Refinements of the Ziegler–Nichols tuning formula

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Abstract: The accuracy of the Ziegler–Nichols tuning formula is reviewed in the context of PID and PI autotuning. For PID autotuning, it will be shown that, for excessive overshoot in the set-point response, set-point weighting can reduce the overshoot to specified values, and the original Ziegler–Nichols tuning formula can be retained. It will also be shown that set-point weighting is superior to the conventional solution of reducing large overshoot by gain detuning or set-point filtering. However, for excessive set-point undershoot, the tuning formula will have to be modified. For PI autotuning, it will be shown that the Ziegler–Nichols tuning formula is inadequate and has to be completely revised. The application of set-point weighting and the modification of the tuning formula can be based simply on the knowledge of the normalised gain or normalised dead-time of the process. These heuristic refinements, when incorporated, will give appreciable improvement in the performance of autotuners.

1 Introduction

The Ziegler–Nichols ultimate-cycle or closed-loop tuning [1, 2] has been widely known as a fairly accurate heuristic method to determine good settings of PID and PI controllers, for a wide range of common industrial processes. However, this manual tuning method is not often applied in practice because it is laborious and time-consuming, particularly for a process with large time constant. It also requires the close attention of the instrument/control engineer and the operator, because the process has to be operated near instability to measure the ultimate gain and period.

Several commercial products for automatic tuning and adaptive control [3–8] have been available since 1981. Recently, a simple method of autotuning has been developed. It is based on the automatic measurement of the ultimate gain and period, from which the optimal PID and PI controller parameters can be computed using the Ziegler–Nichols tuning formula. Various techniques, such as the relay feedback [5, 9], approximate system identification [10] and crosscorrelation [11, 12], have been developed to determine the ultimate gain and ultimate period without the need to drive the system to the verge of instability.

The accuracy of the Ziegler–Nichols tuning formula has been found to be quite adequate in manual tuning, because it can be supplemented by fine-tuning based on experience. With the automation of the controller-tuning procedure, it becomes attractive to explore the possibility of modifying or augmenting the tuning formula by incorporating heuristic knowledge to replace manual fine-tuning. Heuristics are naturally limited to certain classes of problem. The heuristic rules presented in this paper are applicable to the same class of process considered in the work of Ziegler and Nichols [1]. Processes with integrators or resonant poles are not considered in this study.

An extensive simulation study has indicated that a simple means of developing heuristic rules for assessing the performance of a Ziegler–Nichols tuned PID controller is to characterise the process dynamics by the normalised process gain or the normalised dead-time [13]. This paper will go a step further by using these two parameters to refine the Ziegler–Nichols tuning formula.

The idea of using normalised dead-time to improve controller tuning has been around for a long time. One of the early proposals was made by Cohen and Coon [14]. The earlier tuning formulas based on normalised dead-time are however not very good. A reason is probably that they were developed using process models that were too simple. Furthermore, the formula tends to produce very oscillatory set-point response because it was derived to give quarter damping for load-disturbance response.

2 Accuracy of Ziegler–Nichols formula

The Ziegler–Nichols tuning formula [1, 2] is based on the empirical knowledge of the ultimate gain k_u and ultimate period t_u, as shown in Table 1.

Table 1: Ziegler–Nichols tuning formula

	PID	PI
Proportional gain	$k_c = 0.6k_u$	$k_c = 0.45k_u$
Integral time	$T_i = 0.5t_u$	$T_i = 0.85t_u$
Derivative time	$T_d = 0.125t_u$	

The PID controller is usually implemented as follows:

$$u_c = k_c\left(e + \frac{1}{T_i}\int e\,dt - T_d\frac{dy_f}{dt}\right) \tag{1}$$

$$e = y_r - y \tag{2}$$

$$y_f = \frac{1}{1 + sT_d/N}\,y \tag{3}$$

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The controller output, process output and set-point are u_c, y and y_r, respectively.

A PID controller in the form of eqn. 1 avoids 'derivative kick' and provides noise filtering [5]. The noise filtering constant N is usually in the range of 3–10. Without loss of generality, $N = 10$ is used throughout this study. For simplicity, the continuous-time formulation will be used, as the discretisation for digital implementations is straightforward [2].

To illustrate the accuracy of the Ziegler–Nichols tuning formula and to explore appropriate solutions, the performance of the following process, tuned using the equations in Table 1, is first analysed:

$$\text{Process I} \quad \frac{Y(s)}{U_c(s)} = \frac{e^{-\theta_d s}}{(1+s)^2} \tag{4}$$

2.1 Ziegler–Nichols tuned PID control

In general, the performance of the Ziegler–Nichols tuned control system varies signficantly with the process dead-time. For PID control, when the dead-time is small, both the set-point and load-disturbance responses are well tuned in terms of speed of response and damping, as shown in Fig. 1 for $\theta_d = 0.4$. The first overshoot in the set-point response is, however, excessive, and it often needs to be reduced to 10% or 20%, depending on applications. The load disturbance seen in Fig. 1 acts on the process input. This assumption is made throughout the paper.

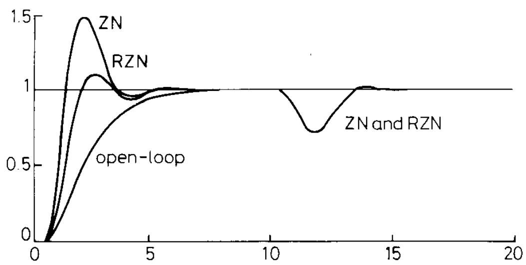

Fig. 1 *Excessive overshoot in the set-point response for PID control*
$\theta_d = 0.4$
ZN = Ziegler–Nichols tuning: $k_c = 3.43$; $T_i = 1.44$; $T_d = 0.36$
RZN = refined Ziegler–Nichols tuning: $k_c = 3.43$; $T_i = 1.44$; $T_d = 0.36$; $\beta = 0.45$

The problem of excessive set-point response overshoot can be solved in several ways. A simple solution is to detune the gain. This will, however, decrease the speed of both the set-point and load-disturbance responses. An alternative is to filter the set-point. The time constant t_f of the filter is usually set to a fraction of the integral time, to achieve a reasonable compromise between overshoot and rise time. The merit of this solution is that the load disturbance response will not be affected, as the PID controller settings are not changed.

In a study on dominant pole designs [15], a new method of reducing overshoot by means of set-point weighting has been proposed. In this case, the control law (1) is modified to include a constant weighting factor β on the set-point in the proportional term. The control law (1) is thus modified to

$$u_c = k_c\left[(\beta y_r - y) + \frac{1}{T_i}\int e\,dt - T_d\frac{dy_f}{dt}\right] \tag{5}$$

Set-point weighting has the same merit as set-point filtering, since the load response is also not affected. It is also better than the extreme solution of totally removing the set-point from the proportional term [16] (corresponding to $\beta = 0$ in eqn. 5), because the responses may then become very sluggish. The introduction of β provides a mean of adjusting the zeros of the closed-loop transfer function, which in turn affects the overshoot [17]. An example is given in Fig. 1.

It is found from extensive simulation studies of the process in eqn. 4 and other common process models that set-point weighting is superior to set-point filtering because the speed of response is sacrificed to a far smaller degree for the same reduction in overshoot. The examples in Fig. 2*a* and *b* use the same process model studied in Fig. 1 to show the effect of set-point weighting. In summary, there is no need to modify the Ziegler–Nichols tuning formula for a process with small dead-time, because the associated large overshoot can be overcome by set-point weighting.

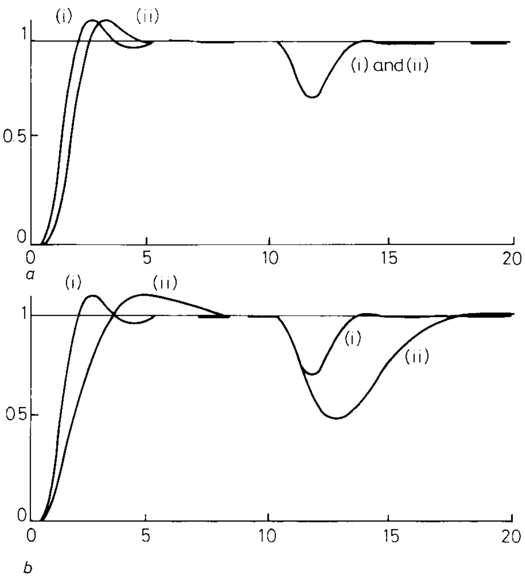

Fig. 2 *Comparison of methods for reducing overshoot*
$\theta_d = 0.4$
a (i) $\beta = 0.45$
(ii) Set-point filtering, $t_f = 1.1$
b (i) $\beta = 0.45$
(ii) Detuning gain, $k_c = 0.9$

The overshoot in the set-point response reduces as the dead-time increases, because the loop gain reduces. It has been found, in practice and in simulations, that a new problem of excessive undershoot would occur in the later part of the transient response when the dead-time is large. In an application where very high performance is required, such as a quality control loop, dead-time compensation using a Smith predictor [2] or other advanced control technique would be necessary.

In less demanding applications, or when the dynamic model of the process is poorly known, PID control may still be used. Therefore it is useful to resolve the problem of excessive undershoot. One possible solution is to apply set-point weighting with $\beta > 1$. However, this is found to be effective only if the undershoot is moderate. When the dead-time is large and undershoot serious, the load disturbance response is also poor. An example is shown in Fig. 3 for $\theta_d = 2.5$. Fine-tuning experience has indicated that the problem can be overcome by increasing the integral action, that is, by reducing the integral time. This results in improvements in both the set-point and load disturbance responses as shown in Fig. 3. The trade-off then may be a larger overshoot that can be compensated as shown by the use of set-point weighting. The automatic setting of the set-point weighting and the modification of the tuning formula will be studied in Section 4.

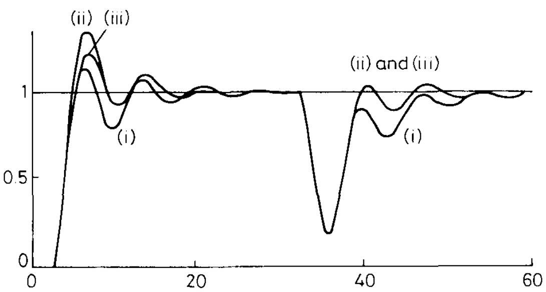

Fig. 3 *Excessive undershoot in the set-point response, and poor load disturbance response for PID control*

$\theta_d = 2.5$
(i) Ziegler–Nichols tuning: $k_c = 0.93$; $T_i = 4.25$; $T_d = 1.1$
(ii) Improvement using $T_i = 2.92$
(iii) $T_i = 2.92$; $\beta = 0.79$

2.2 Ziegler–Nichols tuned PI control

The Ziegler–Nichols tuned PI controller for a process with small dead-time produces a set-point response with both large overshoots and undershoots. This is unlike the Ziegler–Nichols tuned PID controller for a process with small dead-time, where the undershoot is acceptable and only the first overshoot is excessive. For example, using the same plant studied in Fig. 1 for PID control, the Ziegler–Nichols tuned PI controller as shown in Fig. 4 produces a set-point response that is more oscillatory. As set-point weighting would only affect the first overshoot but not the damping, it cannot be used to solve the underdamped response of the Ziegler–Nichols tuned PI controller.

For large dead-time, the Ziegler–Nichols tuned PI controller produces sluggish set-point and load-disturbance responses. An example is given in Fig. 5.

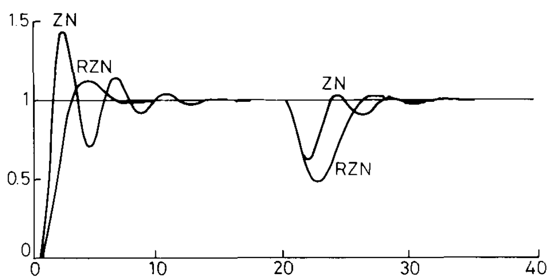

Fig. 4 *Poor damping and excessive overshoot in the set-point response for PI control*

$\theta_d = 0.4$
ZN = Ziegler–Nichols tuning: $k_c = 2.57$; $T_i = 2.45$
RZN = refined Ziegler–Nichols tuning: $k_c = 0.89$; $T_i = 1.45$

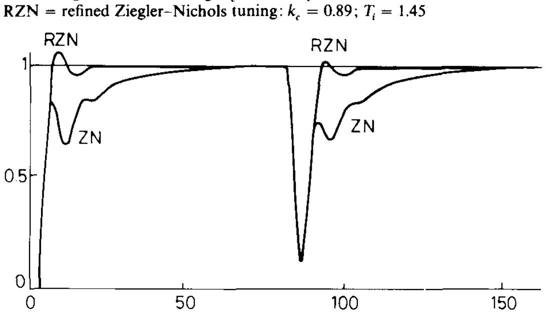

Fig. 5 *Sluggish set-point and load-disturbance responses for PI control*

$\theta_d = 2.5$
ZN = Ziegler–Nichols tuning: $k_c = 0.7$; $T_i = 7.22$
RZN = refined Ziegler–Nichols tuning: $k_c = 0.48$; $T_i = 2.4$

Integral action has to be increased to overcome this problem.

It has been found necessary to revise completely the Ziegler–Nichols tuning formula for PI control. The purpose of the modification is to improve the damping of the set-point response for a process with small dead-time, and to speed up the set-point response as well as load-disturbance response for a process with large dead-time. The improved responses are also shown in Figs. 4 and 5. In Fig. 4, the larger load-disturbance response can be expected, as the system is now tuned to give a greater stability margin.

3 Process characterisation

The recent interests in artificial intelligence [4, 6] and expert control [18] have motivated the study of heuristics used by expert human operators and designers. One such heuristic is the broad characterisation of process dynamics by means of normalised process gain and normalised dead-time [13], from which an order-of-magnitude prediction of the achievable controller performance can be made. An extensive simulation study has shown that the accuracy of the Ziegler–Nichols tuning formula, as discussed in the previous section, is strongly correlated with the normalised gain and normalised dead-time of the process.

The normalised process gain κ is defined as the product of the ultimate gain k_u and the process steady-state gain k_p. Formally

$$\kappa = k_p k_u \tag{6}$$

The steady-state gain of the process can be determined easily in closed-loop by introducing a small set-point change. It is then computed simply as the ratio of the steady-state change in the process output to the steady-state change in the control variable. Hence, the normalised process gain is a parameter that is easily determined once the ultimate gain is known.

The normalised dead-time Θ is defined as the ratio of the dead-time θ_d or apparent dead-time θ_a to the apparent time constant T_p of the open-loop step response of the process. Formally,

$$\Theta = \frac{\theta_a}{T_p} = \frac{a}{k_p} \tag{7}$$

It can be simply determined using a pretuning pulse test [1, 4], as shown in Fig. 6*a*, which is well defined for a stable and damped process. The same definition can also be used to characterise a process with inverse dynamics caused by nonminimum-phase zeros, as shown in Fig. 6*b*. The physical pulse testing to determine Θ is not needed for the relay-feedback autotuner, if the available measurements are further employed to estimate a low-order plus dead-time model [19]. It can also be avoided for the

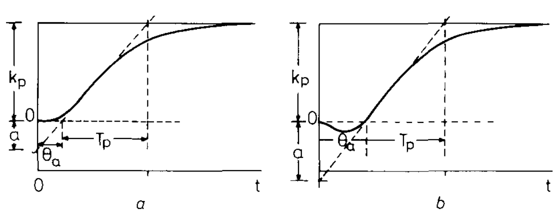

Fig. 6 *Step responses*

$\theta = \theta_a/T_p = a/k_p$
a Stable and damped process
b Stable and nonminimum-phase process

correlation-based autotuner [11, 12], because the normalised dead-time can be readily computed from the impulse-response estimates generated automatically by the correlator.

Besides the process given by eqn. 4, two other processes that exhibit different dynamics, namely, a multiple-lag process and a nonminimum-phase process, are also used in the correlation exercise:

$$\text{Process 2} \quad \frac{Y(s)}{U_c(s)} = \frac{1}{(1+s)^n} \tag{8}$$

$$\text{Process 3} \quad \frac{Y(s)}{U_c(s)} = \frac{1-\alpha s}{(1+s)^3} \tag{9}$$

For convenience, the processes of eqns. 4, 8 and 9 will be referred to as process 1, 2 and 3, respectively. The normalised process gain κ and normalised dead-time Θ will be varied in process 1 by changing the dead-time θ_d, in process 2, the order n, and in process 3, the numerator coefficient α.

The setpoint overshoot and undershoot are correlated to the normalised process gain and the normalised dead-time, in Fig. 7, for PID control, and in Fig. 8, for PI control. The correlation results provide a fairly good quantitative measure of the adequacy of the Ziegler–Nichols tuning formula. For PID control, when the normalised process gain is large, in the range of $\kappa > 4$, or equivalently when the normalised dead-time is small, in the range of $\Theta < 0.3$, the overshoot is about four times the undershoot. This is better than the well known quarter damping ratio [1]. The overshoot, however, is clearly excessive. In the next range of $2 < \kappa < 4$ or $0.3 < \Theta < 0.6$, the damping ratio reduces. For $\kappa < 2$ or $\Theta > 0.6$, the undershoot increases and exceeds the overshoot.

For PI control, Fig. 8 shows that when $\kappa > 4$ or $\Theta < 0.3$, the damping is clearly poor, as the magnitude of the undershoot approaches that of the overshoot. Furthermore, at very large κ or very small Θ, the overshoot and undershoot even exceed 100%, which means that the

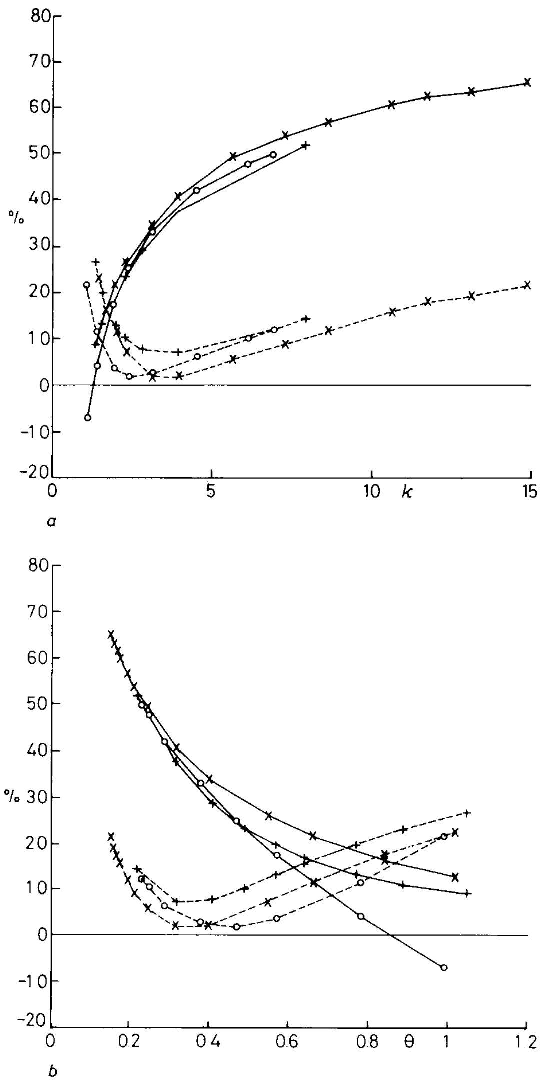

Fig. 7 *Percentage overshoot and undershoot for PID control*
a Against κ
b Against θ
- - - - % undershoot
——— % overshoot
× × × process 1; + + + process 2; ○○○ process 3

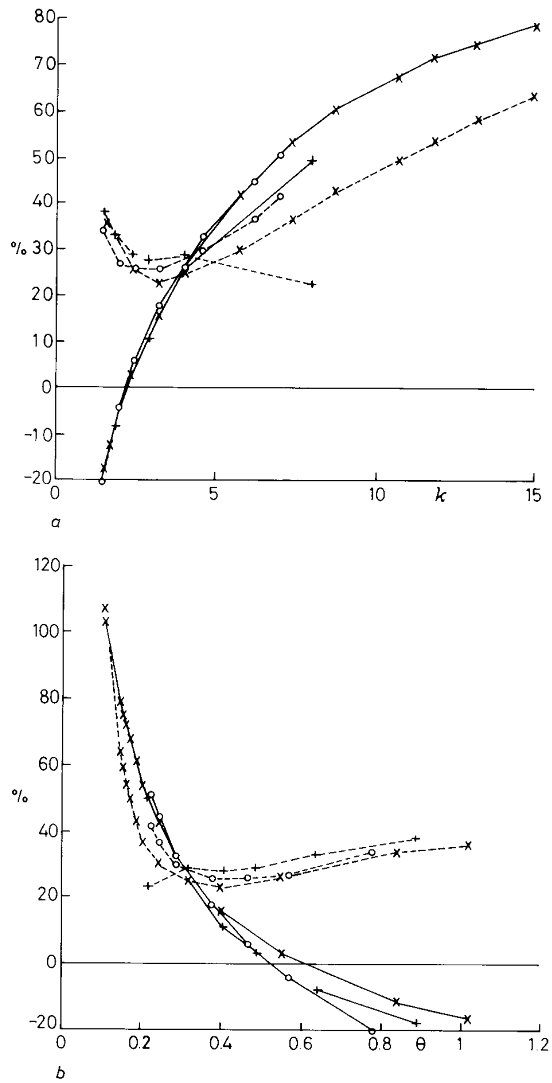

Fig. 8 *Percentage overshoot and undershoot for PI control*
a Against κ
b Against θ
——— % overshoot
- - - - % undershoot
× × × process 1; + + + process 2; ○○○ process 3

system is unstable. In the next range of $2 < \kappa < 4$ or $0.3 < \Theta < 0.6$, the undershoot exceeds the overshoot, and the load-disturbance response becomes sluggish. Finally, for $\kappa < 2$ or $\Theta > 0.6$, the overshoot becomes negative, which means that the response is overdamped and very sluggish. An example of this type of response is shown in Fig. 5. It is thus evident that, for PI control, the existing Ziegler–Nichols tuning formula has to be completely revised.

The normalised process gain κ and the normalised dead-time Θ are closely related. Fig. 9 shows the relation between κ and Θ for the three processes. It can be approximated by the following equation:

$$\kappa = 2\left(\frac{11\Theta + 13}{37\Theta - 4}\right) \tag{10}$$

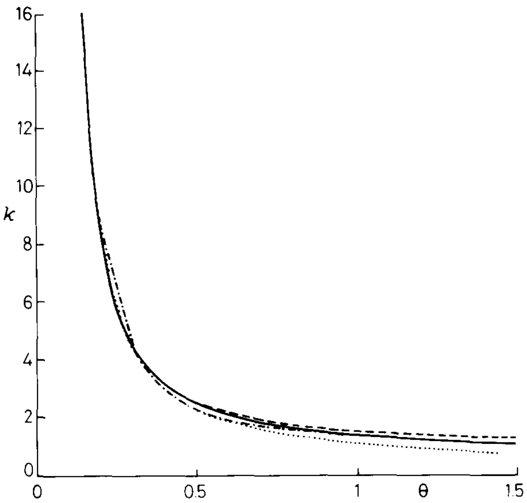

Fig. 9 *Relation between κ and θ*

- - - - process 1
—·—· process 2
······ process 3
——— approximation

In this paper, we shall correlate to give the refined Ziegler–Nichols formula in terms of the normalised process gain κ. The formula also can be expressed in terms of Θ, by using eqn. 10 to substitute for κ. Alternatively, in an earlier study [20], we have correlated and have given the refined Ziegler–Nichols directly in terms of Θ.

4 Refined Ziegler–Nichols tuning for PID control

The fact that the normalised process gain and the normalised dead-time correlate well with the set-point overshoot and undershoot produced by Ziegler–Nichols tuning immediately suggests that these two parameters can also be used to refine the tuning formula. Based on the above empirical observations, the following heuristic criterion for refining the PID tuning formula is recommended. When $\kappa > 2.25$ or $\Theta < 0.57$ (using eqn. 10 for conversion), retain the Ziegler–Nichols tuning formula and apply set-point weighting in the proportional term, as shown in eqn. 5, to reduce the excessive overshoot. When $\kappa < 2.25$ or $\Theta > 0.57$, modify the Ziegler–Nichols tuning formula by a suitable reduction of the integral time to improve both the set-point and load disturbance responses.

4.1 Large normalised process gain or small normalised dead-time ($2.25 < \kappa < 15$; $0.16 < \Theta < 0.57$)

In this section, we will establish a simple correlation formula for the set-point weighting factor β, in terms of the normalised process gain κ, for $2.25 < \kappa < 15$, or $0.16 < \Theta < 0.57$, to obtain an acceptable overshoot in the set-point response. Depending on applications, a 10% or 20% overshoot may be a suitable criterion. An extensive simulation study has been carried out using processes 1, 2 and 3. The results are shown in Fig. 10.

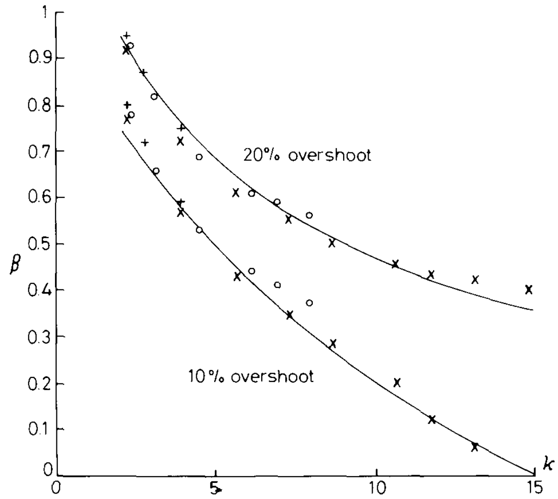

Fig. 10 *Set-point weighting factors for PID control*
× × × process 1; + + + process 2; ○○○ process 3

The following empirical formulas

$$\beta = \frac{15 - \kappa}{15 + \kappa} \quad 2.25 < \kappa < 15 \tag{11}$$

and

$$\beta = \frac{36}{27 + 5\kappa} \quad 2.25 < \kappa < 15 \tag{12}$$

approximate the simulation results for 10% and 20% overshoot, respectively. The graphs of β based on the above formulas are shown as solid lines in Fig. 10. Response *RZN* in Fig. 1 is an example of the improvement in performance that is obtained using this formula.

Notice that when κ is large, or Θ small, the application of set-point weighting reduces not only the overshoot but also the undershoot, hence achieving faster settling time. The effectiveness of set-point weighting may reach its limit when $\kappa > 15$ or $\Theta < 0.16$. This is acceptable because these ranges of κ and Θ signify a low-order process, where other design methods may be more appropriate than heuristics. Otherwise, it is straightforward to use a combination of set-point weighting and set-point filtering to satisfy a given overshoot criterion.

4.2 Small normalised process gain or large normalised dead-time ($1.5 < \kappa < 2.25$; $0.57 < \Theta < 0.96$)

It has been established, in Section 2, that, for large dead-time, the Ziegler–Nichols tuning formula needs to be modified to reduce the undershoot in the set-point response and to improve the load-disturbance response. As in set-point weighting, an extensive simulation study of processes 1, 2 and 3 is used to obtain the appropriate modifications.

It is found that a suitable integral time can indeed be determined without changing the proportional gain and derivative time. However, the improvement is accompanied by an excessive overshoot, and hence the application of set-point weighting with $\beta < 1$ is required. The new integral time is given as

$$T_i = 0.5\mu t_u \tag{13}$$

where μ is defined as the ratio of the modified integral time to the Ziegler–Nichols integral time.

As the control performance in this region of κ and Θ cannot be expected to be very tight, the criterion for adjusting the integral time and set-point weighting is set at set-point response of 20% overshoot and 10% undershoot. The empirical results for all three processes are shown in Fig. 11. The following formulas can be used to approximate the simulation results:

$$\mu = \tfrac{4}{9}\kappa \qquad 1.5 < \kappa < 2.25 \tag{14}$$

$$\beta = \tfrac{8}{17}(\tfrac{4}{9}\kappa + 1) \quad 1.5 < \kappa < 2.25 \tag{15}$$

The graphs of μ and β, based on the formulas, are plotted in Fig. 11 as solid lines. An example of the accuracy of the above formulas has already been shown in Fig. 3. Notice that, especially at small normalised process gain, the results are not very accurate for process 3. However, the computed settings still give good responses. This is demonstrated by the extreme case example shown in Fig. 15*a*.

5 Refined Ziegler–Nichols tuning for PI control

In this section, the Ziegler–Nichols tuning formula for the PI controller will be revised according to the analysis given earlier in Section 2. To ensure good set-point and load disturbance responses over a wide range of κ and Θ, it is found from an extensive simulation study that a suitable performance criterion is to tune for a set-point response with 10% overshoot and 3% undershoot. Set-point weighting is not used here, since tight control is normally not required from a PI controller. If tight control is required, then set-point weighting can likewise be applied to the PI controller, as was done for the PID controller, and the extension is straightforward.

The simulation results, over a wide range of κ and Θ for the three processes, are shown in Fig. 12*a* and *b* for

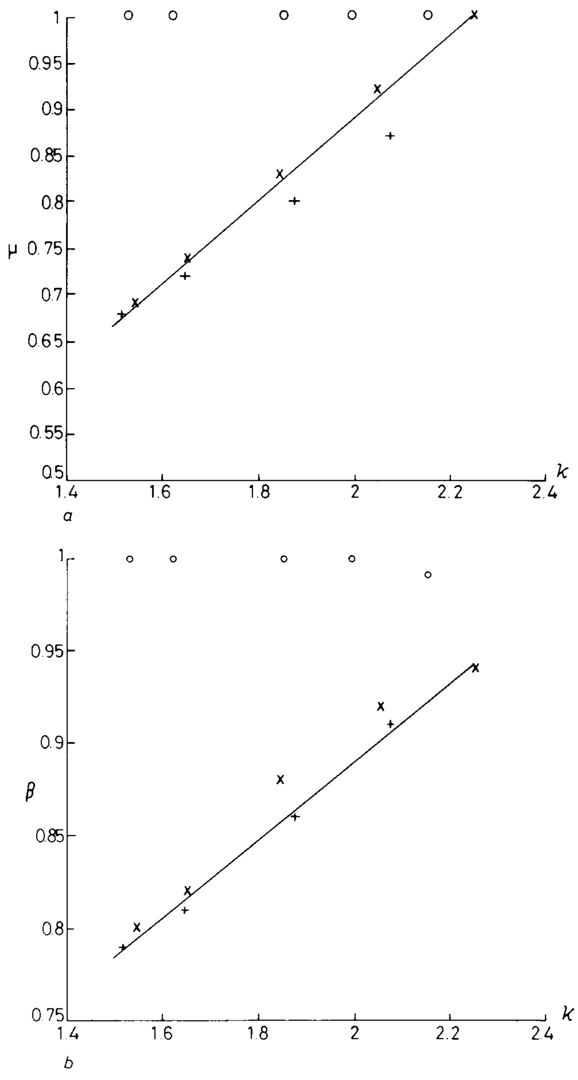

Fig. 11 *Optimal values for PID control*
a Values of μ
b Values of β
× × × process 1; + + + process 2; ○○○ process 3

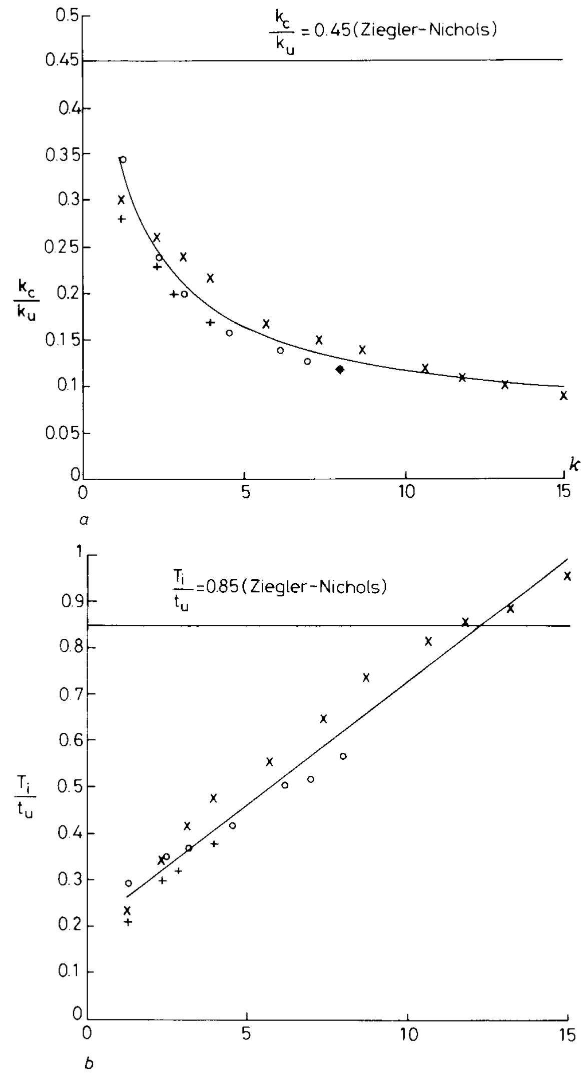

Fig. 12 *Refined Ziegler–Nichols tuning for PI control*
a k_c/k_u against κ
b T_i/t_u against κ
× × × process 1; + + + process 2; ○○○ process 3

k_c/k_u and T_i/t_u, respectively. The following formulas

$$\frac{k_c}{k_u} = \frac{5}{6}\left(\frac{12 + \kappa}{15 + 14\kappa}\right) \quad 1.2 < \kappa < 15 \tag{16}$$

$$\frac{T_i}{t_u} = \frac{1}{5}\left(\frac{4}{15}\kappa + 1\right) \quad 1.2 < \kappa < 15 \tag{17}$$

correspond to the solid curves in the figures. Examples of the excellent performance that can be expected of the formulas have already been given in Figs. 4 and 5.

6 Comparative studies

In this section, the performance of the refined Ziegler–Nichols formula will be compared with the performance of the Ziegler–Nichols and the Cohen–Coon formulas [14]. A popular model in process control, the first order plus dead-time model

$$\frac{Y(s)}{U_c(s)} = \frac{e^{-\theta_d s}}{(1 + s)} \tag{18}$$

where $\theta = 0.2$, 2.0 will be used in the comparative studies. To extend to other process dynamics, the performance of the tuning formulas on a nonminimum phase process

$$\frac{Y(s)}{U_c(s)} = \frac{1 - \alpha s}{(1 + s)^3} \tag{19}$$

where $\alpha = 1.4$, will also be compared.

The result of the comparison for the first order plus dead-time model with PID control is shown in Fig. 13. The superiority of the refined formula is obvious. The result of the comparison for the first order plus dead-time model with PI control is shown in Fig. 14. The larger load-disturbance response produced by the refined formula for the process with $\theta_d = 0.2$ is to be expected, as the system is now tuned to give a greater stability margin. It is also not surprising that the Cohen–Coon formula produces very oscillatory set-point response. The formula was derived to give quarter damping for the load disturbance response for a first order plus dead-time process.

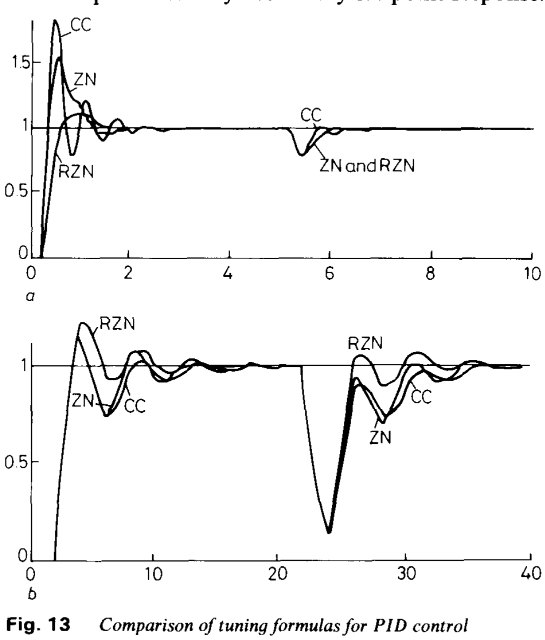

Fig. 13 *Comparison of tuning formulas for PID control*
a $\theta_d = 0.2$
RZN = refined Ziegler–Nichols: $k_c = 5.1$; $T_i = 0.37$; $T_d = 0.09$; $\beta = 0.28$
ZN = Ziegler–Nichols: $k_c = 5.1$; $P_i = 0.37$; $T_d = 0.09$
CC = Cohen–Coon: $k_c = 6.92$; $T_i = 0.45$; $T_d = 0.07$
b $\theta_d = 2$
RZN = refined Ziegler–Nichols: $k_c = 0.91$; $T_i = 1.86$; $T_d = 0.68$; $\beta = 0.79$
ZN = Ziegler–Nichols: $k_c = 0.91$; $T_i = 2.75$; $T_d = 0.68$
CC = Cohen–Coon: $k_c = 0.91$; $T_i = 3.03$; $T_d = 0.53$

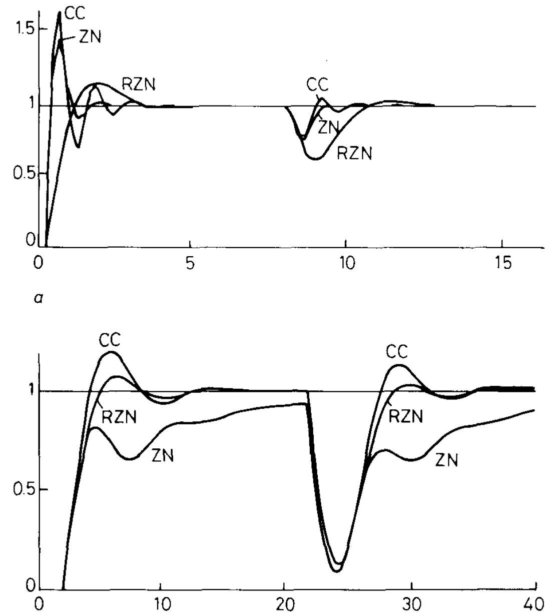

Fig. 14 *Comparison of tuning formulas for PI control*
a $\theta_d = 0.2$
RZN = refined Ziegler–Nichols: $k_c = 1.08$; $T_i = 0.48$
ZN = Ziegler–Nichols: $k_c = 3.83$; $T_i = 0.63$
CC = Cohen–Coon: $k_c = 4.58$; $T_i = 0.47$
b $\theta_d = 2$
RZN = refined Ziegler–Nichols: $k_c = 0.47$; $T_i = 1.55$
ZN = Ziegler–Nichols: $k_c = 0.68$; $T_i = 4.68$
CC = Cohen–Coon: $k_c = 0.52$; $T_i = 1.47$

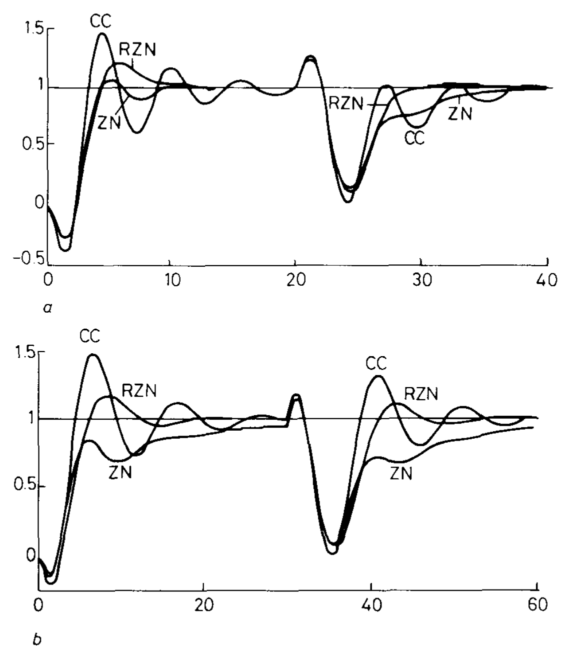

Fig. 15 *Comparison of tuning formulas, where $\alpha = 1.4$*
a PID control
RZN = refined Ziegler–Nichols: $k_c = 0.92$; $T_i = 2.33$; $T_d = 0.85$; $\beta = 0.79$
ZN = Ziegler–Nichols: $k_c = 0.92$; $T_i = 3.41$; $T_d = 0.85$
CC = Cohen–Coon: $k_c = 1.24$; $T_i = 4.09$; $T_d = 0.71$
b PI control
RZN = refined Ziegler–Nichols: $k_c = 0.48$; $T_i = 1.93$
ZN = Ziegler–Nichols: $k_c = 0.69$; $T_i = 5.81$
CC = Cohen–Coon: $k_c = 0.75$; $T_i = 2.3$

The comparisons show that the refined Ziegler–Nichols formula works well for the first order plus dead-time process, even though it was derived using higher-order process models. The performance of the refined Ziegler–Nichols formula for the second order model

$$\frac{Y(s)}{U_c(s)} = \frac{e^{-\theta_a s}}{(1 + s)(1 + \tau s)} \tag{20}$$

commonly used to approximate industrial processes, can be expected to be sandwiched in between process 1 and the first order plus dead-time process.

The result of the comparison for the nonminimum phase process is given in Fig. 15*a* for PID control and in Fig. 15*b* for PI control. The dead-time and time constant parameters for the Cohen–Coon formula are obtained by using least square to fit the step response of a first order plus dead-time model to that of the nonminimum phase process. The response of Cohen–Coon tuning is highly oscillatory, as the first order plus dead-time model does not model the nonminimum phase process adequately. The good performance of the refined formula is again obvious.

7 Conclusions

From the normalised gain or normalised dead-time of the process, an accurate prediction of the performance, and hence refinements, of the Ziegler–Nichols tuning formula can be made. With the proposed refinements of the Ziegler–Nichols tuning formula, PID and PI autotuning will be accurate over a much larger range of process dynamics. Consequently, the need for manual fine-tuning and human expertise will be largely eliminated.

The refinements have several merits. Firstly, they require only a small set-point change in closed-loop to determine the normalised process gain, or a simple pulse test for the measurement of the normalised dead-time. The pulse test can even be avoided for the relay-feedback autotuner and the correlation-based autotuner, because the normalised dead-time can be calculated from available measurements. Secondly, the computations of the tuning factors are also very simple. Finally, through set-point weighting, the PID controller is tuned to give both good set-point response as well as load-disturbance response.

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