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Sensor Fault Detection in Coupled Liquid Tank System

F. Kousar, M. Abid, A. Q. Khan
Department of Electrical Engineering, Pakistan Institute of Engineering and Applied Science,
P. O. Nilore 45650, Islamabad, Pakistan
engr.farzanakousar@gmail.com, mabid@pieas.edu.pk, aqkhan@pieas.edu.pk

Abstract — ***As the field of automation is progressing, safer and reliable systems are highly desirable. Any malfunction in the plant result in reduced efficiency of the plant, reduced quality of the product and sometimes may result into fatalities. Therefore fault detection and process monitoring is becoming an integral part of modern control systems. The coupled liquid tank system is an experimental setup with nonlinear dynamics. The objective of this paper is to develop and implement fault detection techniques for coupled liquid tank system. The proposed scheme makes use of observer based residual generation and norm based residual evaluation. First system was linearized by Jacobian linearization and fault diagnosis system has been designed for the linearized system. Then this algorithm has been implemented on real plant and satisfactory results have been obtained.***

*Keywords—**Sensor fault detection, Observer based fault detection, Model based, Couple liquid tank syste.***

I. INTRODUCTION

The widely accepted concept is that a fault is an unexpected change of system function although it may not represent physical failure or breakdown [1]. This paper discusses sensor faults. This is shown in figure below and can be described mathematically as described in [1].

$$\boldsymbol{y(t) = y_R(t) + f_S(t)} \qquad (1)$$

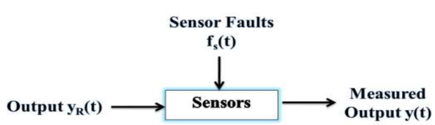

Figure 1: Sensors, outputs and measured outputs [1]

In other words, sensor faults disturb output measurements. These faults can create problems if we use measured output to further control some device.

There are many different approaches which are used for the purpose of fault diagnosis. There are many survey papers that describe the details of these techniques [2-8]. In [9], a structured residual generation approach has been used for fault detection as well as fault isolation purpose for a coupled tank system. Among all these approaches, model based fault diagnosis techniques are most commonly used. Model based fault diagnosis can be defined as the detection, isolation and characterization of faults in components of a system from the comparison of the system's available measurements, with a priori information represented by the system's mathematical model. Faults are detected by setting a (fixed or variable) threshold on a residual quantity generated from the difference between real measurements and estimates of these measurements using the mathematical model [1].

This paper is organized as follows: Section 2 contains description about the plant, In Section 3; Mathematical model of the system has been derived, in section 4, a brief overview of fault diagnosis has been given. Section 5 deals with residual generation step and section 6 deals with residual evaluation stage. In section 7 contains results after implementing the algorithm on real plant. Section 8 concludes the work.

II. DESCRIPTION OF PLANT

Coupled Liquid Tank System (CLTS) is an experimental setup with highly nonlinear dynamics and is quite useful to test nonlinear control and fault detection algorithm. It consists of the cylindrical tanks with equal cross sectional area as shown in figure 2. Detailed description of CLTS can be found in [10]. Each tank is equipped with a sensor to measure water level. The plant can be used to simulate several kinds of faults; these include

- Sensor faults
- Component faults: can be simulated by opening or closing of valves
- Actuator faults

There is seepage in tanks which can be taken as disturbance. In this paper, we shall discuss the detection of sensor faults.

Figure 2: Coupled liquid tank system

III. MATHEMATICAL MODELING

Cross sectional areas tank 1 and 3 do not vary with water level. While cross sectional area of tank 2 varies with the water level. Each tank basically acts as an integrator. Free flow out of each tank is a non linear function of the level in the tank and orifice discharge coefficient C_n. Different notations which are used throughout the paper are described in Table 1.

$$\frac{dV_3}{dt} = q_i - C_3\sqrt{H_3} \quad (2)$$

$$\frac{dV_2}{dt} = C_3\sqrt{H_3} - C_2\sqrt{H_2} \quad (3)$$

$$V_3 = H_3A_3 \quad (4)$$

$$V_2 = L\left[r^2\cos^{-1}\left(\frac{r-H_2}{r}\right) - (r-H_2)\sqrt{r^2-(r-H_2)^2}\right] \quad (5)$$

Table 1: Notations with descriptions and values

Notations	Description	Value
V2, V3	volumes of tank 2 and 3	Variable
C2, C3	orifice discharge coeffiecients	C2=C3=3.44
H2, H3	levels in tank numbers 2 and 3	Variable
A1, A3	Cross sectional areas of tank 1 and 3	283.5cm2
qi	pump flow rate	Variable
R	Cylinder 2 radius	9.5cm
L	Cylinder 2 length	18.5cm

We need water level as state variables. So by applying chain rule

$$\frac{dV_3}{dH_3}\frac{dH_3}{dt} = q_i - C_3\sqrt{H_3} \quad (6)$$

$$\frac{dV_2}{dH_2}\frac{dH_2}{dt} = C_3\sqrt{H_3} - C_2\sqrt{H_2} \quad (7)$$

$$\frac{dV_3}{dH_3} = A_3 = \pi r^2 \quad (8)$$

$$\frac{dV_2}{dH_2} = 2L\sqrt{r^2-(r-H_2)^2} = \alpha(H_2) \quad (9)$$

Final non-linear equations are

$$\frac{dH_3}{dt} = \frac{q_i - C_3\sqrt{H_3}}{A_3} \quad (10)$$

$$\frac{dH_2}{dt} = \frac{C_3\sqrt{H_3} - C_2\sqrt{H_2}}{\alpha(H_2)} = \frac{C_3\sqrt{H_3} - C_2\sqrt{H_2}}{2L\sqrt{r^2-(r-H_2)^2}} \quad (11)$$

For tank 1

$$\frac{dV_1}{dt} = C_2\sqrt{H_2} - q_i \quad (12)$$

So

$$\frac{dH_1}{dt} = \frac{C_2\sqrt{H_2} - q_i}{A_1} \quad (13)$$

Now mathematical model of this system is complete.

Values of different parameters are listed in the Table 1. Replace these value and then these equations become

$$\frac{dH_1}{dt} = 0.012\sqrt{H_2} - 0.0035q_i \quad (14)$$

$$\frac{dH_2}{dt} = \frac{0.093(\sqrt{H_3} - \sqrt{H_2})}{\sqrt{19H_2 - H_2{}^2}} \quad (15)$$

$$\frac{dH_3}{dt} = 0.0035q_i - 0.012\sqrt{H_3} \quad (16)$$

A. Linearized Model

The mathematical model of CLTS is linearized by applying Jacobian linearization. For a system of differential equations with n components

$$\frac{d}{dt}\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} f_1(t, y_1, y_2, \dots y_n, u) \\ f_2(t, y_1, y_2, \dots y_n, u) \\ \vdots \\ f_n(t, y_1, y_2, \dots y_n, u) \end{bmatrix} \quad (17)$$

$$J_1 = \left.\begin{bmatrix} \frac{df_1}{dy_1} & \frac{df_1}{dy_2} & \dots & \frac{df_1}{dy_n} \\ \frac{df_2}{dy_1} & \frac{df_2}{dy_2} & \dots & \frac{df_2}{dy_n} \\ \vdots & \vdots & & \vdots \\ \frac{df_n}{dy_1} & \frac{df_n}{dy_2} & \dots & \frac{df_n}{dy_n} \end{bmatrix}\right|_{(\bar{y}_1, \bar{y}_2, \dots, \bar{y}_n, \bar{u})} \quad (18)$$

$$J_2 = \left.\begin{bmatrix} \frac{df_1}{du} \\ \frac{df_2}{du} \\ \vdots \\ \frac{df_n}{du} \end{bmatrix}\right|_{(\bar{y}_1, \bar{y}_2, \dots, \bar{y}_n, \bar{u})} \quad (19)$$

The above mentioned matrices J_1 and J_2 are called Jacobians. Where $(\bar{y}_1, \bar{y}_2, \dots, \bar{y}_n)$ is the point about which we want to linearize the model. Then linearized model is given by

$$\dot{y} = J_2y + J_2u \quad (20)$$

Linearizing about L_1, L_2, L_3 and flow rate at Q_i.

$$H_i = h_i + L_i \ , i = 1,2,3 \quad (21)$$

$$q_i = Q_i + u \quad (22)$$

Linearized model is given as

$$\frac{dh_1}{dt} = \frac{C_2 h_2}{2\sqrt{2L_2}A_1} - \frac{U}{A_1} \quad (23)$$

$$\frac{dh_2}{dt} = \frac{C_3 h_3}{8L\sqrt{2L_2L_3(r-L_2)}} - \frac{C_2 h_2}{4\sqrt{2}L(r-L_2)^{\frac{3}{2}}} \quad (24)$$

$$\frac{dh_3}{dt} = \frac{U}{A_3} - \frac{C_3 h_3}{2\sqrt{2L_3}A_3} \quad (25)$$

With $L_1 = 5, L_2 = 4, L_3 = 15$, linearized model becomes

$$\frac{dh_1}{dt} = 0.00214h_2 - 0.0035u \quad (26)$$

$$\frac{dh_2}{dt} = 0.0009h_3 - 0.0025h_2 \quad (27)$$

$$\frac{dh_3}{dt} = 0.0035u - 0.0011h_3 \quad (28)$$

Denote h1=x1 h2=x2 and h3=x3 as state variables, linearized model in state space form becomes

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (29)$$

$$y(t) = Cx(t) + Du(t) \quad (30)$$

Where

$$A = \begin{bmatrix} 0 & 0.00214 & 0 \\ 0 & -0.0025 & 0.0009 \\ 0 & 0 & -0.0011 \end{bmatrix} B = \begin{bmatrix} -0.0035 \\ 0 \\ 0.0035 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{And } D = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

B. Discretized Model

Above mentioned model has been discretized using MATLAB with sampling interval of 0.01 sec. Tustin method has been used. Discretized model is

$$x(k+1) = Ax(k) + Bu(k) \quad (31)$$

$$y(k) = Cx(k) + Du(k) \quad (32)$$

With

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} -0.0035 \\ 0 \\ 0.0035 \end{bmatrix}$$

$$C = \begin{bmatrix} .01 & 0 & 0 \\ 0 & .01 & 0 \\ 0 & 0 & .01 \end{bmatrix}, D = 1.04 \times 10^{-4} \begin{bmatrix} -.1750 \\ 0 \\ 0.1750 \end{bmatrix}$$

IV. FAULT DIAGNOSIS

System model may change due to different faults. Actuator and component faults affect state equations of the system while sensor faults affect output equations because sensor faults act directly on output of the system. So we can write system's model under faulty conditions as

$$\dot{x}(t) = Ax(t) + B[u(t) + f_a(t)] + f_c(t) \quad (33)$$

$$y(t) = Cx(t) + D[u(t) + f_a(t)] + f_s(t) \quad (34)$$

C. Fault Detectability

When faults occur in the system, the response of residual vector is

$$r(s) = G_{rf}(s)f(s) = \sum_{i=1}^{g} [G_{rf}(s)]_i f_i(s) \quad (35)$$

Where G_{rf} is fault transfer matrix and it represents the relation between residuals and faults. In order to detect i_{th} fault f_i in the residual $r(s)$, the i_{th} column $[G_{rf}(s)]_i$ should be nonzero.

$$[G_{rf}(s)]_i \neq 0 \quad (36)$$

This condition is defined as the fault detectability condition. But this condition alone is not sufficient to detect faults so we define strong detectability condition.

$$[G_{rf}(0)]_i \neq 0 \quad (37)$$

If this condition is satisfied, we define that the i_{th} fault f_i is strongly detectable in the residual r. This condition can also be defined as the strong fault detectability condition of the residual r to the fault f_i [1].

V. OBSERVER BASED RESIDUAL GENERATION

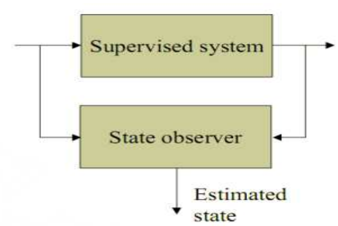

Figure 3. Schematics of observer

We can design observer for the system as designed in [11]. Let process model is

$$\dot{x}(t) = Ax(t) + Bu(t), \quad with \quad x(0) = x_o \quad (38)$$

$$y(t) = Cx(t) \quad (39)$$

Then for this model, Luenberger observer is given by

$$\dot{\tilde{x}} = A\tilde{x} + Bu + L(y - C\tilde{x}) \quad , \quad \text{with} \quad \tilde{x}(0) = \tilde{x}_o \quad (40)$$

We can find state estimation error as

$$e_x = x - \tilde{x} \quad (41)$$

And it can be seen that

$$\dot{e}_x = (A - LC)e_x \quad \text{with } e_x(0) = x_o - \tilde{x}_o \quad (42)$$

Output estimation error

$$e_y = y - \tilde{y} = y - C\tilde{x} = Ce_x \quad (43)$$

Under normal conditions, e_x and e_y should decay to zero exponentially.

When there is fault in the system then

$$\dot{x}(t) = Ax(t) + Bu(t) + Ef(t) \quad , \text{with } x(0) = x_o \qquad (44)$$

$$y(t) = Cx(t) + Gf(t) \qquad (45)$$

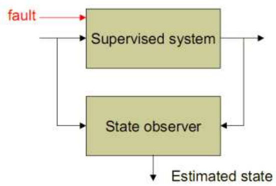

Figure 4. Observer with fault

Luenberger observer is given by equation (40). By differentiating equation (41) and replacing equation (44) and (40), we get

$$\dot{e}_x = (A - LC)e_x + (E - LG)f \,, \text{with } e_x(0) = x_o - \tilde{x}_o \quad (46)$$

Output estimation error

$$e_y = y - \tilde{y} = y - C\tilde{x} = Ce_x + Gf \qquad (47)$$

Now e_x and e_y does not decay to zero exponentially in the presence of a fault and at least exhibit significant transient upon occurrence of a fault. So it can be used to detect faults.

D. Observer Design for CLTS

For fault diagnosis of this system, an observer has been designed for the system. As the system is completely observable, so we can arbitrarily place the observer poles. As we have to use discrete time system, so system was discretized and poles were selected such that its response matches the continuous time case. Then selected values of observer poles (DT case) were 0.5, 0.8 and 0.7. With these eigen values, observer gain matrix is calculated to be

$$Ld = \begin{bmatrix} 50 & 0.0019 & 0 \\ 0 & 20 & 0.0008 \\ 0 & 0 & 30 \end{bmatrix} \qquad (48)$$

This observer has been simulated in case when there is no fault. Residuals are almost zero. For simulation, a sensor noise of variance 0.0008 has been added. Figure 5 shows the residuals which are obtained in fault free case. Variation from zero is due to effect of noise.

VI. Residual Evaluation

Despite the fact that a fault detection system consists of a residual generator, a residual evaluator together with a threshold and a decision maker, in the observer based FDI framework. There exist two major residual evaluation strategies. The statistic testing is one of them, which is well established in the framework of statistical methods. Another one is the so-called norm based residual evaluation. Besides of less on-line calculation, the norm based residual evaluation allows a systematic threshold computation using well-established robust control theory [12]. Standard evaluation functions are

- Peak value
- Average value evaluation
- RMS value evaluation

In this paper, RMS based residual evaluation will be discussed.

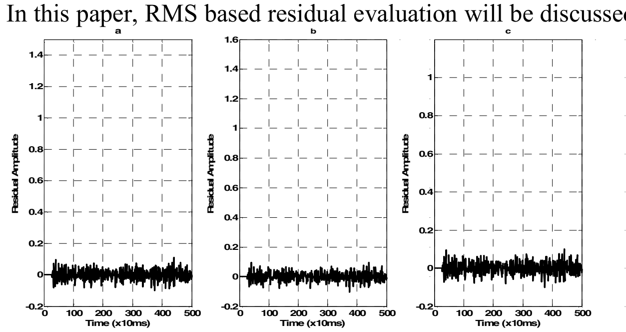

Figure 5. Residuals in fault free case (a) residual 1 (b) residual 2 (c) residual 3

E. RMS Evaluation

RMS residual evaluation has been used. The RMS value of r is defined by

$$J_{RMS} = \|r(k)\|_{RMS} \quad = \left(\frac{1}{N} \sum_{j=k}^{k-N} \vec{r}(k) \times \vec{r}(k)^T \right)^{1/2} \qquad (49)$$

JRMS measures the average energy of $\vec{r}(k)$ over time interval (k, k - N).In our case, $\vec{r}(k)$ is a vector with 3 signals. So expression becomes

$$J_{RMS} = \|r(k)\|_{RMS}$$

$$= \left(\frac{1}{N} \sum_{j=k}^{k-N} (r_1(k)^2 + r_2(k)^2 + r_3(k)^2) \right)^{1/2} \qquad (50)$$

Let

$$J_{th,RMS} = \sup_{fault-free} \|r(k)\|_{RMS} \qquad (51)$$

be the threshold, then the detection logic becomes

JRMS > Jth,RMS **alarm, a fault is detected**

JRMS ≤ Jth,RMS **no alarm, fault-free** [12]

Then we apply residual evaluation technique on the residuals obtained in Figure 5. Norm base technique has been used and threshold has been computed as described above. RMS signal in fault free case is shown in Figure 6.

Threshold computed is 0.063.

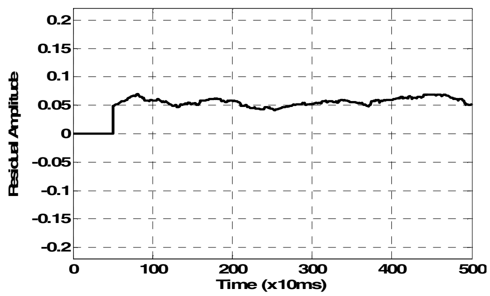

Figure 6. RMS signal in fault free case

F. *Introducing Sensor Faults*

Now we introduce a step fault at k=100 in sensor 1 and then observe residual signals.

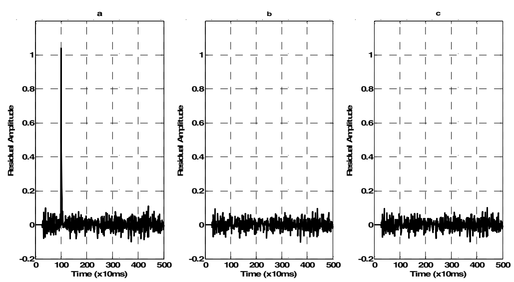

Figure 7. Residuals with fault in sensor1 (a) residual 1 (b) residual 2 (c) residual 3

Similarly this observer has been tested for faults in sensor 2 and 3 and similar results have been obtained. Thus the proposed observer based fault diagnosis scheme is successful in detecting sensor faults.

RMS signal in faulty situation is shown in figure below

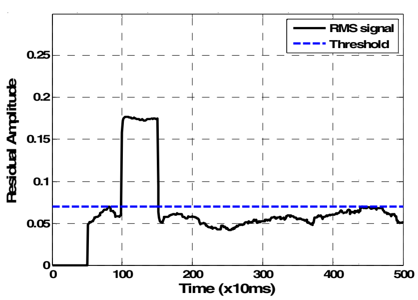

Figure 8. RMS signal in faulty case

Now we can apply decision logic that if RMS signal is greater than indicate fault otherwise there is no fault. We can see that residual signal is above threshold for some time interval. So fault will be indicated at t=100 (x10msec).

VII. Implementation on Real Plant

Model based fault diagnosis scheme for linearized system has been successfully implemented on real system. The system is capable of detecting online as well as offline faults. Data has been taken with the help of C++ program and this algorithm has been tested for various kinds of faults. This section discusses results of introducing sensor faults.

For residual evaluation, norm based technique has been used. Period of 50 was used. Residual has been evaluated by computing RMS value. RMS signal in fault free case is shown in Figure 9.

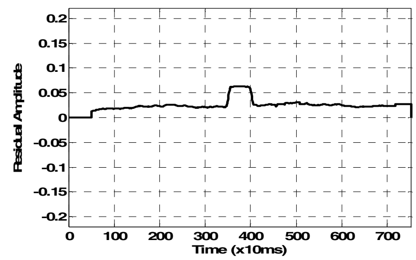

Figure 9: RMS signal in fault-free case

Then threshold was calculated and it comes out to be0.0632.

G. *Sensor Fault Detection in Tank 1*

A sensor fault was introduced in sensor 1. Residuals in case of fault in sensor 1 are shown in figure 10.

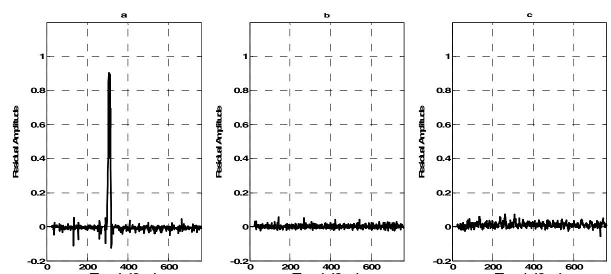

Figure 10: Residuals in case of sensor 1 fault (a) Residual 1 (b) Residual 2 (c) Residual 3

Then residual evaluation has been performed. Norm based technique has been used. RMS signal is shown in Figure 11.

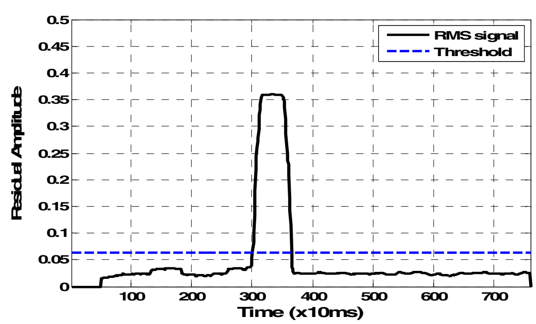

Figure 11. RMS signal for fault in sensor 1

We can see that fault is clearly visible in RMS signal. We can see that signal is greater than threshold after t=300 (x10ms).

H. Sensor Fault Detection in Tank 2

Residual signals after introducing fault in sensor 2 are shown in the figure below.

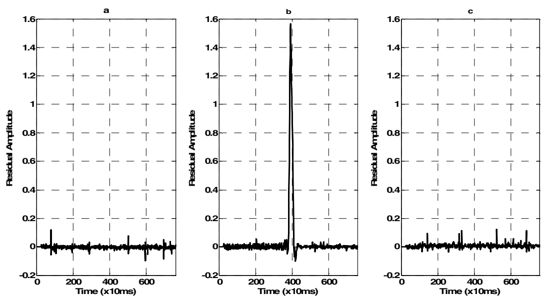

Figure 12. Residuals in case of fault in senor 2 (a) Residual 1 (b) Residual 2 (c) Residual 3

Fault in sensor 2 is clearly visible. RMS signal is shown in figure below.

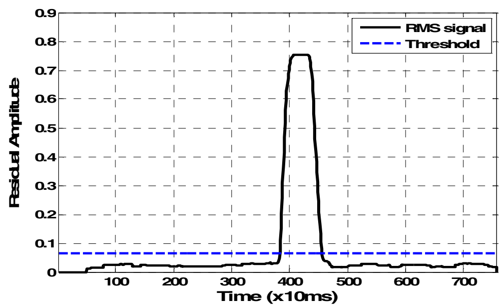

Figure 13. RMS signal for fault in sensor 2

Fault is visible in RMS signal as well. So fault in sensor 2 has also been detected successfully.

I. Sensor Fault Detection in Tank 3

A fault was added in sensor 3. Residuals in this case are shown in Figure 14.

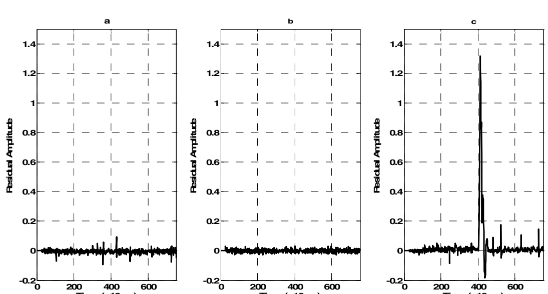

Figure 14. Residuals in case of fault in sensor 3

We can see that fault residual corresponding to sensor 3 is showing remarkable deviation. RMS signal has been computed for this case and result is shown in Figure 15.

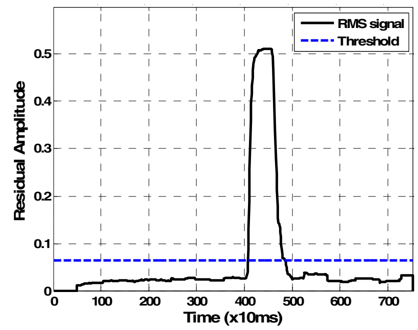

Figure 15. RMS signal in case of fault in sensor 3

It is seen that fault was clearly detected.

VIII. CONCLUSIONS AND FUTURE WORK

First of all the system was linearized by using Jacobian linearization. Then an observer based fault diagnosis scheme was designed for this system. This scheme is able to detect faults in all three level sensors. Residual evaluation was also performed. RMS evaluation was used for this purpose. Then it was implemented on real plant. Sensor faults were introduced into the system. All faults were detected successfully. Online fault diagnosis was performed.

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