Sampled-Data Systems Revisited:

Reflections, Recollections, and Reassessments*

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SUMMARY

A review of the progress made in sampled-data systems during the last thirty years is presented in this paper. In particular the impact of the discrete theory on the continuous counterpart is mentioned. Additionally, the limiting process of discrete system theory when the discrete interval (or the sampling period) goes to zero, is discussed. Recent emergence of digital signal processing and digital filters as an aftermath of sampled-data systems is brought into focus as well as the technological developments which aided in this new development.

The paper concludes with a critical view of the past achievements in this field as well as indications of possible future developments.

I. Introduction

During the last three decades, because of the tremendous progress achieved in the field of sampled-data systems (or discrete systems) several classified bibliographies [1-4] have been published. The last of such a series of articles appeared in [1970] [5].

The purpose of this paper is not to add to those survey papers, but to evaluate and assess the progress made in this important field which played and is still playing an important role in the technological developments of control systems. This is, of course, due to the importance of digital computers (or processors) used as part of the control system.

In the next section, we will present a historical review of the development of the field in the early days of this writer's graduate studies at Columbia University (1949-1953). In Section III, the impact and use of discrete theory on its continuous counterpart is discussed. In the following section a discussion of the problems involved in obtaining the continuous theory concepts from its discrete counterpart as a limiting process (i.e., when T, the sampling period, goes to zero) is presented.

In Section V, we will present the technological advance made in the implementation of digital systems. In particular, the role which microprocessors and process computers are playing in recent years. In Section VI, a discussion of the presently emerging field of multidimensional discrete systems is presented. Finally, in the conclusion, Section VII, several important areas of research in this field are suggested.

II. Historical Review

Applications of sampled-data systems appeared much earlier than the development of the theory. Indeed, the first such application [6] was developed by Abraham-Louis Breguet in Paris in about 1793. This scientist invented the "Pendule Sympathetique" whereby he found an ingenious way to control the rate of revolution of the crudely made watch by means of an impulsive signal injected once every 12 hours by an accurate chronometer, thus making a cheap watch run like an expensive one. An early application of discrete control system was also developed by Gouy [7] in 1897, when he designed an oven regulator that is based on intermittent applications of information. Other early applications were the use of sampling in electronic feedback amplifiers in 1941 [8], radio transmitters, or receivers for improving stability. It is of historical interest to note that Hazen [9], in 1934 in an early classic article, pointed out the usefulness of sampled-data systems to industrial applications.

The theory of sampled-data systems was vigorously developed during the second World War (1939-1945) in connection with radar applications. During this period, scientists in USA and in England worked on developing such a theory. As a result of this work, the two main references on this subject were Chapter 5, vol. 25, of the MIT Radiation Series [10] by Hurewicz' approach later motivated the z-Transform developments and the MacColl approach motivated the infinite frequency approach for impulse-modulation. Independently, of this work Barker [12], Lawden [13] and others in England [14] developed an approach similar to the z-Transform. While in the west the development of the theory of sampled-data systems originated as mentioned earlier, by radar applications, scientists in USSR and in particular, Ya. Z. Tsypkin, developed the theory [15] from his famous work on relay control systems [16]. In a later section, we will indicate how sampled-data systems play an important role in the stability of relay systems. As a supplement to the above, it is also pertinent to mention the discussion of discrete systems in the classic book of Oldenbourg and Sartorious [17] and in the French paper by J.M. Raymond.† A historical survey of some of the early applications can be found in the article of Jury-Tsypkin[5].

Three decades ago, I started working on sampled-data systems as a graduate student at Columbia University in New York under the guidance of Professor John

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† The pioneering work of the late W.K. Linvill is of much importance. In particular, his concept of the sampler as an "impulse modulator," was an enlightening idea.

Ragazzini. At that time the only two references on the subject available to me, were the MIT Radiation Series, Vol. 25 and MacColl's book. The works of Barker and others in England and that of Tsypkin in USSR became known to me after completing my Ph.D. dissertation in 1953. Also, during the last stages of my Ph.D. research, I read Professor William Linvill's Navy report which was based on his Ph.D. Thesis. While my approach was based on the z-Transform [18], that of Linvill's was based on the infinite frequency approach [19]. Hence, there was no overlapping in our independent researches.

At Columbia in the early fifties, John Ragazzini in colloboartion with John Mulligan wrote a manuscript on Control Systems which for some reason, was never published. In that manuscript, I discovered a chapter on sampled-data systems in which the z-Transform was introduced. In 1952, Ragazzini and Zadeh published the contents of this work, where a formal definition of the z-Transform was introduced [20]. In analogy to the z-Transform, Tsypkin independently introduced the Discrete-Laplace Transform. They are both equivalent with notational change of variables, $e^{Tp} = z$.

In a similar and independent work, Barker introduced the sequence transform, similar to the z-Transform method. For instance the Discrete Transform is defined by MacColl and Tsypkin as,

$$D\{f(nT)\} = \sum_{n=0}^{\infty} e^{-pnT} f(nT) = F^*(p) \qquad (1)$$

and the z-Transform is defined as,

$$f(z) = \sum_{n=0}^{\infty} f(nT) z^{-n}, \quad z = e^{Tp}. \qquad (2)$$

In the works of MacColl, Linvill and Tsypkin, the infinite series in terms of the Laplace transform is defined as

$$F^*(s) = \frac{1}{T} \sum_{k=-\infty}^{\infty} F(s+jk\omega_r) + \frac{1}{2} f(0^+) \qquad (3)$$

where

$$F(s) \triangleq \mathcal{L}[f(t)], \; T \triangleq \text{The sampling} \qquad (4)$$

period and $\omega_r = 2\pi/T. \qquad (5)$

In the Ph.D. dissertation of Jury [18], the equivalence of (2) and (3) is shown in terms of a convolution integral of two functions, where one of them included the impulse function, with path of integration in the right and left half planes respectively.

While the z-Transform method yields the system response at the sampling instants, its modification yields the response at all times. This extension was formally introduced by Jury [21], 1956, as an extension of Barker's modified sequence approach, as the Modified-z-Transform Method. The basis of this method was introduced earlier by Barker, using a fictitious delay; and Tsypkin, by introducing the fictitious advance time shift (or negative delays). A detailed discussion of the z-Transform and modified z-Transform was published in book form in 1964 [22], and reprinted in 1973[22a]. The z-Transform with various modifications is applied nowadays in many diverse areas of scientific work. As a historical note, it is pertinent to mention that the Modified z-Transform is a general discrete transform, since the z-Transform is obtainable from it as a special case when the fictitious delay (Δ=1-m), approaches zero. For instance, if the Modified z-transform is defined as:

$$F(z,m) = \sum_{n=0}^{\infty} f(n,m) T z^{-n}, \quad 0 \leq m \leq 1, \qquad (6)$$

then

$$F(z) = \lim_{m \to 0} zF(z,m) \qquad (7)$$

Early books on Sampled-data Systems in the west were published by Ragazzini and Franklin [23], in 1958 and by Jury [24] six weeks later. Many other books followed in the ensuing years. Tyspkin in the USSR published his first book on discrete systems in 1951 [25] and in 1958 [26], published a second extensive text on this subject.

It is not the intent of this article, as mentioned earlier, to survey the field from these early days up to the present, for this has been occasionally done in various reviews at various periods. In the following sections, we will discuss the influence of discrete systems in the technological advances in Control System Theory and Applications.

In concluding this brief historical background, it is pertinent to mention the following:

a. As indicated earlier, most of the early work on the theory of Sampled-data Systems was done independently by several authors in different countries. Because of the war and post war periods, communication between the various scientists was not possible. Hence one notices many duplications of various topics in the theory.

b. The development of Sampled-data theory relating to the z-Transform or its modification is done by various authors so that no one or two authors can claim any monopoly on the theory. Each of the various scientists mentioned earlier contributed in one way or another in various degrees to the theory. It is the collective work of many over several years that consolidated or formed the basis of discrete systems theory. Indeed the z-Transform in equation (1) is closely related to the generating function used by Laplace two centuries ago in probability theory and the infinite summation in equation (3) was obtained by Poisson and is known as the Poisson summation.

c. This Section was written by the author more-or-less from memory of the last thirty years. It is quite possible that some of these recollections are either not precise or not in chronological order. Furthermore, it is quite likely that some other early contributors to the theory were left out, not because of ill will, but because of ignorance. For both of these shortcomings, the author wishes to offer the readers his apologies and regrets.

Finally, it is pertinent to mention the significant contributions of the many doctoral students under the supervision of Professor John Ragazzini at Columbia University. This was, in part, recognized by the AACC last year when, at the JACC meeting, it presented the first education award to Dr. Raggazini. Also he received the IEEE model.

III. Impact of Discrete System Theory on the Continuous Counterpart

In this section, we will indicate applications to continuous systems of the methods developed for the discrete theory, i.e., the z-Transform and the modified z-Transform. Also, we will discuss the impact on continuous systems theory of the notion of inners, which was developed from the discrete theory. Finally, we will discuss the applications of Point mapping as first developed by Poincare to solve problems in nonlinear continuous systems. Some of these methods are used as approximations to continuous systems while there are also exact methods for certain other applications. The discrete methods discussed here represent only a few of the available methods.

A. The Z-Transform and Modified Z-Transform Theory

Although the development of the z-Transform theory was applied mainly to sampled-data systems with considerable ease, it became apparent that this method can also be applied to continuous systems with and without delay to obtain the response for deterministic inputs. Hence, the sampled-data model was used as an approximation to continuous systems [22,24]. Also the z-Forms [27] were applied for such an approximation. Since for continuous systems one is interested in the output at all times, the modified z-Transform or modified z-Forms [28], were also applied in such approximation.

Needless to say, the z-transform and the modified z-transform are now used in all facets of digital filtering problems. Any issue of the "ASSP" attests to this fact.

In certain cases where continuous systems are subjected to periodic inputs, the modified z-Transform was applied with much success for both the steady state [29] and the actual response [24]. Because of the availability of extensive tables of modified z-Transforms [24], the analysis of such systems becomes routine.

The modified z-Transform is applied to exact analysis of relay control systems in order to obtain the periodic solutions or limit cycles. Such analysis was carried out by Hsu and Meyer [30] for various forms of relay systems.

The modified z-Transform has also been used for summing up convergent infinite series, [31] in the solution of mixed linear difference-differential equations [32], in the study of eye movement in biological control systems [32a] and in several other applications as discussed in [24].

It is also of historical interest to note that following the discussion of hidden oscillations and instability [33] in sampled-data systems, the notion of controllability and observability as affected by the sampling period became evident [34]. Such discussions were based on the modified z-Transform.

B. Inners Notion

The notion of inners of a square matrix was first introduced by this writer a decade ago. It was motivated by studying the stability of discrete systems. Soon, it became apparent that this notion offers a feature unifying many of the stability criteria and other related problems for both the continuous and discrete theory. It also sheds some light on many continuous theory concepts. During the last decade a book [35] and many articles have been written on this notion. A review paper on this subject appeared in [36].

In summarizing the impact of inners, we itemize the following:

a. Unification of discrete and continuous stability criteria root-distribution, index of performance and others [36].

b. Achievement of new results in mathematics such as Shur-Complement [39]. Also generalization of Sylvester's Theorems.

c. Realization of another form of the Lyapunov second method based on inners.

d. Foundation for obtaining simplified stability criteria [38] and shed more light on existing criteria [36].

e. Use of one computational algorithm (double triangularization) for both continuous and discrete systems.

It is of interest to note that Tsypkin in his researches on discrete systems was able to obtain formulae and algorithms that unified many concepts and relationships in the fields of adaptive, identification, and learning systems [40]. Thus, the discrete theory in the formulations mentioned had an impact on the continuous counterpart. Another impact of discrete time formulation is in the concept of Point Mapping, as discussed in the next section.

C. Point Mapping Applications

A method for tackling a class of systems governed by periodic differential equations is to formulate the motion of the system as events in discrete-time instants. By doing this, the dynamics of the system will be given in terms of difference equations rather than in terms of non-authonomous differential equations. Obtaining the corresponding difference equations (called Point mapping) can be carried out exactly for those systems of impulsive excitation, as done by Professor Hsu and noted in several important and timely publications [41 - 43]. Also, this method is closely related to the modified z-Transform when applied to certain nonlinear systems (relay systems), as discussed earlier.

It is of interest to note that Poincare [44] was the first to introduce the ideas of point mapping when he showed that the study of an n-dimensional dynamic system (autonomous case) is reduced to the study of an (n-1) dimensional autonomous mapping of the surface onto itself. Since then, point mapping has been pursued by many scientists and mathematicians throughout the world. In 1973, this writer was privileged to participate in a symposium on "Point Mapping and Its Applications," in Toulouse, France. The proceedings of this symposium have since been published, where applications to astronomy, particle physics, automatic control, applied mechanics and applied mathematics are discussed [45].

The advantage of the discrete-time approach in studies of periodic systems is that by introducing this formulation one eliminates the explicit dependence on time. This features allows for an easier determination of the various solutions and also an investigation of their stability properties and in particular the bifurcation and global regions of asymptotic stability phenomenon. Furthermore, searching for periodic solutions reduces to a search for a solution of a set of algebraic equations which is easier than looking for a function of time in a function space. Such ease of solution was also tackled by the modified z-Transform for linear continuous systems subjected to periodic inputs as discussed earlier. Another advantage for discretization is that difference equations are easy to use on digital computers.

In more general dynamic systems, it is difficult to obtain the exact difference equation, hence one has to resort to approximation methods in the mapping. Methods for obtaining such approximations are available [43], [46], where approximation to any desired degree is obtainable [47]. Hence, by examining the behavior of the difference equations one can obtain information on the behavior of the continuous systems.

To briefly illustrate the ideas of point mapping, consider a dynamical system governed by

$$\dot{x}(t) = F(t,x(t)) \tag{8}$$

where x is a real-valued N vector and F is a real valued vector function periodic in "t" of period one, but in general nonlinear in x. Suppose now that we are able to express the state of the systems at the end of the period in terms of the state at the beginning of the period. In that case we can recast the problem in the form of a system of difference equations

$$x(n+1) = G(x(n)], \quad n = \text{integer} \tag{9}$$

where G is a real-valued vector function of x(n). The task is to study the solution of the difference equation, and from this, to obtain the continuous-time history of the original equation (8). For a given F, the vector function G can be determined exactly for some cases and for others only approximately as mentioned earlier. In recent years the development of the above mapping has taken the form of the theory of "diffeomorphism," which is founded on topology and differential geometry [48]. Recently, Hsu [49] has advanced a theory of Index (similar to the Poincaré Theory of Index) for Point Mapping of dynamical systems of order two and higher. The importance of this theory lies in its use to help search and locate periodic solutions for the form given in (9).

IV. Continuous Theory as a Limiting Case of Discrete Theory as the Sampling Period Goes to Zero

In this section, the limiting case of discrete theory as the sampling period T goes to zero, is presented. In particular three cases will be discussed. The first is related to the stability of nonlinear systems, where Popov's Criterion [50] is obtained as a limiting case of the Jury-Lee Criterion [51] for discrete systems. The second is related to the Pontryagin Maximum Principle and the Discrete Maximum Principle. The limiting case in this discussion fails because of divergent methods of developing these principles. The third is related to stochastic systems. In this case, we will indicate both the validity and lack of it for obtaining the continuous theory as a limiting case of the discrete theory. Though the discussion in this section is brief, it will, nevertheless, point out the difficulties in attempting to obtain continuous theory from discrete theory by a limiting process and also to suggest the research needed to fill the gaps in this area. The reader is referred to the main text of the paper for the details.

V. Technological Advances in Discrete Systems

Interest in discrete systems is in great part motivated by the feasibility of practical applications. These applications have increased in number and importance due to technological advances in the field of digital computers.

The recent advances made in the field of microprocessors will eventually (if they have not already) revolutionize the control field and in particular process control. Also its impact on communication systems in the field of digital filters is of paramount importance. The real revolution is in the structural changes made possible by the availability of low cost computing power in small packages. Furthermore, the introduction of microprocessors is a big step toward reducing the theory-practice gap because of their flexibility in computing which permits the application of any control and learning algorithms to real processes.

The areas of application of the digital computer is quite exhaustive one cannot discuss them in detail in such a brief review. However, it is pertinent to mention the area of Direct Digital Control (DDC) which has been applied to several industrial processes. In this case, one can write digital controller programs, i.e. digital PID (Proportional, Integral, Derivative) algorithms, with more flexibility than analog "PID" control. The P, I, and D control gains can be made independent of each other, but, of course, subject to the length of the sampling period.

Since information is lost in the sampling process, it is natural that the digital version of the PID algorithms will not perform as well as the analog ones. On the other hand, digital "PID" control allows flexibility in the types of algorithms to be used, so that one may apply so-called "modern control theory" to fully exploit its software flexibility, while keeping the disadvantage of loss of information to a minimum. A case in point is the "stand-along digital controllers," introduced by Auslander-Tahahashi and Tomizuka, in their recent important paper [56]. In this far-reaching study, the authors have successfully applied both z-Transform theory and state-space methods in the analysis.

The impact of innovation in digital technology is now appearing everywhere, especially in the process instrumentation field. This trend will undoubtedly increase in importance and impact in this new decade.

Another area of practical development which affected discrete systems is in the "Switched-Capacitor Networks." Such networks are particularly attractive in the light of high circuit density possible with "MOS" integrated circuit technology and hybrid integrated circuits using thin-film and silicon technology.

Microprocessors have introduced a major perturbation in many fields and in particular the control field. They have opened the door to further innovative algorithms; and also, with distributed control, it is possible to "tinker" with part of a large process without disturbing the rest of the system. For a survey of results and literature in the area of decentralized control and estimation of large processes, the work of Siljak [57] is noteworthy. Hence, with the rapid introduction of these technological advances, we notice that the history of automatic control is undergoing another major change and certainly with better achievements than in the past.

VI. Recent Advances in Discrete System Theory

In recent years, we notice the emergence of concentrated research activities in two and multidimensional digital filters. Such filters, have found widespread applications in many fields, such as image processing, digitized photographic data, and geophysics for processing seismic, gravity and magnetic data. The advanced technology of recent years, especially in imagery, has motivated the theoretical study connected with these applications. The purpose of this section is to single out those theoretical advances made in recent years.

The theory of one dimensional digital filters is, of course, based mainly on the theory of the z-Transform and state-space techniques, the former of which was discussed in preceding sections. Several books have been written on this subject. It is of historical interest to note that in the survey of Jury-Tsypkin [5] a decade ago, the English version, Jury [5] had included a section on digital filters as one applications of discrete system theory, while in the Russian version of Tsypkin [58], this application was left out because it was considered then that digital filters were more-or-less the discrete compensators used in digital control systems. This indicates the rapid advances made in the field in the last decade.

The multidimensional theory, in contrast to the one dimensional theory, is mainly tackled extensively from the discrete version rather than the continuous one. Hence, we notice many new results in the discrete theory developed before its continuous counterpart.

The linear multidimensional discrete theory is mainly based on the multidimension z-Transform as follows:

$$Z[\{f(n,m,k,\ldots,\ell)\}] = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty}\sum \cdots \sum_{k=-\infty}^{\infty} \qquad (10)$$

$$f(n,m,k,\ldots,)z_1^{-n}z_2^{-m}z_3^{-k}\ldots$$

Historically, such forms have been used in probability theory as the multivariate generating function [59]. The theory of multidimensional z-Transform and modified z-Transform were discussed in the context of sampled-data systems studies. Several articles have been written on this transform [60-63] analogous to the multidimensional Laplace Transform [65].

Similar to the development of the theory for the one-dimensional case, multidimensional state space techniques have recently been advanced [64-67].

The extension of the theory from the one dimensional to the multidimensional case is not straightforward.

Many new concepts arise which considerably complicate the analysis and design. In a recent article by Jury [68], several counterexamples have been discussed which relate to extending the results from the I-D to the N-D case.

One major problem in the study of multidimensional digital filters is that of stability. This has been surveyed and discussed in detail by Jury [69]. Other related and important problems to multidimensional theory are discussed by Bose [70]. The relationships between digital signal processing and control and estimation theory are discussed by Willsky [71, 72]. Other applications to control theory are mentioned by Posthelwaite and MacFarlane [73,74].

In this brief survey, one cannot discuss all the above problems in detail. However, their availability in the literature would aid the reader to pursue this interesting and important area of recent research by mathematicians and engineers alike.

In the future, we will undoubtedly notice many application multidimensional theory to the control field, in particular multiparameter stability and distributed parameter control systems; this would enlarge the horizon of control theory and its applications.

VII. Conclusions

Having studied, taught and done research in the many facets of discrete systems during the past thirty years, I find it worthwhile to reflect upon the assess the work done in this area.

In this paper, I have commented on certain topics which dominated much of the activities in this field. In particular, the discussion of the impact of discrete theory on continuous theory in Section III is of importance. During the past decade, we notice the publications of many texts in control and allied fields which more and more emphasize the discrete theory. This trend will undoubtedly become more emphatic as new developments in digital technology emerge, as discussed in Section V.

The discussion in Section IV on the limiting cases indicates the need of some research in this area in order to further elucidate and overcome the difficulties which arise in certain situations. The need for modifying the optimality constraints in the Pontryagin maximum principle arises in order to obtain it as a limiting case from the discrete counterpart. Furthermore, the discussion of this section focuses on the fact that, in future years, it might be possible to treat continuous theory as a limiting case of the discrete theory, rather than in parallel as at present. Of course, such a perturbation in teaching automatic control theory will require more research work in this area.

The contents of Section VI bring into focus the new theoretical developments in the area of multidimensional discrete theory. This area is still in its infancy and much research work needs to be done to bring it to full fruition. In particular the impact of this theory on control systems is yet to be assessed.

My emphasis on the above three points is only a partial evaluation of the field and need not be construed as claiming that only those points are of importance. My discussion of these topics merely reflects my work and study in this area. Unlike, many of my contemporaries in the field of automatic control, I learned most of the continuous control theory from the discrete theory and not vice versa. This is reflected in my publications as well as in this write up.

I have been fortunate to have grown up professionally in the discrete system area and hope that my modest contributions in this field have been of some use to the profession and to the members of this Society.

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