Decoupling Control of Three-tank Liquid Level Systems Based on Feedforward Compensation

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Abstract: By considering couplings between loops as a kind of measurable disturbance, a steady-state decoupling method based on feedforward compensation is proposed for a three-tank liquid level system often encountered in practical process control. In addition, the three-tank liquid level system's dynamic model with structure of two-input and two-output is presented according to its working principle. Finally simulation experiments given in C++Builder language demonstrate the effectiveness of the proposed method.

Key Words: Three-tank System, Decoupling Control, Feedforward Compensation

1 INTRODUCTION

The three-tank liquid level control system is a typical multi-input and multi-output (abbr. MIMO) system with couplings often encountered in practical process control. In order to achieve effective control, many MIMO systems are decoupled into several two-input and two-output (abbr. TITO) subsystems. However, in view of the couplings among loops, it is difficult for the coupled subsystems to make full use of the conventional controllers widely used in single-loop systems, so the decoupling control is paid great attention to both control theorists and engineers in process control fields. Narendra[1] used multiple adaptive models to identify the unknown system with disturbance, but got poor performance. LIU[2] gave a analytical design of decoupling control for TITO processes in theory, not in practice. WANG[3] adopted auto-turning algorithm to decoupling also only in theory. In Refs.[4~6] multiple models adaptive decoupling controllers were designed for the minimum phase, non-minimum phase system and nonlinear system respectively, but their structures are not suitable for the three-tank liquid level control system. In this paper, considering couplings between loops as a kind of measurable disturbance, a steady-state decoupling method based on feedforward compensation is proposed for a three-tank liquid level control system. The decoupled system is divided into two single loop subsystems with no couplings, and each of the subsystems is controlled by a PI controller. Compared with other methods mentioned above, though the proposed method is based on a known model, it has the advantage of easy implementation in practical process control. Simulation experiments given in C++Builder language illustrate the effectiveness of the proposed method.

2 DESCRIPTION OF SYSYEM

2.1 Presentation of Experimental Equipment

The experimental equipment of three-tank liquid level control system is a MIMO object with characters of strong coupling and nonlinear, it is composed of three columnar tanks (T1,T2,T3), two electromagnetic valves (V7,V8), two water pumps (P1,P2) and one trough as shown in Fig.1. By adjusting electromagnetic valves (V7,V8) to control the flux feeding into tanks (T1,T2) simultaneously, and to achieve the liquid level control of tanks T1 and T2 . Water in tank T3 only get from T1 or T2 and it's liquid level is not controlled, obviously, the three-tank liquid level control system is a TITO system with interactions.
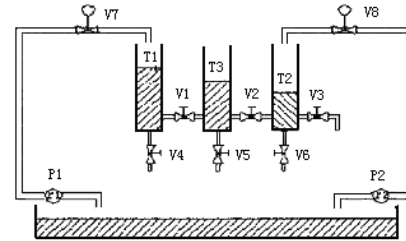

Fig. 1: Schematic diagram of the three-tank system

2.2 Foundation of System Model

According to the dynamic material-balance law, equations for tank T1, T2 and T3 are established as follows

$$\frac{dh_1}{dt} = \frac{1}{S}(q_1 - q_{13}) \tag{1}$$

$$\frac{dh_3}{dt} = \frac{1}{S}(q_{13} - q_{32}) \tag{2}$$

$$\frac{dh_2}{dt} = \frac{1}{S}(q_2 + q_{32} - q_{20}) \tag{3}$$

Where S is the cross section of tank T1,T2 and T3, h_i is the liquid level of tank i, $i \in \{1,2,3\}$, q_1 denotes the flux via V7 to T1, q_2 denotes the flux via V8 to T2, q_{13} denotes the

flux from T1 to T3 via V1, q_{32} denotes the flux from T3 to T2 via V2, q_{20} denotes the flux from T2 to trough via V3. Subject to the Torricelli law which is defined as

$$q = a_z S_n \operatorname{sgn}(\Delta h)(2g\sqrt{|\Delta h|})$$

q_{13}, q_{32} and q_{20} satisfy the following form

$$q_{13} = a_{z1} S_n \operatorname{sgn}(h_1 - h_3)\sqrt{2g|h_1 - h_3|}$$

$$q_{32} = a_{z3} S_n \operatorname{sgn}(h_3 - h_2)(2g\sqrt{|h_3 - h_2|})$$

$$q_{20} = a_{z2} S_n \sqrt{2g h_2}$$

Define vectors $H = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$, $Q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$, Eqs.(1)~(3) can be conveniently rewritten as

$$\frac{dH}{dt} = CQ + D(h) \tag{4}$$

where $D(h) = \frac{1}{S}\begin{bmatrix} -q_{13} \\ q_{32} - q_{20} \end{bmatrix}$, $C = \frac{1}{S}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Eqs.(4) is the state equation of this three-tank liquid level control system. According to Eqs.(4), the system's output variables h_1 and h_2 are coupled with input variables q_1 and q_2. It is obvious that this three-tank liquid level control system is a TITO system with couplings.

3   ANALYSIS   AND   DESIGN   OF DECOUPLING   METHOD

3.1 Analysis of Decoupling Method Based on Feedforward Compensation

Considering the couplings between loops, this three-tank liquid level control system can be described as Fig.2
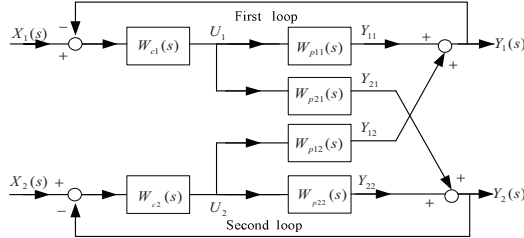

Fig. 2: Structure of system before decoupling

where $W_{c1}(s)$, $W_{c2}(s)$ are PI controllers, $U_1(s)$, $U_2(s)$ are control variables, respectively. Matrix $W_p(s)$ with form of $\begin{bmatrix} W_{p11}(s) & W_{p12}(s) \\ W_{p21}(s) & W_{p22}(s) \end{bmatrix}$ denotes transfer function of the controlled object.

Generally speaking, transfer functions $W_{p11}(s)$ and $W_{p22}(s)$ have the following form[7]

$$W_{p11}(s) = \frac{k_{11}}{T_{11}s+1}e^{-\tau_{11}s}, W_{p22}(s) = \frac{k_{22}}{T_{22}s+1}e^{-\tau_{22}s}$$

For each single-loop, the couplings between loops can be viewed as a measurable disturbance, namely the control variable U_2 may be viewed as a disturbance to first loop

while U_1 may be viewed as a disturbance to second loop too. Because feedforward control[8] can eliminate system's measurable but uncontrolled disturbance, for each single loop, we adopt a feedforward-feedback compound control structure shown in Fig.3, where $W_{ff2}(s)$ denotes the transfer function of the constructed feedforward compensator.
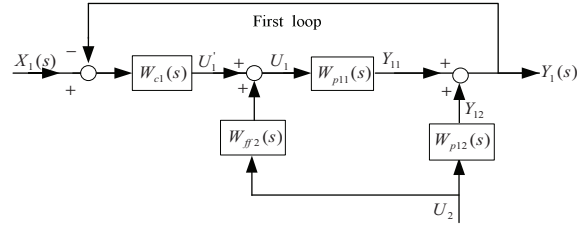

Fig. 3: Structure of feedforward-feedback compound control

According to Fig.3, the first loop's output can be derived as following

$$Y_1(s) = U_2(s)W_{p12}(s) + U_2(s)W_{ff2}(s)W_{p11}(s) - W_{c1}(s)W_{p11}(s)Y_1(s)$$

Namely

$$\frac{Y_1(s)}{U_2(s)} = \frac{W_{p12}(s) + W_{ff2}(s)W_{p11}(s)}{1 + W_{c1}(s)W_{p11}(s)} \tag{5}$$

According to Eqs.(5), the condition of realizing full compensation is $\frac{Y_1(s)}{U_2(s)} = 0$,

namely

$$W_{p12}(s) + W_{ff2}(s)W_{p11}(s) = 0$$

so the constructed feedforward compensator's transfer function in first loop is derived as following

$$W_{ff2}(s) = -\frac{W_{p12}(s)}{W_{p11}(s)} \tag{6}$$

Similar to above discussions, the second loop's feedforward compensator transfer function is

$$W_{ff1}(s) = -\frac{W_{p21}(s)}{W_{p22}(s)} \tag{7}$$

3.2 Design of Decoupling Method Based on Feedforward Compensation

According to Fig.2, the system's outputs are of the form

$$Y_1(s) = U_1(s)W_{p11}(s) + U_2(s)W_{p12}(s)$$

$$Y_2(s) = U_2(s)W_{p22}(s) + U_1(s)W_{p21}(s)$$

For the first loop, we will construct a compensator $W_{ff2}(s)$ which can eliminate the coupling caused by the second loop. According to Eqs.(6), if the first loop achieves full compensation, the constructed compensator's transfer function will have a form of $W_{ff2}(s) = -\frac{W_{p12}(s)}{W_{p11}(s)}$. Similarly,

the second loop's feedforward compensator model is

$W_{ff1}(s) = -\frac{W_{p21}(s)}{W_{p22}(s)}$, which also can eliminate the coupling

caused by the first loop, hence, the decoupled three-tank liquid level control system is illustrated as Fig.4.
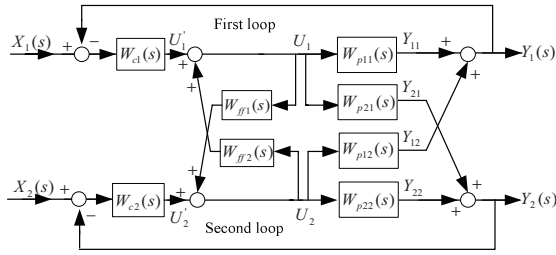
Fig. 4: Structure of the decoupled system

According to Fig. 4, the system's outputs are of the form

$$Y_1(s) = U_1 W_{p11}(s) + U_2 W_{p12}(s) + U_2 W_{ff2}(s) W_{p11}(s) \quad (8)$$

$$Y_2(s) = U_2 W_{p22}(s) + U_1 W_{p21}(s) + U_1 W_{ff1}(s) W_{p22}(s) \quad (9)$$

Since $W_{ff2}(s) = -\dfrac{W_{p12}(s)}{W_{p11}(s)}$, $W_{ff1}(s) = -\dfrac{W_{p21}(s)}{W_{p22}(s)}$, now by

substituting $W_{ff2}(s), W_{ff1}(s)$ into Eqs.(8) and (9) respectively, finally, we obtain

$$Y_1(s) = U_1(s) W_{p11}(s) \quad (10)$$

$$Y_2(s) = U_2(s) W_{p22}(s) \quad (11)$$

According to Eqs.(10) and (11), it is obvious that the decoupled system is divided into two single-loop subsystems without couplings.

4 IMPLEMENTATION OF DECOUPLED SYSTEM

Steady-state decoupling and dynamic decoupling are two means of implementing decoupling, but steady-state decoupling method is suitable for those systems whose variables are paid great attention in steady-state. In practical process control, in order to simplify the processes, it is advisable to substitute the transfer function's ratio with the transfer functions' gain coefficient ratio, namely, the constructed feedforward compensators may be written as $W_{ff1}(s) = -\dfrac{k_{21}}{k_{22}}$, $W_{ff2}(s) = -\dfrac{k_{12}}{k_{11}}$. In this system, because we concern the liquid level of tanks in steady-state conditions, so we adopt steady-state decoupling means to this system. Not only couplings but also time delays often exist in practical processes, however, because all the pipelines in this system are very short, the time delays are very small and may be out of consideration. Because the single tank's model has a form of first order inertia, after experiments, the single tank's transfer function without couplings is chosen as

$$W_{p11}(s) = W_{p22}(s) = \frac{2}{5s+1}$$

The system's control indexes are $h_1 = 30\,cm$, $h_2 = 15\,cm$, since $h_1 > h_2$, the coupling in second loop caused by input variable q1 must be taken into account while the coupling in first loop caused by input variable q2 may be overlooked. Considering the above cases, we let the coupled part transfer function as

$$W_{p21}(s) = \frac{8}{125s^3 + 75s^2 + 15s + 1}, W_{12}(s) \approx 0$$

In the following simulation experiments, we let the decoupling coefficients $W_{ff1} = 1$, $W_{ff2} = 0$. Each of the decoupled subsystems is controlled by PI control algorithm.

Hereto, the decoupled three-tank liquid level control system is described clearly.

5 SIMULATION EXPERIMENTS

To demonstrate the decoupling ability of the proposed method, simulation experiments are done in C++Builder language, the realization steps are as follows

5.1 Parameters Definition

Variables used in program are defined as following

y11, y22 and y12 are subsystems' discrete liquid level, Y1, Y2 and Y3 are liquid level of tanks T1, T2 and T3, respectively. U11,U22 denote the outputs of each PI controllers, U1,U2 denote the outputs of the decoupled system.

5.2 Decoupling Programming

The following is a segment of source code to get the decoupled system's output values U1 and U2.

```
RIN=1;// step length
y11=0.3625*U1_1+0.8187*y11_1;
    //discrete equation of transfer function Wp11(s)
y12=2.456*y12_1-2.011*y12_2+0.5488*y12_3+0.0092
    *U1_1+0.03166*U1_2+0.0068*U1_3;
y22=0.3625*U2_1+0.8187*y22_1;
    // discrete equation of transfer function Wp22(s)
Y1=y11;
Y2=y22+y12;
Y3=1.637*Y3_1-0.6703*Y3_2+0.071*U1_1+0.0613
    *U1_2;
EK1=RIN-Y1;
EI1=EI1+EK1;
EK2=RIN-Y2;
EI2=EI2+EK2;
U11=KP1*EK1+KI1*EI1;
U22=KP2*EK2+KI2*EI2;
U1=U11+U22*0; //Wff2=0
U2=U22+U1*Wff1;
    // U1 and U2 are the decoupled system's output variables
```

5.3 Simulation Experiments

During simulation experiments, the most ideal control effect is gotten under the conditions of $KP_1=3$, $KI_1=0.3$, $KP_2=0.5$ and $KI_2=1$. The simulations are shown in Fig.5~ Fig.6.
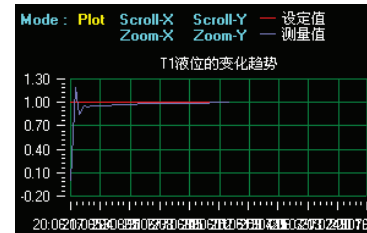

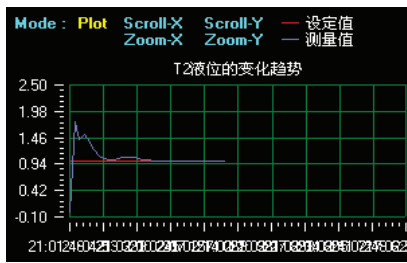Fig. 5: Liquid level control simulation results of tank T1

Fig. 6: Liquid level control simulation results of tank T2

It is obvious that the proposed steady-state decoupling method based on feedforward compensation has a effective control effect.

6   CONCLUSIONS

The couplings between loops considered as a measurable disturbance, a steady-state decoupling control method based on feedforward compensation is presented for three-tank liquid level systems in this paper, simulation experimental results derived from a practical engineering illustrate effectiveness of the proposed method. Compared with others, this method has advantages of relatively simple structure and easy implementation, but it neglects the system's time delays. In the future, fuzzy logic technologies will be considered to develop this method, thus, it can be extended to the processes with time delays[9].

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