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Modeling, Simulation and Decentralized Control of a Nonlinear Coupled Tank System

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Abstract-**This work describes the modeling, simulation and decentralized control of a Coupled Three Tank System. The system is modeled mathematically using Bernoulli's law, mass balance and energy balance. Then the model is simulated in MATLAB using script file and in Simulink using S-functions. This system is linearized using Control and Estimation Tool Manager from Simulink model and using Jacobian method form mathematical model. The model is analyzed using step response, eigen values, real part of poles, number of Right Half Plane (RHP) zeros, Relative Gain Array (RGA) and controllability. The controllers are designed and tuned iteratively using Sequential Loop Closing (SLC). Controllers for flow are designed by specifying the closed-loop response while PI controllers for temperature are tuned iteratively. These controllers are also applied to nonlinear Simulink model where they give a good stability remarkable performance.**

Keywords-Sequential Loop Closing (SLC), Proportional Integral (PI), Relative Gain Array (RGA), Decentralized Control System (DCS)

I. Introduction

In industrial control applications, it is often required to control a MIMO distributed system. Such systems are preferred to control with separate controllers for each individual process and these controllers are linked together with some master controller rather than a central controller. So each controller controls a single process and master controller controls them. It has an advantage that each and individual loop can be controlled separately. In literature, this type of control systems in which control elements has its distributed nature and not central are called Distributed Control Systems. The condition for a master controller is not necessary always. These systems are commonly used in electrical and industrial engineering. A very special example of distributed control systems is a nuclear reactor used for serving mankind through energy production. In a nuclear reactor, all process has its individual controllers linked through a control room in which control is done manually and through computers. A number of control techniques like PID and Fuzzy Control and Neural Network control have been implemented on the 3-Tank system present in DEE PIEAS [1] but this system is not considered for Decentralized Control. The design, implementation, and testing of a distributed monitoring system for a Three-Tank System configuration is performed in [2] and that project was funded by NASA, but they did not consider the control of temperature.

In this work a coupled three tank system is taken as a plant. This system consists of three tanks, one reservoir, three pumps, seven on-off valves, seven level sensors, four temperature sensors and five flow meters. It gives the flexibility that the system can be configured for different experiments like level control or temperature control in one or more tanks or both level and temperatures simultaneously. Another flexibility is that delay can be introduced in both level and temperature control experiments.

In the present work, a laboratory three tank system is modeled, simulated, linearized and scaled for proper units. The controllers are designed and tuned iteratively using SLC. The controllers for flow are designed by specifying the closed-loop response while PI controllers for temperature are tuned iteratively. The complete sketch of system is shown below in fig (1-1).

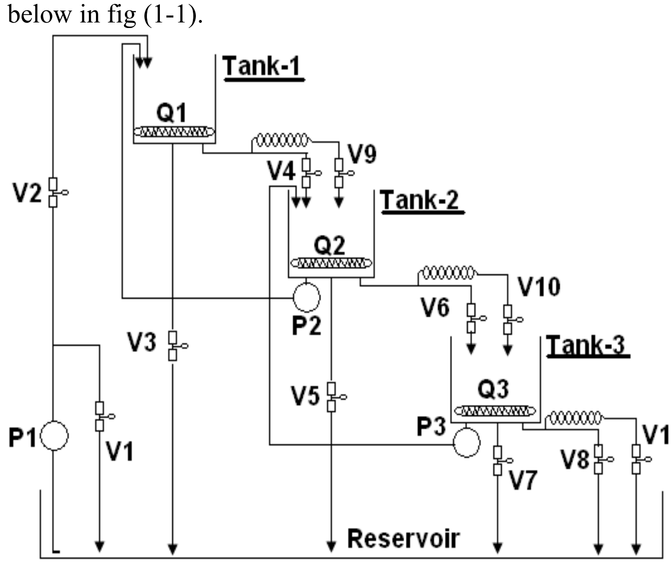

Figure I-1Schematic of Laboratory Coupled Three Tank System

Legends used are given below.

L_1, L_3, L_5 = Lower Level Switches for Tanks -1, 2 and 3 respectively.

L_2, L_4, L_6 = Higher Level Switches for Tanks -1, 2 and 3 respectively.

L_7 = Reservoir Low Level Switch.

P_1, P_2, P_3 = Pumps.

F_1, F_2, F_3, F_4, F_5 = Flow meters.

V_1 = Pump to reservoir valve.

V_2, V_4, V_6 =Solenoid Feed Valves for Tanks -1, 2 and 3 respectively.

V_3, V_5, V_7 = Manual Drain Valves for Tanks -1, 2 and 3 respectively.

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V_8=Solenoid Drain Valve for Tank- 3.
V_9, V_{10}, V_{11}=Solenoid Drain Valves through spiral for delays Tanks -1, 2 and 3 respectively.

II. MODELING AND SIMULATION

A. System Modeling

The system is considered to have six inputs. Three input voltages v_1, v_2 and v_3 are applied to three pumps and three heat inputs Q_1, Q_2 and Q_3 are considered due to three heaters. Three output levels due to height of water in each tank h_1, h_2 and h_3 and three outputs due to temperatures T_1, T_2 and T_3 for each tank are considered. Let d_1, d_2 and d_3 be the disturbances added to the system due to fall of water into tank-1, 2 and 3 respectively with constant gains K_{d1}, K_{d2} and K_{d3} respectively. Similarly let d4, d_5 and d_6 be the disturbances added to the system due to environment temperature difference into tank-1, 2 and 3 respectively with constant gains K_{d4}, K_{d5} and K_{d6} respectively. Let H_{11}, H_{12}, H_{21}, H_{22} and H_{31} are the vertical heights of pipe connected through valves V_3, V_4, V_5, V_6 and V_7 respectively. The system is modeled on the following basis.

(1) No thermal losses in the system.
(2) Perfect mixing in the system.
(3) No thermal delays.
(4) No pump delays.
(5) Hydraulic and friction losses of pipes and tanks are neglected.
(6) Flow out of a pump is directly proportional to voltage applied to it.
(7) Disturbances added to the system are multiple of some constant gains.

According to Bernoulli's law [3], the flow F_0 out of outlet orfine having area of cross-section a of a tank having height of liquid h is given by

$$F_o = a\times\sqrt{2\times g\times h}$$

Here g is gravitational acceleration. If V is a valve having value 0 or 1 according to it's ON or OFF state then the effect of valve can be included as follow.

$$F_o = V\times a\times\sqrt{2\times g\times h}$$

Consider a pipe ho height H is attached to orefin. We can also add a term H_l due to hydraulic losses to it as follow.

$$F_o = V\times a\times\sqrt{2\times g\times\;\; h+H+H_l}$$

As Hydraulic losses are considered small, so it can be neglected. So we neglect H_l according to our assumption. i.e

$$H_l \approx 0$$

So
$$F_o = V\times a\times\sqrt{2\times g\times\;\; H+h} \tag{2.1}$$

According to our assumption, if v is the applied voltage to a pump, then flow F_p out of a pump is given by

$$F_p = K\times v \tag{2.2}$$

Here K is constant of proportionality. Let A be the area of tank and h be the level of water in tank. Here the change of volume is related to level only. If F_i denotes the input flows and F_j denotes the output flows then according to mass balance we can write an equation about the rate of change of volume in a tank as follow.

$$A\times\frac{dh}{dt}=\sum_{i=1}^{n}F_i-\sum_{j=1}^{m}F_j \tag{2.3}$$

According to Energy Balance, rate of change of energy is equal to the energy of incoming flow minus energy of outgoing flow. Let ℓ be the density of water, C_p is the thermal capacity. Let T, T_i and T_j are the temperatures of tank, input flows and output flows respectively, then according to Energy Balance.

$$A\frac{d\;\; hT}{dt}=\sum_{i=1}^{n}F_iT_i-\sum_{j=1}^{m}F_jT$$

By expanding the derivative following expression is obtained.

$$AT\frac{dh}{dt}+Ah\frac{dT}{dt}=\sum_{i=1}^{n}F_iT_i-\sum_{j=1}^{m}F_jT \tag{2.4}$$

Putting equation (2.3) into (2.4) and after some short mathematics following general result as obtained

$$Ah\frac{dT}{dt}=\sum_{i=1}^{n}F_i\;\; T_i-T \tag{2.5}$$

By using Bernoulli's law, Mass Balance and Energy Balance, [3] following nonlinear equations are obtained. For Tank-1 level and temperature equations are

$$A_1\frac{dh_1}{dt}=\begin{bmatrix} -V_3a\sqrt{2g\;\; H_{11}+h_1}-V_4a\sqrt{2g\;\; H_{12}+h_1} \\ +V_2K_1v_1+K_2v_2+K_{d1}d_1 \end{bmatrix}$$

$$\frac{dT_1}{dt}=\begin{bmatrix} -K_2v_2\;\; T_1-T_2\;\; - \\ V_2K_1v_1\;\; T_1-T_0\;\; +\dfrac{Q_1}{\ell C_p} \end{bmatrix}\times\frac{1}{A_1h_1}+K_{d4}d_4$$

Similarly equations for Tank 2 can be found. For Tank-2 level and temperature equations are given below

$$A_2\frac{dh_2}{dt}=\begin{bmatrix} -V_5a\sqrt{2g\;\; H_{21}+h_2}-V_6a\sqrt{2g\;\; H_{22}+h_2} \\ +K_3v_3-K_2v_2+V_4a\sqrt{2g\;\; H_{12}+h_1}\;\; +K_{d2}d_2 \end{bmatrix}$$

$$\frac{dT_2}{dt}=\begin{bmatrix} -K_3v_3\;\; T_2-T_3 \\ -V_4a\sqrt{2g\;\; H_{12}+h_1}\;\; T_2-T_1 \\ +\dfrac{Q_2}{\ell C_p} \end{bmatrix}\times\frac{1}{A_2h_2}+K_{d5}d_5$$

Similarly equations for Tank 3 can be found. For Tank-3 level and temperature equations are

$$A_3\frac{dh_3}{dt}=\begin{bmatrix} -V_7a\sqrt{2g\;\; H_{31}+h_3}-K_3v_3 \\ +V_6a\sqrt{2g\;\; H_{22}+h_2}\;\; +K_{d3}d_3 \end{bmatrix}$$

$$\frac{dT_3}{dt} = \begin{bmatrix} -V_6 a\sqrt{2g \;\; H_{22} + h_2} \\ \times \;\; T_3 - T_2 \;\; + \dfrac{Q_3}{\ell C_p} \end{bmatrix} \times \frac{1}{A_3 h_3} + K_{d6} d_6$$

B. Model Simulation

First the model is simulated in MATLAB using script file. Then model is simulated in Simulink using S-function. An S-function is used to model a dynamic system in SIMULINK using MATLAB programming. Three things are modeled in that work.

(1) Differential Model of Three tank System.
(2) Logic Levels for upper and lower level of each tank.
(3) Implementation of dead zone due to pumps.

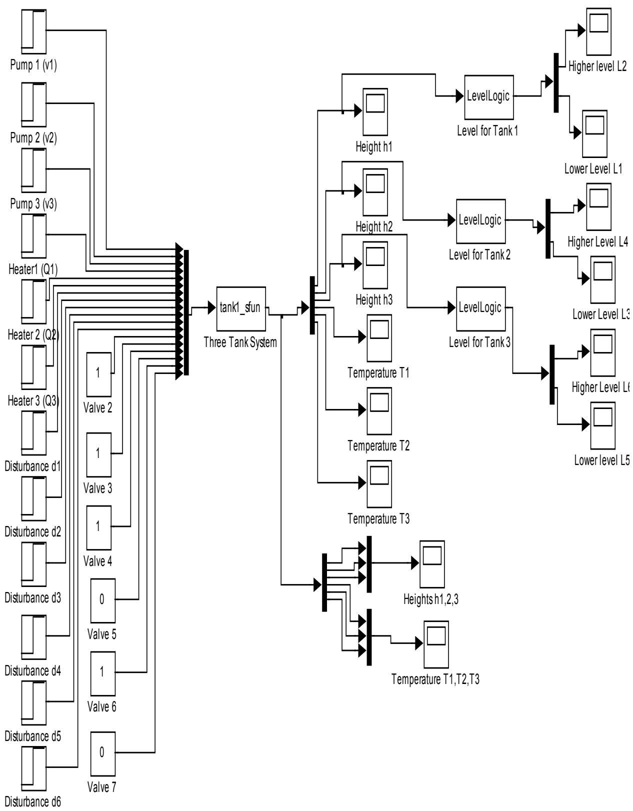

Figure II-1 Simulink Model Implementation of Coupled Tank System

When a voltage is applied to a pump, some of power is used to lift the pump. If the applied voltage is small then pump can not transfer the water up to the tank because in this case power will be small as compared to power required for the lift of water to a tank. So there exists a dead zone related with every pump. Hence each pump must operate within a lower and upper level of input voltage due to dead zone and upper threshold of pump. That is

$$v_{Lower} \leq v_{Operation} \leq v_{Upper}$$

The model in Simulink is considered to show the dead zone. S-Function first take the inputs from Simulink, then check it for the Dead Zone. Dead zone is implemented just by an if-condition. Then after passing through the dead zone, inputs are passed to non linear differential equations. The equations are solved recursively to create a signal for all iteration values. The temperatures are returned to Simulink while levels are checked for upper and lower level logics and then returned to the Simulink. The Simulink model is shown in fig (2-1).

III. MODEL LINEARIZATION AND ANALYSIS

A. Linearization

The system is linearized using Jacobian Method and results are verified using MATLAB Control and Estimation Tool Manager. Constants are defined in CGS system and input output scaling is performed to make a simple model and to avoid complicated controllers. The operating point is taken as $[h_1, h_2, h_3, T_1, T_2, T_3] = [15.24\text{ cm}, 12.7\text{ cm}, 10.16\text{ cm}, 40\ ^oC, 50\ ^oC, 55\ ^oC]$. Following model is obtained.

$$A = \begin{bmatrix} -0.0063 & 0 & 0 & 0 & 0 & 0 \\ 0.0063 & -0.0064 & 0 & 0 & 0 & 0 \\ 0 & 0.0064 & -0.006 & 0 & 0 & 0 \\ -0.0081 & 0 & 0 & -0.0173 & 0.0086 & 0 \\ -0.0049 & 0.0197 & 0 & 0.0450 & -0.0554 & 0.0104 \\ 0 & -0.0032 & 0.0086 & 0 & 0.0547 & -0.0547 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0110 & 0.0110 & 0 & 0 & 0 & 0 \\ 0 & -0.0110 & 0.0110 & 0 & 0 & 0 \\ 0 & 0 & -0.0110 & 0 & 0 & 0 \\ -0.0072 & 0.0072 & 0 & 0.1238 & 0 & 0 \\ 0 & 0 & 0.0043 & 0 & 0.1485 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1857 \end{bmatrix}$$

$$C = I_6$$

$$D = 0_{6\times 6}$$

B. Analysis

Model is analyzed using step response, eigen values, real part of poles, number of RHP zeros, RGA and controllability. The step response, negative real parts of poles and eigen values shows that the model is stable. The determinant of the MIMO model also exists for the required frequency range so system is controllable. The RGA at initial frequency can be found as follow [4].

$$RGA = G(jw = 0) \times (G^{-1}(jw = 0))^T \qquad (3.1)$$

The RGA calculated for our system is shown here.

$$RGA = \begin{bmatrix} 0.0000 & 1.0000 & 0 & 0 & 0 & 0 \\ -0.0000 & 0.0000 & 1.0000 & 0 & 0 & 0 \\ 1.0000 & 0.0000 & 0.0000 & 0 & 0 & 0 \\ 0.0000 & -0.0000 & -0.0000 & 2.0000 & -1.0000 & -0.0000 \\ 0.0000 & -0.0000 & -0.0000 & -1.0000 & 2.4602 & -0.4602 \\ -0.0000 & 0.0000 & -0.0000 & -0.0000 & -0.4602 & 1.4602 \end{bmatrix}$$

The Control strategy using RGA is shown in Table (I).

Frequency dependent RGA is very vital to analyze the system and can be found by using following relation [4], [5].

$$RGAfreq. = G(jw) \times (G^{-1}(jw))^T \qquad (3.2)$$

TABLE I
CONTROL STRATEGY FOR COUPLED TANK SYSTEM

Output	**Input**
1 (Level 1)	2 (Pump 2)
2 (Level 2)	3 (Pump 3)
3 (Level 3)	1 (Pump 1)
4 (Temperature in tank 1)	4(Heater 1)
5 (Temperature in tank 2)	5 (Heater 2)
6 (Temperature in tank 3)	6 (Heater 3)

RGA do not change sign, so there are no RHP zeros in plant. Also RGA are not going larger, so plant is not ill conditioned. The plots are shown here in fig (3.1) and (3.2).

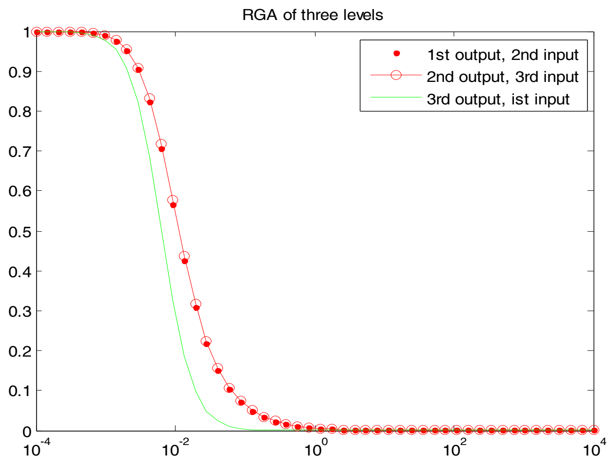

Figure 3-1 RGA of Three Levels for the Linearized Model

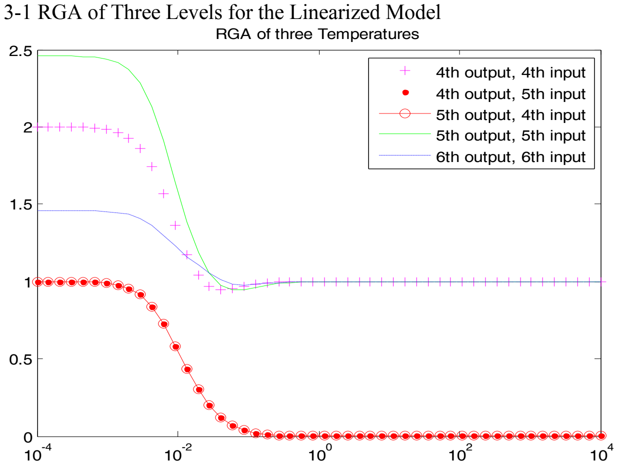

Figure 3-2 RGA of Temperatures for the Linearized Model

The frequency dependent RGA number can be found for different pairings say ‘m’. The best pairing gives the lowest plot of RGA number. Here a plot is shown for three control strategies in fig (4). The best selected pairing has given the lowest plot. The plots are taken by using the following relation [4], [5].

$$RGAno. = \left\| G(jw) \times (G^{-1}(jw))^{T} - m_k \right\|_{SUM}$$

Where k=1, 2, 3…… (Up to Number of control strategies tested).

The plots are shown here. The lowest one is best for control, strategy.

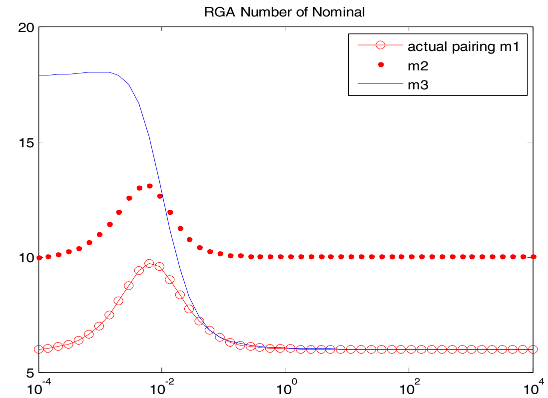

Figure 3-3 RGA number for Different Control Strategies

IV. DECENTERILIZED CONTROLLER DESIGN

Decentralized control [6], [7] is a special form of distributed control. For a MIMO system, if we have diagonal controller then it is called decentralized controller. Hence each controller in diagonal is used for an output input pair. As control of level is independent of control of temperature (but reverse is not true). So techniques are applied to control Level-1 and Level-2 using the Controllers ‘K_1’ and ‘K_2’ with Pump-2 and Pump-3 respectively. The scheme for control is shown in fig (4-1).

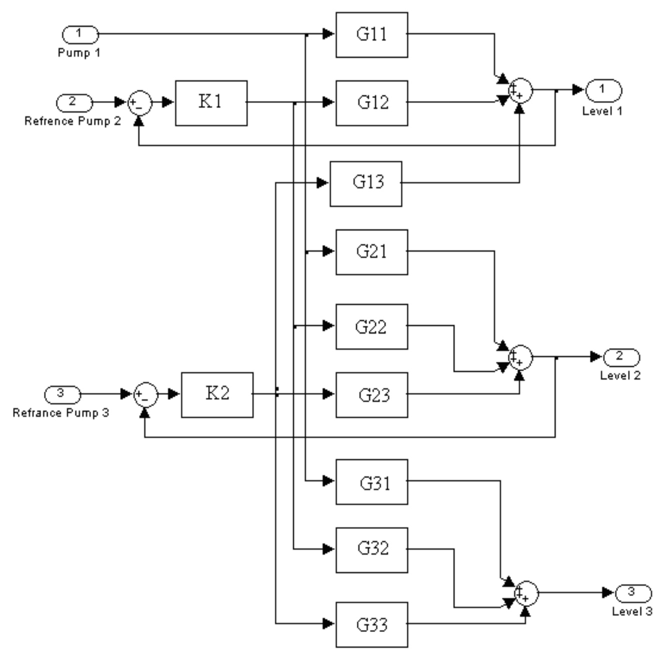

Figure 4-1 Level Control Block Diagram for the System

The transfer functions ‘G_1’ and ‘G_2’ as seen by ‘K_1’ and ‘K_2’ controller are derived by seeing the figure are as follow [6], [7]. Here is a need to solve for the controllers iteratively.

$$G_1 = G_{12} - \frac{G_{22}K_2G_{13}}{1+K_2G_{23}} \tag{4.1}$$

$$G_2 = G_{23} - \frac{G_{13}K_1G_{22}}{1+K_1G_{12}} \tag{4.2}$$

It is seen that to design ‘K_1’ we require ‘K_2’ and similarly to design ‘K_2’ we require ‘K_1’. So we can solve such controller problems iteratively. Let 'Cl_1' and 'Cl_2' be the required close loop responses for controlling Levels-1, 2 respectively. So we can write them as [8], [9]

$$Cl_1 = \frac{G_1 K_1}{1 + G_1 K_1} \tag{4.3}$$

$$Cl_1 = \frac{G_1 K_1}{1 + G_1 K_1} \tag{4.4}$$

The relations can be derived for both controllers as follow

$$K_1 = \frac{Cl_1}{G_2(1 + Cl_1)} \tag{4.5}$$

$$K_2 = \frac{Cl_2}{G_2(1 + Cl_2)} \tag{4.6}$$

By choosing close loop responses, controllers can be designed. Hit and trail methodology is used to tune the controllers so that they give batter responses. To control the temperatures, Sequential Design Method is used [7]. Two PI-Controllers are tuned iteratively for this purpose. The controllers 'PI_1' and 'PI_2' will se the responses as follow.

$$G_4 = G_{44} - \frac{G_{54} PI_2 G_{45}}{1 + PI_2 G_{55}} \tag{4.7}$$

$$G_5 = G_{55} - \frac{G_{45} PI_1 G_{54}}{1 + PI_1 G_{44}} \tag{4.8}$$

Finally the controllers are given to control the h1, h2, T1 and T2 respectively.

$$K_1 = \frac{3.041\ s + 0.01901}{s^2 + 1.033\ s}, \quad K_2 = \frac{-2.28\ s - 0.02893}{s^2 + 1.025\ s}$$

$$PI_1 = \frac{10\ s + 0.5}{20\ s}, \quad PI_2 = \frac{12\ s + 0.5}{24\ s}$$

It is observed that changing of one reference does not affect the settled operating points (steady state) of the other controlled outputs. Here the reference of level 1 is changed but it does not affect the steady state of others. Change in one level causes a disturbance to other but finally it is settled to same reference. Here it is seen that when h2 is at their steady state, the change in reference of h1 causes a disturbance but its steady state does not change. This is shown in fig (4-2).

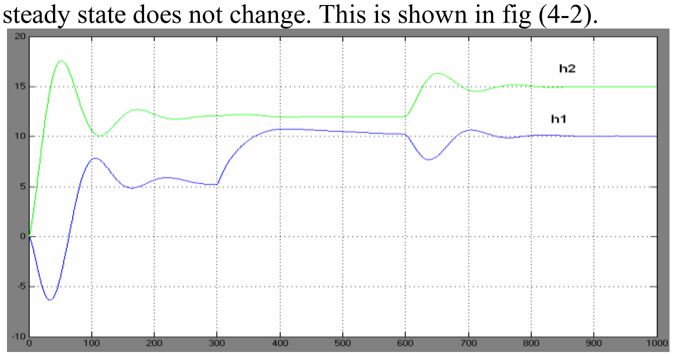

Figure 4-2 Levels Tracking the Reference for Decentralized Control

Similarly change in one temperature causes a disturbance to other but finally it is settled to same reference. Here we see that when T2 is at their steady state, the change in reference of T1 causes a disturbance but its steady state does not changed. Hence the finally designed controllers are distributed controllers and give good results but these are designed without including robustness in the model as shown below in fig (4-3).

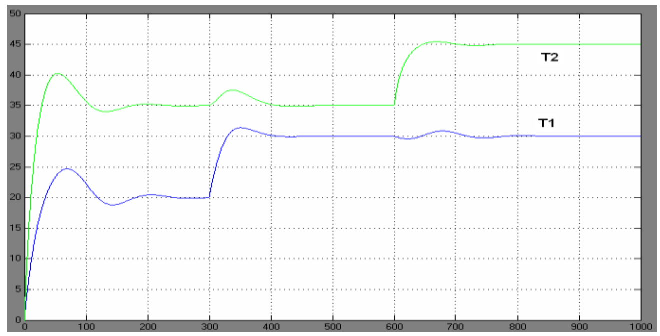

Figure 4-3 Temperatures Tracking the Reference for Decentralized Control

V. CONTROLLER IMPLEMENTATION ON NON-LINEAR MODEL

Till now the controllers are designed for a linear model. But actual model is nonlinear. So it is required to check these controllers on non-linear model as shown in fig (5-1).

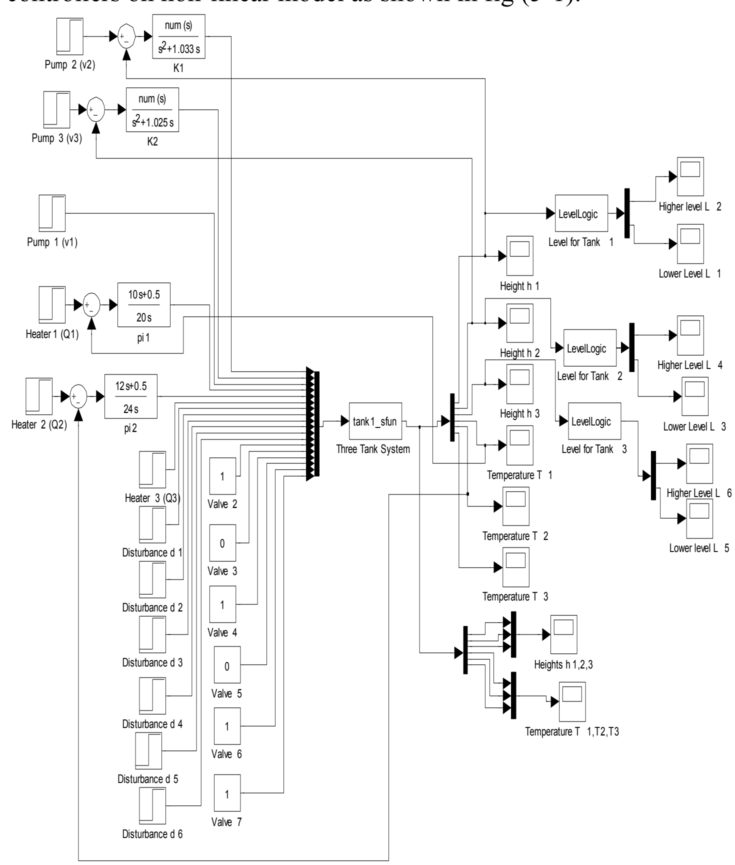

Figure 5-1 Nonlinear Close Loop Model

The step responses of close loop nonlinear system for h1, h2, T1 and T2 are shown here in fig (5-2), (5-3), (5-4) & (5-5). The close loop responses are stable. The level h1 has no overshoot while h2, T1 and T2 has small overshoots.

Nonlinear system is following the operating point. The rise time of h1 is little bit higher otherwise the rise time of h2, T1 and T2 are better. The reason is that it is required to control h1 with very little overshoot so that it can not cross the height of tank-1. In SLC, overshoot of h1 is improved at the cost rise time.

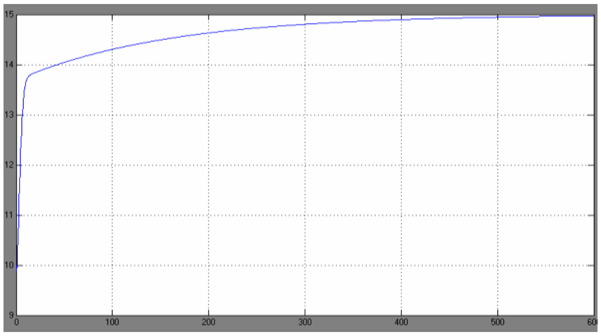

Figure 5-2 h1 step response of nonlinear close loop model

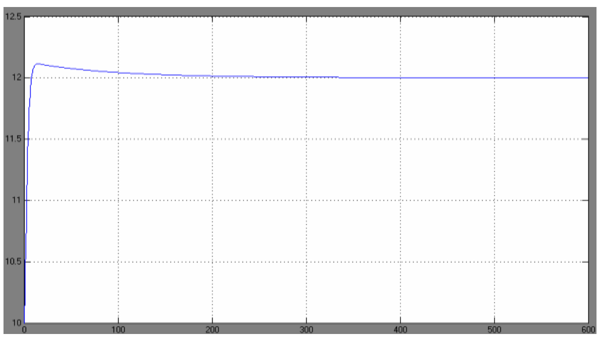

Figure 5-3 h2 step response of nonlinear close loop model

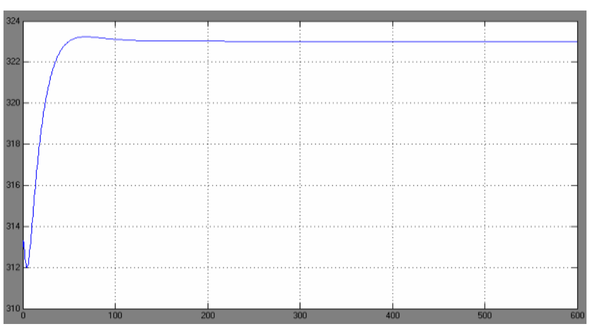

Figure 5-4 T1 step response of nonlinear close loop model

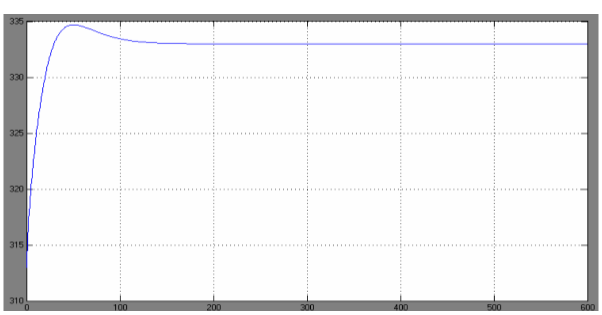

Figure 5-5 T2 step response of nonlinear close loop model

VI. CONCLUSION

In this present work, a coupled three tank system is considered as a distributed MIMO system. The system is modeled using Bernoulli's Law, Mass Balance and Energy Balance for levels and temperatures. The model was simulated in MATLAB using script file and in Simulink using S-Function. The nonlinear model was linearized using Control and Estimation Tool Manager and Jacobin Method and results are compared. The model is analyzed for stability and best control strategy. Controllers are designed using SLC and by solving close loop response .Finally a decentralized controller is designed for four states of h1, h2, T1 and T2. This controller is also used in non-linear model to check the performance of the controller. A good performance is achieved for the nonlinear model in neighbor hood of operating point. Overshoots are reasonably small. Rise time are also very good except h1 which also not bad. The overshoot of h1 is improved at the cost of its rise time.

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