The Composition of Heterogeneous Control Laws*

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Abstract

To design a control system to operate over a wide range of conditions, it may be necessary to combine control laws which are appropriate to the different operating regions of the system. The fuzzy control literature, and industrial practice, provide certain non-linear methods for combining heterogeneous control laws, but these methods have been very difficult to analyze theoretically. We provide an alternate formulation and extension of this approach that has several practical and theoretical benefits. First, the elements to be combined are classical control laws, which provide high-resolution control and can be analyzed by classical methods. Second, operating regions are characterized by fuzzy set membership functions. The global heterogeneous control law is defined as the weighted average of the local control laws, where the weights are the values returned by the membership functions, thereby providing smooth transitions between regions. Third, the heterogeneous control system may be described by a qualitative differential equation, which allows it to be analyzed by qualitative simulation, even in the face of incomplete knowledge of the underlying system or the operating region membership functions. An example of heterogeneous control is given for level control of a water tank and two alternate analysis methods are presented.

1 Introduction

Much control theory is based on linear models. This works very well for steady state regulation at a fixed operating point. To make a control system that can operate over wide regions it is however necessary to introduce nonlinearities. There are several ways to do this. Linear feedback control can be combined with logic for switching between several linear feedback laws. Selectors that choose between different control laws depending on signal levels can be introduced. Systems of these types are common in industry. Due to their complexity they are however poorly understood theoretically. Their design is based on engineering experience combined with extensive simulation.

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Fuzzy control [Zadeh, 1973; Mamdani, 1974] is another approach to obtain nonlinear control systems. In this approach the measured variables are represented as fuzzy variables. A representation of the control signal as a fuzzy variable is computed from the measurements using fuzzy logic. The fuzzy variable is converted to a real variable using some type of "defuzzification."

In this paper we take a new look at the problem of switching between different control strategies. A heterogeneous control problem is decomposed into multiple, possibly overlapping, operating regions. The domain of each operating region is characterized by a fuzzy set membership function. This makes it possible to express smooth transitions between adjacent regions. Each operating region is associated with a qualitative description of the system state, e.g. the low, normal, or high level of water in a tank. The fuzzy set membership functions may be regarded as a measure of the *appropriateness* of applying a given qualitative description to the system state. It is assumed that, for any given system state, the operating region membership functions sum to 1.0.

Each region is associated with a control law, and the control signal applied to the plant is a weighted average of the control signals for each region, where the weights are provided by the membership functions of each region.

This approach makes it possible to decompose the design of a heterogenous controller into two relatively independent decisions: (1) the specification of natural, qualitatively distinct operating regions, and (2) the specification of a control law for each region. The weighted sum combination method provides smooth transitions from one region to another, and facilitates local and global analysis. The idea of combining simple linear feedback units with operations such as average, min, max, etc, is widely used industrially. The intent of this paper is to provide a mathematical basis for the design of such systems, and for local and global analysis of their properties.

Heterogeneous control is also related to gain scheduling. There are however some differences. In gain scheduling a specific control law is selected for a given operating region and the parameters of the controller are changed with the region. In heterogeneous control the values of the control signal for different regions are computed and averaged.

Classical control theory [Franklin, et al, 1986] provides a rich set of methods for local analysis of the individual control laws and for describing their behavior. A global analysis of the behavior of a heterogenous control system is expressed as a transition graph, where the nodes correspond to operating regions and the directed edges correspond to possible transitions between regions. We provide a methodology for deriving this global analysis from the individual control laws and the membership functions of the operating regions, even on the basis of an incomplete, qualitative description of the structure of the system and its controller.

The global analysis methodology makes it possible to derive the assumptions under which a discrete transition-graph abstraction captures the essential properties of a continuous, heterogenous dynamic control system. It also identifies situations where a proposed abstraction may fail, and helps identify the additional constraints required to guarantee the desired global behavior.

The basic concepts of heterogeneous control will be introduced with a simple level controller for a water tank.

2 Qualitative Descriptions of Incomplete Knowledge

There are at least two fundamentally different types of qualitative descriptions of incomplete knowledge of scalar quantities.

- **"Fuzzy" Values** are qualitative descriptions without precise boundaries.
- **Landmark Values** are precise "natural joints" that break a continuum into qualitatively distinct regions.

2.1 The Fuzzy-Set Representation

Fuzzy sets were originally developed by Zadeh [Zadeh, 1965; Yager, et al, 1987] to formalize qualitative concepts without precise boundaries. For example, the level of water in a tank might be characterized by the qualitative descriptive terms, *low*, *normal*, and *high*. There are no meaningful landmark values representing the boundaries between *low* and *normal*, or *normal* and *high*.

Zadeh [1965] formalizes linguistic terms such as these as referring to *fuzzy sets* of numbers, in this case, levels of water in the tank. A fuzzy set, S, within a domain, D, is represented by a *membership function*, $s : D \rightarrow [0,1]$. For our purposes, we will interpret the value $s(x)$, for an element $x \in D$, as a measure of the *appropriateness* of describing x with the descriptor S. Figure 1 includes three membership functions defining the appropriateness of applying the qualitative descriptors $\{low, medium, high\}$ to quantitatively-defined levels.[1]

A *fuzzy controller* consists of a collection of simple control laws whose inputs and outputs are both fuzzy values [Zadeh, 1973; Mamdani, 1974]. For example,

If water level is *high*, then set drain opening to *wide*;

where *high* and *wide* are qualitative terms described by fuzzy sets over their quantitative domains.

All controller rules are fired in parallel, and the recommended actions are combined according to fuzzy value combination rules, weighted by the degree of satisfaction of the antecedent. Some process of "defuzzification" is required to convert the resulting fuzzy set description of an action into a specific value for a control variable.

[1] *Appropriateness measure* is technically synonymous with the terms *membership function* and *possibility measure* as used in the fuzzy research community. However, the English connotation of *appropriateness measure* seems better to capture the relationship between a linguistic term and a quantitative measure.

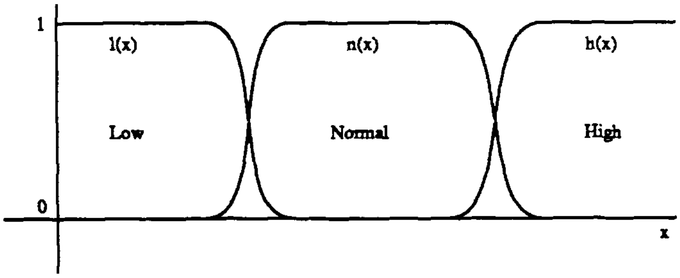


Figure 1: Appropriateness measures (i.e. fuzzy set membership functions) for qualitative terms describing levels.

2.2 Landmark-Based Representation

Frequently, qualitative categories are defined by *landmark values*: precise boundary points separating qualitatively distinct regions of a continuum. For example, water temperature can be described qualitatively with respect to the landmarks

$$\cdots\cdots Freezing \cdots\cdots Boiling \cdots\cdots$$

and angles in a triangle can be described in terms of the landmarks

$$Zero \cdots\cdots Right \cdots\cdots Straight.$$

A value can be described qualitatively either as equal to a landmark value or in the open interval between two landmark values, even when numerical information is unavailable. It is often easier to obtain or justify the qualitative description of a quantity than its numerical value, particularly when knowledge is incomplete. Fortunately, landmark-based descriptions support *qualitative simulation*, to derive qualitative descriptions of the possible behaviors of a system from a qualitative description of its structure [Kuipers, 1986].

An ordinary differential equation describes a system in terms of a set of variables which vary continuously over time, along with constraints such as addition, multiplication, and differentiation, on the relationships among those variables. A *qualitative differential equation* (QDE) describes a system in much the same terms, except that (1) the values of variables are described qualitatively, and (2) certain functional relationships between variables may be incompletely known and qualitatively described. For example, air resistance on a moving body increases monotonically with velocity, and flow of water through an orifice increases monotonically with pressure. Both of these relations are non-linear, but useful qualitative conclusions can be drawn purely from monotonicity. It is useful to define the class M^+ of monotonic functions, and the class S^+ of monotonic functions with saturation.

- M^+ is the set of continuously differentiable functions $f : \Re \rightarrow \Re$ such that $f'(x) > 0$ for all $x \in \Re$. In a QDE, we may write $M^+(pressure, outflow)$ or $outflow = M^+(pressure)$ to mean that there is some $f \in M^+$ such that $outflow = f(pressure)$. M_0^+ is the subset of M^+ such that $f(0) = 0$, and M^- is the set of f such that $-f \in M^+$.

- S^+ is the set of continuously differentiable functions $f : \Re \rightarrow \Re$ such that, for specified pairs of landmark values (x_1, y_1) and (x_2, y_2),
  - $f(x) = y_1$ for all $x \leq x_1$,
  - $f(x) = y_2$ for all $x \geq x_2$,
  - $f'(x) > 0$ for all $x_1 < x < x_2$.

  The turning points (x_1, y_1) and (x_2, y_2) must be specified as landmark values whenever the S^+ constraint is used. Notice, in figure 1, that the fuzzy set membership functions $h(x) \in S^+$ and $l(x) \in S^-$. We can also treat $n(x)$ as belonging to a composite set $S^+ \cdot S^-$. This qualitative description expresses a state of incomplete knowledge where we know only that the membership functions behave monotonically when they are between the landmarks 0 and 1.

Qualitative simulation using the QSIM algorithm [Kuipers, 1986, 1989] takes a QDE and a qualitative description of an initial state, and derives a tree of qualitative state descriptions, where the paths from the root to the leaves of the tree represent the possible behaviors of the system. This set of behaviors is guaranteed to include every solution of every ordinary differential equation consistent with the given QDE and initial state. Thus, a result which follows from a qualitative description of a system, must apply to every fully-specified instance of that description.

The tree of possible behaviors of a qualitatively described system can be a powerful analytical tool. In particular, if a qualitative property (e.g. stability or zero-offset) holds on every branch of the tree, it must hold for every behavior of the system. The importance of the *qualitative* level of description is that the tree of behaviors for a given QDE may be finite, whereas the corresponding set of ordinary differential equations and their solutions is typically uncountably infinite.

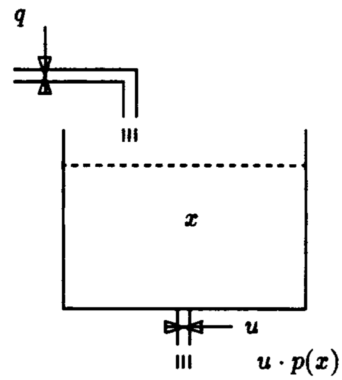
In particular, we will use fuzzy set membership functions to define a non-linear, heterogeneous controller with smooth transitions between operating regions. The system, the control laws, and the operating region definitions can all be represented as a qualitative differential equation, supporting analysis both by classical means and by qualitative simulation.

3 A Heterogeneous Controller for the Water Tank

3.1 The Water Tank

Consider control of the amount x of water in a tank, where the inflow rate q may vary, and the area u of the drain opening is the control variable. The function $p(x)$, representing the influence of x via pressure on outflow, is a monotonically increasing function of x; for a cylindrical tank, $p(x)$ is proportional to the square root of the pressure. The dynamic behavior of the system is described by:

$$\dot{x} = f(x, u) = q - u \cdot p(x).$$



$$\dot{x} = f(x, u) = q - u \cdot p(x).$$

Figure 2: The Water Tank

3.2 Overlapping Operating Regions

The system has separate control laws in three operating regions, *Low*, *Normal*, and *High*, with overlapping membership functions, as shown in figure 3.

Note that there is a "pure" region over the intervals $[0, a]$, $[b, c]$, and $[d, \infty)$, and overlapping regions on (a, b) and (c, d). We assume that the setpoint x_s is in (b, c).

The membership functions $l(x)$, $n(x)$, and $h(x)$, for the three operating regions are not known completely. All that is known is that they rise or fall smoothly and monotonically between their plateaus, where the boundaries of the plateaus are characterized by the landmark values, a, b, c, and d. They are normalized, so that $l(x)+n(x)+h(x) = 1$. This state of knowledge can be expressed in terms of the QSIM S^+ constraint by introducing the two functions,

$$s_1(x) \in S^+_{(a,0),(b,1)} \qquad s_2(x) \in S^+_{(c,0),(d,1)}$$

such that

$$\begin{aligned} l(x) &= 1 - s_1(x) \\ n(x) &= s_1(x)(1 - s_2(x)) \\ h(x) &= s_2(x) \end{aligned}$$

Because we specify the membership functions qualitatively, and depend only on properties of the qualitative class, the properties we derive apply to every member of the class.

3.3 Heterogeneous Control Laws

The control laws[2] for the three regions are:

$$u(x) = \begin{cases} 0 & \text{if } x \in Low \\ k(x - x_s) + u_s & \text{if } x \in Normal \\ MAX & \text{if } x \in High \end{cases}$$

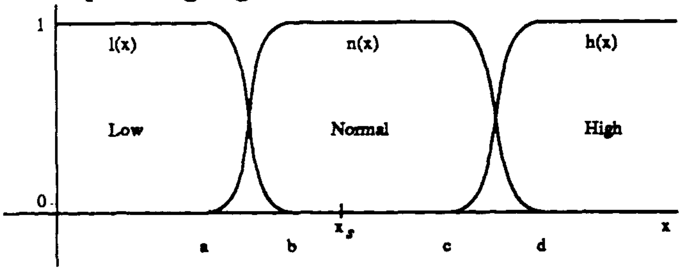
where the bias term u_s is adjusted to give the desired set point x_s for a nominal inflow q_s.

[2]We are assuming here that the state variable x is directly observable, rather than separating out measurements, $y = g(x, u)$. In that case, a measurable variable such as level is linearly, or at least monotonically, related to x, so omitting it simplifies the presentation without reducing generality.

The water tank:

x : the level in the tank (sensed)
u : the drain opening (controlled)

The operating regions:



The local control laws:

$$\begin{aligned} Low &\Rightarrow u_l(x) = 0 \\ Normal &\Rightarrow u_n(x) = k(x - x_s) + u_s \\ High &\Rightarrow u_h(x) = MAX \end{aligned}$$

The global control law:

$$\bar{u}(x) = l(x)u_l(x) + n(x)u_n(x) + h(x)u_h(x).$$

The discrete abstraction:

$$\boxed{\text{Low}} \longrightarrow \boxed{\boxed{\text{Normal}}} \longleftarrow \boxed{\text{High}}.$$

Figure 3: A heterogenous controller for the water tank.

The global heterogenous control law is the average of the individual control laws, weighted by the membership functions of their regions; hence

$$u(x) = l(x) \cdot 0 + n(x) \cdot [k(x - x_s) + u_s] + h(x) \cdot MAX.$$

Figure 3 summarizes the heterogeneous controller for the water tank.

4 Guarantees

We want to prove that the heterogeneous controller brings the system back to the *Normal* operating region under some range of disturbances, and that an equilibrium in the region is obtained for constant disturbances. More importantly, we want to determine any quantitative constraints on the design of the controller (e.g. the value for MAX), and the range of possible disturbances on q, that the controller can handle.

There are two methods for doing this (fig. 4), which are elaborated on below.

1. (a) Determine the qualitative behavior of the system within each operating region.
   (b) Combine the qualitative descriptions.
2. (a) Combine the local laws into a single global law using the weighted average combination rule.
   (b) Determine the qualitative behavior of the global system.

$$\begin{array}{ccc} & \text{Local} & \text{Global} \\ \text{Control Law} & \bullet \xrightarrow{\;2(a)\;} & \bullet \\ & \Big\downarrow 1(a) & \Big\downarrow 2(b) \\ \text{Behavior} & \bullet \xrightarrow[\;1(b)\;]{} & \bullet \end{array}$$

Figure 4: Two approaches to analyzing a heterogeneous controller.

4.1 Qualitative Combination of Local Properties

The direction of motion of the system as specified by each control law individually is determined first. The properties of the membership functions are not required for this analysis. Then, in the regions of overlap, if the directions of change agree, the global law for the heterogenous controller must give motion in the same direction. If the different control laws give motion in opposite directions, qualitative simulation provides the possible behaviors. Alternatively, order-of-magnitude or semi-quantitative analysis may be able to clarify the system's behavior in the overlap regions.

In order to guarantee that the system always ends up within the "pure" operating region, (b, c) of the *Normal* controller, we need to impose constraints on (1) the range of inflow perturbations to be handled, and (2) the magnitude of the *High* response.

1. From the *Normal* model:

   $$q_b \leq q \leq q_c$$

   where q_b (resp. q_c) is the value of q that results in steady state at $x = b$ (resp. $x = c$).

2. From the *High* model:

   $$q/MAX \leq p(c).$$

These conditions simply require that the drain area can be made sufficiently large so that the outflow at the desired level can be made to match the disturbance inflow. This guarantee does *not* depend on other constraints, in particular on the shapes of the membership functions.[3]

Once we have established these qualitative properties of the system and its heterogeneous controller, they can be expressed as a finite transition graph in which the nodes correspond to the operating regions.

$$\boxed{\text{Low}} \longrightarrow \boxed{\boxed{\text{Normal}}} \longleftarrow \boxed{\text{High}}$$

where the double box signifies that the *Normal* region includes a steady state, and so can persist indefinitely, while the other regions can persist only for a finite time.

The abstraction relation is defined as follows:

[3]The individual steps of this analysis can be established automatically by QSIM simulation of the individual controllers.

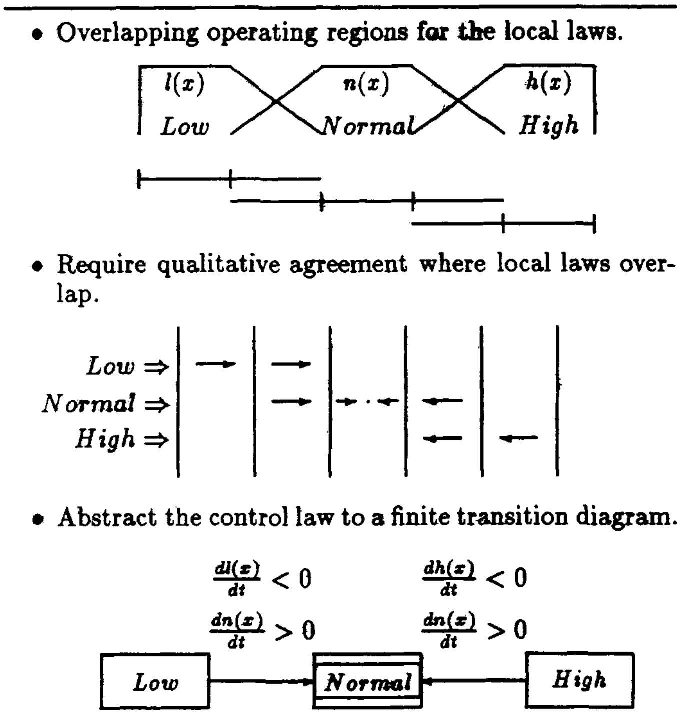


Figure 5: Qualitative combination of properties of local laws.

- The state of the system corresponds to a node of the transition graph if it is in the ***interior*** of the corresponding "pure" operating region, where its membership function is equal to 1.
- The links between nodes correspond to the overlap between operating regions. The system must move from one pure region to another in finite time.

Figure 5 summarizes this analysis.

4.2 Qualitative Analysis of the Global Control Law

A global analysis of the heterogeneous system is possible when we can establish suitable relations among the individual control laws.

Suppose we can establish that the global control law $u(x)$ is a monotonic function of x. Then the closed-loop system can be described as

$$\dot{x} = q - u(x)\,p(x) = q - f(x), \text{ for some } f \in M^+.$$

Since this is a first-order system, the analysis is straightforward. An equilibrium exists if q is in the range of f. The solution is unique since f is monotone. The solution is stable because $f' > 0$, since $f \in M^+$.

It is necessary to introduce some compatibility conditions in order to avoid pathological behavior of the system. To see this, consider the case where only two controllers are combined (e.g., the *Normal* and *High* controllers over the range (b, ∞) in the water-tank example). The control signal is then

$$u(x) = n(x)\,u_n(x) + h(x)\,u_h(x).$$

It is natural to have controllers such that

$$\frac{du_n}{dx} \geq 0 \text{ and } \frac{du_h}{dx} \geq 0.$$

Unfortunately, these conditions do not guarantee that u is monotone. To obtain this, some auxiliary conditions are required.

Consider

$$u' = n\,u_n' + n'\,u_n + h\,u_h' + h'\,u_h$$

$$\begin{aligned} n + h &= 1 \\ n' + h' &= 0 \end{aligned}$$

The problem is that n' is negative. However, we can conclude:

$$u' = n\,u_n' + h\,u_h' + h'(u_h - u_n)$$

This assures us that $u' > 0$, and hence that $f(x) = u(x)\,p(x)$ is in M^+, if we impose the natural condition

$$u_n(x) \leq u_h(x).$$

This condition needs to hold only for x where the two regions overlap. The argument obviously extends to more complex heterogeneous controllers, such as the water tank, where no more than two regions overlap at any point.

Consider the case where three regions overlap.

$$\begin{aligned} l + n + h &= 1 \\ l' + n' + h' &= 0 \end{aligned}$$

The analysis produces a similar result:

$$\begin{aligned} u' &= l\,u_l' + l'\,u_l + n\,u_n' + n'\,u_n + h\,u_h' + h'\,u_h \\ &= l\,u_l' + n\,u_n' + h\,u_h' + n'(u_n - u_l) + h'(u_h - u_l) \end{aligned}$$

Thus the constraint,

$$u_l(x) \leq u_n(x) \leq u_h(x),$$

guarantees that $u' \geq 0$ even when all regions overlap.

Notice that this constraint does not require that the local control laws be linear. Furthermore, a local control law needs to satisfy this constraint only where its membership function is non-zero.

5 Simulation Results

We can illustrate the performance of a heterogeneous controller on a water tank, in comparison with a proportional controller.

The capacity of the tank is 1000 liters of water. The nominal inflow rate is 100 liters/minute. The setpoint, x_s, is 700 liters. The offset u_s in the *Normal* control law u_n is set so that the steady state is at the setpoint when the inflow is nominal. The gain k is set so that $u_n(0) = 0$. The proportional controller simply uses u_n as the global control law.[4]

The operating regions for the HC controller are specified as in figure 3, with $a = 600$, $b = 650$, $c = 750$, $d = 800$, and $MAX = 50$.

Figure 6 shows the two control laws, and contrasts the behavior of the two controllers at constant nominal inflow, starting from initial states with the tank full and empty. Figure 7 shows the response of the two controllers to random variation in inflow.

[4]This comparison is for illustration only, since the proportional controller has an unrealistically low gain. With a higher gain, however, the physical limits on the valve make the proportional controller behave like a heterogeneous controller, but without smooth transitions or explicit design and validation.

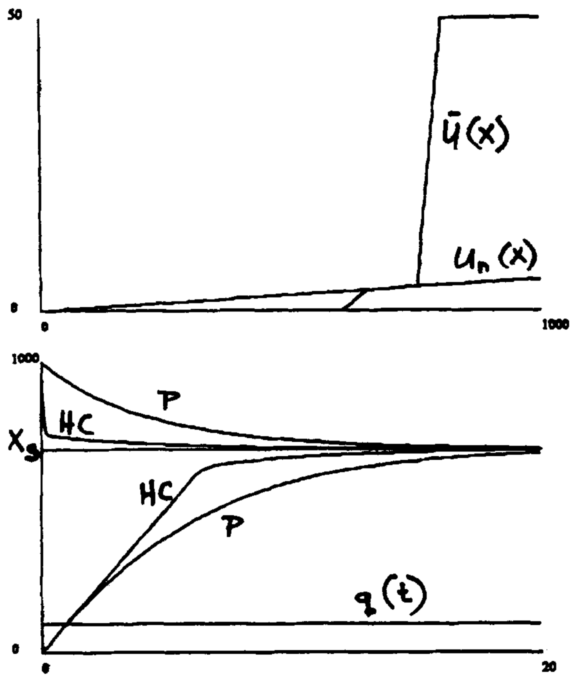


Figure 6: Comparison between P and HC controllers.

(a) The heterogeneous control law $\bar{u}(x)$, and the proportional controller $u_n(x)$ are identical in the Normal region.

(b) The behaviors, $x(t)$, of the P- and HC-controllers, starting with the tank empty or full, with constant q at the nominal rate, so that steady state is at the setpoint.

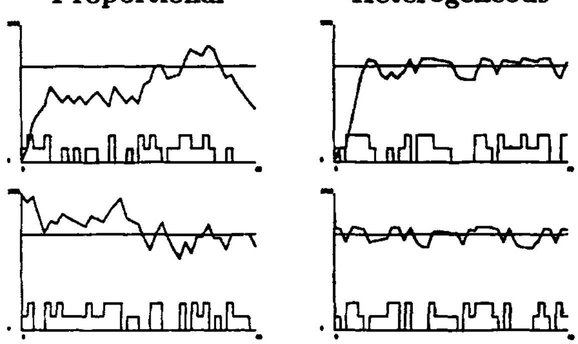


Figure 7: The effect of random inflow variation on P and HC controllers.
Inflow, plotted at the bottom of each graph, varies randomly between zero, nominal, and twice nominal. This figure shows the proportional controller (left column) and the heterogeneous controller (right column), with the tank initially empty (top row) or full (bottom row).

6 Integral Action

The bias term in the proportional controller was introduced to make it possible for the controller to keep the level at the set point. Integral action may be viewed as an automatic adjustment of the bias term [See Figure 2.2 in Åström and Hägglund, 1988]. For a simple PI controller the bias is generated as

$$T\frac{du_s}{dt} + u_s = ke + u_s$$

or

$$T\frac{du_s}{dt} = u_p = ke \qquad (1)$$

where u_p is the output of the PI controller, e the error $x - x_s$, k the proportional gain and T is the integration time. For a composite controller like the one used in heterogeneous control u_p should be replaced by the output of the heterogeneous controller.

Analysis of a controller with integral action is more complicated because the closed loop system is described by a second order differential equation and a simple monotonicity argument like the one used previously does not apply directly.

Just as there were two alternative approaches to the qualitative analysis of the heterogeneous "proportional" controller, there appear to be three basic approaches to analyzing the "integral" component of a heterogeneous controller.

1. The bias term u_s is adjusted, at a slower time-scale, by a heterogeneous P-controller as a function of the steady-state error, $x_s - x(\infty)$ as discussed below.
2. Local control laws, even with integral action, can be analyzed qualitatively, and associated with overlapping operating regions in the phase plane. If the directions of flow in the overlap regions are compatible, the qualitative descriptions can be combined into a discrete transition-graph representing behavior in the phase plane [Sacks, 1990].
3. The local laws may be combined into a global control law using the weighted average combination rule, which may then be analyzed qualitatively.

One possibility is to exploit the fact integral action is a slow process. The idea of time scale separation introduced in [Kuipers, 1987] can then be applied. The full details will be given elsewhere. Let us just outline the ideas of the reasoning. Provided that the integration time T is sufficiently small the closed loop system can be decomposed into a fast system, where the bias term is considered constant, and a slow system, where the fast system is considered as a static system. The previous analysis then applies to the fast system. It follows from this analysis that the level goes to an equilibrium which may be different from the set point. At equilibrium the fast system can be described by

$$u_p = -f(u_s) \qquad (2)$$

where the function f belongs to M^+. The slow system is described by (2) and (1), i.e.

$$T\frac{du_s}{dt} = u_p = -f(u_s)$$

Since f is monotone this equation has a unique stable equilibrium

$$u_p = ke = 0$$

which implies that the error e must be zero when the slow system reaches equilibrium.

7 Relationship to Fuzzy Control

Our approach to heterogeneous control shares many goals with, and draws much inspiration from fuzzy control [Zadeh, 1973; Mamdani, 1974]. First, both approaches provide the ability to express and use incomplete knowledge of the system being controlled and the control law itself. Second, both approaches allow one to specify a complex control law as the composition of simple components. Third, both use fuzzy set membership functions to provide smooth transitions from region to region.

However, there are important differences between our approach and fuzzy control. Within the framework of fuzzy control, it is difficult to exploit, or even relate to, the methods or results of traditional control theory. Our approach uses landmark-based qualitative reasoning to combine the benefits of fuzzy control with the analysis methods of traditional control theory.

Granularity. A fuzzy controller is typically specified as a relatively fine-grained set of (fuzzy) regions, with a constant (fuzzy) action associated with each region. Within our framework, the design for a controller specifies a smaller set of qualitatively distinct operating regions, but with a classical control law associated with each region.

The net result of these two differences is that an HC controller requires a simpler specification, while providing the higher-precision control characteristic of classical control laws.

Ontology. We do not treat "linguistic values" or "linguistic variables" as objects in either the domain or range of our functions. Rather, the fundamental objects in heterogeneous control are real-valued, continuously differentiable functions, and sets of such functions defined by qualitative constraints.

Linguistic terms are treated simply as names for the operating regions of the mechanism. The specifications for the operating regions are tested for soundness by the qualitative analysis methods.

"Defuzzification." The output of a fuzzy control law is typically a constant action with a fuzzy magnitude. The fuzzy magnitude must then be mapped to a real value for output. Since control laws in our framework are classical control laws, they provide real, not fuzzy, outputs, and "defuzzification" is not necessary.

Compatability. The concepts underlying fuzzy control are relatively difficult to map into the classical framework, making it difficult to exploit existing methods for providing guarantees for the properties of fuzzy controllers.

In HC control, the individual regions can be analyzed using classical methods, and we have demonstrated qualitative methods for combining analyses of the individual regions into a global analysis.

Specificity. Although both approaches have the goal of representing incomplete knowledge, fuzzy set membership functions must be represented as specific real-valued functions.

While an appropriateness measure in an HC controller must also be fully specified, the *analysis* of the controller relies only on a qualitative description (e.g. S^+, S^-, or $S^+ \cdot S^-$) of the measure. This makes explicit the fact that a single guarantee applies to a whole class of appropriateness measures, allowing additional degrees of freedom for implementation decisions. The goal of qualitative analysis is to define the least restrictive description of the controller which provides a given performance guarantee.

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