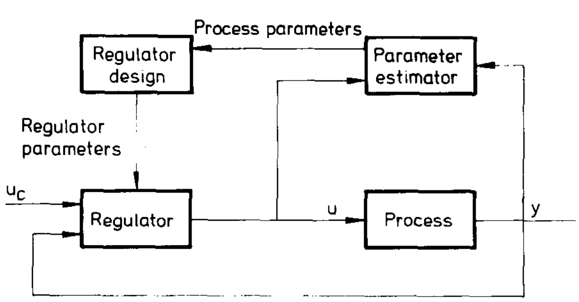

FIG. 3. Block diagram of a self-tuning regulator (STR).

Relations between MRAS and STR

The MRAS was obtained by considering a deterministic servo-problem and the STR by considering a stochastic regulation problem. In spite of the differences in their origin it is clear from Figs. 2 and 3 that the MRAS and the STR are closely related. Both systems have two feedback loops. The inner loop is an ordinary feedback loop with a process and a regulator. The regulator has adjustable parameters which are set by the outer loop. The adjustments are based on feedback from the process inputs and outputs. The methods for design of the inner loop and the techniques used to adjust the parameters in the outer loop may be different, however.

The direct MRAS is closely related to the implicit STR and the indirect MRAS to the explicit STR. The relations between STR and MRAS are further elaborated in the next Section. More details are found in Egardt (1979b, 1980a); Landau (1979); Åström (1980a).

3. AN EXAMPLE

To give a better insight into the different approaches the algorithms will be worked out in detail. The example also serves to introduce some notation needed for the theory in Section 4. A design problem for systems with known parameters is first described. Different adaptive control laws are then given. A pole-placement design is chosen as the underlying design problem. This is useful in order to discuss similarities and differences between self-tuners and model reference adaptive systems. It is also a convenient way to unify many algorithms. Additional details about the design procedure are given in Åström and Wittenmark (1984).

The underlying design problem for known systems

Consider a single input–single output, discrete time system described by

$$Ay = Bu \tag{2}$$

where u is the control signal and y the output signal. The symbols A and B denote relatively prime polynomials in the forward shift operator. Assume that it is desired to find a regulator such that the relation between the command signal and the desired output signal is given by

$$A_m y_m = B_m u_c \tag{3}$$

where A_m and B_m are polynomials.

A general linear control law which combines feedback and feedforward is given by

$$Ru = Tu_c - Sy \tag{4}$$

where R, S, and T are polynomials. This control law represents a negative feedback with the transfer function $-S/R$ and a feed forward with the transfer function T/R.

Elimination of u between (2) and (4) gives the following equation for the closed-loop system

$$(AR + BS)y = BTu_c. \tag{5}$$

The process zeros, given by $B = 0$, will thus be closed-loop zeros unless they are cancelled by corresponding closed-loop poles. Since unstable or poorly damped zeros cannot be cancelled the polynomial B is factored as

$$B = B^+B^- \tag{6}$$

where B^+ contains those factors which can be cancelled and B^- the remaining factors of B. The zeros of B^+ must of course be stable and well damped. To make the factorization unique it is also required that B^+ is monic.

It follows from (5) that the characteristic polynomial of the closed-loop system is $AR + BS$. This polynominal may be designed to have three types of factors: the cancelled process zeros, the desired model poles and the desired observer* poles. Let B^+, A_m and A_0 denote these factors then

$$AR + BS = B^+ A_m A_0. \tag{7}$$

Since B^+ divides B it follows from this equation that B^+ divides R. Hence

$$R = B^+ R_1. \tag{8}$$

Equation (7) can then be written as

$$AR_1 + B^- S = A_0 A_m. \tag{7'}$$

Requiring that the relation (5) between the command signal and the process output should be equal to the desired relation (3) gives

$$B_m = B^- B_m^+ \tag{9}$$

$$T = A_0 B_m^+. \tag{10}$$

The specifications must thus be such that B^- is a factor of B_m otherwise there is no solution to the design problem.

To complete the solution of the problem it remains to give conditions which guarantee that there exist solutions to (7) which give a causal control law. The feedback transfer function S/R is causal if

$$\deg S \leq \deg R.$$

It is thus desirable to find a solution S to (7) which has as low degree as possible. There is always a solution with

$$\deg S \leq \deg A - 1.$$

It then follows from (7) that

$$\deg R = \deg A_0 + \deg A_m + \deg B^+ - \deg A.$$

The condition

$$\deg A_0 \geq 2 \deg A - \deg A_m - \deg B^+ - 1 \tag{11}$$

thus guarantees that the feedback transfer function S/R is causal. Similarly the inequality

$$\deg A_m - \deg B_m \geq \deg A - \deg B. \tag{12}$$

implies that the feed forward transfer function T/R is causal.

To solve the pole-placement design problem which give the selected poles A_m and A_0, with A and B given, the equation (7′) is first solved to obtain R_1 and S. The desired feedback is then given by (4) with R and T given by (8) and (10).

There may be several solutions to the Diophantine equation (7) which satisfy the causality conditions (11) and (12).

All solutions will give the same closed-loop transfer function. The different solutions give however different responses to disturbances and measurement errors.

It follows from (2), (3), (7′) and (8)–(10) that the control law (4) can be written as

$$u = G_m G_p^{-1} u_c - \frac{S}{R} [y - y_m]$$

where

$$G_p = B/A, G_m = B_m/A_m, \text{ and } y_m = G_m u_c.$$

*The introduction of the observer polynomial A_0 is guided by the state-space solution to the problem which is a combination of state feedback and an observer. For details see Åström and Wittenmark (1984).

This shows that the pole-placement design can be interpreted as model following. This is important in order to establish the relations between the STR and the MRAS. Equation (4) is, however, preferable in realizations.

Parameter estimation

The control law (4) is not realizable if the parameters of the model (2) are unknown. However, the parameters may be estimated. There are many ways to do this (Ljung and Söderström, 1983). Many estimators can be described by the recursive equation

$$\theta(t) = \theta(t-1) + P(t)\varphi(t)\varepsilon(t) \tag{13}$$

where the components of the vector θ are the estimated parameters, the vector φ is a regression vector, and ε is the prediction error. The quantities φ and ε depend on the identification method and the model structure. For example if the least-squares method is applied to the model (2) the prediction error is given by

$$\begin{aligned} \varepsilon(t) &= q^{-\deg A}[A(q)y(t) - B(q)u(t)] \\ &= y(t) - \varphi^{\mathrm{T}}(t)\theta(t-1) \end{aligned}$$

where

$$\varphi(t) = [-y(t-1), \ldots, -y(t - \deg A)u(t-d), \ldots, u(t - \deg A)]$$

and

$$d = \deg A - \deg B.$$

The elements of the vector φ are thus delayed values of the input u and the output y.

The quantity P in (13) depends on the particular estimation technique. It may be a constant which gives an updating formula similar to the MIT-rule (1).

Another method (Kaczmarz, 1937) can be viewed as a recursive solution of a set of linear equations. This method is described by (13) with P as the scalar

$$P(t) = \frac{1}{\varphi^{\mathrm{T}}(t)\varphi(t)}. \tag{14}$$

In stochastic approximation methods P is a scalar given by

$$P(t) = \left[\sum_{k=1}^{t} \varphi^{\mathrm{T}}(k)\varphi(k)\right]^{-1}. \tag{15}$$

The recursive least-squares method is given by (13) with

$$P(t) = \left[\sum_{k=1}^{t} \varphi(k)\varphi^{\mathrm{T}}(k)\right]^{-1}. \tag{16}$$

Some minor modifications have to be made if the denominator in (14) and (15) are zero or if the matrix in (16) is singular.

The properties of the estimates depend on the model and the disturbances. In the deterministic case, when there are no disturbances, estimates converge to the correct value in a finite number of steps. The algorithms with P given by (16) have this property, for example. Algorithms with a constant P converge exponentially provided that there is persistent excitation. When data is generated by (2) with independent random variables added to the right-hand side it is necessary to have algorithms where $P(t)$ goes to zero for increasing t in order to get estimates which converge to the correct value. This is the case when P is given by (15) or (16). These algorithms are said to have decreasing gain. An algorithm with decreasing gain is, however, useless when the process parameters are changing. For such a case (14) can be used or (16) can be replaced by

$$P(t) = \left[\sum_{k=1}^{t} \lambda^{t-k}\varphi(k)\varphi^{\mathrm{T}}(k)\right]^{-1} \tag{17}$$

where $0 \le \lambda \le 1$ ia a *forgetting factor or a discounting factor*. This choice of P corresponds to a least-squares estimate with an exponential discounting of past data.

An explicit self-tuner

An explicit self-tuner based on the pole-placement design can be expressed as follows.

Algorithm 1.

Step 1. Estimate the coefficients of the polynomials A and B in (2) recursively using (13) with (14), (15), (16), or (17).

Step 2. Substitute A and B by the estimates obtained in step 1 and solve (7′) to obtain R_1 and S. Calculate R by (8) and T by (10).

Step 3. Calculate the cntrol signal from (4).

Repeat the steps 1–3 at each sampling period. □

An implicit self-tuner

In the implicit self-tuner the design calculations are eliminated and the regulator parameters are updated directly. The algorithm can be derived as follows. One has

$$A_m A_0 y = AR_1 y + B^- Sy = BR_1 u + B^- Sy = B^-[Ru + Sy] \quad (18)$$

where the first equality follows from (7′), the second from (2), and the third from (8). Notice that (18) can be interpreted as a process model, which is parameterized in B^-, R, and S. An estimation of the parameters of the model (18) gives the regulator parameters directly. A solution to the bilinear estimation problem is given in Åström (1980c). In the special case of minimum-phase systems when $B^- = b_0$ the implicit algorithm can be expressed as follows.

Algorithm 2.

Step 1. Estimate the coefficients of the polynomials R, S in (18) recursively using (13) with

$$\begin{aligned}\varepsilon(t) &= q^{-\deg A_0 A_m}[A_0 A_m y(t) - b_0\{Ru(t) + Sy(t)\}] \\ &= q^{-\deg A_0 A_m} A_0 A_m y(t) - \varphi^T(t)\theta(t-1)\end{aligned}$$

where

$$\begin{aligned}\varphi(t) = [&-y(t-d), \ldots, -y(t-d-\deg S)\; b_0 u(t-d), \\ &\ldots, b_0 u(t-d-\deg R)]\end{aligned}$$

with

$$d = \deg A - \deg B$$

and (14), (15), (16), or (17).

Step 2. Calculate the control signal from (4), with R and S substituted by their estimates obtained in step 1.

Repeat the steps 1 and 2 at each sampling period. □

The simple self-tuner in Åström and Wittenmark (1973) corresponds to this algorithm with P given by (17).

Other implicit self-tuners

Algorithm 2 is based on a re-parameterization of the process model (2). The re-parameterization is nontrivial in the sense that (18) has more parameters than (2). The parameterization (18) has the drawback that the model obtained is not linear in the parameters. This makes the parameter estimation more difficult. It is thus natural to investigate other parameterizations. One possibility is to write the model (18) as

$$A_0 A_m y = \mathscr{R} u + \mathscr{S} y \quad (19)$$

where

$$\mathscr{R} = B^- R$$

$$\mathscr{S} = B^- S.$$

The estimated polynomials will then have a common factor B^- which represents the unstable modes. To avoid cancellation of such modes it is then necessary to cancel the common factor before calculating the control law. The following control algorithm is then obtained.

Algorithm 3.

Step 1. Estimate the coefficients of the polynomials $\mathscr{R}$ and $\mathscr{S}$ in the model (19).

Step 2. Cancel possible factors in $\mathscr{R}$ and $\mathscr{S}$ to obtain R and S.

Step 3. Calculate the control signal from (4) where R and S are those obtained in step 2.

Repeat the steps 1–3 at each sampling period. □

This algorithm clearly avoids a nonlinear estimation problem. There are, however, more parameters to estimate than in Algorithm 2 because the parameters of the polynomial B^- are estimated twice.

There are several other possibilities. For the case $B^+ = \text{const}$ it is possible to proceed as follows. Write the model (2) as

$$\begin{aligned} Az &= u \\ y &= Bz. \end{aligned} \quad (20)$$

If the polynomials A and B are coprime there exist two polynomials U and V such that

$$UA + VB = 1 \quad (21)$$

it follows from (10), (20), and (21) that

$$A_0 A_m z = A_0 A_m (UA + VB) z = (RA + SB) z.$$

Equation (20) gives

$$A_0 A_m Uu + A_0 A_m Vy - Ru - Sy = 0.$$

or

$$U(A_0 A_m u) + V(A_0 A_m y) - Ru - Sy = 0. \quad (22)$$

Notice that this equation is linear in the parameters. An adaptive algorithm similar to Algorithm 3 can be constructed based on (22). This was proposed by Elliott (1982). Difficulties with this algorithm have been reported by Johnson, Lawrence and Lyons (1982).

Relations to other algorithms

The simple example is useful because it allows a unification of many different adaptive algorithms. The simple self-tuner based on least-squares estimation and minimum variance control introduced in Åström and Wittenmark (1973) is the special case of Algorithm 2 with $A_0 A_m = z^m$ and $B^- = b_0$.

The model reference adaptive algorithm in Monopoli (1974) is the special case of Algorithm 2 with $B^- = b_0$ and an estimator (13) with P being a constant scalar. The self-tuning controller of Clarke and Gawthrop (1975, 1979) corresponds to the same design with least-squares estimation.

The self-tuning pole placement algorithms of Wellstead, Prager and Zanker (1979) are equivalent to Algorithm 1.

A class of algorithms called IDCOM have been introduced by Richalet and co-workers (1978). These algorithms are based on the idea of estimating an impulse response model of the process and using a control design which gives an exponential decay of disturbances. A simple version of IDCOM may be viewed as the special case of Algorithm 2 with $A_0 A_m = z^{m-1}(z-a)$ and $R = r_0 z^k$.

4. THEORY

The closed-loop systems obtained with adaptive control are nonlinear. This makes analysis difficult, particularly if there are random disturbances. Progress in theory has therefore been slow and painstaking. Current theory gives insight into some special problems. Much work still remains before a reasonably complete theory is available. Analysis of stability, convergence, and performance are key problems. Another purpose of the theory is to find out if control structures like those in Section 2 are reasonable, or if there are better ways to do adaptive control.

Stability

Stability is a basic requirement on a control system. Much effort has been devoted to stability analysis of adaptive systems. It is important to keep in mind that the stability concepts for nonlinear differential equations refer to stability of a particular solution. It may thus happen that one solution is stable and that another solution is unstable.

Stability analysis has not been applied to systems with gain scheduling. This is surprising since such systems are simpler than MRAS and STR.

The stability theories of Liapunov and Popov have been extensively applied to adaptive control. The major developments of MRAS were all inspired by the desire to construct adjustment mechanisms, which would give stable solutions. Parks (1966) applied Liapunov theory to the general MRAS problem for systems with state feedback and also output feedback for systems, whose transfer functions are strictly positive real. Landau (1979) applied hyperstability to a wide variety of MRAS configurations. The key observation in all these works is that the closed-loop system can be represented as shown in Fig. 4. The system can thus be viewed as composed of a linear system and a nonlinear passive system. If the linear system is strictly positive real, it follows from the passivity theorem that the error e goes to zero. See Desoer and Vidyasagar (1975), for example.

To obtain the desired representation it is necessary to parameterize the model so that it is *linear in the parameters*. The model should thus be of the form

$$y(t) = \varphi^{T}(t)\theta.$$

This requirement strongly limits the algorithms that can be considered.

The general problem with output feedback poses additional problems, because it is not possible to obtain the desired representation by filtering the model error. Monopoli (1974) showed that it is necessary to augment the error by adding additional signals. For systems with output feedback the variable ε in Fig. 4 should thus be the augmented error.

There are some important details in the stability proofs based on Fig. 4. To ensure stability it must be shown that the vector φ is bounded. This is easy for systems which only have a variable gain, because the regression vector φ has only one component which is the command signal. The components of the vector φ are, however, in general functions of the process inputs and outputs. It is then a nontrivial problem to ensure that φ is bounded. It should also be noticed that it follows from the passivity theorem that ε goes to zero. The parameter error will not go to zero unless the matrix $\Sigma\varphi\varphi^{T}/t$ is always larger than a positive definite matrix.

When the transfer function $G(s)$ is not positive real there is an additional difficulty because the signal ε is the augmented error. It thus remains to show that the model error also goes to zero.

Several of these difficulties remained unnoticed for many years. The difficulties were pointed out in Morgan and Narendra (1977); Feuer and Morse (1978). Stability proofs were given recently by Egardt (1979a); Fuchs (1979); Goodwin, Ramadge and Caines (1980); Gawthrop (1980); de Larminat (1979); Morse (1980); Narendra and Lin (1979); Narendra, Lin and Valavani (1980). The following result is due to Goodwin, Ramadge and Caines (1980).

Theorem 1. Let the system (2) be controlled by the adaptive Algorithm 2 with $A_0A_m = z^m$, $B^- = b_0$ and Kaczmarz method of parameter estimation, i.e. (14). Assume that

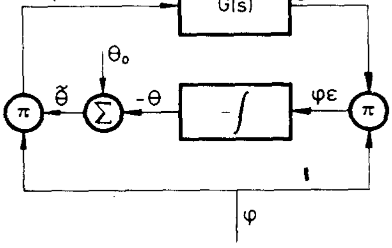

FIG. 4. Block diagram of a MRAS redrawn for the purpose of applying hyperstability theory. The variable ε is the (filtered) model error, φ is a vector of regression variables, θ are the adjustable parameters and θ_0 their true values.

(A1) the pole excess $d = \deg A - \deg B$ is known;
(A2) the estimated model is at least of the same order as the process;
(A3) the polynomial B has all zeros inside the unit disc.

The signals u and y are then bounded and $y(t)$ approaches the command signal $y_c(t)$ as time goes to infinity. □

The proof is not based on hyperstability theory. It is an analysis based on the particular structure of the problem. Notice that the theorem does not say that the parameter estimates converge to their true values.

Theorem 1 is important, because it is a simple and rigorous stability proof for a reasonable adaptive problem. The assumptions required are however very restrictive.

The Assumption A1 means for discrete systems that the time delay is known with a precision, which corresponds to a sampling period. This is not unreasonable. For continuous time systems the assumption means that the slope of the high frequency asymptote of the Bode diagram is known. If this is the case, it is possible to design a robust high gain regulator for the problem (Horowitz, 1963).

Assumption A2 is very restrictive, since it implies that the estimated model must be at least as complex as the true system, which may be nonlinear with distributed parameters. Almost all control systems are in fact designed based on strongly simplified models. High frequency dynamics are often neglected in the simplified models. It is therefore very important that a design method can cope with model uncertainty (Bode, 1945; Horowitz, 1963). The parameters obtained for low-order models depend critically on the frequency content of the input signal (Mannerfelt, 1981). In adaptive control based on low order models it is important that parameters are not updated unless the input signal has sufficient energy at the relevant frequencies.

Assumption A3 is also crucial. It arises from the necessity to have a model, which is linear in the parameters. It follows from (8) that this is possible only if $B^- = b_0$. In other words the underlying design method is based on cancellation of all process zeros. Such a design will not work even for systems with known constant parameters if the system has an unstable inverse.

Notice that Theorem 1 requires that there are no disturbances. The analysis by Egardt (1979a, 1980b, c) also applies to the case when there are disturbances. Egardt has given counterexamples which show that modifications of the algorithms or additional assumptions are necessary if there are disturbances. One possibility is to bound the parameter estimates *a priori* for example by introducing a saturation in the estimator. Another possibility is to introduce a dead zone in the estimator which keeps the estimates constant if the residuals are small. These results also hold for continuous time systems as has also been shown by Narendra and Peterson (1981).

Convergence analysis

Determination of convergence conditions, possible convergence points and convergence rates are the essential problems in convergence analysis.

Convergence analysis reduces to analysis of equations such as (13). Such problems are extensively discussed in the literature on system identification (Eykhoff, 1981). However, there are two complications in the adaptive case. Since the process input is generated by feedback the excitation of the process depends on the process disturbances. It is then difficult to show that the input is persistently exciting, a necessity for convergence of the estimate. The input is also correlated with the disturbances because it is generated by feedback, the regression vector φ in (13) will then also depend on past estimates. This means that (13) is not a simple state equation.

For the purpose of convergence analysis it is commonly assumed that the system is driven by disturbances. It is then possible to apply the powerful results from ergodic theory and martingale theory to establish convergence of some adaptive schemes. There are also some other methods which give more of a system theoretic insight.

Martingale theory

A very general proof for convergence of the least squares algorithm was given by Sternby (1977) by applying a martingale convergence theorem. The convergence condition is simply that $P(t) \to 0$ as $t \to \infty$. An extension of this result was applied to adaptive systems (Sternby and Rootzen, 1982). The results are limited to the model

$$Ay = Bu + e$$

where e is white noise. The approach is Bayesian which means that the parameters are assumed to be random variables. This poses some conceptual difficulties because nothing can be said about convergence for particular values of the parameters.

A convergence theorem for the simple self-tuner based on modified stochastic approximation estimation, and minimum variance control was given by Goodwin, Ramadge and Caines (1981). This case corresponds to the special case of Algorithm 2. A system described by the model

$$Ay = Bu + Ce \tag{23}$$

where e is white noise was investigated. Application of a martingale convergence theorem gave the following result.

Theorem 2. Let the process (23) be controlled by Algorithm 2 with $A_0A_m = z^m$, $B^- = b_0$ and $d = 1$ about a modified stochastic approximation identification, i.e. (13) with

$$P(t) = a_0/t$$

Let Assumptions A1–A3 of Theorem 1 hold and assume moreover that the function

$$G(z) = C(z) - a_0/2 \tag{24}$$

is strictly positive real. Then the inputs and the outputs are mean-square bounded and the adaptive regulator converges to the minimum variance regulator for the system (23). □

Various extensions to larger pole excess and different modifications of the least-squares estimation have been given. A convergence proof for the general Algorithm 2 with least-squares estimation is, however, still not available.

Averaging methods

The algorithms in Section 2 are motivated by the assumption that the parameters change slower than the state variables of the system. The state variables of the closed-loop system can thus be separated into two groups. The variables θ and P in (13) will thus change slowly and the input u and the output y will change rapidly. It is then natural to try to describe the parameters approximately by some approximation of $P\varphi\varepsilon$ in (13). One possibility is to replace $P\varphi\varepsilon$ by its mean value, for example. Such ideas were used by Krylov and Bogoliubov (1937) in deterministic analysis of nonlinear oscillations.

The method of averaging was used by Åström and Wittenmark (1973) to determine possible equilibrium points for the parameters in a tuning problem. For the simple self-tuner based on minimum variance control and least squares or stochastic approximation it was shown that the equilibria are characterized by the equation

$$E\varphi\varepsilon = 0. \tag{25}$$

This equation implies

$$\begin{aligned}&\lim_{t\to\infty}\frac{1}{t}\sum_{k=1}^{t} y(k+\tau)\, y(k) = 0,\ \tau = d,\ d+1,\ldots,d+\deg S\\ &\lim_{t\to\infty}\frac{1}{t}\sum_{k=1}^{t} y(k+\tau)u(k) = 0,\ \tau = d,\ d+1,\ldots,d+\deg R\end{aligned} \tag{26}$$

where $d = \deg A - \deg B$. This characterizes the possible equilibria even if the process is nonlinear or of high order. Åström and Wittenmark (1973) also showed the surprising result that the minimum variance control is an equilibrium even when $C \neq 1$ in (23) and the least-squares estimates are biased.

The method of averaging was developed extensively by Ljung (1977a, b) for the tuning problem. Since the estimator gain $P(t)$ goes to zero as time goes to infinity it can be expected that the separation of fast and slow modes will be better and better as time increases. It was shown by Ljung that asymptotically as time goes to infinity the estimates are described by an ordinary differential equation.

For example, consider an algorithm based on stochastic approximation, i.e. (13), with P given by (15). Introduce the transformed time defined by

$$\tau(t) = c\sum_{k=1}^{t}|P(k)| \tag{27}$$

where $|\cdot|$ denotes some norm and c is a constant. Ljung (1977a) showed that the estimates will approximatively be described by the solutions to the ordinary differential equation

$$\frac{d\theta}{d\tau} = f(\theta) \tag{28}$$

where

$$f(\theta) = E\varphi\varepsilon \tag{29}$$

and E denotes the mean value calculated under the assumption that the parameter θ is constant. Ljung also showed that the estimates converge to the solution of (28) as time increases. The approach can be used for a wide class of problems. Since the estimates are approximately described by ordinary differential equations the technique is referred to as the 'ODE-approach'.

The method is useful for determination of possible equilibrium points and their local stability as well as for determination of convergence rates. The method is however difficult to apply because the function f in (28) is complicated even for simple problems. Another drawback is that the method is based on the assumption that the signals are bounded. It is thus necessary to establish boundedness by some other technique.

Ljung (1977b) applied the ODE-approach to the simple self-tuner based on stochastic approximation and minimum variance control. Theorem 2 was proven under the additional assumption of bounded signals. A similar analysis of the self-tuner based on minimum variance control and least squares estimation showed that the condition corresponding to (24) is that the function

$$G(z) = 1/C(z) - \tfrac{1}{2}$$

is strictly positive real.

Holst (1979) made a detailed analysis of the local behaviour of (28) for the self-tuner based on minimum variance control and least-squares estimation. He showed that the equilibrium corresponding to minimum variance control is locally stable if the function $C(z)$ is positive at the zeros of $B(z)$.

Ljung and Wittenmark (1974) have constructed a counter-example where (28) has a limit cycle. Because of the transformation (28), the estimates will oscillate with ever increasing period. Since the convergence of the parameters depend on the polynomial C, it is clear that convergence can be lost if the characteristics of the disturbances change.

Averaging techniques have also been applied to the tracking problem. In this case the parameters will not converge because the gain $P(t)$ of the estimator does not go to zero. Kushner (1977) and Kushner and Clark (1978) used weak convergence theory to approximate $P\varphi\varepsilon$ in (13) by its mean value and a random term which describes the fluctuations. Singular perturbation methods have also been used to investigate (13).

Stochastic control theory

Regulator structures like MRAS and STR are based on heuristic arguments. It would be appealing to obtain the regulators from a unified theoretical framework. This can be done using nonlinear stochastic control theory. The system and its environment are then described by a stochastic model. The criterion is formulated as to minimize the expected value of a loss function, which is a scalar function of states and controls.

The problem of finding a control, which minimizes the expected loss function, is difficult. Conditions for existence of optimal controls are not known. Under the assumption that a solution exists, a functional equation for the optimal loss function can be derived using dynamic programming. This equation, which is called the *Bellman equation*, can be solved numerically only in very simple cases. The structure of the optimal regulator obtained is shown in Fig. 5 (Bellman, 1961; Bertsekas, 1976). The controller can be thought of as composed of two parts: an estimator and a feedback regulator. The estimator generates the conditional probability distribution of the state from the measurements. This distribution is called the *hyperstate* of the problem. The feedback regulator is a nonlinear function, which maps the hyperstate into the space of control variables.

The structural simplicity of the solution is obtained at the price of introducing the hyperstate, which is a quantity of very high dimension. Notice that the structure of the regulator is similar to the STR in Fig. 3. The self-tuning regulator can be regarded as an approximation where the conditional probability distribution is replaced by a distribution with all mass at the conditional mean value. It will be shown by an example that many interesting properties are lost by such an approximation.

Notice that there is no distinction between the parameters and the other state variables in Fig. 5. This means that the regulator can handle very rapid parameter variations.

The optimal control law has an interesting property. The control attempts to drive the output to its desired value, but it will also introduce perturbations when the parameters are uncertain. This will improve the estimates and the future controls. The optimal control gives the correct balance between maintaining good control and small estimation errors. This property is called *dual control* (Feldbaum, 1965; Florentin, 1971; Jacobs and Patchell, 1972; Bar-Shalom and Tse, 1974, 1976; Tse and Bar-Shalom, 1975). Optimal stochastic control theory also offers other possibilities to obtain sophisticated adaptive algorithms (Saridis, 1977).

Example. Dual control of an integrator. A simple example is used for illustration. Consider a system described by

$$y(t+1) = y(t) + bu(t) + e(t) \tag{30}$$

where u is the control, y the output, e white noise normal $(0, \sigma_e)$. Let the criterion be to minimize the mean square deviation of the output y.

If the parameter b is assumed to be a random variable with a Gaussian prior distribution, the conditional distribution of b, given inputs and outputs up to time t, is Gaussian with mean $\hat{b}(t)$ and standard deviation $\sigma(t)$ (Åström and Wittenmark, 1971). The hyperstate can then be characterized by the triple $(y(t),\hat{b}(t),\sigma(t))$. The equations for updating the hyperstate are the same as the ordinary Kalman filtering equations (Åström, 1970).

Introduce the loss function

$$V_N = \min_u E\left\{ \sum_{k=t+1}^{t+N} y^2(k) \,|\, Y_t \right\} \tag{31}$$

where Y_t denotes the data available at time t, i.e. $\{y(t), y(t-1), \ldots\}$. By introducing the normalized variables

$$\eta = y/\sigma_e, \qquad \beta = \hat{b}/\sigma, \qquad \mu = -u\hat{b}/y \tag{32}$$

it can be shown that V_N depends on η and β only. The Bellman equation for the problem can be written as

$$\begin{aligned} V_N(\eta,\beta) = \min_\mu \Big\{ & 1 + \eta^2[(1-\mu)^2 + \mu^2\beta^{-2}] \\ & + \int_{-\infty}^{\infty} V_{N-1}\,(\eta(1+\mu) + \varepsilon\sqrt{(\beta^2+\mu^2\eta^2)}/\beta, \\ & \sqrt{(\beta^2+\mu^2\eta^2)} - \varepsilon\mu\eta/\beta)\varphi(\varepsilon)\,\mathrm{d}\varepsilon \Big\} \end{aligned} \tag{33}$$

where φ is the normal (0, 1) probability density (Åström, 1978). When the minimization is performed, the control law is obtained as the function $\mu_N(\eta,\beta)$. When the Bellman equation is solved numerically it turns out that the control law is independent of N for large N. A graph of the control law for $N = 30$ is shown in Fig. 6. With the accuracy of the graph there is no difference between the control laws obtained for $N = 20$ and 30.

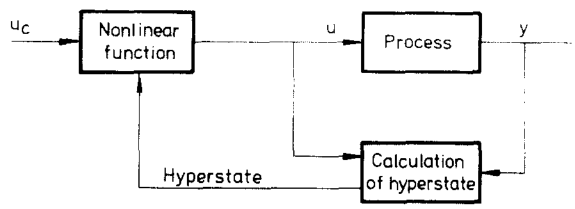

FIG. 5. Block diagram of an adaptive regulator obtained from stochastic control theory.

Some approximations to the optimal control law are also given. The *certainty equivalence control*

$$u(t) = -y(t)/\hat{b} \tag{34}$$

is obtained simply by solving the control problem in the case of known parameters and substituting the known parameters with their estimates. The self-tuning regulator can be interpreted as a certainty equivalence control. Using the normalized variables the control law becomes

$$\mu = 1. \tag{34'}$$

The control law

$$u(t) = -\frac{1}{\hat{b}(t)} \frac{\hat{b}^2(t)}{\hat{b}^2(t) + \sigma^2(t)}\, y(t) \tag{35}$$

is another approximation, which is called *cautious control*, because it hedges and uses lower gain when the estimates are uncertain. In normalized variables this control law can be expressed as

$$\mu = \frac{\beta^2}{1+\beta^2}. \tag{35'}$$

Notice that all control laws are the same for large β, i.e. if the estimate is accurate. The optimal control law is also close to the cautious control for large control errors. For estimates with poor precision and moderate control errors the dual control gives larger control actions than the other control laws. This can be interpreted as the introduction of probing signals to achieve better estimates. Also notice that in a large region the certainty equivalence control is closer to the dual control law than the cautious control is. □

If the Bellman equation can be solved as in the example it is possible to obtain reasonable approximations to the dual control law. Notice, however, that cautious and certainty equivalence controls do not have the dual property.

Unfortunately the solution of the Bellman equation requires substantial computations. In the example the normalized states

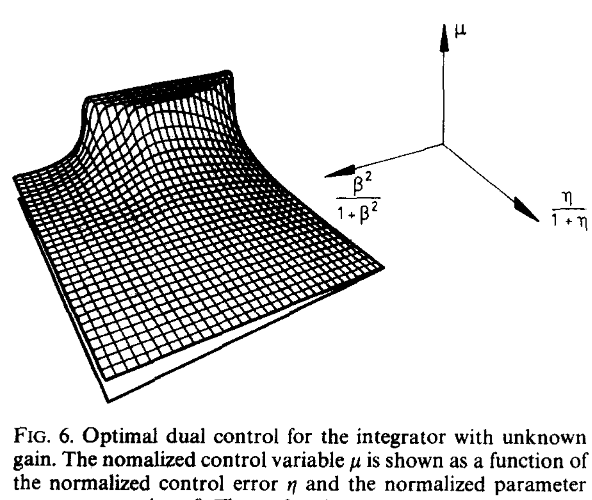

FIG. 6. Optimal dual control for the integrator with unknown gain. The nomalized control variable μ is shown as a function of the normalized control error η and the normalized parameter uncertainty β. The surface is truncated at $\mu = 3$.

were quantized in 64 levels. The solution of the problem for $N = 30$ required 180 CPU-hours on the DEC VAX 11/780. It is not possible to do the calculations in reasonable time for a realistic problem with four or more parameters. To appreciate the difficulties consider the model (23) with $C = 1$ and e white Gaussian noise. Since the conditional distribution of the parameters is Gaussian the triple (φ, θ, P) where φ is the regression vector, θ the estimates and P the conditional covariance matrix is a hyperstate. For a second order model φ and θ have four components and P has 10 different elements. Eighteen real numbers are thus needed to characterize the hyperstate. If each variable is quantized into 32 levels $2^{90} \approx 10^9$ memory locations are needed to store the loss function. When $C \neq 1$ in (23) the complexity increases drastically because the conditional distribution is no longer Gaussian and many more variables are required to characterize the distribution.

5. USES AND ABUSES OF ADAPTIVE TECHNIQUES

Before going into the details of applications of adaptive control some different ways to use adaptive techniques will be discussed.

Auto-tuning

It is possible to tune regulators with three to four parameters by hand if there is not too much interaction between adjustments of different parameters. For more complex regulators it is however necessary to have suitable tuning tools. Traditionally tuning of more complex regulators have followed the route of modelling or identification and regulator design. This is often a time-consuming and costly procedure which can only be applied to important loops or to systems which are made in large quantities.

Both the MRAS and the STR become constant gain feedback controls when the estimated parameters are constant. Compare Figs. 2 and 3. The adaptive loop can thus be used as a *tuner* for a control loop. In such applications the adaptation loop is simply switched on, perturbation signals may be added. The adaptive regulator is run until the performance is satisfactory. The adaptation loop is then disconnected and the system is left running with fixed regulator parameters. Auto-tuning can be considered as a convenient way to incorporate automatic modelling and design into a regulator. It widens the class of problems where systematic design methods can be used cost effectively. Auto-tuning is particularly useful for design methods like feed forward which critically depend on good models.

Automatic tuning can be applied to simple PID controllers as well as to more complicated regulators. It is useful to combine auto-tuning with diagnostic tools for checking the performance of the control loops. For minimum variance control the performance evaluation can be done simply by monitoring the covariances of the inputs and the outputs (Åström, 1970). Tuning can then be initiated when there are indications that a loop is badly tuned.

It is very convenient to introduce auto-tuning in a DDC-package. One tuning algorithm can then serve many loops. Since a good tuning algorithm only requires a few kbytes of memory in a control computer substantial benefits are obtained at marginal cost. Halme (1980) describes a tuning algorithm based on least squares estimation and minimum variance control which has been incorporated in a commercial DDC-package.

Auto-tuning can also be included in single-loop regulators. For example, it is possible to design regulators where the mode switch has three positions: manual, automatic, and tuning. The tuning algorithm represents however a major part of the software of a single loop regulator. Memory requirements will typically be more than doubled when auto-tuning is introduced. A single loop adaptive regulator has been announced by the Leeds & Northrup Co. (Andreiev, 1981).

Automatic construction of gain schedules

The adaptive control loop may also be used to build a gain schedule. The parameters obtained when the system is running in one operating condition are then stored in a table. The gain schedule is obtained when the process has operated at a range of operating conditions, which covers the operating range.

There are also other ways to combine gain scheduling with adaptation. A gain schedule can be used to quickly get the parameters into the correct region and adaptation can then be used for fine tuning.

Adaptive regulators

The adaptive techniques may of course also be used for genuine adaptive control of systems with time varying parameters. There are many ways to do this.

The operator interface is important, since adaptive regulators may also have parameters which must be chosen. It has been my experience that regulators without any externally adjusted parameters can be designed for specific applications, where the purpose of control can be stated *a priori*. The shipsteering autopilot discussed in Section 7 is such an example.

In many cases it is, however, not possible to specify the purpose of control *a priori*. It is at least necessary to tell the regulator what it is expected to do. This can be done by introducing dials that give the desired properties of the closed-loop system. Such dials are called *performance related*. New types of regulators can be designed using this concept. For example, it is possible to have a regulator with one dial, which is labelled with the desired closed-loop bandwidth. Another possibility would be to have a regulator with a dial, which is labelled with the weighting between state deviation and control action in a LQG problem. A third possibility would be to have a dial labelled with the phase margin or the amplitude margin. The characteristics of a regulator with performance related knobs are shown by the following example.

Example. The bandwidth self-tuner. The bandwidth self-tuner is an adaptive regulator which has one adjustable parameter on the front panel which is labelled with the *desired closed-loop bandwidth*. The particular implementation is in the form of a pole-placement self-tuner. The details of the algorithm are given in Åström (1979).

The response of the servo to a square wave command signal is shown in the Fig. 7. It is clear from the figure that the servo performs very well after the first command step. It is seen in Fig. 7(a) that the control signal gives a moderate 'kick' after a command step when the requested bandwidth is low (1.5 rad s^{-1}). The control signal then decreases gradually to a steady-state level. When a larger bandwidth (4.5 rad s^{-1}) is demanded as in Fig. 7(b) the value of the control signal immediately after the

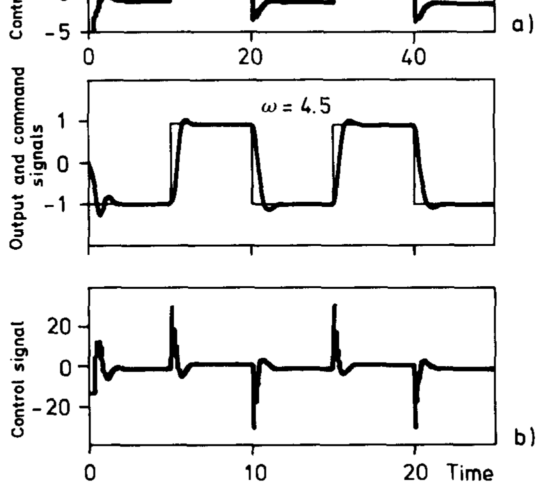

FIG. 7. Simulation of the bandwidth self-tuner. The process has the transfer function $1/(s+1)^2$. The requested bandwidth is 1.5 rad s^{-1} in (a) and 4.5 rad s^{-1} in (b).

step is more than 30 times larger than its steady-state value. A low bandwidth gives a slow response, small control signals and low sensitivity to measurement noise. A high bandwidth gives a fast response. The control actions will however be large and the system will be sensitive to noise. It is easy for an operator to determine the bandwidth which is suitable for a particular application by experimentation. Notice that apart from the specifications in terms of the bandwidth all necessary adjustments are handled automatically by the self-tuner. □

Abuses of adaptive control

An adaptive regulator, being inherently nonlinear, is more complicated than a fixed gain regulator. Before attempting to use adaptive control it is therefore important to first examine if the control problem cannot be solved by constant gain feedback. Problems of this type have only rarely been investigated. Two exceptions are Åström (1980b) and Jacobs (1980). In the vast literature on adaptive control there are many cases where a constant gain feedback can do as well as an adaptive regulator. A typical example is the very ambitious feasibility study of adaptive autopilots for aircrafts (IEEE, 1977). The aircraft used in the experiments could easily be controlled with conventional methods.

Notice that it is not possible to judge the need for adaptive control from the variations of the open loop dynamics over the operating range. Many cases are known where a constant gain feedback can cope well with considerable variations in system dynamics (Åström, 1980d). There are also design techniques for constant gain feedback that can cope with considerable gain variations (Horowitz, 1963).

6. PRACTICAL ASPECTS

Since there is no complete theory for adaptive control, there are in applications many problems which must be solved intuitively with support of simulation. This situation is not unique for adaptive control. Similar problems occur in many other areas.

Consider for example conventional PID control. The system obtained when a PID regulator is connected to a linear system is well understood and can be analysed with great precision. When implementing a PID regulator it is necessary, however, to consider many issues like hand/automatic transfer, bumpless parameter changes, reset windup and nonlinear output, etc. Several PID regulators may also be connected via logical selectors. The systems obtained are then nonlinear and the linear analysis of the ideal case is of limited value. Since the nonlinear modifications give substantial improvements in performance they are widely used although they are poorly understood theoretically and not widely publicised.

The situation is similar for adaptive control. The major difference is that the adaptive systems are inherently nonlinear and more complex. For example in an adaptive regulator windup can occur in the inner loop as well as in the outer loop. A few of these problems will be discussed in this Section. Additional details are found in Clarke and Gawthrop (1981); Schumann, Lachmann and Isermann (1982); Åström (1980d).

Parameter tracking

Since the key property of an adaptive regulator is its ability to track variations in process dynamics, the performance of the parameter estimator is crucial. A fundamental result of system identification theory is that the input signal to the process must be *persistently exciting* or *sufficiently rich* (Åström and Bohlin, 1966). In the adaptive systems the input signal is generated by feedback. Under such circumstances there is no guarantee that the process will be properly excited. On the contrary, good regulation may give a poor excitation. Consequently there are inherent limitations unless extra perturbation signals are introduced, as is suggested by dual control theory. If perturbation signals are not introduced the parameter updating should be switched off when the system is not properly excited. Examples of what happens if this is not down are given by Rohrs and co-workers (1982).

To track parameter variations it is necessary to discount old data. This will involve compromises. If data are discounted too fast the estimates will be uncertain even if the true parameters are constant. If old data are discounted slowly the estimates of constant parameters will be good but it is impossible to track rapid parameter variations.

Exponential discounting is a simple way to discard old data. In the case of least squares estimation this leads to the recursive formula (13) with P given by (17). If the time constant of the exponential discounting is T the forgetting factor is given by

$$\lambda = 1 - h/T$$

where h is the sampling period. The choice of a suitable T is a compromise between the precision of the estimate and the ability to track parameter variations.

The simple exponential discounting works very well if the process is properly excited. Practical experiences which report good results with this method are reported by Borisson and Syding (1976); Åström and co-workers (1977); Westerlund, Toivonen and Nyman (1980).

Estimator windup and bursts

There are severe problems with exponential discounting when the excitation of the process changes. A typical example is when the major excitation is caused by set point changes. There may then be long periods with no excitation at all. When the set point is kept constant the relevant data will be discounted even if no new data is obtained.

The effect can be seen analytically. It follows from (17) that

$$P(t) = [\varphi(t)\varphi^T(t) + \lambda P^{-1}(t-1)]^{-1}. \qquad (34)$$

If the process is poorly excited the input u and the output y are small. Since the components of the new vector φ are delayed values of y and u the vector φ is also small. In the extreme case when the regression vector φ is zero it follows from (34) that the gain matrix $P(t)$ grows exponentially. When the gain becomes sufficiently large the estimator becomes unstable. Small residuals then give very large changes in the parameters, and the closed loop system may become unstable. The process will then be well excited, and the parameter estimates will quickly achieve good values. Looking at the process output there will be periods of good regulation followed by bursts (Morris, Fenton and Nazer, 1977; Fortescue, Kershenbaum and Ydstie, 1981). There will be bursts because the estimator with exponential forgetting is an open-loop unstable system. The phenomenon is therefore also called *estimator windup* in analogy with integrator windup in simple regulators. A typical case of estimator windup is shown in Fig. 8. Notice the nearly exponential growth of the diagonal element of P in the intervals when the command signal remains

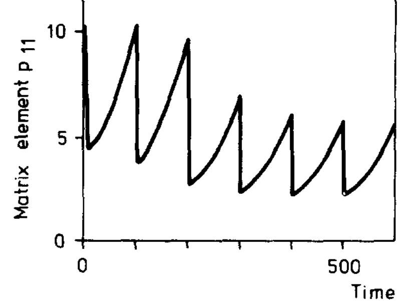

FIG. 8. Estimator windup. The reference signal, the output signal and one diagonal element of the matrix P are shown. Notice that P increases exponentially when the set point is constant.

constant. The variations in P have little effect on the regulation because they are not large enough. They will be much larger if the command changes less frequently. Without further excitation the adaptive loop may then even become unstable.

Notice that φ need not be zero for windup to occur. If φ lies in a given subspace for a period then P will grow exponentially in directions orthogonal to the subspace.

There are many ways to avoid estimator windup and bursts. Since the problem is caused by poor excitation in combination with discounting of old data there are two possibilities to avoid the difficulty, to make sure that the process is properly excited or to eliminate the discounting. The condition for persistent excitation can be monitored and perturbation can be introduced when the excitation is poor. This is in the spirit of dual control theory.

In some cases it is not feasible to introduce extra perturbations to obtain good excitation. Covariance windup can then be avoided by discounting old data only when there is proper excitation. Several ad hoc procedures have been proposed. It has been proposed to update the estimates only when $|\varphi|$ or $|P\varphi|$ are larger than a given quantity. Fortescue, Kershenbaum and Ydstie (1981) have proposed a method for varying the forgetting factor automatically. This decreases the probability for bursts but does not eliminate them. A simple scheme to limit the estimator gain has been proposed by Irving (1979). A forgetting factor $\lambda < 1$ is used if the trace of the matrix P is sufficiently small. The forgetting factor is set to unity if tr P exceeds a critical value. Hägglund (1983) has proposed to forget data only in the directions where new information is obtained.

Robustness

In practice it is necessary to make sure that the regulator works well under extreme conditions. There are many *ad hoc* features that have proven useful. It is often useful to limit the control signal and its rate of change (Wittenmark, 1973). To avoid that a single large measurement error upsets the recursive estimation it is useful to use robust estimation (Huber, 1981; Poljak and Tsypkin, 1976).

Equation (13) is then modified to

$$\theta(t) = \theta(t-1) + P(t)\varphi(t)f(\varepsilon(t)) \tag{35}$$

where the function f is a saturation function or some similar nonlinearity. The function f can be determined from the probability distribution of the disturbances.

Numerics and coding

It is very important that no numerical problems occur in any operating modes. There are potential problems both in the parameter estimation and in the control design. All estimation methods are poorly conditioned if the models are over-parameterized. The conditioning will also depend on the excitation of the process which may change considerably. The least-squares estimation problem is poorly conditioned for high signal to noise ratios. It is particularly harmful if estimation is based on signals which have a high superimposed d.c. level. In such cases it is useful to remove the d.c.-level and to use square root algorithms instead of the ordinary algorithms (Lawson and Hanson, 1974; Peterka, 1975; Bierman, 1977).

Many control design methods are also poorly conditioned for some model parameters. The singularities are typically associated with loss of observability or controllability of the model. Since parameter estimation algorithms may give any values it is important to guard against the difficulties that may arise in explicit or indirect schemes. This problem is discussed in Åström (1979).

Adaptive regulators are easily implemented using microcomputers (Bengtsson, 1979; Clarke and Gawthrop, 1981; Fjeld and Wilhelm, 1981; Glattfelder, Huguenin and Schaufelberger (1980). The simple self-tuner (Algorithm 2) is easily coded in less than 100 lines in a high-level programming language (much less if readabilty is sacrificed). Other adaptive algorithms may require considerably longer codes. The adaptive algorithms are in any case at least an order of magnitude more complex than fixed gain regulators.

Integral action

It is very important that a control system has the ability to maintain small errors in spite of large low frequency disturbances. For fixed gain regulators this is ensured by introducing integral action which gives a high-loop gain at low frequencies. There are several different possibilities to obtain similar effects in adaptive systems. Some adaptive schemes introduce integral action automatically when needed. Integral action can also be forced by choosing special regulator structures. It can also be introduced indirectly via the parameter estimation. A discussion of some alternatives and their pros and cons is given in Åström (1980d).

Supervisory loops

The adaptive systems in Figs. 2 and 3 can be regarded as hierarchical systems with two levels. The lower level is the ordinary feedback loop with the process and the regulator. The adaptation loop which adjusts the parameters represents the higher level. The adaptation loop in typical STR or MRAS requires parameters like the model order, the forgetting factor and the sampling period. A third layer can be added to the hierarchy to set these parameters. It has already been mentioned that the forgetting factor can be determined by monitoring the excitation of the process. By storing process inputs and outputs it is also possible to estimate models having different sampling periods, different orders and different structures. Experimentation with systems of this type are given by Saridis (1977); Schumann, Lachmann and Isermann (1982).

7. APPLICATIONS

There are over 1500 papers on adaptive control, several hundred simulations have been performed, and several hundred experiments on laboratory processes have been made. Adaptive techniques are also starting to be used in commercial products. An overview of the applications is given in this section.

Laboratory experiments

Over the past ten years there have been extensive laboratory experiments with adaptive control mostly in universities but also to an increasing extent in industrial companies. Schemes like gain scheduling, MRAS, and STR have been investigated. The goal of the experiments has been to understand the algorithms and to investigate many of the factors, which are not properly covered by theory.

Industrial feasibility studies

There have been a number of industrial feasibility studies of adaptive control. The following list is not exhaustive but it covers some of the major studies.

autopilots for aircrafts and missiles
- IEEE (1977)
- Young (1981)

autopilots for ships
- Källström and co-workers (1979)
- van Amerongen (1981, 1982)

cement mills (grinding and mixing of raw material)
- Csaki and co-workers (1978)
- Keviczky and co-workers (1978)
- Kunze and Salaba (1978)
- Westerlund, Toivonen and Nyman (1980)
- Westerlund (1981)

chemical reactors
- Harris, MacGregor and Wright (1980)
- Clarke and Gawthrop (1981)
- Buchholt, Clement and Bay Jorgensen (1979)

diesel engines
- Zanker (1980)

digesters
- Sastry (1979)

glass furnaces
- Haber and co-workers (1979)

heat exchangers
- Jensen and Hänsel (1974)
- Zanker (1980)
- Kurz, Isermann and Schumann (1980)

heating and ventilation
- Clarke and Gawthrop (1981)
- Jensen and Hänsel (1974)
- Schumann (1980)

motor drives
- Courtiol (1975)
- Courtiol and Landau (1975)
- Speth (1969)
- Negoesco, Courtiol and Francon (1978)

optical telescope
- Gilbart and Winston (1974)

ore crusher
- Borisson and Syding (1976)

paper machines
- Borisson and Wittenmark (1974)
- Cegrell and Hedqvist (1975)
- Fjeld and Wilhelm (1981)

pH-control
- Shinsky (1974)
- Buchholt and Kümmel (1979)
- Bergmann and Lachmann (1980)
- Jacobs, Hewkin and White (1980)

power systems
- Irving (1979)
- Irving and van Mien (1979)
- Isermann (1980b)

rolling mills
- Bengtsson (1979)
- Seyfried and Stöle (1975)

titanium oxide kiln
- Dumont and Bélanger (1978)

The references above and the recent books on applications of adaptive control, Narendra and Monopoli (1980) and Unbehauen (1980), contain more details and many additional references.

The feasibility studies have shown that there are indeed cases, where adaptive control is very useful. They have also shown that there are cases where the benefits are marginal.

Signal processing applications

Algorithms similar to the ones discussed in this paper have found extensive applications in the communications field. There has in fact been almost parallel development of adaptive algorithms in the control and communications communities. The signal processing problems are mostly concerned with adaptive filtering. Among the problems considered in communications we can mention adaptive prediction in speech coding and adaptive noise cancellation. Adaptive echo cancellation, based on VLSI technology with adjustment of over 100 parameters, is, for example, beginning to be introduced in telephone systems. A survey of these applications are given in Falconer (1980)

Industrial products

Adaptive control is now also finding its way into industrial products. There appears to be many different ways of using adaptive techniques.

Gain scheduling is the predominant technique for design of autopilots for high performance aircrafts. It is considered as a well established standard technology in the aerospace industry. Gain scheduling is also starting to be used in the process industry. A regulator for process control Micro Scan 1300 which uses gain scheduling has been announced by Taylor Instruments (Andreiev, 1977). It is also easy to implement gain scheduling using the modern hardware for distributed process control.

There are also products based on the STR and the MRAS, both general purpose self-tuners as stand alone systems and software in DDC-packages. A self-tuning PID regulator has been announced by Leeds & Northrup Co. The Swedish company ASEA AB makes a general purpose self-tuner NOVATUNE and a general purpose adaptive optimizer NOVAMAX. The Finnish cement factory Lohja Corporation has announced a cement kiln control system with a self-tuner. There are more commercial adaptive regulators which are on their way to the market. Two examples are chosen to illustrate typical industrial products.

Example. A ship steering autopilot. A ship has to operate in a varying environment: wind, waves, and currents may change considerably. The dynamics of a ship may also vary with trim, loading, and water depth. An adaptive autopilot based on recursive estimation and LQG control is manufactured by Kockumation AB in Sweden. Fig. 9 compares the performances of this autopilot with a conventional autopilot under comparable conditions. It is clear from the figure that the performance of the adaptive autopilot is superior to the conventional system. The reason for this is that the control law is more complex. If the operating conditions were known it would be possible to redesign the conventional autopilot so that it gives the same performance as the adaptive system. In the particular case this would, however, require a regulator with at least eight parameters. It would not be practical to tune such a regulator manually. □

The following example illustrates the benefits of self-tuning in a typical process control problem.

Example. Paper machine control. The feasibility of minimum variance control as a design method for control of basis weight and moisture content of a paper machine was established by Åström (1967). The application of a self-tuner for adjusting both feedback and feed forward parameters was suggested in Åström and Wittenmark (1973). The industrial feasibility of such a scheme was demonstrated by Borisson and Wittenmark (1974) and Cegrell and Hedqvist (1975). Five years later the concept was taken up by one of the manufacturers of paper machine controls (Fjeld and Wilhelm, 1981). The histogram in Fig. 10 shows a comparison between the self-tuner and a conventional system. The figure indicates that the ability of the self-tuner to adjust the parameters in response to a changing environment is beneficial. □

8. CONCLUSIONS

The adaptive technique is slowly emerging after 25 years of research and experimentation. Important theoretical results on

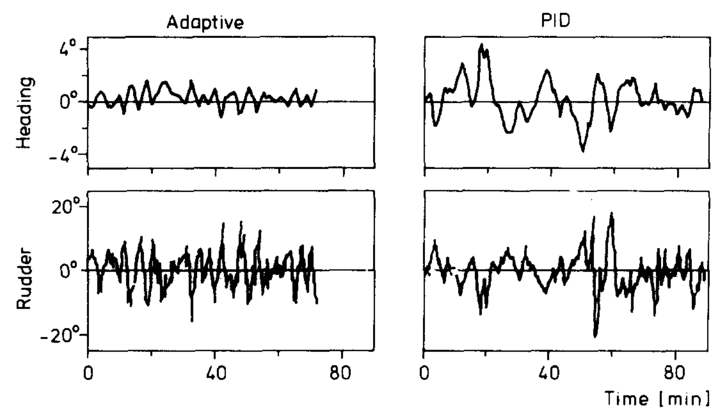

FIG. 9. Heading and rudder angles for a conventional and an adaptive autopilot. From Källström and co-workers (1979).

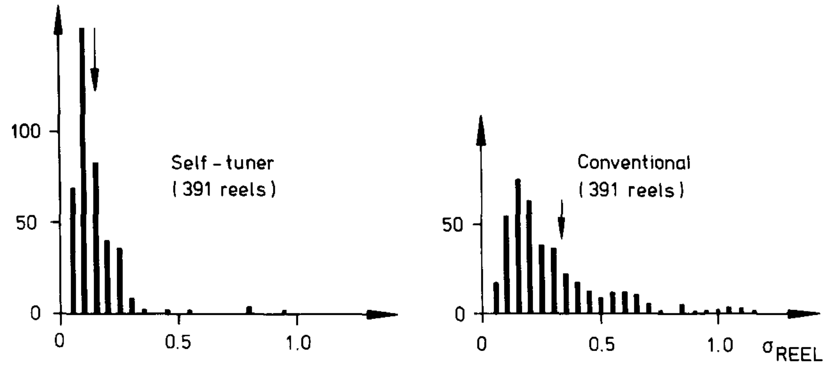

FIG. 10. Histograms for standard deviation in moisture for a self-tuner and a conventional fixed gain regulator. From Fjeld and Wilhelm (1981).

stability and structure have been established. Much theoretical work still remains to be done. The advent of microprocessors has been a strong driving force for the applications. Laboratory experiments and industrial feasibility studies have contributed to a better understanding of the practical aspects of adaptive control. There are also a number of adaptive regulators appearing on the market.

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