SLIDING MODE CONTROL FOR COUPLED-TANK LIQUID LEVEL CONTROL SYSTEM

HUR ABBAS
Faculty of Engineering and Technology
International Islamic University,
Islamabad, Pakistan
Hur_abbas70@yahoo.com

SAJJAD ASGHAR
Faculty of Engineering and Technology
International Islamic University,
Islamabad, Pakistan
Sajadoon@hotmail.com

SHAHID QAMAR
Department of Electrical Engineering
COMSATS Institute of Information Technology,
Abbottabad, Pakistan
Shahidqamar@ciit.net.pk

Abstract: This paper presents the mathematical modeling of coupled-tank system and designing of sliding mode control (SMC) for liquid level control in the system. The non-linear single-input single-output (SISO) mathematical model of plant is developed initially. A simulation is carried out using MATLAB/SIMULINK to control the non-linear model of coupled-tank system for a number of different conditions. The commanded inputs such as that of step, random, square, saw tooth and sinusoidal input signals are fed to the system to test the tracking performance of SMC. Some realistic situations are also included in the plant to examine the robustness of controller. The controller showed robustness for disturbance in the plant and produced an appropriate control signals for the sake of controlling the liquid level in the coupled-tank system. The tracking performance of SMC is also compared with conventional PID controller in terms of performance index ITAE. The SMC showed excellent tracking results than PID controller for time varying command signals.

Keywords: Coupled-tank, non-linear system, Sliding mode control,

TABLE I. NOMENCLATURE

Symbol	Name	Symbol	Name
A_1	Cross-sectional Area of Tank 1	a	Cross-sectional area of outlet
A_2	Cross-sectional Area of Tank 2	g	Gravitational constant
H_1	Liquid height in Tank 1	H_D	Desired height
H_2	Liquid height in Tank 2	λ	Slope of sliding surface
U_{i1}	Inflow rate to Tank 1	U_D	Discontinuous control
U_{i2}	Inflow rate to Tank 2	U_c	Continuous control
β	Coefficient of discharge	δ	Chattering suppression factor

I. INTRODUCTION

In process industries, often it is essential that the fluid to be supplied in tanks, may be store up in tanks and afterwards transferred to other tanks as per requirement. But mostly it is required that the fluid must be maintained at a specific height or in a certain range. For the sake of achieving this goal, the fluid supplied to the tanks must be controlled in a proficient manner. The applications of liquid level control in industries includes, boilers, food processing, dairy, beverage, spray coating, filtration, effluent treatment, pharmaceutical industries, nuclear power generation plants, water purification systems, automatic liquid dispensing and refilling devices and industrial chemical processing, [1]. Coupled-tank is regarded as an important plant model to be studied as it represents the basic problem occurring in process industries and it is a benchmark problem for testing and analyzing the tracking performance of designed controllers.

Fixed gain controllers are often not proficient in controlling a plant whose parameters are time varying and a number of disturbances acting upon the system during the mode of operation. There are also other types of control schemes, either categorized as intelligent or adaptive used in controlling the liquid level in the coupled-tank system. Application of PI Model Reference Adaptive Controller for coupled-tank system in order to control the level of liquid in tanks is investigated by Boonsrimuang, Numsomran and Kangwanrat [2]. By using the Model Reference Adaptive control approach, the PI controller gains are updated every time when any disturbance acts on the plant and changes the behavior of plant. It has been found that the control scheme proved to be good for controlling the level in the coupled-tank plant. Ivan Holic and Vojtech Vesely [3] designed PID controller for the level control in coupled-tank system in the frequency domain. The controller is tested for different set points tracking and proposed controller gave satisfactory results for these reference heights.

The use of robust control technique such as SMC in the process control system brings new solution to overcome nonlinearities, disturbances and measurement noise.

Sliding Mode Control (SMC) is a technique which is derived from Variable Structure Control (VSC). This control technique was originally studied by V. I. Utkin [4] in late 1960s. The most important characteristic of SMC is to deal with non-linear and time varying systems. [5-6]. SMC is a robust control strategy and it has very simple procedure to design the controllers for linear and non-linear plants. The main advantage of SMC is that, it is robust to plant uncertainties and insensitive to disturbances acting on the system. Because of its excellent robustness to uncertainties, SMC has been recognized as a proficient technique for robust control of uncertain systems. In the near past, most of the researchers paid their attention for designing the robust controllers. The SMC is one of them [7].

In this research work, we have developed the non-linear mathematical model of coupled-tank system and designed the SMC for controlling liquid level in the system. The simulation results are also discussed by taking different command signals. The controller tracking performance is also observed by considering the disturbances acting on the system. In order to examine the robustness of SMC, a brief comparison of SMC is made with conventional PID Controller in the presence of disturbance and non-linearity.

II. MATHEMATICAL MODEL OF COUPLED-TANK

The coupled-tank system consists of two cylindrical tanks with an internal baffle in between as shown in Figure 1. The internal baffle can be adjusted to vary the flow between the two tanks. Both tanks have an outlet whose opening can also be varied by means of adjustable clamps. These clamps vary the discharge coefficient of the liquid flowing out of the respective tank. The control objective is to control the liquid level in 2nd tank by manipulating the flow rate of the liquid into the 1st tank. The liquid used in the plant is assumed to be steady, non-viscous and incompressible which leads to the use of Bernoulli's equation to obtain a set of non-linear state equations:

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$$A_1 \frac{dH_1}{dt} = U_{i1} - \beta_1\sqrt{H_1} - \beta_{12}\sqrt{|H_1 - H_2|}\ [\mathrm{sgn}(H_1 - H_2)] \quad (1)$$

$$A_2 \frac{dH_2}{dt} = U_{i2} - \beta_2\sqrt{H_2} + \beta_{12}\sqrt{|H_1 - H_2|}\ [\mathrm{sgn}(H_1 - H_2)] \quad (2)$$

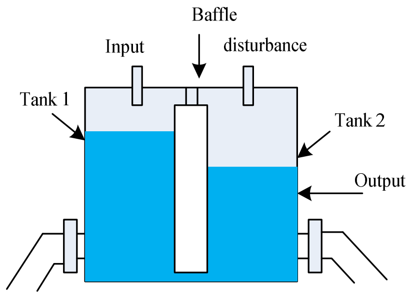


Figure 1: Schematic diagram of coupled-tank system

H_1 and H_2 are the heights of liquid in 1st tank and 2nd tank respectively. The cross-sectional area of corresponding tanks is A_1 and A_2. The volumetric flow rates into the 1st tank and 2nd tank are represented by U_{i1} and U_{i2} respectively. For single-input single-output (SISO) coupled-tank plant, the term U_{i2} will be considered as disturbance. Each outlet drain can be modeled as a simple orifice. The parameters β_1, β_2, β_{12} are the proportionality constants and are dependent on the coefficients of discharge, gravitational constant and the cross-sectional area of each outlet. Mathematically,

$$\beta_i = a_i\sqrt{2g} \qquad i=1,2 \qquad (3)$$

Where "a" denotes the cross-sectional area of outlet pipe. The subscript i denote the respective tank. Where a_{12} is the cross-sectional area of outlet pipe between the two tanks. The coupled-tank plant parameters are given in Table II and are taken from [15].

TABLE II. PLANT PARAMETERS

Name	**Symbol**	**Parametric Value**		
Cross Sectional Area of each tank	A_1 & A_2	32 cm^2		
Proportionality constant which depends upon the cross sectional area of tank and gravitational constant	β_i Where subscript i denotes the respective tank	β_1	β_2	β_{12}
		14.30 $cm^{5/2}$/sec	14.30 $cm^{5/2}$/sec	20 $cm^{5/2}$/sec
Maximum allowable volumetric flow rate	U_{i_max}	1000cm^3/s		
Gravitational constant	g	981cm/sec^2		

III. OPEN LOOP RESPONSE OF PLANT

The open loop response of non-linear coupled-tank plant is shown in figure 2. The input to the plant is 90 cm^3/s flow rate and output is the liquid level (cm) in 2nd tank. It is clear from the plot that the plant is stable but has large settling time. For maintaining the liquid at 8cm in 2nd tank, almost 80 seconds are required. Hence the control objective is to reduce the settling time. The open loop response is stable but it is unable to handle the uncertainties and disturbances acting on the system.

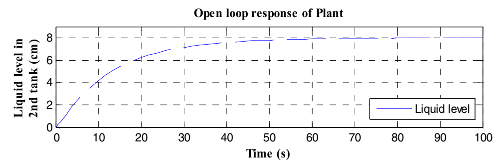


Figure 2: Coupled-tank open loop response

IV. SLIDING MODE CONTROL

The idea behind SMC is to choose a sliding surface along which the system can slide to its desired final value. Figure 3 is representing the principle of SMC.

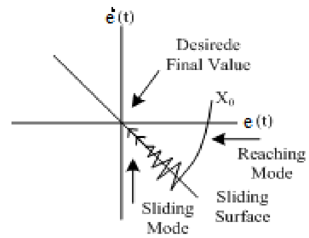


Figure 3: Principle of SMC

The structure of SMC is deliberately changed when the system state trajectory crosses the sliding surface in accordance with a given control law. Hence first of all a sliding surface is selected for the designing of SMC [4]. The system becomes insensitive to unknown external disturbances and certain parameters variations when it slides on the switching surface [8].

By designing the SMC, a sliding surface has been selected at first, then a suitable control law is designed so that the control variable is being driven to its reference value. The structure of SMC law U(t) is based on two main parts; a continuous part U_C (t) and a discontinuous part U_D (t) [9]. That is

$$\mathrm{U(t)} = \mathrm{U_c(t)} + \mathrm{U_D(t)} \qquad (4)$$

$U_c(t) = U_{eq}(t)$, is the dominated equivalent control, represents the continuous part of the controller that maintains the output of the system restricted to the sliding surface. The continuous part of SMC is given by,

$$\mathrm{U_c} = \mathrm{f[R(t), Y(t)]} \qquad (5)$$

It is a function of the reference value R(t) and controlled variable Y(t).

The part U_D (discontinuous) of SMC comprises a non-linear element that contains the switching element of the control law. This part (U_D) of the controller is discontinuous across the sliding surface.

A. CONTROLLER DESIGN

In SMC, the objective is to make the error and derivative of error equal to zero. As the system error is defined as the difference between actual height and desired height, mathematically

$$e(t) = H_D(t) - H_2(t) \quad (6)$$

$H_D(t)$ is desired height while $H_2(t)$ is the actual height in 2nd tank.

The expression for the nth order sliding function [10, 11, 12] is given by

$$S(t) = (d/dt + \lambda)^{n-1} e \quad (7)$$

A 2nd order sliding function (n=2) can be written as

$$S = \dot{e} + \lambda e \quad (8)$$

Where λ>0 is the slope of sliding surface.

Theorem: *Stability Condition*

Consider a Lyapunov function:

$$V = \frac{1}{2} S^2 \quad (9)$$

From Lyapunov theorem we know that if $\dot{V}$ is negative definite, the system trajectory will be driven and attracted toward the sliding surface and remain sliding on it until the origin is reached asymptotically [4].

$$\dot{V} = S\dot{S} \quad (10)$$

A sufficient condition for the stability of the system is

$$\frac{1}{2}\frac{d}{dt} S^2 \leq -|S| \quad (11)$$

Where is a positive constant. The equation (11) is called reaching condition or sliding condition.

The basic discontinuous control law of SMC is given by

$$U_D = Ksgn(S) \quad (12)$$

Where the parameter K is the constant manual tuning parameter and is responsible for the reaching mode. The main disadvantage of SMC is the Chattering phenomena.

Chattering is a high-frequency oscillation around the desired equilibrium point [5, 9, 13]. The chattering problem could be solved satisfactorily if the control U_D is designed according to [14]:

$$U_D = K \frac{s}{|s| + \delta} \quad (13)$$

The parameter δ is chattering suppression factor and is manually adjusted to eliminate the unwanted chattering. When system remains on sliding surface that means *e (t)* is zero all times [8]. Hence it is desired to make

$$\frac{dS(t)}{dt} = 0 \quad (14)$$

The continuous part of the control law will be

$$u_c = \frac{1}{b}\left[(a_1 - \lambda)\frac{dH_2(t)}{dt} + a_0 H_2(t)\right] \quad (15)$$

This procedure in which the continuous part of the controller derived is known in the SMC theory and the equivalent control procedure [4]. The parameters b, a_0 and a_1 are calculated from the approximated model of the system and these values are 0.006989, 0.02785 and 0.5539 respectively.

Then, the complete SMC law can be represented as follows

$$U = \frac{1}{b}\left[(a_1 - \lambda)\frac{dH_2(t)}{dt} + a_0 H_2(t)\right] + K\frac{s}{|s| + \delta} \quad (16)$$

The closed loop schematic diagram of coupled-tank plant controlled by SMC is shown in figure 3.

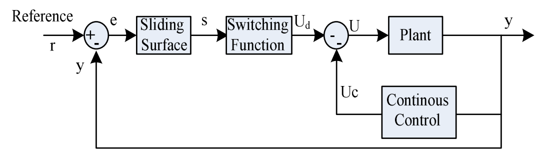
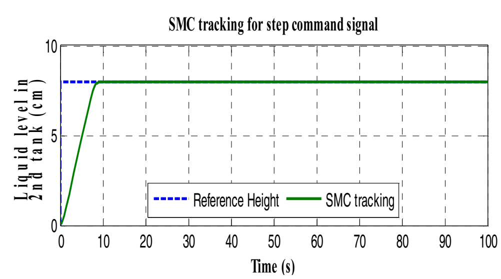


Figure 3: Closed loop schematic diagram of Sliding mode control

V. SIMULATION RESULTS OF SMC

The simulated coupled-tank plant is non-linear single-input single-output (SISO) system. The plant is simulated by the non-linear equations (1) and (2) which are representing the linear and non-linear dynamics of coupled-tank system. The liquid height in the 2nd tank is the output while the inflow to the 1st tank is control variable. Hence the objective of the control is to adjust the inflow rate to the 1st tank so that the desired liquid height should be achieved in 2nd tank. The saturation is used in the simulation to ensure that the control signal always remain within the bound. The value of tuning parameters used in the simulation results is $K = 358$ and δ=0.21. The different command inputs are taken, such as step, random, square, saw tooth and sinusoidal. These command inputs are the desired liquid heights (in cm) in 2nd tank. The tracking of controller for step, random, square, saw tooth and sinusoidal command signals and their control signals are shown in figure 4 to figure 8.



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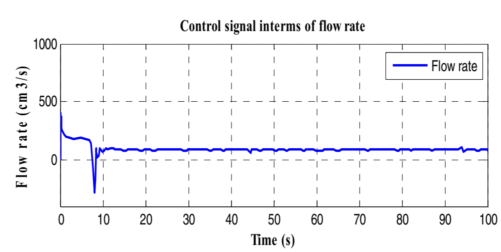
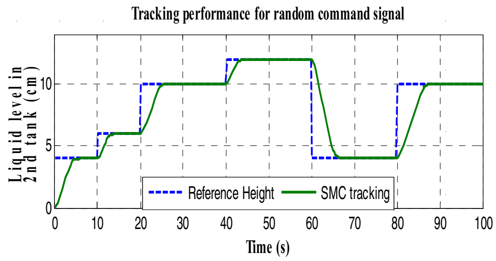


(b)

Figure 4: (a) SMC tracking for step command signal (b) Control effort



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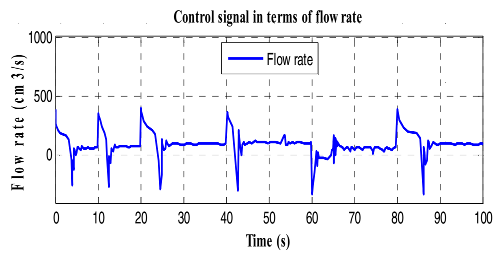
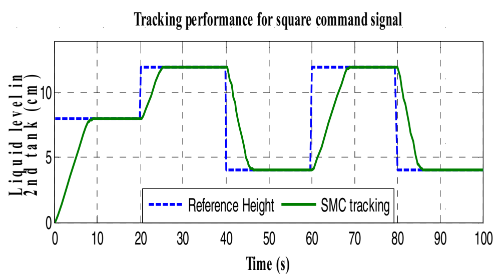


(b)

Figure 5: (a) SMC tracking for Random command signal (b) Control effort



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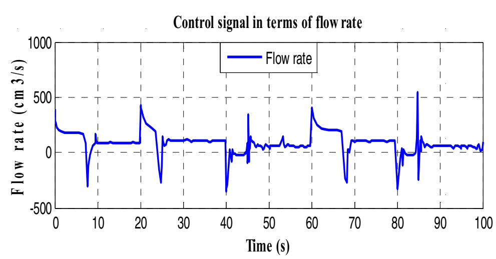
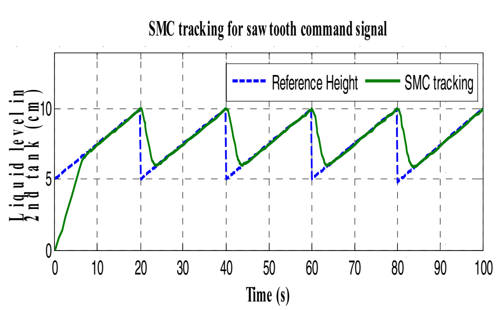


(b)

Figure 6: (a) SMC tracking for square command signal (b) Control effort



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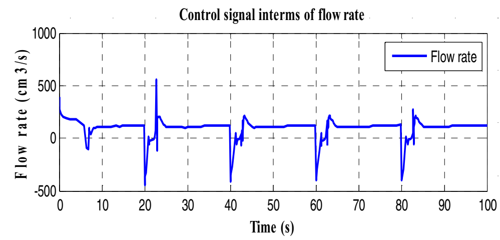
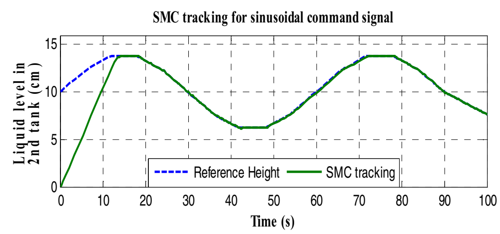


(b)

Figure 7: (a) SMC tracking for saw tooth command signal (b) Control effort



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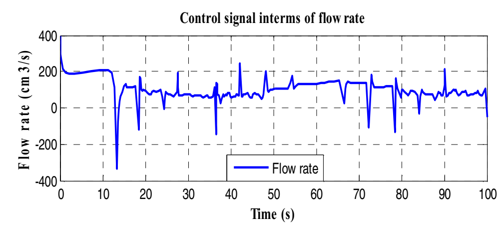


(b)

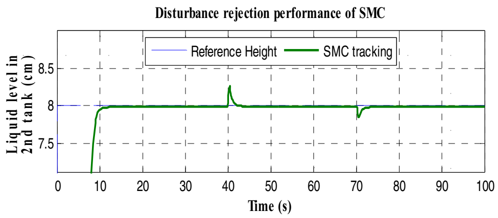
Figure 8: (a) SMC tracking for sinusoidal command signal (b) Control effort

VI. DISTURBANCE REJECTION PERFORMANCE

The extra inflow to the 2nd tank and leakage in 2nd tank are considered as disturbances acting on the system. These disturbances directly affect the output of the plant, i.e the liquid height in 2nd tank. In the next section we have discussed both of them separately.

A. Disturbance In 2nd Tank

The term U_{i2} in (2) is representing the disturbance in the 2nd tank. This disturbance has the resemblance with real plant when another viscous fluid is mixed in the tank that causes to decrease the outflow from tank. As a result, the liquid level in the 2nd tank increases. The disturbance is added in the system by supplying liquid at the rate of $50cm^3/s$ into the 2nd tank for 30 seconds (40 to 70 sec of total simulation time of 100 sec). The disturbance in the 2nd tank causes to increase the liquid level and affects the output of the plant. The SMC adjusts the flow rate into the 1st tank so as to maintain the liquid level at 8cm reference height in 2nd tank. Disturbance rejection performance of the SMC is shown in figure 9.



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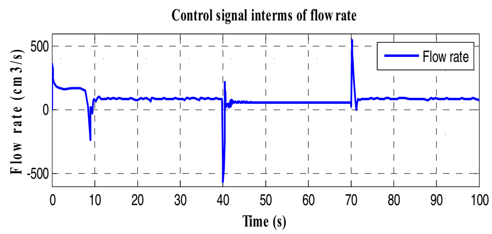
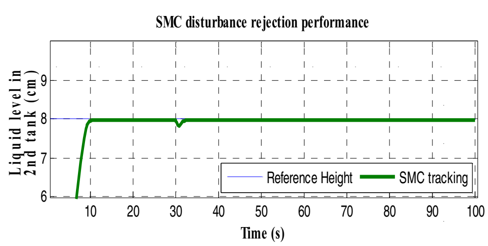


(b)

Figure 9: (a) SMC disturbance (extra inflow to 2nd tank) rejection performance (b) Control effort

B. Leakage In The 2nd Tank

The leakage condition has the resemblance of increased outflow from tank in case of increased temperature in physical plants which affects the liquid viscosity and hence the outflow. The leakage (20cm^3/s extra liquid outflow at 30 sec simulation time) in the 2nd tank is introduced and the performance of SMC is observed and shown in figure 10. The SMC showed robustness under this uncertain condition. Because of the leakage in the 2nd tank, the liquid level falls a bit but the SMC recovered that error within 3 seconds.



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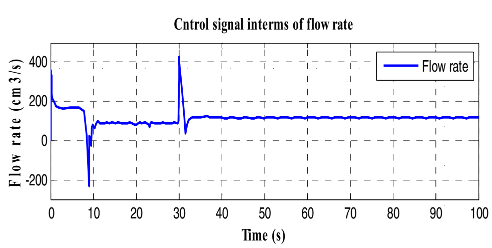

(b)

Figure 10: (a) SMC disturbance (leakage in 2nd tank) rejection performance (b) Control effort

VII. PERFORMANCE COMPARISON

The tracking performance of designed SMC for different time varying command signals is compared with conventional PID controller (which is designed by Zeigler Nichols method for same plant model) in terms of ITAE (Integral Time Absolute Error). The performance comparison is made in table III.

TABLE III. PERFORMANCE INDEX

PERFORMANCE TEST	SMC	PID
Step command signal	208	428
Random command signal	3986	4186
Square command signal	5163	6017
Saw tooth command signal	2268	3775
Sinusoidal command signal	558	607
Disturbance Rejection Performance (50 cm^3/sec inflow to 2nd tank)	210	442
Disturbance Rejection Performance (20 cm^3/sec leakage to 2nd tank)	277	416

The performance index comparison suggests that SMC has much better tracking and disturbance rejection performance than PID controller in controlling the liquid level in coupled-tank system.

VIII. CONCLUSION

Sliding mode control scheme has been successfully implemented on nonlinear coupled-tank plant model with various types of commanded input signals through simulation. It is shown that SMC is able to control the non-linear system with different set points given by the time varying commanded input signals. The SMC exhibits robust behavior even in the presence of significant measurement noise. Furthermore, the performance of SMC is compared with conventional PID control scheme and is proved to be much better in all aspects.

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