

Controller design using second order sliding mode algorithm with an application to a coupled-tank liquid-level system

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Abstract— A method for the design of second-order sliding mode controllers is developed for a coupled two tank system. The Standard Sliding Mode (SSM) controller produces oscillations in the closed-loop system. The newly invented Second Order Sliding Mode Controller (SOSMC) not only retains the robustness properties of the classical Sliding Mode Controller (SMC) but also eliminates the drawback of the SMC that is high frequency chattering due to high frequency switching. The high frequency switching can excite the unmodelled dynamics and makes the system unstable. The work presented in this paper is the super twisting controller design for a coupled two tank system. Robustness of the controllers is analyzed and dynamic bounds for the uncertainty are found for a class of this system. The application of this technique to the two-tank system to solve a robust tracking problem is presented.

I. INTRODUCTION

The control problem of dynamical systems in presence of bounded uncertainties is one of the most common problems to deal with when considering real plants. In fact, the mathematical models of real systems provide only approximate representations of the real phenomena.

The sliding mode control (SMC) approach seems to be an effective tool to manage such class of dynamic systems by virtue of its simplicity and robustness [3], [8], [16]. Furthermore, any non ideal behavior, leading to finite frequency switches of the control, implies that the system trajectories are confined within a boundary layer of the sliding manifold. Its size is related to the switching frequency, and any real sliding mode trajectory differs from the ideal one by small perturbing terms [16].

When considering the chattering phenomenon, that is, the highfrequency finite amplitude control signal generated by the sliding mode method, some authors (see, for instance, [9] and [18]) appear to relate this behavior to the discontinuity of the sign function on the sliding manifold. In other terms, they simply propose to replace this function with a smooth approximation in order to counteract the chattering effect at the price of a small deterioration in performances. However, the use of smoothing devices, which are characterized by a high gain to have a small approximation error, does not guarantee that oscillations will disappear. Indeed, the approximate sliding motion so originated is guaranteed

to lay in a small vicinity of the sliding manifold, but nothing can be told about the behavior inside this vicinity.

Another approach, which is effective in the presence of unmodeled dynamics, is based on the introduction of observers for the modeled part of the system with a sliding manifold defined in terms of the observer states [16]. The almost ideal high-frequency observer control signal is filtered by the high-gain fast dynamical part of the system so that a smooth control is actually applied [7], [6]. In [1], two different dynamic sliding mode control schemes are proposed.

Second order sliding mode control (2-SMC) is characterized by a discontinuous control acting directly on the second total derivative of the sliding variable, whose vanishing defines the sliding manifold. This property can be exploited in order to avoid chattering [15], [4], [10], [13] or to control systems with relative degree two between the sliding variable and the plant input [2], [10] or to implement robust differentiation devices [5], [14]. Khan et al. [11] applied a 2-sliding control algorithm to stabilise the dynamics of the sliding variable, this algorithm produces a dynamic control and does not require the derivative of the sliding variable, thus eliminating the requirement to design observers or peak detectors.

In this paper, the simplified robust algorithm known "super twisting" have been implemented for stabilizing the liquid level of the two tank system, the advantage of this algorithm is not only for chattering attenuation but also for the robust control of uncertain system with relative degree one and higher. Also the advantage of the latter is that it does not need any knowledge of the derivative of the sliding variable s , therefore, it does not need to implement a robust differentiator.

This paper is organized in the following way. In section 2, the model of two tank system is presented. The application of the high order controller scheme to the model of the two tank is presented in section 3. Experimental results are finally provided in Section 4.

II. COUPLED-TANKS SYSTEM

A. Process description

The three tank system consists of three cylindrical tanks with identical section A supplied with distilled water, which

are serially interconnected by two cylindrical pipes of identical sections S_n . The pipes of communication between the tanks T_1 and T_2 are equipped with manually adjustable; the flow rates of the connection pipes can be controlled using ball valves a_{z1} and a_{z2} . The plant has one outlet pipe located at the bottom of tank T_3 . There are three other pipes installed at the bottom of each Tank, they are provided with a direct connection (outflow rate) to the reservoir with ball valves b_{z1} , b_{z2} and b_{z3} , respectively, it can only be manipulated manually. The pumps 1 and 2 are supplied by water from a reservoir below the three tank with flow rates $q_1(t)$ and $q_2(t)$, respectively. The necessary level measurements $h_1(t)$, $h_2(t)$ and $h_3(t)$ are carried out by the piezo-resistive differential pressure sensors.

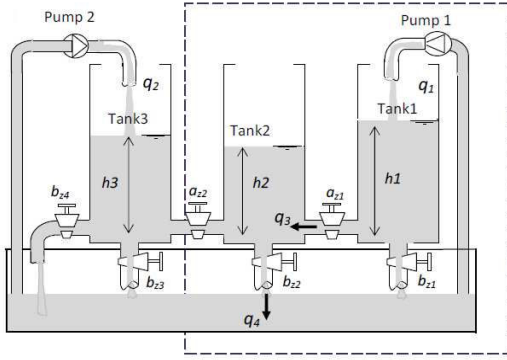


Fig. 1. Three tank system

B. Dynamic model for the two tanks system

By using the above physical principal, which is based on the conservation of mass law, the rate of the level of fluid in each tank is related directly to the difference between inflow and outflow rates and inversely to tank cross section area.

Then the two-tank system can be modeled by the following two differential equations

$$\begin{aligned}\frac{dh_1}{dt} &= \frac{1}{A}(q_1 - q_3) \\ \frac{dh_2}{dt} &= \frac{1}{A}(q_3 - q_4)\end{aligned}\quad (1)$$

The exit of the tanks is defined by:

$$q_3 = a_{z1} S_n \sqrt{2g|h_1 - h_2|} \quad \text{for} \quad h_1 > h_2 \quad (2)$$

$$q_4 = b_{z2} S_L \sqrt{2gh_2} \quad \text{for} \quad h_2 > 0 \quad (3)$$

where A , q_1 , q_3 , q_4 and g are the cross-sectional area, inflow rate, outflow rate and gravitational constant respectively. S_n denotes the cross section areas of the connecting pipes between the tanks, S_L means the maximal cross section area of the leakage, a_{z1} is the flow coefficient of the interconnecting pipe, b_{z2} is the leak flow coefficient, h_1 is the liquid level of tank 1 and h_2 is the liquid level of tank 2.

Valves a_{z2} and b_{z1} were fully closed during the experiments, valve a_{z1} was fully opened and valve b_{z2} was

partially opened. The valve positions did not change during the experiments. The controlled signal (y) was the height of the liquid level in the middle tank ($y = h_2$). This level was controlled by the control voltage of the pump $P_1(u)$. The system can be considered as a single input single output system (SISO) where the input is inflow q_1 and output is liquid level h_2 . Then the two-tank system can be modeled by the following two differential equations:

$$\begin{aligned}\frac{dh_1}{dt} &= -C_1 \text{sign}(h_1 - h_2) \sqrt{|h_1 - h_2|} + \frac{q_1}{A} \\ \frac{dh_2}{dt} &= C_1 \text{sign}(h_1 - h_2) \sqrt{|h_1 - h_2|} - B_2 \sqrt{h_2}\end{aligned}\quad (4)$$

where the parameters are defined by

$$C_1 = \frac{1}{A} a_{z1} S_n \sqrt{2g} \quad (5)$$

$$B_2 = \frac{1}{A} b_{z2} S_L \sqrt{2g} \quad (6)$$

C. System constraints

For the coupled tanks apparatus, the fluid flow, q_1 , into tank 1, cannot be negative because the pump can only pump water into the tank. Therefore, the constraint on the inflow rate is given by

$$q_1 \geq 0 \quad (7)$$

At equilibrium, for constant water level set point, the derivatives must be zero, i.e.,

$$\dot{h}_1 = \dot{h}_2 = 0 \quad (8)$$

Therefore, using Eq.(4) in the steady state, the following algebraic relationship holds

$$-C_1 \text{sign}(h_1 - h_2) \sqrt{|h_1 - h_2|} + \frac{q_1}{A} = 0 \quad (9)$$

$$C_1 \text{sign}(h_1 - h_2) \sqrt{|h_1 - h_2|} - B_2 \sqrt{h_2} = 0$$

where q_1 is the equilibrium inflow rate given by

$$q_1 = AC_1 \text{sign}(h_1 - h_2) \sqrt{|h_1 - h_2|} \quad (10)$$

From Eq. (10), and to satisfy the constraint in Eq. (7) on the input flow rate, we should have $C_1 \text{sign}(h_1 - h_2) \geq 0$, which implies

$$h_1 \geq h_2$$

$u = q_1$, $h_1 = x_1$ and $h_2 = x_2$, in which case the dynamics can be rewritten

$$\begin{aligned}\dot{x}_1 &= -C_1 \sqrt{x_1 - x_2} + \frac{u}{A} \\ \dot{x}_2 &= C_1 \sqrt{x_1 - x_2} - B_2 \sqrt{x_2} + w(t) \\ y(t) &= x_2\end{aligned}\quad (11)$$

where $w(t)$ is the external disturbance. The objective of the control scheme is to regulate the output $y(t) = x_2(t)$ to a desired value x_{2d} .

III. SECOND ORDER SLIDING MODE CONTROL

Sliding mode control laws for nonlinear systems have been widely studied since they were introduced in [16]. The objective of this method is, by means of a discontinuous control, to constrain the system to evolve and stay, after a finite time, on a sliding manifold where the resulting behavior has some prescribed dynamics. Sliding mode control exhibits relative simplicity of design and some robustness properties with respect to matching perturbations.

Emel'yanov et al [12] generalized the basic sliding mode idea to higher order sliding modes (HOSM). They are characterized by a discontinuous control acting on the higher time derivatives of the sliding constraint (instead of the first time derivative in first order sliding mode (FOSM)). For example, the case of second order sliding modes corresponds to the control acting on the second derivative of the sliding variable, namely \ddot{s} and the sliding set is defined as $s = \dot{s} = 0$.

Higher order sliding mode control has the advantage, when compared to FOSM control, that it removes chattering effects, providing a smooth or at least piecewise smooth control, and provides better performance with respect to switching delays in the control implementation.

A. Second order sliding mode

Consider an uncertain SISO nonlinear system which is affine in the control u :

$$\begin{aligned}\dot{x} &= f(x, t) + b(x, t)u \\ s &= s(x, t)\end{aligned}\quad (12)$$

with $x \in \mathcal{X} \subset \mathbb{R}^n$ the state variable and $u \in \mathcal{U} \subset \mathbb{R}$ the input, such that $\mathcal{X} = \{x \in \mathbb{R}^n \mid |x_i| \leq x_{iMAX}, 1 \leq i \leq n\}$ and $\mathcal{U} = \{u \in \mathbb{R} \mid |u| \leq u_{MAX}\}$. $s(x, t)$ is the output function, called *sliding variable*. f , b and s are smooth uncertain functions. The objective is to enforce, possibly in a finite time, the zeroing of the measurable sliding (or constraint) variable $s = s(x, t)$. By differentiating twice s , under the assumption that system (12) has relative degree versus s equal to 2, it leads to the following relationship

$$\ddot{s}(t) = \varphi(x, t) + \gamma(x, t)u(t) \quad (13)$$

The dynamics in Eq. (13) are assumed to satisfy the following bounding conditions

$$0 < K_m \leq \gamma(x, t) \leq K_M \quad |\varphi(x, t)| < C_0 \quad (14)$$

where K_m , K_M and C_0 are some positive constants. Essentially this is a requirement that the uncertainty levels in the process are bounded and that some worse case bounds on the uncertainty can be assumed.

Let us set $y_1(t) = s(x, t)$, it has been shown that, under sensible conditions, apart from a possible initialization phase, the second order sliding mode problem is equivalent to the finite time stabilization problem for the following uncertain second order system [10]

$$\begin{cases} \dot{y}_1 = y_2(t) \\ \dot{y}_2 = \varphi(x, t) + \gamma(x, t)v(t) \end{cases} \quad (15)$$

If system (12) has relative degree $r = 2$ with respect to $y_1 = s$, then $v = u$, while, if $r = 1$, $v = \dot{u}$.

Note that (14) is formulated in inputoutput terms. These conditions are satisfied at least locally for any smooth system (1) having a well-defined relative degree at a given point with $s = \dot{s} = \dots = s^{r-1} = 0$.

Then, it is then possible to generate different kinds of algorithms (ideal twisting, sampled twisting, super twisting, sub-optimal...) such that the system evolve featuring a second order sliding mode, after a finite time, i.e. the trajectories lie in the second order sliding set defined by:

$$S^2 = \{x \in \mathbb{R}^n \mid s = \dot{s} = 0\} \quad (16)$$

The second order sliding mode (SOSM) approach [10] solves the stabilization problem for (15) by requiring the knowledge of y_1 and just the sign of y_2 . Some algorithms, which propose a solution to the above control problem, have been presented in the literature [14], [2], [3], [10]. In the present paper the focus is on the so-called super twisting second order sliding mode algorithm. The "Super Twisting" algorithm has the advantage of not require any knowledge of the derivative of the sliding variable \dot{s} . The class of algorithm can be defined by the following control law

$$u(t) = u_1(t) + u_2(t) \quad (17)$$

$$\dot{u}_1 = \begin{cases} -u & \text{if } |u| > 1 \\ -W \text{sign}(s) & \text{if } |u| \leq 1 \end{cases} \quad (18)$$

$$u_2 = \begin{cases} -\lambda_1 |s|^\rho \text{sign}(s) & \text{if } |s| > s_0 \\ -\lambda_1 |s|^\rho \text{sign}(s) & \text{if } |s| \leq s_0 \end{cases} \quad (19)$$

This algorithm defines the control law, as a combination of two terms. The first is defined in terms of a discontinuous time derivative while the second is a continuous function of the sliding variable. The corresponding sufficient conditions for the finite time convergence to the sliding manifold are [14].

$$W > \frac{C_0}{K_m}, \quad \lambda_1^2 \geq \frac{4C_0}{K_m^2} \frac{K_M(W + C_0)}{K_m(W - C_0)}, \quad 0 < \rho \leq 0.5 \quad (20)$$

where W , ρ and λ_1 are variable controller parameters, C_0 is positive norm bound on the smooth uncertain function, s_0 is the boundary layer thickness, the choice of $\rho = 0.5$ assures that sliding order 2 is achieved [14].

B. Sliding variable and control design

Let x_{2d} be the desired liquid level for the x_2 state variable.

Let us define the sliding constraint s as:

$$s = \dot{x}_2 + \lambda_{st}\tilde{x}_2 \quad (21)$$

where λ_{st} is a positive parameter such that $P(z) = \dot{z} + \lambda_{st}z$ is Hurwitz polynomial and $\tilde{x}_2 = x_2 - x_{2d}$.

The sliding surface variable, s ; in Eq. (21) has relative degree one with respect to the control, v . The task is to generate a second order sliding mode on the second order sliding manifold given by the equalities: $s = \dot{s} = 0$. The first time derivatives of s can be written as

$$\begin{aligned}\dot{s} &= (\ddot{x}_2 - \ddot{x}_{2d}) + \lambda_{st}(\dot{x}_2 - \dot{x}_{2d}) \\ &= a_{st} + b_{st}u\end{aligned}\quad (22)$$

where

$$\begin{aligned}a_{st} &= -C_1^2 + \frac{B_3^2}{2} + \frac{C_1 B_3}{2} \frac{2x_2 - x_1}{\sqrt{x_2(x_1 - x_2)}} + \dot{w} \\ &+ \lambda_{st}(C_1\sqrt{x_1 - x_2} - B_3\sqrt{x_2} - \dot{x}_{2d}) - \ddot{x}_{2d} + \lambda_{st}w \\ &= \hat{a}_{st} + \tilde{a}_{st}\end{aligned}\quad (23)$$

$$b_{st} = \frac{C_1}{2A} \frac{u}{\sqrt{x_1 - x_2}} = \hat{b}_{st} + \tilde{b}_{st}\quad (24)$$

\hat{a}_{st} and \hat{b}_{st} are the known nominal expressions whereas the expressions \tilde{a}_{st} and \tilde{b}_{st} contain all the uncertainties due to parameters variations. External disturbance is supposed to be bounded as well as its first time derivatives.

$$\begin{aligned}\hat{a}_{st} &= -\hat{C}_1^2 + \frac{\hat{B}_3^2}{2} + \frac{\hat{C}_1 \hat{B}_3}{2} \frac{2x_2 - x_1}{\sqrt{x_2(x_1 - x_2)}} \\ &+ \lambda_{st}(\hat{C}_1\sqrt{x_1 - x_2} - \hat{B}_3\sqrt{x_2} - \dot{x}_{2d}) - \ddot{x}_{2d}\end{aligned}\quad (25)$$

$$\begin{aligned}\tilde{a}_{st} &= -\tilde{C}_1^2 - 2\tilde{C}_1\hat{C}_1 + \frac{\tilde{B}_3^2}{2} + \tilde{B}_3\hat{B}_3 + \lambda_{st}\tilde{C}_1\sqrt{x_1 - x_2} \\ &+ \frac{\tilde{B}_3\hat{C}_1 + \tilde{C}_1\hat{B}_3 + \hat{B}_3\tilde{C}_1}{2} \frac{2x_2 - x_1}{\sqrt{x_2(x_1 - x_2)}} + \dot{w} \\ &- \lambda_{st}\tilde{B}_3\sqrt{x_2} + \lambda_{st}w\end{aligned}\quad (26)$$

$$\hat{b}_{st} = \frac{\hat{C}_1}{2A} \frac{1}{\sqrt{x_1 - x_2}}\quad (27)$$

$$\tilde{b}_{st} = \frac{\tilde{C}_1}{2A} \frac{1}{\sqrt{x_1 - x_2}}\quad (28)$$

Remark 1: Note from (25)-(27) that there is a control singularity $x_1(t) = x_2(t)$. Furthermore, if $x_1(t) = 0$ no control will be applied to the system, which eventually causes $x_1(t) - x_2(t) = 0$ thus creating a singularity.

Assume that the reference level x_{2d} and their first and second time derivatives are bounded and that the function $\tilde{a}_{st}(\cdot)$ is bounded such that

$$0 < |\dot{\tilde{a}}_{st}(\cdot)| \leq C'_0\quad (29)$$

where C'_0 is positive constant.

We assume that x_1 and x_2 are bounded, and $|\tilde{a}_{st}| < |a|$, $|\tilde{b}_{st}| < |b|$.

Let us apply the following control laws, called the simplified super twisting algorithm:

$$\dot{u} = v = v_1 + \dot{v}_2\quad (30)$$

Then

$$u = v_2 + \int v_1\quad (31)$$

with

$$v_1 = -W \text{sign}(s)\quad (32)$$

$$v_2 = -\kappa |s|^\rho \text{sign}(s)\quad (33)$$

where W , κ and ρ are positive constants that satisfy the following conditions:

$$0 < C'_0 < W\quad (34)$$

$$2^{\frac{1}{\rho}-2}(W + C'_0) < \rho\kappa^{\frac{1}{\rho}}\quad (35)$$

$$0 < \rho < 0.5\quad (36)$$

IV. EXPERIMENTAL RESULTS

The application used to illustrate the above design techniques and demonstrate the performances of the system, is a three tank system which is in our laboratory (Fig. 2). the SOSM control proposed is applied to a physical laboratory plant consisting of the two-tank system, the objective is to control the liquid level of tank two by introducing a leakage (external disturbance) in the outflow pipe of tank 2.

The experimental schemes have been done under Matlab/Simulink, using Real-Time Interface, and run on the DS1102 DSPACE system, which is equipped by a power PC processor. The control algorithm is implemented on DSP (TMS 320C31).

The bidirectional information flux between the physical



Fig. 2. real system

part and the computer is supported by a data acquisition interface. Control Desk is used to visualize the functional parameters of the simulated three-tank-system and to acquire data in real time. The numerical values of the parameters for the three tank, liquid level system are provided in TABLE 1.

The second order sliding mode control strategy is applied to a coupled-tank liquid-level as shown in Fig.2. The control law is obtained from equation (30), where $u_{max} =$

TABLE I
NUMERICAL VALUES FOR PHYSICAL PARAMETERS OF THE TWO TANK
SYSTEM

Symbol	Value	Meaning
A	$0.0154m^2$	tank section of valve
S_n	$5.10^{-5}m^2$	cross section
a_z	$0 < a_z \leq 1.7$	flow coefficient of the interconnecting pipe
b_z	$0 < b_z \leq 1.2$	leak flow coefficient
g	$9.81ms^{-2}$	gravity constant
h_{max}	$0.6m$	maximum water level in each tank
Q_{imax}	$1.17.10^{-4}m^3/s$	maximum inflow through pump $i(i = 1, 2)$

$1.17.10^{-4}m^3/s$, $A = 0.0154m^2$, $S_n = S_L = 5.10^{-5}m^2$ and $g = 9.81m/s^2$.

The objective consists in minimizing the liquid level tracking error in presence of model uncertainties and leakage in tank 2.

The parameters for the second order sliding mode controller (30) are $W = 3.5.10^{-4}$, $\lambda_{st} = 30$, $\rho = 0.5$, $\kappa = 0.005$.

Figure 3 displays the tracking liquid level without external disturbances (the valve a_{z2} is closed). It can be seen that SOSM controller can track the liquid level very well.

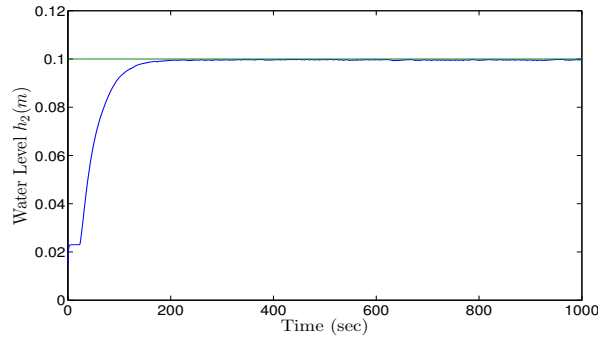


Fig. 3. Implementation results for stabilisation of water level in tank 2 using SOSMC

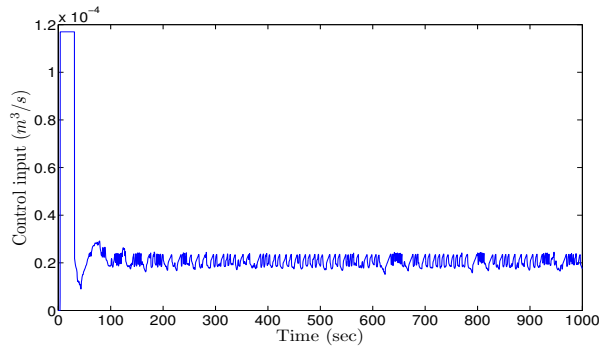


Fig. 4. The control input

Figure 4 displays the control input which is not affected

by the chattering effect. From these experiment results, good tracking responses are obtained for the level owing to the robust control characteristics of the controller. These two curves are obtained without a leakage in thanks. Fig.5

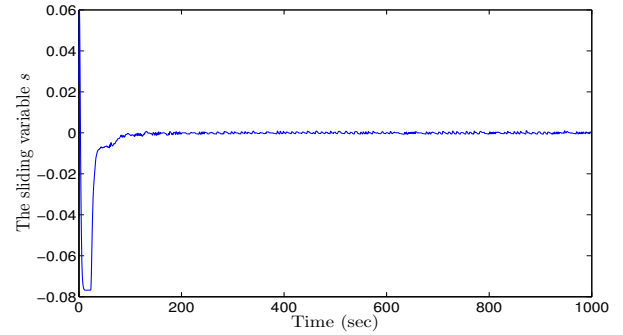


Fig. 5. The sliding variable

portrays the sliding surface variable when the super twisting SOSMC is applied.

The Phase trajectory is shown in Fig.6. The trajectories are characterized by twisting around the origin on the phase portrait of sliding variable.

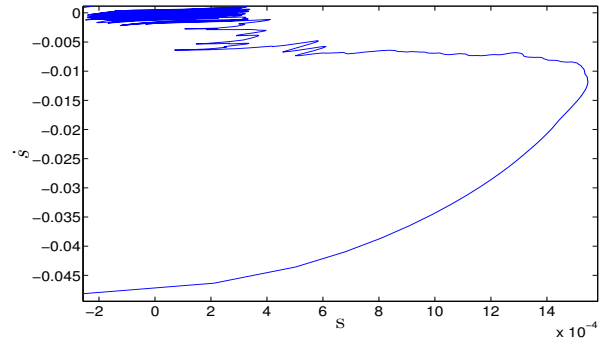


Fig. 6. The tracking error

To show the robustness of the controlled system to disturbance acting on the system, the output pipe with admittance coefficient equal to a_{z2} is opened. In this case, the proposed

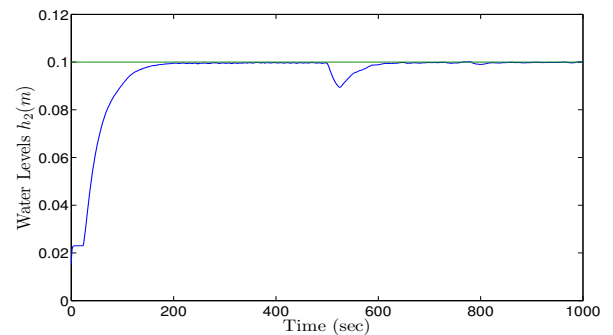


Fig. 7. Stabilisation of level h_2 with leak in Tank 2

second order sliding mode controller performs very well and there is no discernable drop in the water level due to the leak, which was introduced at around 500sec on the time scale.

The experimental results are shown in Fig. 7. It can be seen

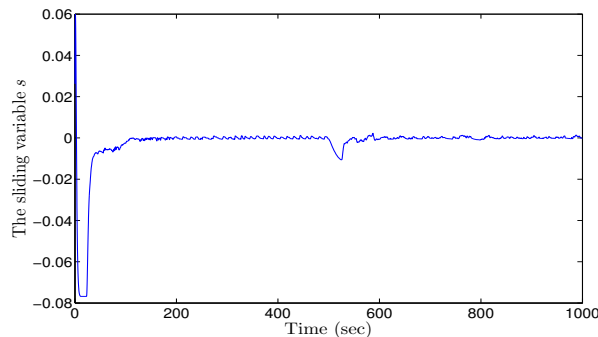


Fig. 8. The sliding variable

from these responses that the output converges to its desired value. The figure 9 shows how the control signals adjusted themselves so that the output converges to its desired value. Therefore, it can be concluded that the proposed control schemes are robust to disturbances acting on the system. The presented results are obtained without changing the control gains value.

The performances of the controlled system are studied under variations in system parameters and in the presence of external disturbance. The robust control characteristics of this controller versus the external disturbance can be observed in Figure 9.

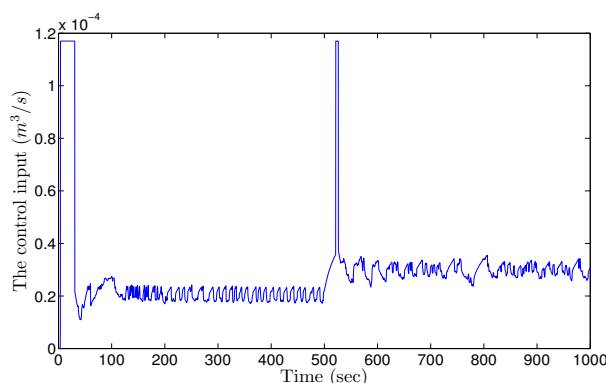


Fig. 9. The control input

From these experimental results, we can conclude that the robust second order sliding mode control strategy, which only uses measured outputs and requires minimal tuning of a small number of parameters, has the ability to provide robust performance across a wide range of system operation.

V. CONCLUSION

In this paper, a 2-sliding super-twisting liquid level control scheme for a class of nonlinear systems subjected

to parameters uncertainties and external disturbance has been developed. The proposed control scheme is robust against parameter and leakage variations. The advantage of such control strategy is to reduce chattering phenomena along with inflow rate control via sliding mode. Stability and performance criterions are proved. The experimental results show that the proposed second order sliding mode control strategy is preserved even if parameters variations and leakage disturbance occur.

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