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A Natural Approach to Modeling Physical Systems†

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Abstract.

Models are useful for design of control systems and they are also an integral part of high performance controllers. Although modeling is an important aspect of most engineering activities, the control engineer faces special problems because he has to deal with strongly heterogeneous systems which have mechanical, electrical, and chemical parts. This paper describes a general approach that can be applied to a wide range of systems. It is based on constraints and object orientation and leads naturally to behavioral systems. The approach is in this paper illustrated with modeling of a thermal boiler.

1. Introduction

State models are commonly used in control theory. One of the key drawbacks of the state models is that they are causal. When modeling physical systems it is a severe restriction to assign causality in physical modeling because causality depends on the way systems are interconnected. This difficulty has been observed when developing model libraries. Behavioral models or constraint models is another modeling paradigm which is better suited to physical modeling. In this case the model is simply a set of differential or algebraic equations which introduces constraints among the variables that describe a system. This approach is very convenient when modeling complex systems. It leads to a system description in terms of differential-algebraic equation systems. Recent advances in numerical analysis have given algorithms for integrating such equations. There is also an emerging control theory for such systems.

2. An Approach to Modeling

The notion of decomposition is very useful when dealing with a large system. By decomposing it into smaller components that can be analyzed and understood separately, it is possible to get an understanding of the total system. The physical structure of the system often suggests suitable subdivisions. The inverse concept, composition, is naturally applied to construct large systems from simple components. A library of good model components is indeed a good way of supporting use of mathematical models.

We distinguish between primitive and structured behavior descriptions. A structured behavior description consists of connected submodels, which in turn can be structured. The model concept is hierarchical.

A primitive description gives behavior in terms of equations. The behavior of the subsystems can be captured in terms of balance equations and constitutive equations. Variables that account for storage of mass, energy and momentum have to be introduced as well as parameters. There may also be discrete events that change the behavior of the subsystems. The interaction between the subsystems seldom is unidirectional, this means that causal input-output models are inappropriate. An equation oriented or a behavioral approach where a model is viewed as constraints between variables is much more natural.

The concepts of object orientation are convenient tools for describing system decomposition. All subsystems are represented as classes. Inheritance and specialization support easy modification. A class can be defined as a subclass of another class, called the super class or the base class. By inheritance, all attributes of the base class are available in the subclass. New attributes can be added to the subclass to extend or specialize the definition.

The modeling approach outlined above is supported by the modeling language Omola [3, 1] and the simulation environment OmSim. Omola supports behavioral descriptions in terms of differential-algebraic equations, difference equations and discrete events. The primitives for describing discrete events allow implementation of high level descriptions as Petri nets and Grafcet. Matrix notation is supported.

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3. An Example

As an illustration we will consider a model of a drum boiler. See Figure 1. We will only discuss the downcomer-riser-drum section. This subsystem which is composed of a mixture of steam, water and metal has a quite complicated behavior. Joint balance equations for water and steam eliminate the need to model the transfer between steam and water explicitly. The global mass and energy balances are

$$\frac{d}{dt}\,(\rho_s V_s + \rho_w V_{wt}) = q_{fw} - q_s \tag{1}$$

$$\frac{d}{dt}\,(\rho_s h_s V_s + \rho_w h_w V_{wt} + m c_p T) = P + q_{fw} h_{fw} - q_s h_s \tag{2}$$

where ρ denotes specific density, h enthalphy, V volume, and q mass flow rate. The indices s, w and fw refer to steam, water and feedwater respectively. The total mass of the metal tubes is m, the specific heat is c_p, the average metal temperature is T, and the input power from the fuel is denoted by P. The total steam volume is given by

$$V_s = V_{drum} - V_w + a_m V_r \tag{3}$$

where V_{drum} is the drum volume, V_w the volume of water in the drum, V_r the riser volume, and a_m the average steam-water volume ratio in the risers. The total water volume is

$$V_{wt} = V_w + V_{dc} + (1 - a_m) V_r \tag{4}$$

The distribution of steam and water along the risers is described by partial differential equations. To obtain a finite dimensional model we assume that the shape of the distribution is known. The assumed shape is based on solving the partial differential equations in the steady state. This gives a linear distribution of the steam-water mass ratio along the risers. Assuming that the steam-water mass ratio is also linear during dynamic changes the dynamics can be captured by a finite dimensional system characterized by a_m.

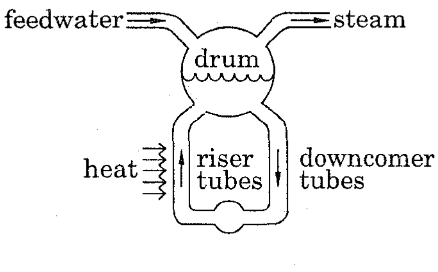

Figure 1 Schematic diagram of a drum boiler.

Equations similar to (1) and (2) can be derived for the drum. For this system it is necessary to use the flows in and out of the drum as terminal variables. The steam flow into the drum can be characterized by x_r, the steam-water mass ratio at the riser outlet. It follows from the linear steady state distribution of steam that

$$a_m(x_r) = \frac{\rho_w}{\rho_w - \rho_s}\Big(1 - \frac{\alpha}{x_r}\ln(1 + \alpha x_r)\Big) \tag{5}$$

where $\alpha = \rho_s/(\rho_w - \rho_s)$.

The mass flow rate through the downcomers, q_{dc}, is modeled by a momentum balance

$$a_m V_r (\rho_w - \rho_s) = \frac{1}{2} k\, q_{dc}^2 \tag{6}$$

where k is a friction coefficient. Since all parts that are contact with steam are in thermal equilibria, their behavior can be represented by one variable, which we choose as the steam pressure. Using steam tables the variables T, ρ_s, ρ_w, h_s, and h_w can then be expressed as functions of steam pressure p. The system can be represented by four differential equations (1), (2), and two similar equations for the drum volume, four algebraic equations (3), (4), (5), (6) and steam tables. Notice the simplicity of the basic physical equations. Attempts to eliminate variables manually to obtain traditional state equations are very tedious and error prone. The basic equations can be entered into the modeling language Omola. OmSim can simulate them as they are or manipulate them to explicit state space form first.

4. Conclusions

A methodology for modeling physical systems has been described. The modeling language Omola is described in [3, 1] and the model in [2]. For information on Omola and access to OmSim see WWW at URL: `http://www.control.lth.se/~cace`.

5. References

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