PID Controller Design using Characteristic Ratio Assignment Method for Coupled-Tank Process.

Jutarut Chaorai-ngern, Arjin Numsomran, Taweepol Suesut, Thanit Trisuwannawat and Vittaya Tipsuwanporn.
Faculty of Engineering, King Mongkuts Institute of Technology Ladkrabang, Bangkok 10520, Thailand.
(Tel: 66-2739-2407: Ext.128; knarjin@kmitl.ac.th)

Abstract— **This paper presents the PID controller design for coupled-tank process using characteristic ratio assignment (CRA). The coupled-tank process is a challenge plant for testing the performance of PID controller because it is a high uncertainty nonlinear model. For this paper, the characteristic ratio assignment (CRA) which is satisfied specification of performance of control system. Furthermore, it is very convenient as a fast adjustment damping ratio as well as a high speed response. Finally, the simulation results can be illustrated the validity of our approach by MATLAB.**

Keywords : PID Controller, CRA, Robust Control.

I. INTRODUCTION

Generally, the performance analysis of control system focuses on time domain response such as percent overshoot, rise time, setting time and steady state error. Although there are many methods to design the controller, it is a few approach can be achieved the satisfied response. The characteristic ratio assignment (CRA) is one techniques based on defined parameter of characteristic equation that is a famous method at present.[1][2].

To design controller by CRA method [3], adjustment of speed response and the damping ratio can be done by only one parameter. Therefore, this technique is convenient and suitable for tuning controller under the requirement of the system.

In this paper, the coupled-tank process, the interactive process is controlled by PID controller based on CRA method. The coefficient of characteristic (α_i) and time constant (τ) are the parameters to determine the characteristic equation of the control system that is necessary to design the time domain system.

II. THE COUPLED-TANK PROCESS

According to fig.1, the input $u(t)$ is the input pressure which is taken to the pump, and the output $h_2(t)$ is the water level in tank1. The nonlinear equation can be obtained by mass balance equation and Bernoullis law is given by.

$$\begin{aligned}\frac{dh_1(t)}{dt} &= -\frac{\beta_{12}a_{12}}{A_1}\sqrt{2g\left(h_1(t)-h_2(t)\right)}+\frac{k}{A_1}u(t)\\ \frac{dh_2(t)}{dt} &= -\frac{\beta_2 a_2}{A_2}\sqrt{2gh_2(t)}+\frac{\beta_{12}a_{12}}{A_2}\sqrt{2g\left(h_1(t)-h_2(t)\right)}\end{aligned} \quad (1)$$

Where A_i is the cross section area of tank i (cm^2), a_2 is the cross section area of outlet of tank2 (cm^2) , β_{12} a the cross section area of jointed pipe between tank1 and tank2 (cm^2) , β_2 the value ratio at the outlet of tank2, is the value ratio

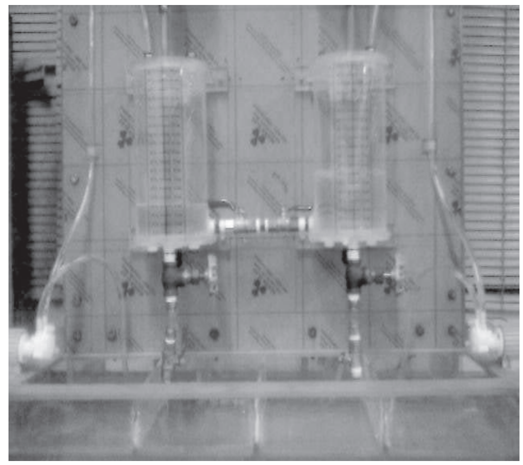

Fig. 1. The Coupled-Tank Process.

between tank1 and tank2, g is the gravity (cm^2/s) and k is the gain of pump $(cm^3/V \bullet s)$ according to (1) is linearized as (2).

$$\begin{aligned}\frac{dH_1(t)}{dt} &= \frac{1}{T_{12}}\left(-H_1(t)+H_2(t)\right)+\frac{k}{A_1}U(t)\\ \frac{dH_2(t)}{dt} &= -\frac{1}{T_2}H_2(t)+\frac{1}{T_{12}}\left(H_1(t)-H_2(t)\right)\end{aligned} \quad (2)$$

Where
$T_{12} = \frac{A_1}{\beta_{12}a_1}\sqrt{\frac{2(\bar{h}_1-\bar{h}_2)}{g}}$, s and $T_2 = \frac{A_2}{\beta_2 a_2}\sqrt{\frac{2\bar{h}_2}{g}}$, s
$\bar{h}_1$ and $\bar{h}_2$ is the water level at operating point of this process, T_{12} is the time constant between tank1 and tank2 ,and T_2 is the time constant of tank2 and k is gain of pump. For the (2) can be modeled as the (3). This is the transfer function for designing this controller.

$$\frac{H_2(s)}{U(s)} = G(s) = \frac{K}{T_{12}T_2s^2+(T_{12}+2T_2)s+1} \quad (3)$$

Where

$K = \frac{kT_2}{A_2}$, cm/V

III. THE CONTROL SYSTEM STRUCTURE.

The control system structure (figure 2) is two-degree of freedom that its parameters are determined by CRA method. Where $B_p(s)$ and $A_p(s)$ is the polynomial equation. The block diagram in fig. 2 can be formulated to transfer

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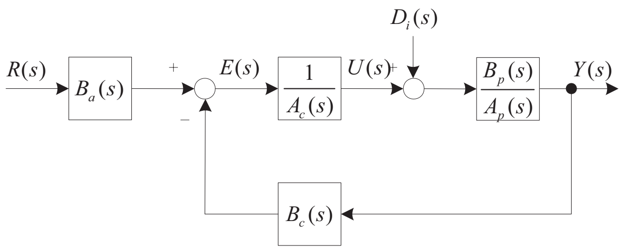

Fig. 2. The block diagram of control system

function as.

$$\frac{Y(s)}{R(s)} = \frac{B_a(s)B_p(s)}{A_c(s)A_p(s)+B_c(s)B_p(s)} \tag{4}$$

In this case

$$\begin{aligned} B_p(s) = K, A_p(s) = T_{12}s^2 + (T_{12}+2T_2)s+1 \\ B_c(s) = K_d s^2 + K_p s + K_i, A_c(s) = s, B_a(s) = K_i \end{aligned} \tag{5}$$

To substitute equation (5) into equation (4) is given a characteristic equation as.

$$P(s) = T_{12}T_2s^3 + (T_{12}+2T_2+KK_d)s^2 + (1+KK_p)s + KK_i \tag{6}$$

Equation (6) is the characteristic equation to design the PID controller by CRA method.

IV. THE CRA METHOD

Naslin[4] has studied the problems to optimize a damping ratio of control systems, and he found that the damping ration relates with characteristic ratio. The relation of characteristic equation can be illustrated as follow.

$$p(s) = a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0, \forall a_i > 0 \tag{7}$$

Where, the characteristic ratio is given by.

$$\alpha_1 = \frac{a_1^2}{a_0 a_2}, \alpha_2 = \frac{a_2^2}{a_1 a_3}, \ldots, \alpha_{n-1} = \frac{a_{n-1}^2}{a_{n-2}a_n} \tag{8}$$

And the inverse of characteristic equation is given as.

$$b_0 = \frac{a_0}{a_1}, b_1 = \frac{a_1}{a_2}, \ldots, b_{n-1} = \frac{a_{n-1}}{a_n} \tag{9}$$

Otherwise, it is able to depict the relation of coefficient ratio and characteristic pulsatances as.

$$\alpha_1 = \frac{b_1}{b_0}, \alpha_2 = \frac{b_2}{b_1}, \ldots, \alpha_{n-1} = \frac{b_{n-1}}{b_{n-2}} \tag{10}$$

Thus the time constant is given by.

$$\tau = \frac{a_1}{a_0} \tag{11}$$

Given equation (8)-(10), it is able to describe in another form by the coefficient of characteristic equation.

$$A = \begin{bmatrix} a_n & a_{n-1} & \ldots & a_1 & a_0 \end{bmatrix} \tag{12}$$

$$B = \begin{bmatrix} b_{n-1} & b_{n-2} & \ldots & b_1 & b_0 \end{bmatrix} \tag{13}$$

$$C = \begin{bmatrix} c_{n-2} & c_{n-3} & \ldots & c_1 & c_0 \end{bmatrix} \tag{14}$$

Where

$$b_i = \frac{a_i}{a_{i+1}}, i = 0,1,2,\ldots,n-1 \tag{15}$$

$$c_i = \frac{b_{i+1}}{b_i}, i = 0,1,2,\ldots,n-2 \tag{16}$$

Design controller by CRA method, assigns value of time constant (τ) and characteristic pulsatances (α_i) up to system requirement. To define the value of α_i , it is under the rule of Lipatov and Sokolov [5] in order to retain the system stability which is given by.

$$\sqrt{\alpha_i \alpha_{i+1} > 1.4656}, i = 1,2,\ldots,n-2 \tag{17}$$

$$\alpha_i \geq 1.12374\alpha_i^*, i = 2,3,\ldots,n-2 \tag{18}$$

$$\alpha_i^* = \frac{1}{\alpha_{i+1}} + \frac{1}{\alpha_i - 1}, \alpha_n = \alpha_0 = \infty \tag{19}$$

A. Adjustment speed response of control system

To adjust a speed response of control system, the CRA method can be tuned by changing a value of time constant as shown in the next equation. Assume, the transfer function is given as.

$$G(s) = \frac{a_0}{a_n s^n + a_{n-1}s + \ldots + a_1 s + a_0} \tag{20}$$

Then it is arranged in new format.

$$G(s) = \frac{a_0/a_n}{s^n + \frac{a_{n-1}}{a_n}s + \ldots + \frac{a_1}{a_n}s + \frac{a_0}{a_n}} \tag{21}$$

From equation (21) is able to construct the coefficient as inverse form of characteristic equation.

$$A = \begin{bmatrix} 1 & \prod_{i=n-1}^{n-1} b_i & \ldots & \prod_{i=2}^{n-1} b_i & \prod_{i=1}^{n-1} b_i \end{bmatrix} \tag{22}$$

When increasing a gain with equivalent ratio by k, it is obtained by.

$$A = \begin{bmatrix} 1 & k\prod_{i=n-1}^{n-1} b_i & \ldots & k^{n-1}\prod_{i=2}^{n-1} b_i & k^n\prod_{i=1}^{n-1} b_i \end{bmatrix} \tag{23}$$

Equation (22) and (23), coefficient ratio is unchanged, but the time constant is able to change as shown in equation (24)(25).

$$G_k(s) = \frac{k^n a_0}{a_n s^n + k a_{n-1}s^{n-1} + \ldots + k^{n-1}a_1 s + k^n a_0} \tag{24}$$

$$\tau = \frac{1}{k}\left(\frac{a_1}{a_0}\right) \tag{25}$$

Where $0 < k < 1$

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TABLE I

PARAMETERS OF COUPLED-TANK PROCESS

A_1, A_2;cm^2	a_2, a_{12};cm^2	β_2	β_{12}
66.25	0.1963	0.3	0.56

TABLE II

OPERATING POINT OF COUPLED-TANK PROCESS.

$\bar{h}_1$;cm	$\bar{h}_2$;cm	$\bar{u}$;V	k;cm^3/V$\cdot$s
9	7	3	2.3

B. Adjustment damping ratio of control system

To adjust the damping ratio, if a system is high order then there are many parameters (α_i) to be adjusted. Thus the CRA method is designed for tuning only one parameter that follows as.

$$G(s) = \frac{1}{\frac{a_n}{a_0}s^n + \frac{a_{n-1}}{a_0}s^{n-1} + \ldots + \frac{a_1}{a_0}s + 1} \tag{26}$$

$$A = \begin{bmatrix} \frac{1}{\prod_{i=0}^{n-1} b_i} & \frac{1}{\prod_{i=0}^{n-2} b_i} & \cdots & \frac{1}{\prod_{i=0}^{1} b_i} & \frac{1}{\prod_{i=0}^{0} b_i} \end{bmatrix} \tag{27}$$

$$A = \begin{bmatrix} \frac{1}{\left[\prod_{i=0}^{n-2} c_i \prod_{i=0}^{n-3} c_i \ldots \prod_{i=0}^{0} c_i\right] b_0^n} \\ \frac{1}{\left[\prod_{i=0}^{n-3} c_i \prod_{i=0}^{n-4} c_i \ldots \prod_{i=0}^{0} c_i\right] b_0^{n-1}} & \cdots & \frac{1}{b_0} & 1 \end{bmatrix} \tag{28}$$

When the coefficient ratio is increased by k then the new characteristic equation is given by.

$$A = \begin{bmatrix} \frac{1}{k^{\frac{1}{2}n^2-\frac{1}{2}} \left[\prod_{i=0}^{n-2} c_i \prod_{i=0}^{n-3} c_i \ldots \prod_{i=0}^{0} c_i\right] b_0^n} \\ \frac{1}{k^{\frac{1}{2}n^2-\frac{1}{2}-1} \left[\prod_{i=0}^{n-3} c_i \prod_{i=0}^{n-4} c_i \ldots \prod_{i=0}^{0} c_i\right] b_0^{n-1}} & \cdots & \frac{1}{b_0} & 1 \end{bmatrix} \tag{29}$$

Finally, equation (28) and (29) are formulated to the equation (30) When $k > 1$; damping ratio will be increased then $0 < k < 1$; damping ration will be decreased.

$$G_k(s) = \frac{k^{\frac{1}{2}n^2-\frac{1}{2}n} a_0}{a_n s^n + k^{n-1} a_{n-1} s^{n-1} + \ldots + k^{\frac{1}{2}n^2-\frac{1}{2}n-1} a_1 s + k^{\frac{1}{2}n^2-\frac{1}{2}n} a_0} \tag{30}$$

V. THE SIMULATION RESULTS

In this paper, the simulation results of the coupled-tank, the interactive process is given by MATLAB. The experiment of PID control based on CRA illustrates the adjustment of speed response and damping ratio. Parameters and operating point show detail in Table I and Table II.

From this data formed by equation (31) then it is obtained the transfer function as.

$$G(s) = \frac{4.662}{5177s^2 + 307.2s + 1} \tag{31}$$

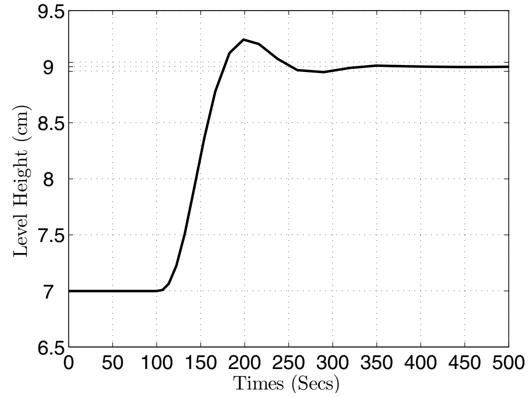

Fig. 3. The step response of nominal parameter.

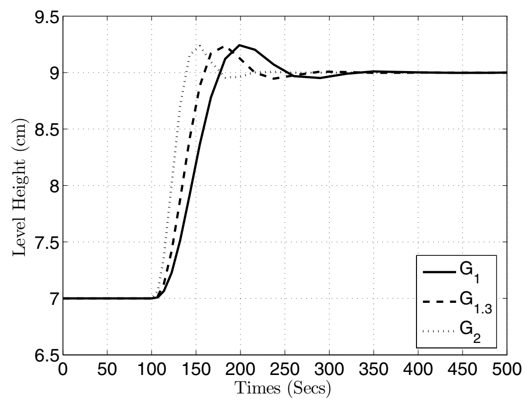

Fig. 4. The result adjustment speed of step response by k.

A. The Step Response of nominal parameter

According to equation (31), design of PID controller based on CRA, is assigned parameter follow as $\alpha_1 = 1.8, \alpha_2 = 2$ and $\tau = 40$then the characteristic equation is,

$$P(s) = 5177s^3 + 465.925s^2 + 20.966s + 0.5242$$

And the parameter of PID controller is given by.

$$K_d = 34.052, K_p = 4.2825, K_i = 0.1124$$

The step response in fig.3, the overshoot is over 12 percent and rise time is at 296 sec.

B. The Adjustment Speed of Response

By using CRA method, the adjustment of speed response can be produced by adjusting value of k according to equation (24).

$$G_k(s) = \frac{k^3 0.5242}{5177s^3 + k465.925s^2 + k^2 20.966s + k^3 0.5242}$$

Then the solution is given as the PID parameter as shown in Table3.

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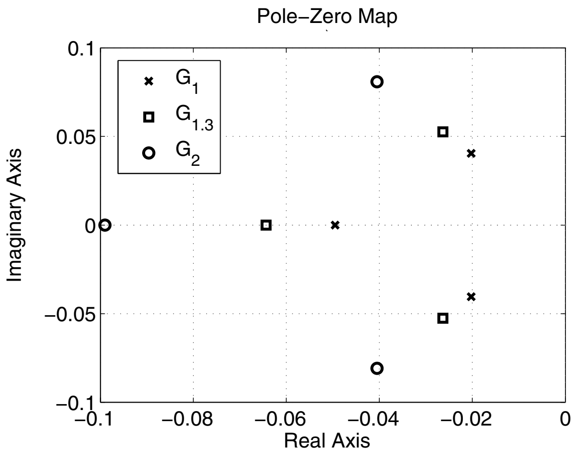

Fig. 5. The position of each closed-loop pole (Table III)

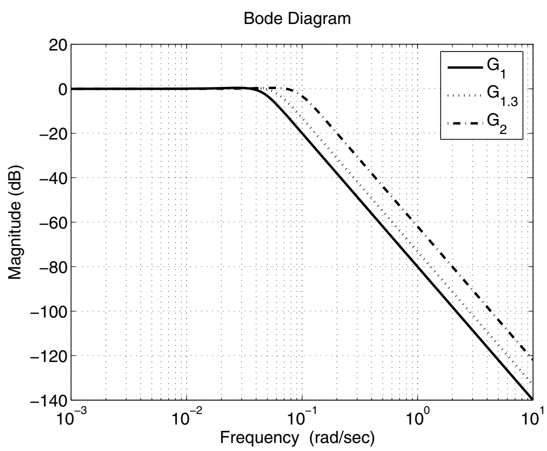

Fig. 6. Frequency response when adjust parameter k (Table III)

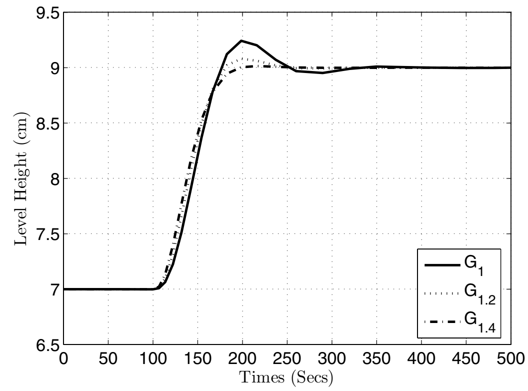

Fig. 7. The response when adjustment Damping Ratio by k. (Table IV)

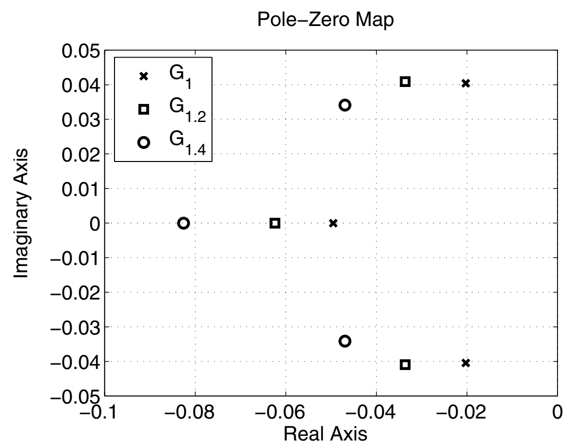

Fig. 8. The position of each closed-loop pole when adjust damping ratio. (Table IV)

When k is adjusted from 1.3 to 2 respectively, setting time is decreased from 148 sec. to 93 sec. therefore; the system response is faster. Fig 6 shows the position of close loop pole and frequency response that changes during adjustment parameter k.

C. The adjustment of damping ratio

By using CRA method, the adjustment of damping ratio can be done by formulating the feedback transfer function (equation 30) is given as.

$$G_k(s) = \frac{k^3 0.5242}{5177s^3 + k^2 465.925s^2 + k^2 20.966s + k^3 0.5242}$$

Fig 7 shows the step response, when k is changed to 1.2 and 1.4 respectively then the damping ratio is also increased but the overshoot is decreased from 4 and 0.5 , and the setting time is faster (118 sec to 97 sec)

TABLE III

OPERATING POINT OF COUPLED-TANK PROCESS.

	K_d	K_p	K_i
$k=1$	34.052	4.282	0.112
$k=1.3$	64.031	7.385	0.247
$k=2$	133.984	17.773	0.899

TABLE IV

PARAMETERS OF PID CONTROLLER VERSUS K.

	K_d	K_p	K_i
$k=1$	34.052	4.282	0.112
$k=1.2$	78.021	7.556	0.1942
$k=1.4$	129.986	12.125	0.3084

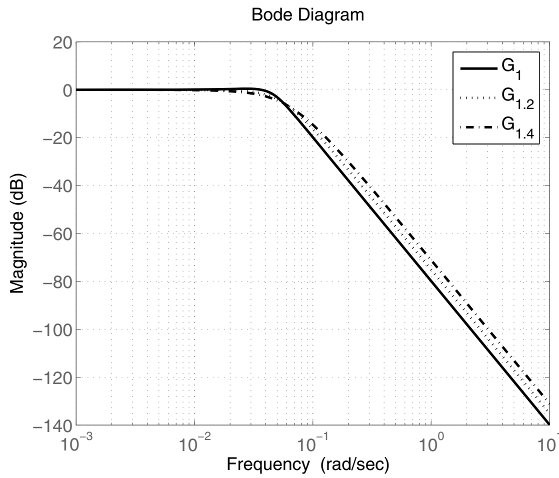

Fig. 9. Frequency response when adjust parameter k (Table IV)

VI. CONCLUSIONS

In this paper, the design of PID controller using CRA is presented. The simulation results from MATLAB are able to illustrate the advantage which only one parameter is to be adjusted. Thus, our scheme is convenient and suitable for designing and tuning the controller.

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