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Fault Tolerant Control Based on Sliding Mode Control Approach with Application to Water Tank System*

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Abstract: Recent advanced application technologies have appeal to fault tolerant control due to the crucial consequence that might be caused by faulted system. This paper is subjected to fault tolerant control under sliding mode technique and present the tool needed to achieve the design controller when the fault information is inserted into equivalent control part.
Key Words: Fault tolerant control, Sliding mode control, water tank system

1 INTRODUCTION

In recent research, variable structure systems (VSS) and sliding mode control (SMC) have received great potential for practical applications. The aspects of SMC are well documented in many books and articles [1-2]. The discontinuous nature of the control action in SMC could reach robustness features for both systems stabilization and output tracking problems. SMC has been applied in many control fields which include robot control[3], motor control [4-5], flight control[6], control of power system[7-8], and process control[9]. The efficiency of SMC to lighten the problems caused by uncertain or changing system dynamics or parameters[10-11], push many researcher to implement it as tool for reconfiguration purpose, by that, deal with fault tolerant control system while reconfigurable control is a critical technology[12-13] with its objectives to detect the fault and recover the functionality of the faulty system as same as that of the nominal system[14].

In this paper, we propose a sliding mode control schemes for the coupled tanks system under the framework of fault tolerant control system.

The paper is organized as follows. In second section, we will exhibit the proposed way to handle the fault tolerant control system by means of sliding mode technique, the implementation of the benchmark of two water tank control system as application to the mentioned proposal come in the third section, finally conclusion.

2 PROBLEM STATEMENTS

Typical description for the system uncertainty caused by system faults[16] can be represented with

$$\dot{x}(t) = Ax(t) + Bu(t) + Rf(t)$$
$$y(t) = Cx(t) \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $u(t) \in \mathbf{R}^l$ is the control input vector and $y(t) \in \mathbf{R}^m$ is the measurement vector and $f(t) \in \mathbf{R}^g$ represents the fault vector which is considered as an unknown function of time.

Assume $\{C, A\}$ is observable, R matrix is the distribution matrix and $Rf(t)$ the uncertainty caused by system or actuator fault that could be inserted into A and B, then

$$A_{\text{fault}} = A + \Delta A \quad \text{and} \quad B_{\text{fault}} = B + \Delta B \tag{2}$$

with $Rf(t) = \Delta Ax(t)$ system fault or $Rf(t) = \Delta Bx(t)$ actuator fault.

By means of an observer, the residual can be generated as,

$$\dot{\hat{x}}(t) = (A - LC)\hat{x}(t) + Bu(t) + Ly(t) \tag{3}$$
$$\hat{y}(t) = C\hat{x}(t)$$

with $$r(t) = y(t) - \hat{y}(t) = C[x(t) - \hat{x}(t)] \tag{4}$$

where $r(t) \in \mathbf{R}^p$ the residual vector, $\hat{x}(t)$ and $\hat{y}(t)$ are state and output estimates. L is any matrix such that $A + LC$ is stable, then estimation error as: $e(t) = x(t) - \hat{x}(t)$ where error dynamic can be writing as

$$\dot{e}(t) = (A - LC)e(t) + Rf(t) \tag{5}$$

note that $$r(t) = Ce(t) \tag{6}$$

As it is mentioned above controller structure subject to sliding mode should consider the fault information which might occur in the system, so the aim here is to find a relation between sliding mode controller and faults information, (6) and (4) could help to extract the fault information with the assumption that C is unit matrix

$$\dot{r}(t) = \dot{e}(t) = (A - L)e(t) + Rf(t) \tag{7}$$

then finally $$Rf(t) = \dot{r}(t) - (A - L)e(t) \tag{8}$$

This relation of extracted fault information could be inserted into sliding mode controllers to adjust the corrective control part of sliding-mode like this we could obtain reconfiguration of controller which is robust to any system faults.

Fault embedded in control side subjected to sliding mode:

Let us introduce some useful definition for this approach: the sliding manifold as defined in [15] to achieve at the same time all state in (1) and track desired trajectories:

$$s(t) = \tilde{x}(t) + \delta \int \tilde{x}(t)\mathrm{d}t \tag{9}$$

then $$\dot{s}(t) = \dot{\tilde{x}}(t) + \delta \tilde{x}(t) \tag{10}$$

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where s is the sliding surface and δ is a diagonal matrix defining the slope of the sliding surfaces, $\tilde{x}$ is the state error vector and defined as

$$\tilde{x} = x - x_d \tag{11}$$

where x_d is desired state.

In order to guarantee stability, a candidate Lyapunov function according to sliding manifold given in (9) is

$$V(s) = \frac{s^{\mathrm{T}} s}{2} \tag{12}$$

In way to consider fault information extracted in (8) to update the equivalent part of sliding mode control, from (1) and (10)

$$\begin{aligned} \dot{s}(t) &= Ax(t) + Bu(t) + Rf(t) \\ &\quad -\dot{x}_d(t) + \delta\tilde{x}(t) \end{aligned} \tag{13}$$

Let B^{+} be pseudo-inverse of B and its columns are linearly independent:

$$B^{+} = (B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}$$

Hence the Equivalent control term is obtained as:

$$u_{\mathrm{eq}}(t) = (B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}\left[-Ax(t) - Rf(t) + \dot{x}_d(t) - \delta\tilde{x}(t)\right] \tag{14}$$

when $\dot{s}(t)$=0. In (14) the only unknown term is δ and to satisfy the sliding mode condition a corrective control term is used for sliding mode controllers.

The overall controller:

$$u(t) = u_{\mathrm{eq}} - B^{+}\left[K \cdot \mathrm{sat}\,(\frac{s}{\Phi})\right] \tag{15}$$

where $sat(\frac{s}{\Phi}) = \mathrm{sgn}(s)$ when $|s| \prec \Phi$

and $\mathrm{sat}(\frac{s}{\Phi}) = \frac{s}{\Phi}$ when $|s| \succ \Phi$

with $\Phi \succ 0$ extremity of saturation function. K is parameter design that will be chosen

Remark1. Since the fault distribution vector is included into the equivalent control part of controller then the reconfiguring adaptive manner is established. To guarantee stability of overall system (12) should be negative by that ensure the state trajectory is always moving towards $\dot{s}(t) = 0$ whenever s is not zeros, that's mean

$$\dot{V}(s) \prec 0 \tag{16}$$

Remark2. In case that the fault not presented in (1) the fault distribution information, from (2) of the system and actuator as follow:

$$\dot{x}(t) = A_{\mathrm{fault}}x(t) + B_{\mathrm{fault}}u(t) \tag{17}$$

then the sliding surface function becomes

$$\dot{s}(t) = A_{\mathrm{fault}}x(t) + B_{\mathrm{fault}}u(t) - \dot{x}_d(t) + \delta\tilde{x}(t) \tag{18}$$

From (18) give us the following:

$$\hat{u}(t) = B^{+}{}_{\mathrm{fault}}\left[-A_{\mathrm{fault}}x(t) + \dot{x}_d(t) - \delta\tilde{x}(t)\right] \tag{19}$$

then the overall control together with corrective control gain become **:**

$$u = \hat{u} - B^{+}{}_{\mathrm{fault}}K\mathrm{sat}(\frac{s}{\Phi}) \tag{20}$$

with sat(.) is saturation function and K parameter design.

Consider the case then we have $s\dot{s} = -s^2 \prec 0, \forall s \neq 0$ thisgives the solution of $s(t) = \mathrm{e}^{-t}s(0) \Rightarrow s(t) = 0, t \to \infty$ that shows $s\dot{s} < 0$ is not enough by that (16) is not enough as condition for stability, then we should introduce η-reachability term as strong condition for finite time convergence subjected to K. The condition for K can be obtained by inserting (20) and (19) into (13):

$$\begin{aligned} \dot{s}(t) &= Ax(t) + BB^{+}{}_{\mathrm{fault}}\left[-A_{\mathrm{fault}}x(t) + \dot{x}_d(t) - \delta\tilde{x}(t)\right] \\ &\quad -\dot{x}_d(t) + \delta\tilde{x}(t) - BB^{+}{}_{\mathrm{fault}}k\,\mathrm{sgn}(s) + Rf(t) \\ \dot{s}(t) &= (Ax(t) - BB^{+}{}_{\mathrm{fault}}A_{\mathrm{fault}}x(t)) + Rf(t) \\ &\quad +(-BB^{+}{}_{\mathrm{fault}} + I)(-\dot{x}_d(t) + \delta\tilde{x}(t)) - BB^{+}{}_{\mathrm{fault}}k\,\mathrm{sgn}(s) \end{aligned} \tag{21}$$

The reachabilty condition is $s\dot{s} \leqslant -\eta|s|$ with $\eta \succ 0$ then k must verify

$$\begin{aligned} BB^{+}{}_{\mathrm{fault}}k &\geqslant \|(Ax(t) - BB^{+}{}_{\mathrm{fault}}A_{\mathrm{fault}}x(t)) + Rf(t) \\ &\quad +(-BB^{+}{}_{\mathrm{fault}} + I)(-\dot{x}_d(t) + \delta\tilde{x}(t))\| + \eta \end{aligned} \tag{22}$$

Then in case that no uncertainty in the input matrix，i.e. B equal to B_{fault}

Then (22) become:

$$k \geqslant \|(A - A_{\mathrm{fault}})x(t) + Rf(t)\| + \eta I \tag{23}$$

With bounded uncertainty $\|\Delta A\| \leqslant \varepsilon_1$ and bounded state vector $\|x(t)\| \leqslant \varepsilon_2$ we could get finally from (23):

$$k_{\mathrm{fault}} \geqslant \|Rf(t)\| + \varepsilon_1\varepsilon_2 + \eta \tag{24}$$

or $k_{\mathrm{fault}} \geqslant \|Rf(t)\| + \eta_1$ with $\varepsilon_1\varepsilon_2 + \eta = \eta_1$

From (24) we can conclude that for faulty case k_{fault} could take $\|Rf(t)\| + \eta_1$ value and in nominal case k_{fault} will take the value of η .

3 APPLICATIONS

We will try to apply this proposal to the benchmark consisting of the two tank level control system connected as shown in Fig. 1.

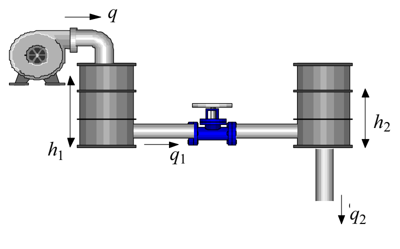

Fig.1 Two water tank benchmark

By that we assume that our system consist of just two tank coupled by an orifice, the input is supplied by a variable speed pump which supplies water to the first tank. The orifice allows the water to flow into the second tank and hence out to a reservoir. The objective of the control problem is to adjust the inlet flow rate $q(t)$ so as to maintain the level in the second tank $h_2(t)$ close to a desired set point level H . The dynamic model of the coupled tanks can be written as

$$\frac{dh_1}{dt}=\frac{1}{c}(-q_1+q)+\frac{\omega}{c} \quad \text{and} \quad \frac{dh_2}{dt}=\frac{1}{c}(q_1-q_2) \qquad (25)$$

where $q_1=c_{12}\sqrt{2g(h_1-h_2)}$, for $h_1 \succ h_2$

and $q_2=c_2\sqrt{2gh_2}$, for $h_2 \succ 0$ (26)

and ω is actuator fault

$h_1(t)$ the level in the first tank;

$h_2(t)$ the level in the second tank;

$q_1(t)$ the flow rate from Tank 1 to Tank 2;

$q_2(t)$ the flow rate out of Tank 2;

$q(t)$ the inlet flow rate;

g the gravitational constant;

c the cross-section area of Tank 1 and Tank 2;

c_{12} the area of the coupling orifice;

c_2 the area of the outlet orifice.

Since $q \succ 0$ because the pump delivers just in a positive direction then the general model for this system will be

$$\begin{aligned}\dot{h}_1&=-\frac{c_{12}}{c}\sqrt{2g|h_1-h_2|}\,\mathrm{sgn}(h_1-h_2)+\frac{1}{c}q\\ \dot{h}_2&=\frac{c_{12}}{c}\sqrt{2g|h_1-h_2|}\,\mathrm{sgn}(h_1-h_2)-\frac{c_2}{c}\sqrt{2gh_2}\end{aligned} \qquad (27)$$

At the equilibrium state the derivative should be zeros by that $\dot{h}_2=\dot{h}_1=0$ then hold

$$-\frac{c_{12}}{c}\sqrt{2g|h_1-h_2|}\,\mathrm{sgn}(h_1-h_2)+\frac{1}{c}Q+\frac{\omega}{c}=0$$

and $\frac{c_{12}}{c}\sqrt{2g|h_1-h_2|}\,\mathrm{sgn}(h_1-h_2)-\frac{c_2}{c}\sqrt{2gh_2}=0$ (28)

where Q is the equilibrium inflow rate and since that $q \succ 0$ and following (28) $\mathrm{sgn}(h_1-h_2)$ should be positive which implies $h_1 \geqslant h_2$

Following [16] that let us reduce the model system to $z_1=h_2 \succ 0$ and $z_2=h_1-h_2 \succ 0$ with $z=\begin{pmatrix} z_1\\ z_2\end{pmatrix}$

$\alpha_1=\frac{c_2\sqrt{2g}}{c}$, and $\alpha_2=\frac{c_{12}\sqrt{2g}}{c}$, the output of the coupled tanks system is taken to be the level of the second tank. Therefore, the dynamic model in (32) and (33) could be writing as

$$\begin{aligned}\dot{z}_1&=-\alpha_1\sqrt{z_1}+\alpha_2\sqrt{z_2}\\ \dot{z}_2&=-\alpha_1\sqrt{z_1}-2\alpha_2\sqrt{z_2}+\frac{1}{c}u\\ y&=z_1\end{aligned} \qquad (29)$$

The objective of the control scheme is to regulate the output $y(t)=z_1(t)=h_2(t)$ to a desired value. But the system is highly non linear and need to transform the system equation (29) to an easier one for design control.

Let $x=\begin{pmatrix} x_1\\ x_2\end{pmatrix}$ and the transformation matrix be $x=Tz$ such that

$$\begin{aligned}x_1&=z_1\\ x_2&=-\alpha_1\sqrt{z_1}+\alpha_2\sqrt{z_2}\end{aligned} \qquad (30)$$

The inverse transformation will be $z=T^{-1}x$ then (30) will be

$$\begin{aligned}z_1&=x_1\\ z_2&=\left(\frac{\alpha_1\sqrt{x_1}+x_2}{\alpha_2}\right)^2\end{aligned}$$

It can be checked that we can write the dynamic model in Eq. (29)

$$\begin{aligned}\dot{x}_1&=x_2\\ \dot{x}_2&=\frac{\alpha_1\alpha_2}{2}\left(\frac{\sqrt{z_1}}{\sqrt{z_2}}-\frac{\sqrt{z_2}}{\sqrt{z_1}}\right)+\frac{\alpha_1^2}{2}\\ &-\alpha_2^2+\frac{\alpha_2}{2c}\frac{1}{\sqrt{z_2}}(u+\omega)\end{aligned} \qquad (31)$$

where the value of z_1 and z_2 are function of x_1 and x_2. Hence, the dynamic model of the system can be written in a compact form as:

$$\begin{aligned}\dot{x}_1&=x_2\\ \dot{x}_2&=f+\Phi u\\ y&=x_1\end{aligned} \qquad (32)$$

where $f=\frac{\alpha_1\alpha_2}{2}\left(\frac{\sqrt{z_1}}{\sqrt{z_2}}-\frac{\sqrt{z_2}}{\sqrt{z_1}}\right)+\frac{\alpha_1^2}{2}-\alpha_2^2+\frac{\alpha_2}{2c}\frac{\omega}{\sqrt{z_2}}$

and $\Phi=\frac{\alpha_2}{2c}\frac{1}{\sqrt{z_2}} \succ 0$ are nonlinear function. We will use (32) in control design.

Design of sliding mode controller:

Let H be the desired level output value, assume its variation is small, with δ and K be positive scalar, we choose σ, the sliding surface as

$$\begin{aligned}\sigma&=\dot{s}=\dot{x}_1+\delta(x_1-H)\\ &=-\alpha_1\sqrt{z_1}+\alpha_2\sqrt{z_2}+\delta(z_1-H)\end{aligned} \qquad (33)$$

$$\begin{aligned}\dot{\sigma}&=\ddot{s}=\ddot{x}_1+\delta\dot{x}\\ &=\frac{\alpha_1\alpha_2}{2}\left(\frac{\sqrt{z_1}}{\sqrt{z_2}}-\frac{\sqrt{z_2}}{\sqrt{z_1}}\right)+\frac{\alpha_1^2}{2}-\alpha_2^2+\frac{\alpha_2}{2c}\frac{\omega+u}{\sqrt{z_2}}\\ &+\delta(-\alpha_1\sqrt{z_1}+\alpha_2\sqrt{z_2})\end{aligned} \qquad (34)$$

then the sliding mode controller is given by

$$\begin{aligned}u=\frac{2c\sqrt{z_2}}{\alpha_2}&\left[-\frac{\alpha_1\alpha_2}{2}\left(\frac{\sqrt{z_1}}{\sqrt{z_2}}-\frac{\sqrt{z_2}}{\sqrt{z_1}}\right)-\frac{\alpha_1^2}{2}+\alpha_2^2-\frac{\alpha_2}{2c}\frac{\omega}{\sqrt{z_2}}\right.\\ &\left.-\delta\left[(-\alpha_1\sqrt{z_1}+\alpha_2\sqrt{z_2})-K\,\mathrm{sgn}(\sigma)\right]\right]\end{aligned} \qquad (35)$$

Let $\hat{f}$ be an estimation value of f, assume that the estimation error is bounded by some known function $F=F(x,\dot{x})$ as follows $|f-\hat{f}|\leqslant F$ (36)

In order to have the system track $x_1(t)=x_{1d}(t)$ we define a sliding surface $\sigma=\dot{s}(t)=\dot{x}_1(t)+\delta(x_1-H)$

then $\dot{\sigma}=\ddot{s}(t)=\ddot{x}_1(t)+\delta\dot{x}_1$

$$= f + \Phi u + \delta \dot{x}_1 \tag{37}$$

To achieve $\dot{\sigma} = 0$ we choose control law

$$\Phi u = -f - \delta \dot{x}_1 \tag{38}$$

Because f is unknown and replaced by its estimation $\hat{f}$, the Control is chosen as :

$$u \to \Phi \hat{u} = -\hat{f} - \delta \dot{x}_1 \tag{39}$$

$\hat{u}$ can be as the accepted estimate of the equivalent control. In order to stratify the sliding condition

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t} s^2 \leqslant -\eta |s| \tag{40}$$

Despite the uncertainty on the dynamics f, we add to $\hat{u}$ a term discontinuous across the surface $\sigma = 0$

$$u = \hat{u} - K \operatorname{sgn}(\sigma) \tag{41}$$

By choosing $K = K(x_1, \dot{x}_1)$ in (40) to be large enough, we can now guarantee that (39) is verified. Indeed, from (37) and $\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\sigma^2 = \dot{\sigma}\sigma = \left[\left|f - \hat{f}\right| - k\Phi \operatorname{sgn}(s)\right]\sigma$

$$= \left|f - \hat{f}\right||\sigma| - k\Phi |\sigma| \tag{42}$$

$-\eta |\sigma| \geqslant \left|f - \hat{f}\right||\sigma| - k\Phi |\sigma|$ then easy to check

$$\eta + \left|f - \hat{f}\right| \geqslant k\Phi \tag{43}$$

then we could take

$$k\Phi = F + \eta \tag{44}$$

4 CONCLUSIONS

This paper presents techniques to achieve fault tolerance control using sliding mode control approach.

The fault could be introduced in the modeling part of the system model before proceeding to phase of controller design.

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