Digital Control of a Tank System

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***Abstract*— This paper describes the laboratory environment for experiments in digital control of a coupled tank system. The control process consists of upper and lower containers with draining orifices. As the cross section of the lower container varies with the level, the dynamic of the process changes extensively with the selected steady-state operating level. Measurements of the tank levels are available, while the flow rate into the upper tank is forced by a pump driven by an electric motor. A nonlinear model is employed to study various features of the process. The control strategy is to stabilize the level in the lower tank by adjustment of the flow rate into the upper tank. An IBM AT, equipped with an universal analog input/output board is used to control the process. The interactive real-time software frame, available under MS-DOS, uses selectable operating modes to analyze the performance of different control algorithms. Digital state-feedback algorithm is presented as an illustration.**

I. Introduction

A COMMON CONTROL PROBLEM in process industries is the control of fluid levels and temperatures in storage tanks, chemical blending, and reaction vessels. A typical situation is one that requires fluid to be supplied to a chemical reactor at a constant rate. An upper tank can be used for filtering the variations in the upstream supply flow.

Coupled tanks systems are favorite equipment in control engineering laboratories for research and educational purposes [1], [2], [3].

A variety of configurations have been proposed, including hot-water inputs and temperature stabilization [4]. Although the dynamics of the fluid level control systems are relatively straightforward, sometimes they involve challenging problems. In particular, for tanks where the cross section varies with the fluid level, the dynamics of the flow can change extensively with the operating conditions of the process due to the nonlinearity between level and fluid volume in the tank.

The main objective in developing the experimental system for the tanks in the Control Laboratory of the Institute of Automatics in Krakow was to offer the students an environment for a complete engineering design. Starting from modeling and ending with experimental verification of a wide range of digital control strategies. A standard technique for modeling a nonlinear system is the linearization of equations near some predetermined operating point and digitalization of the state equations. On the basis of a digital model, control algorithms can be developed, simulated, and installed in the control software. The proposed model is an excellent tool for investigation of the nonlinear effects which influence the sensitivity of the proposed algorithms to changes in the steady-state operating point. In the case of large magnitude set point changes, more complex algorithms are necessary, providing an alternative to classical design of discrete controllers.

The laboratory offers the educational possibilities in other fields connected with real-time digital control. A student can write his own control algorithm and link it to the system without a deep knowledge of the process interface, but the selection of the sample periods can be carried out as a laboratory experiment. The effects of sampling can be demonstrated clearly. Digital filtering algorithms can be included. Using the experimental set-up, other experiments can be carried out: D/A and A/D conversion, signal transmission, and sensor calibration.

Applications of IBM PC computers equipped with A/D, D/A interface boards have been reported from several control laboratories [5], [6]. They provide a low-cost alternative to expensive industrial controllers. The various attributes of the tanks control station provide students with a full range of process mimics, trends, fault diagnostics, and alarms. A user-friendly interface and a large-screen color display, create an environment close to that of an industrial application.

II. Flow Process and Instrumentation

Fig. 1 shows in schematic form the model used to illustrate fluid-level control problems. The model consists of the upper tank I having constant cross section, and the lower tank II of a cylindrical shape having variable cross section. Water is pumped into the first tank from the auxiliary compensatory tank ZW by a constant speed pump driven by an electric motor. The motor dynamics are neglected. The water flows from tank I into the tank II and finally out to the tank ZW. The limitations of the flows come from the orifices C_1 and C_2. The size of the orifice can be varied by plugging and unplugging the holes with stops.

The levels in the tanks (H_1, H_2) are measured with pressure transducers. The appropriate converters, amplifiers, and signal conditioning modules are installed, enabling transmission of the signals to the control computer. An IBM AT equipped with the multifunction data acquisition and control board [7] is used to control the process. This configuration offers relevant computing power, sufficient graphics capabilities (SVGA graphics

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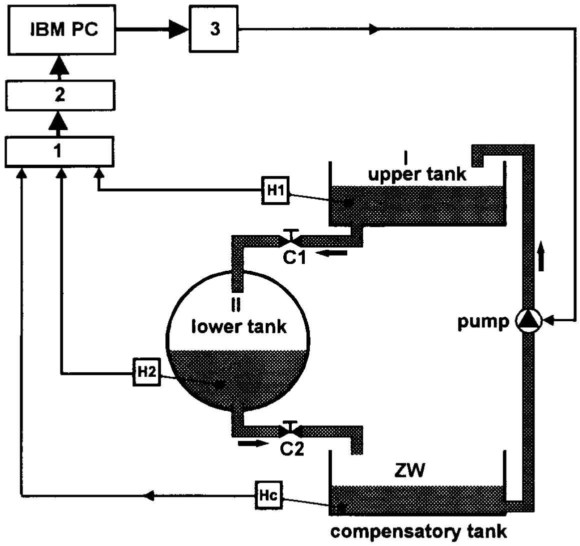


Fig. 1. Process and instrumentation : 1-amplifiers, 2-signal conditioning, 3-drivers.

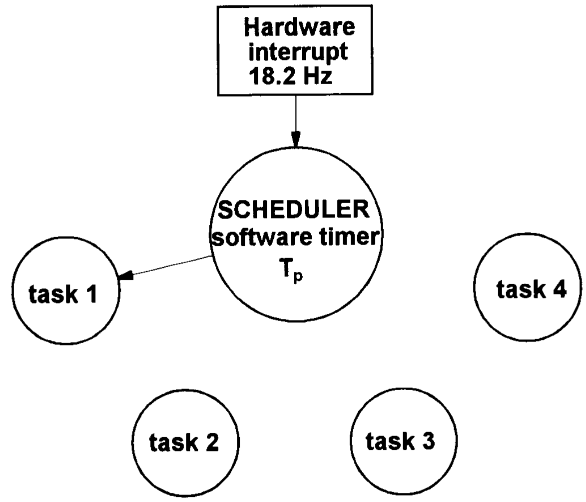


Fig. 2. Sequential real-time system.

card), up to 16 analog inputs and 16 digital inputs/outputs. The multifunction data acquisition board is configurable to handle a variety of different input signals. A 12-bit A/D converter offers a good representation of the original analog signals. Input/output operations can be easily programmed in standard high-level languages.

The output signals from a digital port of the board are processed by an amplifier to provide control signals to the driver of the pump. In the driver, the Triac turns on a fraction of the subintervals in a fixed sampling period. This method allows the application a linear action using a relay-type power controller.

The general control strategy is to stabilize the level in the lower tank by adjustment of the pump operation. The control algorithms are resident in IBM-AT computer as the modules of MS-DOS–based real-time system.

III. Software Organization

The software is portioned into two units—one containing tasks which have to be carried out in real time, the other containing nonreal-time activities. To choose the desired part of the software and to give its parameters, user interfaces are available with a uniform screen layout (Fig. 3).

The software has been developed under the MS-DOS operating system. The real-time unit has an interrupt controlled organization, based on the internal MS-DOS clock working with frequency 18.2 Hz. As the system controls a slow-varying process this limitation is not the essential one. For data exchange a common memory is used.

The real-time unit is divided into tasks. A task is a segment of code which is treated by a scheduler as a program unit that can be executed or omitted. The scheduler of the real-time tank control system supports the sequential task organization [8] (Fig. 2). The basic parameters for the real-time unit are: DOS interrupt period (1/18.2 s), scheduler interrupt period ($T_p = L * 1/18.2$), and sampling period ($T_o = N * T_p$), where L and N are integer constants.

The scheduler interrupt period is the time assumed to complete each single task. The sampling period is a standard notion used in digital control models [9]. A sequence of scheduler interrupt periods can be used to collect all input data, i.e., to create a single image of the process.

Fig. 3 shows the display of the screen where one writes '1' if a task should be active during the interrupt period. In this example $N = 6$, and two interrupt periods are used to collect input data for the digital model.

For each sampling period the control output is approximated by a fraction of "on" pump states in the sampling period. In Fig. 3 the pump is active during 66% of the sampling period.

The tasks to be performed are:

1) "SENSORS" task—reads inputs from A/D port,
2) "FILTER" task—filters input data,
3) ..6. "ALG1," "ALG2," "ALG3," "ALG4" tasks— compute control outputs,
7) "CONTROL" task—transmits outputs to actuators,
8) "KEYBOARD" task—checks operator keyboard,
9) "SCREEN" task—updates operator display or graphical user interface,
10) "END" task—checks task realization time.

It is assumed for sequential organization that the time taken for various paths through the tasks does not vary greatly from cycle to cycle. "END" task puts into the memory the state of the clock counter at the beginning of each task and compares it with its value at the end of the task activity. An error message is generated if the difference between these two numbers is greater then the assumed interrupt period Tp. In this case one solution is to increase the interrupt period. Another approach is to divide a control algorithm into parts carried out in sequence during two or three successive interrupt periods.

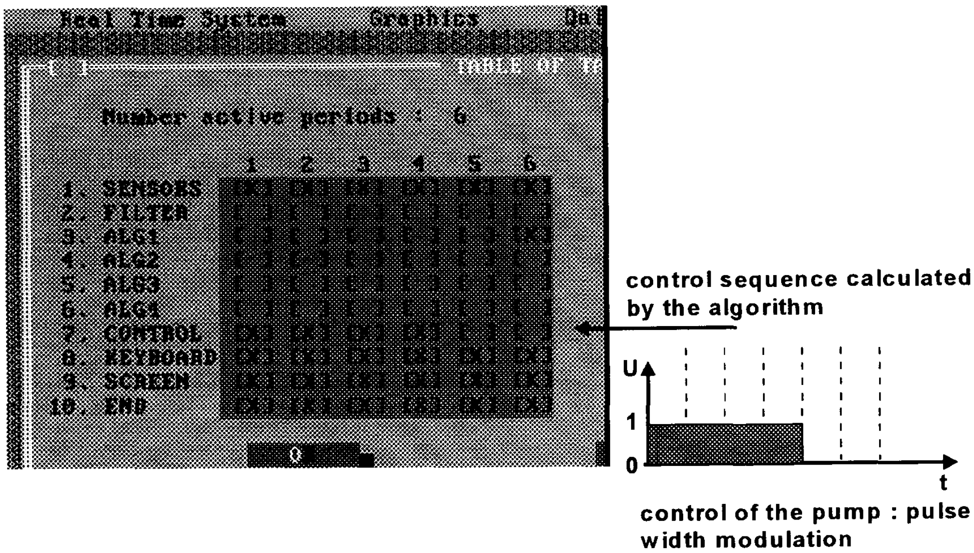


Fig. 3. Table of tasks.

IV. Process Model

The model of the flow process is obtained by means of a mass balance:

$$\left.\begin{aligned} \frac{dV_1}{dt} &= q_i - C_1 \cdot \sqrt{H_1} \\ \frac{dV_2}{dt} &= C_1 \cdot \sqrt{H_1} - C_2 \cdot \sqrt{H_2} \end{aligned}\right\} \tag{1}$$

where:

V_1, V_2—volumes of water in upper and lower tanks

C_1, C_2—orifice discharge coefficients,

H_1, H_2—levels of water in tanks,

q_i— flow rate forced by the pump.

One can obtain from elementary geometry:

$$\left.\begin{aligned} V_1 &= H_1 \cdot S \\ V_2 &= w \cdot \left[r^2 \cdot \arccos\frac{r-H_2}{r} - (r-H_2) \cdot \sqrt{r^2-(r-H_2)^2}\right] \end{aligned}\right\}$$

where:

S – area of upper tank,

r – radius of lower tank,

w – length of the lower tank.

The levels of the water in the tanks can be selected as the state variables:

$$\left.\begin{aligned} \frac{dV_1}{dH_1} \cdot \frac{dH_1}{dt} &= q_i - C_1\sqrt{H_1} \\ \frac{dV_2}{dH_2} \cdot \frac{dH_2}{dt} &= C_1\sqrt{H_1} - C_2\sqrt{H_2} \end{aligned}\right\} \tag{2}$$

where:

$$\begin{aligned} \frac{dV_1}{dH_1} &= S \\ \frac{dV_2}{dH_2} &= 2 \cdot w \cdot \sqrt{r^2 - (r - H_2)^2} = \alpha(H_2) \end{aligned} \tag{3}$$

Finally, the nonlinear state equations are:

$$\left.\begin{aligned} \frac{dH_1}{dt} &= \frac{1}{S}q_i - \frac{1}{S}C_1 \cdot \sqrt{H_1} \\ \frac{dH_2}{dt} &= \frac{1}{\alpha(H_2)}C_1 \cdot \sqrt{H_1} - \frac{1}{\alpha(H_2)}C_2 \cdot \sqrt{H_2} \end{aligned}\right\} \tag{4}$$

A standard technique for modeling a nonlinear system is the linearization of equations near some predetermined operating point.

The steady-state operating points are denoted by H_{10}, H_{20}, q_i:

$$\left.\begin{aligned} H_1 &= H_{10} + \Delta H_1 \\ H_2 &= H_{20} + \Delta H_2 \\ q_i &= Q_i + \Delta q_i \end{aligned}\right\} \tag{5}$$

yielding:

$$\frac{d}{dt}\begin{bmatrix}\Delta H_1 \\ \Delta H_2\end{bmatrix} = \begin{bmatrix} a_1 & 0 \\ a_3 & a_4 \end{bmatrix}\begin{bmatrix}\Delta H_1 \\ \Delta H_2\end{bmatrix} + \begin{bmatrix} b_1 \\ 0 \end{bmatrix} \cdot \Delta q_i \tag{6}$$

where:

$$\begin{aligned} a_1 &= \frac{-C_1}{2S\sqrt{H_{10}}} \\ b_1 &= \frac{1}{S} \\ a_3 &= \frac{C_1 \cdot \sqrt{H_{10}}}{4H_{10} \cdot w \cdot \sqrt{r^2 - (r - H_{20})^2}} \\ a_4 &= \frac{-C_2 \cdot \sqrt{H_{20}}}{4H_{20} \cdot w \cdot \sqrt{r^2 - (r - H_{20})^2}} \end{aligned}$$

A discrete model is easily obtained from (6) for the assumed sampling period T_0 and with the input supplied through a zero-order hold:

$$\Delta H(T_0(k+1)) = A_D \Delta H(T_0 k) + B_D \Delta q_i(T_0 k) \tag{7}$$

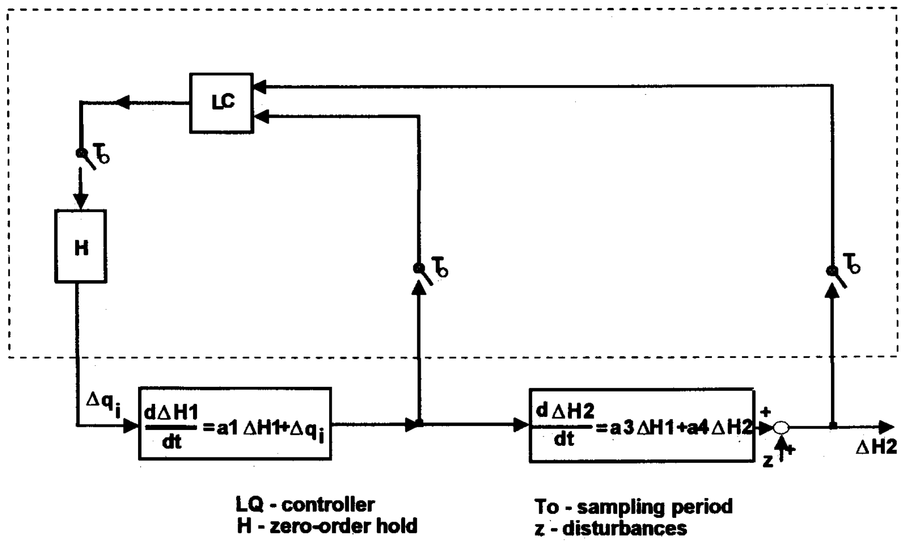


Fig. 4. Model of digital stabilization system with state feedback.

where:

$$\left.\begin{array}{ll} A_D & = e^{AT_0} \\ B_D & = A^{-1}[e^{AT_0} - 1]B \end{array}\right\} \tag{8}$$

A, B are corresponding matrices in (6). Combining (6) with (8) we have:

$$A_D = \begin{bmatrix} e^{a1T_0} & 0 \\ \frac{a_3}{a_4 - a_1}(e^{a1T_0} - e^{a_4T_0}) & e^{a_4T_0} \end{bmatrix}$$

$$B_D = \begin{bmatrix} \frac{1}{a_1}(e^{a_1T_0} - 1)b_1 \\ \left(\frac{a_3(2a_1 - a_4)}{a_1a_4(a_4 - a_1)}e^{a_1T_0} - \frac{a_3}{a_4(a_4 - a_1)}e^{a4T_0} + \frac{a_3}{a_1a_4}\right)b_1 \end{bmatrix}$$

Models (6) and (7) have locally stable eigenvalues, but the conditions for global stability of the nonlinear model (4) can be discussed.

V. Control System Design

The discrete sequence providing control signals to the driver of the pump is an approximation of the control generated by the control algorithm:

$$\Delta q_i(kT_0) \approx \sum_{n=m+1}^{N} x_n(kT_0)q_cT_p \tag{9}$$

where q_c is the constant flow rate of the pump, and $x_n(kT_0)$ is a (0, 1)—sequence establishing the fraction of pump operation in the sampling period (T_0).

First "m" interrupts are not taken into account, modeling a pump delay (τ_p):

$$\left\{m : \sum_{n=1}^{m} x_n(kT_0)T_p \approx \tau_p, x_1(kT_0) = 1, \ldots x_m(kT_0) = 1\right\} \tag{10}$$

A complete control engineering design starts from simulation and identification. Experiments are carried out by routines of PC-MATLAB packages. Students have to analyze the dynamical properties of the process in order to define possible control constraints. The controller is synthesized by combining the model with the control constraints and demands. The following control methods were selected as being representative:

1) Cascade PID control [9],
2) Minimum-time control [10],
3) State feedback control [11].

Finally, the control methods are to be applied using the instrumentation and real-time software described in the previous sections. The relative influence of the controller parameters can be inspected since the software allows the following controller parameters to be changed on-line:

1) Set point (linearization effect),
2) Sampling period (discretization effect),
3) Saturation integral limits (prevention of integral wind-up).

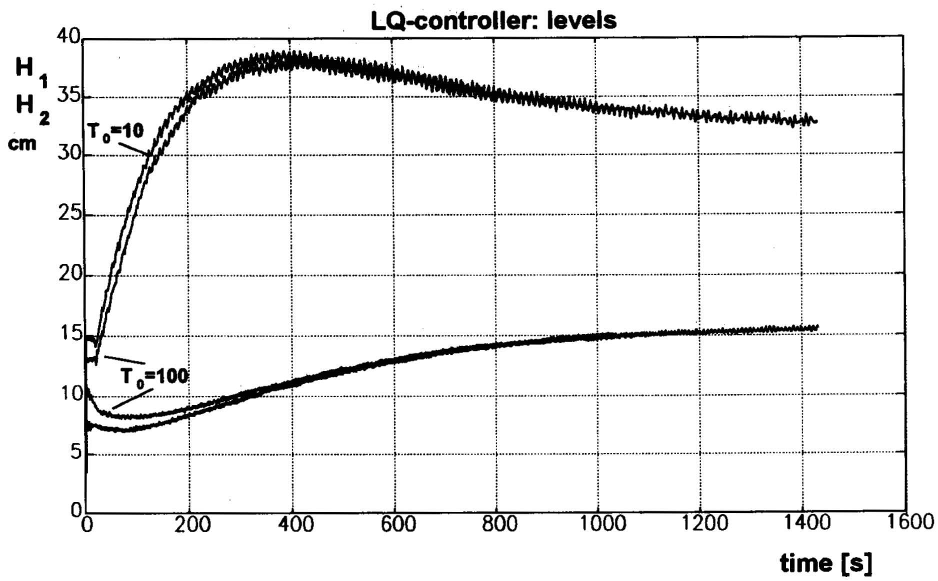
As an example consider the linear state feedback method (Fig. 4), in the form [11]:

$$\Delta q_i(kT_0) = -K\Delta H(kT_0) \tag{11}$$

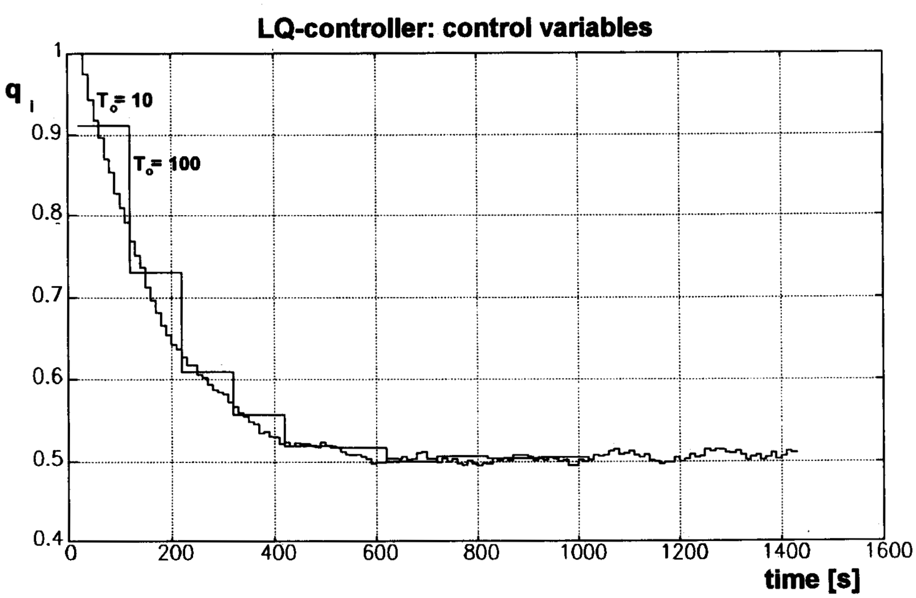
The objective function is:

$$J = \sum_{0}^{K-1} \Delta H^T(kT_0)Q\Delta H(kT_0) + \Delta {q_i}^T(kT_0)R\Delta q_i(kT_0) \tag{12}$$

where Q, R are symmetric, positive definite matrices. In order for a unique positive definite solution to exists, the (A_D, B_D) pair must be controllable. The calculation of the optimal



(a)



(b)

Fig. 5. Control experiment with the tank system: levels in the tanks and control variables.

feedback matrix requires the solution of a discrete version of matrix Riccati equation [11].

Fig. 5 shows the step response of the digital control system to a change of the set point. The parameters of the controller were:

$$Q = \begin{bmatrix} 0 & 0 \\ 0 & 80 \end{bmatrix};$$

$R = 1$; $T_p = 1.0$ s, $T_0 = 10$ s, 100 s (two cases).

VI. Conclusion

The laboratory environment for experiments in digital control of a coupled tank system was described. The experimental system offers educational possibilities in many fields connected with real-time digital control. Because of the rather complex model of the system there are other applications which can be tested with the tank system. For example, a temperature stabilization in the lower tank may be applied on the basis of an extended model of the process.

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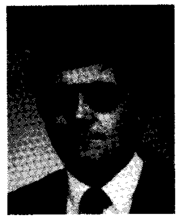

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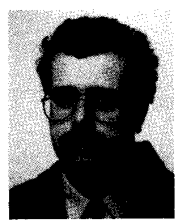

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