Assessment and Application of Adaptive Controllers

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ABSTRACT: A number of adaptive controllers based on the non-dual certainty equivalent principle are assessed and their implementation aspects discussed based on the application of these adaptive controllers to a coupled tanks system.

INTRODUCTION

In the last few years a number of adaptive controllers have been proposed. There are relatively few papers(1,2) that have assessed the different strategies. This paper investigates a number of non-dual certainty equivalent adaptive controllers with a view to comparing their basic design and examining aspects of their implementation. The adaptive controllers considered are:-

(i) Minimum Variance (STR)/Generalised Minimum Variance (STC)
(ii) Pole Assignment (PA)
(iii) Generalised Minimum Variance with pole assignment (GSTC)
(iv) Dead Beat(DB)/Extended Dead Beat (EDB)

When implementing these controllers, a number of assumptions have to be made such as system order or time delay. It is also assumed that the system can be model by a difference equation of the form:-

$$A(q^{-1})y(t) = q^{-k}B(q^{-1})u(t)+C(q^{-1})e(t)+d' \qquad 1$$

where (q^{-1}) is the backward shift operator and $y(t)$, $u(t)$, $e(t)$, d' and k are respectively the system output, control input, zero mean random variables, d.c. value and time delay in integer multiples of the sampling time.

$$\begin{aligned} A(q^{-1}) &= a_0 + a_1q^{-1}+\dots\dots+a_nq^{-n} \\ B(q^{-1}) &= b_0 + b_1q^{-1}+\dots\dots+b_nq^{-n} \\ C(q^{-1}) &= c_0 + c_1q^{-1}+\dots\dots+c_nq^{-n} \end{aligned} \qquad 2$$

and without loss of generality it is assumed that $a_0 = c_0 = 1$.

2 ADAPTIVE CONTROL ALGORITHMS

Table 1 presents the control algorithms and their design criterion together with the parameters that have to be chosen before implementation.

The following may be stated from reference to Table 1.

STR(3): The controller parameters are estimated directly. This is clear if the controller design equation,

$$C = AE + q^{-k}F \ ; \qquad E(q^{-1}) = 1 + e_1q^{-1}+e_2q^{-2}+\dots \qquad 3$$

is introduced to system equation (1) but first multiplied by E.

$$AEy = q^{-k}\ BEu + CEe + Ed' \qquad 4$$

By introducing (3) into (4) and assuming C = 1 for simplicity, a modified model resulted.

$$y = q^{-k}Fy + q^{-k}\ Gu + d + Ce \qquad 5$$

and $G = BE$; $d = Ed'$

This controller is restricted to minimum phase system only.

STC(4): This controller extends the criterion of STR to include weightings, P Q and R, which may be polynomials or transfer functions, on $y(t)$, $u(t)$ and $w(t)$ respectively, $w(t)$ is the desired set-point. This extension enables the control of non-minimum phase systems. A number of interpretations of this controller can be derived, however the interpretation used in this paper is:

$$Q = \frac{Q_1(1-q^{-1})}{1 - Q_2q^{-1}} \quad \text{and } P = R = 1 \qquad 6$$

This interpretation gives zero steady state error. The d.c. value can be estimated as d and employed in the controller or an incremental version of the controller can be used (5). Note that the controller parameters are obtained through the extended controller equation:

$$PC = AE + q^{-k}\ F \qquad 7$$

and if P=R=1, Q=0 and w(t)=0, the controller is reduced to STR.

PA(6): The controller parameters are obtained via the controller equation:

$$AG + q^{-k}BF=TC; \quad H = \frac{TC}{B} \qquad 8$$

by first estimating the system parameters A, B and C. This control law will give an offset if d'= 0 even if d' is explicitly estimated and employed in the control law. Methods of eliminating this offset are discussed in reference 1.

GSTC(7): The objective of the controller is to combine the criterions of STC and PA. It can be readily shown (7) that the following control law tries to minimise the variance of $\phi(t)$:

$$Fy(t) + Gu(t) + Hw(t) + d = 0 \qquad 9$$

where F is obtained through equation (7); $G = BE + QC$; $H = -CR$ and $d = Ed'$.

The closed loop equation is obtained by substituting for u(t) from equation (9) into the system equation (1) to give after some manipulation:

$$y(t) = \frac{BR}{T} w(t-k) + \frac{G}{T} e(t) + \frac{Qd'}{T} \qquad 10$$

and T is the prespecified pole location. To ensure that the closed loop poles are at their pre-specified location:

$$BP + QA = T \qquad 11$$

The polynomials P, Q and R are self tuned through a modified equation of (11) and since Q may not necessarily have a factor $(1 - q^{-1})$ zero offset is not guaranteed even if A has a factor $(1 - q^{-1})$ if $d' \neq 0$.

DB/EDB(2): The simplest of the approaches considered. The system parameters A and B are estimated and directly used in the control laws. These control algorithms have integral action and the d.c. values have only to be considered for parameter estimation and not in the control laws.

3 APPLICATION CONSIDERATIONS

The adaptive controllers considered have the same basic structure : an adaptation level where the system or controller parameters are estimated and a control level where the parameters are used in the control laws. To meet application requirements coordination level may be included to manage the adaptation and control levels. Some of the functions in the coordination level are (i) Freezing of estimation (ii) Estimation and control (iii) Reinitialisation of estimation (iv) Rate limit on control signal (v) Transfer of control from adaptive to non-adaptive controller and vice versa (vi) Linearisation.

These functions can be triggered by some ad hoc methods such as:

Freezing of estimation: $|y(t)-w(t)|<x_1$ for n_1 samples

Estimation and Control: $|y(t)-w(t)|>x_1$

Reinitialisation: $|y(t)-w(t)|>x_2$ for n_2 samples

Immediate Reinitialisation: $|y(t)-w(t)|>x_3$

and $x_1 < x_2 < x_3$

The values of x_1,x_2,x_3 and n_1,n_2 are chosen through familiarity with the system. Slowly time varying parameters can be tracked by incorporating a variable forgetting factor. There are alternative methods, however if the plants are fairly deterministic, a variable forgetting factor

$$\lambda = \lambda min + (1 - \lambda min) \exp(-1\Sigma(t)1) \qquad 12$$

where $0.8 < \lambda\ min < 1$; $\Sigma(t)$ is the model error at time t have so far been successfully used in our experiments.

4 APPLICATION

The adaptive controller considered are applied to a coupled tanks system. The system is shown in Figure 1. The system consists of 2 transparent tanks which are linked by an orifice. Water is pumped into tank 1 at a rate determined by the speed of a positive displacement rotary pump. This in turn is determined by the control signal voltage v applied to the pump drive socket on the instrumentation box. The water flows from tank 1 to tank 2 and finally through an outlet tap. The objective of the control is to regulate the level H_2 in tank 2 to follow a varying step changes in set-point.

The system when linearised at specific operating points has a transfer function;

$$G(s) = \frac{K}{(1 + sT_1)(1+sT_2)} \qquad 13$$

The system time constants vary with the selected steady state operating level so that operating point dependent non-linearity can be simply demonstrated. The characteristics of the pump (v vs rate of flow) is non-linear. The linearisation process is included in all the adaptive algorithms.

The water level set-point in tank 2 changes between 10 cm and 13 cm and λmin was chosen at 0.8. A sampling time of 3 seconds was used for all the controllers.

The user-specified parameters of each controllers were set as:

STC: $Q = \dfrac{Q1(1-q^{-1})}{1-0.7q^{-1}}$

PA: $T = 1 - 0.7q^{-1}$

EDB: $q_0 = \frac{1}{2}\left[\dfrac{1}{\Sigma bi} + \dfrac{1}{(1-a_1)\Sigma bi}\right]$

The objective of using self-tuning control is the possibility of reducing time consuming design and tuning. With this in mind, these designer parameters are chosen relatively 'arbitrarily'.

5 CONCLUSIONS

The approaches to adaptive control are reviewed and methods of eliminating offset for each controllers are discussed ranging from no guarantee (GSTC), complex method of PA, to relatively simple approaches like STC and the inherent ability of DB/EDB.

The controllers are successfully applied to the coupled tanks system except GSTC. Experimental results (8) have shown that GSTC is extremely sensitive to initial set-up parameters. Good performance is achieved by other controllers. Improvement is further achieved by including a linearisation process in the adaptive controllers. Harsh control action can be remedied by including rate-limit or introducing different designer parameters. Tuning of these designer parameters will affect the estimated parameters except STC where Q is not a factor of the controller parameters. STC therefore allows an extra degree of freedom and was found extremely useful in this application.

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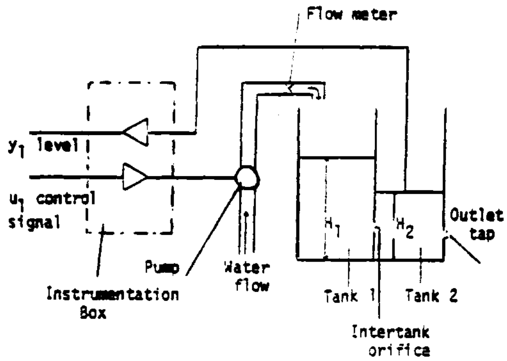

Fig.1 : Coupled Tanks System

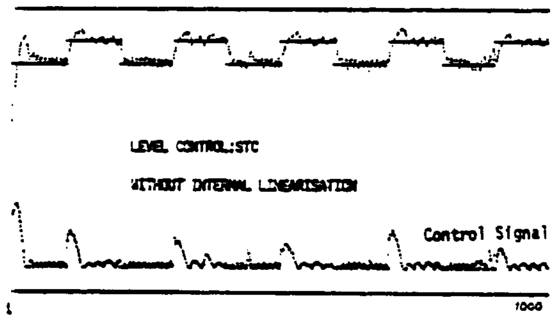

Fig.2

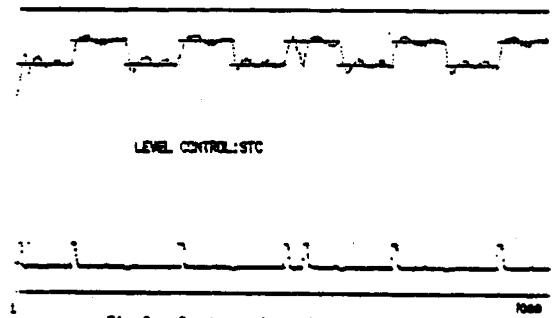

Fig.3. Readaptation after disturbance between samples 600-700

Notation	Design Criterion	Weightings	Control Law
STR	$I = E\{y(t)^2\} = \min$		$Fy(t) + Gu(t) + d = 0$
STC	$I = E\{\phi(t)^2\} = \min$ $\phi(t) = Py(t)+Qu(t-k) - Rw(t-k)$	P, Q and R	$U(t) = \frac{Rw(t)-\phi^* y(t+k/t)}{Q}$ and $C\phi^* y(t+k/t)=Fy(t)+Gu(t) + d$
PA	Poles of closed loop transfer function given	T	$Fy(t) + Gu(t) = Hw(t)$
GSTC	Objective of STC and PA	T	$Fy(t)+Gu(t)+Hw(t)+d = 0$
DB	Finite Settling Time		$U(t)=F[w(t)-y(t)]+Gu(t)$ and $F = q_o A; G=q_o q^{-k}B$ $q_o = 1/\Sigma bi$
EDB	Similar to DB	$r_o<q_o < r_1$ $r_o = 1/(1-a_1)\Sigma bi$ $r_1 = 1/\Sigma bi$	$U(t)=F[w(t)-y(t)]+Gu(t)$ and $F = q_o A(1-\alpha q^{-1})$ $G = q_o q^{-k}B(1-\alpha q^{-1})$ $\alpha = 1 - 1/q_o\Sigma bi$

TABLE 1

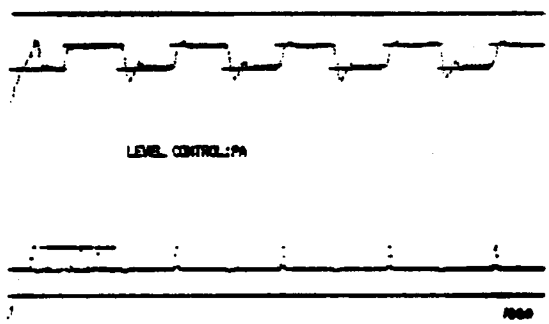

Fig.4 : Without control signal rate limit on first 200 samples

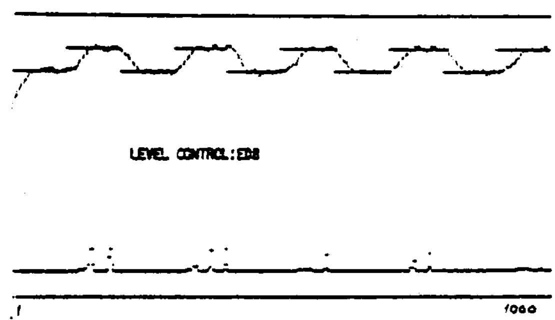

Fig.6.

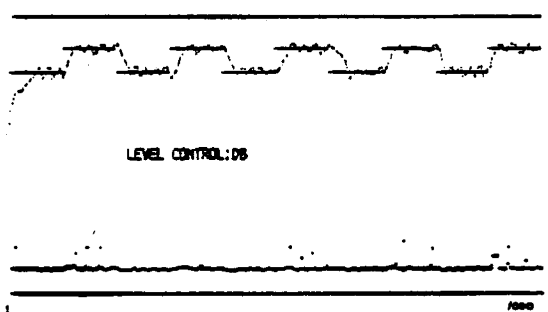

Fig.5.

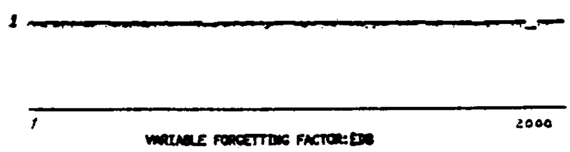

Fig.7 Typical performance of λ