Level Control using a Feed-Forward Structure

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Abstract—**This paper presents an alternative for level control in three coupled tanks, using a feed-forward control structure. The mathematical model for the lab installation is developed and the model is tuned by comparing its output with the real output of the lab rig. A comparative analysis of PI control and feed-forward control is presented.**

Keywords—level control, feed-forward, three coupled tanks

I. Introduction

There are numerous examples of industrial installations that use an assembly of three coupled tanks. The chemical industry uses tank assemblies for groups of chemical reactors, combined with heat exchangers and mixing tanks. Another use of coupled tanks is for balance control of commercial ships or oil tankers. A different use may be found in the assembly of the reservoirs for multiple hydro-power plants along a single watershed. In all these cases, we deal with a complex non-linear system described by fluid flow equations. These systems require an exact control of the fluid level in all tanks, with a short response time for the level control system [1],[2],[3,],[4].

In order to verify the mathematical models of the system and the response of the level control systems, we used an experimental lab rig, consisting of three coupled tanks (see figure 1). The rig was built using the model Ammira, 1998 [5].

Our experimental lab rig is a fluid system composed of three identical tanks. The first two tanks are each connected to a variable speed pump. In addition, the three tanks are connected to a common transfer pipe and a common drain pipe. Six manual-operated valves control the transfer of water through each of the tanks, and three pressure-level sensors provide measurements of the water level in each tank.

The system consists of three modules (figure. 1):

* the System with three tanks, including the execution (pumps) and measurement elements (sensors),

* the Control Module with the power gains PG for execution elements (actuators) and Signal Converter SC for sensors and

* the Quanser MultiQ PCI / MQ4 NI-E Series DAQ board. The WinCon-Simulink-RTX program is provided with the driving board, which enables the design of level control program (Real-Time Control Software) [6].

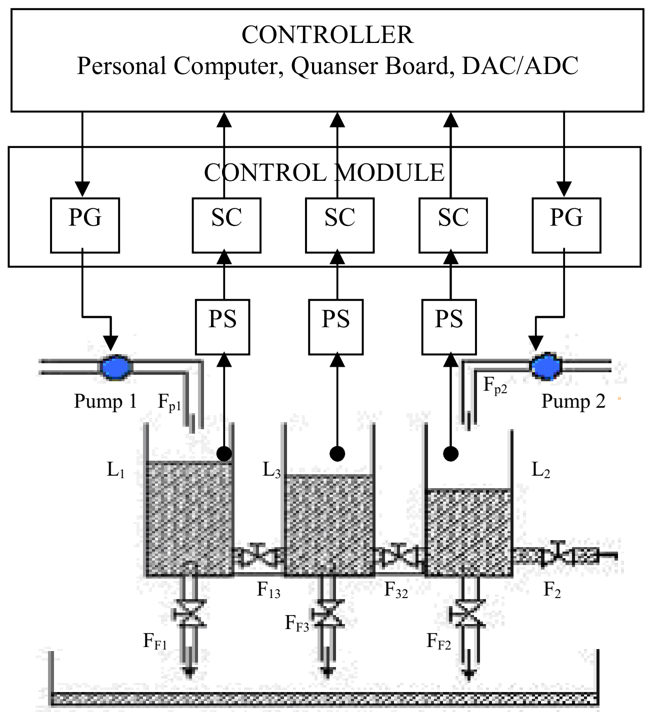

Fig. 1. Experimental lab rig for three tank level control

The maximum level that can be reached is 45cm, after which the pumps are automatically disconnected. Pumps are driven by DC-motors. To provide disturbances in the system simulation, interconnecting pipes as well as the nominal drain are equipped with valves. At the same time, each tank has a direct exit to the fluid collector tank.

II. Mathematical Model of The Process

A. System Equations

In the lab rig, pump 1 feeds into tank 1 and pump 2 feeds into tank 2. Therefore, the inputs of the process are the pump voltages V_{p1} and V_{p2} and the outputs are water levels L_1 in tank 1 and L_2 in tank 2. The plant has the possibility to simulate the presence of faults, consisting in leakages from

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one or two tanks by flows F_{F1}, F_{F2} or F_{F3}, which influence the tank levels L_1, L_2, L_3.

Considering that the system is not under fault conditions (there is neither clogging nor leakages $F_{F1} = F_{F2} = F_{F3} = 0$) from the conservation of the mass in the tanks we obtain the differential equations:

$$\begin{aligned} A\frac{dL_1}{dt} &= F_{p1}(u_1) - F_{13}(L_1, L_3) \\ A\frac{dL_2}{dt} &= F_{p2}(u_2) + F_{32}(L_2, L_3) - F_2(L_2) \\ A\frac{dL_3}{dt} &= F_{13}(L_1, L_3) - F_{32}(L_2, L_3) \end{aligned} \quad (1)$$

where, F_{13}. F_{32} and F_2 are the flows from tanks [13, 14]:

$$\begin{aligned} F_{13} &= \mu_1 S_{13}\sqrt{2g}.sign(L_1 - L_3).\sqrt{|L_1 - L_3|} \\ F_{32} &= \mu_2 S_{32}\sqrt{2g}.sign(L_3 - L_2).\sqrt{|L_3 - L_2|}, \\ F_2 &= \mu_2 S_{32}\sqrt{2g}.sign(L_2).\sqrt{|L_2|} \end{aligned} \quad (2)$$

The parameters in equations (2) are presented in Table 1.

The volumetric inflow rates to the tanks are assumed to be directly proportional to the applied pumps voltages: $F_{p1}=K_{p1}V_{p1}$, $F_{p2}= K_{p2}V_{p2}$.

The pump flow constants K_{p1} and K_{p2} were evaluated by experiments on the lab plant:

$K_{p1}= 1.28.10^{-5}$ [m^3/secV], $K_{p2}= 0.45.10^{-5}$ [m^3/secV].

The numerical values of the parameters for the lab three-tank water level control system are provided in Table 1.

TABLE I. NUMERICAL VALUE OF PHYSICAL PARAMETERS

Physical quantity	Symbol	Value	Units
Tank 1,2,3 sections	A_1, A_2, A_3	0.0154	m^2
Valves V_{13}, V_{32}, V_2 section	S_{13}, S_{32}, S_2	5.10^{-5}	m^2
Gravitational constant	g	9.81	m/sec^2
Pump constant	k_1, k_2	$1.28\ 10^{-5}$ $0.45\ 10^{-5}$	m^3/V.sec
Friction coeficient	μ_1, μ_2	0.5	

The equations modeling the flow coefficients μ_1, μ_2 are difficult to determine. We have adjusted the coefficients to obtain model responses identical to those of laboratory installation (see Figure 2). Equations (1) and (2) can be represented in the numerical form (3), with parameter values given by Table 1, using real variables L_1 for tank 1, L_2 for tank 2 and L_3 for tank 3.

$$\begin{aligned} &1.54\dot{L}_1 = 1.28.10^{-3}V_{p1} - 1.1073\cdot 10^{-2} sign(L_1 - L_3)\sqrt{|L_1 - L_3|} \\ &1.54\dot{L}_2 = 0.45.10^{-3}V_{p2} - 1.1073\cdot 10^{-2} sign(L_2)\sqrt{|L_2|} + \\ &+1.1073\cdot 10^{-2} sign(L_3 - L_2)\sqrt{|L_3 - L_2|} \\ &1.54\dot{L}_3 = 1.1073\cdot 10^{-2} sign(L_1 - L_3)\sqrt{|L_1 - L_3|} - \\ &-1.1073\cdot 10^{-2} sign(L_3 - L_2)\sqrt{|L_3 - L_2|} \end{aligned} \quad (3)$$

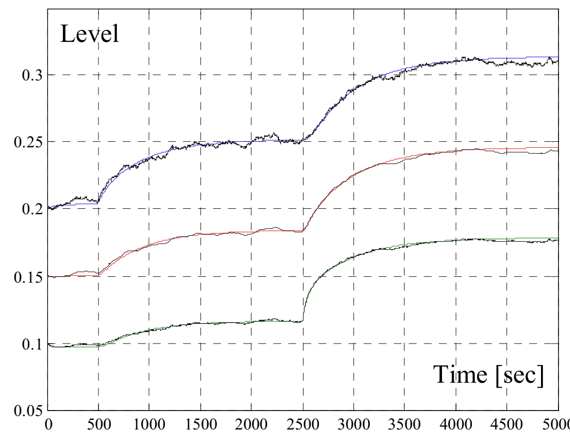

Fig. 2. Comparison between the model and the response of the plant

B. Linear System Equations

The purpose of the present modeling section is to provide the system open-loop transfer function matrix H(s), which in turn will be used to design an appropriate level controller.

The transfer function can only represent the system's dynamics from a linear differential equation. Therefore, the nonlinear equations of motion (1) and (2) should be linearized around a steady-state operation point.

In the case of the waters levels in each tank, the liquid level corresponds to small level variations x_1, x_2, and x_3 from the desired equilibrium point (L_{10}, L_{20}, L_{30}):

$$L_1=L_{10}+x_1,\ L_2=L_{20}+x_2,\ L_3=L_{30}+x_3$$

$$V_{p1}=V_{p10}+u_1, \qquad V_{p2}=V_{p20}+u_2$$

For steady state values of levels in each tank L_{10}=0.25 m, L_{20}=0.15 m and L_{30}=0.2 m the linear model of system has the following form:

$$\begin{aligned} \dot{x}_1 &= a_{11}x_1 + a_{13}x_3 + b_1u_1 \\ \dot{x}_2 &= a_{22}x_2 + a_{23}x_3 + b_2u_2 \\ \dot{x}_3 &= a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{aligned} \quad (4)$$

The coefficients of linearized equations are functions of L_{10}, L_{20} and L_{30}. The form of these coefficients is as follows:

$$a_{11} = -\frac{7.19.10^{-3}}{2\sqrt{L_{10}-L_{30}}}; \quad a_{13} = \frac{7.19.10^{-3}}{2\sqrt{L_{10}-L_{30}}}$$

$$a_{22} = -\frac{7.19.10^{-3}}{2\sqrt{L_{10}-L_{30}}} - \frac{7.19.10^{-3}}{2\sqrt{L_{20}}}; \quad a31 = -\frac{7.19.10^{-3}}{2\sqrt{L_{10}-L_{30}}}$$

$$a_{32} = \frac{7.19.10^{-3}}{2\sqrt{L_{10}-L_{30}}}; \quad a_{23} = -\frac{7.19.10^{-3}}{2}\left[\frac{1}{\sqrt{L_{10}-L_{30}}} + \frac{1}{\sqrt{L_{30}-L_{20}}}\right]$$

Equations (4) can be written in matrix form

$$\dot{\bar{x}} = A \cdot \bar{x} + B \cdot u \quad y = C \cdot \bar{x} + D \cdot u \tag{5}$$

were $\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}; A = \cdot 10^{-2} \begin{bmatrix} -1.6077 & 0 & 1.6077 \\ 0 & -2.536 & 1.6077 \\ -1.6077 & 1.6077 & -3.2177 \end{bmatrix};$

$$B = \begin{bmatrix} 0.8312 & 0 \\ 0 & 0.2922 \\ 0 & 0 \end{bmatrix}, C^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}; \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Transfer function models are popular in the design of control systems, therefore we also write the transfer function matrix. This has the following form:

$$Y(s) = \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \tag{6}$$

where

$H_{11}(s)=8.3117.10^{-4}(s^2+0.0575.s+0.000557)/N(s)$,

$H_{12}(s)=2.1484.10^{-7}/N(s)$, $H_{21}(s)=0.755.10^{-7}/N(s)$,

$H_{22}(s)=2.922.10^{-4}(s^2+0.0482s+0.0002528)/N(s)$,

$N(s)=(s+0.050163)(s+0.02117)(s+0.00226)$.

Note that the system has a 3rd-order behavior with time constants T_1=19.4 sec, T_2=44.24 sec, T_3=442.6 sec. Transfer factors for the control channels have the values:

$K_1=Y_1/U_1=0.197$[V/m], $K_2=Y_2/U_2=0.0312$ [V/m].

These items will be used to determine the tuning parameters for PI controllers. To simplify the design of control laws, the control channels H11 and H22 are approximated with transfer functions of order 1, with coefficients determined by identification:

$$H_{11}(s) = \frac{K_1}{1+T_{11}s} = \frac{0.197}{1+390s}; H_{22}(s) = \frac{K_2}{1+T_{22}s} = \frac{0.03121}{1+320s} \tag{7}$$

C. Pole Placement

For closed loop control system with PI controller we impose the normalized characteristic polynomial:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \tag{8}$$

$$s^2 + \frac{1+K_i K_{pi}}{T_{ii}} s + \frac{K_i K_{Ii}}{T_{ii}} = 0 \tag{9}$$

Solving (8) and (9) for two unknown K_{pi} and K_{Ii} , the PI controller parameters can be expressed as follows:

$$K_{pi} = \frac{2\xi_i \omega_{ni} T_{ii} - 1}{K_i}, \quad K_{Ii} = \frac{\omega_{ni} T_{ii}}{K_i} \tag{10}$$

The minimum damping ratio ξ to meet the maximum overshoot σ requirement can be obtained by:

$$\sigma = 100\exp\left(-\frac{\xi_i \pi}{\sqrt{1-\xi_i^2}}\right) \Rightarrow \quad \xi_i = \frac{\ln(\pi/100)}{\sqrt{(\ln(\pi/100))^2 + \pi^2}} \tag{11}$$

$$\omega_{ni} = 4/(\xi_i T_i)$$

The response to a desired step level set point from both tanks operating level positions, the water height behavior should satisfy the following design performance requirements:

-The equilibrium height: L_{10}=0.25m, L_{20}=0.15m;

-The percent overshoot should be: $\sigma_1 \le 10\%$, $\sigma_2 \le 10\%$;

-The settling time should be: $T_{i1} \le 390$sec, $T_{i2} \le 320$sec;

-The steady state error: $e_{s1} = 0$, $e_{s2} = 0$.

Evaluating (8) and (9) we have obtained the following PI controller parameters:

K_{p1}=35.53[V/m], K_{I1}=5.96[V/ms], K_{p2}=224.53[V/m], K_{I1}=4.6[V/ms]

III. Level Controller Design: Feed Forward

The feed forward action is necessary since the PI control system is designed to compensate for small variations from the linearized operating point (L_{i0}, V_{pi0}). The feed forward action compensates for the water withdrawal through valves and tubes from tank 1 to tank 3 and respectively from tank 3 to tank 2 and the PI controllers compensate for dynamic disturbances.

Operation of real plants that contain an assembly of three coupled tanks (e.g. tank assembly from hydroelectric reservoirs with pumping system or assembly of coupled tanks to maintain ship's horizontality to wave or wind action) require periodic changes of level in tank 1 and 2 . In this case, the level control system should be as small phase. This can be achieved by adjusting the feed forward structure.

For lab installation, the feed forward structure is presented in figure 3 [6].

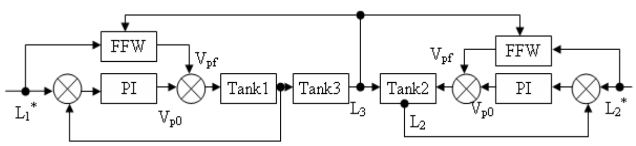

Fig. 3. Feed Forward structure

This implementation is possible because the variable L_3 is measurable and variables L_1^* and L_2^* are accessible through the control algorithm. Entire PI + feed forward control structure is implemented in software and does not require additional physical equipment.

At equilibrium, from (3), we calculate the feed forward commands:

$$
\begin{aligned}
U_{p1} &= V_{pf1} = 8.651 sign(L_{10}^* - L_3)\sqrt{\left|L_{10}^* - L_3\right|} \\
U_{p2} &= V_{pf2} = 24.61\left[\sqrt{L_{20}^*} - sign(L_3 - L_{20}^*)\sqrt{\left|L_3 - L_{20}^*\right|}\right]
\end{aligned} \tag{12}
$$

Where L_{10}^* is the set point for closed loop control for tank1, L_{20}^* is the set point for closed loop control for tank2 and L_3 is measurement value of the level in tank 3, which is accessible from the installation. The detailed feed forward structure for tank 1 is presented in figure 4.

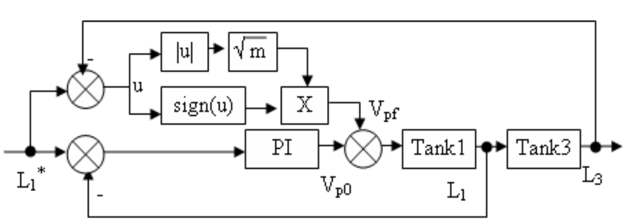

Fig. 4. Feed Forward structure (detail)

We implemented the PI-plus feed forward control loops for the Simulink model of three coupled tanks and compared the actual response with PI control loops. The results are presented in figure 5.

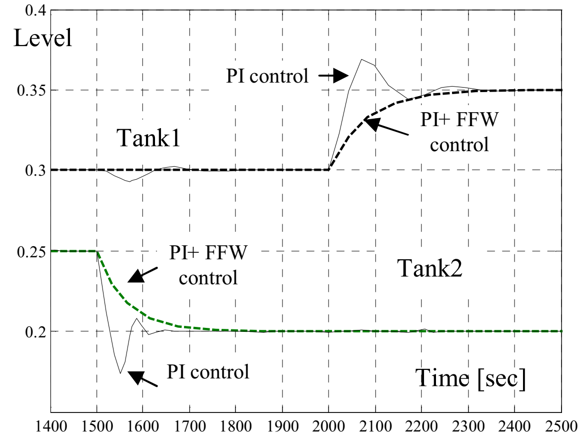

Fig. 5. Results for PI controllers and PI-plus feedfoeward controllers

Figure 5 shows that the structure of feed forward control adjustment eliminates oscillations in response to step variations of the set point. It provides no-oscillations answer required by high capacity hydraulic systems. Liquid level oscillations can produce very high local dynamic pressure, which can cause breakage of pipes.

IV. MODELLED RESULTS

The experiment aims to study the behavior of control systems with PI control law + feed forward compared with PI control law for a set point corresponding to continuous value L^*_{10} =0.3 m and L^*_{20}=0.2 m then add a square wave with the amplitude 0.05m.

The results are presented in figure 6.

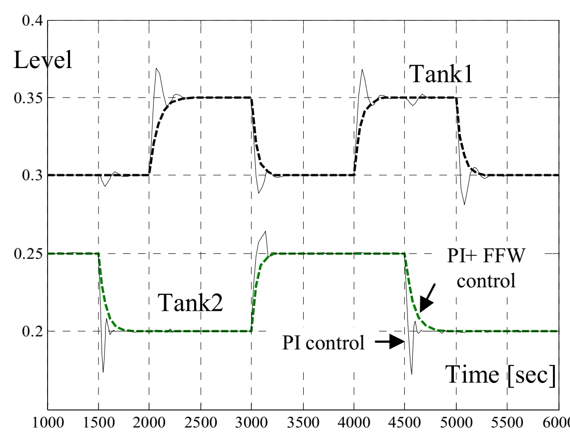

Fig. 6. Results for squar wave set point

In the same experiments we aimed to control how the structure compensates the influences that occur between the two control loops.

Figure 6 shows the powerful influence of level variation in tank 2 at time points t = 1500 sec and t = 4500 sec on the performance of control loop 1 using PI control for both loops.

Also we aimed to discover how the two control loops respond in case of faults occurring in the system, consisting in leakages of pipes.

Figure 7 shows the response of the control system for a break fault (leakage) in tank 3 at the time t=3500 sec. Observe that the defect does not affect the response of the control system.

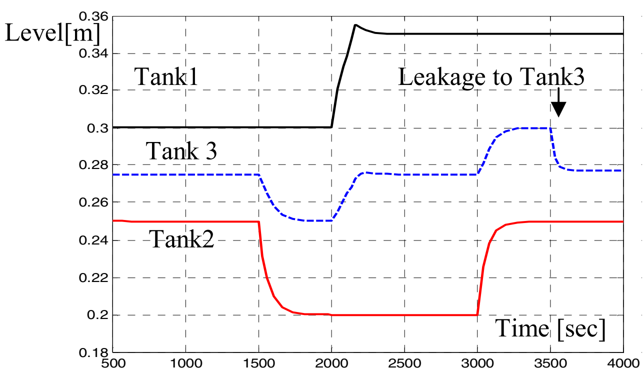

Fig. 7. System with leakage to tank 3 controled by PI+Feed Forward law

Figure 8 presents the response of the system to a break fault in the tank 1, at the time t=1000 sec. Note that the defect is not compensated by the control system Also the fault affects the control system performances in fact that the next step of set point at time t=2000sec the amortized system response becomes oscillatory. If the defect is larger than the prescribed size, the system cannot provide the desired result (it exceeds the control)

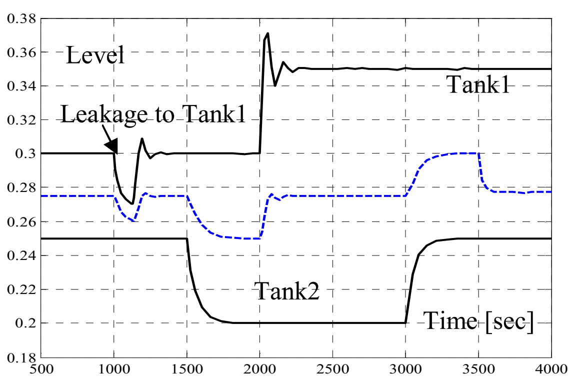

Fig. 8. System with leakage to tank 1 controled by PI+Feed Forward law

The same experiments were performed with the PI control law.

The results are shown in Figure 9. Note that the system is oscillating with large overshoot.

In exchange, the control system compensates better the fault effects (leakage at the tank at t=1000 sec).

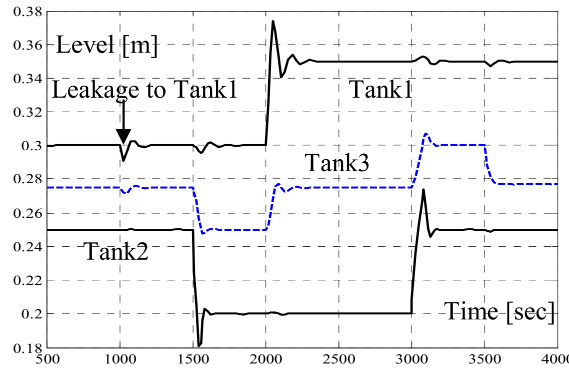

Fig. 9. System with leacage to tank 1 at t=1000sec controled by PI Controller

The last experiment studied the behavior of the sinusoidal set point adjustment. Behavior of PI + feed forward control is shown in Figure 10.

This can be used for the ship stability control system. The set point can be used from the external transducer of the ship that follows oscillations of the waves.

Note that the control system seeks imposed sinusoidal set point and it compensates well the effect of disturbances mentioned in other experiments.

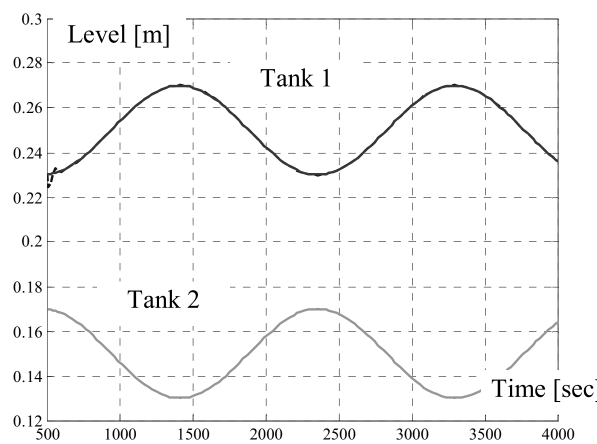

Fig. 10. Control system response to the sinusoidal set point

V. CONCLUDING DISCUSSION

In this paper, a new control scheme based on the feed forward reaction has been proposed. Using this scheme, the parameters' perturbations induced by perturbations or faults are estimated using information provided by the state/output trajectory deviation and the linear transfer matrices. The component with unacceptable perturbation is declared as faulty [12], [16].

The scheme has been illustrated using a three tank control system developed for a laboratory rig. We tried to find equivalence with an assembly of reservoirs for multiple hydro-power plants along a single watershed.

In the operation of hydro-power plants it is required to maintain the liquid level in the reservoirs for maximum energy conversion.

In these hydro-power plants it is very important to detect pipe breakages or obstruction of the supply lines. These defects cannot be determined by measurements in the plants. They can however be determined based on a mathematical model of the system with a structure suitable for fault detection [1].

Therefore, in this paper, it was paid particular attention to develop a more accurate mathematical model and control structure to compensate the effect of perturbation on the response of the system.

Sudden variations in level produce sudden variations of hydrostatic pressure and can cause pipe breakage.

The structure of PI + Feed forward control ensures elimination of oscillations and perturbation compensation simplifies the task of PI controller.

The next step is the development of a structure for faults detection and localization, using a method based on model and residues methods similar to those developed by the author in other works [17].

Fault detection of pipe breakages is very important in the case of hydro power stations. These bursts can occur in pipes located underground and can not be detected by measurements or by visual inspection. But the breakage may be detected using residues methods based on actual model plants.

Another use of coupled tanks is for balance control of commercial ships or oil tankers. In this case it is necessary to adjust the liquid level in equilibrium tanks so that she would follow the oscillation of the waves. PI or PD control alone cannot provide fast enough answer. This paper has demonstrated that the structure of PI + feed forward control provide rapid operation of the control system and mitigate oscillations.

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