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Robust PI Controller Design for Coupled-Tank Process: LMI Approach

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Abstract— **This paper proposes an approach to a robust PI controller design for real uncertain Coupled-Tank process in time domain. Only the first independent tank is considered (single-input single-output system). Polytopic model of uncertain system is considered as a plant model to be controlled. Stability and performance of the closed loop system is determined through the LMI region, where the closed loop poles of whole uncertainty domain are placed. A robust stability condition based on the parameter-dependent Lyapunov function (PDLF) is used.**

Keywords— ***Robust stability; PI controller; LMI region; output feedback; Lyapunov function***

I. INTRODUCTION

Control of real processes always includes uncertainties (perturbations about the nominal linear dynamics), which have to be considered in the adequate control design. Therefore robustness belongs to an important control design qualities: closed loop system stability and performance should be guaranteed over the whole uncertainty domain, [10].

There exist various approaches to robust stability analysis and robust control design for uncertain linear systems. Many different problems in control can be reduced to the same linear algebra problem using algebraic approach. Robust control problem is formulated in algebraic framework and solved as an optimization problem, preferably in the form of Linear Matrix Inequalities (LMI). LMI techniques enable to solve a large set of convex problems in effective way [8]. This approach is applicable when control problems for linear uncertain systems with a convex uncertainty domain are solved.

In this paper the time domain PI controller design for real Coupled-Tank process is considered. Robust stability and performance of the closed loop system with PI control algorithm from [3] is determined through the LMI regions, where the closed loop poles of uncertain system are placed as in [1] and [2]. Liquid tank processes play important role in industrial application such as in food processing, filtration, pharmaceutical industry, industrial chemical processing and spray coating [5]. Many industrial applications are concerned with level of liquid control, may it be a single loop level control or sometimes multi loop level control [6]. In this paper only the first tank with liquid is used (SISO).

The paper is organized as follows. The next section gives details about Coupled-Tank process used for education and research. Section 3 introduces a PI controller design using LMI approach with special LMI regions. In section 4 some simulation results are presented. Several step responses of closed loop system with proposed robust output feedback PI controller are plotted there. Finally, conclusion is given in section 5.

Notation: Throughout this paper, the following notation is used. For a real matrix M, $M \geq 0$ (respectively $M > 0$) means that matrix M is symmetric and positive semi-definite (respectively positive definite); the transpose of a matrix M is denoted as M^T. For a complex matrix N, N^* denotes its complex conjugate transpose. $\otimes$ is the Kronecker product of two matrices. It is known that $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$.

II. COUPLED-TANK PROCESS

The industrial Coupled-Tank process is one of the real processes built for control education and research at Institute of Control and Industrial Informatics. The apparatus consists of two tanks (T1 and T2 in Fig. 1), which can be coupled using valve V12 (the manual valve). Therefore the Coupled-Tank process with two tanks represents a multi-input multi-output (MIMO) system for opened valve V12 or two independent single-input single-output (SISO) systems for closed valve V12. Both tanks are made of Plexiglas. These two tanks are mounted on a platform with a metering scale before each tank indicating the approximate liquid level in tank. Exact liquid level in each tank is measured using an electronic sensor. Other components of system are liquid basin (reservoir), two pumps (Pump1 and Pump2 in Fig. 1), two outlet valves (V1 and V2 in Fig. 1) and electronic circuit communicating with *LABREG* software in computer. This software is made for identification and control of real processes. The *LABREG* operates in MATLAB using toolboxes *SIMULINK*, *Ident*, *Control* and *Real Time*.

Cooperation between Coupled-Tank process and computer and *LABREG* software is ensured using Advantech data acquisition card of type PCI 1711. More about *LABREG* and mentioned toolboxes can be found in [9].

The paper deals with design of robust PI controller for SISO system (valve V12 is closed), consequently there can be used one or two independent tanks. Only one tank process is considered, therefore the purpose is to control liquid level in the first tank by the inlet liquid flow from the first electronic DC pump (Pump1). The process input is $u_1(t)$ (voltage input to Pump1) and the output is $h_1(t)$ (liquid level in the first tank – T1). Input power is bounded by interval $\langle 0,10\rangle$ volts and output signal is measured using electronic sensor. Q_{i1} and Q_{o1} in Fig. 1 denote the inlet and outlet flow rates for T1 respectively. Outlet flow is affected by electronic outlet valve (V1), which can be set manually from 0 to 10 volts (for 0 [V] is closed, for 10 [V] fully opened) and represent perturbation.

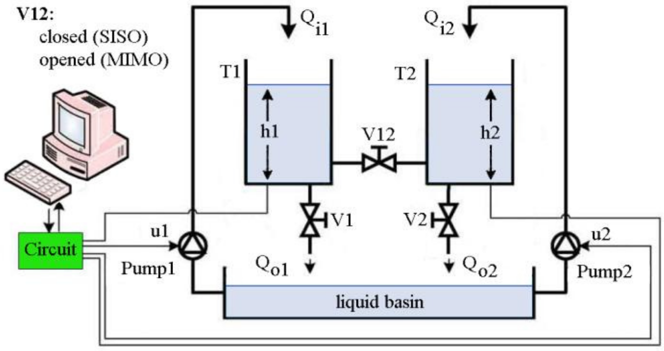

Fig. 1. Coupled-Tank process

III. PRELIMINARIES AND PROBLEM FORMULATION

In this paper we apply our recent results on robust PI controller design, [3], aiming at subsystem pole placement into prescribed region in the complex plane, [2].

Consider a linear time invariant uncertain dynamical system

$$\begin{aligned}\dot{x}(t) &= A(\alpha)x(t) + B(\alpha)u(t)\\ y(t) &= Cx(t)\end{aligned} \tag{1}$$

where $x(t)\in R^n$, $u(t)\in R^m$, $y(t)\in R^l$ are state, control and output vectors respectively. The system dynamics matrix $A(\alpha)$ and matrix $B(\alpha)$ belong to a convex polytopic box defined as

$$\begin{aligned}A(\alpha) &\in \left\{\sum_{k=1}^{K}\alpha_k A_k,\ \sum_{k=1}^{K}\alpha_k = 1,\ \alpha_k \ge 0\right\}\\ B(\alpha) &\in \left\{\sum_{k=1}^{K}\alpha_k B_k,\ \sum_{k=1}^{K}\alpha_k = 1,\ \alpha_k \ge 0\right\}\end{aligned} \tag{2}$$

Matrices A_k, B_k and C are known constant matrices of corresponding dimensions.

For system (1), we consider the following PI control algorithm

$$u(t) = K_P y(t) + K_I\int_0^t y(t)dt \tag{3}$$

Consider $z(t)=\int_0^t y(t)dt$ and $X(t)=\begin{bmatrix}x^T(t) & z^T(t)\end{bmatrix}^T$, the PI control algorithm (3) can be written as

$$u(t) = FC_nX(t) \tag{4}$$

where $F=\begin{bmatrix}F_P & F_I\end{bmatrix}$ and $C_n = diag\{C,I\}, I\in R^{lxl}$.

The system (1) can be expanded as the following form

$$\dot{X}(t) = A_n(\alpha)X(t) + B_n(\alpha)u(t) \tag{5}$$

where

$$\begin{aligned}A_n(\alpha) &\in \left\{\sum_{k=1}^{K}\alpha_k A_{nk},\ \sum_{k=1}^{K}\alpha_k = 1,\ \alpha_k \ge 0\right\},\ A_{nk} = \begin{bmatrix}A_k & 0\\ C & 0\end{bmatrix}\\ B_n(\alpha) &\in \left\{\sum_{k=1}^{K}\alpha_k B_{nk},\ \sum_{k=1}^{K}\alpha_k = 1,\ \alpha_k \ge 0\right\},\ B_{nk} = \begin{bmatrix}B_k\\ 0\end{bmatrix}\end{aligned}$$

The respective system (5) with the PI control algorithm (4) can be rewritten as the closed loop output feedback system

$$\dot{X}(t) = A_C(\alpha)X(t) \tag{6}$$

where

$$\begin{aligned}A_C(\alpha) &\in \left\{\sum_{k=1}^{K}\alpha_k A_{Ck},\quad \sum_{k=1}^{K}\alpha_k = 1,\quad \alpha_k \ge 0\right\}\\ A_{Ck} &= A_{nk} + B_{nk}FC_n\end{aligned} \tag{7}$$

Since our control aim is to place the closed loop system poles into the prescribed region, we start with defining the appropriate regions in convex plane. We consider LMI regions guaranteeing stability and performance. Let us consider $\mathcal{D}_{\mathcal{R}}$ – stability region defined as in [1]

$$\mathcal{D}_{\mathcal{R}} = \left\{z\in C: R_{11} + R_{12}z + R_{12}^T z^* + R_{22}zz^* < 0\right\} \tag{8}$$

where $R_{11}=R_{11}^T \in \mathrm{R}^{d\times d}$, $R_{12}\in \mathrm{R}^{d\times d}$, $R_{22}=R_{22}^T \in \mathrm{R}^{d\times d}$ and $R_{22}\geq 0$ (for convex $\mathcal{D}_{\mathcal{R}}$). Without any assumption on the matrix R_{22}, $\mathcal{D}_{\mathcal{R}}$ regions are generally not convex domains.

Standard choices of $\mathcal{D}_{\mathcal{R}}-$ stability region are left half plane ($R_{11}=0,\ R_{12}=1,\ R_{22}=0$) for continuous-time systems and the unit disc ($R_{11}=-1,\ R_{12}=0,\ R_{22}=1$) for discrete-time systems. Some special forms of LMI regions can be seen in [2]. More complicated region can be constructed as the intersection of some individual LMI regions. The region of our interest is shown in Fig. 2 and is constructed as an intersection of these individual sectors:

1. Conic sector with apex at the origin and inner angle 2δ with characteristic function

$$f(z)=\begin{bmatrix}\sin\delta(z+z^*) & \cos\delta(z-z^*)\\ \cos\delta(z^*-z) & \sin\delta(z+z^*)\end{bmatrix}<0 \tag{9}$$

from where $R_{11}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$, $R_{12}=\begin{bmatrix}\sin\delta & \cos\delta\\ -\cos\delta & \sin\delta\end{bmatrix}$ and $R_{22}=0$.

2. Moved half-plane of complex plane $\mathrm{Re}(z)<-\varepsilon$ with characteristic function

$$f(z)=2\varepsilon+z+z^*<0 \tag{10}$$

from where $R_{11}=2\varepsilon$, $R_{12}=R_{12}^T=1$, $R_{22}=0$.

The basic transient response characteristics of interest: a minimum damping ratio $\varsigma=\cos\delta$ and minimum decay rate ε (stability degree) are ensured with confining the closed loop poles to this specific region [2].

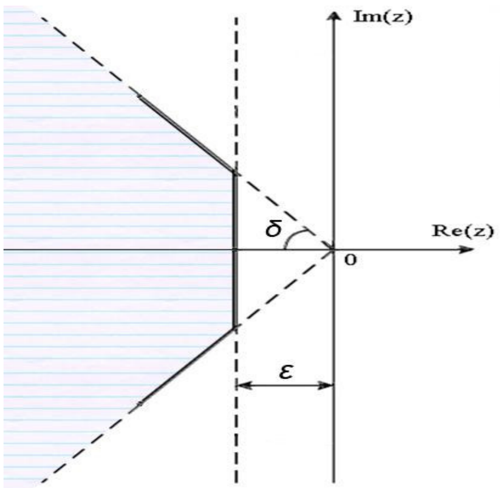

Fig. 2. Intersection of individual LMI sectors

And finally, the intersection of mentioned subregions from Fig. 2 is defined by matrices

$$R_{11}=\begin{bmatrix}R_{11,1} & 0_{2\times 1}\\ 0_{1\times 2} & R_{11,2}\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 2\varepsilon\end{bmatrix} \tag{11}$$

$$R_{12}=\begin{bmatrix}R_{12,1} & 0_{2\times 1}\\ 0_{1\times 2} & R_{12,2}\end{bmatrix}=\begin{bmatrix}\sin\delta & \cos\delta & 0\\ -\cos\delta & \sin\delta & 0\\ 0 & 0 & 1\end{bmatrix} \tag{12}$$

and R_{22} is zero matrix with dimensions 3×3. $R_{11,1}$ and $R_{12,1}$ represent matrices of conic sector defined in (9), $R_{11,2}$ and $R_{12,2}$ are matrices of moved half-plane of complex plane (10).

To develop a robust stability condition respective to the above described LMI regions we use a robust stability notion based on the parameter-dependent Lyapunov function (PDLF).

$$P(\alpha)=\sum_{k=1}^{K}\alpha_k P_k \tag{13}$$

where $P_k=P_k^T>0$.

A new condition for robust $\mathcal{D}_{\mathcal{R}}-$ stability using PDLF is defined in the following *Lemma.*

Lemma 3.1 [1]

Consider the uncertain linear system (1). For given matrices R_{11}, R_{12}, R_{22}, if there exist a gain matrix F, matrices $H\in R^{nxn}$, $G\in R^{nxn}$ and K symmetric positive definite matrices $P_k\in R^{nxn}$ such that for all $k=1,\dots,K$

$$\begin{bmatrix}\mathcal{M}_{11} & \mathcal{M}_{12}\\ \mathcal{M}_{12}^T & \mathcal{M}_{22}\end{bmatrix}<0 \tag{14}$$

where

$$\mathcal{M}_{11}=R_{11}\otimes P_k+(I_d\otimes A_{Ck}^T)H^T+H(I_d\otimes A_{Ck})$$
$$\mathcal{M}_{12}=R_{12}\otimes P_k-H+(I_d\otimes A_{Ck}^T)G$$
$$\mathcal{M}_{22}=R_{22}\otimes P_k-G-G^T$$

then the closed loop output feedback system (6) is robustly $\mathcal{D}_{\mathcal{R}}$ stable.

IV. Design Of Robust PI Controller For Coupled-Tank Process

In this case, system step response is examined. Transfer functions of the system in all three working points (15), (16) and (17) are obtained from the output step response of open loop system using BJ (Box-Jenkins) method of identification with different outlet valve voltage (perturbation). We consider transfer functions of a liquid level in the first tank (see Fig. 1).

In all three working points these parameters were set:

- water pump voltage (input voltage) is set on $4\,[V]$ (approximately $17.58\ [cm]$),
- step of water pump voltage in time t to $5\,[V]$ (approximately $21.85\ [cm]$).

WP1 (working point 1):

$$outlet\ valve\ voltage\ (perturbation) = 8\ [V]$$

$$G_{WP1}(s) = \frac{0.0452s + 0.9092}{32.5254s^2 + 7.8585s + 1} \tag{15}$$

WP2 (working point 2):

$$outlet\ valve\ voltage\ (perturbation) = 9\ [V]$$

$$G_{WP2}(s) = \frac{0.046s + 0.9279}{18.6493s^2 + 7.6896s + 1} \tag{16}$$

WP3 (working point 3):

$$outlet\ valve\ voltage\ (perturbation) = 10\ [V]$$

$$G_{WP3}(s) = \frac{0.0367s + 0.7385}{19.8166s^2 + 5.7141s + 1} \tag{17}$$

As can be seen, system is with one affine uncertainty (outlet valve voltage in volts). Corresponding polytopic model has 2 vertices and is defined by following matrices

$$\begin{aligned}
&A_1 = \begin{bmatrix} 0 & -0.0505 \\ 1 & -0.2884 \end{bmatrix} \qquad B_1 = \begin{bmatrix} 0.0374 \\ 0.0018 \end{bmatrix} \\
&A_2 = \begin{bmatrix} 0 & -0.0537 \\ 1 & -0.4124 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 0.0498 \\ 0.0024 \end{bmatrix} \\
&C = \begin{bmatrix} 0 & 1 \end{bmatrix}
\end{aligned} \tag{18}$$

Using the robust $\mathcal{D}_{\mathcal{R}}$ stability condition (14) with matrices R_{11} (11) and R_{12} (12), robust output feedback PI controller (19) was designed. The LMI region of our interest is shown in Fig. 2 and represents intersection of conic sector with apex at the origin and inner angle $\delta = 65^{\circ}$ (damping factor limit) and moved half-plane to amount -0.08 (stability degree of lower limit). Numerical calculations have been realized by using YALMIP toolbox and BMI solver.

The results of calculation for this case are:

$$F = \begin{bmatrix} F_P & F_I \end{bmatrix} = \begin{bmatrix} -0.3861 & -0.1167 \end{bmatrix} \tag{19}$$

$$MaxEig_{closed\ loop} = \text{-}0.093003$$

The poles of closed loop polytopic system with the output feedback PI controller are shown in Fig. 3. As can be seen all closed loop poles are placed within the prescribed LMI region. Step responses of polytopic model of system in all vertices are depicted in Fig. 4. Step responses of real process in all three working points are depicted in Fig. 5.

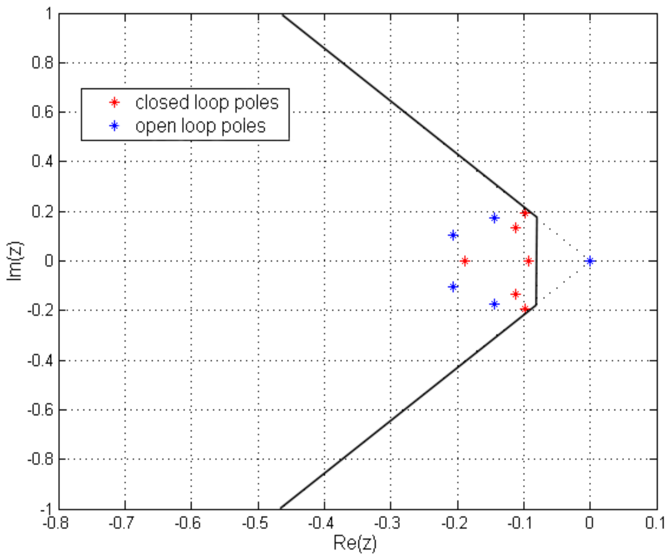

Fig. 3. Poles of open loop (blue stars) and closed loop (red stars) polytopic system in all vertices

The region of closed loop poles cannot be chosen arbitrarily, because the output feedback PI control algorithm is applied. The parameters of the LMI region are chosen by consideration of process dynamics.

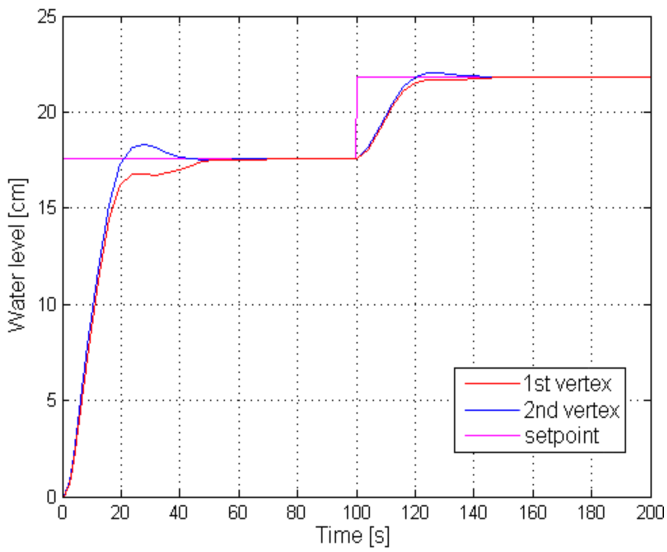

Fig. 4. Step responses of polytopic system in all vertices – simulation results

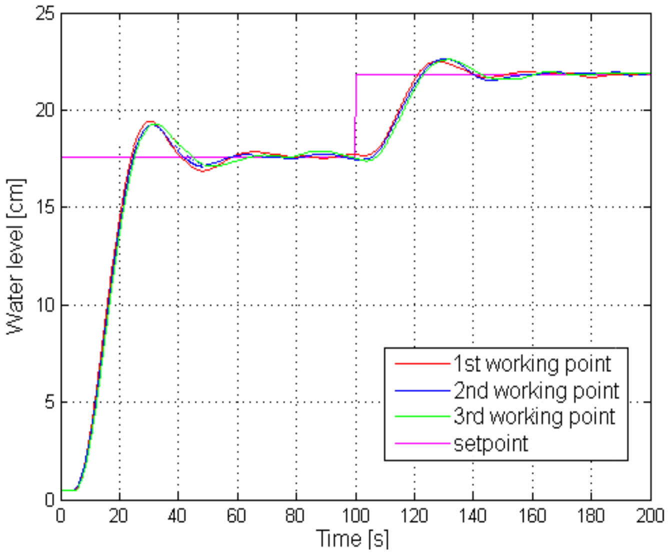

Fig. 5. Step responses of real process in all working points

V. CONCLUSIONS

In this paper the robust output feedback PI controller design based on LMI regions, formulation and solution of the corresponding BMI has been proposed. Recent results are applied to real uncertain Coupled-Tank process with polytopic uncertainty domain, where the required closed loop poles region is determined by the prescribed damping factor. Finally, simulations and numerical results illustrate the effectiveness of the proposed algorithm. Results obtained in the paper will be used for control education at Institute of Control and Industrial Informatics.

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