The Two Tank Experiment: A Benchmark Control Problem

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Abstract

This paper describes an experimental process control system under development at Caltech. It is intended to be a source of benchmark control and identification problems. A first principles theoretical model is developed and compared to preliminary experimental data.

1 Introduction

1.1 Experimental Control Research

There is no substitute for real applications when evaluating control system design and identification methodologies. Of course, the design of any given control system involves many factors, and success (or failure) on one application problem is not a complete measure of a methodology's value. Furthermore, "applications" can be abused in the same way as the academic examples that are carefully constructed to illustrate the effectiveness of everyone's favorite approach. Nevertheless, much of the debate surrounding the benefits of various methodologies would simply disappear if there were more honest attempts at applications.

Since experimenting with full-scale systems can be expensive and time-consuming, the control theoretician seeking application opportunities is naturally led to consider the use of laboratory systems, with the hope that they will allow experiments with new methodologies at a much earlier stage in their development than is typically possible with even prototype industrial systems. The design of laboratory experiments in control is a challenge because many interesting control problems do not scale well. For example, it is difficult to recreate both the physical and dynamic characteristics of proposed large flexible space structures in a laboratory scale and 1-g environment. An example in process control is the high-purity distillation column [1], where many of the interesting control problems are difficult to mimic with laboratory scale distillation columns. As with experiments in any discipline, it is important to design control experiments to achieve specific goals.

The approach taken in the control laboratory at Caltech is to design experiments which do not try to mimic the physical details of practical engineering systems, but focus on providing similar control design problems in a more generic sense. The goal is to build laboratory systems which are inexpensive, easy to understand, operate, and expand, and yet provide interesting control design challenges of the type found in practical engineering systems. Since much more is learned from failure than success, an additional goal is to build experiments which do not neatly fit any theoretical framework, thus suggesting new lines of research. To this end, a simple two tank process control experiment has been constructed at Caltech. This paper presents a physical description and, to the extent that it has been developed, a mathematical model of the system. The experiment's construction and instrumentation were only recently completed and this paper reports on work in progress

The experimental system consists of two water tanks; the first feeding the second. A computer based real time control system is used to control hot and cold water flows into the first tank. A variety of level and temperature control problems can then be studied. The details of the experimental arrangement are given in Section 2. In Section 3 a nonlinear model of the two tanks is developed from first principles along with an experimentally determined actuator and sensor models. Several identification and control problems of varying difficulty are described in Section 4. This experiment is typical of process control systems, but has several issues that are important in many other problem domains as well. The features that are expected to be most critical are: input uncertainty and saturation, a high condition number plant, uncertain components throughout the system, and operating regimes dominated by a nonlinear function of a measurable quantity.

Preliminary open loop identification experiments have been performed and a sample of the results is included in Section 5. Considerable discrepancies exist between the theoretical model of the system and the observed data and some potential sources of these discrepancies are discussed. Further experimental work is in progress to determine which of the observed effects can be explicitly included in the model and which must be included in an uncertainty description.

We are eager to discuss the design, construction, and instrumentation of experimental control facilities with other researchers and engineers. We believe that experimentation is a highly underrated activity in our field with substantial challenges and equally substantial rewards. We plan eventually to make this experimental facility available in some way to other researchers. We hope that the ease with which experimental data and controllers can be transmitted by electronic mail will facilitate this, but the details remain to be worked out.

2 Experimental System Description

The system, shown schematically in Figure 1, consists of two water tanks. The upper tank, referred to as tank 1, is fed by hot and cold water via computer controllable valves. The lower tank, referred to as tank 2, is fed by water from an exit at the bottom of tank 1. A constant level is maintained in tank 2 by means of an overflow. A fixed stream of cold water also feeds tank 2. It is the presence of this bias stream that allows the tanks to be maintained at different temperatures.

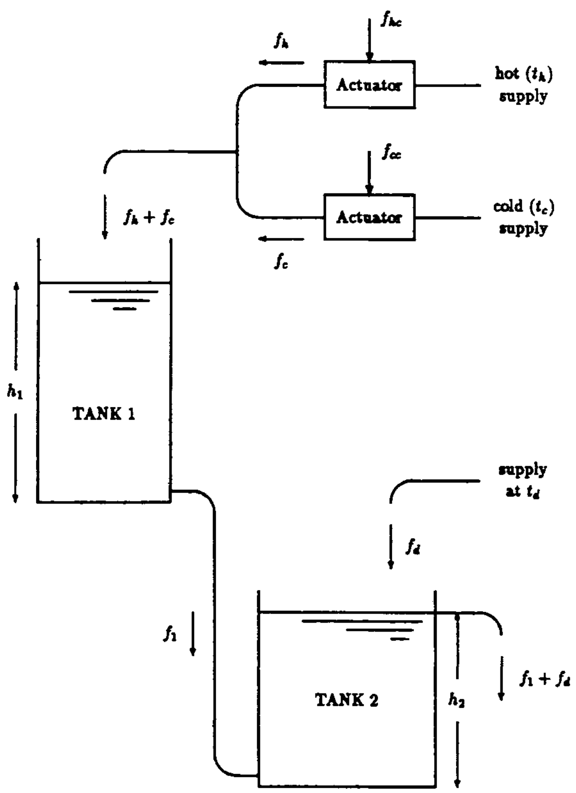

Figure 1: Schematic Diagram of the Two Tank System

Tank 1 is $5\frac{3}{8}$ inches in diameter and 30 inches in height. Tank 2 is $7\frac{1}{2}$ inches in diameter. Four overflows are provided at $5\frac{1}{4}$, $7\frac{1}{4}$, $9\frac{1}{4}$, and $11\frac{1}{4}$ inches. For the experiments described in this paper the overflow at $7\frac{1}{4}$ inches was used. This configuration maintains the water level in tank 2 at $4\frac{3}{4}$ inches below the base of tank 1.

The hot and cold water supplies are filtered through a 5 micron filter and regulated to 20 psig. Half inch piping is used for the main flow lines and the connection between tanks 1 and 2. Two half inch Kämmer valves (30 000 series) with electropneumatic actuators are used to control the flows. These have a linear characteristic and a C_v of 1.0. Variable area flowmeters measuring from 0.2 to 2.0 gpm provide a means of calibrating the actuators. Approximately 72 inches of pipe connects the actuators to the flowmeters. The hot and cold flows are combined in a tee junction 10 inches from the flowmeters and the mixed flow is piped a further 18 inches to tank 1. The pipe outlet is 31 inches above the base of tank 1. Approximately 36 inches of pipe connects the tanks, entering at the base of tank 2. The tank 2 bias stream is fed from the cold supply via a needle valve and flowmeter. This arrangement allows manual adjustment of the bias stream flow from 0.015 to 0.3 gpm.

Isolated E type thermocouples are inserted into each of the tanks approximately $\frac{1}{4}$ inch above the base. Omega MCJ thermocouple connectors provide the ice point reference. The temperature signals are amplified by Omega Omni-Amp amplifiers (gain: 1000). There is also provision to measure the temperature of the hot and cold flows prior to the valves and the mixed flow just before it enters tank 1.

A pressure sensor (0 to 5 psig) provides a measurement of the water level in tank 1. Both tanks are stirred with laboratory stirrers. Tank 1 has a shaft running the full length of the tank with three 2 inch propellers mounted on it. A single propeller stirs tank 2.

A Masscomp 5400 is used for the data acquisition and control. All signals from the experiment are filtered using fourth order butterworth filters each with a cutoff frequency of 2.25 Hz. The Masscomp is equipped with analog/digital (AD12FA) and digital/analog (DA08F) boards, each with 12 bit resolution, and a floating point processor board. Software allows the designer to provide arbitrary control signals to the experiment and implement linear state space and nonlinear controllers. A sample time of 0.1 seconds has been used.

The experimental arrangement makes it possible to formulate a variety of control problems of different complexities.

3 Modeling

3.1 A Model of the Two Tanks

The following assumptions have been made in the derivation of the model in this section:

- No thermal losses in the system.
- Perfect mixing occurs in both tanks.
- The flow out of tank 1 is related only to the height of tank 1.
- There are no delays.

As will be seen in later sections, there are some significant discrepancies between the model based on these assumptions and experimental data. A preliminary discussion of more refined assumptions and a corresponding model is given in Section 5.2.

The system variables are given the following designations.

f_{hc}	command to hot flow actuator.
f_h	hot water flow into tank 1.
f_{cc}	command to cold flow actuator.
f_c	cold water flow into tank 1.
f_1	total flow out of tank 1.
A_1	cross-sectional area of tank 1.
h_1	tank 1 water level height.
t_1	temperature of tank 1.
t_2	temperature of tank 2.
A_2	cross-sectional area of tank 2.
h_2	tank 2 water level.
f_d	flow rate of tank 2 bias stream.
t_d	temperature of tank 2 bias stream.
t_h	hot water supply temperature.
t_c	cold water supply temperature.

Tank 1 is considered first. Conservation of mass gives:

$$\frac{\mathrm{d}}{\mathrm{dt}}(A_1 h_1) = f_h + f_c - f_1 \quad (1)$$

It is assumed that the flow out of tank 1 (f_1) is a memoryless function of the height (h_1). This is reasonable because of the incompressibility of the media. As the exit from tank 1 is a pipe with a large length to diameter ratio, the flow is proportional to the pressure drop across the pipe and thus to the height in the tank. With a constant correction term for the flow behavior at low tank levels the height and flow can be related by an affine function.

$$h_1 = \alpha f_1 - \beta \quad \text{where} \quad \alpha, \beta > 0 \quad \text{and} \quad f_1 \geq \beta/\alpha. \quad (2)$$

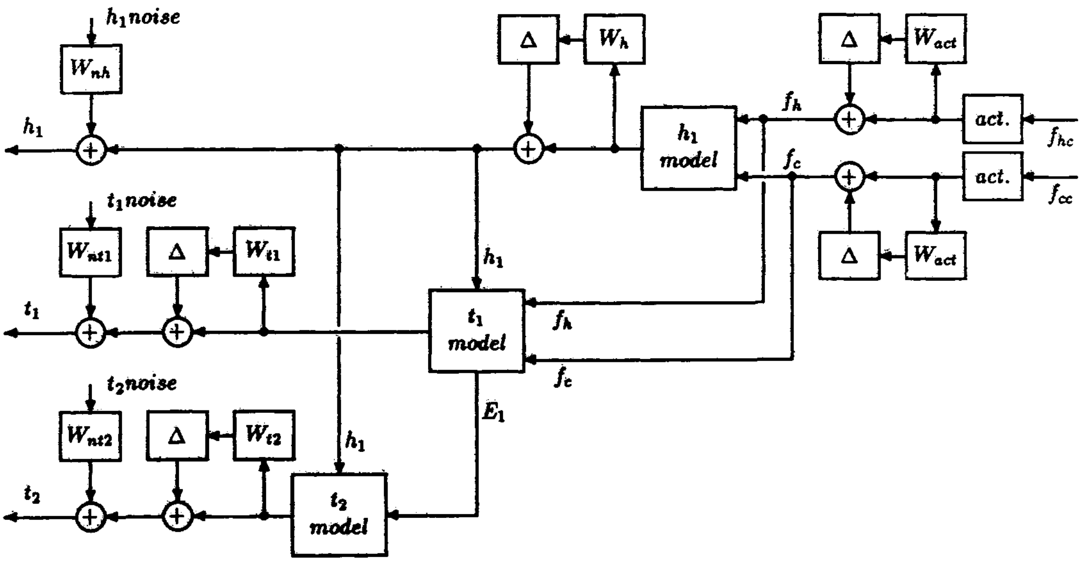

Figure 2: Schematic Diagram of the System Model.

Therefore

$$A_1\alpha\frac{d}{dt}f_1 = f_h + f_c - f_1. \tag{3}$$

Defining f_1 as a state variable leads to a linear state equation and an affine output equation for h_1 (in the allowable range of f_1).

$$\dot{f}_1 = \frac{-1}{A_1\alpha}f_1 + \frac{1}{A_1\alpha}f_h + \frac{1}{A_1\alpha}f_c. \tag{4}$$

$$h_1 = \alpha f_1 - \beta. \tag{5}$$

Conservation of energy will lead to a model for the temperature of tank 1 (t_1) as

$$\frac{d}{dt}(A_1h_1t_1) = f_ht_h + f_ct_c - f_1t_1. \tag{6}$$

It is useful to define a variable

$$E_1 = h_1t_1 \tag{7}$$

which can loosely be thought of as the energy in tank 1. Now

$$\begin{aligned} f_1t_1 &= \left(\frac{h_1+\beta}{\alpha}\right)t_1 \\ &= \frac{1}{\alpha}\left(1+\frac{\beta}{h_1}\right)E_1. \end{aligned} \tag{8}$$

Defining E_1 as a state variable gives a nonlinear state equation and a nonlinear output equation for t_1.

$$\dot{E}_1 = \frac{-1}{A_1\alpha}(1+\frac{\beta}{h_1})E_1 + \frac{t_h}{A_1}f_h + \frac{t_c}{A_1}f_c. \tag{9}$$

$$t_1 = \frac{1}{h_1}E_1. \tag{10}$$

Note that for a fixed h_1 the above equations are linear. This will aid in the identification.

In tank 2 the height(h_2) is fixed and the input flow from tank 1 (f_1) is of temperature t_1. This gives only one equation:

$$\frac{d}{dt}(A_2h_2t_2) = f_1t_1 + f_dt_d - (f_1+f_d)t_2. \tag{11}$$

Using the previous definition of E_1 and the relationship between h_1 and f_1 gives

$$A_2h_2\frac{d}{dt}t_2 = \frac{1}{\alpha}(1+\frac{\beta}{h_1})E_1 + f_dt_d - (\frac{h_1+\beta}{\alpha})t_2 - f_dt_2 \tag{12}$$

Defining t_2 as a state variable leads to the following state equation and a trivial output equation.

$$\dot{t}_2 = -\left(\frac{h_1+\beta+\alpha f_d}{\alpha A_2h_2}\right)t_2 + \frac{f_dt_d}{A_2h_2} + \frac{1}{\alpha A_2h_2}(1+\frac{\beta}{h_1})E_1. \tag{13}$$

For a fixed h_1 the t_2 state equation is affine.

Equations 4, 5, 9, 10, and 13 provide a simple model for the two tanks. To complete the description of the system a model must be obtained for the actuators and sensors. The uncertainty associated with elements of the model must also be identified.

The structure of the full system model is shown in Figure 2. The full system has five inputs: two control inputs (f_{hc} and f_{cc}) and three noise inputs (h_1noise, t_1noise, and t_2noise). A weighting function is included on each noise input (W_{nh}, W_{nt1}, and W_{nt2}). The uncertainty is provided by the five Δ blocks, where $|\Delta| \leq 1$. Each uncertainty block is weighted by one of five uncertainty weights: one on the output of each actuator (W_{act}), and one on the output of each of the models for h_1 (W_h), t_1 (W_{t1}), and t_2 (W_{t2}).

3.2 Choice of Units

To quantify the above model the units are defined by:

temperature:	$t_h = 1.0$ tunit
	$t_c = 0.0$ tunit
height:	tank 1 full = 1.0 hunit
	tank 1 empty = 0.0 hunit.

These two definitions define all of the other units in the problem. Note from Section 2 that the input flows range from 0 to 2.0 gallons/minute. It is convenient to define a flow unit at the input by 2.0 gpm = 1.0 funit. Using the above units the system dimensions are now:

A_1	0.0256	hunits^2
A_2	0.0477	hunits^2
h_2	0.241	hunits
f_d	7.4×10^{-5}	hunits^3/sec
t_d	0.0	tunits
flow scaling	0.00028	hunits^3/sec/funit

The flow scaling factor converts the input (0 to 1 funits) to flow in hunits^3/second.

3.3 Actuator and Sensor Models

In the frequency range of interest the actuators can be modeled as a single pole system with rate and magnitude saturations. Software is used to impose a magnitude saturation at the input to the actuator which is smaller than the actual actuator magnitude limit. This is done in order that the flows are maintained in the calibrated range. The rate saturation has been estimated from observing the effect of triangle waves of different frequencies and magnitudes. The following model has been estimated for the actuators.

$$f_h = \left[\frac{1}{(1+0.05s)}\right] f_{hc} \tag{14}$$

with:	magnitude limit:	1.0	funits
	rate limit:	3.5	funits/sec

Note that the rate limit determines the actuator performance rather than the pole location. For a linear model some of the effects of rate limiting can be included in an uncertainty model.

Except for a possible small time delay in the thermocouples, we expect the sensor dynamics to be insignificant relative to the dynamics of the rest of the system. This will not be true of the sensor noise. Although the sensor noise was small enough to allow the experiments described later, we were unable to get a consistent estimate of the sensor noise level. The potential sources of noise include electronic noise in thermocouple compensators, amplifiers, and filters, radiated noise from the stirrers, and poor grounding (this is, after all, an inexpensive laboratory experiment).

There are additional disturbances or noises associated with the measurements that are not, strictly speaking, sensor noise. For example, in tank 2 the imperfect mixing of the bias stream causes variations in the temperature measurement. How these various noises and disturbances are treated depends on one's philosophy. The philosophy we take for this experimental system is that our performance objectives will be in terms of the measured quantities, since that is all that can be verified. Thus distinguishing between sensor noise and other disturbances will not effect the control design or the closed loop performance.

4 The Identification and Control Problems

Only preliminary identification experiments have been performed. It will be seen in subsequent sections that the above theoretical model predicts responses that have significant differences from the experimental data. Not surprisingly, the model fits the data very well at low frequencies but not at higher frequencies.

Our modeling philosophy is that all the experimental responses must be accounted for explicitly. Some of the differences between the theoretical differential equation model and the observed data may be attributed to a more sophisticated model which includes additional phenomena. After refining the theoretical model, a residual difference between model and data will remain and must be included in an explicit "uncertainty" model. This uncertainty may be thought of as arising in several distinct ways:

- external disturbances and noise
- unmodeled dynamics
- parametric variations in the differential equation model

It could be argued that these distinctions are artificial, and reflect as much the restrictions of our modeling methodologies as the exigencies of physical reality. After all, uncertainty in the sense used here is in the model, not in the physical system being modeled. We are not trying to make any statements about physical reality. Nevertheless, within the context of available modeling techniques we must distribute the residual uncertainty among these mechanisms. A systematic methodology for doing this does not currently exist. A major goal for this experiment is to provide a testbed for the development of an identification methodology to be used in conjunction with robust control synthesis techniques.

Several interesting control problems can be studied. The most simple is the control of h_1. Our model for the dynamics of this system is linear. The multivariable problems are more interesting. Control of h_1 and t_1 is not difficult, and this problem is one of the most common process control experiments. As the equations for t_1 are a function of h_1 this problem lends itself to testing gain scheduled or nonlinear controllers to achieve high performance. The most difficult problem is the control of t_1 and t_2. In addition to the nonlinearities, there is a condition number problem [1]. This will prove particularly troublesome in the presence of input uncertainty. Input saturation will also have a large effect on the achievable performance in this problem [2].

In any of the above cases an operating point can be selected and a linear controller can be designed for the linearized system. To cover a range of operating points the parameters of the linearized model could be considered as varying or uncertain. Finally, the full nonlinear model may be used to consider problems involving large signals.

5 Experimental Results

5.1 Identification Experiments

The parameters α and β have been estimated from static h_1 and f_1 measurements and are:

α	4876
β	0.59

Open loop experiments have been conducted to test the validity of the model described in Section 3. Band limited noise, in several frequency bands, is used for the input signals. Data records were 8192 samples in length with sample rates of 1.0 Hz and 10.0 Hz. The transfer function estimates, presented below, have been obtained by the Welch method, using Hanning windows on sections of the data [3]. The data plotted in the figures comes from several window lengths, typically 1024 and 4096. Only the points with good coherency are plotted.

Figure 3 shows the estimated transfer function between $f_{hc} + f_{cc}$ and h_1, and the transfer function predicted from the

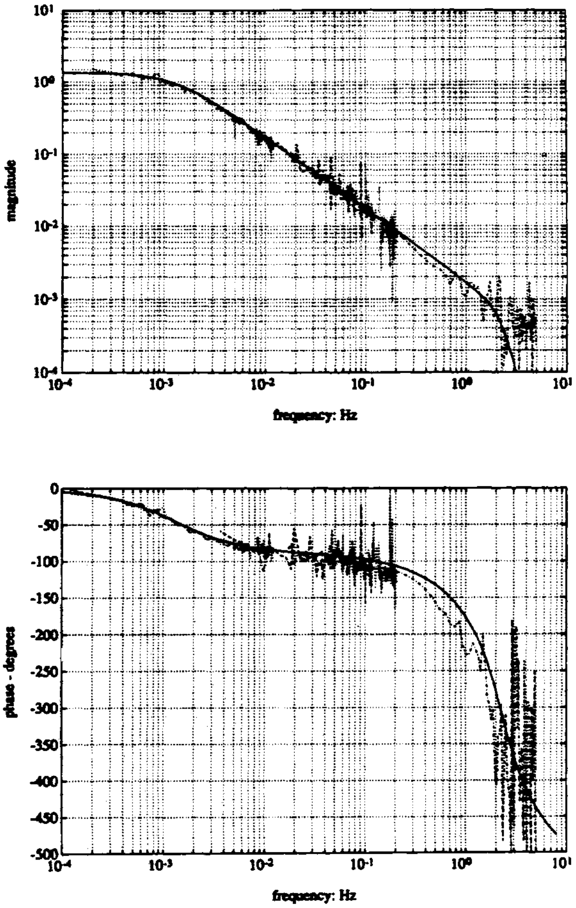

Figure 3: Transfer function between $f_{hc} + f_{cc}$ and h_1. Experimental data and theoretical model (solid line).

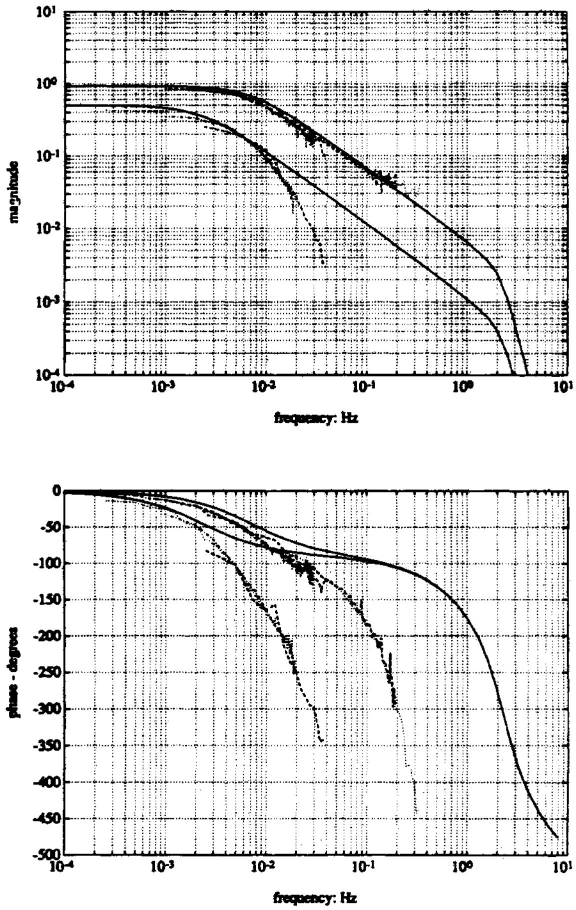

Figure 4: Transfer function between $f_{hc} - f_{cc}$ and t_1. Experimental data and models (solid lines). $h_1 = 0.13$ and $h_1 = 0.76$. (The system with the higher gain is $h_1 = 0.13$.)

model (equations 4 and 5), including the nominal actuator and butterworth filter. The experimental data comes from five experiments at three different levels. For frequencies greater than 0.2 Hz, the plotted data comes from an experiment at $h_1 = 0.47$.

For h_1 fixed, the E_1 state variable equation (9) and the t_1 output equation (10) are linear. Experiments have been performed at $h_1 = 0.13, 0.25, 0.47$, and 0.76. The input waveforms were generated such that $f_{hc} = -f_{cc}$ which maintains a constant h_1. Figure 4 shows the transfer function between $f_{hc} - f_{cc} (= 2f_{hc})$ and t_1 calculated from the experimental data and estimated from the model (equations 9 and 10). For the data shown $h_1 = 0.13$ and $h_1 = 0.76$. The other cases lie between the two curves shown.

We have not yet performed extensive experimental verification of the tank 2 model.

5.2 Towards a System Model

Figures 3 and 4 show sample results from the above series of experiments. Several discrepancies are immediately obvious between the theoretical model and the the smoothed empirical transfer function estimates (SETFE) [4] . The latter are derived from data that is averaged, so much of the information on the uncertainty has been smoothed out and there are biases. The remaining discrepancies may be due to consistent linear effects that are as yet unmodeled or to the averaging of nonlinear or time-varying effects. We will refine the model to at least account for these averaged effects, with the remainder to be included in an uncertainty model. This must be done on an ad hoc basis as we develop more systematic identification methods.

The theoretical model for h_1 fits the SETFE quite well (refer Figure 3), but we have observed some consistent height dependent phenomena at high frequencies (around 1 Hz.) that might be modeled by several possible mechanisms. There is some time delay in the stream between the pipe exit and the water surface in tank 1. Splashing may also have some effect at high frequencies. A less likely source could be nonlinear sensor dynamics. Experimentally determining these high frequency characteristics is made difficult because the signals are small relative to noise and discretization levels. Note that the response at 1 Hz. is down 3 decades from its dc level. It is unlikely that a more refined model at these frequencies will lead to better closed loop performance.

For the t_1 transfer functions the discrepancies from the model are observed at lower frequencies (refer Figure 4), due mostly to imperfect mixing in tank 1 and thermal delays. The flow dependent thermal delay is due to the hot and cold streams mixing before being piped to tank 1 and to the drop from the pipe exit to the water surface. At $h_1 = 0.13$ the delay is approximately 1 second. Combined with a possible small thermocouple delay, this accounts for much of the discrepancy in the phase. Imperfect mixing probably accounts for most of the rest.

At $h_1 = 0.76$ the effects of mixing dominate the response and the discrepancies occur at much lower frequencies. The theoretical thermal delay is approximately 0.4 seconds, which is swamped by the higher order mixing dynamics. This has been corroborated by visually observing the mixing using dyes. Thus the modeling discrepancies are dominated by imperfect mixing at high flow levels and heights and by thermal delays at low flows and heights.

The problem is not as simple as modifying the model to make it fit the data of Figure 4. Linear temperature controllers, designed for fixed h_1, based on best fit data, have been found to be unstable on the actual system. Some of the observed effects must therefore be attributed to nonlinearities or uncertainty. We are currently working on this problem from both a theoretical and experimental point of view.

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