Genetic-based Fuzzy Sliding Mode Control of an Interconnected Twin-Tanks

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Abstract—**In this paper, an improvement of sliding mode control design is investigated. A fast reaching velocity into the switching hyperplane in the hitting phase and little chattering phenomena in the sliding phase is desired. A Fuzzy Sliding Mode Control (FSMC) in cooperation with Genetic Algorithms (GAs) in coupled tanks problem is studied. A fuzzy logic controller is used to replace the discontinuity in the signoum function of the reaching law in the Sliding Mode Control (SMC). Parameters of FSMC are adjusted by GAs. Finally, the performance and the significance of the controlled system are investigated under variation in system parameters and also in presence of an external disturbance. The simulation results indicate performance of genetic-based FSMC controller.**

Keywords—**Nonlinear control, Sliding Mode control, Fuzzy logic, Genetic Algorithms, Coupled Tank**

I. INTRODUCTION

During the last three decades, Variable Structure Systems (VSS) and Sliding Mode Control (SMC) have received significant interest and have become well-established research areas with great potential for practical applications. Due to its good robustness to uncertainties, sliding mode control has been accepted as an efficient method for robust control of uncertain systems. Being limited only by practical constraints on the magnitude of control signals, the sliding mode controller, in principle, can treat a variety of uncertainties as well as bounded external disturbances. The SMC guarantees the stability and robustness of the resulting control system, which can be systematically achieved but at the cost of chattering effect. Unfortunately, an ideal sliding mode controller inevitably has a discontinuous switching function. Due to imperfect switching in practice it will raise the issue of chattering, which is highly undesirable. To suppress chattering in sliding mode controllers several techniques have investigated in the literatures. A second order sliding mode control algorithm for a class of MIMO nonlinear systems in input-output (I/O) form is proposed in [1]. In [2] a static and two dynamic sliding mode control schemes for the coupled tanks system are proposed. A Chattering-free fuzzy sliding-mode control strategy is given in [3], which can control the uncertain chaotic behaviors to a desired state without oscillator very fast; also the switching function is smooth without chattering. Also a novel adaptive fuzzy PI Sliding Mode Control is proposed in [4]. In [5] a parameter selection method by using the genetic algorithms in the sliding mode control is given which can increase the speed of system response in the reaching phase and reduce the chattering in the sliding phase. A fuzzy sliding mode control for nonlinear system is proposed in [6] and in [7] a fuzzy sliding mode based on genetic algorithms for the tracking problem of two-degree-of-freedom rigid robot manipulator is given. The development and application to experimental equipment of fast constrained predictive control algorithms based on feedback linearization for coupled-tanks apparatus is proposed in [8].

Chattering can be made negligible if the width of the boundary layer is chosen large enough; the guaranteed tracking precision will deteriorate if the available control bandwidth is limited.

To reach a better compromise between small chattering and good tracking precision in the presence of parameter uncertainties, various compensation strategies have been proposed.

To tackle these difficulties, a fuzzy logic control is applied to deal with the discontinue sign function in the reaching law in SMC. The fuzzy logic control (FLC) schemes have widely been developed for almost 40 years, and have been successfully applied to many applications.

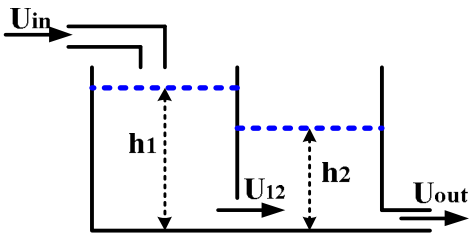
The objective of this paper is to propose a genetic-based fuzzy sliding mode controller for coupled Tanks which has been introduced as an example of a nonlinear system. A set of the fuzzy linguistic rules based on expert knowledge are used to design the switching control law of FSMC. The FSMC is a hybrid controller, which combines the advantages of the fuzzy controller and the sliding mode controller. FSMC Parameter determination and optimization procedure have been performed by GA.

Simulation results illustrate the effectiveness of the proposed controller.

II. INTERCONNECTED TWIN-TANKS MODEL

The control of liquid level in tanks and flow between tanks is a basic problem in the process industries. Process industries require liquid to be pumped, stored in tanks, and then pumped to another tank. Level and flow control in tanks are at the heart of all chemical engineering systems.

The twin-tank system consists of two small tanks coupled by an orifice and a pump that supplies water to the first tank. The pump only increases the liquid level and is not responsible for pumping the water out of the tank. It is assumed that the back pressure created by the water-head does not affect the flow rate of the pump significantly [1]. The twin-connected tanks system is a nonlinear dynamical system which the governing dynamical equations can be written as [1], [2]:



Tank 1 Tank 2

Fig.1. Schematic of Interconnected Twin-Tanks

$$\dot{h}_1 = (U_{in} - U_{12})/A$$
$$\dot{h}_2 = (U_{12} - U_{out})/A \tag{1}$$

Where

$$U_{12} = a_{12}\sqrt{2g(h_1 - h_2)} \quad \text{for } h_1 > h_2 \tag{2}$$
$$U_{out} = a_2\sqrt{2gh_2} \quad \text{for } h_2 > 0$$

Where h_1 and h_2 are the total water heads in Tank 1 and Tank 2, respectively, U_{in} is the inlet flow rate, U_{12} is the flow rate from Tank 1 to Tank 2, A is the cross-section area of Tank 1 and Tank 2, a_{12} is the area of the coupling orifice, a_2 is the area of the outlet orifice and g is the gravitational constant. Moreover $U_{in} \geq 0$ means that pump can only force water into the tank. Let

$$z_1 = h_2 > 0 \ , \ z_2 = h_1 - h_2 > 0$$
$$c_1 = a_2\sqrt{2g}/A \ , \ c_2 = a_{12}\sqrt{2g}/A$$

Also the output of the coupled tanks is taken $h_2(t)$, hence the dynamic model in (1) and (2) can be written as:

$$\dot{z}_1 = -c_1\sqrt{z_1} + c_2\sqrt{z_2}$$
$$\dot{z}_2 = c_1\sqrt{z_1} - 2c_2\sqrt{z_2} + U_{in}/A \tag{3}$$
$$y_1 = z_1$$

Then the goal is to regulate the system output ($h_2(t)$) to the desired value (H) using a Sliding Mode Controller.

III. SLIDING MODE CONTROLLER DESIGN

Sliding Mode Control is considered to be a robust control methodology and therefore able to handle changes in the plant and external disturbances without significant performance degradation. The structure of a Sliding Mode controller is composed of a nominal part plus an additional term aimed at providing additional control effort for dealing with disturbances. The control problem is to make the system response track a specified and desired trajectory utilizing a Sliding Mode controller.

Sliding Mode control is a robust nonlinear Lyapunov-based control algorithm in which an *n*th order nonlinear and uncertain system is transformed to a 1st order system. Consider a SISO system [9]:

$$x^{(n)} = f(x) + b(x)u \tag{4}$$

Where u is the scalar input and x is the state vector. For this system the nonlinear functions f and b are not exactly known. The control objective is state must follow a desired vector trajectory $x_d(t)$. Furthermore consider the surface S(t) in the state-space:

$$S(x,t) = (d/dt + \lambda)^{n-1}\tilde{x} \tag{5}$$

Where $\tilde{x} = x - x_d$ is the tracking error and λ is a positive constant. It can be shown that tracking of the desired state vector is equivalent to keep them remaining on the surface S(t). Therefore the control law should be selected in a way that the distance to this surface decreases along with all system trajectories (sliding condition).

$$\frac{1}{2}\frac{d}{dt}S^2 \leq \eta S \tag{6}$$

Where η is a positive constant. As a better result, all the trajectories are forced to reach the sliding surface in finite time and stay on the surface for all future times. S(t) is called the sliding surface and the systems behavior once on the surface is called sliding mode ($\dot{s} = 0$). It can be shown that the reach time from the initial state to the surface is

$$t_{reach} \leq S(t=0)/\eta \tag{7}$$

Thus a typical motion under sliding mode control consists of reaching phase and sliding phase during which the motion is confined to the sliding surface [9].

Consequently, the aim is to design a Sliding Mode Controller as a height regulator for the coupled Tank in Fig.1 with the dynamic model in (3). Let H be the desired constant value of the output of the system ($h_2(t)$), the sliding mode controller as follows:

$$S = \dot{z}_1 + \lambda(z_1 - H) \tag{8}$$

By taking the time derivative of both sides of (8), one can obtain

$$\dot{S} = \ddot{z}_1 + \lambda\dot{z}_1 \tag{9}$$

Using (3)

$$\dot{S} = (-c_1\dot{z}_1/2\sqrt{z_1}) + (c_2\dot{z}_2/2\sqrt{z_2}) + \lambda\dot{z}_1 \tag{10}$$

Again using (3) into (10), it follows that:

$$\dot{S} = (c_1^2 - 2c_2^2)/2 + (c_1c_2/2)(\sqrt{z_1}/\sqrt{z_2} - \sqrt{z_2}/\sqrt{z_1}) + (c_2/2A\sqrt{z_2})U_{in} + \lambda(-c_1\sqrt{z_1} + c_2\sqrt{z_2}) \tag{11}$$

The sliding mode controller from (11) can be obtained:

$$U_{in} = (2A\sqrt{z_2}/c_2)[-(c_1^2 - 2c_2^2)/2 - (c_1c_2/2)(\sqrt{z_1}/\sqrt{z_2} - \sqrt{z_2}/\sqrt{z_1}) - \lambda(-c_1\sqrt{z_1} + c_2\sqrt{z_2}) - K\,\mathrm{sgn}(S)] \tag{12}$$

Where K is the positive switch gain and

$$\mathrm{sgn}(S) = \begin{cases} +1 & if \ \ S > 0 \\ 0 & if \ \ S = 0 \\ -1 & if \ \ S < 0 \end{cases} \tag{13}$$

Using (12) in to (11), results

$$\dot{S} = -K\,\mathrm{sgn}(S) \tag{14}$$

The system states are now approaching to the hyperplane, and the error vector asymptotically reduces to zero once the system states reaches S=0, and $h_2(t)$ will asymptotically converges to its desired value H. Hence the static sliding mode controller guarantees the asymptotic convergence of the output to its desired value. If the control law U_{in} is chosen as (12), chattering phenomena will occur after the first time state hits the sliding surface.

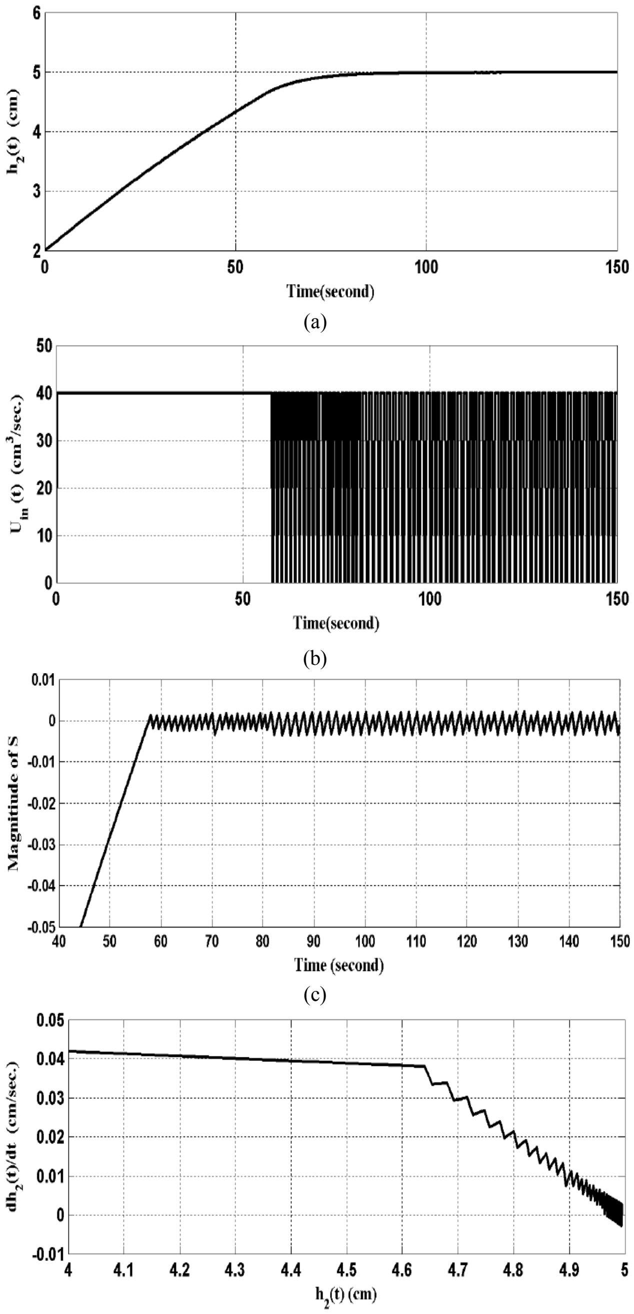


Fig.2. (a): Water level in Tank2 (b): The inflow rate into Tank1 (c): The time response of S (d): The state trajectory in the phase plane using SMC.

An example is presented to clarify the discussion. Let A= 200 cm^2, a_2= 0.25 cm^2, a_{21}=0.6 cm^2, g= 981 cm/sec.^{2}, H= 5cm and (0 (cm^3/sec) <U_{in}< 40 (cm^3/sec.)). The simulation results for this controller with K=20 and $\lambda = 0.1$ has shown in Fig. 2. The results show the conventional sliding mode controller produces serious chattering phenomena as shown in Fig 2.c. The chattering is due to the inclusion of the sign function in the switching term. It causes the control input to start oscillating around the zero sliding surface, resulting in unwanted wear and tear of the actuators.

In order to overcome the problem, a fuzzy logic control (FLC) is suggested and applied in the next section. The main advantage of the fuzzy controller is its heuristic design procedure and it is a model-free approach. The sliding mode control approach guarantees the robustness and stability of the resulting control system, which can systematically be achieved at the cost of the chattering side effect. The combination of fuzzy control strategy and sliding-mode control method becomes a feasible approach to preserve the advantages of these two approaches. A fuzzy sliding surface is introduced to develop a sliding mode controller. The IF-THEN rules of fuzzy sliding mode controller can be described as [6]:

R_1: If S is NB then K is PB
R_2: If S is NM then K is PM
R_3: If S is ZE then K is ZE (15)
R_4: If S is PM then K is NM
R_5: If S is PB then K is NB

where NB, NM, ZE, PM, PB are the linguistic terms of antecedent fuzzy set, they mean negative big, negative medium, zero, positive medium, and positive big, respectively. The fuzzy membership function for each fuzzy term would be a proper design factor in the fuzzy sliding-mode control problem. The more fuzzy terms are defined, the more fuzzy rules will be generated for completeness.

A general form is used to describe these fuzzy rules:

R_i: If S is A_i , then K is B_i , i =1,2,3,4,5 (16)

Where A_i has a triangle membership function (depicted in Fig. 3.) and B_i is a fuzzy singleton.

Using (12), the control law U_{in} is:

$$U_{in} = (2A\sqrt{z_2}/c_2)[-(c_1^2 - 2c_2^2)/2 - (c_1c_2/2)(\sqrt{z_1}/\sqrt{z_2} - \sqrt{z_2}/\sqrt{z_1}) - \lambda(-c_1\sqrt{z_1} + c_2\sqrt{z_2}) - K.sat(S/\Phi)] \quad (17)$$

Where

$$sat(S/\Phi) = \begin{cases} 1 & if \quad (S/\Phi) > 0 \\ S/\Phi & if \quad S = 0 \\ -1 & if \quad (S/\Phi) < 0 \end{cases} \quad (18)$$

From the control point of view, the parameters of structures should be modified automatically by evaluating the results of fuzzy control. Hitting time and chattering phenomenon are two important factors that influence the performance of SMC. The width of boundary layer will influence the chattering magnitude of sliding mode controller, and K will influence how soon the state reaches to the sliding surface. We will introduce the GAs to the problem of determining and optimizing these parameters of FSMC for the system. The advantage of the GAs is that they don't need extra professional knowledge or mathematics analysis. During the execution of the GAs, only the fitness function of the strings is evaluated. The performance surface doesn't need to be differentiated with respect to the change of control parameters and no derivatives, gradient calculations or other environment knowledge is necessary by GAs. Since GAs can consider multiple objective problem and the selected control parameters by GAs is based on the direction of the fitness function, we choose the tracking error and the chattering of the controlled system's response as the performance measures for selecting the parameters.

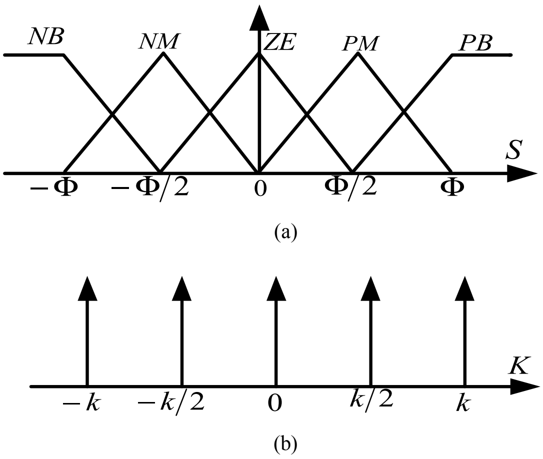


Fig.3. (a): The input membership function of the FSMC (b): The output membership FSMC

The proposed fitness function is defined in such a way that the selected parameters can drive the state to hit the sliding surface fast and then keep the state slide along the surface with less chattering and tracking error.

IV. GENETIC ALGORITHMS

Genetic algorithms are searching algorithms based on the mechanism of natural selection and genetics [7]. In this optimization scheme, the solution space (the design parameter space) will be coded as strings, like genes in nature. An objective function is used to evaluate the fitness of the string to the environment. In GAs, there are three basic operators: (1) reproduction, (2) crossover and (3) mutation. Reproduction is a process by which individual strings are selected and copied to a mating pool according to their objective function value (fitness). By the reproduction operator, strings with better fitness can have more copies. In contrast, strings with poor fitness may be eliminated. Crossover is performed over the mating pool to generate offspring in the next generation. The simple crossover may proceed into two steps, i.e., mating and crossover processes. The mating process is a random procedure to select two strings from the mating pool. As a mating pair is selected, the strings crossover each other at a particular crossover point. The mutation process plays the role as an insurance policy. In other words, GAs may lose some important message when executing reproduction and crossover procedures and may reach a local optimum. In this case, the mutation process will hopefully recover the GAs to continue the search [10].

Consequently, GA is used to search the parameter space to find appropriate values of the FSMC parameters, Φ and K in (17). The definition of the fitness function is defined as follows:

$$y = \int [W_1 \times (S(x,t))^2 + W_2 \times (h_2(t) - H)^2]dt \tag{19}$$

Where $S(x,t)$ is the desired sliding surface and $(h_2(t)\text{-}H)$ is the error between the real water heads in Tank 2 and the desired value H, and W_1 and W_2 are the weight factors. The design parameters of the FSMC based on the GAs associated with the above control rules are specified as follows:

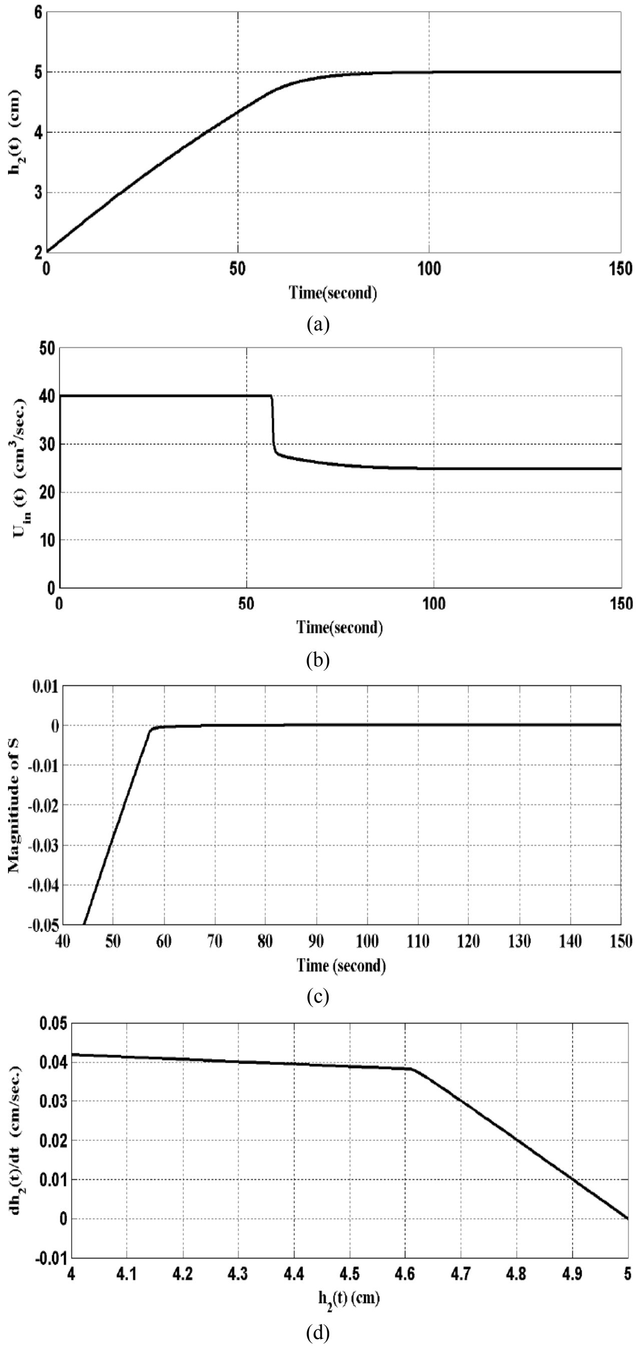


Fig.4 (a): Water level in Tank2 (b): The inflow rate into Tank1 (c): The time response of S (d): The state trajectory in the phase plane for Genetic based FSMC.

Population size = 50,
Crossover probability = 0.8,
Mutation probability = 0.02,
Generations = 60,
k belongs to [0, 30],
Φ belongs to [0, 20].

Let W_1=2 And W_2=1, the optimal parameters of the FSMC are obtained with GAs, k=14.57947 and Φ=0.19723. From Fig.4 can be seen that $h_2(t)$ converges to its desired value H, and chattering is also greatly reduced. The FSMC with GAs control signal in Fig. 4 is smoother than SMC control signals.

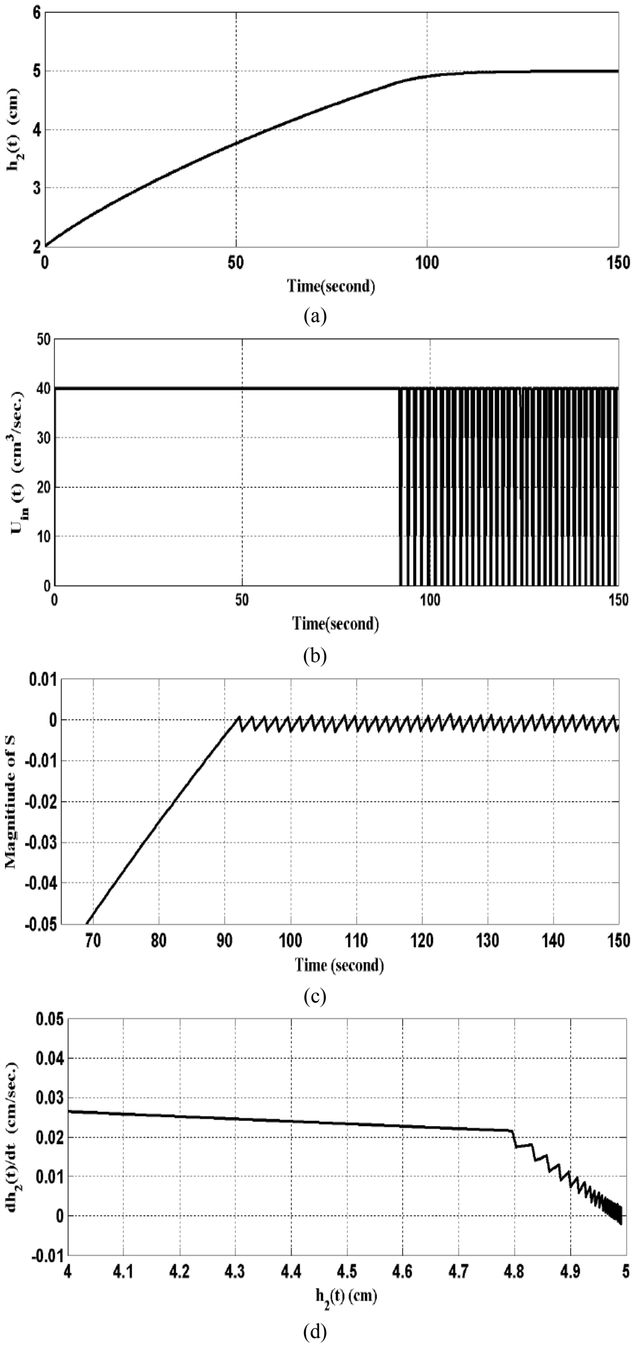


Fig. 5 (a): Water level in Tank2 (b): The inflow rate into Tank1 (c): The time response of S (d): The state trajectory in the phase plane for SMC, In the presence of a step disturbance of -5 (cm^3/sec.) applied at t=125 (sec.) and with 20% variation in system parameters.

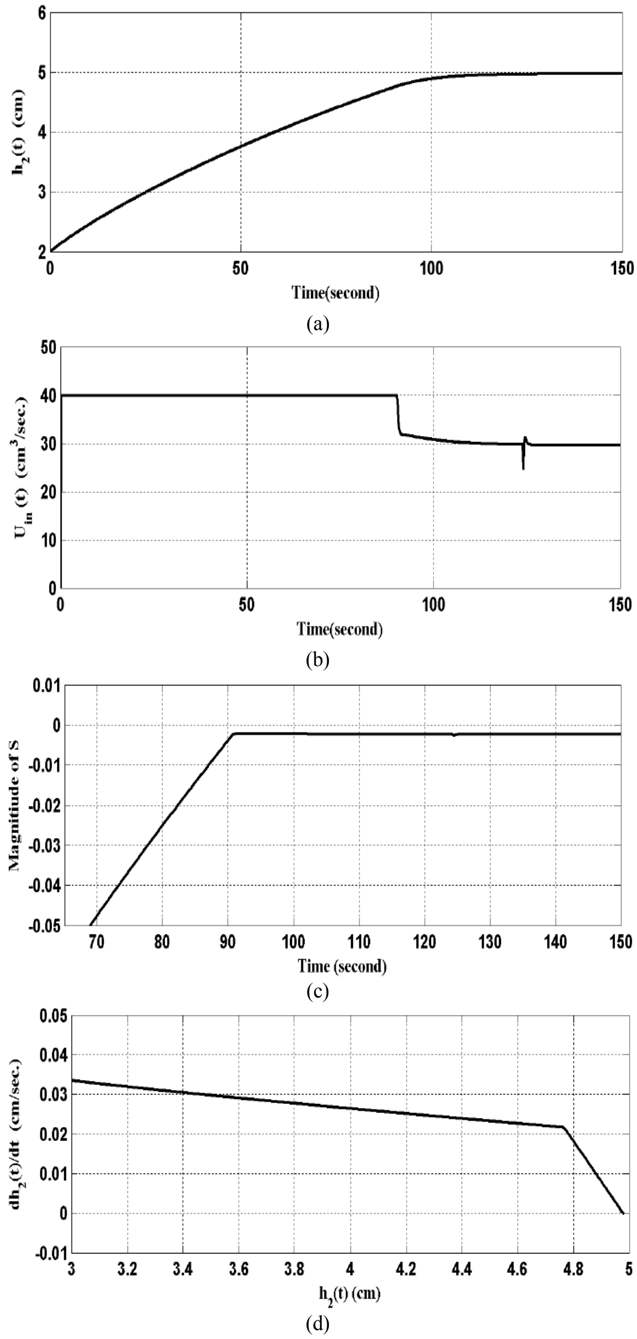


Fig. 6 (a): Water level in Tank2 (b): The inflow rate into Tank1 (c): The time response of S (d): The state trajectory in the phase plane for Genetic based FSMC, In the presence of a step disturbance of -5 (cm^3/sec.) applied at t=125(sec.) and with 20% variation in system parameters.

Also the performance of the system is carried out for the traditional SMC and Genetic-based FSMC when the parameters of the system are varied and disturbances are acting on the system. In the presence of a step disturbance of -5 (cm^3/sec.) applied at t=125 (sec.) and with 20% variation in system parameters the system responses have shown in Fig. 5 and Fig.6. The simulation results indicate that the genetic-based FSMC controller is more robust and has the smaller chattering than the traditional SMC.

Therefore, it can be concluded that the proposed control scheme is robust to changes in the parameters and to disturbances acting on the system.

V. CONCLUSION

An improvement of the sliding mode control design for coupled tanks is investigated in this paper. The main objective is to propose an effective method to reduce the hitting time and to attenuate the chattering such that a high overall performance of small hitting time and small chattering can be achieved. First, a SMC controller for coupled tanks is designed and then a Genetic-based Fuzzy Sliding Mode Controller is proposed.

Generally, the FSMC approach is classified into the anti-chattering-type sliding-mode control law. The parameters of proposed FSMC are adjusted by GAs. The advantage of the GAs is that they don't need extra professional knowledge or mathematics analysis. The proposed control scheme is insensitive to the parameter variations (Uncertainties) and capable of being against disturbance and noise too.

It is therefore concluded that the integrated performance of FSMC based on GAs is superior to the traditional SMC. The former is more responsive, stable and robust than the SMC. It also provides smoother control signals and therefore reduces wear and tear of actuators.

REFRENCES

[1] M. Kh. Khan and S. K. Spurgeon, "Robust MIMO water level control in interconnected twin-tanks using second order sliding mode control", Control Engineering Practice Vol.14 , Issue 4, April 2006, pp. 375–386.

[2] N. B. Almutairi, and M. Zribi, "Sliding mode control of coupled tanks", Mechatronics, Vol. 16, Issue 7, September 2006, pp. 427-441.

[3] H. T. Yau, Ch. Li Chen, "Chattering-free fuzzy sliding-mode control strategy for uncertain chaotic systems", Chaos, Solitons and Fractals, Vol. 30, Issue 3, November 2006, pp. 709-718.

[4] R. Shahnazi, H. Shanechi and N. Pariz, "Position Control of Induction and DC Servomotors: A Novel Adaptive Fuzzy PI Sliding Mode Control", Power Engineering Society General Meeting, 2006 IEEE, pp 1-9.

[5] Ch. Ch. Wong and Sh. Yu Chang, "Parameter Selection in the Sliding Mode Control Design Using Genetic Algorithms", Tamkang Journal of Science and Engineering, vol. 1, No. 2, 1998, pp. 115-122.

[6] T. Wang, Sh. Ch. Tong, "Fuzzy sliding mode control for nonlinear systems", International Conference on Machine Learning and Cybernetics, IEEE 2004, Vol. 2, pp. 839 – 844.

[7] B. Moshiri, M. Jalili-Kharaajoo, F. Besharati, "Application of fuzzy sliding mode based on genetic algorithms to control of robotic manipulators", Emerging Technologies and Factory Automation, Vol. 2, 16-19 Sept. 2003 IEEE, pp. 169 – 172.

[8] N.K. Poulsen, B. Kouvaritakis and M. Cannon, "Nonlinear constrained predictive control applied to a coupled-tanks apparatus", IEE Proc. Control Theory Appl., Vol. 148, No.3, May 2001

[9] J. J. E. Slotine, and W. Li, *Applied Nonlinear Control,* New Jersey, USA: Prentice Hall International Inc., 1991.

[10] D.E. Goldberg, *Genetic Algorithms in Search, Optimization and Machine Learning*, Addison-Wesley Publishing Co. ISBN 0-201-15767-5, San Mateo, CA, 1989.