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Robust Almost Time-Optimal Fuzzy Control of a Two-Tank System

Thomas Heckenthaler and Sebastian Engell

Process Control Group
Department of Chemical Engineering
University of Dortmund
D-44221 Dortmund, Germany
e-mail: ast@astaire.chemietechnik.uni-dortmund.de

Abstract — The paper deals with the level controls in a laboratory two-tank system. The plant is strongly nonlinear due to the basic dynamic equations and the characteristics of the valves. We developed a nonlinear control law which achieves robust almost time-optimal control over the full range of operation conditions. The development is based on ideas from fuzzy control, but in contrast to usual fuzzy controller designs, most of the rules are not derived from heuristics but rather are mathematical formulae which, together with the standard fuzzy quantization of the system's variables, approximate the time-optimal control law. This approximation is improved by heuristic rules which were gained from the observation of the behaviour of the controlled plant. The resulting nonlinear control law exhibits a performance which is not attainable with standard linear control nor with classical time-optimal control.

1. Introduction

Fuzzy control is at present being extensively explored as a means to develop controllers for plants with complex and often not precisely known dynamics. In almost all cases, the basis of a fuzzy control law is heuristic knowledge about a suitable control strategy. This knowledge may be acquired by interviewing operators who have a lot of experience in the manual control of the plant, or it may stem from basic "common sense" engineering reasoning. If the dynamics of the plant are complex, however, basic reasoning is often insufficient for the derivation of suitable controls. Furthermore, even if there are experienced operators available, the formalization of their knowledge is usually difficult, and doubts about the quality of the solution may remain even if the implemented controller behaves reasonably.

We decided therefore to take another approach to the derivation of a fuzzy control law. Our starting point was to assume that there exists a - possibly crude - mathematical model of the plant dynamics. Then instead of asking "What does the operator do?" we ask "How would the ideal operator perform?". One answer to this question, and in many cases a very reasonable one, is that large deviations of the actual state of the plant from the desired state should lead to control actions which fully use the available range of actuator movements to bring the plant state to the vicinity of the desired state as quickly as possible, whereas around this desired state, a behaviour similar to that of a linear controller or a linear controller with a dead zone is appropriate. In other words, we want almost time-optimal control for large deviations and robust, not too hectic control for small ones.

Thus as a first step, time-optimal controls are derived and simulated for a plant model. Then the quantitative system variables are classified into qualitative values and for each of these values, a formula for the control law is computed. The classification is fuzzy with significant overlap, so that several control laws are usually activated. The resulting controller output is the weighted mean of the output of these local control laws. Fuzzification is used here to provide a smooth interpolation of the local control laws.

The controller which we thus obtained contains 8 rules for the control of the level of one tank. It approximated the time-optimal controller very well, including its unattractive features, particularly its high sensitivity to noise in the measurements and to modelling errors. At this point, after the theoretically founded methods had been exhausted and not before, we brought in heuristics. The trajectories of the system were observed and additional rules were formulated to improve the robustness. One obvious and simple measure was to introduce a linear control band for small regulation errors. In our case, the decisive step was to add two more simple rules which modify the time-optimal control law depending on the state of the system. The implementation of the complete control law in the form of rules provides a flexible and uniform representation and allows for the easy implementation of further heuristic knowledge, constraints, etc..

2. Description of the two-tank system

The plant which we want to control consists of two tanks T1 and T2 connected as shown in Fig. 1. Their levels h_1 and h_2 are controlled by the actuators V1 and V2. The control valves can only be opened and closed rather slowly. It takes about 80 seconds for a complete movement from fully open to closed and vice versa. Due to this speed limitation, a three-term controller is applied, so that the valves are either opened or closed at maximal speed or remain in their present position.

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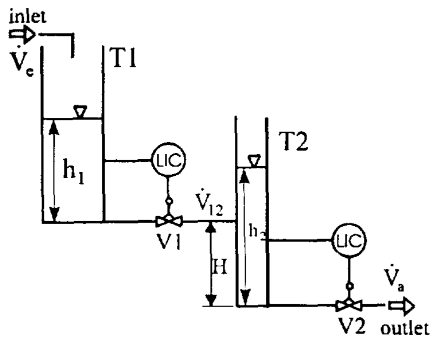

Fig. 1: Laboratory two-tank system

The mathematical model results from the mass balance equations for the two tanks:

$$\dot{h}_1 = (\dot{V}_e - \dot{V}_{12}) / A_1 \quad \text{and} \quad \dot{h}_2 = (\dot{V}_{12} - \dot{V}_a) / A_2 . \tag{1}$$

A_1 and A_2 are the cross-sectional areas, and $\dot{V}_e$ is the volumetric inflow of water into T1. The outgoing volumetric flows $\dot{V}_{12}$ and $\dot{V}_a$ are determined by Toricelli's law:

$$\dot{V}_{12} = \begin{cases} K_1(P_1)\cdot\sqrt{h_1 - (h_2 - H)} & \forall\, h_2 > H \\ K_1(P_1)\cdot\sqrt{h_1} & \forall\, h_2 < H \end{cases}$$
$$\dot{V}_a = K_2(P_2)\cdot\sqrt{h_2} . \tag{2}$$

The acceleration in the tubes can be neglected due to the small mass of water in the tubes. The coefficients K_1 and K_2 are determined by the respective valve positions P_1 and P_2. Fig. 2 shows the relation between the coefficient K_1 and the valve position for various ingoing volumetric flow rates, as determined experimentally. The resulting curve was determined by interpolation and is significantly inaccurate at the extreme positions of the valve. If this curve is used to calculate model-based control strategies, this inaccuracy can lead to unsatisfactory system performance, especially when sensitive time-optimal control is applied.

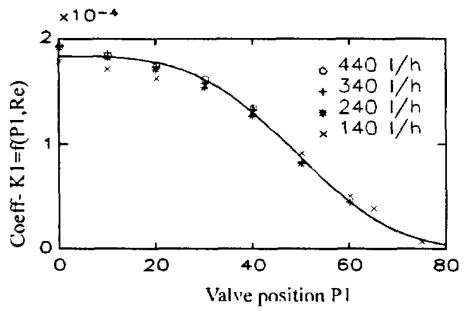

Fig. 2: Valve flow characteric

The plant exhibits a strongly nonlinear behaviour because of the nonlinear flow characteristics, Toricelli's outflow equation, actuator saturation, and the three-term controller. The complexity of this seemingly simple plant became obvious as we first tried to control it manually.

The time needed for manual control experiments is very long because of the slow system dynamics. For this reason, the development of heuristics based on exhaustive experiments which could eventually be used in a fuzzy rule base did not seem promising.

Columns T1 and T2 have different diameters. Because the time constant of tank 1 is 6 times larger than that of tank 2, the level in T1 is much more difficult to control. Therefore we started with the development of a controller for the level in tank 1.

a) Conventional linear control

As one might expect, a fixed-gain linear controller cannot adequately cope with the nonlinearities of this system. A PD-controller must be used because of the double integrator which is present in the plant. The simulated system performance for two consecutive set-point steps at a constant inflow rate $\dot{V}_e$ is shown in Fig. 3. The level follows the first step very well, but an undesirable overshoot occurs when the set point is reset. In more extreme cases, the tank gets out of control because the transients overshoot so much that the overflow alarm is triggered.

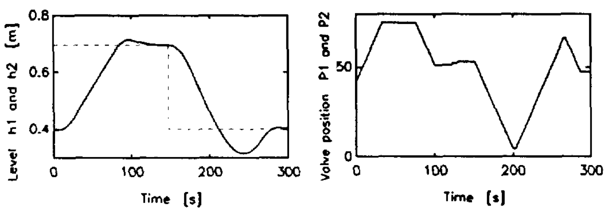

Fig. 3: Control of level 1 with a PD-controller (simulation)

b) Time-optimal control

As we mentioned already in the introduction, we used the time-optimal controller as a reference for the desirable behaviour. Fig. 4 shows the simulated system using the time-optimal controller.

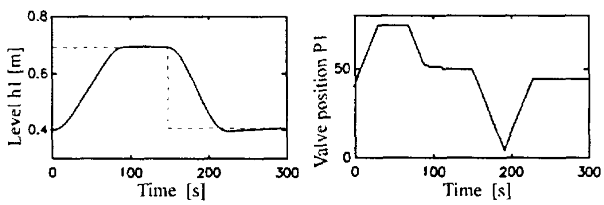

Fig. 4: Time optimal control of level 1 (simulation)

Although this controller performs well in situations without measurement noise, it is not a good choice at the real plant. Modelling inaccuracies and measurement noise lead to inefficient valve movements, and this causes settling times which are not only not optimal but worse than those obtained with PD control (see Fig. 5).

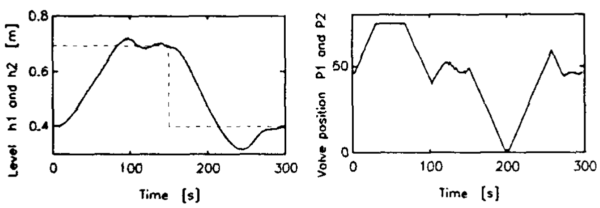

Fig. 5: Time-optimal control of level 1 at real plant

3. Approximation of time-optimal control law by a fuzzy controller

a) Motivation

The application of fuzzy logic in a controller typically results in a controller which consists of 3 components. In the fuzzification component, the controller inputs are classified into certain fuzzy variables, i.e. they are converted from real numbers into qualitative variables with fuzzy membership functions. This is followed by the evaluation of the rule base in the inference component. Here the actuator movement is determined by means of linguistic rules. The results of these are combined into a unique, but still fuzzy, output using fuzzy logic. Since the final output of the controller has to be a crisp real number or one of a number of alternative discrete outputs, retranslation by the defuzzification component is necessary. Whereas the fuzzification of the input variables and the qualitative rules in the rule base usually can be understood more or less easily, the application of one of the various possible inference and defuzzification methods gives rise to a behaviour of the controller which is in general not transparent to the user. A graphical user interface which shows which rules are active in special cases and the resulting control surface can be helpful. Still, the standard fuzzy controller design is characterized by "trial and error" and not by clear systematics.

b) General approach

Our approach differs from the one described above, in that the identification of the fuzzy rule base was automated and the rules are formulae for computations with real variables. The method we used to determine an optimal fuzzy rule base is by and large the one proposed by Sugeno [1]. In this approach, the conclusions of the fuzzy rules take a special form. The output is determined by a parametrized function of the controller inputs and not by fuzzy values. This means each rule returns a pair of values - a truth value which is determined by the truth values (membership degrees) of the variables in the precedent, and a crisp value for the control output according to the consequence. If the fuzzy sets of the qualitative input variables have sufficient overlap, a soft interpolation results with the advantage that the number of implications is reduced, and a smooth transition between the different control laws is guaranteed [1,2,3]. With A_m^n being the linguistic values , the format of the fuzzy rule base is:

Rule 1: IF x_1 is A_1^1, x_2 is A_2^1, ..., x_m is A_m^1 THEN

$$y^1 = p_0^1 + p_1^1 \cdot x_1 + p_2^1 \cdot x_2 + \ldots + p_m^1 \cdot x_m$$

$\vdots$

Rule n: IF x_1 is A_1^n, x_2 is A_2^n, ..., x_m is A_m^n THEN

$$y^n = p_0^n + p_1^n \cdot x_1 + p_2^n \cdot x_2 + \ldots + p_m^n \cdot x_m \quad .$$

The consequence functions are thus characterized and determined by the parameter vector p_i. The final output y is computed as the weighted average of the individual outputs y_i of the rules:

$$y^* = \frac{\sum_{i=1}^{n} \min\left[\mu A_1^i(x_1),\ldots,\mu A_m^i(x_m)\right]\cdot\left(p_0^i + p_1^i \cdot x_1 + \ldots + p_m^i \cdot x_m\right)}{\sum_{i=1}^{n} \min\left[\mu A_1^i(x_1),\ldots,\mu A_m^i(x_m)\right]} \quad . \tag{3}$$

$\mu A_m^i(x_m)$ is the degree of membership of the (crisp) input x_m to the linguistic variable A_m^i. With the notation from [1], (3) can be arranged into the equation:

$$y^* = X \cdot P. \tag{4}$$

The matrix X represents the fuzzification and the values of the inputs and vector P contains the unknown parameters of the consequence. By using singletons as output values in the consequence, the normally required, complicated defuzzification step is no longer needed. In order to approximate the time-optimal control strategy for the full range of operating conditions, simulations were performed over the entire feasible region of the state space of the process. These computations yield, for each point considered, a pair of controller inputs and the desired time-optimal control output. For k ordered sets, (4) becomes:

$$Y_{k,1} = X_{k,n(m+1)} \cdot P_{n(m+1),1} \; . \tag{5}$$

The matrix X on the right-hand side is given by the controller inputs x_i and their respective fuzzy values A_m^i. The parameter-vector P is unknown. For k=n·(m+1) ordered pairs, the solution of equation (5) is uniquely determined. But we used far more points at which the optimal control is computed than there are free parameters. Thus (5) is an overdetermined linear system of equations. The parameter vector P can be calculated using the pseudoinverse of X, such that the root mean square error F

$$F = \sqrt{\frac{1}{k-1}\sum_{j=1}^{k}\left(y_{j,1}^* - X_{j,n(m+1)} \cdot P_{n(m+1),1}\right)^2} \tag{6}$$

is minimized

$$P = (X^T \cdot X)^{-1} \cdot X^T \cdot Y \; . \tag{7}$$

The magnitude of the minimal value of F is determined by the structure of the fuzzy rule base. The important factors here are the number of rules and the choice of the membership functions. For a given number of rules, the membership functions can be optimized so that the minimal error F_{min} is minimized. Usually one will start with the smallest reasonable number of fuzzy partitions and increase it until the error is sufficiently small. The neccessary accuracy must be determined by simulations or tests of the resulting controller.

c) Application to the control of level 1

Now we show how our approach was applied to the tank system described in the previous section. The controller inputs considered in the premises of the fuzzy rules have to be determined first. These inputs are the same as the inputs of the time-optimal controller. In the case of the decoupled first tank, the inputs are the regulation error e_1, the ingoing volumetric flow rate $\dot{V}_e$, and the actual level h_1. Next the state space of the process has to be discretized into physically reasonable partitions for which time-optimal control switching points have to be computed. Taking into account the fact that for larger errors in a given flow rate interval the valve always drives into saturation, and for small errors ($|e|<0.01$) a robust linear controller is applied, about 500 ordered pairs were obtained with a reasonable discretization.

Several fuzzy rule bases with different numbers of rules have been identified. The minimal error F_{min}, which is a measure of the deviation from the time-optimal valve-position, decreases with an increasing number of rules as one would expect. The membership functions were arranged symmetrically with linear functions and a significant overlap. To determine the optimal fuzzifaction one can use optimization algorithms (e.g. the Quasi-Newton method [4]). In our case it was found that a fuzzy rule base containing 8 rules yields an already satisfactory, smooth interpolation of the given time optimal control data. For this the level h_1 and the inlet flow $\dot{V}_e$ were divided into fuzzy partitions *small* and *big*, and the regulation error e_1 into *positive* and *negative*. The result can be seen in Fig. 6, which shows that the simulated system performance is almost time optimal.

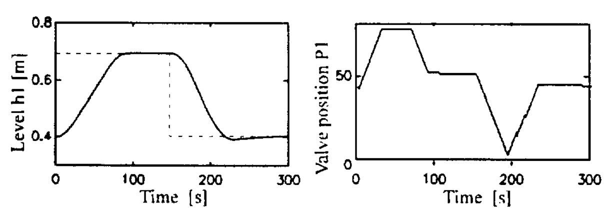

Fig. 6: Fuzzy-Controller with 8 rules (simulation)

The given simulation proves that the time-optimal strategy can be successfully approximated by a fuzzy controller with a sufficient number of rules. Next the controllers were tested for robustness on the real system.

4. Modification and experimental results for the control of decoupled tank 1

While the simulations so far were made under the assumption that the existing model is correct, we now have to deal with noise and modelling errors, and their effect on the system performance. The inlet flow $\dot{V}_e$ is not measured online, but can only be estimated on the basis of the value of the actual valve position, the corrupted signal h_1, and the crude model for the valve characteristic, and is therefore inexact. In Fig. 3 we showed already the classical time-optimal controller responses to reference step changes. Even though for small errors ($|e_1|<1$cm) a less sensitive PD-controller was used, significant overshoot and therefore a longer regulation time results.

It cannot be expected that a fuzzy controller which is determined as an approximation of the ideal time-optimal controller would provide a significant improvement. Despite frequent claims by advocats of fuzzy control, fuzzy control is not more robust per se.

Indeed, the fuzzy controller exhibits the same sensitivity to measurement noise and modelling errors. But observations of the transients gave additional insight which was then represented by additional rules. In order to suppress overshooting, it is obvious that the ingoing volumetric flow rate should be manipulated. At positive level errors, the volumetric rate has to be increased, and at negative errors it has to be decreased. Manipulating the estimated inflow rate in this manner leads to earlier movements of the valve to its final steady state position. Experiments have shown that an overshoot can be avoided by a 20%-change in both directions. An almost time-optimal system performance with the real plant is now achieved over the full range of operation conditions (see Fig. 7).

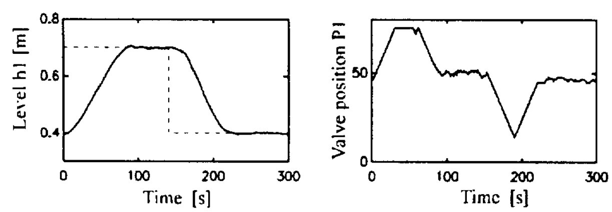

Fig. 7: Fuzzy-Controller at the real plant

5. Control of the coupled system

In the previous sections, only the first tank without coupling to the second one was considered, i.e. the case $h_2 < H$. We now turn to the case of coupled tanks and the control of both levels simultaneously. The complexity of the control of the coupled system is much higher because now the system has not only nonlinear dynamics, but its structure also changes during operation when h_2 exceeds H=0.4m. Fig. 8

shows that the performance deteriorates considerably when applying two fixed linear PD-controllers for both levels.

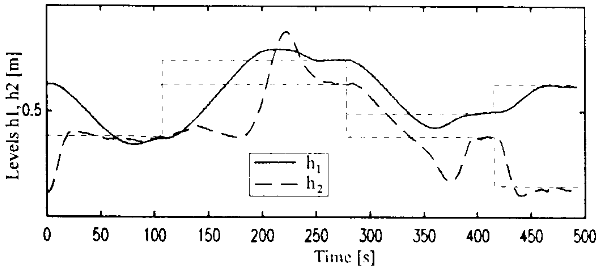

Fig. 8: PD-Controllers for the coupled system

For the same reasons as discussed above, we want to achieve a time-optimal behaviour, i. e. the smallest possible settling time without overshoot for both levels. However, as can be seen from the simulation of the globally time-optimal controller in Fig. 9, to merely specify the smallest possible settling time of the overall system produces trajectories which would not be considered as acceptable from a practical point of view. This is a common feature of standard time-optimal control laws for multivariable systems with different maximal speeds of the subsystems [5]. As the settling time of level 1 is always much longer than the settling time of level 2 and hence determines the overall settling time which is to be minimized, the globally time-optimal controller manipulates level 2 in a manner which yields the maximal "support" for level 1 (i. e. reduces the settling time of level 1) before level 2 itself tends to the desired reference value. This indeed produces a minimal overall settling time but at the expense of large deviations of level 2 from the desired value in the transient period. (The global time-optimal control was determined by backward integration of the system dynamics as in the case of level 1 alone).

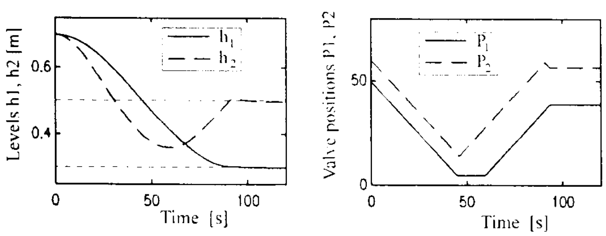

Fig. 9: Global time-optimal control of both levels (simulation)

This observation led us to the conclusion that globally time-optimal behaviour might be mathematically nice but of little practical interest. Another disadvantage is that the mathematical effort to calculate global time-optimal controls for this strongly nonlinear system is huge. This is also true in principle if one would try to approximate the global time optimality by a fuzzy model of the format described above, because 7 inputs enter into the computation of the time optimal control law. If each input is partitioned into at least two fuzzy sets, $2^7=128$ rules with 1024 parameters altogether have to be identified.

Our idea then was to restrict the time-optimal behaviour to isolated subsystems. We first identified a fuzzy controller (10 rules) for tank 2 in the same manner as for tank 1. The fuzzy-controller for level 1 was expanded by one reasonable rule which is due to the backwards coupling of level 2 into tank 1: *if* $h_2>H$ *then* reduce input h_1 by (h_2-H). From Fig. 10, we can see that the performance of the fuzzy-control of level 1 is almost unaffected in the coupled case. On the other hand, the performance of the fuzzy controller for level 2 is unsatisfactory. This is due to the lack of the input $\dot{V}_{12}$ which is not a measured variable. Thus the inlet flow $\dot{V}_e$ in tank 1 is used as an input in fuzzy controller 2 as well. This means that the strongly variable flow into tank 2 due to the actions of the controller actions for level 1 is neglected. The consequence is seen in Fig. 10 which exhibits significant overshoot of level 2.

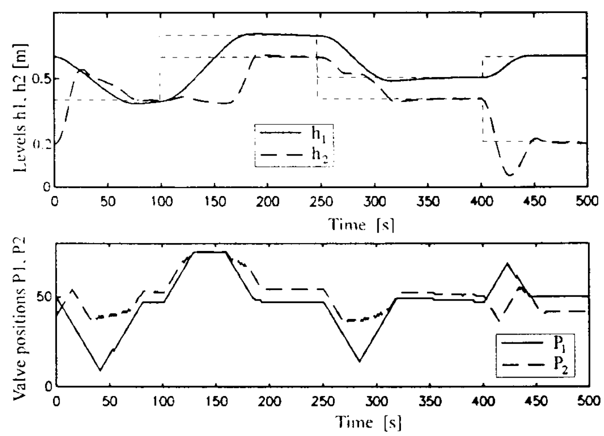

Fig. 10: Coupled system with 2 fuzzy controllers (simulation)

As one examines the transients it can be seen that the simultaneous control in tanks 1 and 2 can be classified into 3 situations. These are determined by combinations of the signs of the regulation errors e_1 and e_2. According to these combinations one is able to formulate rules to take the effect of the actuator movement of valve V1 into account for tank 2 by means of a correction of the volumetric flow into tank 2:

1) If $\mathrm{sign}(e_1)=\mathrm{sign}(e_2)$ then $\Delta\dot{V}_{12}=K_1\cdot e_1$
2) If $e_1<0$ and $e_2>0$ then $\Delta\dot{V}_{12}=K_2\cdot e_1$
3) If $e_1>0$ and $e_2<0$ then $\Delta\dot{V}_{12}=K_3\cdot e_1$

$$\Rightarrow \dot{V}_{12}=\dot{V}_e+\Delta\dot{V}_{12}$$

The values of the parameters K_1 through K_3 were found in a short time by optimizing the transients from Fig. 10. The resulting system behavior of the real plant with the expanded fuzzy controller for tank 2 is shown in Fig. 11. The movements of the actuators prove that the control of the global system can be characterised *almost time-optimal.*

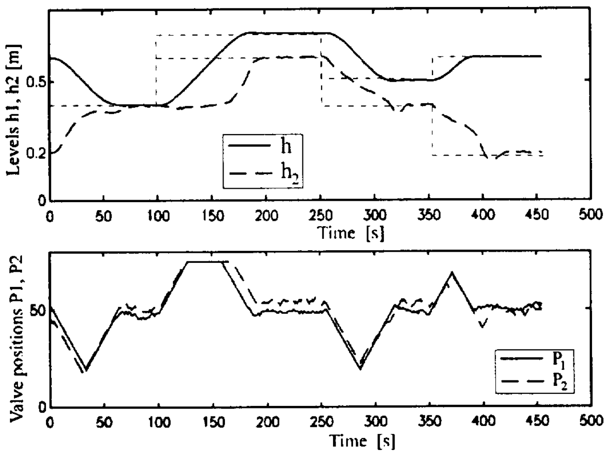

Fig. 11: System behavior of the coupled system with expanded fuzzy controllers at real plant

In Fig. 12 the reference steps from Fig. 9 are repeated with the expanded fuzzy controllers. It can be seen that in comparison to the global time-optimal control, the (fuzzy) rule-based approach only marginally increases the settling time of level 1 and leads to very reasonable transients of both controlled variables. Another benefit is the much smaller computational effort for the fuzzy controllers compared to the global time-optimal control, which makes real-time application much easier.

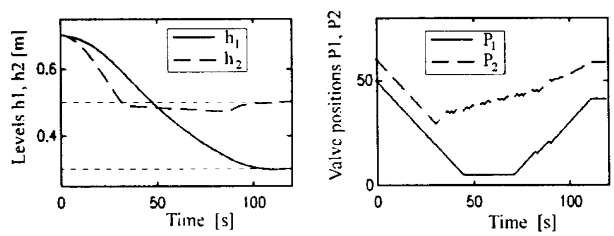

Fig. 12: System behavior of the coupled system with expanded fuzzy controllers (simulation)

Conclusions

With this contribution, we want to support two main statements:

1. With unconventional methods, it is possible to obtain controllers which are superior both to conventional fixed time-invariant controllers (PI, PD, PID) and to classical optimal controllers. We have demonstrated this for the control of the levels in a tank system which has considerable nonlinearities and poses a nontrivial but still relatively transparent control problem. Our controller combines the low sensitivity of the standard control algorithms with the performance of the time-optimal control law.

2. The standard approach to fuzzy control, the implementation of heuristic knowledge, is in many situations of limited value. For plants with complex dynamics, heuristic knowledge is hard to acquire and equally hard to assess with respect to the resulting quality of control. Stability and performance of the resulting control laws have to be tested by extensive simulations or by trials at the real system. In most cases, there will be some model of the plant dynamics available. It therefore makes much more sense to use the available analytical tools first and then eventually modify and improve the resulting controller using heuristic knowledge gained by experiments or further physical insight. In this process it is beneficial to represent the control law in the form of rules with analytical formulae as conclusions. The parameters of the rules can be determined by optimization. This representation provides a basis for the additional representation of heuristic knowledge and for an efficient implementation of complex nonlinear control laws which might be impossible to compute on-line, as in the case of time-optimal control of high-order linear or nonlinear systems [5].

Further work in this area will concentrate on the systematic simplification of the rule base in the case of a large number of variables which enter into the rules, as in the case of the coupled tank system, to allow for easy modification and implementation of the controller.

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