Optimised Multivariable Nonlinear Predictive Control for Coupled Tank Applications

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Abstract

This paper presents the design of a novel nonlinear model predictive control (NMPC) strategy using a stochastic genetic algorithm (GA) to control highly nonlinear, uncertain and complex multivariable process with significant cross coupling effects between the process input and output variables. Raw multi-input and multi-output (MIMO) data from an experimental setup were collected and analysed. Both a GA and a backpropagation gradient descent based approach known as Levenberg–Marquardt Algorithm (LMA) are employed to train artificial neural network (ANN) nonlinear model. Real time practical experimental implementation on a MIMO coupled tank system is performed and the results show the effectiveness of the strategy. The approach can easily be adapted to other industrial processes.

1. Introduction

The coupled tank system is a miniature laboratory equipment that depicts many complex processes such as fluid supply to a chemical reactor at constant rates and maintaining certain fluid quantity as could be established in oil and gas production, oil refineries, process plants and batch processes [1]. Coupled tank system is very prominent and many researchers have explored this area over many years [1–8]. Multi-input and multi-output systems are more complex than single-input and single-output (SISO) due to the coupling effects that exist in the interactions between multiple variables in the plant system configuration. Despite the many complex interactions in chemical processes or mixing treatment in the industrial tanks, the control of fluid level is still very important and the flow between tanks must be regulated [9]. Efficient and effective controls of these processes have immense economical advantage and its success depends on the type of control strategy. Coupled tank system is highly nonlinear due to the feature characteristics of the valves and the fundamental dynamic equations [9].

Model Predictive Control (MPC) is an advanced method of process control strategy that has been in use in the process industries such as chemical plants and oil refineries for the past decades and has now been extended its use to many other fields [2], [10]. MPC has had a significant impact in its application because of its ability to control and optimise complex multivariable processes with constraints [11]. Model predictive controllers can employ the use of either linear or nonlinear dynamic models of the process.

Many designs have been utilised on SISO systems [2], [3], [12] while few researchers focused on MIMO systems [2], [4]. Coupled tank systems come in different forms; apart from the works undertaken on one, two and three tanks configurations, there are still few researches on the four tanks setup [8], [13]. Recently, an improved hybrid GA-AIS (genetic algorithm - artificial immune system) for tuning PID (proportional-integral-derivative) controller was introduced for coupled tank system [1] for proper tuning of PID controller for coupled tank. PID alone was used as coupled tank controller in [3], [5], [7].

Some researchers employed fuzzy systems as logic control strategy [4], [5], [14]. Neural network controllers are known to give better control strategy performance than PID controllers [7]. Linear models were also implemented on model predictive control [2], [6], [8]. Majstorovic *et al* [7] use ANN model in MPC on a SISO system configuration. Most researchers implemented work on coupled tank in simulations except for [4] and [8] who experimented with PID control strategies. Badreddine *et al* [15] however implemented MIMO MPC on three-tank system using polynomial nonlinear representation in input-output variables. Little work had been done in the area of implementing an ANN model with MPC for MIMO coupled tank systems. ANN structure functions by mimicking the behaviour of the human brain which processes good approximation capabilities and are employed here to model the MIMO coupled tank plant from experimental data set. The method of optimising the manipulated variables within model predictive controllers at every sampling instant is also key issue that determines the effectiveness of the strategy while genetic algorithm (GA) stands as the optimisation approach at every sampling instant. Gradient descent algorithm is known to easily converge and getting locked in a local minimum during training process [16]. The combination of both GA and LMA are explored and adopted for training in this work to avoid been trapped in a local minimum and also to achieve a high performance in the control strategy.

The focus of this paper is on nonlinear MIMO systems since most researchers that used neural network for modelling and control for nonlinear model predictive control are for SISO systems [12]. This is a novel approach whereby the salient features of both LMA and GA taken into advantage to produce an efficient nonlinear neural network model of coupled tank for MIMO system configuration.

2. Coupled Tank System

Liquid level control is probably the most common control problem in practical process systems [9]. The picture from the real time experiment practical setup at the control laboratory of Plymouth University is given in figure 1 while its key features and the schematic diagram of two-input and two-output (TITO) tank can be seen in the figure 2. There are two tanks placed side by side, between them at the bottom is an adjustable valve ***A*** that can be used to change the flow structure or vary the coupling between the tanks. At the bottom of each tank is an adjustable valve ***B*** and ***C*** that can be used to change the flow out of the tank. These provide direct discharge into the basement reservoir from left and right tanks respectively. It must be noted that the position of the valves is significant and determines the system configuration.

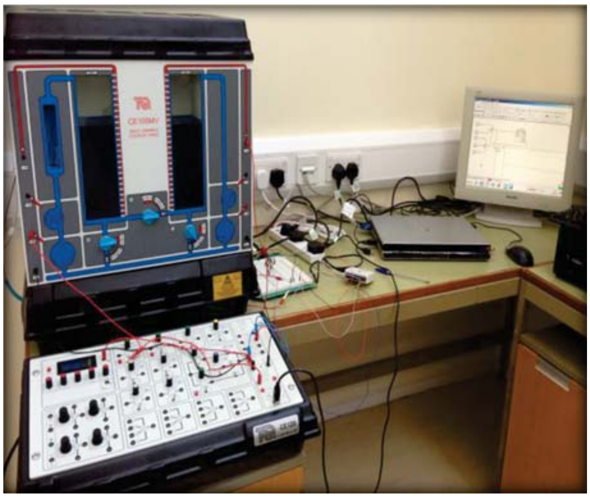

Figure 1: Coupled tank setup at Plymouth University.

At any given time, the height of the water in either of the two tanks, which is to be controlled in figure 1 is related to the water inlet rate, outlet rate and the interactions that exist between them. The dynamic equations (1) and (2) was derived using the first principles mass balance of flows equation on the tanks and whereas equation (3) gives the nonlinear characteristics equation of the fluid leaving either of the tanks.

$$A_1 \frac{dh_1}{dt} = K_1 V_1(t) - \beta_1 \alpha_1 \sqrt{2g} h_1 \pm \beta_{12} \alpha_{12} \sqrt{(2g(h_1(t) - h_2(t)))} \quad (1)$$

$$A_2 \frac{dh_2}{dt} = K_2 V_2(t) - \beta_2 \alpha_2 \sqrt{2g} h_2 \pm \beta_{12} \alpha_{12} \sqrt{(2g(h_1(t) - h_2(t)))} \quad (2)$$

$$Q_x = \beta_{cd} \alpha_x \sqrt{2g} h_x \quad (3)$$

In this work, subscripts 1 and 2 refer to tanks 1 and 2 respectively. $\boldsymbol{A}$ is the cross sectional area of the two tanks, $\boldsymbol{\alpha}$ is the cross sectional area of the small outlet orifice, $\boldsymbol{V}$ is the voltage of the pump, $\boldsymbol{h}$ is the height of the liquid in the tank, $\boldsymbol{\beta}$ is the valve ratio of tanks, $\boldsymbol{g}$ is the gravitational constant, $\boldsymbol{K}$ is the pump gain and $\boldsymbol{Q}$ is the rate of flow of fluid out of the tanks.

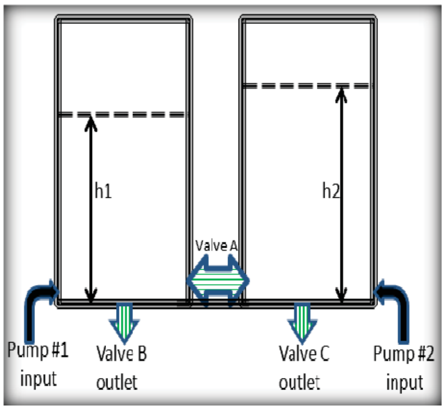

Figure 2: Schematic Diagram of TITO Tank.

The physical parameters of the TQ CE105MV coupled tank apparatus are given in Table 1.

System Parameter of Coupled Tank Apparatus	
Tank 1	Cross Sectional Area=9350mm^2
Tank 2	Cross Sectional Area=9350mm^2
Valves A, B and C	Cross Sectional Area=78.5mm^2 (10mm Valve Orifice) Valves range from 0 (close) to 5 (open)
Liquid Level Sensors	0 to 10V DC Output (0 to 250mm Height)
Pump Flow Sensors	0 to 10V DC Output (0 to 4400cm^3/min)

Table 1: Physical parameter of coupled tank apparatus.

The real time implementation is performed on CE105MV multi-variable coupled tanks apparatus from TQ TecQuipment (see figure 1). Data acquisition (DAQ) device (NI 6009) from National Instrument with LabView® software driver is configured to acquire real time sensor data and to send the multivariable input to control the fluid levels in the tank. The 14-Bit DAQ is only capable of handling voltages in the range of 0 to 5 volts whereas pumps input voltage ranges between 0 and 10 volts. A Pentium IV computer system with central processing unit (CPU) of $2.80Ghz$ and $3GB$ of random-access memory (RAM) was used as an interface with the other equipment. The CE105MV unit comprises of two variable speed pumps, two tanks connected by a variable area channel and drain valves to a sump located in the base of the equipment. There are two calibrated pressure type level sensors, an electronic flow meter and a variable area gap flow meter to provide visual indication of flow rate. The control strategy is designed in a way that the rate of change of the control input is controlled in small steps to avoid major fluctuations.

3. System Identification

In order to derive a model for the plant, system identification is performed. Good model representation is important for the success of any control strategy. One of the most significant aspects of system identification is the data collection. The data gives the characteristics and the dynamic behaviour of the system and care must be taken to capture the salient features especially in the MIMO case. The input used is a uniformly distributed signal with combination of different amplitudes and frequencies so as to properly excite the plant. An experiment was performed to determine the sampling time and this ensures that the sampling frequency is greater than twice the maximum frequency of the signal to be sampled. Three different input-output data of 2445 sample each were collected from real open loop practical experiments on the MIMO coupled tank system with a sampling rate (T_s) of 0.2 seconds with all valves in midway positions while putting the system parameters and limits into consideration. The reasons for collecting different samples rather than dividing a particular sample into three parts are to ensure generalisation and prevention of overfitting. The efficacy of derived model is determined not by its performance on the training data but by its ability to perform well on unseen data (validation and testing data sets). The collected samples details and features are tabulated in table 2.

Performance Function (OUTPUTS)	Data One (Training)		Data Two (Validation)		Data Three (Testing)	
RMSE (m)	1.4081		0.1201		0.7403	
NSSE (m^2)	9.8267e-08		1.5571e-07		3.2832e-07	
INPUTS	**I/P 1**	**I/P 2**	**I/P 1**	**I/P 2**	**I/P 1**	**I/P 2**
Mean (volts)	4.0851	5.0091	4.8336	4.9532	4.5705	5.3594
Variance (volts)	9.1842	6.9189	11.5329	14.3033	11.0491	14.5067

Table 2: The performance functions for raw input and output data.

Table 2 gives the root mean square error (RMSE) of the outputs, the normalised sum of square error (NSSE) of the outputs with the mean and variance values of the input signals. The first data set is for training; data set two is for validation while data set three is for testing. The three data sets: training, validating and testing collected from the coupled tank MIMO systems are shown in figures $3a, 3b$ and $3c$ respectively.

During experiments, noise is undesirable and must be removed so as not to hamper the quality of the data to be trained. The inputs signal vector $u_1(t)$ and $u_2(t)$ are the voltages applied to pumps 1 and 2 respectively which generate flow into both tanks while outputs $y_1(t)$ and $y_2(t)$ are the heights of the fluid in the tank. The structure for the ANN with two neurons, 2 delays and giving a total of eight regressed inputs are: $[\ u_1(t-1),\ u_1(t-2),\ u_2(t-1),\ u_2(t-2),\ y_1(t-1),\ y_1(t-2),\ y_2(t-1)\ and\ y_2(t-2)\ ]$ and is as shown in figure 4.

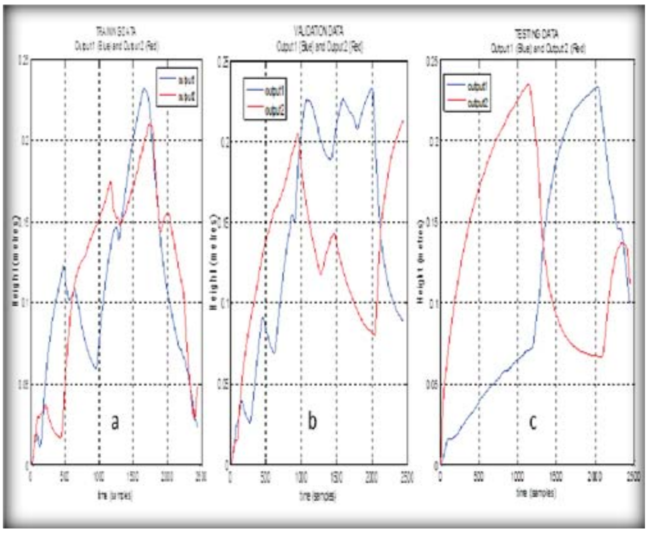

Figure 3: (a) Training, (b) Validation, and (c) Testing data sets.

Figure 4 also have bias weight term vectors in both hidden and the output layer. These weights are usually initialised randomly using no prior information and which invariably increases the offline training computation time. An initial trial and error computation is carried out to determine the optimal number of two neurons in the hidden layer and two delays in both input and output in order to decrease the overall computation time.

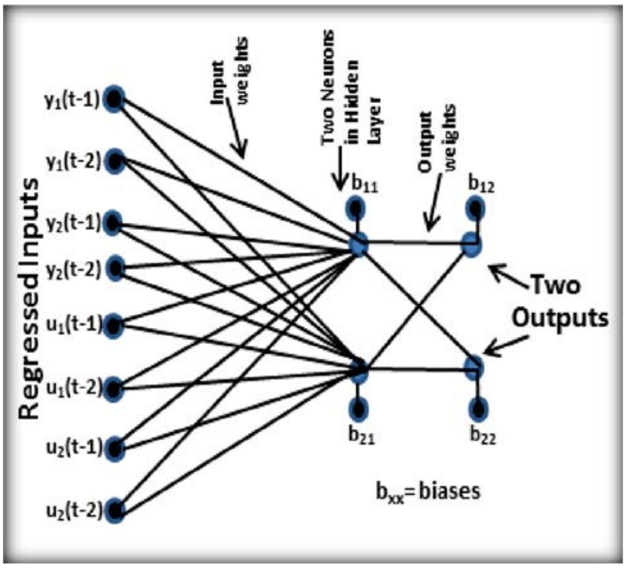

Figure 4: Structure of TITO Neural Network.

ANN training is carried out in two stages. A GA was used for initial training to ensure that a local minimum solution is avoided and to provide a good initial weight for the LMA algorithm to minimise the error between the trained output and target output, which subsequently leads to weights and biases update during computations. The performance index is calculated by using sum of squared errors (SSE). The training process constantly validates the result with a validation data set two to ensure good correlation. Hyperbolic tangent activation function is used in both hidden and output layer. The output is always confined between the ranges of -1 and $+1$ and is expressed in equation (4).

$$y = 1 - \frac{2}{e^{2x}+1} \qquad (4)$$

4. GA-Model Predictive Control

Model Predictive Control (MPC) is a form of an advanced control strategy in which the current manipulated control input applied to the real plant. A finite prediction horizon open-loop optimal control problem is derived by obtaining a real time solution online at each sampling instant. The optimisation yields an optimal control sequence and the first control in this sequence is applied to the plant.

A model predictive control strategy was implemented by using a GA as the optimisation approach while the predictor is the nonlinear artificial neural network model. The schematic picture of the process is given in figure 5. The predictor's task is to predict the plant's outputs based on the regressed inputs at every instant. This is done for different control moves within a prediction range. Figure 6 shows the prediction control process at every sampling time. The value of the control horizon should always be less than the prediction horizon. Genetic algorithm is used to solve and minimise the complex optimisation cost function (see equation 5) at every sampling time to determine the best optimum control inputs that give the least error between the predicted output and the trajectories reference signals and minimise the controller efforts:

$$\left\{\sum_{i=1}^{p}\left(\sum_{j=1}^{n_y}|w_{i+1,j}^{y}\left(y_j(k+i+1|k)\right)-r_j(k+i+1)|^2+\sum_{j=1}^{n_u}|\,w_{i,j}^{\Delta u}\Delta u_j(k+i|k)|^2\right)\right\} \qquad (5)$$

The first terms in equation 5 represents the error in prediction value and the reference valve while the second term denotes the change in the previous and the present control effort.

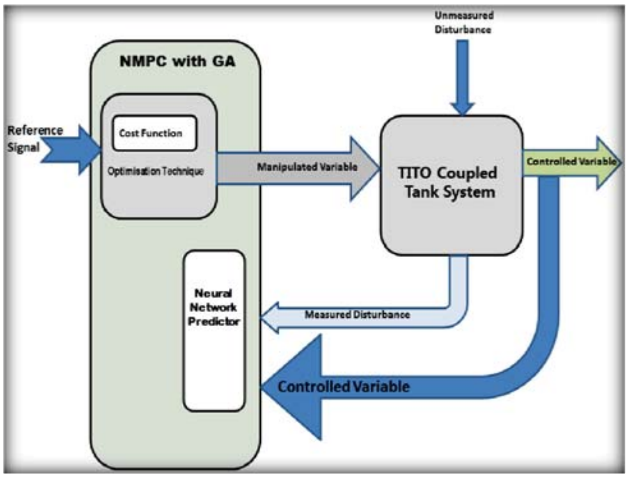

Figure 5: Structure of NMPC with GA Optimisation.

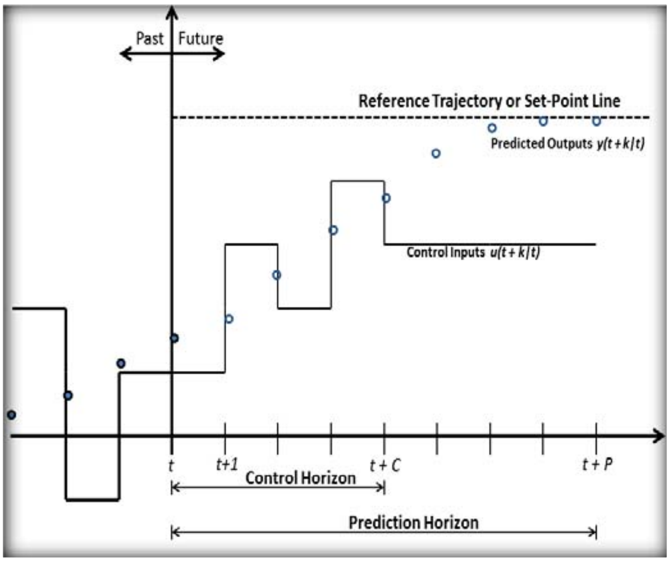

Figure 6: Predictive control at every sampling time.

In order to deal with real-time implementation constraints, termination measures were implemented to abort the optimisation once a defined sampling time is passed. This invariably might leads to convergence to some sub-optimal/optimal solution within the sampling time period.

Two performance indexes are considered here to evaluate the performance of the NMPC strategy: the total mean square error (MSE) and the average control energy (ACE). The total mean square error is the addition of all the squares of the error differences between the reference and the plant output for the two outputs divided by the total number of samples. This is expressed in equation (6).

$$\text{MSE} = \frac{\sum_{j=1}^{N}\left(y1_j^r - y1_j^p\right)^2 + \sum_{j=1}^{N}\left(y2_j^r - y2_j^p\right)^2}{N} \qquad (6)$$

In equation (6), superscripts ***r*** and ***p*** stand for reference value and plant output respectively while N stands for the total number of samples. The average control energy is defined as the addition of the squares of all the manipulated variables input to the plant divided by the total number of samples and expressed as:

$$\text{ACE} = \frac{\sum_{j=1}^{N}u1_j^2 + \sum_{j=1}^{N}u2_j^2}{N} \qquad (7)$$

In this work, the TITO coupled tank system block in NMPC (figure 5) is represented first in simulation dynamic equations of the coupled tank. Figure 7 uses equation (1) , equation (2) and the parameters from table 1 to create a MIMO coupled tank plant for NMPC. In the second case, NMPC was tested with the actual coupled tank in loop.

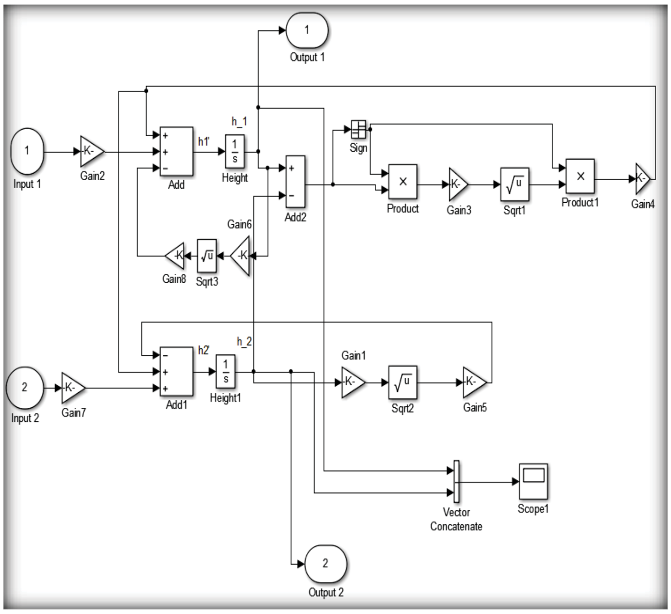

Figure 7: Coupled tank system in SIMULINK® design.

5. Results

The MPC control strategy first shown in MATLAB® simulation (using figure 7) and the response is presented in figure 8. After trial and error, the optimal parameters that give fewer computations were selected as: population size of 50, prediction horizon of 5, control horizon of 2, generation number of 4, crossover ratio of 0.5 and mutation ratio of 0.5.

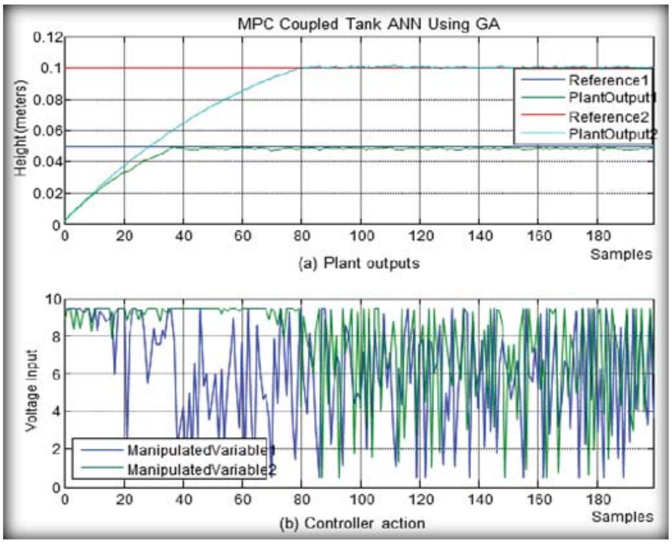

Figure 8: Result of NMPC strategy in Simulation using figure 7 as the plant.

The total MSE value calculated is $1.10\ X\ 10^{-3}\ m^2$ whereas the ACE $115.0085\ volts^2$.

The same parameter was used for the second case with the real plant in loop and figure 9 shows the response from NMPC.

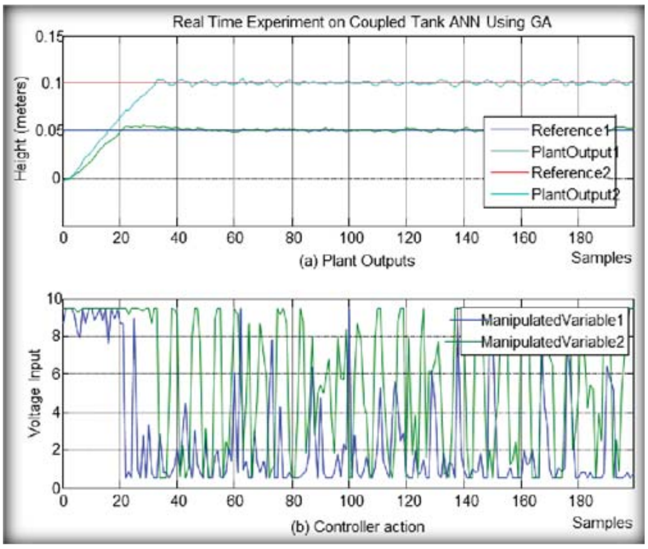

Figure 9: Response of NMPC strategy using actual plant.

The total MSE value calculated is $6.4797\ X\ 10^{-4}\ m^2$ whereas the ACE $66.7982\ volts^2$. The results of the NMPC strategy are tabulated in table 3. It is observed from the table that the real time experiment result (figure 9) gave better response than the simulation (figure 8). This is because of the imperfection in the calculation of the physical system parameters (cross sectional area of the tanks, pump gains, heights and valve coefficients) used for the equations in figure 7. The computing time for the 200 samples in this work is the time taken from the beginning to the end of the computer codes written in MATLAB® is 66.0524 seconds for the simulation case while the elapsed time is 139.9375 seconds for the real time case with the actual plant. The actual plant took longer time because of the delays involved in the duplex data acquisition processes and obtaining sensor readings from the coupled tank system.

	Figure 8	Figure 9
Mean Square Error (m^2)	1.10e-3	6.4797e-4
Average Control Energy (v^2)	115.0085	66.7982
Prediction Horizon	5	5
Control Horizon	2	2
Population Size	50	50
Crossover Ratio	0.5	0.5
Mutation Ratio	0.5	0.5
Number of Generations	4	4
Elapsed Time (s)	66.0524	139.9375

Table 3 : The Results of the response of NMPC control strategy.

6. Conclusion

This work has demonstrated both in simulation and in real time a nonlinear model predictive control strategy for a MIMO coupled tank system. In order to handle the problem of becoming trapped in a local minimum solution using the LMA, a GA was employed for initial network training to give an initial weight for the LMA. The results showed that the effectiveness of the system identification and it allows a wide range prediction which is well suited for chemical process with varying interaction rates and model predictive control strategy. It has the ability to perform very well in the area of set point tracking. This shows the strength of the ANN in handling difficult problems especially when properly trained. Most importantly, the NMPC strategy could easily handle nonlinear MIMO task efficiently. There is always a coupling effect whenever two tanks are linked together during experiments which make the design of a NMPC challenging task.

Acknowledgements

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