

# Adaptive Feedback Control

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*Invited Paper*

*Adaptive control is now finding its way into the marketplace after many years of effort. This paper reviews some ideas used to design adaptive control systems. It covers early ideas which primarily attempt to compensate for gain variations and more general methods like gain scheduling, model reference adaptive control, and self-tuning regulators. It is shown that adaptive control laws can be obtained using stochastic control theory. Techniques for analyzing adaptive systems are discussed. This covers stability and convergence analysis. Issues of importance for applications like parameterization, tuning, and tracking, as well as different ways of using adaptive control are also discussed. An overview of applications which includes feasibility studies as well as products based on adaptive techniques concludes the paper.*

## 1. INTRODUCTION

In everyday language "adapt" means to change a behavior to conform to new circumstances. Intuitively, an adaptive regulator can change its behavior in response to changes in the dynamics of the process and the disturbances. Since ordinary feedback was introduced for the same purpose, the question of the difference between feedback control and adaptive control immediately arises. Over the years there have been many attempts to define adaptive control. Truxal [1] proposed to define an adaptive system as a physical system which has been designed with an adaptive viewpoint. Other definitions were proposed by early workers in the field [2], [3]. A committee of the IEEE tried to reconcile the different views and proposed a new vocabulary [4] based on words like *self-organizing control* (SOC) parameter adaptive SOC, performance adaptive SOC, and learning control system. These efforts have, however, not received much following. A meaningful definition of adaptive control which makes it possible to look at a regulator and decide if it is adaptive or not is still missing. There appears, however, to be a consensus that a constant gain feedback is not an adaptive system. In this paper we will, therefore, take the pragmatic approach that adaptive control is simply a special type of nonlinear feedback control, where the states of the process can be separated into two categories, which change at different rates. The slowly changing states are viewed as parameters.

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Research on adaptive control was very active in the early 1950s. It was motivated by design of autopilots for high-performance aircrafts. Such aircrafts operate over a wide range of speeds and altitudes. It was found that ordinary constant gain, linear feedback can work well in one operating condition. Difficulties can, however, be encountered when operating conditions change. A more sophisticated regulator which can work well over a wide range of operating conditions is therefore needed. The work on adaptive flight control is summarized in [5] and [6]. It was characterized by a lot of enthusiasm, poor hardware, and nonexistent theory. Interest in the area diminished due to lack of insight and a disaster in a flight test; see Taylor and Adkins [7].

In the 1960s, there were many contributions to control theory, which were important for the development of adaptive control. State space and stability theory were introduced. There were also important results in stochastic control theory. Dynamic programming, introduced by Bellman [8], increased the understanding of adaptive processes [9]. It was used by Feldbaum to introduce the notion of dual control [10], [11]. Fundamental contributions were also made by Tsytkin [12], [13], who showed that many schemes for learning and adaptive control could be described in a common framework as recursive equations of the stochastic approximation type. There were also major developments in system identification and in parameter estimation, see Åström and Eykhoff [14], which contributed to gaining insight into the adaptive problem.

In the late 1970s and the early 1980s correct proofs for stability of adaptive systems appeared albeit under very restrictive assumptions. Investigation of the necessity of the assumptions has sparked new interesting research into the robustness of adaptive control as well as into controllers that are universally stabilizing.

The rapid and revolutionary progress in microelectronics has made it possible to implement adaptive regulators simply and cheaply. There is now a vigorous development of the field both at universities and in industry. A number of commercial adaptive regulators based on different ideas are appearing on the market and the industrial use of adaptive control is growing slowly but surely. In the spring of 1986 there were several thousand adaptive regulators in industrial use.

There are several surveys on adaptive control. The early

work was surveyed in [15]–[17]. Surveys of special areas in the field are given in [18]–[23]. An extensive bibliography which covers more than 700 papers published before 1976 is given in [24]. The survey paper [25] reviews process control applications. The textbooks [26] and [27] describe model reference adaptive systems and self-tuning regulators, respectively, the books [28]–[30] contain representative collections of papers dealing with recent applications, and [31] and [32] give recent theoretical results.

This paper is organized as follows. Different adaptive schemes are reviewed in Section II. This includes a number of early heuristic schemes. Three general approaches: gain scheduling, model reference adaptive control, and self-tuning regulators; are discussed in more detail. The section ends with a presentation of dual control which unifies several approaches and suggests improvements to the conventional approaches. Adaptive control theory is covered in Section III. The issues discussed include a discussion of concepts and generic models, stability and convergence. To discuss these topics it is also necessary to introduce the notion of persistency of excitation. The issues of parameterization and robustness are also discussed. Section IV deals with the practical aspects of adaptive control. The section begins with a presentation of four commercial products. Some speculations on future trends based on the characteristics of the products discussed are also given.

There are many aspects of adaptation that are not covered in this paper. There is a very vigorous development in signal processing [33], [34] that parallels the development in automatic control. This is not discussed in this paper. Neither are self-optimizing controls [35] or more esoteric forms of adaptation [36]–[38].

## II. ADAPTIVE SCHEMES

This section gives an overview of some adaptive concepts. Early heuristic schemes are first discussed. Many of them tried to compensate for variations in the process gain only. The early schemes contain several ideas which also appear in later systems; they also display a considerable engineering ingenuity. It is therefore of interest to know about them. They have been thoroughly explored by simulation and flight tests. There is, however, very little analysis of the early systems. Three schemes, gain scheduling, model reference adaptive control, and self-tuning regulators, are discussed in more detail. Finally, we discuss the systems that arise from stochastic control theory. Such systems are of interest even if they cannot easily be realized because they allow fast adaptation and they indicate the need for new functions which are not present in the previous systems based on heuristic ideas.

### The General Electric Autopilot

An adaptive autopilot, proposed and built by General Electric [39], is based on the idea that the frequency content of the error signal in a feedback system changes with the loop gain. High frequencies dominate if the gain is high and vice versa. A block diagram of the GE system is shown in Fig. 1. The gain of the system is changed by a parameter adjustment circuit which operates on the error signal. A block diagram of the parameter adjustment mechanism is shown in Fig. 1(b). The GE system has been successfully flight tested [40]. It is critical for the operation of the system that it is excited by wide-band disturbances. The gain of the

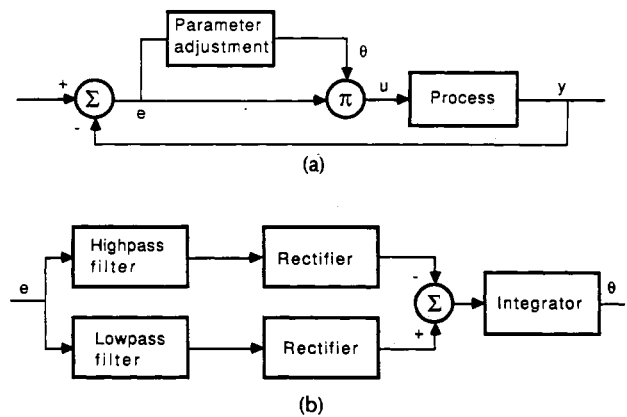


Fig. 1. Block diagram of General Electric's adaptive regulator.

system can be driven to its limits by sinusoidal command signals.

### Marsik's System

Another simple adaptive system [41] which also intends to compensate for variations in the process gain is shown in Fig. 2. The system is based on the fact that many closed-

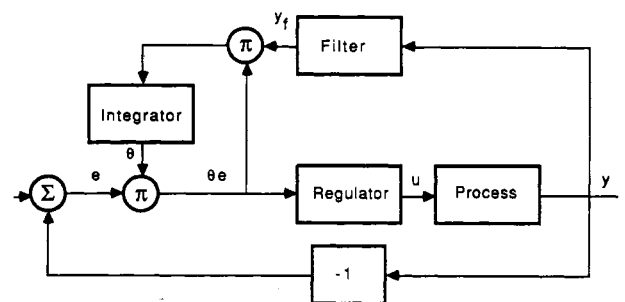


Fig. 2. Block diagram of Marsik's adaptive regulator.

loop systems have a resonance peak that increases with increasing open-loop gain. The peak occurs approximately at the crossover frequency where the open-loop system has a phase shift of  $180^\circ$ . Assuming that the command signal has a frequency content which covers a reasonably wide range, the output signal  $y$  has a significant frequency component which is out of phase with the error  $e$ . Similarly, the output  $y$  will have a significant component in phase with the error  $e$  if the gain is low. Marsik's system sets the gain simply from the correlation of the error and the output. The system can be improved by filtering the output  $y$  before introducing it to the correlator.

A more mathematical explanation is as follows. Assume that the updating loop is so slow that the parameter  $\theta$  varies much slower than the error  $e$  and the output  $y$  in the system. The parameter  $\theta$  is then given as follows:

$$\frac{d\theta}{dt} = k(\theta e) \cdot y = k[r - y]y\theta \quad (2.1)$$

where  $r$  is the reference value. The system thus attempts to adjust the gain so that the correlation between the error  $e$  and the output  $y$  is zero. Marsik's system works well under some circumstances and poorly under other.

Notice that the parameter adjustment mechanism in the system shown in Fig. 3 has two multipliers and an integrator. This is a generic part of the parameter adjustment in

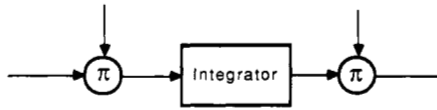


Fig. 3. Generic part of the parameter adjustment mechanism in Marsik's systems.

many adaptive systems. Also notice that the adjustment mechanism can be interpreted as if it attempted to drive the correlation between two signals in the system to zero. The system contains a filter which improves the performance of the system. The system attempts to adjust the parameter  $\theta$  while it is in closed-loop operation.

#### Measurement of Transient Response Features

There are many adaptive schemes where the regulator parameters are determined from some features of the transient response of the system. Zero-crossings, overshoot, damping ratio, etc., are typical features that are considered. Process control systems are, e.g., commonly adjusted so that the amplitudes of successive peaks of the impulse response have a ratio of 1:4. This is called quarter-amplitude damping.

The advantage of these schemes is that they are simple to implement. They have, however, some drawbacks because they depend heavily on the disturbances being isolated steps or impulses. The algorithms are typically based on detection of peaks and zero-crossings which may be noise-sensitive. There are improved methods which are based on area calculations. Such methods have been used to tune PID regulators automatically [42]. A related method is based on the fact that second-order systems have the property that

$$\frac{A^+}{A^-} = e^{\pi\sqrt{1-\zeta^2}}$$

where  $A^+$  is the positive impulse response area and  $A^-$  the negative impulse response area. An adaptive system which is based on adjustment of the regulator gain to give a constant  $A^+/A^-$  ratio has been developed by the Ford Company in 1959 [43]. A similar system has recently been successfully tested by Leigh [44]. Foxboro has for a long time been experimenting with a system which determines characteristic features of the response to disturbances [45]. An adaptive regulator for process control based on the concept has recently been announced. This system will be discussed further in Section V.

#### Self-Oscillating Adaptive Systems

An interesting approach for compensation of variations in the process gain was used in an autopilot proposed by Minneapolis-Honeywell [46]. A block diagram of this system is shown in Fig. 4. The basic idea is to have a feedback loop whose gain is as high as possible combined with a feedforward compensation to give the desired response to command signals. The high loop gain is maintained by introducing a relay in the feedback loop. This will create a limit cycle oscillation. It can be shown [47]–[50] that for sig-

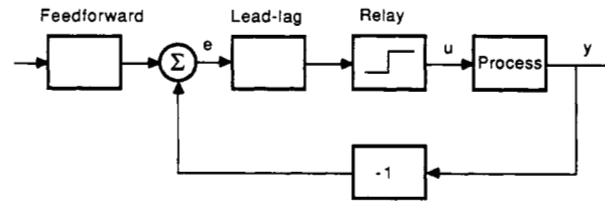


Fig. 4. Block diagram of self-oscillating adaptive system.

nals whose frequencies are much lower than the limit cycle oscillation, the equivalent amplitude margin is approximately  $A_m = 2$ . The system with relay feedback thus automatically adjusts itself to give a reasonable amplitude margin.

Notice that the system will always be excited because of the limit cycle oscillation. The frequency of this oscillation can be influenced by the lead-lag filter shown in Fig. 4. Its amplitude can be adjusted by changing the relay amplitude. The limit cycle oscillation is sometimes acceptable, e.g., in a missile application. For piloted aircrafts, however, it has been subject to much discussion because experience has shown that pilots will always notice the limit cycle and they often will object to it.

There are many variations of the basic self-oscillating adaptive control system. Attempts have been made to adjust the limit cycle amplitude by feedback. However, if the relay amplitude is too small the response to command signals will be too slow. Attempts have also been made to quench the relay oscillations by a dither signal. A comparison of a self-oscillating system to a fixed gain controller is given in [51].

The externally excited adaptive system (EEAS) is closely related to the self-oscillating adaptive system. In these systems, a sinusoidal perturbation signal is introduced into the system. The process gain is determined by measuring the signal propagation through the system and the regulator gain is adjusted appropriately [52].

Different versions of the self-oscillating adaptive system have been flight-tested on piloted aircraft. The approach is being used successfully in flight control systems for many different missiles.

#### Gain Scheduling

In some systems there are auxiliary variables which correlate well with the characteristics of the process dynamics. If these variables could be measured it would be possible to use them to change the regulator parameters. This approach is called gain scheduling because the scheme was originally used to accommodate changes in process gain. Fig. 5 shows a block diagram of a system with gain scheduling.

Gain scheduling is an open-loop compensation. There is

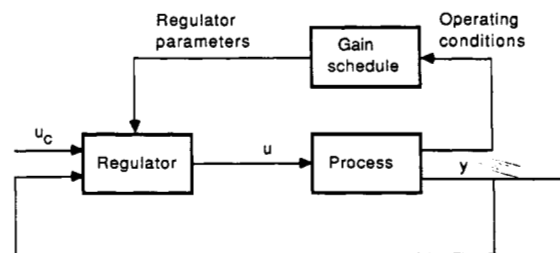


Fig. 5. Block diagram of a system with gain scheduling.

no feedback which compensates for an incorrect schedule. It can be viewed as a system with feedback control where the feedback gains are adjusted by feedforward compensation.

The idea of gain scheduling originated in connection with development of flight control systems [5]. In this application, the Mach number and the dynamic pressure are measured by air-data sensors and used as scheduling variables. A key problem in the design of systems with gain scheduling is to find suitable scheduling variables. This is normally done based on knowledge of the physics of a system. For process control, the production rate can often be chosen as a scheduling variable since time constants and time delays are often inversely proportional to production rate.

When scheduling variables have been obtained, the regulator parameters are determined at a number of operating conditions using a suitable design method. Stability and performance of the system are typically evaluated by simulation where particular attention is given to the transition between different operating conditions.

A drawback of gain scheduling is that the design is time-consuming. The regulator parameters must be determined for many operating conditions. The performance must be checked by extensive simulations. The design can sometimes be simplified by introducing normalized dimension-free parameters in such a way that the normalized model does not depend on the operating conditions. The auxiliary measurements are used together with the process measurements to calculate the normalized measurement variables. The normalized control variable is calculated and retransformed before it is applied to the process.

An example illustrates this approach.

*Example 2.1:* Consider the system

$$\begin{aligned}\frac{dx_1}{dt} &= f(x_1, x_2) \\ \frac{dx_2}{dt} &= g(x_1, x_2, u).\end{aligned}\quad (2.2)$$

Assume that the state variables can be measured and that it is desired to find a feedback such that the response of the variable  $x_1$  to the command signal is given by the transfer function

$$G(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}. \quad (2.3)$$

Introduce new coordinates  $y_1$  and  $y_2$  defined by

$$\begin{aligned}y_1 &= x_1 \\ y_2 &= \frac{dx_1}{dt} = f(x_1, x_2)\end{aligned}\quad (2.4)$$

and the new control signal  $v$  defined by

$$v = F(x_1, x_2, u) = \frac{\partial f}{\partial x_1} f + \frac{\partial f}{\partial x_2} g. \quad (2.5)$$

These transformations result in the linear system

$$\begin{aligned}\frac{dy_1}{dt} &= y_2 \\ \frac{dy_2}{dt} &= v.\end{aligned}\quad (2.6)$$

It is easily seen that the linear feedback

$$v = \omega^2(r - y_1) - 2\zeta\omega y_2 \quad (2.7)$$

gives the desired closed-loop transfer function (2.3) for the linear system (2.6). It remains to transform back to the original variables. It follows from (2.5) and (2.7) that

$$F(x_1, x_2, u) = \omega^2(r - x_1) - 2\zeta\omega f(x_1, x_2). \quad (2.8)$$

Solving this equation for  $u$  gives the desired feedback. It follows from the implicit function theorem that a condition for local solvability is that the partial derivative  $\partial F/\partial u$  is different from zero.  $\square$

The generalization of the example requires a solution to the general problem of transforming a nonlinear system into a linear system by nonlinear feedback changes of coordinates control variables. This problem was originally discussed by Krener [53] and Brockett [54] using geometric control theory and has later been subject to much research, [55]–[60].

A neat application for design of a flight control system for a helicopter was made by Meyer [57]. The idea has also been applied to process control [61], [62] and to robotics [63]–[65]. In robotics, the transformed equation can be written as

$$J_n \frac{d^2\varphi_n}{dt^2} = T_n$$

where  $J_n$  is the moment of inertia,  $\varphi_n$  a joint angle, and  $T_n$  a torque which depend on the motor current, the torque angles, and their first two derivatives. The equations are thus in the desired form and the nonlinear feedback is obtained by computing the current which gives the desired torque. The scheme is, therefore, called the computed torque method [66].

Gain scheduling has the advantage that the parameters can be changed very quickly in response to process changes. The limiting factors depend on how quickly the auxiliary measurements respond to process changes.

There is a controversy in nomenclature whether gain scheduling should be considered as an adaptive system or not because the parameters are changed in open loop. Irrespective of this discussion, gain scheduling is a very useful technique to reduce the effects of parameter variations. It is the standard method for design of flight control systems for aircrafts which operate over a wide range of altitudes and speeds [67]. The technique is used in some control systems for industrial robots. It is also used increasingly in process control [68], [69]. It is easy to implement gain scheduling using modern hardware for distributed process control. It can be expected that applications of the technique will increase in the future because of its potential benefits.

#### Model Reference Adaptive Systems (MRAS)

The model reference adaptive system (MRAS) was originally proposed by Whitaker at MIT [70]–[72]. He considered a problem where the specifications were given in terms of a reference model which tells how the process output ideally should respond to the command signal. A block diagram of the system is shown in Fig. 6. Notice that the reference model is part of the control system. The regulator can be thought of as consisting of two loops. An inner loop,

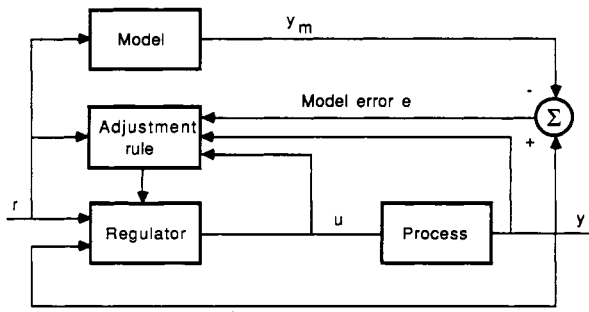


Fig. 6. Block diagram of model reference adaptive system (MRAS).

which is an ordinary feedback loop composed of the process, and a regulator. The parameters of the regulator are adjusted by the outer loop in such a way that the error  $e$  between the model output  $y_m$  and the process output  $y$  becomes small. The outer loop is thus also a regulator loop. The key problem is to determine the adjustment mechanism so that a stable system which brings the error to zero is obtained. This problem is nontrivial. It cannot be solved with a simple linear feedback from the error to the controller parameters. This is illustrated in Example 3.1. The following parameter adjustment mechanism, called the "MIT-rule," was used in the Whitaker's original MRAS:

$$\frac{d\theta}{dt} = -ke \text{grad}_\theta e. \quad (2.9)$$

In this equation,  $e$  denotes the model error. The components of the vector  $\theta$  are the adjustable regulator parameters. The components of the vector  $\text{grad}_\theta e$  are the sensitivity derivatives of the error with respect to the adjustable parameters. The sensitivity derivatives can be generated as outputs of a linear system driven by process inputs and outputs. The number  $k$  is a parameter which determines the adaptation rate.

Whitaker motivated the rule as follows. Assume that the parameters  $\theta$  change much slower than the other system variables. To make the square of the error small it seems reasonable to change the parameters in the direction of the negative gradient of  $e^2$ .

Notice that the parameter adjustment mechanism described by (2.9) can be thought of as composed of three parts: a linear filter for computing the sensitivity derivatives from process inputs and outputs, a multiplier, and an integrator; compare with Fig. 3. Also notice that the MRAS attempts to adjust the parameters so that the correlation between the error  $e$  and the sensitivity derivatives becomes zero. The MRAS shown in Fig. 6 is called a *direct* scheme because the regulator parameters are updated directly.

A simple example illustrates how the MIT-rule can be used.

**Example 2.2—Adaptation of a Feedforward Gain:** Consider the problem of adjusting a feedforward gain. Let the model and the process have the transfer function  $G(s)$ . The error is

$$e = y - y_m = G(p)\theta r - G(p)r$$

where  $r$  is the command signal,  $y_m$  the model output,  $y$  the process output,  $\theta$  the adjustable parameter, and  $p = d/dt$  the differentiation operator. The sensitivity derivative is

$$\frac{\partial e}{\partial \theta} = G(p)r = y_m.$$

The MIT-rule (2.9) then gives

$$\frac{d\theta}{dt} = -ky_m e. \quad (2.10)$$

The rate of change of the parameter should thus be made proportional to the product of the error and the model output. A block diagram of the model reference adaptive system is shown in Fig. 7.  $\square$

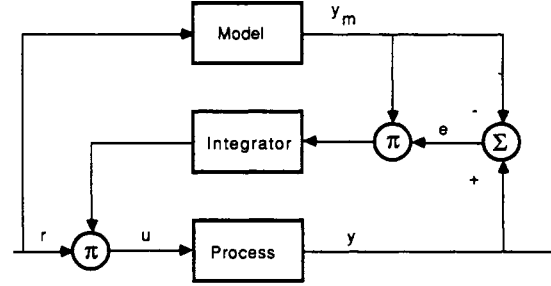


Fig. 7. Block diagram of an MRAS for adjustment of a feedforward gain based on the MIT-rule.

Notice that no approximations were needed in Example 2.2. When the MIT-rule is applied to more complicated problems it is necessary to use approximations to obtain the sensitivity derivatives. This is illustrated by an additional example.

**Example 2.3—MRAS for a First-Order System:** Consider a system described by the model

$$\frac{dy}{dt} = -ay + bu \quad (2.11)$$

where  $u$  is the control variable and  $y$  the measured output. Assume that it is desirable to obtain a closed-loop system whose input-output model is described by the relation

$$\frac{dy_m}{dt} = -a_m y_m + b_m r. \quad (2.12)$$

This goal can be achieved with the feedback

$$u = k_r r - k_y y \quad (2.13)$$

provided that the feedforward gain  $k_r$  and the feedback gain  $k_y$  are chosen as

$$k_r = \frac{b_m}{b} \quad (2.14)$$

$$k_y = \frac{a_m - a}{b}.$$

To use Whitaker's rule introduce the error

$$e = y - y_m$$

where  $y$  is the closed-loop output. This is obtained by eliminating  $u$  between (2.11) and (2.13). Hence

$$\frac{dy}{dt} = -ay + bu = -ay - bk_y y + bk_r r$$

or

$$y = \frac{bk_r}{p + a + bk_y} r.$$

The sensitivity derivatives are given by

$$\frac{\partial e}{\partial k_r} = \frac{b}{p + a + bk_y} r$$

$$\frac{\partial e}{\partial k_y} = -\frac{b^2 k_r}{(p + a + bk_y)^2} r = -\frac{b}{p + a + bk_y} y.$$

The MIT-rule, (2.9), gives the following equations for updating the regulator parameters:

$$\frac{dk_r}{dt} = -k \left\{ \frac{b}{p + a + bk_y} r \right\} e$$

$$\frac{dk_y}{dt} = k \left\{ \frac{b}{p + a + bk_y} y \right\} e.$$

These formulas cannot be used because the process parameters  $a$  and  $b$  are not known. Some approximations are, therefore, required in order to obtain realizable parameter adjustment rules. First observe that the parameter  $b$  may be absorbed in the adaptation gain  $k$ . Furthermore, observe that for the equilibrium values of the gain it follows that

$$p + a + bk_y = p + a_m.$$

It is thus reasonable to approximate the quantity on the left-hand side by the quantity on the right-hand side. The following parameter adjustment rule is then obtained:

$$\frac{dk_r}{dt} = -k \left\{ \frac{b_m}{p + a_m} r \right\} e \quad (2.15)$$

$$\frac{dk_y}{dt} = k \left\{ \frac{b_m}{p + a_m} y \right\} e. \quad \square$$

The example shows how the MIT-rule may be applied to obtain a parameter adjustment rule. Although special examples were considered, there are some characteristics that are worth noticing. The procedure can be applied to nonlinear systems. The structure of Fig. 3 appears again. Some approximations are necessary in order to obtain a realizable parameter adjustment control law.

The MIT-rule will perform well if the parameter  $k$  is small. The allowable size depends on the magnitude of the reference signal. Consequently, it is not possible to give fixed limits which guarantee stability. The MIT-rule can thus give an unstable closed-loop system. Modified adjustment rules can be obtained using stability theory. These rules are similar to the MIT-rule. The sensitivity derivatives are, however, replaced by other functions.

The model reference adaptive system represents an important line of development in adaptive control. It has been subject to much theoretical analysis which will be discussed in more detail in Section III. The algorithms have been extended to nonminimum phase systems [73]–[78], multivariable systems [79]–[82], and nonlinear systems [83]. The MRAS have also been applied to a wide range of different problems [18], [26], [84]–[93].

### Self-Tuning Regulators STR

All schemes discussed so far are called direct methods because the adjustment rules tell directly how the regulator parameters should be updated. The self-tuning regulator

is a different scheme where process parameters are updated and the regulator parameters are obtained from the solution of a design problem. A block diagram of such a system is shown in Fig. 8. The adaptive regulator can be thought

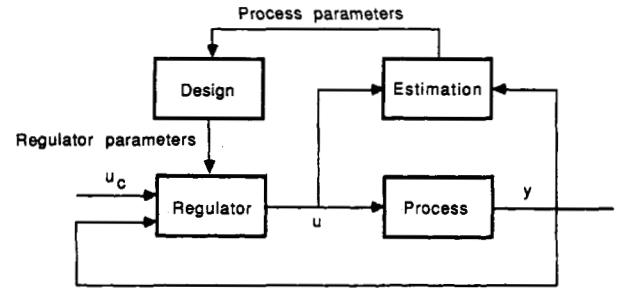


Fig. 8. Block diagram of a self-tuning regulator (STR).

of as composed of two loops. The inner loop consists of the process and an ordinary linear feedback regulator. The parameters of the regulator are adjusted by the outer loop, which is composed of a recursive parameter estimator and a design calculation. To obtain good estimates it may also be necessary to introduce perturbation signals. This function is not shown in Fig. 8 in order to keep the figure simple. Notice that the system may be viewed as an automation of process modeling and design where the process model and the control design are updated at each sampling period.

The block labeled "design" in Fig. 8 represents an on-line solution to a design problem for a system with known parameters. This is called the *underlying design problem*. Such a problem can be associated with most adaptive control schemes. However, the problem is often given indirectly. To evaluate adaptive control schemes it is often useful to find the underlying design problem because it will give the characteristics of the system under the ideal conditions when the parameters are known exactly.

The self-tuner also contains a recursive parameter estimator. Many different estimation schemes have been used, for example stochastic approximation, least squares, extended and generalized least squares, instrumental variables, extended Kalman filtering, and the maximum likelihood method.

The self-tuner shown in Fig. 8 is called an *explicit STR* or an STR based on estimation of an explicit process model. It is sometimes possible to reparameterize the process so that it can be expressed in terms of the regulator parameters. This gives a significant simplification of the algorithm because the design calculations are eliminated. In terms of Fig. 8, the block labeled design calculations disappears and the regulator parameters are updated directly. An example illustrates the idea.

*Example 2.4:* Consider the discrete time system described by

$$y(t+1) + ay(t) = bu(t) + e(t+1) + ce(t),$$

$$t = \dots -1, 0, 1, \dots \quad (2.16)$$

where  $\{e(t)\}$  is a sequence of zero-mean uncorrelated random variables. If the parameters  $a$ ,  $b$ , and  $c$  are known, the proportional feedback

$$u(t) = -\theta y(t) = \frac{a-c}{b} y(t) \quad (2.17)$$

minimizes the variance of the output. The output then becomes

$$y(t) = e(t). \quad (2.18)$$

This can be concluded from the following argument. Consider the situation at time  $t$ . The variable  $e(t+1)$  is independent of  $y(t)$ ,  $u(t)$ , and  $e(t)$ . The output  $y(t)$  is known and the signal  $u(t)$  is at our disposal. The variable  $e(t)$  can be computed from past inputs and outputs. Choosing the variable  $u(t)$  such that the terms underlined in (2.16) vanish makes the variance of  $y(t+1)$  as small as possible. This gives (2.18) and (2.17). For further details, see [94].

Since the process (2.16) is characterized by *three* parameters, a straightforward explicit self-tuner would require estimation of three parameters. Notice, however, that the feedback law is characterized by *one* parameter only. A self-tuner which estimates this parameter can be obtained based on the model

$$y(t+1) = \theta y(t) + u(t). \quad (2.19)$$

The least squares estimate of the parameter  $\theta$  in this model is given by

$$\theta(t) = \frac{\sum_{k=1}^{t-1} y(k) [y(k+1) - u(k)]}{\sum_{k=1}^t y^2(k)} \quad (2.20)$$

and the control law is then given by

$$u(t) = -\theta(t) y(t). \quad (2.21)$$

The self-tuning regulator given by (2.20) and (2.21) has a remarkable property which can be seen heuristically as follows. Equation (2.20) can be written as

$$\begin{aligned} \frac{1}{t} \sum_{k=1}^t y(k+1) y(k) &= \frac{1}{t} \sum_{k=1}^t [\theta(t) y^2(k) + u(k) y(k)] \\ &= \frac{1}{t} \sum_{k=1}^t [\theta(t) - \theta(k)] y^2(k). \end{aligned}$$

Assuming that  $y$  is mean square bounded and that the estimate  $\theta(t)$  converges as  $t \rightarrow \infty$  we get

$$\lim_{t \rightarrow \infty} \frac{1}{t} \sum_{k=1}^t y(k+1) y(k) = 0. \quad (2.22)$$

The adaptive algorithm (2.20), (2.21) thus attempts to adjust the parameter  $\theta$  so that the correlation of the output at lag one is zero. If the system to be controlled is actually governed by (2.16) it follows from (2.18) that the estimate will converge to the minimum variance control law under the given assumption. This is somewhat surprising because the structure of (2.19) which was the basis of the adaptive regulator is not compatible with the true system (2.16).  $\square$

The property illustrated in the example, namely, that the simple self-tuner based on a simplified model structure may give the correct minimum variance control law when applied to a model with correlated disturbances, was explored more fully in [95].

The self-tuning regulator was proposed by Kalman [96], who built a special-purpose computer to implement the regulator. Several experimental investigations were done as digital computers became available, see [97] and [98]. Self-tuners based on least squares estimation and minimum variance regulation were explored in [95], where it was shown

that they will give the correct equilibrium even for colored noise, as was illustrated in Example 2.4. The self-tuning regulator has received considerable attention since then, because it is flexible, easy to understand, and easy to implement with microprocessors. Various modifications to the simple self-tuner have been proposed [99]–[102]. Since the approach is very flexible with respect to the underlying design method, many different extensions have been made. Self-tuners based on phase and amplitude margins are discussed in [103]–[105]. The pole placement design method has been used as described in [73], [78], [85], [106]–[119]. Linear quadratic Gaussian self-tuners are described in [120]–[134]. It has been shown in [135] that self-tuning does not necessarily occur for an LQG problem unless the criterion is modified.

### Relations Between MRAS and STR

The MRAS originated from a continuous time, deterministic servo-problem and the STR from a discrete time, stochastic regulation problem. The two approaches are quite similar in spite of their different origins. This can be seen superficially from the comparison of Fig. 6 and Fig. 8. Both systems have two feedback loops. The inner loop is an ordinary feedback loop with a process and a regulator. The regulator has adjustable parameters which are set by the outer loop. The adjustments are based on feedback from the process inputs and outputs. The methods for design of the inner loop and the techniques used to adjust the parameters in the outer loop are, however, different.

The regulator parameters are updated directly in the MRAS in Fig. 6. In the STR in Fig. 8, they are updated indirectly via parameter estimation and design calculations. This difference is, however, not fundamental because the STR may be modified so that the regulator parameters are updated directly as was shown in Example 2.4.

The model reference adaptive regulator can also be modified so that the parameters are updated indirectly. Such an MRAS scheme is called *indirect*, see [136]. The direct MRAS is closely related to the implicit STR and the indirect MRAS to the explicit STR. More details are found in [112] and [137]–[140].

### Adaptive Schemes Derived from Stochastic Control Theory

The adaptive schemes discussed so far are based on purely heuristic arguments. It would be appealing to obtain adaptive systems from a unified theoretical framework. This can be done using nonlinear stochastic control theory where the system and its environment are described by a stochastic model. To do so, the parameters are introduced as state variables and the parameter uncertainty is modeled by stochastic models. An unknown constant is thus modeled by the differential equation

$$\frac{d\theta}{dt} = 0$$

with an initial distribution that reflects the parameter uncertainty. This corresponds to a Bayesian approach where unknown parameters are viewed as random variables.

Parameter drift is captured by adding random variables to the right-hand sides of the equations. A criterion is formulated as to minimize the expected value of a loss function, which is a scalar function of states and controls.

The problem of finding a control, which minimizes the expected loss function, is difficult. Under the assumption that a solution exists, a functional equation for the optimal loss function can be derived using dynamic programming; see [8], [9], and [141]. The functional equation, which is called the *Bellman equation*, can be solved numerically only in very simple cases. The structure of the optimal regulator obtained is shown in Fig. 9. The controller can be thought

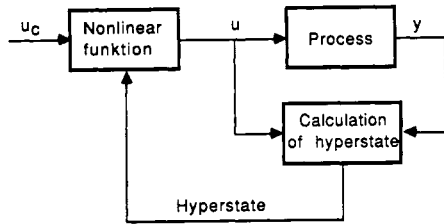


Fig. 9. Block diagram of an adaptive regulator derived from stochastic control theory.

of as composed of two parts: a nonlinear estimator and a feedback regulator. The estimator generates the conditional probability distribution of the state from the measurements. This distribution is called the *hyperstate* of the problem. The feedback regulator is a nonlinear function, which maps the hyperstate into the space of control variables. This function can be computed off-line. The hyperstate must, however, be updated on-line. The structural simplicity of the solution is obtained at the price of introducing the hyperstate, which is a quantity of very high dimension. Updating of the hyperstate requires, in general, the solution of a complicated nonlinear filtering problem. Notice that there is no distinction between the parameters and the other state variables in Fig. 9. This means that the regulator can handle very rapid parameter variations. Notice, however, that it is necessary to have prior information about the stochastic properties of the variations of states and parameters.

The optimal control law has interesting properties which have been found by solving a number of specific problems. The control attempts to drive the output to its desired value, but it will also introduce perturbations (probing) when the parameters are uncertain. This improves the quality of the estimates and the future controls. The optimal control gives the correct balance between maintaining good control and small estimation errors. The name *dual control* was coined by Feldbaum [11] to express this property, see [142]–[144]. Optimal stochastic control theory also offers other possibilities to obtain sophisticated adaptive algorithms, see [37].

It is interesting to compare the regulator in Fig. 10 with the self-tuning regulator in Fig. 8. In the STR, the states are

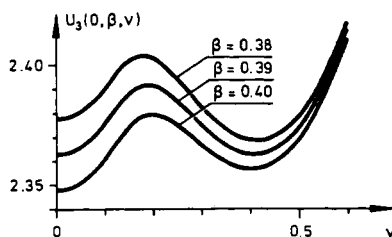


Fig. 10. Graph of the function  $U_3(0, \beta, \nu)$ .

separated into two groups, the ordinary state variables of the underlying constant parameter model and the parameters which are assumed to vary slowly. In the optimal stochastic regulator there is no such distinction. There is no feedback from the variance of the estimate in the STR although this information is available in the estimator. In the optimal stochastic regulator there is feedback from the full conditional distribution of parameters and states. The design calculations in the STR are made in the same way as if the parameters were known exactly; there are no attempts to modify the control law when the estimates are uncertain. In the optimal stochastic regulator, the control law is calculated based on the hyperstate which takes full account of uncertainties. This also introduces perturbations when estimates are poor. The comparison indicates that it may be useful to add parameter uncertainties and probing to the STR.

A simple example illustrates the dual control law and some approximations.

*Example 2.5—From [145]:* Consider a discrete time system described by

$$y(t+1) = y(t) + bu(t) + e(t) \quad (2.23)$$

where  $u$  is the control,  $y$  the output, and  $e$  normal  $(0, \sigma_e)$  white noise. Let the criterion be to minimize the mean square deviation of the output  $y$ . This is a special case of the system in Example 2.4 with  $a = 1$  and  $c = 0$ . When the parameters are known, the optimal control law is given by (2.17), i.e.,

$$u(t) = -\frac{y(t)}{b}. \quad (2.24)$$

If the parameter  $b$  is assumed to be a random variable with a Gaussian prior distribution, the conditional distribution of  $b$ , given inputs and outputs up to time  $t$ , is Gaussian with mean  $\hat{b}(t)$  and standard deviation  $\sigma(t)$ . The hyperstate can then be characterized by the triple  $(y(t), \hat{b}(t), \sigma(t))$ . The equations for updating the hyperstate are the same as the ordinary Kalman filtering equations, see [94] and [146].

Introduce the loss function

$$V_N = \min_u E \frac{1}{\sigma^2} \left\{ \sum_{k=t+1}^{t+N} y^2(k) | Y_t \right\} \quad (2.25)$$

where  $Y_t$  denotes the data available at time  $t$ , i.e.,  $\{y(t), y(t-1), \dots\}$ . The loss function is thus the conditional mean square error of the control errors  $N$  steps ahead. By introducing the normalized variables

$$\eta = y/\sigma_e \quad \beta = \hat{b}/\sigma \quad \mu = -\hat{b}/y \quad (2.26)$$

it can be shown that  $V_N$  depends on  $\eta$  and  $\beta$  only. The Bellman equation for the problem can be written as

$$V_N(\eta, \beta) = \min U_N(\eta, \beta, \mu) \quad (2.27)$$

where

$$V_0(\eta, \beta) = 0$$

and

$$U_N(\eta, \beta, \mu) = 1 + \eta^2(1 - \mu^2) + \frac{\mu^2 \eta^2}{\beta^2} + \int_{-\infty}^{\infty} V_{N-1}(y, b) \varphi(\epsilon) d\epsilon \quad (2.28)$$

where  $\varphi$  is the normal probability density and



$$y = \eta + \beta\mu + \epsilon\sqrt{1 + \mu^2}$$

$$b = \epsilon\mu + \beta\sqrt{1 + \mu^2}$$

see [146]. When the minimization is performed, the control law is obtained as

$$\mu_T(\eta, \beta) = \arg \min U_N(\eta, \beta, \mu). \quad (2.29)$$

The minimization can be done analytically for  $N = 1$ . We get

$$\mu_1(\eta, \beta) = \arg \min [(1 - \beta\mu)^2 + 1 + \mu^2] = \frac{\beta^2}{1 + \beta^2}.$$

Transforming back to the original variables we get

$$u(t) = -\frac{1}{\hat{b}(t)} \cdot \frac{\hat{b}^2(t)}{\hat{b}^2(t) + \sigma^2(t)} y(t). \quad (2.30)$$

This control law is called *one-step control* or *myopic control* because the loss function  $V_1$  only looks one step ahead. It is also called *cautious control* because in comparison with the certainty equivalence control it hedges by decreasing the gain when the estimate of  $b$  is uncertain.

For  $N > 1$ , the optimization can no longer be made analytically. Instead, we have to resort to numerical calculations. The solution has some interesting properties. Fig. 10 shows the function  $U_3(0, \beta, \mu)$  for different values of  $\beta$ . Notice that the function has several local minima with respect to  $\mu$ . For  $\beta = 0.40$  the minimum at  $\mu = 0$  is the smallest one but for  $\beta = 0.38$  the minimum at  $\mu = 0.42$  is the smallest one. The control  $\mu_3(\eta, \beta)$  is thus discontinuous in  $\beta$ . For  $\eta = 0$ , the control signal is zero if  $\beta$  is sufficiently large, i.e., the estimates are reasonably accurate. When the estimates become sufficiently poor,  $\beta < 0.39$ , the control signal  $\mu$  jumps to about 0.4. The discontinuity of the control law corresponds to the situation that a probing signal is introduced to improve the estimates.

Some approximations to the optimal control law will also be discussed. The *certainty equivalence control*

$$u(t) = -y(t)/\hat{b} \quad (2.31)$$

is obtained simply by taking the control law (2.24) for known parameters and substituting the parameters by their estimates. The self-tuning regulator can be interpreted as a certainty equivalence control. Using normalized variables, the certainty equivalence control law becomes

$$\mu = 1. \quad (2.31')$$

Using normalized variables the cautious control law can be expressed as

$$\mu = \frac{\beta^2}{1 + \beta^2}. \quad (2.30')$$

Notice that all control laws are the same for large  $\beta$ , i.e., if the estimate is accurate. The optimal control law is close to the cautious control for large control errors. For estimates with poor precision and moderate control errors the dual control gives larger control actions than the other control laws. A graphical representation of the control laws for time horizons  $N = 1, 3, 6$ , and  $31$  are given in Fig. 11.

### III. THEORY

Theory has different roles in analysis and design of adaptive control systems. Analysis aimed at understanding specific algorithms is one goal. Creation of new adaptive control laws is another role. Adaptive systems are inherently nonlinear. Their behavior is also quite complex which makes them difficult to analyze. Progress in theory has been slow and much work remains before a reasonably complete coherent theory is available. In this section it is attempted

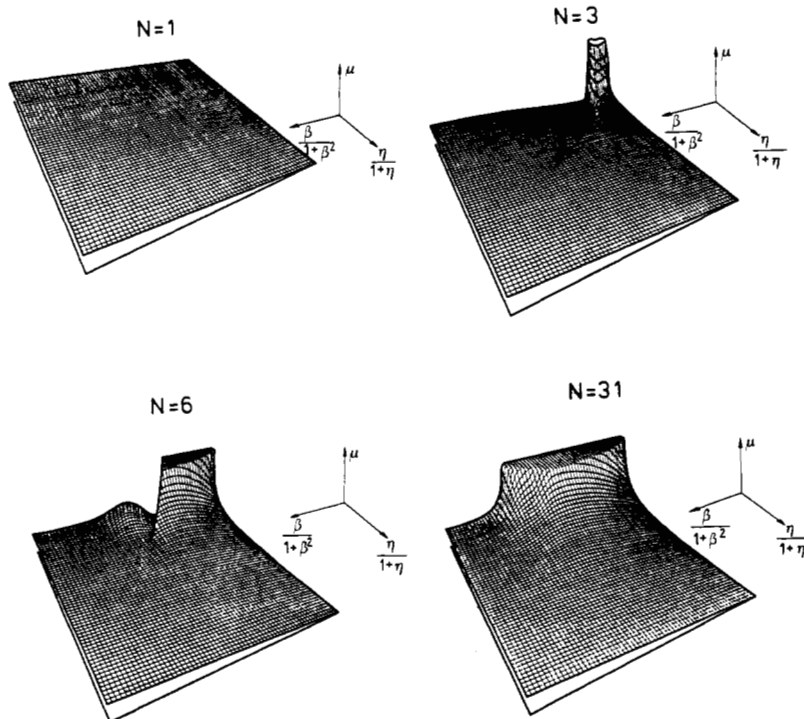


Fig. 11. Illustration of the cautious control  $\mu_1$  and the dual control laws  $\mu_3$ ,  $\mu_6$ , and  $\mu_{31}$ . The graphs show the normalized control variable as a function of the normalized parameter uncertainty  $\beta^2/(1 + \beta^2)$  and the normalized control error  $\eta/(1 + \eta)$ .

to discuss a number of different issues and results that are available.

Because of the complex behavior of adaptive systems it is necessary to consider them from several points of view. Theories of nonlinear systems, stability, system identification, recursive estimation, convergence of stochastic algorithms, and optimal stochastic control all contribute to the understanding of adaptive systems.

### Generic Problems

A considerable effort has been devoted to construction of models which can serve as prototypes for general adaptive problems. The early work concentrated on systems where there was only a variation in the process gain. The system shown in Example 2.2 is a typical example. This example can be used to show that adaptive systems are inherently nonlinear.

**Example 3.1—Adaptive Systems are Not Linear:** Consider the system in Example 2.2 with  $G(p) = 1$ . Assume that we attempt to use a parameter adjustment rule which is linear in the model error. The equation for updating the parameters is then given by

$$\frac{d\theta}{dt} = -ker = k\epsilon(1 - \theta). \quad (3.1)$$

This equation is stable only if the product  $k\epsilon$  is positive. With a fixed adaptation gain  $k$  it is thus impossible to obtain a stable solution if the reference signal can assume both positive and negative values.  $\square$

Much attention has been given to single-input single-output systems described by the equation

$$A(q)y(t) = B(q)u(t) + v(t). \quad (3.2)$$

In this model  $u$  is the control variable,  $y$  is the measured output, and  $v$  is a disturbance.  $A$  and  $B$  are polynomials in the forward shift operator for discrete time systems and polynomials in the differential operator for continuous time systems. Multivariable systems where  $u$  and  $y$  are vectors and  $A$  and  $B$  are matrix polynomials have also been explored.

The model (3.2) represents a system where the system dynamics is totally unknown. In many applications the situation is quite different because the system is partially known. This situation has not been investigated much perhaps because each problem has a special structure. We believe, however, that the problem is of significant practical interest; see [147] and [148].

It is customary to separate the tuning and the adaptation problems. In the tuning problem it is assumed that the process to be controlled has constant but unknown parameters. In the adaptation problem it is assumed that the parameters are changing. Many issues are much easier to handle in the tuning problem. The convergence problem is to investigate if the parameters converge to their true values. The corresponding problem is much more difficult in the adaptive case because the targets are moving. The estimation algorithms are the same in tuning and adaptation. They can be described by

$$\theta(t+1) = \theta(t) + P(t)\psi(t)[y(t+1) - \varphi^T(t)\theta(t)]. \quad (3.3)$$

Compare with the Appendix, where several algorithms are described in detail. The gain matrix  $P$  behaves, however,

very differently in tuning and adaptation. It goes to zero as  $t$  increases in the tuning case but it does not converge to zero in the adaptive case.

### Stability

Much effort has been devoted to analysis of stability of adaptive systems. It is important to keep in mind that the stability concepts for nonlinear differential equations refer to stability of a particular solution. It is thus often the case that one solution is stable while another unstable.

Stability theory has been the major inspiration for the development of model reference adaptive systems. It has, however, not been applied to systems with gain scheduling. This is surprising since such systems are simpler than MRAS and STR. When experimenting with Whitaker's system it was found that the system could become unstable in certain circumstances. This inspired Donelson and Leondes [149], Shackcloth and Butchart [150], and Parks [151] to apply stability analysis. This was followed by a lot of research where stability theories of Lyapunov and Popov were applied to the analysis of adaptive control systems.

The stability conditions for Whitaker's scheme have been explored in detail for the simple case of adjustment of a feedforward gain shown in Example 2.2 [152]. When the reference signal  $r$  is as sinusoid, (2.10) becomes a linear time-varying differential equation similar to the classical Mathieu equation. Such an equation can be investigated using Floquet theory. Stability conditions were determined by James [152] using a combination of analytical and numerical methods. Fig. 12 shows the results. It shows that the stability conditions can be very complicated even in simple cases.

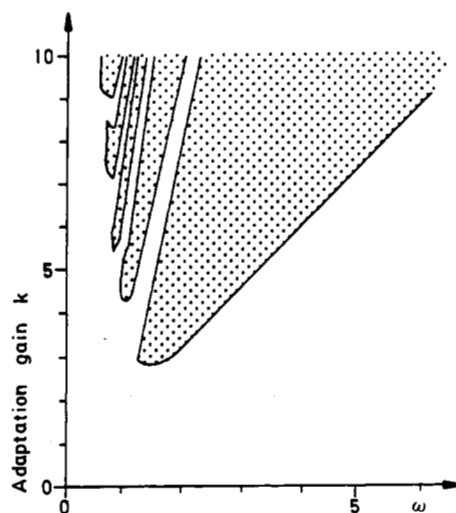


Fig. 12. Stability conditions for (2.10) when  $r = \cos \omega t$ , and  $G(s) = 1/(s+1)$ . The system is unstable in the dotted areas.

Stability theory has also been used to construct adjustment mechanisms, which give stable closed-loop systems; see [18], [26]. The example below shows how this can be done.

**Example 3.2—A Parameter Adjustment Rule Derived from Stability Theory:** Consider the same problem as in Example 2.3 and assume that the parameters  $a$  and  $b$  are constant. When the parameters of the process are known, the control

law (2.13), (2.14) gives the desired result. A model reference adaptive system which can find the appropriate gains  $k_r$  and  $k_y$  when the parameters  $a$  and  $b$  are not known is obtained as follows. Introduce the error

$$e = y - y_m.$$

Taking derivatives and using (2.11) and (2.12) to eliminate the derivatives of  $y$  and  $y_m$  gives

$$\frac{de}{dt} = -a_m e + (a_m - a - bk_y) y + (bk_r - b_m)r. \quad (3.4)$$

Notice that the error goes to zero if the gains are given by (2.14). It will now be attempted to construct a parameter adjustment mechanism which will drive the parameters  $k_r$  and  $k_y$  to the values of (2.14). For this purpose, the Lyapunov function

$$V(e, k_r, k_y) = \frac{1}{2} \left[ e^2 + \frac{1}{bk} (bk_y + a - a_m)^2 + \frac{1}{bk} (bk_r - b_m)^2 \right] \quad (3.5)$$

is introduced. The derivative of  $V$  is

$$\begin{aligned} \frac{dV}{dt} &= e \frac{de}{dt} + \frac{1}{k} (bk_y + a - a_m) \frac{dk_y}{dt} \\ &\quad + \frac{1}{k} [bk_r - b_m] \frac{dk_r}{dt} \\ &= -a_m e^2 + \frac{1}{k} (bk_y + a - a_m) \left[ \frac{dk_y}{dt} - kye \right] \\ &\quad + \frac{1}{k} (bk_r - b_m) \left[ \frac{dk_r}{dt} + kre \right]. \end{aligned}$$

If the parameters are updated as

$$\begin{aligned} \frac{dk_r}{dt} &= -kre \\ \frac{dk_y}{dt} &= kye \end{aligned} \quad (3.6)$$

we get

$$\frac{dV}{dt} = -a_m e^2. \quad (3.7)$$

The function  $V$  will thus decrease as long as the error  $e$  is different from zero and it can thus be concluded that the error will go to zero. Notice, however, that it does not follow that the parameters  $k_r$  and  $k_y$  will converge to the equilibrium values unless more conditions are imposed. This will be discussed in more detail later.

The parameter adjustment rule then becomes

$$\frac{d\theta}{dt} = -k\varphi e \quad (3.8)$$

where

$$\theta = \begin{bmatrix} k_r \\ k_y \end{bmatrix} \quad \text{and} \quad \varphi = \begin{bmatrix} r \\ -y \end{bmatrix}.$$

The rule is thus similar to the MIT-rule (2.9) but sensitivity derivatives  $\partial e / \partial \theta$  are replaced by the regression vector  $\varphi$ .  $\square$

An interesting result is obtained by applying stability theory to the problem of adjusting a feedforward gain.

**Example 3.3:** The following parameter adjustment law can be obtained by applying Lyapunov theory to the problem in Example 2.2:

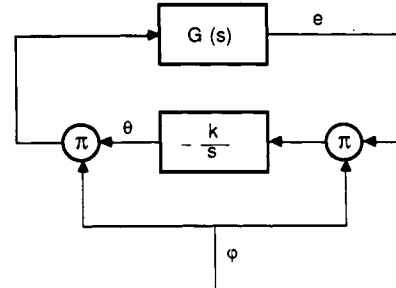
$$\frac{d\theta}{dt} = -kre. \quad (3.9)$$

The rate of change of the parameter is proportional to the product of the error  $e$  and the reference  $r$ . Notice that the adjustment law in Example 2.2 obtained by the MIT-rule gives a rate of change proportional to  $y_m e$ . In the case of  $r = \cos \omega t$  and  $G(s) = 1/(s+1)$  which corresponds to Fig. 12 it can be shown that the system is stable for all  $k$  and  $\omega$ ; see [151].  $\square$

Example 3.3 shows that a minor modification of the MIT-rule can give a much larger stability region. A direct generalization of the method used in the examples shows that it is possible to derive stable parameter adjustment rules for systems where all the state variables are measurable. The key step is to derive an equation corresponding to (3.4) which expresses the error  $e$  in terms of the parameter errors. This equation is called *the error equation*.

### Output Feedback

It is much more difficult to construct parameter adjustment rules when all state variables are not measured directly. Other methods are then required. Hyperstability theory [18], [26], [153], is a useful tool. Consider, e.g., the system in Example 3.3. The block diagram of the system can be redrawn as shown in Fig. 13. The closed-loop system is



**Fig. 13.** Block diagram of a model reference adaptive system.

thus composed of a linear system with the transfer function  $G(s)$  and a nonlinear system. This is a generic system configuration for which there are several stability conditions. To express these we need two concepts, positive-real and passive. Notice that Fig. 13 is an abstract representation which is useful for analysis but that it does not represent an implementation of the adaptive system. The notions positive-real and strictly positive-real originated in circuit theory to characterize driving-point functions for networks.

#### Definition:

A transfer function  $G(s)$  is called positive-real (PR) if  $\operatorname{Re} G(s) \geq 0$  for  $\operatorname{Re} s \geq 0$ . It is strictly positive-real (SPR) if  $G(s) - \epsilon$  is PR for some positive  $\epsilon$ .

#### Definition:

A system with input  $u$  and output  $y$  is passive if

$$\int_0^T y(t) u(t) dt = \geq 0.$$

A feedback loop which is composed of a linear part and a nonlinear part, as in Fig. 13, is stable if the linear part is strictly positive-real (SPR) and the nonlinear part is passive, see [154].

The nonlinear system which represents the parameter adjustment mechanism in Fig. 13 is passive because it follows that

$$\begin{aligned} \int_0^T y(t) u(t) dt &= \int_0^T \varphi(t) \left[ \int_0^t \varphi(s) u(s) ds \right] u(t) dt \\ &= \frac{1}{2} \left[ \int_0^T \varphi(t) u(t) dt \right]^2 \geq 0. \end{aligned}$$

To ensure stability it then remains to ensure that the transfer function  $G(s)$  is SPR. If this is not the case it can be attempted to filter the error with a linear system  $G_f$  such that the combination  $GG_f$  is SPR. In this way it is possible to obtain adjustment rules for systems which are minimum phase and have relative degree one. To obtain the representation shown in Fig. 13, it is necessary to parameterize the model so that it is linear in the parameters. The model should thus be of the form

$$y(t) = \varphi^T(t)\theta \quad (3.10)$$

where  $\varphi$  is a vector of signals and  $\theta$  a vector of parameters. This requirement strongly limits the algorithms that can be considered. Notice that if a system is linear in the parameters then the error can be represented as the output of a dynamical system driven by the signal  $\varphi^T[\theta_0 - \theta]$  where  $\theta_0$  are the true parameters, i.e.,

$$e = G(p) \{ \varphi^T[\theta - \theta_0] \} = G(p) \{ \varphi^T x \} \quad (3.11)$$

where  $x$  is the parameter error

$$x = \theta - \theta_0. \quad (3.12)$$

This equation is an error model of the type discussed in Example 3.2. Compare also with Fig. 13.

#### The Augmented Error

The general problem with output feedback poses additional problems, because the desired representation cannot be obtained by filtering the model error. A clever idea was given by Monopoli [155]. He suggested to replace the error (3.11) by the quantity

$$\epsilon = e + [x^T G(p)\varphi - G(p)\varphi^T x] = x^T G(p)\varphi \quad (3.13)$$

where the second equality follows from (3.11). The signal  $\epsilon$  is called the augmented error and the correction signal within brackets is called the error augmentation. This signal can be generated by realizable operations. Notice that the augmented error is equal to the error when the parameters are constant. The correction signal is then zero. Monopoli suggested using the parameter adjustment rule

$$\frac{d\theta}{dt} = -k \{ G(p)\varphi \} \epsilon \quad (3.14)$$

where  $\epsilon$  is defined by (3.13). This differs from (3.8) in two

ways: The regression vector has been filtered by  $G$  and the error  $e$  in (3.8) is replaced by the augmented error.

For systems where only a gain is adjusted it is straightforward to show that Monopoli's approach leads to a stable closed-loop system. It follows from (3.12) and (3.13) that

$$\frac{dx}{dt} = \frac{d\theta}{dt} = -kx[G(p)\varphi]^2. \quad (3.15)$$

Notice that  $\varphi$  is equal to the reference signal. Compare with Example 3.1.

#### Stability Conditions for a Direct MRAS

The stability problem is much more difficult in the general case because the  $r$  is replaced by the regression vector  $\varphi$  whose components are functions of the inputs and outputs of the system, these depend on old parameter values, see the Appendix. It is essential to show that the regression vector is bounded. Many attempts were made to provide stability proofs during the 1970s. It was not until the dependence of the regressive vector on the parameters was explicitly taken into account that correct proofs were given. This difficulty which remained unnoticed for many years was pointed out in [156]. Stability proofs were given by [137], [157]–[166]. The key elements of the proof is to show that the parameter error is bounded and to use estimates of the growth rates of the signals in the system to show that the regressor is bounded. An elegant formalism for estimating the growth rates is given in [167].

The following assumptions are required to prove stability of the system obtained when applying a direct MRAS based on gradient estimation to the system (3.2).

- A1 the relative degree  $d = \deg A - \deg B$  is known,
- A2 the sign of the leading coefficient  $b_0$  of the polynomial  $B$  in (3.2) is known,
- A3 the polynomial  $B$  in (3.2) is stable,
- A4 the estimated model is at least of the same order as the process.

The stability theorems are important because they give simple and rigorous analysis of an idealized adaptive system. The assumptions required are, however, very restrictive.

Assumption A1 for discrete systems means that the time delay is known with a precision which corresponds to a sampling period. This is not unreasonable. For continuous time systems the assumption means that the slope of the high-frequency asymptote of the Bode diagram is known. Together with assumption A2 it also means that the phase is known at high frequencies. If this is the case, it is possible to design a robust high-gain regulator for the problem, see [168]–[170]. For many systems, such as a flexible aircraft, electromechanical servos, and flexible robots, the main difficulty in control is the uncertainty of the dynamics at high frequencies, see [67]. A comparison between robust and adaptive control is given in [171].

Assumption A3 is also crucial. It arises from the necessity to have a model, which is linear in the parameters. The underlying design method is, in fact, based on cancellation of all process zeros. Such a design will not work even for systems which known constant parameters if the system has an unstable inverse.

Assumption A4 is very restrictive, since it implies that the estimated model must be at least as complex as the true system, which may be nonlinear with distributed parameters. Almost all control systems are, in fact, designed based on strongly simplified models. High-frequency dynamics are often neglected in the simplified models. It is, therefore, very important that a design method can cope with model uncertainty; compare [172] and [168].

### Bounded Disturbances

The stability results given above require that there are no disturbances. The signal  $v$  in (3.2) thus must be zero. The analysis in [157], [159], [160], [173]–[176] also applies to the case when the disturbance  $v$  in (3.2) is bounded. Egardt has constructed examples which show that modifications of the algorithms are necessary. One possibility is to bound the parameter estimates *a priori* by projecting the estimates into a bounded region of parameter space. Another possibility is to introduce a dead zone in the estimator which keeps the estimates constant if the residuals are small.

### Persistency of Excitation

Notice that even if the error  $e$  goes to zero there is no guarantee that the parameters will converge to their true values. Some conditions on the input signal are needed when the parameters of a system are determined from input-output data. An example illustrates what may happen.

**Example 3.4:** Consider the simple adaptive system in Example 3.2. Let  $G(s) = 1$ . The parameter adjustment law then becomes

$$\frac{d\theta}{dt} = kr^2[1 - \theta]. \quad (3.16)$$

Assume that the reference signal  $r$  is an exponential

$$r(t) = e^{-\alpha t}.$$

Equation (3.16) then has the solution

$$\theta(t) = 1 + (\theta_0 - 1)e^{-(k/2\alpha)[1 - e^{-2\alpha t}]},$$

As time increases the solution converges to

$$\theta_\infty = 1 + (\theta_0 - 1)e^{-(k/2\alpha)}.$$

Notice that the limit value depends on the initial condition and that it is different from the equilibrium value if  $\theta_0 \neq 1$ .  $\square$

The reason why the parameter does not converge to the equilibrium point is that the input signal  $r$  does not excite the system sufficiently. In the example it is easily seen that any input such that

$$\int_0^t r^2(s) ds$$

going to infinity as  $t$  increases will drive the parameter error to zero.

The notion of persistent excitation was introduced in [177] in connection with system identification. An input signal  $u$  is called persistently exciting (PE) of order  $n$  if its sample covariance function

$$C_u(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T u(t + \tau) u(t) dt = \overline{u(t + \tau) u(t)} \quad (3.17)$$

exist and if the matrix

$$\begin{bmatrix} C_u(0) & \cdots & C_u(n-1) \\ \vdots & & \vdots \\ C_u(n-1) & \cdots & C_u(0) \end{bmatrix} \quad (3.18)$$

is positive-definite.

When estimating parameters in linear models as in (3.2), the input signal must be persistently exciting of order  $\deg A + \deg B + 1$ . If it is not, the parameters cannot be estimated. A step signal is persistently exciting of order 1 and a sinusoidal signal of order 2.

The notion of persistent excitation can also be expressed in the frequency domain. Generalized harmonic analysis can be applied to a signal with the property (3.17) and a spectrum can then be defined. It can be shown that if the spectrum is different from zero at  $n$  positive frequencies then it is also persistently exciting of order  $2n$ . A signal is persistently exciting of any order if the spectrum is different from zero in any interval, see [178]–[180].

The notion of persistent excitation is important in order to understand the behavior of adaptive control systems [181]. If the process input is not persistently exciting, it is not possible to determine the parameters. This means that the parameters may drift on hypersurfaces in the parameter space. The importance of the condition has been noticed in many papers [182]–[186]. The difficulties with a model reference system noticed in [187] can also be explained due to the lack of persistent excitation, see [188] and [189].

In an adaptive system, the input signal is created by feedback from the output. It is then a nontrivial task to guarantee that it is persistently exciting unless external perturbations are introduced. It is, however, always possible to monitor the process input and to detect if it is not persistently exciting. Such a monitoring device can be used to control injection of perturbation signals or to switch off adaptation when the input signal is not exciting. This device was suggested in [126]. Simpler versions are also used in several commercial systems. This is discussed in Section IV.

### Averaging Methods

Many adaptive algorithms are motivated by the assumption that the parameters change slower than the state variables of the system. We can make sure that the parameters change slowly by choosing a small adaptation gain. The variables describing the adaptive system can then be separated into two groups which change at different rates. The adjustable parameters are the slow parameters and the state of the controlled dynamical system are the fast variables. It is then natural to derive approximations from this fact.

Consider, e.g., the parameter adjustment laws (3.3) and (3.8). The left-hand side of these equations are the slowly varying parameters. The right-hand sides contain both slow and fast variables. One possibility is to approximate the right-hand sides by taking averages of the fast variables. The averages are calculated under the assumption that the slow variables are constant. This approximation is called averaging. The idea originated in analysis of nonlinear oscillations, [190]–[192]. The method has been used extensively in many branches of applied mathematics, [193], [194].

There are averaging theorems for both deterministic and

stochastic systems. For deterministic systems it must be required that the rapidly varying signals are periodic or almost periodic, see [193], [194]. The averaging approximations will be better for smaller adaptation gains. In several cases it can be shown that the difference between the true and the averaged equations is proportional to the adaptation gain. There are averaging results for stochastic systems when the signals are ergodic [195].

Averaging methods have been used to analyze adaptive systems [180], [196]–[201]. The simple self-tuner [95] based on stochastic approximation and minimum variance control was analyzed by Ljung [200]. It was shown that the parameter estimates converge if the signals are bounded and if the disturbances are such that  $C(z)$  is strictly positive-real. A similar analysis of the self-tuner based on minimum variance control and least squares estimation requires the condition that the function

$$G(z) = 1/C(z) - \frac{1}{2}$$

is strictly positive-real. Holst [201] made a detailed analysis of the local behavior. He showed that the equilibrium corresponding to minimum variance control is locally stable if the function  $C(z)$  is positive at the zeros of  $B(z)$ .

Åström [202] used averaging theory to understand strange behavior of MRAS reported by Rohrs [203]. Averaging methods have recently received much attention and they may lead to the unification of analysis of adaptive systems. The monograph [32] gives a lucid exposition of recent applications of averaging theory to analysis of adaptive systems.

### Instability Mechanisms

Apart from the stability proofs it is also useful to have an understanding of the mechanisms that may create instability. To develop this insight we will consider a model which is linear in the parameters with the parameter adjustment rule (3.8) derived from stability theory as in Example 3.2. The system can be described by the equations

$$\begin{aligned} y &= G(p)u \\ u &= \theta^T \varphi \\ \frac{d\theta}{dt} &= -k\varphi e \\ e &= y - y_m \end{aligned} \quad (3.19)$$

where  $u$  is the process input,  $y$  the process output,  $y_m$  the desired model output,  $e$  the error, and  $\theta$  a vector of adjustable parameters. The transfer function of the process is  $G$ . The components of the vector  $\varphi$  are functions of the command signal. In Example 3.2 the vector  $\varphi$  is given by

$$\varphi = [r - y]^T$$

where  $r$  is the reference signal. It follows from (3.19) that

$$\frac{d\theta}{dt} + k\varphi[G(p)\varphi^T\theta] = k\varphi y_m. \quad (3.20)$$

This equation gives insight into the behavior of the system. Notice that  $G(p)$  is a differential operator which operates on the time function  $\varphi^T\theta$ .

Assume first that the adaptation loop is much slower than the process dynamics. The parameters then change much slower than the regression vector and the term  $G(p)\varphi^T\theta$  in (3.3) can then be approximated by its average

$$G(p)\varphi^T\theta \approx \overline{[G(p)\varphi^T(\theta)]}\theta. \quad (3.21)$$

Notice that the regression vector depends on the parameters. The following approximation to (3.20) is obtained:

$$\frac{d\theta}{dt} + k\varphi(\theta)\overline{[G(p)\varphi^T(\theta)]}\theta \approx k\varphi y_m. \quad (3.22)$$

This is the normal situation because the adaptive algorithm is motivated by the fact that the parameters change slower than the other variables in the system under this assumption. Notice, however, that it is not easy to guarantee this.

Equation (3.22) is stable if  $k\varphi[G(p)\varphi^T]$  is positive. This is true, e.g., if  $G$  is SPR and if the input signal is persistently exciting.

The approximation (3.21) is based on the assumption that the parameters  $\theta$  change much slower than the other system variables. If the parameters  $\theta$  change faster than  $\varphi$  then (3.20) can be approximated by

$$\frac{d\theta}{dt} + k\varphi\varphi^T G(p)\theta \approx k\varphi y_m. \quad (3.23)$$

A linearization for constant  $\varphi_0$  shows that the stability is governed by the algebraic equation

$$\det[sI + k\varphi_0\varphi_0^T G(s)] = s^n - 1[s + KG(s)] = 0$$

where  $I$  is the identity matrix and  $K$  given by

$$K = k\varphi_0^T\varphi_0$$

is the equivalent adaptive loop gain. The stability can then be determined by a simple root-locus argument. Equation (3.23) was originally derived by Shackcloth and Butchart [150]. It was also used by Parks [151] under the name “adaptive step response” and by Rohrs et al. [187] who called it “ $d^*$ -root locus.”

For sufficiently large  $k\varphi_0^T\varphi_0$  the system will always be unstable if the pole-excess of  $G(s)$  is larger than or equal to 2. Also notice that the equivalent gain  $K$  is proportional to  $\varphi_0^T\varphi_0$ . The equivalent gain can thus be made arbitrarily large by choosing the command signal large enough. It thus seems intuitively clear that the adaptive system can be made unstable by making the command signal large enough. Once the source of the difficulty is recognized it is easy to find a remedy. Since the equivalent gain  $K$  in the adaptive loop is too large because of its signal dependence, one possibility is simply to modify the parameter updating law to

$$\frac{d\theta}{dt} = -\frac{d}{1 + \varphi^T\varphi} \varphi e. \quad (3.24)$$

The equivalent gain in the adaptation loop is then bounded and the parameters  $\theta$  will change arbitrarily slow at all signal levels. A suitable value of the adaptation gain  $k$  can be chosen based on a simple root-locus argument.

The modification of the parameter updating law is also essential for the stability proofs such as those in [162], [165], and [166]. It is also worthwhile to note that a law of this type is obtained automatically when adaptive laws are derived from recursive estimation, see [188].

### Universal Stabilizers

After the appearance of the stability proofs much research was initiated to find out if the assumptions given above are necessary in order to obtain a stable adaptive system. It was conjectured by Morse [204] that assumption A2 was nec-

essary. Morse also suggested the following test problem. Consider the system

$$\frac{dy}{dt} = ay + bu.$$

Find a feedback law of the form

$$u = f(\theta, y)$$

$$\frac{d\theta}{dt} = g(\theta, y)$$

which stabilizes the system for all  $a$  and  $b$ . Morse also conjectured that there are no rational  $f$  and  $g$  which stabilize the system. Morse's conjecture was proven by Nussbaum [205], who also showed that there exist nonrational  $f$  and  $g$  which will stabilize the system, e.g., the following functions:

$$f(\theta, y) = y\theta^2 \cos \theta$$

$$g(\theta, y) = y^2.$$

This problem has been explored further in [206] and [207]. Mårtensson [208], [209] has shown that the order of any stabilizing regulator is the necessary and sufficient *a priori* knowledge needed for adaptive stabilization.

#### Parameter Convergence

The behavior of the parameters is an important issue in adaptive control. This has also been the subject of much theoretical investigation [210]. A typical approach is to assume that the system to be controlled is known and to investigate the behavior of the estimated parameters. The key problems are determination of convergence conditions, possible convergence points, and convergence rates. These problems have also been investigated in connection with system identification, see, e.g., [14] and [211]. The results depend in a complicated way on the process model, the disturbances, and the estimation algorithm. There is, however, one additional complication in the adaptive case because the input to the process is generated by feedback.

A few simple observations can be made. If there are no disturbances, if the process input is persistently exciting, and if the model structure is appropriate then the parameters can be determined exactly in a finite number of sampling periods. If the recursive least squares method given by (3.3) is used the matrix  $P$  will then decrease rapidly. It becomes zero when the parameters are known exactly. Algorithms with constant gain such as the gradient method or (3.3) with constant  $P$  or constant trace, or algorithms with covariance resetting have exponential convergence.

There are considerable differences between the convergence rates obtained by different algorithms. This is illustrated in Fig. 14 which shows the parameter estimates obtained when the standard model reference algorithm of Example 3.2 is applied to a first-order system where the command signal is a square wave. The estimates obtained with a recursive least squares algorithm are also shown. Notice the drastic difference in convergence rates. The least squares estimate converges in a few steps to the correct values since the model structure is correct and there are no disturbances. The conventional MARS which is based on a gradient method has a much slower convergence rate.

When discussing convergence rates it is also important to keep in mind that performance measures are approxi-

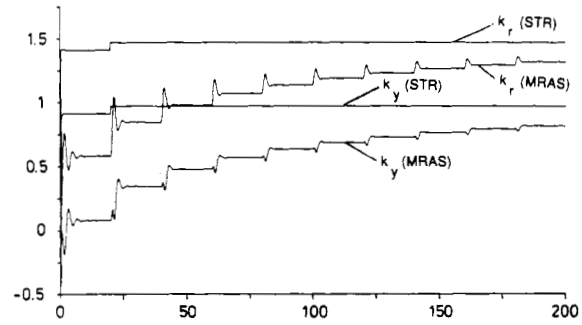


Fig. 14. Comparison of the parameter convergence rates obtained with an MRAS based on the gradient method and on STR based on least squares estimation.

mately quadratic functions of the parameter errors. This means that the differences in performance can be very small for moderate parameter errors.

When the system has constant parameters and there are random disturbances the parameters of the adaptive systems will fluctuate. The magnitude of the fluctuations decreases with decreasing adaptation gain. Selection of suitable gains in adaptive control algorithms is thus a compromise between the tracking rate and precision. It is necessary to have algorithms with decreasing gain in order to obtain estimates that converge. The gain will typically decrease as  $1/t$ .

A very general proof for convergence of the least squares algorithm is given in [212] and [213] by applying a Martingale convergence theorem. The algorithm is given in the Appendix by (A13) (with  $\psi = \varphi$ ) and (A16). The model is given by

$$Ay = Bu + e$$

where  $e$  is white noise. The convergence condition is simply that  $P(t) \rightarrow 0$  as  $t \rightarrow \infty$ . The approach is Bayesian which means that the parameters are assumed to be random variables. This poses some conceptual difficulties because nothing can be said about convergence for particular values of the parameters.

A convergence theorem for the simple self-tuner based on modified stochastic approximation estimation, where the adaptation gain is given by

$$P(t) = a_0/t$$

and the minimum variance control, was given in [214]. A system described by the model (3.2) with

$$v = Ce \quad (3.25)$$

where  $e$  is white noise was investigated. To prove convergence it was required that the function

$$G(z) = C(z) - a_0/2$$

were strictly positive-real. Parameter convergence in adaptive systems are also discussed in [215]–[219].

#### Equilibrium Conditions

In many cases it is possible to calculate possible equilibrium values of the parameters. Such calculations give insight into possible behavior of the systems. The simple self-tuner based on minimum variance control and least squares or stochastic approximation was investigated in [95]. It was shown that the equilibria were characterized by



the equation

$$E\varphi\epsilon = 0.$$

This equation implies

$$\lim_{t \rightarrow \infty} \frac{1}{t} \sum_{k=1}^t y(t + \tau) y(t) = 0,$$

$$\tau = d, d + 1, \dots, d + \deg S$$

$$\lim_{t \rightarrow \infty} \frac{1}{t} \sum_{k=1}^t y(t + \tau) u(t) = 0,$$

$$\tau = d, d + 1, \dots, d + \deg R \quad (3.26)$$

where  $d = \deg A - \deg B$ . This characterizes the possible equilibria even if the process is nonlinear or of high order. It was also shown in [95] that surprisingly the minimum variance control is an equilibrium even when  $C \neq 1$  in (3.25) and the least squares estimates are biased.

Other examples of calculation of equilibrium values are given in [188], [189]. The property that the parameters converge to values that correspond to optimal controls for a specified criteria is called self-tuning. The property of self-tuning was investigated by Kumar [135], [196] who showed that several different algorithms are self-tuning. Kumar also showed that there are algorithms which are not self-tuning. Ideas for modifying criteria to obtain self-tuning were also proposed.

### Robustness

Several of the stability proofs for adaptive systems are based on the assumption that the model structure used in the adaptive regulator is correct. The system in Example 2.2 where a feedforward gain is adapted will be stable for all values of the adaptation gain if the transfer function  $G$  is SPR. The adaptive control law derived in Example 3.2 is based on the assumption that the system to be controlled is actually a first-order system. Condition A4 for stability of the direct MRAS requires that the model used to design the adaptive control law has the same complexity as the real system. These assumptions are, of course, highly unrealistic because in most real problems the plants that are controlled will be very complex while the adaptive regulators like most controls will be designed based on comparatively simple models.

A simple example gives some insight into the consequences of neglecting high-order dynamics.

**Example 3.5:** Consider a system as in Example 3.3 where only a feedforward gain is adjusted. Such a system will be stable if the transfer function  $G$  is SPR. To see what happens when this assumption is violated assume that the command signal is a sum of sinusoids, i.e.,

$$r(t) = \sum_{k=1}^n a_k \sin(\omega_k t).$$

Using the model reference algorithm developed in Example 3.3 the parameter estimate is given by

$$\frac{d\theta}{dt} = kr[1 - \theta] G(p)r.$$

Assuming that the adaptation gain is small and using averages we find that the estimates are approximately given by

$$\frac{d\bar{\theta}}{dt} = ka[1 - \bar{\theta}] \quad (3.27)$$

where

$$a = \frac{1}{2} \sum_{k=1}^n a_k^2 \cos[\arg G(j\omega_k)]. \quad (3.28)$$

Equation (3.27) is stable if  $a$  is positive. This is the case when the transfer function  $G$  is SPR because each term in the sum (3.28) is then positive. If  $G$  is not SPR there may be negative terms in (3.28) which can be made to dominate by choosing the frequency of the reference signal appropriately.

The example thus shows that if  $G$  is not SPR then it is essential to make sure that the excitation is in a frequency range where the phase of  $G(j\omega)$  is less than  $90^\circ$ .  $\square$

The consequences of neglecting high-frequency dynamics have been illustrated in [220] and [203] where it is shown that assumption A4 is essential for the simple adaptive algorithms to work. This work has inspired much research to understand the mechanisms involved and to find appropriate modifications of the algorithms. This is currently an area of intensive research in adaptive control theory, see [32], [202], [188], [189], [221]–[246].

To explain some of the ideas first notice that the effect of unmodeled dynamics can be described by modifying the model (3.2) to

$$Ay = Bu + \delta Ay + \delta Bu + v = Bu + w \quad (3.29)$$

where  $A$  and  $B$  are the models used in the adaptive design and  $\delta A$  and  $\delta B$  represent the unmodeled high-frequency dynamics. The effect of unmodeled dynamics can thus formally be described as a disturbance. Notice, however, that the disturbance  $w$  is no longer bounded even if  $v$  is bounded because  $w$  also contains internal signals which may be potentially unbounded.

The unmodeled dynamics can be reduced by filtering of the signals. This is particularly noticeable for sampled data systems where anti-aliasing together with sampling gives a significant reduction of the high-frequency content of the signals; see [247]. Even if filtering reduces the unmodeled dynamics in the estimated model it does not imply that the signal  $w$  in (3.29) will be bounded.

The idea of normalization was introduced in [248]. It can be described as follows. Introduce the variable  $\rho$  defined by

$$\frac{d\rho}{dt} = \mu(|u| + |y| - \rho)$$

and normalize the signals as follows:

$$\bar{y} = \frac{y}{\rho} \quad \bar{u} = \frac{u}{\rho} \quad \bar{\varphi} = \frac{\varphi}{\rho} \quad \bar{w} = \frac{w}{\rho}.$$

The normalized signals are bounded and satisfy (3.29). The parameter adjustment law is then given by

$$\frac{d\theta}{dt} = -\frac{k}{\rho + \varphi^T \varphi} \varphi e. \quad (3.30)$$

The stability analysis is reduced to a problem with bounded disturbances and the previous results can then be applied. Stability results for systems with unmodeled dynamics have been developed along these lines in [31], [235], [236], [246], [248]–[250]. It should be noticed that a normalization similar to (3.30) is obtained automatically if a method based on recursive parameter estimation is used. See the Appendix.

One possibility to obtain stability for systems with



bounded disturbances is to bound the parameters. This can be done by using projections. Another possibility has been suggested in [228], [251], where the parameter adjustment rule (3.30) is changed to

$$\frac{d\theta}{dt} = -\frac{k}{\rho + \varphi^T \varphi} \varphi e - \sigma \theta. \quad (3.31)$$

This is called  $\sigma$ -modification or leakage. The added term  $-\sigma \theta$  will drive the parameters towards the origin. The term will, however, also give a shift in the equilibrium values. A stability analysis for this case is given in [251] and [252]. An additional modification proposed in [250] and [31] is

$$\frac{d\theta}{dt} = -\frac{k}{\rho^2 + \varphi^T \varphi} \varphi e - \frac{\gamma |\epsilon|}{\rho + \varphi^T \varphi} \theta. \quad (3.32)$$

In this case the shift in the equilibrium will be much smaller. The analysis in [250] gives stability conditions for this algorithm. The analysis shows an interesting interplay between persistency of excitation and unmodeled dynamics.

Another possibility to deal with unbounded disturbances is to switch off the adaptation when the error is small. This is often referred to as introducing a dead-zone in the estimator [157], [173], [174], [253]. The size of the dead-zone is related to the unmodeled dynamics and to the bound on the disturbance, see [157]. In [254] it is proposed to change the dead-zone adaptively. A stability analysis is also provided. This is equivalent to using a fixed dead-zone with normalized variables. Many of these modifications are used in the commercial adaptive regulators, see [255]. An algorithm that incorporates many of the features has been analyzed in [256].

#### IV. APPLICATIONS

There have been a number of applications of adaptive feedback control over the past 30 years. The early experiments which used analog implementations were plagued by hardware problems. Systems implemented using minicomputers appeared in the early 1970s. The number of applications have increased drastically with the advent of the microprocessor which made the technology cost-effective. Because of this the adaptive regulators are also entering the marketplace even in single loop controllers. This section gives an overview of the applications. Four industrial adaptive regulators are first discussed in some detail. General aspects on the industrial use of adaptive techniques are then given.

##### Feasibility Studies

A number of feasibility studies have been performed to evaluate the usefulness of adaptive control. They cover a wide range of control problems:

- autopilots for aircrafts and missiles [257], [258]
- autopilots for ships [259]–[263]
- ship propulsion [264]
- diesel engines [265]
- motor drives [266]–[272]
- motion control [273]–[275]
- industrial robots [89], [93], [276]–[282]
- machine tools [283]
- power systems [284]–[288]
- process control [289]–[298]

- pulp and paper [299]–[306]
- rolling mills [307]–[309]
- cement mills [310]–[314]
- chemical reactors [101], [315]–[317]
- distillation [318]–[321]
- pH-control [322]–[325]
- glass furnaces [326]
- heating and ventilation [101], [115], [265], [327], [328]
- ore crusher [329]
- biomedical [330]–[332].

The survey papers [25], [188], [333] and books [27]–[29] contain more details and many additional references.

The feasibility studies have shown that there are cases, where adaptive control is very useful and others where the benefits are marginal.

##### Industrial Products

Adaptive techniques are being used in a number of products. Gain scheduling is the standard method for design of flight control systems for high-performance aircrafts, see [67]. It is also used in the process industries, see [68], [69], [334]. The self-oscillating adaptive systems are used in missiles. There are several commercial autopilots for ship steering, see [259], [262], [264]. There are adaptive motor drives and adaptive systems for industrial robots. Some general-purpose adaptive control systems for industrial use are described below in the following subsections.

##### The SattControl Autotuner

Most industrial processes are controlled by PID regulators. A large industrial plant may have hundreds of regulators. Many instrument engineers and plant personnel are used to select, install, and operate such regulators. In spite of this, it is common experience that many regulators are poorly tuned. One reason is that simple robust methods for automatic tuning have not been available. A PID (proportional, integral, and derivative) regulator can be described by

$$u = K \left( e + \frac{1}{T_i} \int_0^t e(s) ds - T_d \frac{dy}{dt} \right) \quad (4.1)$$

where

$$e = r - y$$

and  $u$  is the control variable,  $y$  the measured variable, and  $r$  the set point.

The Swedish company SattControl has developed an auto-tuner that adjusts the parameters of a PID regulator automatically. It is available in two different versions. The auto-tuner is a part of a small (about 45 loops) DDC system (Direct Digital Control system) for industrial process control. The tuner can be connected to tune any loop in the system. The auto-tuner is also available as a stand-alone PID regulator with a mode switch for manual, automatic, and tune.

The auto-tuner is based on a special technique for system identification which automatically generates an appropriate test signal and a variation of the classical Ziegler–Nichols [335] method for control design. An interesting feature is that it has the ability to determine if derivative action is necessary. The auto-tuner is described in [103]–[105].

**The Basic Idea:** The Ziegler-Nichols method is based on the observation that the regulator parameters can be determined from knowledge of one point on the Nyquist curve of the open-loop system. This point is the intersection of the Nyquist curve with the negative real axis. It is traditionally described in terms of the ultimate gain  $k_c$  and the ultimate period  $T_c$ . In the original scheme, described in [335], the critical gain and the critical period are determined in the following way: A proportional regulator is connected to the system. The gain is gradually increased until an oscillation is obtained. The gain  $k_c$  when this occurs is the critical gain and the oscillation has the critical period. It is difficult to perform this experiment automatically in such a way that the amplitude of the oscillation is kept under control.

A block diagram of the SattControl auto-tuner is shown in Fig. 15. It is based on the idea that the critical gain and

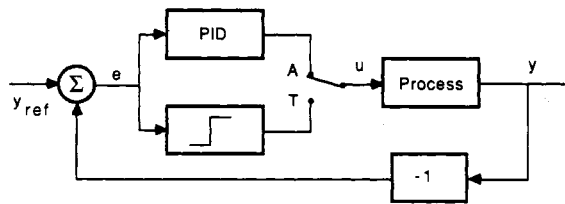


Fig. 15. The SattControl auto-tuner. The system operates as a relay controller in the tuning mode (T) and as an ordinary PID regulator in the automatic mode (A).

the critical frequency can be determined from an experiment with relay feedback. A periodic oscillation is then obtained. The critical period  $T_c$  is simply the period of the oscillation and the critical gain is easily determined from the relay amplitude and the amplitude of the oscillation. The relay amplitude is initially set to 10 percent of the output range. The amplitude is adjusted after one half period to give an oscillation of 2 percent in the output. The modified relay amplitude is stored for the next tuning. The relay has a hysteresis which is set automatically based on measurements of the measurement noise.

**Parameter Estimation:** Fig. 16 shows the input and output signals, under relay feedback. If the process attenuates high frequencies so that the first-harmonic component domi-

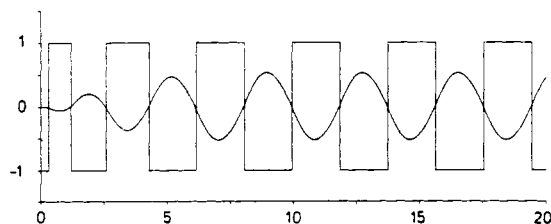


Fig. 16. Process inputs and outputs for a system under relay feedback.

nates the response it follows that the input and the output are out of phase. Furthermore, if the relay amplitude is  $d$  it follows from a Fourier series expansion that the amplitude of the first-harmonic component of the input is  $4d/\pi$ . If the amplitude of the output is  $a$ , then the process gain is  $\pi a/4d$  at the critical frequency. The critical gain is then

$$k_c = \frac{4d}{\pi a}.$$

Exact analyses of relay oscillations are also available; see [48], [103], [336], [337]. The period of an oscillation can easily be determined by measuring the times between zero-crossings. The amplitude may be determined by measuring the peak-to-peak values of the output. These estimation methods are easy to implement because they are based on counting and comparison only. Simulations and extensive experiments on industrial processes have shown that the simple estimation method based on zero-crossing and peak detection works very well in comparison with the more sophisticated estimation methods. The simple methods have also some additional advantages, see [338].

**Control Design:** When the critical gain  $k_c$  and the critical period are known the parameters of a PID regulator can be determined by the Ziegler-Nichols rule which can be expressed as

$$k = \frac{k_c}{2} \quad T_i = \frac{T_c}{2} \quad T_d = \frac{T_c}{8}. \quad (4.2)$$

This rule gives a closed-loop system which is sometimes too poorly damped. Modifications of the basic Ziegler-Nichols rule are, therefore, common.

**Prior Information:** A major advantage of the auto-tuner is that there are no parameters which have to be set *a priori*. To use the tuner, the process is simply brought to an equilibrium by setting a constant control signal in manual mode. The tuning is then activated by pushing the tuning switch. The regulator is automatically switched to automatic mode when the tuning is complete. Different control objectives may be obtained by modifying the parameters in the Ziegler-Nichols rule.

The properties of the auto-tuner are illustrated in Fig. 17 which shows an application to level control in three cascaded tanks. After bringing the system to an equilibrium,

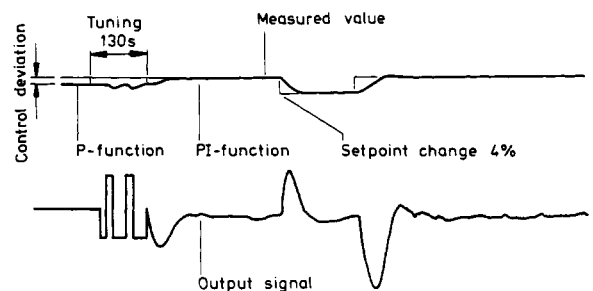


Fig. 17. Results obtained when using the auto-tuner for level control of three cascaded tanks.

the auto-tuner is initiated. The relay oscillation then appears. After 5 half-periods, good estimates of the critical gain and the critical period are obtained and the regulator is switched to normal PID control. A set point change is later introduced manually. This shows that the tuning has resulted in a system with good transient behavior.

Simplicity is the major advantage of the auto-tuner. This has proven particularly useful for plants which do not have qualified instrument engineers and for operation during the night shift when instrument engineers are not available.

It is also easy to explain the principles of operation of the auto-tuner to the instrument engineers.

### EXACT—The Foxboro Adaptive Regulator

This regulator is based on analysis of the transient response of the closed-loop system to set point changes or load disturbances and traditional tuning methods in the Ziegler–Nichols spirit. The regulator is thus similar to those discussed in Section II. The ideas behind it are described in [45] and [339]. Some details about the actual implementation are given in [340].

**Parameter Estimation:** Fig. 18 shows how the control error responds to a step or an impulse disturbance. Heuristic logic

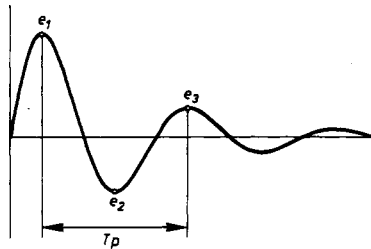


Fig. 18. Typical response of control error to step or impulse disturbance.

is used to detect that a proper disturbance has occurred and to detect the peaks  $e_1$ ,  $e_2$ , and  $e_3$  and the period  $T_p$ . The heuristic logic is outlined in Fig. 19. The estimation process is simple. It is based on the assumption that the disturbances are steps or short pulses. The algorithm can give wrong estimates if the disturbances are two short pulses because  $T_p$  will then be estimated as the distance between them.

**Control Design:** The control design is based on specifications on damping, overshoot, and the ratios  $T_i/T_p$  and  $T_d/T_p$ , where  $T_i$  is the integration time,  $T_d$  the derivative time, and  $T_p$  period of oscillation. The damping is defined as

$$d = \frac{e_3 - e_2}{e_1 - e_2}$$

and the overshoot as

$$z = -\frac{e_2}{e_1}$$

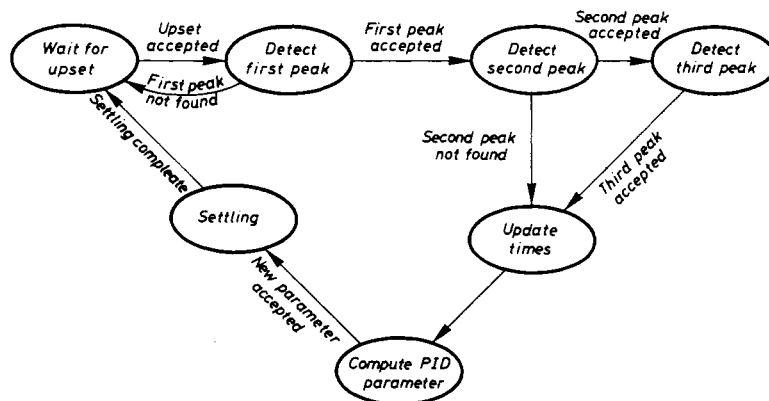


Fig. 19. Heuristic logic used in the EXACT system.

In typical cases, it is required that both  $d$  and  $z$  are less than 0.3. The Ziegler–Nichols tuning rule (4.2) gives

$$\frac{T_i}{T_p} = 0.5 \quad \text{and} \quad \frac{T_d}{T_p} = 0.12.$$

The numbers 0.5 and 0.12 have been modified based on empirical studies. Smaller values are chosen for processes with dominant dead-time and larger values are selected for processes with a dominant lag.

**Prior Information:** The tuning procedure requires prior information on the regulator parameters  $k$ ,  $T_i$ , and  $T_d$ . It also requires knowledge of the time scale of the process. This is used to determine the maximum time the heuristic logic waits for the second peak. Some measure of the process noise is also needed to set the tolerances in the heuristic logic. There are also some parameters which may be set optionally. They are damping  $d$ , overshoot  $z$ , maximum derivative gain, and bounds on the regulator parameters.

**Pretune:** The regulator has a mode called pretune which can be used if the prior information needed is not available. A step test is made where the user specifies the step size. Initial estimates of the regulator parameters are determined from the step. The time scale and the noise level are also determined. The pretune mode can be invoked only when the process is in steady state.

### Electromax V—The Leeds and Northrup Adaptive Regulator

This regulator is an adaptive single-loop controller based on the PID structure. The regulator has an adaptive function as an option. The adaptation is a self-tuning regulator where a second-order discrete time model is estimated. The parameters of a PID regulator are then computed from the estimated model using a pole placement design, see [341]–[343]. The regulator is primarily intended for temperature control. The regulator can operate in three different modes called fixed, self-tune, and self-adaptive. In the fixed mode, the regulator operates like an ordinary fixed gain PID regulator. In the self-tune mode, a perturbation signal is automatically introduced, a model of the process is estimated, and PID parameters are computed from the model. The parameters are displayed to an operator who may accept or reject the new parameters. In the self-adaptive mode the parameters are updated continuously.

**Parameter Estimation:** Parameter estimation is per-

formed in the self-tune and the self-adaptive modes. In both cases, the estimation is performed in closed loop. Set point changes are generated automatically in the self-tune mode to ensure that the estimation is based on good data. The changes are cycles of positive and negative pulses as shown in Fig. 20. The pulse height (acceptable set point upset) is

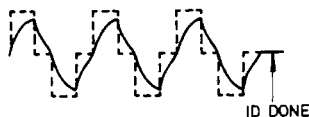


Fig. 20. Set point changes used in the self-tune mode.

set by the operator. The cycle time is computed from the response time which is also set by the operator. The goodness of fit is determined after each cycle. When the fit is good enough a message is given. The operator may view the regulator parameters obtained. If the fit is still poor after five cycles the procedure is aborted and a message is given.

In the self-adaptive mode, the parameter estimate and the computation of PID parameters are repeated at each sampling period. The estimated model is a second-order pulse transfer function. The parameters are estimated using the instrumental variable method where components of the regression vectors are formed from the model output, see the Appendix and [344]. The signals are high-pass filtered before they are fed to the estimator. The parameter updating is discontinued when the control error is below a certain limit, see [343].

**Prior Information:** To use the regulator it is necessary to specify five numbers: nominal values of the PID parameters, the process response time, and the admissible set point upset. The nominal values of the regulator parameters are needed because the estimation is done in closed loop. The process response time is defined as the time it takes for the open-loop step response to reach 90 percent of the steady-state value. This number is used to determine the perturbation signal in the identification phase, the sampling period of the discrete time model and the desired response of the closed-loop system. The performance is quite sensitive to the choice of the response time. The admissible set point upset gives the amplitude of the pulses used in the identification phase. Values of 3 to 5 percent are typical. Tests are performed to make sure that the output does not saturate.

**Pretune:** If good estimates of the prior data is not available it is recommended that an open-loop step response is performed.

### Novatune—The ASEA Adaptive Regulator

The ASEA Novatune is an adaptive regulator which is incorporated as a part of a distributed system for process control. The system is block-oriented which means that the process engineer creates a system by combining blocks of different types. The system has blocks for conventional PID control, logic, and computation. There are three different blocks called STAR1, STAR2, and STAR3 which are adaptive regulators. These are self-tuning regulators based on least squares estimation and minimum variance control. All the regulators use the same algorithm; they differ in the regulator complexity and the prior information which has to be supplied when using them.

The Novatune differs from the other regulators discussed in this section, because it is not based on the PID structure. It is, instead, a general sampled data regulator which is more general. It also admits dead-time compensation and feed-forward control. The system may be viewed as a tool box for solving control problems. There are also Novatune systems that are tailored to particular applications such as heating and ventilation and control of paper rolling.

**Principle:** The Novatune is an implicit self-tuning regulator of the type discussed in [95]. The parameters of a discrete time model are estimated using recursive least squares. The control design is a minimum variance regulator which is extended to admit positioning of one pole and a penalty on the control signal. Fig. 21 shows the different adaptive modules available in the Novatune system. In the block diagram, the input signals are shown on the left and top sides of the box, the output signals on the right, and the parameters are at the bottom. The parameters can be changed at configuration time. The parameters  $PL$ ,  $T$ , and  $PN$  can also be changed on-line. All other signals can be changed instantaneously.

The simplest module, STAR1, has three input signals: the manual input  $UEXT$ , the measured value  $FB$ , and the set point  $REF$ . It has three parameters. The variable  $PY$  is the smallest relevant change in the feedback signal. The adaptation is inhibited for changes less than  $PY$ . The parameters  $MAX$  and  $MIN$  denote the bounds on the control variable, and  $T$  is the sampling period.

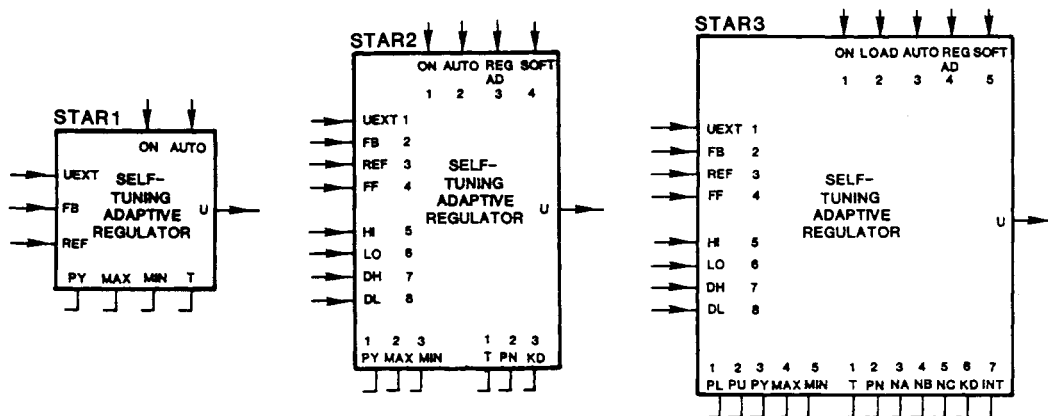


Fig. 21. Block diagram of the adaptive modules STAR1, STAR2, and STAR3 which are available in Novatune.

The module STAR2 has more input signals. It admits a feedforward signal  $FF$ . There are also four signals  $HI$ ,  $LO$ ,  $DH$ , and  $DL$  which admit dynamic changes on the bounds of the control variable and its rate of change. There are also additional parameters:  $PN$  for a penalty on the control variable and  $KD$  which specifies the prediction horizon. The module also has two additional mode switches.  $REGAD$  which turns off adaptation when false and  $SOFT$  which admits a soft start.

The module STAR3 has an additional mode  $LOAD$  which admits parameters stored in an EEPROM to be loaded. It also has several additional parameters which admit positioning of one pole  $PL$  and specification of controller structure  $NA$ ,  $NB$ ,  $NC$ , and  $INT$ . Parameter estimation is suspended automatically when the changes in the control signal and the process output are less than  $PU$  and  $PY$ . The parameter updating may also be suspended on demand through the switch  $REGAD$ . In combination with the other modules in the Novatune system this gives a convenient way to obtain robust estimation.

**Parameter Estimation:** The parameter estimation is based on the model

$$y(t + KD) - PLy(t + KD - 1) - (1 - PL)y(t) = A(q^{-1})\Delta y(t) + B(q^{-1})\Delta u(t) + C(q^{-1})\Delta v(t) \quad (4.3)$$

where  $A$ ,  $B$ , and  $C$  are polynomials in the delay operator,  $y$  is the measure variable,  $u$  the control signal,  $v$  a feedforward signal,  $q^{-1}$  the delay operator, and  $\Delta$  the difference operator. The integers  $NA$ ,  $NB$ , and  $NC$  give the number of coefficients in the polynomials  $A$ ,  $B$ , and  $C$ . The number  $PL$  is the desired pole location for the optional pole. When the parameter  $INT$  is zero a similar model without differences is used. The parameters are estimated using recursive least squares with a forgetting factor  $\lambda = 0.98$ .

**Control Design:** The control law is given by

$$B(q^{-1})\Delta u(t) = (PL - 1)y(t) - A(q^{-1})\Delta y(t) - C(q^{-1})\Delta v(t). \quad (4.4)$$

Since the algorithm is an implicit self-tuner the regulator parameters are obtained directly.

### Industrial Experiences

In 1986 there is moderate experience in industrial use of adaptive control. The autopilots for ship steering were probably the first adaptive systems that were produced. Today there are a few hundred of these systems in continuous operation. In the process control field the Electromax V was introduced in 1981, the ASEA Novatune in 1982, the SattControl auto-tuner and the Foxboro Exact in 1984. There are also a handful of other adaptive systems which have been announced. Today there are several thousand loops under adaptive control. The actual industrial experience of adaptive control is, however, not well documented. There are only a few publications from manufacturers, [340], [343], [345] and from users [346], [347].

Some experiences related to the use of Electromax V, which can operate both as a PID tuner and an adaptive PID regulator, are reported in [343]. The majority of applications are in temperature control. The experiences are generally quite favorable, although it is noted that adaptive control is not a panacea for everything. Most of the benefits are

derived from self-tuning although there are a number of cases where the continuous adaptation has been profitable. Difficulties in using the regulator have been observed with processes which have unsymmetric process response (typically heating and cooling), rapid parameter variations, or strong nonlinearities. The regulator cannot be applied to processes such as silicon crystal growing which do not tolerate the process upsets required in the identification phase. Difficulties with regulators used in the self-adaptive mode have also been found under operating conditions where the measured value is suddenly disconnected. The parameter estimation is then performed on totally irrelevant data. The remedy is to stop the parameter updating when the output is disconnected.

The Novatune applications cover a wide range of industrial processes in steel, pulp, paper, petrochemical, waste water treatment, and climate control, [345]. In the spring of 1986 there have been about 1000 loops adaptively controlled by the Novatune. The ability to deal with systems having time delays, and to use adaptive feedforward have been found particularly useful.

A comparison between the Novatune and the Exact is reported in [346] and [347]. The comparison is based on simulation and field tests.

### Uses of Adaptive Control

Based on the products discussed above and others we can now discuss uses of adaptive control in a more general setting. It is clear that adaptive techniques can be used in several different ways.

**Auto-Tuning:** Simple regulators with two or three parameters can be tuned manually if there is not too much interaction between adjustments of different parameters. Manual tuning is, however, not possible for more complex regulators. Traditionally, tuning of such regulators has followed the route of modeling or identification and regulator design. This is often a time-consuming and costly procedure which can only be applied to important loops or to systems which are made in large quantities.

All adaptive techniques can be used to provide automatic tuning. In such applications the adaptation loop is simply switched on. Perturbation signals may be added to improve the parameter estimation. The adaptive regulator is run until the performance is satisfactory. The adaptation loop is then disconnected and the system is left running with fixed regulator parameters. Adaptive techniques can be used in this way in all four products discussed in the beginning of this section. The auto-tuner has some particularly attractive features because it requires no prior information and it generates an appropriate test signal automatically.

Auto-tuning can be considered as a convenient way to incorporate automatic modeling and design into a regulator. It simplifies the use of simple regulators and it widens the class of problems where systematic design methods can be used cost effectively. This is particularly useful for design methods such as feedforward which critically depend on good models.

Automatic tuning can be applied to simple PID controllers as well as to more complicated structures. It is very convenient to introduce into a DDC-package because one tuning algorithm can serve many loops. Auto-tuning can also be included in single-loop regulators. It is, for example, pos-

sible to obtain standard regulators where the mode switch has three positions: manual, automatic, and tuning.

The available industrial experiences indicate that there is an industrial need for automatic tuning of PID controllers. The industrial regulators are often poorly tuned. Derivative action is seldom used although it can often be beneficial. One reason is that PID regulators are more difficult to tune than PI regulators: see [346], [347]. An auto-tuner should be very simple to use. Some of the available schemes do, however, require *a priori* information which makes them more difficult to use. This is probably the reason for introducing the "pre-tune" mode. The experiences with the SattControl auto-tuner indicated it was particularly useful when commissioning new industrial plants where there is no prior knowledge of regulator parameters. It has been shown that the commissioning time can be shortened considerably by using the auto-tuning, especially for plants that have many slow loops. It also seems clear that auto-tuning will be a very useful feature on more complex regulators.

**Automatic Construction of Gain Schedules:** Auto-tuning or adaptive algorithms may be used to build gain schedules. A scheduling variable is first determined. The parameters obtained when the system is running in one operating condition are then stored in a table together with the scheduling variable. The gain schedule is obtained when the process has operated at a range which covers the operating range.

If a good scheduling variable can be found, automatic generation of gain scheduling could be introduced in the commercial systems discussed in the beginning of this section. A table for storing regulator parameters and appropriate commands for recalling them are the only facilities required. The Novatune has a facility to save and recall one set of parameters. The SattControl auto-tuner has gain scheduling. A discussion of the needs for adaptation in process control is given in [343]. Most of the cases discussed there could be covered by auto-tuning combined with gain scheduling. Systems of this type will be able to follow rapid changes in the operating conditions.

**Adaptive Regulators:** The adaptive techniques may, of course, be also used for genuine adaptive control of systems with time-varying parameters. There are many ways to do this. The operator interface is important, since adaptive regulators may have parameters, which must be chosen. It has been my experience that regulators without any externally adjusted parameters can be designed for specific applications, where the purpose of control can be stated *a priori*. Autopilots for missiles and ships are typical examples, see [259]. In many cases it is, however, not possible to specify the purpose of control *a priori*. It is at least necessary to tell the regulator what the regulator is expected to do. This can be done by introducing dials that give the desired properties of the closed-loop system. Such dials are called *performance related*. New types of regulators can be designed using this concept. For example, it is possible to have a regulator with one dial, which is labeled with the desired closed-loop bandwidth. Another possibility would be to have a regulator with a dial, which is labeled with the weighting between state deviation and control action in an LQG problem. Adaptation can also be combined with gain scheduling. A gain schedule can be used to quickly get the parameters into the correct region and adaptation can then be used for fine tuning.

Feedforward control is very useful when there are measurable disturbances. Feedforward control, being an open-loop compensation, requires, however, good models of process dynamics. Adaptation, therefore, appears to be a prerequisite for effective use of feedforward. The experiences with the Novatune have shown that adaptive feedforward control can be very beneficial [345].

### Expert Control

Algorithms such as the STR and the MRAS may be viewed as local gradient methods. They will drive the regulator parameters to values which give close to optimal performance. The methods will, however, also require substantial prior information. This is illustrated by the pre-tune facility in the Foxboro and Leeds and Northrup autopilot. Systems like the SattControl auto-tuner require very little prior information. The performance obtained with this is, however, limited because of the restrictions on the regulator structure. It seems natural to design systems which combine different types of algorithms. For example, an auto-tuner could be used initially. The prior information required by a self-tuner like the one used in the Novatune can be extracted from the auto-tuner and the self-tuner can then be switched on. If this approach is taken it is natural also to include many different algorithms for parameter estimation and control design and to combine these with algorithms for diagnosis and supervision. Facilities for generation of gain schedules and learning can also be included. Experiments with such systems are described in [348]–[353].

### Abuses of Adaptive Control

An adaptive regulator, being inherently nonlinear, is more complicated than a fixed gain regulator. Before attempting to use adaptive control it is, therefore, important to first examine if the control problem cannot be solved by constant gain feedback. Problems of this type have only rarely been investigated. Two exceptions are [354] and [355]. In the vast literature on adaptive control there are many cases where a constant gain feedback can do well as an adaptive regulator. A typical example is the very ambitious feasibility study of adaptive autopilots for aircrafts [257]. The aircraft used in the experiments could easily be controlled with conventional methods.

Notice that it is not possible to judge the need for adaptive control from the variations of the open-loop dynamics over the operating range. Many cases are known where a constant gain feedback can cope well with considerable variations in system dynamics [356]. There are also design techniques for constant gain feedback that can cope with considerable gain variations, see [168] and [171]. It must, however, also be kept in mind that for large classes of problems it requires very little effort to get a system like the Novatune to work well.

### V. CONCLUSIONS

Adaptive techniques are emerging after a long period of research and experimentation. Important theoretical results on stability and structure have been established. Much theoretical work still remains to be done and the field is currently in a state of rapid development. The advent of microprocessors has been a strong driving force for the

applications. Laboratory experiments and industrial feasibility studies have contributed to a better understanding of the practical aspects of adaptive control. Also a number of adaptive regulators are now appearing on the market. Several thousand loops are now under adaptive control.

## APPENDIX

### EXAMPLES OF ADAPTIVE ALGORITHMS

Some adaptive control laws are given in this Appendix. A design problem for systems with known parameters is first described. Different adaptive control laws are then given. A pole-placement design is chosen as the underlying design problem. This is useful in order to discuss similarities and differences between self-tuners and model reference adaptive systems. It is also a convenient way to unify many algorithms.

#### *The Underlying Design Problem for Systems with Known Parameters*

Consider a single-input, single-output, discrete time system described by

$$Ay = Bu \quad (A1)$$

where  $u$  is the control signal,  $y$  the output signal, and  $A$  and  $B$  relatively prime polynomials in the forward shift operator, i.e.,

$$A = A(q) = q^n + a_1 q^{n-1} + \dots + a_n$$

$$B = B(q) = b_0 q^m + b_1 q^{m-1} + \dots + b_m$$

The relative degree or the pole excess is

$$d = \deg A - \deg B = n - m. \quad (A2)$$

Assume that it is desired to find a regulator such that the relation between the command signal  $r$  and the desired output signal  $y_m$  is given by

$$A_m y_m = B_m r \quad (A3)$$

where  $A_m$  and  $B_m$  are polynomials in the forward shift operator.

The desired control law is given by

$$Ru = Tu_c - Sy \quad (A4)$$

where  $R$ ,  $S$ , and  $T$  are polynomials. This control law represents a negative feedback with the transfer function  $-S/R$  and a feedforward with the transfer function  $T/R$ . To see how  $R$ ,  $S$ , and  $T$  can be determined proceed as follows, see [357]. Elimination of  $u$  in (A1) and (A4) gives the following equation for the closed-loop system:

$$(AR + BS)y = BTu_c \quad (A5)$$

The process zeros, given by  $B(q) = 0$ , will thus be closed-loop zeros unless they are canceled by corresponding closed-loop poles. Since unstable or poorly damped zeros cannot be canceled the polynomial  $B$  is factored as

$$B = B^+ B^- \quad (A6)$$

where  $B^+$  contains those factors which can be canceled and  $B^-$  the remaining factors of  $B$ . The zeros of  $B^+$  must be stable and well-damped. To make the factorization unique it is also required that  $B^+$  be monic.

It follows from (A5) that the characteristic polynomial of

the closed-loop system is  $AR + BS$ . This polynomial has three types of factors, canceled process zeros given by  $B^+$ , desired model poles given by  $A_m$ , and observer poles given by the observer polynomial  $A_0$ . Hence

$$AR + BS = B^+ A_m A_0. \quad (A7)$$

It follows from this equation that  $B^+$  divides  $R$ . Hence

$$R = B^+ R_1. \quad (A8)$$

Equation (A7) can then be written as

$$AR_1 + B^- S = A_0 A_m. \quad (A9)$$

We require that the relation (A5) between the command signal  $u_c$  and the process output  $y$  should be equal to the desired closed-loop response given by (A2). Hence

$$B_m = B^- B_m^+ \quad (A10)$$

$$T = A_0 B_m^+.$$

The specifications must thus be such that  $B^-$  divides  $B_m$ ; otherwise, there is no solution to the design problem.

To complete the solution of the problem it remains to give conditions which guarantee that there exist solutions to (A7) which give causal control law. It is shown in [357] that the condition

$$\deg A_0 \geq 2 \deg A - \deg A_m - \deg B^+ - 1 \quad (A11)$$

guarantees that the feedback transfer function  $S/R$  is causal. Similarly, the inequality

$$\deg A_m - \deg B_m \geq \deg A - \deg B \quad (A12)$$

implies that the feedforward transfer function  $T/R$  is causal.

To solve the design problem, (A9) is first solved to obtain  $R_1$  and  $S$ . The desired feedback is then given by (A4) with  $R$  given by (A8) and  $T$  by (A10). There may be several solutions to the Diophantine equation (A7) which satisfy the causality conditions. All solutions give the same closed-loop transfer function. They may, however, give different responses to disturbances and measurement errors.

It follows from the equations that the control law (A4) can be written as

$$u = G_m G_p^{-1} u_c - \frac{S}{R} [y - y_m]$$

where

$$G_p = B/A \quad G_m = B_m/A_m \quad \text{and} \quad y_m = G_m u_c$$

This shows that the pole-placement design can be interpreted as model following. This is important in order to establish the relations between the STR and the MRAS. Equation (A4) is, however, preferable in realizations.

#### *Parameter Estimation*

The control law (A4) is not realizable if the parameters of the model (A1) are unknown. However, the parameters can be estimated. There are many ways to do this, see [344]. Many estimators can be described by the recursive equation

$$\theta(t) = \theta(t-1) + P(t-1) \psi(t-1) \epsilon(t) \quad (A13)$$

where the components of vector  $\theta$  are the estimated parameters, vector  $\psi$  is a vector of instrumental variables, and  $\epsilon$



is the prediction error  $\epsilon$ . The quantities  $\psi$  and  $\epsilon$  depend on the identification method and the model structure. For example, if the least squares method is applied to the model (A1) the prediction error is given by

$$\begin{aligned}\epsilon(t) &= [A^*(q^{-1}) y(t) - B^*(q^{-1}) u(t-d)] \\ &= y(t) \varphi^T(t-1) \theta(t-1)\end{aligned}$$

where  $A^*$  and  $B^*$  are the reciprocals of the polynomials  $A$  and  $B$ , respectively, i.e.,

$$\begin{aligned}A^*(q^{-1}) &= 1 + q_1 q^{-1} + \dots + a_n q^{-n} \\ B^*(q^{-1}) &= b_0 + b_1 q^{-1} + \dots + b_n q^{-n}\end{aligned}$$

and

$$\begin{aligned}\psi(t-1) = \varphi(t-1) &= [-y(t-1) \dots -y(t-n) \\ &\quad \cdot u(t-d) \dots u(t-n)]^T.\end{aligned}$$

The elements of vector  $\psi$  are thus delayed values of the input  $u$  and the output  $y$ .

The quantity  $P$  in (A13) depends on the particular estimation technique. It may be a constant which gives an updating formula similar to the MIT-rule. Another method due to Kaczmarz [358] can be viewed as the solution of a set of linear algebraic equations using successive projections. This method is described by (A13) with  $\psi = \varphi$  and

$$P(t) = \frac{1}{\varphi^T(t) \varphi(t)}. \quad (\text{A14})$$

In stochastic approximation methods  $P$  is a scalar given by

$$P(t) = \left[ \sum_{k=1}^t \varphi^T(k) \varphi(k) \right]^{-1}. \quad (\text{A15})$$

The recursive least squares method is given by (A13) with  $\psi = \varphi$  and

$$P(t) = \left[ \sum_{k=1}^t \varphi(k) \varphi^T(k) \right]^{-1}. \quad (\text{A16})$$

Some minor modifications have to be made if the denominator in (A14) is zero or when the matrices in (A15) or (A16) are singular.

The properties of the estimates depend on the model and the disturbances. In the deterministic case when the data are generated by (A1) which has no disturbances there are estimates which converge to the correct values in a finite number of steps. The algorithms with  $P$  given by (A16) have, e.g., this property. Algorithms with a constant  $P$  converge exponentially. When data are generated by (A1) with independent random variables added to the right-hand side it is necessary to have algorithms where  $P(t)$  goes to zero for increasing  $t$  in order to get estimates which converge to the correct value. This is the case when  $P$  is given by (A15) or (A16). These algorithms are said to have decreasing gain. An algorithm with decreasing gain is, however, useless when the process parameters are changing. For such a case (A14) can be used or (A16) can be replaced by

$$P(t) = \left[ \sum_{k=1}^t \lambda^{t-k} \varphi(k) \varphi^T(k) \right]^{-1}. \quad (\text{A17})$$

where  $0 < \lambda \leq 1$  is a forgetting factor or a discounting factor.

This choice of  $P$  corresponds to a least squares estimate with an exponential discounting of past data. A disadvantage with exponential forgetting is that data are discarded uniformly in time. Therefore, it has been suggested to change the forgetting factor adaptively [359], [360]. Another approach is directional forgetting which means that data are discarded only in those directions where new information is received [361]–[363].

To obtain numerically stable algorithms it is advantageous to base the algorithms on the square root of the matrix  $P$  instead of  $P$  itself. Numerically stable algorithms are given in [364]–[367].

To obtain an estimator which is insensitive to spurious large errors it is also useful to introduce a nonlinear function which reduces the influence of large errors as is done in robust statistics [368], [369].

#### An Explicit Self-Tuner

An explicit self-tuner based on the pole-placement design can be expressed as follows:

##### Algorithm 1

- Step 1: Estimate the coefficients of the polynomials  $A$  and  $B$  in (A1) recursively using (A13) with (A14), (A15), (A16) or (A17).
- Step 2: Substitute  $A$  and  $B$  by the estimates obtained in step 1 and solve (A10) to obtain  $R_1$  and  $S$ . Calculate  $R$  by (A8) and  $T$  by (A10).
- Step 3: Calculate the control signal from (A4).  
Repeat steps 1, 2, and 3 at each sampling period.  $\square$

#### An Implicit Self-Tuner

In the implicit self-tuner, the design calculations are eliminated and the regulator parameters are updated directly. The algorithm can be derived as follows. We have

$$A_m A_0 y = A R_1 y + B^- S y = B R_1 u + B^- S y = B^- [R u + S y] \quad (\text{A18})$$

where the first equality follows from (A9), the second from (A1), and the third from (A8). Notice that equation (A18) can be interpreted as a process model, which is parameterized in  $B^-$ ,  $R$ , and  $S$ . An estimation of the parameters of the model (A18) gives the regulator parameters directly. A solution to the bilinear estimation problem is given in [74]. In the special case of minimum phase systems when  $B^- = b_0$  the implicit algorithm can be expressed as follows.

##### Algorithm 2

- Step 1: Estimate the coefficients of the polynomials  $R$ ,  $S$  in (A18) recursively using (A13) with

$$\begin{aligned}\epsilon(t) &= A_0^* A_m^* y(t) - b_0 [R^* u(t-d) + S^* y(t-d)] \\ &= A_0^* A_m^* y(t) - \varphi^T(t-d) \theta(t-1)\end{aligned}$$

where

$$\begin{aligned}\varphi(t-d) &= [-y(t-d) \dots -y(t-d-\deg S) \\ &\quad \cdot b_0 u(t-d) \dots b_0 u(t-d-\deg R)]\end{aligned}$$

and (A14), (A15), (A16), or (A17).

- Step 2: Calculate the control signal from (A3), with  $R$  and  $S$  substituted by their estimates obtained in Step 1.

Repeat steps 1 and 2 at each sampling period.  $\square$



The simple self-tuner in [95] corresponds to this algorithm with  $P$  given by (A17).

There are many variations of the algorithm. One simple modification is to replace  $\epsilon$  by

$$\epsilon(t) = y(t) - \varphi^T(t-d) \theta(t-1)$$

where

$$\bar{\varphi}(t) = \frac{1}{A_0^* A_m^*} \varphi(t).$$

#### Other Implicit Self-Tuners

Algorithm 2 is based on a reparameterization of the process model (A1). The reparameterization is nontrivial in the sense that (A18) has more parameters than (A1). The parameterization (A18) has the drawback that the model obtained is not linear in the parameters. This makes the parameter estimation more difficult. It is thus natural to investigate other parameterizations. One possibility is to write the model (A18) as

$$A_0 A_m y = R u + S y \quad (\text{A19})$$

where

$$R = B^- R$$

$$S = B^- S.$$

The estimated polynomials will then have a common factor which represents poorly damped modes. To avoid cancellation of such modes it is then necessary to cancel the common factor before calculating the control law. The following control algorithm is then obtained.

#### Algorithm 3

- Step 1: Estimate the coefficients of the polynomials  $R$  and  $S$  in the model (A19)
- Step 2: Cancel possible common factors in  $R$  and  $S$  to obtain  $R$  and  $S$ .
- Step 3: Calculate the control signal from (A4) where  $R$  and  $S$  are those obtained in step 2.
- Repeat steps 1, 2, and 3 at each sampling period.  $\square$

This algorithm avoids the nonlinear estimation problem. There are, however, more parameters to estimate than in Algorithm 2 because the parameters of polynomial  $B^-$  are estimated twice.

There are several other possibilities. For the case  $B^+ = \text{const}$  it is possible to proceed as follows. Write the model (A1) as

$$\begin{aligned} Az &= u \\ y &= Bz. \end{aligned} \quad (\text{A20})$$

If the polynomials  $A$  and  $B$  are coprime there exist two polynomials  $U$  and  $V$  such that

$$UA + VB = 1. \quad (\text{A21})$$

It follows from (A10), (A20), and (A21) that

$$A_0 A_m z = A_0 A_m (UA + VB)z = (RA + SB)z.$$

Equation (A20) gives

$$A_0 A_m U u + A_0 A_m V y - R u - S y = 0$$

or

$$U(A_0 A_m u) + V(A_0 A_m y) - R u - S y = 0. \quad (\text{A22})$$

Notice that this equation is linear in the parameters. An adaptive algorithm similar to Algorithm 3 can be constructed based on (A22). This was proposed in [75].

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