Self-tuning Controllers for a Coupled Tank Pilot Plant : Evaluation and Comparison

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Abstract - Real-time self tuning control was implemented on a coupled tank pilot plant using a microcomputer. Three different algorithms namely, generalized minimum variance, pole placement and extended horizon were used. The performances of these controllers are evaluated for set-point tracking experiments and load disturbance experiments.

INTRODUCTION

Over the last few decades, there have been extensive laboratory experiments and industrial feasibility studies of self-tuning control. Several design strategies for implementation of self-tuning control have been proposed in the literature. Current interest in self-tuning control systems was largely simulated by the development of the self-tuning regulator (STR) by Astrom and Wittenmark [1]. The feedback controller in STR was designed to minimize the variance of the system output. Although STR has been used successfully in a number of experimental applications, it has few disadvantages; particularly incorrectly specified time delays can result in poor or even unstable performance and it is not directly applicable to nonminimum phase systems.

To overcome the stability problems faced with STR, Clarke and Gawthrop [2] proposed the generalized minimum variance (GMV) self tuning controller. It was designed to minimize the variance of an auxiliary output. The GMV control law provides several important advantages over the minimum variance control law : (a) it is more easily tuned and (b) it can be applied to nonminimum phase systems. However, this controller design still requires an accurate knowledge of the system time delay.

Meanwhile, Ydstie et al. [3] have developed a robust version of STR that requires relatively little knowledge of the system characteristics. This algorithm referred as extended horizon (EH) self-tuning controller is a member of predictive controllers that uses receding horizon technique.

As an alternative to controller design based on optimization, Wellstead et al [4] have proposed self-tuning controller based on pole placement (PP). The key idea of pole placement strategy is to shift the open loop poles of a system to some desired set of closed loop poles. An advantage of the algorithm is that it does not cancel the zeroes of the system, thus enables to control nonminimum phase systems. Furthermore, the pole placement algorithm does not require that the leading coefficients of the numerator dynamics of the system to be nonzero, and thus effectively allow for unknown or varying time delays by overparameterizing the numerator.

In the present study, the above three algorithms namely, generalized minimum variance, extended horizon and pole placement self tuning controllers would be investigated experimentally. The above algorithms were programmed into microcomputer using Turbo Pascal (version 6) programming language and implemented on a coupled tank pilot plant.

MODEL STRUCTURE

The pole placement and GMV algorithms assumes a locally linearized model

$$A(z^{-1})y(t) = z^{-k}B(z^{-1})u(t) + C(z^{-1})\xi(t) \qquad (1)$$

where z^{-1} is the backward shift operator, k is the estimate of the process time delay, y(t) is the process output, u(t) is the input to the process and $\xi(t)$ is an independent noise sequence with zero mean.

Although eqn. (1) can also be used to derive extended horizon controller, a better approach suggested by Ydstie et al. [3] is to use integrating model representation

$$A(z^{-1})\Delta y(t) = z^{-k}B(z^{-1})\Delta u(t) + \xi(t) \qquad (2)$$

where Δ is the differencing operator $(1 - z^{-1})$.

CONTROL ALGORITHMS

A summary of generalized minimum variance, pole placement and extended horizon self-tuning control algorithms are presented below.

The algorithms would be simplified by assuming $C(z^{-1})=1$.

Generalized Minimum Variance (GMV)

System auxiliary output $\phi(t)$ is introduced by defining

$$\phi(t+k) = P(z^{-1})y(t+k) + Q(z^{-1})u(t) - R(z^{-1})r(t) \qquad (3)$$

where P, Q and R are user specified polynomials in the backward shift operator and r(t) is the set-point.

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The cost function to be minimized is the variance of the auxiliary output:

$$J = E[\phi^2(t+k)] \tag{4}$$

The control law is defined as

$$G\Delta u(t) + Fy(t) - Hr(t) = 0 \tag{5}$$

where G, F, H are polynomials in z^{-1} These polynomials are estimated using recursive least square algorithm. The design parameters are the P, Q and R polynomials. The following conditions need to be satisfied for zero steady state error:

$$R = P(1) \text{ and } Q(1) = 0$$

A typical choice for Q is

$$Q(z^{-1}) = \lambda(1 - z^{-1})$$

where λ is a scalar. Increasing λ tends to reduce the control action, while the $(1\text{-}z^{-1})$ term provides integral action to eliminate offsets after load and set-point changes.

Pole Placement (PP) Algorithm

The pole placement control law is expressed as

$$F(z^{-1})\,u(t) = H(z^{-1})\,r(t) - G(z^{-1})\,y(t) \tag{6}$$

Manipulation of eqn. (1) and eqn. (6) yields

$$\left[FA + z^{-k}BG\right]y(t) = z^{-k}\,[BH]r(t) + F\,\xi(t) \tag{7}$$

The roots of the left hand side term are the closed loop poles of the system. These closed loop poles can be assigned to their desired locations, specified by a design polynomial $T(z^{-1})$, by selecting F and G according to the polynomial identity:

$$FA + z^{-k}BG = T \tag{8}$$

Eqn. (8) can be solved using Gaussian elimination method.

The precompensator H is selected to attain zero steady state tracking error, hence:

$$H = \left.\frac{T(z^{-1})}{B(z^{-1})}\right|_{z=1} \tag{9}$$

The desired polynomial (pole set) is typically specified by

$$T(z^{-1}) = 1 + t_1 z^{-1} + t_2 z^{-2} \tag{10}$$

where

$$t_1 = -2\exp(-\xi\omega_n t_s)Cos\sqrt{t_s\omega_n(1-\xi^2)} \tag{10a}$$

$$t_2 = \exp(-2\xi\omega_n t_s) \tag{10b}$$

ξ is the damping ratio and ω_n is the natural frequency of the desired closed loop second order transient response.

Extended Horizon (EH) Controller

The system model of eqn. (2) is used and without loss of generality, polynomials A and B are assumed to have the same number of coefficients. Hence, $n_a = n_b\text{-}1 = n$

New polynomials F and G are chosen to satisfy the polynomial identity

$$1 - z^{-T} = \Delta F(z^{-1})A(z^{-1}) + \Delta z^{-T}G(z^{-1}) \tag{11}$$

in which T is the response horizon. Manipulations of Eqs. (2) and (11) yields

$$(1 - z^{-1})y(t) = G\Delta y(t-T) + z^{-k}FB\Delta u(t) + F\xi(t) \tag{12}$$

Assuming k=1 and $\xi(t+T)=0$, Eq. (12) can be expressed as a T-step ahead prediction model

$$y(t+T) = y(t) + \phi(t)\theta(t) \tag{13}$$

where

$$\phi(t) = [\Delta y(t), \Delta y(t-1), \ldots \Delta y(t-n+1); \\ \Delta u(t+T-1), \ldots, \Delta u(t), \ldots, \Delta u(t-n+1)]$$

$$\theta(t) = [\alpha_0, \alpha_1, \ldots, \alpha_{n-1}; \beta_1, \ldots, \beta_T, \ldots, \beta_{n+T-1}]$$

The solution for control equation that satisfies the extended horizon control criterion and minimizes future control efforts is given by

$$\Delta u(t) = \beta_T[\hat{y}(t+T) - h(t+T)] / \sum_{i=1}^{T}\beta_i^2 \tag{14}$$

where $\hat{y}(t+T)$ is the desired output at time (t+T) and

$$h(t+T) = y(t) + \sum_{i=0}^{n-1}\alpha_i\Delta y(t-i) + \sum_{i=1}^{n-1}\beta_{T+i}\Delta u(t-i) \tag{15}$$

The values of α and β in $\theta(t)$ are estimated on-line using recursive least square algorithm.

SYSTEM DESCRIPTION

The coupled tank pilot plant, shown in Fig. 1, comprises two separate tanks of equal size (Tank 1 and Tank 2) which are connected by a flow tube with an adjustable valve (V1). A larger tank (Tank 3) at the base of the unit acts as a water reservoir for the plant. An AC motor pumps water from Tank 3 into Tank 1 through an inlet tube. The speed of the pump is controlled by a variable speed drive (VSD).

Water flows from Tank 1 to Tank 2 through V1. An outlet tube with an adjustable valve (V2) connected to Tank 2 provides a return path for the water to flow into Tank 3. Therefore, the water is being recirculated continuously.

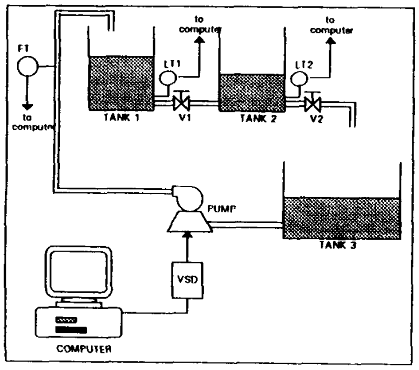


Fig. 1 : Schematic diagram of coupled tank pilot plant

Tank 1 and Tank 2 are fitted with pressure sensing liquid level transducer to measure the level in each tank. A flow transducer is connected across the inlet tube for monitoring water flow into Tank 1. The output signals from transducers are connected to the computer through an interfacing card mounted into an expansion slot inside the computer. The computer displays the measured values, executes control algorithm and then sends a signal to VSD which will in turn control the speed of the pump.

EXPERIMENTAL RESULTS

Two main objectives of a control system are: (i) to track set-point changes (small rise time and minimum overshoot) and (ii) to reject load disturbances. Therefore two sets of experiments were performed.

In the first set of experiments, a rectangular pulse set-point was injected to the system. The set-point changes at every 40 sampling instants. In the second set of experiments, a constant set-point was used, but, the system is disturbed by performing the following sequence of actions on the adjustable valves:

(i) At $t = 0$: V1 and V2 fully open
(ii) At $t = 50$ samples : V1 fully open and V2 fully closed
(iii) At $t = 60$ samples : V1 half open and V2 fully open
(iv) At $t = 80$ samples : V1 and V2 fully open

The performance of the self tuning algorithms are evaluated based on the sum of absolute error index:

$$I = \sum_{i=30}^{N_s} \text{ABS}\left[y(i) - r(i)\right]$$

where N_S is the number of samples. Note that the performance indexes are evaluated after first 30 samples. This is to allow some period for initial tuning.

For parameter estimation, recursive least square algorithm with a variable forgetting factor was used. Sampling period used for all the experiment was 5 seconds.

Fig. 2 shows a typical display of real-time self-tuning control of the plant. There are three distinct features on the display. First is the mimic display of the physical plant. Second is the digital display of the physical values and third is the graphical display of the set-point and the controlled output variable.

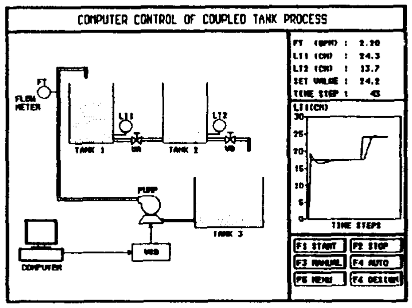


Fig. 2 : Real time self-tuning control

Experiments were conducted for different model structures and design parameters. However, to save space, only a typical result would be presented here. Following are the model structure and design parameters used

<u>Model structure</u>

GMV : $n_a=2$, $n_b=0$, $k=2$
PP : $n_a=2$, $n_b=1$, $k=1$
EH : $n_a=n_b-1=n=2$

<u>Design Parameters</u>

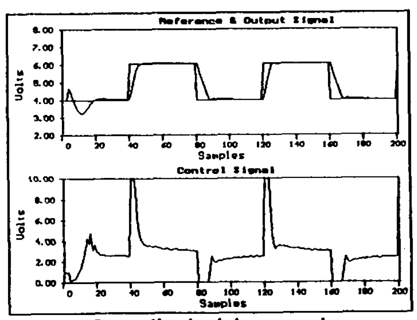
GMV : $P = 1$, $\lambda = 0.05$, $R=1$
PP : $\omega_n=0.35$ $\zeta = 0.9$
EH : $T = 3$

Figure 3(a), 3(b) and 3(c) show the response of set-point tracking experiments for GMV, PP and EH controllers respectively. Table 1 displays the performance index of the results.

Table 1 : Performance index for set-point tracking experiment

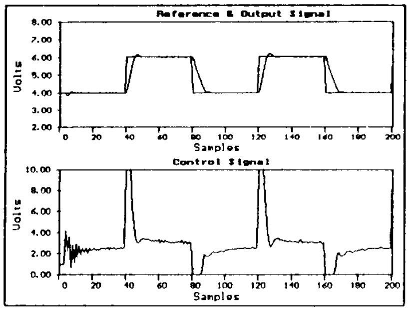
	I
GMV	36.65
PP	33.25
EH	20.00

It can be noted that, EH controller results in minimum error. This is mainly due to the property of EH algorithm that uses future set-point information to calculate current input signal. From further experiments with different design parameter values, it was found that by increasing the time horizon (T) for EH controller causes the index I to increase slightly.

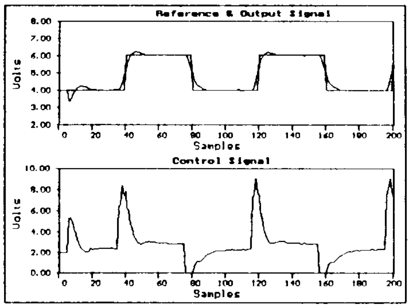


(a) Generalized minimum variance

Fig 3 Set-point tracking response



(b) Pole placement controller



(c) Extended horizon controller

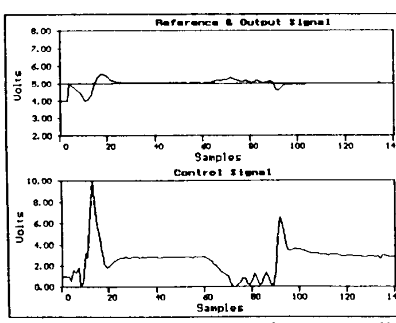
Fig 3 (continued)

For GMV controller the response was quite sensitive to the choice of λ. A suitable range for λ was found to be between 0.01 to 0.3. For PP controller, the rise time of the response can be achieved by selecting an appropriate value for ω_n and it seems that increasing the model structure does not show any significant effect on the response.

Meanwhile, figure 4(a), 4(b) and 4(c) illustrate the ability of the controllers to reject disturbances. The performance index for the results are shown in Table 2.

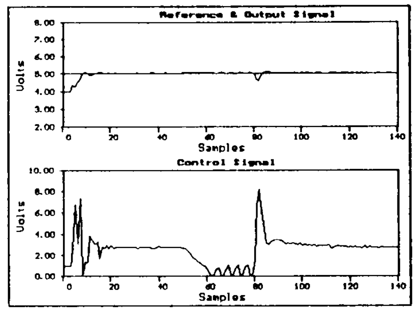
Table 2 : Performance index for load disturbance response

	I
GMV	7.21
PP	2.71
EH	6.92

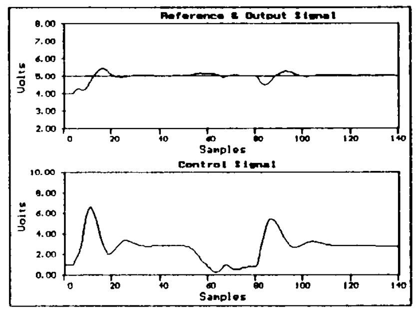


(a) Generalized minimum variance controller

Fig 4 : Load disturbance experiment response



(b) Pole placement controller



(c) Extended horizon controller

Fig 4 : (continued)

The above results indicate that the PP controller provides much faster load disturbance rejection compared to GMV and EH controllers. Further experiments with different values of design parameters follow the same result pattern.

CONCLUSION

This project has experimented some popular self-tuning algorithms on a coupled tank pilot plant using microcomputer. The results indicate that the extended horizon controller exhibits excellent set-point tracking performance while the pole placement controller seems to be better for load disturbance rejection.

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REFERENCES

[1] Astrom, K. J. and B. Wittenmark, "On self-tuning regulators," Automatica, vol. 9, pp 185-199, 1973.

[2] Clarke, D.W. and P.J. Gawthrop, "*Self-tuning controller,*" Proc. IEE, vol 126, pp 633-640, June 1979.

[3] Ydstie, B.E. L.S. Kershenbaum and R.W.H. Sargent, "*Theory and application of an extended horizon self-tuning control,*" AIChE Journal, vol 31, pp 1771-1780, 1985.

[4] Wellstead, P.E., D.L. Prager and P.M. Zanker, "A pole assignment self tuning regulator," Proc. IEE, vol 126, pp 781-787, August 1979.