Recent Advances in Relay Feedback Methods – A Survey

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Abstract

Relay feedback has played a role in adaptive control for a long time. In recent years, they have found a new lease of life in the automatic tuning of PID contiollers and in the initialization of other sophisticated adaptive controllers. These developments have spurred research into extensions of the method to advanced controllers and intelligent systems. This paper surveys the recent results and developments in the field.

1. Introduction

Relay feedback is a classical configuration with many applications. In the fifties, relays were mainly used as amplifiers but such applications are obsolete now, owing to the development of electronic technology. A useful discussion on such relay systems can be found in [1],[2]. In the sixties, relay feedback was applied to adaptive control. A prominent example is the self-oscillating adaptive controller developed by Minneapolis Honeywell which uses relay feedback to attain an amplitude margin of $A_m = 2$. This system which is described in [3] was tested extensively for flight control systems and it is used in several missiles.

Lately, there has been a renewed interest in this subject after Astrom and co-workers have successfully applied the relay feedback technique to the automatic tuning of PID controllers in process control [4]-[6]. The auto-tuning technique has since found its way as a feature in many industrial control products [7]. There are several reasons for the success of the relay feedback method. First, it facilitates single-button tuning since the arrangement automatically extracts the process frequency response at an important frequency and this information is sufficient to tune PID controllers. Secondly, relay feedback is carried out under tight closed-loop control so that with an appropriate choice of the relay parameters, the process can be kept close to the set-point. This keeps the process in the linear region where the frequency response is of interest which is precisely why the method works well on highly nonlinear processes. Thirdly, unlike traditional methods in process identification, the relay feedback technique eliminates the need for a careful choice of the sampling period which has proven to be a very difficult task indeed. This has made it very suitable as an initialization module for more sophisticated adaptive controllers [8].

Following the success for PID auto-tuners and coupled with the control performance standards getting increasingly stringent, research effort to extend the relay feedback technique to diverse applications have been rampant in recent years. Various enhanced auto-tuning techniques for the PID controller were developed [9]-[12] to yield better performance. Hagglund and Tengvall [13] extended the PID auto-tuning procedure to unsymmetrical processes with two different operating modes. Applications have not been restricted to PID controllers only. In [14], the relay auto-tuning method is extended to general digital controllers and in [15], it is used in the design of phase-lead and phase-lag filters for general frequency response compensation. Relay feedback methods have been used for process identification too. In [16],[17], interesting approaches to obtain low-order transfer function models using relay feedback are described. In [18]-[22], auto-tuners for advanced controllers like the Smith-predictor controller and a Finite Spectrum Assignment controller are developed. These controllers are useful for process with complex dynamics, *e.g.* long deadtimes, oscillatory and unstable dynamics. Relay feedback methods have being proposed and incorporated in knowledge-based and intelligent systems as integrated initialization and tuning modules [23]-[29]. There are also more innovative applications of the method to determine stability margins [26],[27] and uncertainty bounds [30]. Lately, there have been attempts to extend the tuning method to PI controllers for multivariables systems [31]-[33] and cascade loops [34]. The paper will take stock of these developments and other related results which have made them possible.

2. The relay feedback method for process identification

Consider the basic relay feedback arrangement as shown in Figure 1. The *critical point*, *i.e* the frequency response of the process at the phase lag of $-\pi$ can be determined automatically from the experiment. By making the observation that the describing function of a relay is the negative real axis, it follows that the system oscillates with a frequency that is close to the ultimate frequency ω_u [6]. The process ultimate gain k_u is approximately given by Astrom

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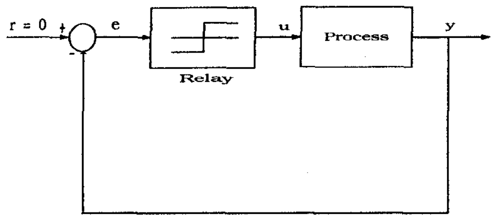

Figure 1: The relay feedback system.

and Hagglund [6] as

$$k_u = \frac{4u_m}{\pi y_m},$$

where u_m and y_m are the amplitudes of relay and process output, respectively.

It may be advantageous to use a relay with hysteresis so that the resultant system is less sensitive to measurement noise. In this case, the oscillation corresponds to the point where the negative inverse describing function of the relay crosses the Nyquist curve of the process. With hysteresis, there is an additional parameter which can be set based on a determination of the measurement noise level. In the presence of a constant load disturbance, a dc bias compensation can be introduced into the relay to prevent an assymetrical oscillation [35].

Existence and stability of limit cycles

There are fundamental problems related to the existence and stability of limit cycles resulting from relay feedback and these problems have to be adequately addressed for effective applications of the relay feedback method. For simple cases, a good analysis of limit cycles and their stability can be found in [36]. Conditions for boundedness of oscillation in relay feedback systems are given in [37] for more general cases, including those containing an integrator. It has also been observed that linear systems with relay feedback may exhibit very complicated behaviour. In [24], it is shown that a second order oscillatory system with time delay can have different stable limit cycles depending on the initial conditions. Cook [38] has found chaotic behaviour for an unstable second order linear system. Another example of chaotic behaviour is given in [39]. However, for systems with non-oscillatory poles which are typical of many industrial processes, unique stable limit cycles are often found [6].

Applications to automatic tuning and initialization of adaptive control

The critical point identified from a relay feedback experiment is sufficient to tune PID controllers, the bread and butter of control engineering. In fact, the idea has already been successfully applied in several industrial products [7]. There are classical tuning rules available, *e.g.* the Ziegler-Nichols rules [40] and a refined version [41]. In [5], tuning rules are developed so that the critical point is moved to a desired position in the complex plane so that the closed-loop satisfy some gain and phase margins considerations. In [11], a PID control system in closed-loop is auto-tuned without having to remove the existing controller from the loop. In [13], the dynamics of a second-order asymmetrical process are estimated from the asymmetrical limit cycle obtained from relay feedback, and two PID controller are tuned based on the dynamics estimation to control the nonlinear process.

Apart from PID controllers, Astrom [42] has extended the relay feedback method to automatic tuning of more general digital controllers and has successfully applied them to HVAC plants (heating, ventilation and air-conditioning). He extracted a sampled data model from the limit cycle oscillation and applied a pole-placement design method to the model to achieve some desired closed-loop properties. Yang *et. al.* [15] has applied the method to design phase-lead and phase-lag filters to satisfy desired frequency response specifications. The method has also being applied to auto-tune a Finite Spectrum Assignment controller for unstable processes [21], [22]. For such processes, the step tuning tests are clearly not applicable and relay feedback experiments which provide an on-off control are more useful. The effectiveness and simplicity of the method has also resulted in the method being applied as an initialization module in many knowledge-based adaptive and intelligent systems [23]-[29]. Lately, there have even been applications beyond auto-tuning single-loop controllers. In [34], the relay method is used to auto-tune a PI controller for a cascade loop and in [31]-[33], it is extended to cover multivariable systems. More on the applications of relat feedback auto-tuning in multi-loop systems will be described in Section 4.

3. Extensions of the relay feedback method

While the auto-tuned PID controllers are sufficient in many cases of processes with benign dynamics, there are more complicated cases (*e.g.* processes with long deadtime or oscillatory and unstable dynamics) where such controllers are ineffective especially when the performance requirements are becoming rather strict with industry standards like the ISO 9000. With the basic relay feedback method, only one crude critical point estimation of the process is obtained which is clearly insufficient to implement advanced control algorithms. Recently, much effort has been geared towards the extension of the basic technique to extract more information from the relay feedback experiment. The following subsections will summarize some of the results obtained.

Improvement on the identification accuracy

While the relay feedback technique will yield sufficiently accurate results for many of the processes encountered in the process industry, there are potential problems associated with such techniques. These arise as a result of the approximations used in the development of the procedures for estimation. In particular, the basis of most existing

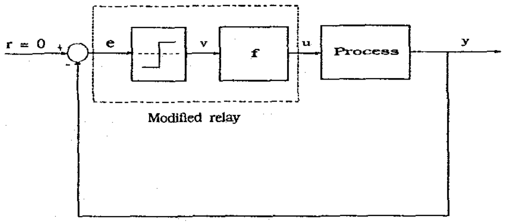

Figure 2: The modified relay feedback system.

relay-based procedures of critical point estimation is the describing function (DF) method [43],[44]. This method is approximate in nature, and under certain circumstances, the existing relay-based procedures could result in estimates of the critical point that are significantly different from their real values. Such problematic circumstances arise particularly in processes with significant deadtime and those with underdamped dynamics. Some theoretical works [45], [46] have investigated the validity and accuracy of the DF method for limit cycle determination but unfortunately, they have not yielded any results of true practical significance. A more acceptable egineering approach is to formulate a distortion criteria which would check the accuracy of earlier assumptions to validate the information obtained [43]. However, such an approach will by no means improve the accuracy of the critical point.

A simple variation of the relay feedback experiment is developed in [47] to achieve improved accuracy. It consists of a relay and a static mapping function f in cascade as shown in Figure 2. The static mapping function f is defined such that

$$u(t) = f(v(t)) = v_1(t) = \frac{4u_m}{\pi}\sin\omega t,$$

where $v_1(t)$ is the fundamental harmonic of $v(t)$ with frequency ω.

With this modified relay feedback arrangement, it is shown in [47] that it is possible to obtain an exact estimate of the process critical point. For a simple implementation, one can obtain a rough estimate of the ultimate frequency from 2 or 3 switches of the relay under normal relay feedback, and then turn on the static function f using the rough estimate. The function f is then updated iteratively using the observed signals. Upon convergence, the process critical point can be obtained directly from the oscillation amplitude and frequency. This technique has been applied to yield tighter control performance [47] especially for processes which yield poor estimates of the critical point.

Identification of points other than the critical point

With the simple relay feedback approach, only one point on the process Nyquist curve is determined. It is possible, for example, to cascade a known linear dynamical system to the system in Figure 1 to obtain a point other than the critical point. For example, an integrator can be cascaded to the system of Figure 1 to obtain the point where the process Nyquist curve crosses the negative imaginary axis. In [20], a first-order lag instead is designed and cascaded to obtain a point with a frequency below the ultimate frequency. However, with these modifications, we cannot specify the frequency of interest; it is fixed by the choice of the linear element cascaded. Besides, the introduction of the linear system affects the amplitude response of the original process and in the case of a gain reduction, we have a smaller signal-to-noise ratio (SNR) which could affect the estimation accuracy adversely.

Holmberg [39] suggested a relay with adaptive hysteresis so that its negative inverse describing function would be a ray through the origin in the third quadrant of the complex plane. With this particular nonlinearity, it is possible to obtain a point on the process frequency response at an arbitrarily specified phase lag in the quadrant. This is an interesting feature especially in auto-tuning applications where it is of interest to find out the system Nyquist curve in the neighbourhood of -1. The nonlinearity can also be used as an indicator of nonlinearities [39]. However, with this method, the relay amplitude may have to be continuously adjusted to sustain the limit cycle oscillation. A variation of the relay feedback system is presented in [12] which extends Holmberg's invention to the third and fourth quadrants and at the same time, removing the need to vary the relay amplitude to sustain the limit cycle oscillation. The arrangement is given in Figure 2 but f is now an adaptive delay element given by

$$u(t) = f(v(t)) = v(t - \frac{\phi_m}{\omega}),$$

where ϕ_m is the specified angle of the ray to the negative real axis and ω is the oscillating frequency of $v(t)$. With additional points on the frequency response, there are simple algorithms to fit low-order transfer function models to the process [29]. This facilitates auto-tuning of model-based controllers like the Smith-predictor controller [18]-[20] and the Finite Spectrum Assignment controller [21],[22].

Identification of multiple points

While the identification technique described in [12] can give a general frequency of interest, tuning time is increased proportionally when more frequency estimations are required especially if high accuracy is desirable. This is particularly true in the case of a process with a long deadtime where tuning time is considerably long. A further extension of the technique is described in [19] which allows multiple frequency identifications in one single relay experiment. The arrangement and implementation is similar to Figure 2 but with the mapping function f defined such that

$$u(t) = f(v(t)) = \sum_{k=1}^{k_m} a_k \sin k\omega t\ , \ \ k \in Z^+$$

where $v_1 = a_1 \sin\omega t$ is the fundamental frequency of the input v, $a_k = \frac{4u_m}{k\pi}$ and $k_m\omega$ is the upper bound of the frequencies injected. For good estimation accuracy from this procedure, the identification procedure can be restricted to two or three frequencies *i.e.* $k_m = 2, 3$. The points can be obtained simultaneously from the output and input of the process via a Fourier series expansion as follow :

$$g_p(jk\omega_u) = \frac{\int_{-\frac{t_u}{2}}^{\frac{t_u}{2}} y(t)e^{-jk\omega_u t}dt}{\int_{-\frac{t_u}{2}}^{\frac{t_u}{2}} u(t)e^{-jk\omega_u t}dt}, \quad k = 1 \cdots k_m,$$

where $t_u = \frac{2\pi}{\omega_u}$ and $g_p(s)$ represents the frequency response of the process. The technique has been applied to the auto-tuning of the Smith-predictor controller using a second-order process model [19].

Identification of the process deadtime

A gross estimation of the process deadtime can be obtained from the detection of "crucial" points on the limit cycle oscillation [48] as

$$L \approx t_s - t_r,$$

where t_s is the time corresponding to the occurrence of the "crucial" point and t_r corresponds to the preceding relay switch. For first-order systems, the "crucial" point corresponds to the oscillation extremas and the estimation is exact. Such deadtime estimates are useful in intelligent systems as they can be used to characterize process dynamics and as a selection guide for control strategies [29].

Identification of the process order

One method of process order identification using relay feedback is proposed by Lundh [49]. He noted that the maximal slope of the frequency response magnitude is a measure of the process complexity and uses the slope in the vicinity of the ultimate frequency to estimate the order of the process. In his method, FFTs are performed on the input and output oscillations of the process, from which the amplitudes of the first and third harmonics of the frequency spectrum are used to compute the amplitude gains at these two frequencies. From these frequencies, the slope of the magnitude response at the geometrical mean of the harmonics can be calculated and served as a measure of the process order.

A simpler method is proposed in [12] which does not require the use of FFTs. In this case, a process model of the following form is assumed,

$$g_p(s) = \frac{K}{(Ts+1)^n}e^{-sL}.$$

Two points of the process frequency response are acquired and a cost function is formulated based on the above process model structure to obtain the optimal process order estimate.

Identification of stability margins

Relay feedback can be applied to closed-loop systems to determine the amplitude margin on-line [26], [27]. Using a describing function argument, it can be shown that the amplitude margin is given by

$$A_m = \frac{a + u_m}{a},$$

where a is the amplitude of the first harmonic of the limit cycle oscillation and u_m is the relay amplitude. This will give an approximation of the system sensitivity. Such information is useful for the self-diagnostic functions of intelligent systems.

Identification of uncertainty bounds

In [30], an interesting idea is proposed to obtain uncertainty bounds for robust control models using a modified relay feedback experiment. These bounds are very useful in practical applications of robust control theory as they are often assumed to be known in theory. By successively introducing a series of phase lead compensators in the feedback loop, the closed-loop will oscillate at progressively higher frequencies. Beyond a certain frequency, consistent limit cycling no longer occurs and this cut-off frequency may be applied to obtain the uncertainty bounds.

4. Applications to multi-loop systems

In SISO cascade systems, relay feedback has been used to tune the controllers in a sequential manner [34]. The secondary loop is tuned followed by the primary loop. Subsequent re-tuning of either controllers can be done in closed-loop. The ratio of the ultimate frequencies obtained also indicate the ratio of the speed of the loops. This serves as a useful quantity to indicate the effectiveness of the cascade set-up.

Recently, there have been some interesting results too on the application of relay feedback methods to auto-tuning of PI controllers for multivariable systems. There are two such approaches proposed in [31]. The first, which combines the relay feedback method with a sequential tuning method, tunes the multivariable system loop by loop using a relay feedback configuration, closing each loop once it is tuned, until all the loops are done. The Ziegler-Nichols rules are used to tune the PI controllers after the ultimate quantities are obtained from relay feedback.

In another approach, all the loops of a MIMO plant are placed on relay feedback in a multi-loop fashion, and the controllers tuned simultaneously. Three modes of oscillation may occur. Mode 1 consists of relay outputs oscillating at the same frequency. In this case, the ultimate gains are the describing functions of the relays and the ultimate period is the oscillating period which is the same for all loops. The PI controllers may be tuned using Ziegler-Nichols rules. Mode 2 is characterized by relay outputs which are square waves of different frequencies in each loop. Here, the ultimate gains are the describing functions of the relays and the ultimate periods correspond to the respective periodic oscillation in each loop. The PI controllers can be tuned according to the ultimate quantities too. Mode 3 is one of periodic complex oscillations consisting of multiple relay switches within one fundamental period. In this case, it is difficult to extract the ultimate quantities and it is suggested in [32] to manipulate the relay switching levels so as to force the oscillations from a Mode 3 to a Mode 1 oscillation. A methodology to set relay switching levels is also proposed in [32].

In [33], relay feedback is used to obtained process models for MIMO systems. The models are either expressed in integrator-plus-time-delay or gain-plus-time- delay forms. The former is attractive for processes with large time-constants, while the latter is useful for those with significant time delays. The identified model makes possible the use of various tuning methods like the BLT and other tuning methods suggested by Chien and Fruehauf [50] and Tyreus and Luyben [51].

5. Conclusions

Relay feedback has come a long way from their classical configurations. Far from being obsolete, they have found new applications in the automatic tuning of PID controllers and the initialization of adaptive controllers. Current research effort has been extending the increasingly popular method to advanced controller and intelligent systems. These developments as well as related issues have been summarized in the paper.

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