A sampled-data level control of nonlinear coupled-tanks

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***Abstract*—The object of this work is to highlight, on a physical coupled tanks system, the performances of a digital design of a continuous-time passivity based controller. The design considers the sampled-data model of the plant and provides better results compared to the emulations of given continuous-time controllers. A two-state continuous-time non-linear model is used to design the digital controller. A feedback passivating controller is developed in continuous-time from a suitable choice of the storage function V. The results show good performances of the controller when increasing the sampling period or the controllers gain.**

***Keywords*—passivity, nonlinear systems, level tank system, sampled-data control, process control**

I. Introduction

The level liquid control is a basic problem with vast and different applications in process industries. Many applications can be met in the case of steam generators, petrochemical plants, storage tanks in gas/oil production industry, paper making and water industries and many others.

Various experimental plants were used for testing and designing control strategies like: coupled tank systems, cascade tanks, three-tank systems [2], conical tanks [9], quadruple-tank process [4], state-coupled tanks [1] and so on. Related to the control strategies used, the majority deals with the design of linear controllers with linear systems. Among these, the PID control proved to be the most used with satisfactory performances.In a non-linear context many algorithms still are based on PID control were the controller parameters have to be continuously adjusted(e.g. [2]) by using different strategies. Other strategies involves predictive control, optimal control and adaptive control [9]. Intelligent control involving fuzzy control, neural network control and genetic algorithms have been also developed for level tanks system (see references in [8]).

In the context of the exact knowledge of the nonlinear model, only fewer nonlinear control strategies have been implemented, as from our knowledge we can cite: sliding mode control in [8],backstepping control in [1], passivity based control in [15], [7]. In this paper we used a passivity based control strategy. The passivity concept (firstly introduced by Popov [10]) has generated many useful tools in designing robust controller in the nonlinear context. Today, this topic is very attractive for many researchers, since not all underlying concepts have been fully comprehended and there are a wide range of applicability in the nonlinear control area. The passivity based controllers are relying on concepts of energy shaping and are more suitable for nonlinear dynamics where frequency mixing is most difficult to deal with. This is the case of many of the practical systems.

Another aspect, that we deal with, is the digital implementation of these strategies. When the exact knowledge of the system model is not available real-time adaptive controllers represent the best option. When a non-linear continuous-time model is available then a sampled-data control scheme should be carried out. In most sampled-data schemes, the analog/digital conversions rely on the use of zero-order holder devices. By our knowledge, in this context of the exact knowledge of the non-linear model of a coupled-tank system, there are no references of suitable sampled-data control designs. This represents in fact the novelty of this article.

Usual, these sampled-data schemes, are designed trough the emulation of the continuous-time controller. This solution is not efficient when increasing the sampling period. Another approach, often used, is to exploit the discrete-time equivalent of the continuous-time model and to design a digital controller, in this way the entire design is carried out in discrete-time. This solution also is not efficient, since for nonlinear models there can not be calculated exact discretization, and also, this solution does not take into account the inter-sampling behavior of the system.

The sampled-data solution that we have implemented on a tank level system, is inspired from [6] and [14]. The solution proposed is superior to other sampled-data approaches, mentioned earlier. According to this approach, the controller design is carried out in continuous-time domain and then a digital controller is computed in order to reproduce a specific continuous-time property of the closed loop, at each sampling instant. In this case we are referring to reproduce the passivity

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property assured by the continuous-time controller. It is a known fact that the passivity is lost under sampling [14]. This controller is proved to better preserve the passivity property with respect with the emulated solution.

The article is organized as follows: section II recalls the theoretical background elements, section III concern the description of the experimental plant and the controllers design. The results are illustrated and commented in section IV.

II. UNDERLYING THEORY

A. Preliminaries

In this paper the input-affine nonlinear systems are considered:

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + u_c(t)g(x(t)) \\ y(t) &= h(x(t)) \end{aligned} \tag{1}$$

where state vector $x \in \mathbb{R}^n$, containing all the information of the plant, input vector $u_c \in \mathbb{R}$ (the index c comes from 'continuous time') and output $y \in \mathbb{R}$. Also f, g are smooth vector fields and h is an analytic function.

1) Lie series: Let consider an analytic function $\lambda : R^n \to R$ and a vector field θ on R^n. The associated differential operator L_θ applied to λ such that $L_\theta \lambda = \frac{\partial \lambda}{\partial x}\theta(x)$ represents the Lie derivative of λ with respect to θ evaluated in a point x. The p-iterative of this operator is defined as $L_\theta^p = L_\theta \circ \ldots \circ L_\theta$, for $p \geq 0$ or $L_\theta^p \lambda = \frac{\partial L_\theta^{p-1}}{\partial x}\theta$ with $L_\theta^0 = 1$ the identity operator. We note e^{L_θ} the Lie series operator associated to L_θ, $e^{L_\theta} = 1 + \sum_{p\geq 1} \frac{L_\theta^p}{p!}$. Given another vector field σ, the Lie bracket is defined as $L_{[\theta,\sigma]} = L_\theta \circ L_\sigma - L_\sigma \circ L_\theta$. The evaluation of a function in x is noted "(x)" or "$|_x$" and the evaluation of an operator applied to identity function Id e.g. $L_\theta Id|_x$ is noted as $\theta(x)$.

2) Sampled models: The exact solution of the continuous plant (1) is given by a differential equation parameterized by the sampling period δ:

$$x_{k+1} = F_c^\delta(x_k, u_c) = e^{\delta(L_{f+u_c g})} x_k \tag{2}$$

This solution can be computed for $\delta \in (0, T^*]$, where T^* is sufficiently small.

Considering now the sampled data system when the control input $u(t)$ is constant over time intervals of amplitudes δ, such that $u(t+\tau) = u(t) = u_k$ for $0 \leq \tau < \delta,\ t = k\delta,\ k \geq 0$, the sampled equivalent is:

$$x_{k+1} = F_d^\delta(x_k, u_k) = e^{\delta(L_{f+u_k g})} x_k \tag{3}$$

In this case the Lie series operator $e^{\delta(L_{f+u_k g})}$ can be rewritten as:

$$\begin{aligned} &e^{\delta(L_f + u_k L_g)} = 1 + \delta(L_f + u_k L_g) + \\ &+ \frac{\delta^2}{2!}(L_f^2 + u_k(L_f L_g + L_g L_f) + u_k^2 L_g^2) + \ldots + \frac{\delta^P}{P!} L_{f+u_k g}^P + \ldots \end{aligned}$$

A sampled model of degree P represent an approximation of the asymptotic series of P order in δ (error in $O(\delta^{P+1})$). Each coefficient of δ is in fact the time derivative of order P of the state $x(t)$ evaluated at $t = k\delta$ instant.

As remark, the sampled model of degree 1 in δ is the same with Euler discretization.

B. Passivity based control

Passive systems are a class of processes that dissipate certain types of physical or virtual energy, described by Lyapunov-like functions [3]. The concept is defined as an input-output property of a system and it is particularly useful in the analysis of the stability. Furthermore, the passivity concept allows us constructing robust control laws even for nonlinear process. It is shown for instance, as the feedback controller adds to the system an "excess of passivity", this controller can handle with important uncertainties of the system parameters.

In the following lines, we recall from [13], the main definitions and results that are necessary to design the continuous-time controller in section III-C.

P1 - Passivity The system (1) is passive if there exists a positive C^1 function V on $\mathbb{R}^n$ (the storage function) such that V(0) = 0 and for all $u \in U,\ t \geq 0$

$$\dot{V}(x(t)) \leq y^T(t)u(t) \tag{4}$$

or its integral equivalent for all $x_0 \in R^n$

$$V(x(t)) - V(x_0) \leq \int_0^t y^T(\tau)u(\tau)d\tau \tag{5}$$

where U is a set in $\mathbb{R}$ of admissible inputs.
Equivalent definitions state that a system is passive if it is *dissipative* and has a bilinear supply rate $w(u,y) = y^T u$ [11]. The interpretation is that the rate of increase of energy is not bigger than the input power.

P2 - zero-state detectability (ZSD) The system (1) is locally zero state detectable if no solution of the uncontrolled dynamics $\dot{x} = f(x)$ can stay in the set $Z = \{x \in \mathbb{R}^n,\ \text{s.t. } h(x) = 0\}$ other than solutions $x(t)$ converging asymptotically to the zero equilibrium.

The main benefice of the ZSD property is that it gives the condition to connect the stability with passivity. It follows the next result.

P3- negative output feedback gain Under the assumptions P1, with $V > 0$ and zero-state detectability, then the storage function V qualifies as a Lyapunov function and the controller $u_c(t) = -Ky(t)$ with $K > 0$ asymptotically stabilizes the initial system. Under these conditions, the closed loop dynamics satisfies

$$\dot{V}(t) \leq -K^2 y(t). \tag{6}$$

With the increase of the gain K, more negativity is added to $\dot{V}$ compared to the free evolution and thus improving the damping performances.

In conclusion, if we look to stabilize the system (1), it becomes a simple task if this system with input u and output y is passive. The property *P3* tell us that we can close the loop with $u = -Ky$, and if the system is ZSD, the equilibrium point is GAS. As consequence, the system has to be stable also when $u = 0$. If this condition is not satisfied, then it is necessary to make the following feedback transformation

$$u_c = \alpha(x(t)) + \beta(x(t))v(t, x(t)), \tag{7}$$

with $\beta(\cdot)$ invertible, such that the new system is passive. In this way, the feedback transformation can render the system passive.

C. Digital design

In this section, we recall the steps for the digital design of a PBC controller in the context of preserving under sampling, the stabilizing properties of the continuous-time plant.

Under sampling, the passivity condition (5) is rewritten as follows, by assuming $u(t) = u_k$ for $t \in [k\delta, (k+1)\delta)$, with δ the sampling period:

$$V(x_{k+1}) - V(x_k) \leq u_k \int_{k\delta}^{(k+1)\delta} y(\tau) d\tau \tag{8}$$

The left side represents the Lyapunov difference of the sampled-data plant for a time period equals to δ. The sampled Lyapunov at instant $k+1$ can be computed as

$$V(x_{k+1}) = e^{L_f + u_k L_g} V(x)|_{t=k\delta} \tag{9}$$

The basic idea for designing a digital controller u_k is that it has to satisfy the inequality (8) for various δ. In this context, two direct digital strategies were proposed in [13], one that render negative the right hand side and the other one the left hand side (L_{G^δ} controller) negative of the inequality (8). These approaches not rely on the controller design that was carried out in continuous-time, and allow to specify the performances of the sampled-data controller in discrete-time.

Another solution, that can be applied, relies on the existence of a continuous-time Lyapunov-based controller. The strategy, called *the input-Lyapunov matching* [13], aims to design a piecewise constant controller of the form (10) that ensures the matching of the sampled-data Lyapunov evolution with the continuous-time one. Such a procedure can be reset in the context of input-output matching at the sampling instants of some target closed loop dynamics under piecewise constant controls as developed in [5], [12].

The sampled data controller has the following expression:

$$u_k = u_k^\delta = u_{d0} + \sum_{i \geq 1} \frac{\delta^i}{(i+1)!} u_{di} \tag{10}$$

This controller is computed to solve term-by-term the equality

$$V(x_{k+1}) - V(x_k) = \int_{k\delta}^{(k+1)\delta} \dot{V}(x(t)) dt \tag{11}$$

where $\dot{V}(x(t)) = (L_f + u_c(t) L_g) V(x(t))$ represents the continuous-time target closed loop behavior of V, while the left hand side is the sampled-data evolution. The existence of such solution is proved in [13] and it relies to some relative degree definitiveness which correspond, in the present context, to require $L_g V \neq 0$ and ZSD property with respect to the output mapping $y = L_g V$. The controller u_k achieves global asymptotic stability for the origin of system (1). As the exact computation of this controller is rarely possible, due to the nonlinearities, the digital implementation requires approximate solutions and the origin can be semi-globally stabilized. According to [13], the first order components of the controller u_k are :

$$\begin{cases} u_{d0} = & u_c(x_k); \\ u_{d1} = & \dot{u}_c(x_k); \\ u_{d2} = & \ddot{u}_c(x_k) + \frac{1}{2}\dot{u}_c(x_k)(L_f L_g - L_g L_f)V(x_k)/L_g V(x_k). \end{cases} \tag{12}$$

Remark 2.1: The first controller component u_{d0} is called the emulated control and represents the usual implementation of a continuous-time control on a numerical device by means of zero order holders. The higher order control components are called *corrective* terms for the emulated solution.
The complexity of the controllers expressions is higher for increasing orders. The second order approximate controller $u_{da}^2 = u_{d0} + \frac{\delta}{2} u_{d1} + \frac{\delta^2}{3!} u_{d2}$ guarantees Lyapunov step-by-step matching up to third order of approximation (error in $O(\delta^4)$).

We can conclude now, this controller render at each sampling instant the same continuous-time Lyapunov evolution and in consequence the passivity condition (8) is assured for $\forall \delta < T^*$, where T^* is the maximum allowable sampling period and it depends on the system characteristics.

III. The experimental plant model and controller design

A. The plant description

The laboratory platform (ELWE Technick) used to test the proposed algorithms, available at Politehnica University Bucharest, consists of 3 main water tanks and a water reservoir. The flows of the water is assured by the means of 6 electro valves and 2 pomps and each liquid level is measured by one of the 3 piezoresistive transducers (VEGABAR 14).

In figure 1 is presented the schematic of a coupled-tank system that was configured on the laboratory platform. The water is pumped into tank T_1 and from there through a connection pipe (with a section area S_1) into the tank T_2. The water flows into reservoir through an electro valve (full open) with a section area S_2. The values of the plant parameters are

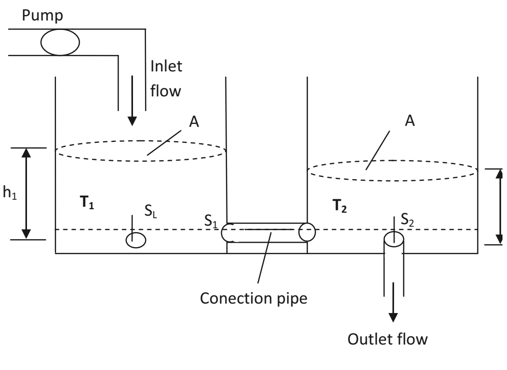


Figure 1. Schematic of coupled-tanks

described next:

- $A = 0.0154\ m^2$ - section area of each tank;
- $S_1 = S_2 = 5 \cdot 10^{-5}\ m^2$ - the section area of each pipe corresponding to electro valves;

- $Q_0 = 10^{-4}\ m^3/s$, $h_0 = 0.6\ m$ - the maximal values of the inlet flow and tank level respectively.

The connection of the plant with a PC is assured by 2 acquisition cards : "Humusoft MF624" for Pump 1 and level transducers and "National Instruments PCI-6503" which controls the electro valves. The real-time interface is configured in Simulink.

B. Dynamic model

The dynamic model of the coupled tanks is described by

$$\begin{aligned} \dot{h_1} &= \frac{1}{A}Q_i(t) - \frac{1}{A}c_1\sqrt{|h_1-h_2|}sign(h_1-h_2) \\ \dot{h_2} &= \frac{1}{A}c_1\sqrt{|h_1-h_2|}sign(h_1-h_2) - \frac{1}{A}c_2\sqrt{h_2} \end{aligned} \quad (13)$$

with $c_1 = 1.0121\cdot10^{-4}$, $c_2 = 1.6854\cdot10^{-4}$ experimentally determined. The parameters $c_i = az_iS_n$ define the product between the flow coefficient az_i and the pipe cross area. The input is the inlet volumetric flow rate $u(t) = Q_i(t)$, the state variables represents the liquid levels in tank T_1 and T_2 and the output is chosen equal to h_1. In the case of coupled tanks, the inequality $h_1 \geq h_2$ holds, in every operating point.

Considering now, the storage function S described by

$$S(t) = \frac{1}{2}A(h_1^2(t)+h_2^2(t)) \quad (14)$$

which represents, in some sens, half of the total energy stored in both tanks. For this system next proposition holds.

Proposition 3.1: The system Σ_c defined by equation (13) is passive with output $y = h_1$ and control $u(t) = Q_i(t)$.

Proof: Looking to property $P1$, the system is passive if condition (4) holds,

$$\frac{dS}{dt} \leq y(t)u(t) \quad (15)$$

Then

$$\begin{aligned} \dot{S} &= Ah_1\dot{h}_1 + Ah_2\dot{h}_2 = \\ &= Q_ih_1 - c_1\sqrt{h_1-h_2}(h_1-h_2) - c_2\sqrt{h_2}h_2 \end{aligned} \quad (16)$$

and

$$y(t)u(t) = Q_ih_1 \quad (17)$$

The inequality (15) holds

$$Q_ih_1 - c_1\sqrt{h_1-h_2}(h_1-h_2) - c_2\sqrt{h_2}h_2 \leq Q_ih_1 \quad (18)$$

since $\forall h_1 \geq h_2 \in \mathscr{R}_+$. ■

At the equilibrium point, a useful relation can be found,

$$h_{1e} = \frac{c_1^2+c_2^2}{c_1^2}h_{2e} = h_{2e}/d_e,\ d_e \cong 0.265. \quad (19)$$

C. Continuous-time controller design

The control objective is to maintain the levels of the coupled-tanks, to desired values h_{1d}, h_{2d} given by an external reference.

The dynamic (13) is passive and also satisfies the *ZSD* property. According to $P3$ property, the negative output controller $u(t) = -Ky(t)$ asymptotically stabilizes the origin of the system. This controller will regulate only to bring the system to the equilibrium $h_1 = 0$ and $h_2 = 0$. To accomplish the objective, a change of coordinates is required in order to construct an error dynamics model.

Let us now to consider the following change of coordinates:

$$\eta_1 = \frac{h_1-h_{1r}}{h_0}, \quad \eta_2 = \frac{h_2-h_{2r}}{h_0}.$$

In this way, $\eta_1 \in [-d, 1-d]$ and $\eta_2 \in [-d_ed, d_e-d_ed]$ with $d = h_1/h_0$, $d \in [0,1]$ and d_e previously defined. The controller is considered as $u(t) = Q_0(u_s+v(t))$ and the error plant dynamics becomes:

$$\begin{aligned} \dot{\eta_1} &= \frac{1}{Ah_0}\left((u_s+v(t))Q_0 - c_1\sqrt{h_0(\eta_1-\eta_2)+h_{1r}-h_{2r}}\right) \\ \dot{\eta_2} &= \frac{1}{Ah_0}\left(c_1\sqrt{h_0(\eta_1-\eta_2)+h_{1r}-h_{2r}} - c_2\sqrt{h_0\eta_2+h_{2r}}\right) \end{aligned} \quad (20)$$

with control input $v(t)$ and now the output is chosen as $y = Q_0\eta_1$. The static component of the controller u_s is designed such that the equilibrium point of the error dynamics to be $\eta_1 = 0$, $\eta_2 = 0$. In this point the dynamics are:

$$0 = \frac{1}{Ah_0}\left(u_sQ_0 - c_1\sqrt{h_{1r}-h_{2r}}\right) \quad (21)$$

$$0 = \frac{1}{Ah_0}\left(c_1\sqrt{h_{1r}-h_{2r}} - c_2\sqrt{h_{2r}}\right) \quad (22)$$

and one gets

$$u_sQ_0 = c_1\sqrt{h_{1r}-h_{2r}} = c_2\sqrt{h_{2r}}.$$

Furthermore the system is now respecting the *ZSD* property. The storage function V can be chosen as:

$$V = \frac{1}{2}Ah_0(\eta_1^2+\eta_2^2). \quad (23)$$

The passivity condition (4) is translated to

$$\begin{aligned} &\eta_1Q_0(u_s+v(t)) - c_1\sqrt{h_0(\eta_1-\eta_2)+h_{1r}-h_{2r}}(\eta_1-\eta_2) \\ &\qquad -c_2\sqrt{h_0\eta_2+h_{2r}} \leq Q_0\eta_1v(t) \end{aligned} \quad (24)$$

The upper condition is satisfied, since, after some computations

$$\begin{aligned} &\eta_1\sqrt{d(1-d_e)} - \sqrt{\eta_1-\eta_2+d(1-d_e)}(\eta_1-\eta_2) \\ - &\eta_2\sqrt{\frac{1}{d_e}-1}\sqrt{\eta_2+d_ed} \leq 0 \end{aligned}$$

holds for every $\eta_1 \in [-d, 1-d]$, $\eta_2 \in [-d_ed, d_e-d_ed]$ and $d \in [0,1]$, with $\eta_1-\eta_2+d(1-d_e) \geq 0$.

We conclude now, that the controller $v(t) = -Ky(t)$ asymptotically stabilizes globally the system to the equilibrium point

$\eta_1 = \eta_2 = 0$, with $K > 0$. The controller $v(t)$, is computed also as $v = -KL_gV$. Going back to initial coordinates, the practical controller $u(t)$ is expressed as:

$$u(t) = c_1\sqrt{h_{1r} - h_{2r}} - KQ_0\frac{h_1(t) - h_{1r}}{h_0},\ K > 0.$$

The parameter K is chosen according to the admissible values of the overshoot and the speed of the systems output.

D. Sampled-data controller design

For the next developments let us consider the error dynamics written in a compact manner:

$$\dot{\eta} \quad = \quad f(\eta) + g(\eta)u_c(t) \tag{25}$$

with

$$f(\eta) = \frac{1}{Ah_0}\left[\begin{array}{c} u_sQ_0 - c_1\sqrt{h_0(\eta_1 - \eta_2) + h_{1r} - h_{2r}} \\ c_1\sqrt{h_0(\eta_1 - \eta_2) + h_{1r} - h_{2r}} - c_2\sqrt{h_0\eta_2 + h_{2r}} \end{array}\right]$$

$$g(\eta) = \frac{Q_0}{Ah_0}\left[\begin{array}{c} 1 \\ 0 \end{array}\right], \quad u_c(t) = -Ky(t)$$

The second order digital controller, according to the design presented in section II-C, has the next structure:

$$u_{da}^2 = u_{d0} + \frac{\delta}{2}u_{d1} + \frac{\delta^2}{6}u_{d2} \tag{26}$$

with

$$\begin{aligned} u_{d0} &= -K\eta_1|_{t=k\delta} \\ u_{d1} &= \frac{Kc_1(\sqrt{h_{1r} - h_{2r} + h_0(\eta_1 - \eta_2)} - \sqrt{h_{1r} - h_{2r}}) + K^2Q_0\eta_1}{Ah_0}|_{t=k\delta} \\ u_{d2} &= \frac{ud2e}{A^2{h_0}^2\eta_1\sqrt{h_{1r} - h_{2r} + h_0(\eta_1 - \eta_2)}}|_{t=k\delta} \end{aligned}$$

with

$$\begin{aligned} u_{d2e} &= (-\tfrac{K}{4})(3{c_1}^2h_0\eta_1\sqrt{h_{1r} - h_{2r} + h_0(\eta_1 - \eta_2)} - 4KQ_0c_1h_{2r}\eta_1 \\ &+ {c_1}^2h_0\eta_2\sqrt{h_{1r} - h_{2r} + h_0(\eta_1 - \eta_2)} - 3KQ_0c_1h_0\eta_1\eta_2 \\ &- {c_1}^2h_0\eta_1\sqrt{h_{1r} - h_{2r}} - {c_1}^2h_0\eta_2\sqrt{h_{1r} - h_{2r}} \\ &+ 4K^2{Q_0}^2{\eta_1}^2\sqrt{h_{1r} - h_{2r} + h_0(\eta_1 - \eta_2)} + 4KQ_0c_1h_{1r}\eta_1 \\ &- 2c_1c_2h_0\eta_1\sqrt{h_{2r} + h_0\eta_2} + 5KQ_0c_1h_0{\eta_1}^2 \\ &- 4KQ_0c_1\eta_1\sqrt{h_{1r} - h_{2r}}\sqrt{h_{1r} - h_{2r} + h_0(\eta_1 - \eta_2)}). \end{aligned}$$

A change of coordinates is required to express this controller into initial physical parameters.

IV. RESULTS

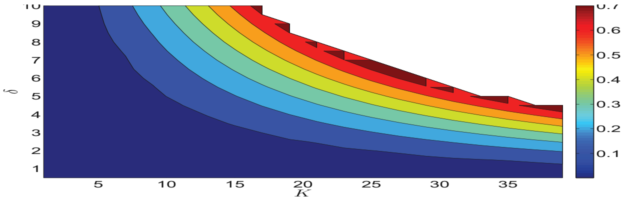
A. Preliminary result

As the sampled-data design concerns the reproduction under sampling of the continuous-time Lyapunov evolution of the closed loop dynamics, some simulations were performed to evaluate the matching error between these evolutions under the action of an emulated or under a second order controller, respectively. These simulations are also necessary to fix the admissible sampling period and the gain of the controller.

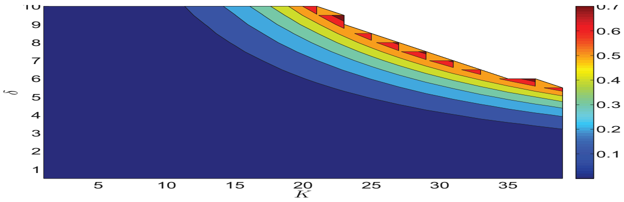
An error indicator, which represents the mean square error between the continuous-time and the sampled Lyapunov evolutions, is defined in order to evaluate the performances of the controls, i.e.

$$E := \sqrt{\sum_{k=1}^{n}\frac{(V(x(t))|_{t=k\delta} - V(x(k\delta)))^2}{n}} \tag{27}$$

As the errors have small values then the reproduction is satisfactory and the properties that rely on it, like stabilization and passivity, are well preserved. In figure 2 are plotted the



(a) Emulated



(b) $2nd$ order control

Figure 2. The matching error under sampled-data control

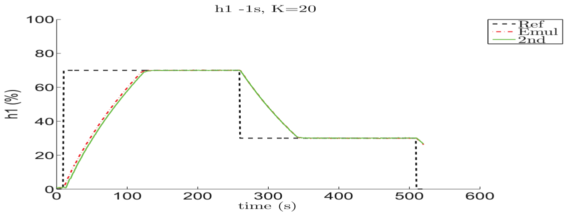
error criteria values for the gain K, of the controller, in the range $[1, 40]$ and for a sampling period between $[0.5, 10]$ seconds. The results confirm that the use of a second order controller reduces the amplitudes of the error and makes available the use of higher sampling periods or of higher gains.

B. Experimental results

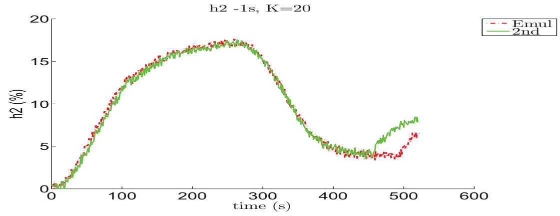
For the practical implementations we have considered a gain $K = 20$, the sampling period was given in the range $[1, 10]$ seconds and the tanks were empty at the start of each experiment. The value of the gain is set to this value, in order to obtain a fast response for the reference changes. Also in this case, the control performances are more sensitive to the amplitudes of the sampling period used.

In figures 3a)-c) are plotted the evolutions of the tank levels and the control input, for a sampling period equal to 1 second. The reference is given as down staircase signal with values of 70%, 30% and 0% from the maximum height of the tank h_0. For important changes in the reference points, the control become saturated. We have imposed this values to reference, to show that the controller regulates, with zero static error, any reference given. What we are interestead also in, is how evolves the system when increasing the sampling period. In figure 4 are plotted the same evolutions for a sampling period equals to 10 seconds.

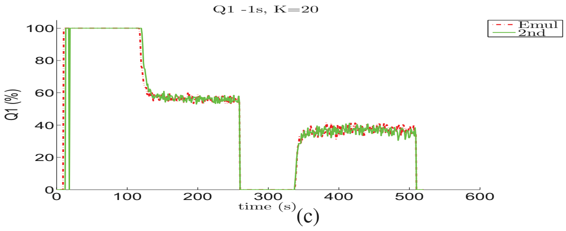
The Figure 4b) is an expanded image of 4a) which shows that the emulated control becomes unstable, with increasing oscillations. The same observation can be made on the control evolution in Figure 4c), where the second order controller acts smoother. For sampling periods smaller than 10 s, for the emulated case the closed loop is stable but the controller is much stressed compared to a second order controller. The unstable behavior is due to the fact that the sampling period



(a)

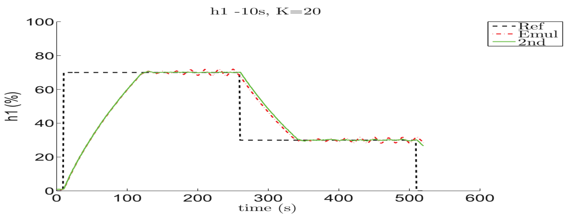


(b)

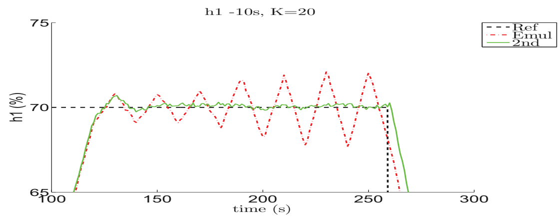


(c)

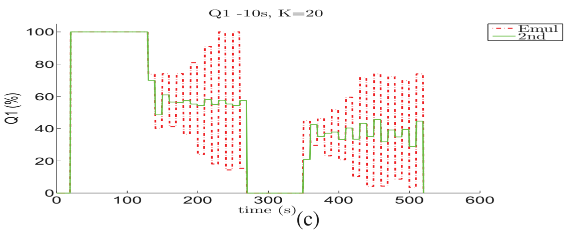
Figure 3. Experimental results for $\delta = 1$s



(a)



(b)



(c)

Figure 4. Experimental results for $\delta = 10$s

used in this case is larger than the admissible value for which the emulated control is still stabilizing.

V. Conclusions

The benefits of the sampled-data design have been proved on a level control system. The passivity property of the system, allowed as to construct a continuous-time stabilizing controller for the non-linear system. The digital controller was specially designed in order to preserve the continuous-time properties of the controller. The results show that the digital design approach used can increase the use of the sampled period and also the use of a bigger gain of the controller. A bigger value of K can increase the speed response of the system.

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