Control of a Nonlinear Coupled Three Tank System using Feedback Linearization

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Abstract – **This paper presents the modeling and control of a nonlinear coupled three tank system (CTTS). In this work, nonlinear state space dynamics of CTTS are derived. The objective is to control the level and temperature of water in each tank. The control is achieved using classical nonlinear feedback linearization technique. Input and output scaling is done according to the interface with a standard PLC. Design steps of nonlinear control and input/output responses are shown.**

Keywords – Multi-Input-Multi-Output (MIMO) system, Coupled Three-Tank System (CTTS), Feedback Linearization (FBL), Programmable Logic Control (PLC)

I. INTRODUCTION

Feedback linearization (FBL) is the most widely used technique for the control of nonlinear systems. It is based on some early work of Krener [1] and Brocket [2] according to which a large class of nonlinear systems can be exactly linearized by a combination of a nonlinear transformation of state coordinates and a nonlinear state feedback control law. A good survey of theory can be found in work done by Isidori [3], Nijmeijer and Van Der Schaft [4], Slotine and Lie [5].

The central idea of this technique is the algebraic transformation of the nonlinear system dynamics into a fully or partly linear system so that conventional linear control techniques can be applied. The main difference between feedback linearization and conventional linearization is that feedback linearization is achieved by exact state transformations and feedback, rather than by linear approximations of the dynamics [5]. It results in cancellation of the nonlinearities present in a nonlinear system in such a way that the closed-loop dynamics of the system is linear.

This paper presents the control of a nonlinear multi-input-multi-output (MIMO) system using the technique of feedback linearization by which the original nonlinear system can be transformed into a linear and controllable system by nonlinear state space change of coordinates and a state feedback control law. The MIMO system under consideration is a coupled three tank system (CTTS).

II. COUPLED THREE TANK SYSTEM AND ITS MODELING

The sketch of CTTS is shown in Fig 1. It consists of one reservoir, three tanks, three pumps P_1, P_2, P_3, seven on-off valves V_1, V_2, V_3, V_4, V_5, V_6, V_7, seven level sensors L_1, L_2, L_3, L_4, L_5, L_6, L_7, four temperature sensors and five flow-meters F_1, F_2, F_3, F_4, F_5 The system has two feedback loops of the flow of water; one between tank-1 and tank-2 and the second between tank-2 and tank-3. Inputs to this system are voltages applied to the pumps and heat inputs. Outputs of the system are level and temperature of water in each tank. Levels are measured in centimeters and temperatures are measured in Kelvin.

To model the system, it is assumed that there are no thermal losses, thermal delays, pump delays in the system, mixing is perfect and hydraulic and friction losses are negligible. The system has been modeled using Bernoulli's law, principle of conservation of fundamental quantities mass and energy.

According to Bernoulli's law, the flow 'F_0' out of an outlet having area of cross-section 'a' of a tank having height of liquid 'h' is given by:

$$F_0 = a \times \sqrt{2 \times g \times h} \tag{1}$$

In (1), 'g' is the gravitational acceleration whose value is equal to $9.8\, m/s^2$. Let 'V' be an on/off valve having binary logic 0 or 1 according to it's OFF or ON state respectively, then the effect of this valve can be introduced as follows:

$$F_0 = V \times a \times \sqrt{2 \times g \times h} \tag{2}$$

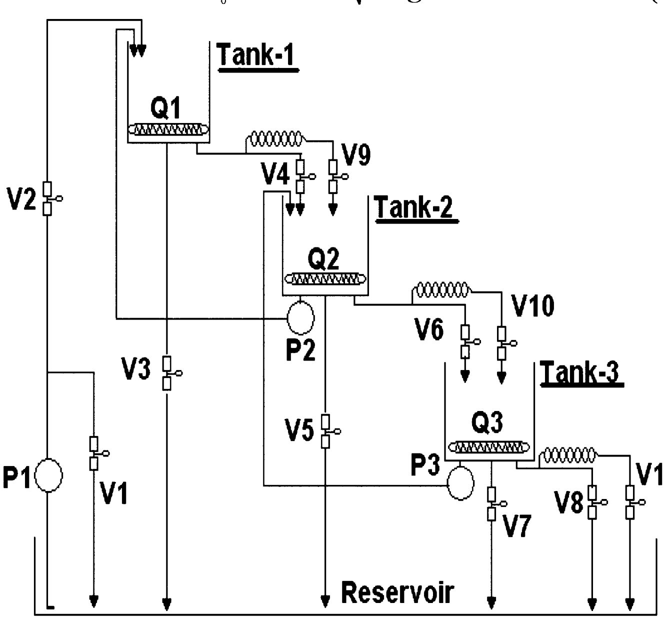

Fig. 1. Sketch of coupled three tank system

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Equation (2) shows that when valve is open (i.e. $V = 1$), the output-flow of the pipe is given by (1) and when valve is closed (i.e. $V = 0$), the output-flow of the pipe is equal to zero.

Consider a pipe of height i.e. $'H'$ attached to the outlet. Then outflow is given by:

$$F_0 = V \times a \times \sqrt{2 \times g \times (h + H)} \tag{3}$$

If $'v'$ is the voltage applied to the pump, then outflow of the pump $'F_p'$ out of a pump is given by:

$$F_p = K \times v \tag{4}$$

where $'K'$ is the constant of proportionality.

The rate of change of volume in a tank is equal to the difference of the inflow and outflow of the tank. Let $'n'$ be the total number of flows into a single tank and $'m'$ be the total number of flows out of the same tank. Let $'A'$ be the area of the tank and $'h'$ be the level of the water in the tank. As area of the tank is constant, so change in volume of water in the tank is dependent only on the level of water in the tank. Let $'F_i'$ be the input flow through i^{th} inlet of the tank and $'F_j'$ be the output flow of the j^{th} outlet of the tank, then according to the law of conservation of fundamental quantity mass:

$$A \times \frac{dh}{dt} = \sum_{i=1}^{n} F_i - \sum_{j=1}^{m} F_j \tag{5}$$

Let $'T'$, $'T_i'$ and $'T_j'$ be the temperatures of water in the tank, water coming into the tank from i^{th} inlet and water leaving the tank via j^{th} outlet respectively. Let $'\ell'$ be the density of water. According to the law of conservation of fundamental quantity energy:

$$\frac{d\left(\ell A h C_p T\right)}{dx} = \ell C_p \sum_{i=1}^{n} F_i T_i - \ell C_p \sum_{j=1}^{m} F_j T \tag{6}$$

Simplifying (6), applying product rule of differentiation and then using (5), we get;

$$Ah \frac{dT}{dx} = \sum_{i=1}^{n} F_i \left(T_i - T\right) \tag{7}$$

Using (1) to (7), the state equations obtained for the three tanks are as follows:

A. State Equations for Tank-1

$$A_1 \frac{dh_1}{dt} = \begin{bmatrix} -V_3 a\sqrt{2g\left(H_{11} + h_1\right)} + V_2 K_1 v_1 \\ -V_4 a\sqrt{2g\left(H_{12} + h_1\right)} + K_2 v_2 \end{bmatrix} \tag{8}$$

$$\frac{dT_1}{dt} = \begin{bmatrix} -K_2 v_2 \left(T_1 - T_2\right) + \dfrac{Q_1}{\ell C_p} \\ -V_2 K_1 v_1 \left(T_1 - T_0\right) \end{bmatrix} \times \frac{1}{A_1 h_1} \tag{9}$$

B. State Equations for Tank-2

$$A_2 \frac{dh_2}{dt} = \begin{bmatrix} -V_5 a\sqrt{2g\left(H_{21} + h_2\right)} - V_6 a\sqrt{2g\left(H_{22} + h_2\right)} \\ +V_4 a\sqrt{2g\left(H_{12} + h_1\right)} + K_3 v_3 - K_2 v_2 \end{bmatrix} \tag{10}$$

$$\frac{dT_2}{dt} = \begin{bmatrix} -K_3 v_3 \left(T_2 - T_3\right) + \dfrac{Q_2}{\ell C_p} \\ -V_4 a\sqrt{2g\left(H_{12} + h_1\right)}\left(T_2 - T_1\right) \end{bmatrix} \times \frac{1}{A_2 h_2} \tag{11}$$

C. State Equations for Tank-3

$$A_3 \frac{dh_3}{dt} = \begin{bmatrix} -V_7 a\sqrt{2g\left(H_{31} + h_3\right)} - K_3 v_3 \\ +V_6 a\sqrt{2g\left(H_{22} + h_2\right)} \end{bmatrix} \tag{12}$$

$$\frac{dT_3}{dt} = \left[-V_6 a\sqrt{2g\left(H_{22} + h_2\right)}\left(T_3 - T_2\right) + \frac{Q_3}{\ell C_p} \right] \times \frac{1}{A_3 h_3} \tag{13}$$

Reference [6] can be seen for more details.

III. INPUT AND OUTPUT SCALING AND OPERATING POINT CONSTANTS

Scaling is very important in practical applications because it makes model analysis and controller design (weight selection) much simpler and easier [7]. The specifications of the system should be considered very thoroughly to have a better scaling. Inputs and outputs should be scaled properly to get a maximum variation of output for a maximum variation of input. Table 1 shows the constants used with description and numerical value. For details, [6] can be seen.

TABLE 1
CONSTANTS WITH DESCRIPTION AND VALUES

Constant	Description	Value with Units
V_2, V_4, V_6, V_7	Valves 2, 4, 6, 7	1
V_3, V_5	Valves 3, 5	0
A_1, A_2, A_3	Cross section Area of Tanks-1, 2, 3	126.6769 cm^2
a	Cross section of pipes	0.2419 cm^2
K_1, K_2, K_3	Gain Constants of Pumps	1.3887
H_{11}	Height of pipe through V_3	135 cm
H_{12}	Height of pipe through V_4	30.48 cm
H_{21}	Height of pipe through V_5	82.55 cm
H_{22}	Height of pipe through V_6	30.48 cm
H_{31}	Height of pipe through V_7	30.48 cm
T_0	Input water Temperature	303 K
$\hat{v}_1$, $\hat{v}_2$, $\hat{v}_3$	Operating Voltages of pumps	12 V
$\hat{Q}_1$, $\hat{Q}_2$, $\hat{Q}_3$	Operating Powers of Heaters	1000 Watt
ℓ	Density of Water	1000 Kg/cm^3
C_p	Specific Heat of Water	4185 J/Kg.K

IV. CONTROLLER DESIGN USING FEEDBACK LINEARIZATION

Consider a MIMO system of the form

$$\dot{\underline{x}} = \underline{f}(\underline{x}) + \sum_{j=1}^{m} \underline{g}_j(\underline{x})u_j \tag{14}$$

$$y_1 = h_1(x) \tag{15}$$

$$y_2 = h_2(x) \tag{16}$$

$$\vdots$$

$$y_m = h_m(x) \tag{17}$$

where $\underline{f}$ and $\underline{g}_j$'s are smooth vector fields and $h_j(x)$'s are smooth functions defined on an open set of R^n. Then for CTTS,

$$x_i = h_i, u_i = v_i, 1 \le i \le 3 \tag{18}$$

$$x_j = T_{j-3}, u_j = Q_{j-3}, 4 \le j \le 6 \tag{19}$$

$$\begin{aligned}\underline{y} = \underline{h}(\underline{x}) &= \left[h_1(\underline{x}) \quad h_2(\underline{x}) \quad h_3(\underline{x}) \quad h_4(\underline{x}) \quad h_5(\underline{x}) \quad h_6(\underline{x})\right]^T \\ &= \left[x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6\right]^T\end{aligned} \tag{20}$$

$$\underline{f}(\underline{x}) = \begin{bmatrix} -0.0845\sqrt{30.48+x_1} \\ 0.0845\left(\sqrt{30.48+x_1} - \sqrt{30.48+x_2}\right) \\ 0.0845\left(\sqrt{30.48+x_2} - \sqrt{30.48+x_3}\right) \\ 0 \\ \dfrac{-0.0845\sqrt{(30.48+x_1)}(x_5-x_4)}{x_2} \\ \dfrac{-0.0845\sqrt{(30.48+x_2)}(x_6-x_5)}{x_3} \end{bmatrix} \tag{21}$$

$$\underline{g}(\underline{x}) = \left[\underline{g}_1(\underline{x}) \quad \underline{g}_2(\underline{x}) \quad \underline{g}_3(\underline{x}) \quad \underline{g}_4(\underline{x}) \quad \underline{g}_5(\underline{x}) \quad \underline{g}_6(\underline{x})\right] \tag{22}$$

where

$$\underline{g}_1 = \begin{bmatrix} 0.011 \\ 0 \\ 0 \\ \dfrac{0.011(x_4-303)}{x_1} \\ 0 \\ 0 \end{bmatrix}, \underline{g}_3 = \begin{bmatrix} 0 \\ 0.011 \\ -0.011 \\ 0 \\ \dfrac{-0.011(x_5-x_6)}{x_2} \\ 0 \end{bmatrix}, \underline{g}_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \dfrac{0.0019}{x_2} \\ 0 \end{bmatrix},$$

$$\underline{g}_2 = \begin{bmatrix} 0.011 \\ -0.011 \\ 0 \\ \dfrac{-0.011(x_4-x_5)}{x_1} \\ 0 \\ 0 \end{bmatrix}, \underline{g}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{0.0019}{x_1} \\ 0 \\ 0 \end{bmatrix}, \underline{g}_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \dfrac{0.0019}{x_3} \end{bmatrix}$$

A. Step 1: Equilibrium Point

To find the equilibrium point of a system, we need to solve the following:

$$\dot{\underline{x}} = 0$$

To which equilibrium point, a system will converge, depends on the initial conditions. Optimization toolbox of MATLAB is used to solve for equilibrium point. $x_{initial}$ is the vector representing initial values of the states.

$$x_{initial} = \left[5 \quad 5 \quad 5 \quad 300 \quad 300 \quad 300\right]^T \tag{23}$$

The technique used to find the solution uses large scale algorithm to solve for the problems with bound constraints. Different options can be set according to the requirements. For CTTS, the upper bound for the levels is 25 cm whereas the lower bound is 0 cm. For temperature of water in each tank, the upper bound is 373 K and lower bound is 273 K. The equilibrium point thus obtained is:

$$\begin{aligned}\underline{x}^0 &= \left[x_1^0 \quad x_2^0 \quad x_3^0 \quad x_4^0 \quad x_5^0 \quad x_6^0\right] \\ &= \left[15.8 \quad 23.47 \quad 23.27 \quad 314.59 \quad 317.76 \quad 320.314\right]^T\end{aligned} \tag{24}$$

This equilibrium point is needed in the verification of the two conditions for a MIMO system to have a particular vector relative degree.

B. Step 2: Vector Relative Degree

Let r_i be the relative degree of y_i, then r_i is equal to the number of times, output y_i needs to be differentiated until any of the inputs appears explicitly in the expression.

$$\Rightarrow r_i = 1, 1 \le i \le 6 \tag{25}$$

If

$$\sum_{i=1}^{m} r_i = n \tag{26}$$

Then nonlinear system can be exactly feedback linearized [3] which is true for CTTS.

There are two conditions relevant to vector relative degree which must be satisfied for the MIMO system.

a. Condition No.1

For each y_i, for at least one $\underline{g}_j$,

$$L_{g_j} L_f^{r_i - 1} h_i(\underline{x}^0) \ne 0, 1 \le i, j \le m \tag{27}$$

For output $y_1 = h_1(\underline{x}) = x_1$,

$$r_1 = 1 \Rightarrow L_f^0 h_1 = h_1 = x_1 \tag{28}$$

$$\begin{aligned}L_{g_1} L_f^0 h_1 &= \frac{\partial(L_f^0 h_1)}{\partial \underline{x}} \underline{g}_1(\underline{x}) = \frac{\partial(x_1)}{\partial \underline{x}} \underline{g}_1(\underline{x}) \\ &= \left[1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\right] \underline{g}_1(\underline{x}) = 0.011 \ne 0\end{aligned} \tag{29}$$

$$L_{g_2} L_f^0 h_1 = \frac{\partial(L_f^0 h_1)}{\partial \underline{x}} \underline{g}_2(\underline{x}) = \frac{\partial(x_1)}{\partial \underline{x}} \underline{g}_2(\underline{x}) = 0.011 \ne 0 \tag{30}$$

$$L_{g_3} L_f^0 h_1 = L_{g_4} L_f^0 h_1 = L_{g_5} L_f^0 h_1 = L_{g_6} L_f^0 h_1 = 0 \tag{31}$$

For output $y_2 = h_2(\underline{x}) = x_2$,

$$r_2 = 1 \Rightarrow L_f^0 h_2 = h_2 = x_2 \tag{32}$$

$$L_{g_2} L_f^0 h_2 = -0.011 \ne 0 \tag{33}$$

$$L_{g_3}L_f^0h_2 = 0.011 \neq 0 \quad (34)$$

$$L_{g_1}L_f^0h_2 = L_{g_4}L_f^0h_2 = L_{g_5}L_f^0h_2 = L_{g_6}L_f^0h_2 = 0 \quad (35)$$

For output $y_3 = h_3(\underline{x}) = x_3$,

$$r_3 = 1 \Rightarrow L_f^0h_3 = h_3 = x_3 \quad (36)$$

$$L_{g_1}L_f^0h_3 = L_{g_2}L_f^0h_3 = L_{g_4}L_f^0h_3 = L_{g_5}L_f^0h_3 = L_{g_6}L_f^0h_3 = 0 \quad (37)$$

$$L_{g_3}L_f^0h_3 = -0.011 \neq 0 \quad (38)$$

For output $y_4 = h_4(\underline{x}) = x_4$,

$$r_4 = 1 \Rightarrow L_f^0h_4 = h_4 = x_4 \quad (39)$$

$$L_{g_1}L_f^0h_4 = -\frac{0.011(x_4^0 - 303)}{x_1^0} \neq 0 \quad (40)$$

$$L_{g_2}L_f^0h_4 = -\frac{0.011(x_4^0 - x_5^0)}{x_1^0} \neq 0 \quad (41)$$

$$L_{g_4}L_f^0h_4 = \frac{0.0019}{x_1^0} \neq 0 \quad (42)$$

$$L_{g_3}L_f^0h_4 = L_{g_5}L_f^0h_4 = L_{g_6}L_f^0h_4 = 0 \quad (43)$$

For output $y_5 = h_5(\underline{x}) = x_5$,

$$r_5 = 1 \Rightarrow L_f^0h_5 = h_5 = x_5 \quad (44)$$

$$L_{g_1}L_f^0h_5 = L_{g_2}L_f^0h_5 = L_{g_4}L_f^0h_5 = L_{g_6}L_f^0h_5 = 0 \quad (45)$$

$$L_{g_3}L_f^0h_5 = -\frac{0.011(x_5^0 - x_6^0)}{x_2^0} \neq 0 \quad (46)$$

$$L_{g_5}L_f^0h_5 = \frac{0.0019}{x_2^0} \neq 0 \quad (47)$$

For output $y_6 = h_6(\underline{x}) = x_6$,

$$r_6 = 1 \Rightarrow L_f^0h_6 = h_6 = x_6 \quad (48)$$

$$L_{g_i}L_f^0h_6 = 0, 1 \le i \le 5 \quad (49)$$

$$L_{g_6}L_f^0h_6 = \frac{0.0019}{x_3^0} \neq 0 \quad (50)$$

Hence first condition for defining relative degree r_i at the point $\underline{x}^0$ is satisfied.

b. Condition No.2

The matrix

$$\underline{A}(\underline{x}) = \begin{bmatrix} L_{g_1}L_f^{r_1-1}h_1(\underline{x}) & . & . & . & . & L_{g_6}L_f^{r_1-1}h_1(\underline{x}) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ L_{g_1}L_f^{r_6-1}h_6(\underline{x}) & . & . & . & . & L_{g_6}L_f^{r_6-1}h_6(\underline{x}) \end{bmatrix}_{6\times 6} \quad (51)$$

should be a non-singular matrix at $\underline{x} = \underline{x}^0$. This condition is also satisfied.

Since both conditions are satisfied, so exact linearization of the system is possible.

C. Step 3: Feedback Law

The control law which converts the nonlinear model of the CTTS into an exact linear representation is given by:

$$\underline{u} = \underline{A}^{-1}(\underline{x})(\underline{w} - \underline{b}) = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{bmatrix}^T \quad (52)$$

where

$$\underline{w} = \begin{bmatrix} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 \end{bmatrix}^T \quad (53)$$

and

$$\underline{b}(\underline{x}) = \begin{bmatrix} L_f^{r_1}h_1(\underline{x}) \\ L_f^{r_2}h_2(\underline{x}) \\ L_f^{r_3}h_3(\underline{x}) \\ L_f^{r_4}h_4(\underline{x}) \\ L_f^{r_5}h_5(\underline{x}) \\ L_f^{r_6}h_6(\underline{x}) \end{bmatrix} = \begin{bmatrix} L_fh_1(\underline{x}) \\ L_fh_2(\underline{x}) \\ L_fh_3(\underline{x}) \\ L_fh_4(\underline{x}) \\ L_fh_5(\underline{x}) \\ L_fh_6(\underline{x}) \end{bmatrix} \quad (54)$$

where

$$L_fh_i(\underline{x}) = \frac{\partial(h_i(\underline{x}))}{\partial \underline{x}}\underline{f}(\underline{x}) \quad (55)$$

$$\Rightarrow \underline{b}(\underline{x}) = \begin{bmatrix} -0.0845\sqrt{30.48 + x_1} \\ 0.0845\left(\sqrt{30.48+x_1} - \sqrt{30.48+x_2}\right) \\ 0.0845\left(\sqrt{30.48+x_2} - \sqrt{30.48+x_3}\right) \\ 0 \\ \dfrac{-0.0845\sqrt{(30.48 + x_1)}(x_5 - x_4)}{x_2} \\ \dfrac{-0.0845\sqrt{(30.48 + x_2)}(x_6 - x_5)}{x_3} \end{bmatrix} \quad (56)$$

$$\Rightarrow \underline{u} = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{bmatrix}^T \quad (57)$$

where

$$u_1 = 90.9091w_1 + 1.5364\sqrt{762 + 25x_1} + 526.316(x_4 - 303)w_4$$

$$u_2 = 90.9091w_1 + 3.0727\sqrt{762 + 25x_1} - 90.9091w_2 - 1.5364\sqrt{762 + 25x_2} + 526.316(x_5 - 303)w_4$$

$$u_3 = 90.9091w_1 + 3.0727\sqrt{762 + 25x_1} - 90.9091w_3 - 1.5364\sqrt{762 + 25x_3} + 526.316(x_5 - 303)w_4 + 526.316(x_6 - x_5)\left(w_5 - 0.0169\sqrt{762 + 25x_1}\,{}^{(x_5 - x_4)}\!/_{x_2}\right)$$

$$u_4 = 526.316x_1w_4$$

$$u_5 = 526.316x_2\left(w_5 - 0.0169\sqrt{762 + 25x_1}\,{}^{(x_5 - x_4)}\!/_{x_2}\right)$$

$$u_6 = 526.316x_3\left(w_6 + 0.0169\sqrt{762 + 25x_2}\,{}^{(x_6 - x_5)}\!/_{x_3}\right)$$

Using the nonlinear feedback control law given by (57), the linear system obtained is:

$$\dot{\underline{z}} = \underline{A}\underline{z} + \underline{B}\underline{w}, \underline{y} = \underline{C}\underline{z} \quad (58)$$

where

$$A = O_6, B = I_6, C = I_6, D = O_{6\times 6} \quad (59)$$

This system is completely state controllable and observable.

D. Step 4: Controller Design using Pole Placement

Since the system linearized using feedback linearization is completely state controllable, so it can be stabilized using pole placement technique [8] according to which

$$\underline{w} = -\underline{K}^T \underline{z} \tag{60}$$

Poles can be placed arbitrarily for a completely state controllable system. Let the arbitrarily selected location for poles be:

$$P = [-1, -2, -3, -4, -5, -6] \tag{61}$$

So, the controller obtained using pole placement technique is as follows:

$$K = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 \end{bmatrix} \tag{62}$$

V. Results and Conclusion

The closed loop transfer function matrix of the overall system is shown in (63) which is same as it should be according to the form given in [3]. Step response of the closed loop system is as shown in Fig 2 which shows that all states are stabilized. This is the response to unit step input, so all outputs can be stabilized to any desired value by applying gain.

$$H(s) = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{s+2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{s+3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{s+4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{s+5} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{s+6} \end{bmatrix} \tag{63}$$

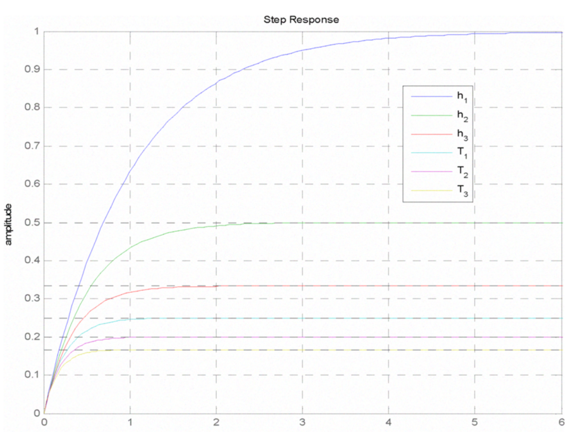

Fig. 2. Step response of the closed loop system

So, using the feedback law given by (57), the resultant overall closed-loop system has linear dynamics given by (59) and (63) and it is completely state-controllable. Let C_1, C_2 and C_3 be the controllers used to control states of tank-1, tank-2 and tank-3 respectively. Then controller C_1 includes control inputs u_1, u_4, w_1, w_4, controller C_2 includes control inputs u_2, u_5, w_2, w_5 and controller C_3 includes control inputs u_3, u_6, w_3, w_6.

Fig 3 shows the flow chart having the three tanks being controlled using C_1, C_2 and C_3 which take states of tank-1, tank-2 and tank-3 respectively as inputs and gives control signals to control the states. Also there is communication between the controllers and interactions are also present. Controllers C_2 and C_3 depend on controller C_1 and states of tank-1. As there is communication and interaction between the controllers, the breaking of a loop can result in the instability.

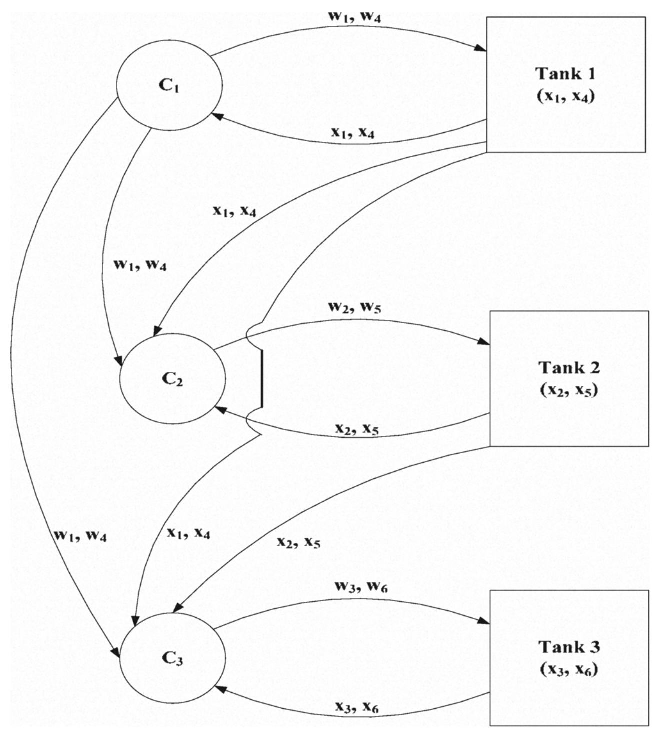

Fig. 3. Flow chart showing controllers with interactions

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