Sliding Mode Control of a Coupled Tank System with a State Varying Sliding Surface Parameter

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Abstract

In general, sliding mode controllers with moving sliding surfaces are achieved by using time-dependent expressions. In this situation, the parameters of the sliding surface have to be rearranged if the initial conditions of the error phase plane are changed. This paper presents a special purpose approach with a state dependent moving algorithm for the control of the coupled tank system where the state variables are defined as the liquid levels of the tanks. Instead of using a time-dependent parameter, sliding surface parameter is adjusted by directly using the state variables of the tank system. The comparisons with a conventional sliding mode controller for different desired liquid levels are presented via simulations. The simulation results show that sliding mode controller with a state varying sliding surface parameter has a better performance with respect to conventional sliding mode controller even when the system initial conditions of the error phase plane are varied.

1. Introduction

Process control techniques always deal with increasing production rate and product quality of industrial enterprises while keeping the costs as low as possible. In process control, precise liquid level control of storage tanks and reaction vessels is essential in many industrial operations and mainly in chemical engineering systems where the liquids are pumped to the tanks, stored and flow through the coupled tanks [1]. Namely, the coupled tank system has wide applications in many commercial and industrial sectors. It is most commonly used in the area of water purification, chemical and biochemical processing, automatic liquid dispensing, food and beverage processing [2], and pharmaceutical industries [3]. The coupled tank system has two vertical tanks joined with an orifice and has inlet liquid pumps and discharge valves [4]. In the literature, different control strategies such as PID type controllers [5], fuzzy logic based methods [6], and genetic algorithms based performance improvement methods [7] are all investigated for liquid level control of coupled tank systems.

Sliding mode control is a type of variable structure control where the dynamics of a nonlinear system is changed by switching discontinuously on time on a predetermined sliding surface with a high speed, nonlinear feedback [8]. Actually, sliding mode controller design has two steps: the first step involves obtaining a sliding surface for desired stable dynamics and the second step is about providing the control law that provides to reach this sliding surface. The system trajectories are sensitive to parameter variations and disturbances during the reaching mode whereas they are insensitive in the sliding mode [9]. Consequently, there are many studies in literature that aim to eliminate or lessen the system sensitivity by minimizing or removing the time to reach the sliding mode. A well-known method is using sliding surfaces with time-dependent parameters which can be obtained with various formulations [10-12]. For instance, discrete time rotation and shifting schemes are introduced in [10]. In this method, sliding surface parameters are initially chosen according to the initial conditions and then sliding surface is moved towards the predetermined sliding surface. Later, the discontinuity effect is discussed and a continuously time-varying sliding surface is obtained [11-12].

Coupled tank systems have nonlinear state equations, parameter variations and external disturbances. Therefore, sliding mode control is also used for coupled tank systems [13, 14]. In [13], a dynamic model of the coupled tank system is obtained and an input-dependent sliding surface is designed. In [14], second order sliding mode control idea is applied for the control of coupled tank systems. In this study, on the other hand, a sliding mode controller with a state-dependent sliding surface parameter is introduced for the coupled tank system. Typical actuators used in liquid level control systems include pumps, motorized valves, on-off valves, *etc*. Because of their simple and economical usage, on-off valves have wide applications in the industry [15]. Therefore, in this study, an on-off valve is considered for the control of the coupled tank system. The control chattering caused by the discontinuity of the control action is undesirable in most applications. However, the on-off valve is naturally controlled with a chattering signal and sliding mode control can be effectively used for this purpose. The performance measures including transient response characteristics and reaching time are compared with a conventional sliding mode controller for different initial conditions to show the effectiveness of the method.

The paper is organized as in the following: after introduction given in section 1, the system dynamics of the coupled tank system is given in section 2. In section 3, the proposed sliding mode controller with a state

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dependent sliding surface is presented. Computer simulations are performed and discussed for the proposed and conventional sliding mode controllers in section 4. Finally, conclusion appears in the last section, section 5.

2. Problem Formulation

The schematic diagram of the coupled tank system is shown in Fig.1 where q is the inlet flow rate to tank 1, q_1 is the liquid flow rate from tank 1 to tank 2 through the orifice, q_2 is the outlet flow rate of tank 2 through a valve, and **h**=[h_1, h_2] denotes the liquid levels of tank 1 and tank 2. In this study, the plant input is q, the plant output is h_2 and the aim of the controller is to adjust the liquid level of tank 2 to the desired set point level h_{2d}. An on-off valve supplies q which is unidirectional and only into tank 1 which means that $q \geq 0$. The liquid flow q_1 is also assumed to be unidirectional from tank 1 to tank 2. To provide this, it is assumed that $h_1 \geq h_2$. System becomes decoupled when h_1 and h_2 levels are equal to each other [13]. Therefore, the steady state characteristics of the coupled tank system are dependent on operating levels. Thus, the sliding mode control law for the coupled and decoupled cases can be considered separately [14].

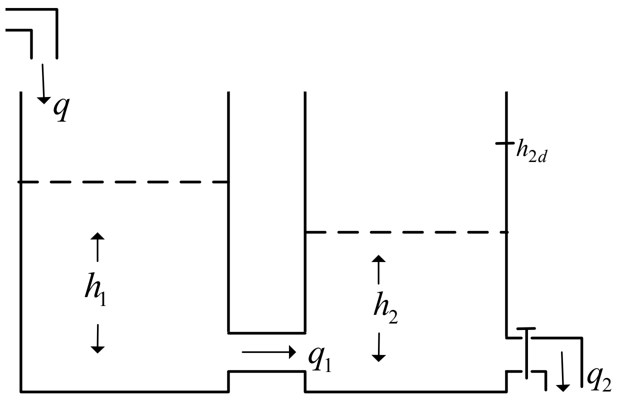

Figure 1 Schematic Diagram of the Coupled Tank System

By assigning the cross section area of both of the two tanks as c, the flow balance equations for both tanks can be given as [1]

$$c\dot{h}_1 = -q_1 + q \tag{1}$$

$$c\dot{h}_2 = q_1 - q_2 \tag{2}$$

The flow rates of valves obey the Bernoulli's equations and depend on system parameters as follows

$$q_1 = c_{12}\sqrt{2g(h_1 - h_2)} \tag{3}$$

$$q_2 = c_2\sqrt{2gh_2} \tag{4}$$

where g is the gravitational constant, c_{12} is the area of coupling orifice, and c_2 is the area of the outlet orifice [1]. Then, using (1)-(4) the dynamic model of the nonlinear system can be obtained as [13, 14]

$$\dot{h}_1 = -a_2\sqrt{(h_1 - h_2)} + \frac{1}{c}q \tag{5}$$

$$\dot{h}_2 = a_2\sqrt{(h_1 - h_2)} - a_1\sqrt{h_2} \tag{6}$$

where a_1 and a_2 depend on system parameters as

$$a_1 = \frac{c_2\sqrt{2g}}{c} \qquad a_2 = \frac{c_{12}\sqrt{2g}}{c} \tag{7}$$

It must be pointed out that (3)-(6) model the system for the assumption that $h_1 \geq h_2$ and $h_2>0$ [1]. When the system is at the equilibrium point, the derivation of liquid levels of both tanks will be equal to zero as $\dot{h}_1 = \dot{h}_2 = 0$ [13, 14]. When the system approaches the steady state, the difference between h_1 and h_2 must be $a_1^2 h_{2d} / a_2^2$ [13]. By letting [6, 13]

$$\begin{aligned} z_1 &\hat{=} h_2 > 0 \\ z_2 &\hat{=} h_1 - h_2 > 0 \\ u &\hat{=} q \end{aligned} \tag{8}$$

and defining the new state variables x_1, x_2 as [13]

$$\begin{aligned} x_1 &\hat{=} z_1 \\ x_2 &\hat{=} -a_1\sqrt{z_1} + a_2\sqrt{z_2} \end{aligned} \tag{9}$$

For the coupled case, the dynamic model of the system can be written as [13, 14]

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{a_1 a_2}{2}\left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right) + \frac{a_1^2}{2} - a_2{}^2 + \frac{a_2}{2c}\frac{1}{\sqrt{z_2}}u \\ y &= x_1 \end{aligned} \tag{10}$$

For the decoupled case $h_1 = h_2$, the nonlinear equations for the liquid level of the tanks can be rewritten from (5)-(6) as

$$\dot{h}_1 = \frac{1}{c}u \tag{11}$$

$$\dot{h}_2 = -a_1\sqrt{h_2} \tag{12}$$

and the transformed state equation becomes

$$\dot{x}_2 = \frac{a_1^2}{2} + \frac{a_2}{2\sqrt{z_2}}\left(\frac{u}{c} + a_1\sqrt{z_1}\right) \tag{13}$$

In the next section, the proposed sliding mode controller will be presented for the dynamic system given in (10), (13) to regulate the system output $y=x_1=h_2$ to the desired value h_{2d}.

3. The Proposed Sliding Mode Controller with a State Varying Sliding Surface

For the nonlinear dynamic system (10), the aim is to design a sliding mode controller as a height regulator for the coupled tank system in Fig.1 to provide the tank 2 level at the desired constant value h_{2d}. For the conventional case, the sliding surface can be chosen as:

$$s_c(\mathbf{e}) \hat{=} \alpha_c e_1(t) + e_2(t) = \alpha_c(x_1 - x_{1d}) + (x_2 - x_{2d}) \tag{14}$$

where $x_{1d} = h_{2d}$ and $x_{2d} = \dot{x}_{1d} = 0$ are the constant desired state values and e_1, e_2 are the state error values. For the proposed sliding mode controller with a state-varying sliding surface, a new moving algorithm is designed that neither depends on time as in [10, 11] nor on the control input as in [13]. New design is adjusted by directly appending the state-variables of the coupled tank system as the sliding surface parameter. For this purpose, the sliding surface is designed as:

$$s_p(\mathbf{e}) \hat{=} \alpha_p(\mathbf{h}) e_1(t) + e_2(t) \tag{15}$$

where

$$\alpha_p(\mathbf{h}) \hat{=} \alpha_{p1} + \alpha_{p2}\left((h_1 - h_2) - (a_1^2 h_{2d} / a_2^2)\right) \tag{16}$$

where α_{p1} and α_{p2} are the constant design parameters of the proposed sliding surface. When the system output reaches the desired value, the difference between h_1 and h_2 will become $a_1^2 h_{2d} / a_2^2$ and thus $\alpha_p(\mathbf{h}) = \alpha_{p1}$. Therefore, if $\alpha_{p1} = \alpha_c$ is chosen, then the sliding surface parameters of both conventional sliding mode controller and sliding mode controller with the state-varying sliding surface will be equal to each other. Using the definitions given in (8), (16) can be rewritten as

$$\dot{\alpha}_p(\mathbf{h}) \hat{=} \alpha_{p2} \dot{z}_2 = \alpha_{p2}\left(a_1 z_1 - 2a_2\sqrt{z_2} + \frac{1}{c}u\right) \tag{17}$$

The derivative of the proposed sliding surface (15) can be written as

$$\dot{s}_p = \dot{\alpha}_p(\mathbf{h}) e_1 + \alpha_p \dot{e}_1 + \dot{e}_2 \tag{18}$$

By substituting the values of error states and their derivatives into (18) and taking $\dot{x}_1 = x_2$, $\dot{x}_{1d} = \dot{h}_{2d} = 0$ and $\dot{x}_{2d} = 0$ one can write

$$\dot{s}_p = \dot{\alpha}_p(\mathbf{h})(x_1 - h_{2d}) + \alpha_p(\mathbf{h}) x_2 + \dot{x}_2 = 0 \tag{19}$$

If the system equations (10) and derivative of the sliding surface parameter (17) are substituted into (19) one can write

$$\begin{aligned} \dot{s}_p &= \alpha_{p2}\left(a_1 z_1 - 2a_2\sqrt{z_2} + \frac{1}{c}u\right)(z_1 - h_{2d}) + \alpha_p(\mathbf{h}) x_2 \\ &+ \frac{a_1 a_2}{2}\left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right) + \frac{a_1^2}{2} - a_2^2 + \frac{a_2}{2C}\frac{1}{\sqrt{z_2}}u = 0 \end{aligned} \tag{20}$$

After carrying out the intermediate steps, the control law of the proposed method for $h_1 \neq h_2$ can be calculated as

$$\begin{aligned} u(t) = k_1\Big[&-\alpha_{p2}(z_1 - h_{2d})\left(a_1\sqrt{z_1} - 2a_2\sqrt{z_2}\right) - \alpha_p(\mathbf{h}) x_2 \\ &- \frac{a_1 a_2}{2}\left(\sqrt{\frac{z_1}{z_2}} - \sqrt{\frac{z_2}{z_1}}\right) - \frac{a_1^2}{2} + a_2^2 - k_2 sign(s_p)\Big] \end{aligned} \tag{21}$$

where

$$k_1 = \frac{2c\sqrt{z_2}}{2\alpha_{p2}\sqrt{z_2}(z_1 - h_{2d}) + a_2} \tag{22}$$

is a parameter that depends on system state variables, sliding surface parameters and desired output value. As already mentioned, for $\alpha_{p1} = \alpha_c$ and $\alpha_{p2} = 0$, the proposed sliding mode control law (17) will be equal to the conventional sliding mode control law given in [8, 9].

For the decoupled case, (13) will be used and the derivative of the sliding surface parameter can be written as

$$\dot{\alpha}_p(h) \hat{=} \alpha_{p2}\dot{z}_2 = \alpha_{p2}\left(a_1 z_1 + \frac{1}{c}u\right) \tag{23}$$

and from the derivative of s_p in (19), the control law for the decoupled case can be reformulated as

$$u(t) = -ca_1\sqrt{z_1} - k_1 k_2 sign(s_p) \tag{24}$$

The discontinuous control rule is $u_N = -k_1 k_2 sign(s_p)$ for both conventional and proposed sliding mode controllers. The discontinuous control gain k_2 is a constant parameter. In this study, a bounded disturbance $d(t)$ is assumed to affect the control input and k_2 is chosen large enough to provide the stability condition and the control law is chosen between

$$u_{min} \leq u(t) \leq u_{max} \tag{25}$$

to obtain an on-off control signal.

In most industrial applications, the liquid level of the second tank must not change suddenly for obtaining a proper production quality. Therefore, in this study, the constant design parameter α_{p1} of the proposed sliding surface (15) is calculated with respect to the initial conditions and the system parameters as

$$\alpha_{p1} = w_1 \frac{\dot{h}_{2\max}}{h_{2d}^2} + w_2 \tag{26}$$

where $\dot{h}_{2\max}$ is the maximum allowable value of $\dot{h}_2$, w_1 and w_2 are constant design parameters that are determined with respect to the system parameters. In (26), w_1, w_2 are design parameters and must be adjusted properly to provide the change at the derivative of the liquid level at the second tank to be always under the level $\dot{h}_{2\max}$. The α_{p1} depends on constant parameters w_1, w_2, $\dot{h}_{2\max}$, h_{2d}^2 and does not need any measurement value.

The other parameter of the proposed sliding surface, α_{p2}, is chosen in order to finally approach a conventional sliding mode controller having a constant sliding surface parameter α_{p1} as the system approach the equilibrium point.

The comparison of the proposed method with the conventional sliding mode controller is given at the next section.

4. Simulation Results

In this study, simulation results of the conventional sliding mode controller (SMC-C) and the proposed sliding mode controller with a state varying sliding surface (SMC-S) for the coupled tank system are presented. The minimum and maximum values of the on-off control law is chosen as u_{min} =0 [cm^3/s] and u_{max}=150 [cm^3/s]. The maximum allowable value of $\dot{h}_2$ is chosen as $\dot{h}_{2\max}$=0.15 [cm/s] and it is used in (26) to obtain the sliding surface parameter α_{p1}. The parameters of the coupled tank system are taken as follows [13]:

- gravitational rate $g = 981$cm/s^2
- cross-section area of tank 1 and tank 2 c=208.2cm^2
- area of the coupling orifice c_{12}=0.58 cm^2
- area of the outlet orifice c_2=0.24 cm^2
- parameter of discontinuous control law k_2= 10

The computer simulations are performed using Matlab® between [0:100]s with a sampling interval taken as 0.01s. The simulations are given for different h_{2d} values to investigate the effect of initial conditions. For the proposed method, the sliding surface parameters w_1, w_2, α_{p2} are chosen once and no more adjustment is performed for different desired water levels. On the other hand α_{p1} is calculated from (26) which changes with the desired liquid level h_{2d}.

The disturbance effect is also added directly to the control signal $u(t)$ by modeling it with a sinusoidal wave depending on time and it is taken as

$$d(t) = 20\sin(0.5\pi t) \tag{27}$$

The SMC-C is simulated for two different cases (SMC-C1 and SMC-C2) at each desired liquid level. In SMC-C1, α_c is adjusted for each desired liquid level in order to provide the $e_2(t)$ at its maximum value $\dot{h}_{2\max}$. SMC-C2 is taken as $\alpha_c = \alpha_{p1}$ to show the effect of using the same parameter value with SMC-S. As stated before, α_c of both SMC-C1 and SMC-C2 could be obtained from (16) by choosing $\alpha_{p2} = 0$ and $\alpha_{p1} = \alpha_c$. For SMC-C1, SMC-C2 and SMC-S, behavior of sliding surface error parameters α_c and α_p are represented in Fig.2 for the desired level h_{2d} =6 cm. The sliding surface s_p for SMC-S depends on system states and therefore is not constant in time.

The error phase planes for the desired liquid level $h_{2d} = 6$ cm are given in Fig. 3. For both SMC-C1 and SMC-S, the error phase plane approaches $\dot{h}_{2\max}$ and then enters the sliding mode regime. SMC-C2 does not provide to stay under $\dot{h}_{2\max}$ as a result of using a relatively large α_c value. Also, the sliding mode behavior of SMC-S is nonlinear whereas SMC-C1 and SMC-C2 have a linear sliding mode behavior in the phase plane as expected.

As seen in Fig.3, SMC-C2 does not provide the constraint on $\dot{h}_{2\max}$. Therefore SMC-C2 is ignored in Fig.4 where the comparison between SMC-C1 and SMC-S for the desired liquid level h_{2d} = 6 is given.

In Fig 5, in order to show the effect of changing the desired liquid level, SMC-C1, SMC-C2 and SMC-S are represented in error phase plane for different desired liquid levels h_{2d} = 5 and h_{2d} = 7. As can be seen from Fig.5, without adjusting any parameter for the new liquid levels, the proposed controller SMC-S is less affected from a change in h_{2d} .The behavior of α_p is similar to Fig.2 and is not plotted again. In Fig 5, the proposed sliding surface parameter is directly adjusted with the desired liquid level h_{2d} .It must be noted that the maximum allowable value $\dot{h}_{2\max}$ is always provided for SMC-S without changing any designed parameter

The liquid level of the second tank (h_2) for both SMC-C1 and SMC-S are given in Fig. 6 for three different desired liquid levels h_{2d} without highlighting the parameter values to show the effectiveness of the proposed method. The improvement in the transient response obtained with SMC-C with respect to SMC-C1 could be easily seen from Fig.6.

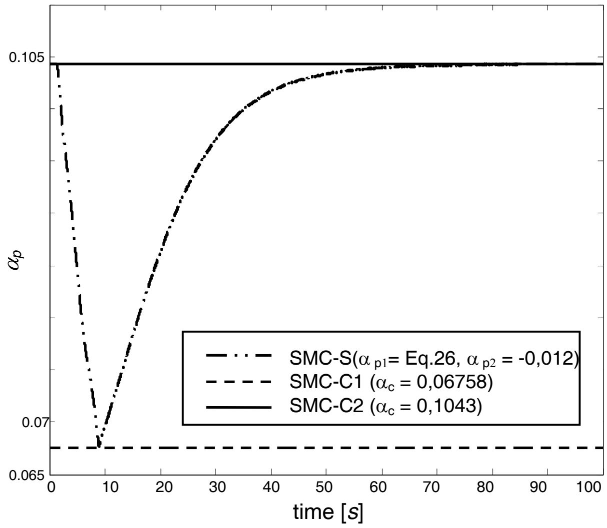

Figure 2. The α_p behavior for h_{2d} =6.

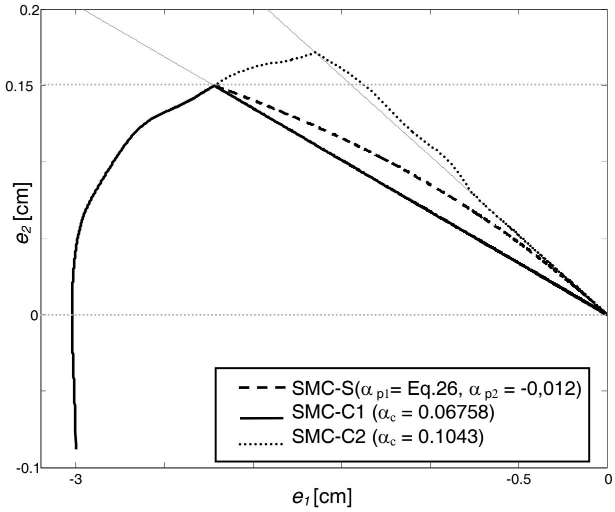

Figure 3. The error phase plane behavior of the related controllers for h_{2d} =6 cm

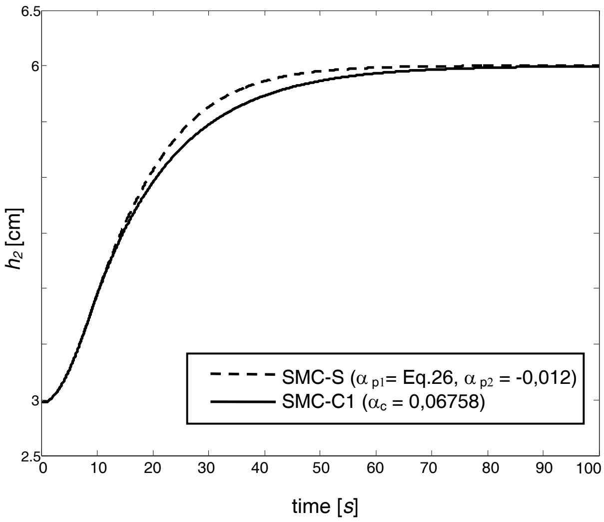

Figure 4. Liquid level in tank 2 for h_{2d} =6 cm.

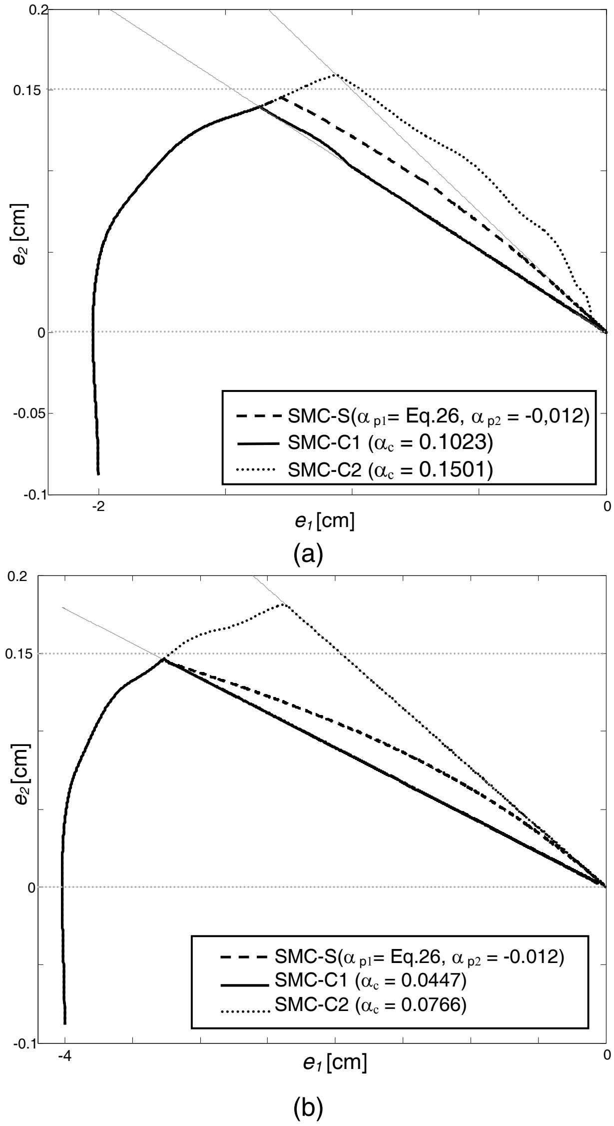

Figure 5. The error phase plane behavior of the related controllers for a)h_{2d} =5 cm, b)h_{2d} =7 cm

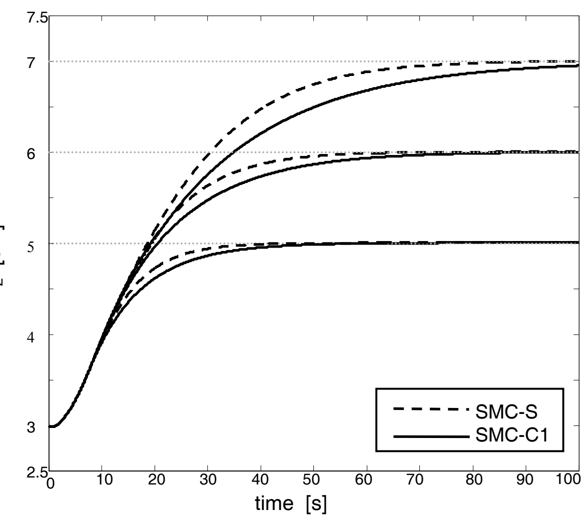

Figure 6. Liquid level in tank 2 for different values of h_{2d}.

The SMC-C1 and SMC-S are also compared according to some performance criteria and presented at Table 1 where t_s is the setting time, t_r is the reaching time, IAE is the integral of absolute error, ISE is the integral of square error, ITAE is the integral of time multiplied absolute error and ITSE is the integral of time multiplied square error. As seen from the table, SMC-S has better performance criteria than SMC-C1.

Table 1 Performance criteria for different h_{2d} values.

	h_{2d}=5		h_{2d}=6		h_{2d}=7	
	SMC-C1	SMC-S	SMC-C1	SMC-S	SMC-C1	SMC-S
t_s	55.55	**42.49**	87.88	**65.04**	100.00	**92.60**
IAE	27.64	**24.88**	57.32	**51.33**	104.40	**90.50**
ISE	35.13	**33.61**	104.34	**99.79**	242.10	**226.30**
ITSE	208.18	**180.43**	870.74	**748.91**	2860.4	**2293.00**
ITAE	288.69	**213.70**	874.40	**633.11**	2251.0	**1532.50**
t_r	4.44	**4.44**	5.38	**5.38**	6.20	**6.20**

To show the on-off structure of the control law, the control signal $u(t)$ is given only for SMC-S with h_{2d}=6 in Fig.7. After reaching the sliding surface, the on-off valve chatters and fills the first tank in a way to reach the desired liquid level in tank 2.

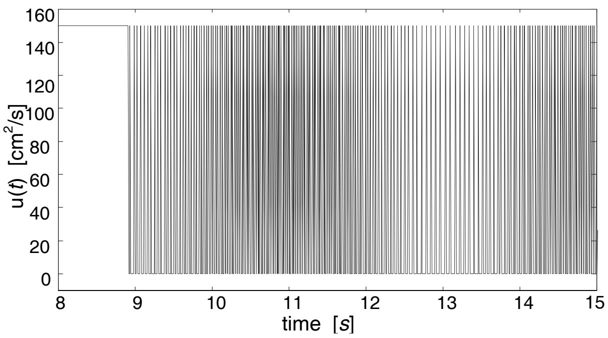

Figure 7. The inlet flow rate into tank 1 for h_{2d}=6 cm for the time interval [8:15] s.

5. Conclusion

In this study, for the coupled tank system, a new sliding mode controller with a state-varying sliding surface is designed. The sliding surface parameter changes as a function of the liquid levels of the two tanks that provides a nonlinear sliding regime in the error phase plane. The proposed method provides a better performance with respect to the conventional sliding mode controller even under different initial conditions without adjusting any parameters and without crossing the predefined maximum value of the second tank liquid level. The simulations are also performed for a conventional sliding mode controller that has the same final sliding surface value with the proposed method. It is seen that it is not possible to provide both the design parameters and better robustness by only changing the sliding surface parameter of the conventional sliding mode controller.

Acknowledgements

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