An LMI Robust Predictive Control Approach Applied in a Coupled Tanks Systems

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Abstract*— This work deals of an on-line control strategy based on Robust Model Predictive Control (RMPC) technique applied in a real coupled tanks system. This process consists of two coupled tanks and a pump to feed the liquid to the system. The process variables (levels) are transmitted to the PLC (Programmable Logic Controller) thought a voltage signal. The control signal, in volts, generated in the PLC, is sent to the pump. The control objective (regulator problem) is to keep the tanks levels in the considered operation point even in disturbance presence. The RMPC is a technique that allows explicit incorporation of the plant uncertainty in the problem formulation. The goal is to design, at each time step, a state-feedback control law that minimizes a 'worst-case' infinite horizon objective function, subject to constraint in the control input. The existence of a feedback control law and satisfying the input constraints is reduced to a convex optimization over linear matrix inequalities (LMIs) problem. It is shown that for the plant uncertainty described by the polytope, the feasible receding horizon state feedback control design is robustly stabilizing. The software implementation of the RMPC is made using Scilab, and its communication with Coupled Tanks Systems is done through the OLE for Process Control (OPC) industrial protocol.

I. Introduction

The theory of model-based predictive control (MPC) originated in the late 1970s and has grown considerably since then [2],[14]. Today there are many successful applications of this technique, not only in the chemical industry but also in other areas [15]. The MPC is characterized by the use of the process model to predict its outputs at a future time and calculate a sequence of control actions that minimize an objective function with the implementation of the first control action calculated from the sequence and update the output signals' measures to perform new calculations of minimization [2], [12], [17].

According to [13] several applications for the formulation of robust predictive control laws began to appear in the literature in the 1990s, focusing on both model uncertainties and disturbances. In general, the formulations of MPC consider a simple linear and time-invariant model to describe the plant. This formulation generates a signal that can result in poor performance when implemented in a physical system that is not exactly described by the model. This fact led the authors of [3] to modify the on-line minimization problem (minimizing some objective functions subject to input and output constraints) to a min-max problem (minimization of the worst-case value of the objective function, where the worst-case is taken over the set of uncertain plants). This problem of robust stability was described by several authors [3], [8], [18]. Applications of min-max problem can be found in [1] and [10].

In this method, a cost function is minimized considering the worst case into all the plants described by the uncertainties. Barriers of robust MPC algorithms include: the computational cost, the applicability depending on the speed and size of the plant on which the control will act. In this regard, the authors in [9] used the formulation in Linear Matrix Inequalities (LMIs) to solve the optimization problem. The basic idea of LMIs is to interpret a control problem as a semidefinite programming (SDP), in other words, an optimization problem with linear objective and positive-definite constraints involving symmetric matrices that are related to the decision variables. The authors in [9] developed this technique of control and called it the Robust Model Predictive Control (RMPC). The RMPC incorporates a broad class of uncertainties, since the main flaw in the design of MPC techniques is their inability to deal explicitly with uncertainty in the plant model.

In [18] they worked with the class of uncertain systems for modeling of non-linear plants and applied the technique in a reactor tank of the industry. The authors in [5] used Lyapunov functions in the vertices of the polytope of uncertain system with the goal of reducing the conservatism. The work [4] uses the Lyapunov functions for systems with output feedback. According to authors in [6] the control law off-line based in states warrants constraints on input and output states of the plant, when online, at each sampling period and it is chosen an appropriate sequence to be applied.

The goal of this paper is to develop a strategy for on-line control technique based on Robust Predictive Controller proposed in [9] based on LMIs applied to a second order real system of coupled tanks of Quanser Consulting. The organization of the paper is as follows. Section 2 provides a brief description of the LMI-based MPC formulation adopted in this work. Section 3 describes the physical structure used for the experiments in the plant. Illustrations of the results of the experiments are found in Section 4. Conclusion remarks are presented in section 5.

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II. LMI-BASED ROBUST MODEL CONTROL PREDICTIVE CONTROL

Consider the following model system represented by the linear time-varying in discrete form [9]

$$x(k+1) = A(k)x(k) + B(k)u(k) \qquad (1)$$
$$y(k) = C(k)x(k)$$
$$[A(k)\ B(k)] \in \Omega$$

where $x(k) \in \Re^n$, $u(k) \in \Re^p$, $y(k) \in \Re^q$ denote the state, control input and output respectively. Assuming that the system is controllable and all states $x(k)$ are evaluated by feedback. Thus, ratings $A(k)$ and $B(k)$ of (1) denote that the matrices of the model can change every sampling time, but that does not mean its variation with time is known (as a function of k). Soon, the whole Ω of (2) represents the convex polytope obtained by describing the polytopic uncertainties.

$$\Omega = C_o\{[A_1\ B_1],[A_2\ B_2],...,[A_L\ B_L]\} \qquad (2)$$

where C_o refers to the convex hull. If $[A\ B] \in \Omega$, then for some nonnegative $\lambda_1, \lambda_2,...,\lambda_L$ summing to one, we have

$$[A\ B] = \sum_{i=1}^{L} \lambda_i [A_i\ B_i] \qquad (3)$$

The vertices of the polytope in the (3) correspond to a set of models at different operating conditions of the process where any point within this set can be represented by the convex sum of a finite number of points L called vertices of the polytope represented by (2). In [9] the goal is to minimize the worst case of the following quadratic objective function of the infinite horizon

$$J_\infty(k) = \sum_{i=0}^{\infty} [x(k+1|k)^T Q_1 x(k+i|k) + u(k+i|k)^T R u(k+1|k)] \qquad (4)$$

where $Q_1 > 0$ and $R > 0$ are symmetric weighting matrices subject to

$$\left|u_j(k+i|k)\right| \le u_{j,\max}, i \ge 0, j = 1,2,...,p \qquad (5)$$

and

$$\left|y_j(k+i|k)\right| \le y_{j,\max}, i \ge 0, j = 1,2,...,q \qquad (6)$$

Hence, accomplish the solution of the following problem result in

$$\min_{u(k+i|k), i\ge 0} \max_{[A(k+i)Bx(k+i)]\in\Omega, i\ge 0} J_\infty(k) \qquad (7)$$

With this min-max problem is computationally complex, authors in [9] proposed as an alternative to minimizing an upper bound γ to $J_\infty(k)$ and enunciated the following theorem:

Theorem 1. Let $x(k) = x(k|k)$ be the state of the uncertain system described in (1) measured at sampling time k. Assume that there are no constraints on the control input and plant output. Suppose that the uncertain set Ω is defined by a polytope as in (2). Then the state feedback matrix F in the control law $u(k+i|k) = Fx(k+i|k), i \ge 0$ that minimizes the upper bound γ of $J_\infty(k)$ the objective function of robust performance at sampling time k is given by:

$$F = YQ^{-1} \qquad (8)$$

where $Q > 0$ and Y are obtained from the solution (if it exists) of the following linear objective minimization problem:

$$\min_{\gamma,Q,Y} \gamma \qquad (9)$$

subject to

$$\begin{bmatrix} 1 & x(k|k) \\ x(k|k)^T & Q \end{bmatrix} \ge 0 \qquad (10)$$

and

$$\begin{bmatrix} Q & QA_j^T + Y^T B_j^T & QQ_1^{1/2} & Y^T R^{1/2} \\ A_j Q + B_j Y & Q & 0 & 0 \\ Q_1^{1/2} Q & 0 & \gamma I & 0 \\ R^{1/2} Y & 0 & 0 & \gamma I \end{bmatrix} \ge 0, j = 1,...,L \qquad (11)$$

The constraints in input showed in (5) and output showed in (6) are respect if for possible to minimize the previous plus the following LMIs

$$\begin{bmatrix} u_{\max}^2 & Y \\ Y^T & Q \end{bmatrix} \ge 0, \qquad (12)$$
$$u_{\max}^2 \ge X_{jj}, j = 1,...,p$$

and

$$\begin{bmatrix} Q & (A_i Q + B_i Y)^T \\ C_j(A_i Q + B_i Y) & Z \end{bmatrix} \ge 0, \qquad (13)$$
$$y_{j,\max}^2 \ge Z_{jj}, j = 1,...,q, i = 1,...,L$$

Authors in [9] pronounce that any feasible solution of the optimization in Theorem 1 at time k is also feasible for all times $t > k$.

Thus, if the optimization problem in Theorem 1 is feasible at time k, then it is feasible for all time $t > k$. For more details, it is recommended to consult the paper [9].

III. SYSTEM DESCRIPTION

The Figure 1 shows the structure used with the equipment located at the Laboratory of Computation and Automation Engineering - LECA, Federal University of Rio Grande do Norte - UFRN.

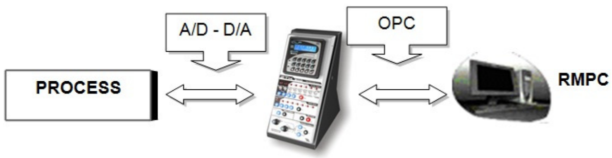

Fig.1. Physical structure of System.

The Equipment and software utilized in the physical structure of Fig.1 were: Process (Hydraulic System), Power Amplifier Module – UPM 2405-240, Training Kit ZTK 900 of *HI Technologia* (PLC ZAP 900) as A/D and D/A converter, OPC communication module (Scilab Toolbox) and the computer with the control strategy RMPC using the platform Scilab (Scientific Laboratory). The following topics will present a brief description of the components of the physical structure.

A. *Hydraulic System*

The hydraulic system used is a coupled tank system feed by a DC pump. Through the power amplifier module, UPM 2405-240, it is possible to perform an interconnection with the training kit ZTK 900 (PLC), Fig. 2.

(A) (B)

Fig. 2. Coupled Tank System of Quanser (A) and Power Amplifier Module - UPM 2405-240 (B).

The operation is as follows: the two tanks are connected in cascade, however, the tank receives a water pump and tank 2 receives water from tank 1 (Fig.3). This system allows working with two configurations; the first one is simply to control the level of tank 1, where it behaves as a first order system, whereas the second configuration aims to control the level of tank 2, in which case it behaves as a second order system.

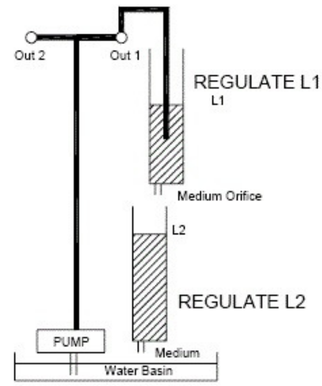

Fig. 3. Tanks System Configuration.

The mathematical model of Equation (14) represents the dynamics of a level system for tank 2. This paper worked with the second configuration of the experimental results in section 4.

$$\dot{L}_2 = \frac{a_1}{A_1}\sqrt{2gL_1} - \frac{a_2}{A_2}\sqrt{2gL_2} \tag{14}$$

The Equation (15) represents the system of (14) linearized at the points of operations $L_{10} = 15cm$ and $L_{20} = 15cm$.

$$\dot{L}_2 = -\frac{a_2}{A_2}\sqrt{\frac{g}{2L_{20}}}L_2 + \frac{a_1}{A_1}\sqrt{\frac{g}{2L_{10}}}L_1 \tag{15}$$

The Table 1 is used to identify each parameter of the equations above.

TABLE I
PARAMETERS OF TANKS SYSTEM

Parameter	Description	Values
L_1 and L_2	Height of the water level in the tank 1 the 2 in cm	0 – 30 cm
A_1 and A_2	Transversal section area of Tank 1 and 2	15.518 cm^2
a_1 and a_2	Transversal section area of outflow orifices 1 and 2	0.1781 cm^2
Km	Pump Constant	4.6 (cm^3/s).V
Vp	Voltage applied to the pump	-22V until +22V
g	Gravitational acceleration	981 cm/s^2

B. *ZTK 900*

The training kit ZTK 900 was used as an A/D and D/A interface for the plant. Since Scilab can't establish direct communication with the drivers from the maker of the plant, the PLC was used to perform this interface. Scilab then communicates with the PLC through the OLE protocol for Process Control (OPC). For communicating with the amplifier module the inputs and the outputs should work with voltages. So the integrated circuit LM 324N was installed to convert the output signal from 0..4 mA to -5 at 5 Volts adapting the system to communicate with the UPM-2405-240 power module.

C. Control of Strategy and OPC

The control strategy RMPC was developed in a computer with the software Scilab 4.2. The implementation of communication with the process was done with the toolbox OLE Process Control (OPC) for Scilab. The OPCServer was the HS1 Power Tool from *HI Tecnologia* and the client was the OPC pack from Scilab. The solver used was LMITOOL which is a package that implements a user friendly interface to solve the optimization problem in the form of linear matrix inequalities. The Quanser Educate Innovate manual adopts a nominal value of 4.6 for the Km (Constant of Pump) parameter. It was verified that it altered with the time of the nominal value due to wear and tear of operation (wearing of pump). It was adopted in the experiments a variation of ± 20% of uncertainty in the constant of the pump for the experimental results. The results were based in the nonlinear models represented by equations (14) and (16), since the goal is to have the simulation as close as possible to the real plant.

IV. Experimental Results

Discretization was performed, with sampling time of 0.5 seconds, the models shown in (16) for a $Km = 3.68$ (-20%) and (17) with a $Km = 5.52$ (+20%).

$$\begin{bmatrix} L_1(k+1) \\ L_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.9677 & 0 \\ 0.03229 & 0.9677 \end{bmatrix} \begin{bmatrix} L_1(k) \\ L_2(k) \end{bmatrix} + \begin{bmatrix} 0.1166 \\ 0 \end{bmatrix} V_p(k) \quad (16)$$

$$\begin{bmatrix} L_1(k+1) \\ L_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.9677 & 0 \\ 0.03229 & 0.9677 \end{bmatrix} \begin{bmatrix} L_1(k) \\ L_2(k) \end{bmatrix} + \begin{bmatrix} 0.175 \\ 0 \end{bmatrix} V_p(k) \quad (17)$$

For the real pump in question it was estimated the value of $Km = 5.2$, in other words, it is located within the zone of uncertainty established.

A. RMPC without disturbances and without constraints

In this experiment the tank 1 behaved near the equilibrium point as shown in Fig (4) and Fig (5). The sensors generated some noise due to its natural wearing and the influence of external effects such as temperature.

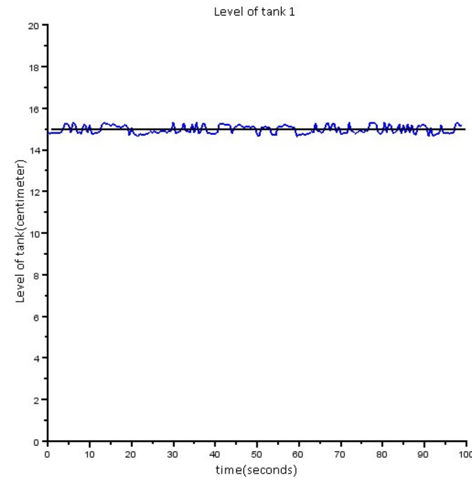

Fig. 4. Level in tank 1 without disturbance and without constraints.

It is observed that the initial level of tank 1 was near 15 and stayed so during the implementation of the process, as well as the level of tank 2.

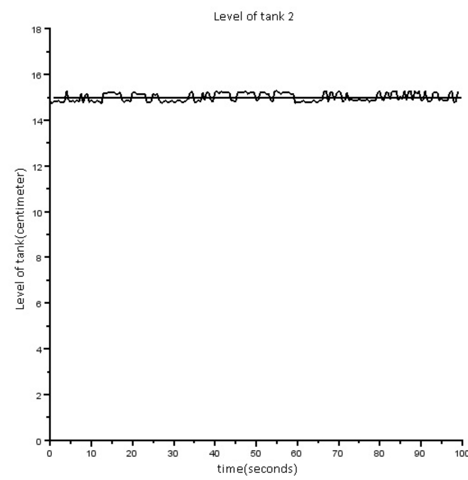

Fig. 5. Level in tank 2 without disturbance and without constraints.

The Figure 6 shows the control signal of RMPC.

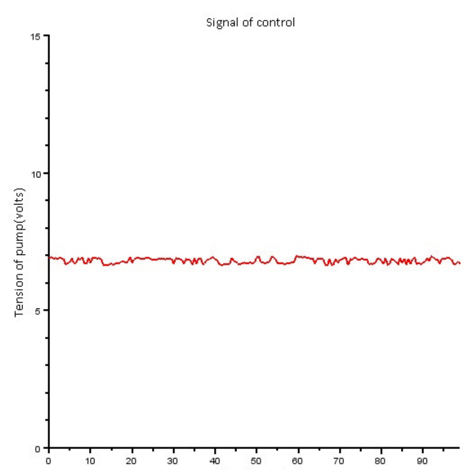

Fig. 6. Control signal of RMPC without disturbance and without constraints.

B. RMPC with disturbances and without constraints

In this experiment the level of tank 1 and tank 2 are shown in Fig (7) and Fig (8) and it shows the responses near the point of equilibrium. The disturbance in this experiment was performed in tank 1 and tank 2, both performed in different moments and due to the coupling (in closed loop) between tanks 1 and 2 an influence among disturbances occurred.

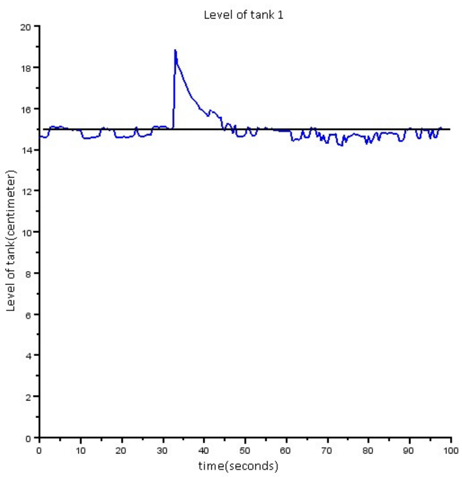

Fig. 7. Level in tank 1 with disturbance and without constraints.

The Figures 7 and 8 shows the behavior of the process with a disturbance like a limited step. This occurred in the time interval [40s to 50s]. The influence of the disturbance occurred in the interval [30s to 40s] in tank 1 can be seen in Fig 9 in the same interval.

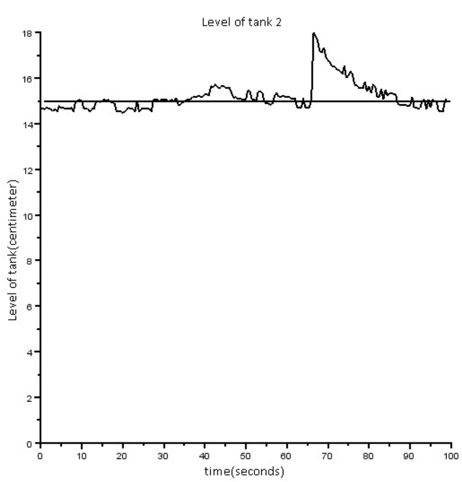

Fig. 8. Level in tank 2 without disturbance and without constraints.

The influence of the disturbance occurred in the interval [70s to 90s] in tank 2 is also observed in Fig. 7 and Fig. 8. The Figure 9 shows the control signal of RMPC.

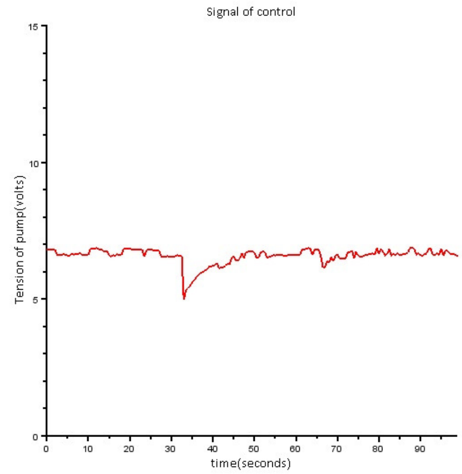

Fig. 9. Control signal of RMPC with disturbance and without constraints.

C. RMPC without disturbances and with constraints

For this experiment it was necessary to initiate the states with a value corresponding to $L_1(0)=-15cm$ and $L_2(0)=-15cm$ which means tanks 1 and 2 were empty so that the control signal exceeded the saturation limit. Figures 10 and 11 shows the behavior of tank 1 and tank 2.

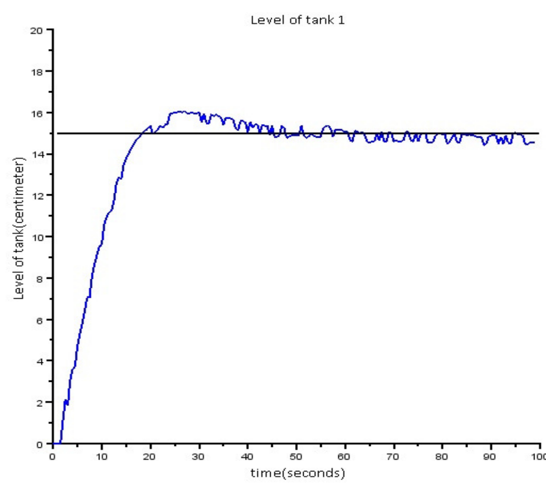

Fig. 10. Level in tank 1 without disturbance and with constraints.

It is observed that the response of the tank 1 showed an Overshoot, which is acceptable, since the scope of this work lies in the regulator problem.

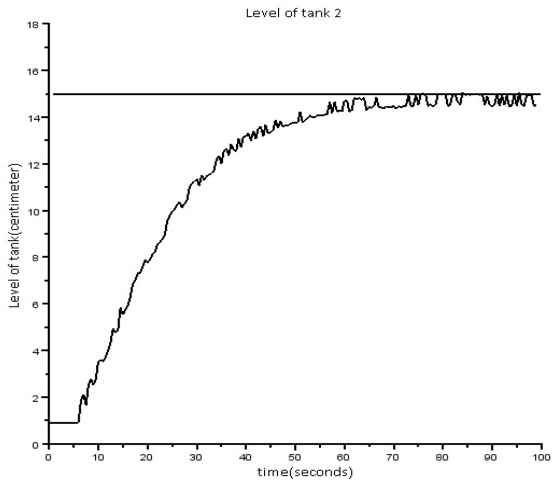

Fig. 11. Level in tank 2 without disturbance and with constraints.

In this experiment, the controller guarantees that the control signal remains into a operational safety range, for example.

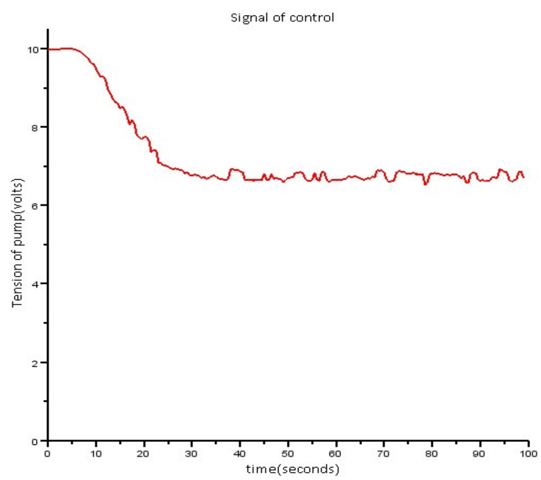

Fig. 12. Control signal of RMPC without disturbance and with constraints.

V. CONCLUSIONS

This paper studied and implemented the robust predictive controller. This control technique was applied in the system of tanks Quanser Consulting. The software Scilab 4.2 was used for the implementation of the algorithm RMPC and for solving optimization problems the package LMITool of that software was used. The experiments in the plant were carried out in various situations and with and without disturbance, no matter the transitional regime. The results of these experiments show that the controller performed well, especially in regard of the disturbance rejection. A deficiency of this formulation is its conservatism, since its formulation solves the optimization problem for the worst case. This problem makes its control law far from reaching the barriers of imposed constraints. Some insights may arise from this work, which we quote: the implementation of algorithms of semi-definite programming in C language and direct communication with the drivers from data Acquisition with the tank system, the experimental study of robust controllers associated with fault-tolerant control and the use of robust controllers developed in empirical models, for example, neural networks.

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