Liquid Level Control of Nonlinear Coupled Tanks System using Linear Model Predictive Control

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Abstract—**This paper investigates the usage of Discrete-time Linear Model Predictive Control in controlling a nonlinear Coupled Tanks System. Two different schemes of Model Predictive control are employed. To begin with, a basic Model Predictive Control based on Generalized Predictive Control is used and then a Model Predictive Control approach based on Laguerre functions. Simulation results have been included which demonstrate the performance of both controllers when used to control Single-Input Single-Output Coupled Tanks System and the performance when Laguerre based Model Predictive Control is applied to Multi-Input Multi-Output Coupled Tanks System.**

Keywords—**Model Predictive Control, Laguerre functions, Nonlinear Coupled Tanks System, Generalized Predictive Control.**

I. INTRODUCTION

Model Predictive Control (MPC) has made a significant impact on modern control engineering. It has found a wide range of applications in the process, chemical and food processing industries. Model Predictive Control refers to a specific procedure in controller design from which many kinds of algorithms can be developed for different systems, linear or nonlinear, discrete or continuous. The main difference in the various methods of MPC is mainly the way the control problem is formulated. One of the most popular methods of MPC is Generalized Predictive Control (GPC). GPC was developed by Clarke [1]. The idea of GPC is to calculate future control signals in such a way that it minimizes a cost function defined over a prediction horizon. GPC is capable of controlling processes with variable dead-time, unstable and non-minimum phase systems.

In this paper Discrete-time Model Predictive Control (DMPC) is used to control the liquid level of a nonlinear Coupled Tanks System (CTS) in MATLAB, Simulink environment. At first, Model Predictive Control based on Generalized Predictive Control [2] which is a restricted model approach, is employed. Then a different approach using Laguerre functions [4] is used. Laguerre functions when used with DMPC have many benefits such as, the number of terms used in the optimization problem can be reduced to a fraction of that required by the basic procedure, allows substantial improvements in feasibility [5], two explicit tuning parameters can be used for tuning the closed loop performance with ease and For Multi-Input and Multi-Output (MIMO) configuration both of these tuning parameters can be selected independently for each input.

Finally, simulation results are given to demonstrate the performance achieved when both approaches are applied to Single-Input and Single-Output (SISO) nonlinear CTS. Also, the DMPC using Laguerre functions approach is applied to MIMO nonlinear CTS.

II. THE COUPLED TANK SYSTEM

The coupled tank system consists of two vertical tanks interconnected by a flow channel (Fig.1) which causes the levels of the two tanks to interact. Each tank has an independent pump for inflow of liquid. The sectional area of the outlets present and the base of each tank and the channel connecting the two tanks can be varied with rotary valves. The coupled tank system can be configured as a SISO or as a MIMO system via manipulation of pumps input and sectional area of rotary valves.

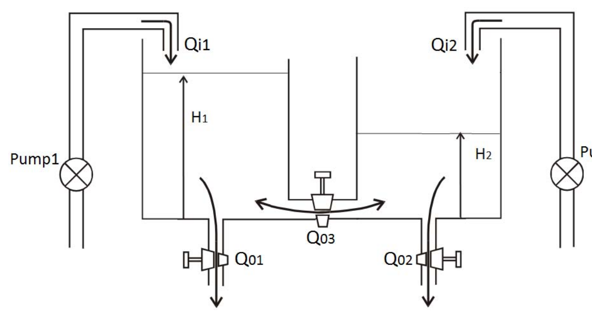

Figure 1. Layout of the coupled tanks system.

Considering mass balance, the dynamic equation of each tank is developed.

$$A_1 \frac{dH_1}{dt} = Q_{i1} - Q_{01} - Q_{03} \tag{1}$$

$$A_2 \frac{dH_2}{dt} = Q_{i2} - Q_{02} - Q_{03} \tag{2}$$

where, H_1 ,H_2 are heights of liquid in tank 1 and tank 2 respectively.A_1 and A_2 are cross-sectional areas of tank 1 and

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tank 2.Q_{03} is the flow rate between tanks.Q_{i1},Q_{i2} are pump flow rate into tank 1 and tank 2 respetively.Q_{01} and Q_{02} are the flow rate of liquid out of tank 1 and tank 2 respectively.

By Bernoulli's equation for a non-viscous, incompressible fluid in steady flow,

$$Q_{01} = s_1 \cdot a_0 \cdot \sqrt{2 \cdot g} \cdot \sqrt{H_1} = \alpha_1 \cdot \sqrt{H_1} \quad (3)$$

$$Q_{02} = s_2 \cdot a_0 \cdot \sqrt{2 \cdot g} \cdot \sqrt{H_2} = \alpha_2 \cdot \sqrt{H_2} \quad (4)$$

$$Q_{03} = s_3 \cdot a_1 \cdot \sqrt{2 \cdot g} \cdot \sqrt{H_1 - H_2} = \alpha_3 \cdot \sqrt{H_1 - H_2} \quad (5)$$

where, α1, α2 and α3 are proportionality constants which depend on the coefficients of discharge, the cross-sectional area of each area and gravitational constant. By using values from (3) to (5) in (1) and (2) the nonlinear equations that describe the dynamics of the multi-input and multi-output system are derived:

$$A_1 \frac{dH_1}{dt} = Q_{i1} - \alpha_1 \sqrt{H_1} - \alpha_3 \sqrt{H_1 - H_2} \quad (6)$$

$$A_2 \frac{dH_2}{dt} = Q_{i2} - \alpha_2 \sqrt{H_2} + \alpha_3 \sqrt{H_1 - H_2} \quad (7)$$

For single-input and single-output configuration Q_{o1} and Q_{i2} are zero, since the outlet valve of tank 1 is closed and liquid supply from pump 2 is also stopped.

$$A_1 \frac{dH_1}{dt} = Q_{i1} - \alpha_3 \sqrt{H_1 - H_2} \quad (8)$$

$$A_2 \frac{dH_2}{dt} = \alpha_2 \sqrt{H_2} + \alpha_3 \sqrt{H_1 - H_2} \quad (9)$$

A. Linearized Perturbation Model

Considering a small variation of q_1 and q_2 in both the control inputs respectively. Let h_1 and h_2 be the resulting change in heights of the two tanks due to this variation. The linearized model can thus be derived:

1) Muli-Input and Multi-Output Linear Model

$$A_1 \frac{dh_1}{dt} = q_1 - \frac{\alpha_1}{2\sqrt{H_1}} h_1 - \frac{\alpha_3}{2\sqrt{H_1 - H_2}} \left(h_1 - h_2\right) \quad (10)$$

$$A_2 \frac{dh_2}{dt} = q_2 - \frac{\alpha_2}{2\sqrt{H_2}} h_2 + + \frac{\alpha_3}{2\sqrt{H_1 - H_2}} \left(h_1 - h_2\right) \quad (11)$$

2) Single-Input and Single-Output Linear Model

$$A_1 \frac{dh_1}{dt} = q_1 - \frac{\alpha_3}{2\sqrt{H_1 - H_2}} h_1 + \frac{\alpha_3}{2\sqrt{H_1 - H_2}} h_2 \quad (12)$$

$$A_2 \frac{dh_2}{dt} = \frac{\alpha_3}{2\sqrt{H_1 - H_2}} h_1 - \left(\frac{\alpha_2}{2\sqrt{H_2}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}} \right) h_2 \quad (13)$$

TABLE I. PARAMETERS OF THE COUPLED TANK SYSTEM

Symbol	Quantity	Value
A_1,A_2	tank section area	9350.10^{-6} m^2
s_1,s_2,s_3	channel sectional area	$78.50.10^{-6}$ m^2
a_0	discharge coefficient of channel 1 and channel 2	1
a_1	discharge coefficient of channel 3	0.5
g	gravitational constant	9.8 m/s^2

III. MPC ALGORITHM BASED ON GPC

This basic MPC approach uses GPC to formulate the control problem [2]. Considering a single-input and single-output system:

$$X_m(k+1) = A_m X_m(k) + B_m u(k) \quad (14)$$

$$y(k) = C_m X_m(k) \quad (15)$$

where, u(k) is the manipulated variable, y(k) is the process output and Xm is the state variable vector having a dimension of 2.

The system has u(k) as input. To design a predictive controller this needs to be altered, thus the model is augmented with an integrator. Hence the state space form becomes:

$$\begin{bmatrix} \Delta X_m(k+1) \\ y(k+1) \end{bmatrix} = \begin{bmatrix} A_m & 0_m^T \\ C_m A_m & 1 \end{bmatrix} \begin{bmatrix} \Delta X_m(k) \\ y(k) \end{bmatrix} + \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix} \Delta u(k) \quad (16)$$

$$y(k) = \begin{bmatrix} 0_m & 1 \end{bmatrix} \begin{bmatrix} \Delta X_m(k) \\ y(k) \end{bmatrix} \quad (17)$$

Or simply:

$$X(k+1) = AX(k) + B\Delta u(k) \quad (18)$$

$$y(k) = CX(k) \quad (19)$$

where, 0m=[0 0…0] is a 1 x n vector, n being the order of the system. $\Delta X_m(k+1) = X_m(k+1) - X_m(k)$ and augmented state vector $X(k) = [\ \Delta X_m(k)^T\ y(k)\ ]^T$.

The future states of the system are calculated by using (18), assuming current sampling instant k_i, with ki >0 :

$$\begin{aligned} X(k_i+1|k_i) &= AX(k_i) + B\Delta u(k_i) \\ X(k_i+2|k_i) &= A^2X(k_i) + AB\Delta u(k_i) + B\Delta u(k_i+1) \\ &\dots \\ X(k_i+N_p|k_i) &= A^{Np}X(k_i) + A^{Np-1}B\Delta u(k_i) + A^{Np-Nc}B\Delta u(k_i+N_c-1) \end{aligned} \quad (20)$$

where, N_c is called the control horizon. It dictates the number of parameters used to capture the future control trajectory and N_p is the prediction horizon. It is also the length of the optimization window.

Using (20) the output prediction can be found as:

$$Y = FX(k_i) + \phi\, \Delta U \quad (21)$$

where, $Y = \left[y(k_i+1|k_i)\ \ y(k_i+2|k_i) \dots y(k_i+N_p|k_i) \right]^T$,

$$\Delta U = \left[\Delta u(k_i)\ \ \Delta u(k_i+1) \dots \Delta u(k_i+N_C-1) \right]^T$$

and

$$F=\begin{bmatrix} CA \\ CA^2 \\ CA^3 \\ : \\ CA^{Np} \end{bmatrix}$$

$$\varphi=\begin{bmatrix} CB & 0 & 0 & ... & 0 \\ CAB & CB & 0 & ... & 0 \\ CA^2B & CAB & CB & ... & 0 \\ : & & & & : \\ CA^{Np-1}B & CA^{Np-2}B & CA^{Np-3}B & ... & CA^{Np-Nc}B \end{bmatrix}$$

Let the data vector containing the set point information be $R_s^T=[1\ 1\ ..\ 1]r(k_i)$. The cost function is now defined as:

$$J=\left(R_s-Y\right)^T\left(R_s-Y\right)+\Delta U^T R_d \Delta U \tag{22}$$

where, $Rd=r_w \times I_{Nc \times Nc}$ with $rw \geq 0$ is used as a tuning parameter for desired closed-loop performance.

IV. MPC Using Laguerre Functions

Laguerre functions are orthonormal functions and can be used to approximate the increments of control signals. First the single-input case for MPC algorithm using Laguerre functions is described and is then extended to multi-input case. The z-transforms of discrete-time Laguerre functions are written as:

$$\Gamma_N(z)=\frac{\sqrt{1-a^2}}{z-a}\left[\frac{1-az}{z-a}\right]^{N-1} \tag{23}$$

where, $0\leq a<1$ is the scaling factor and N=1,2,.. is the number of Laguerre terms.

Discrete-time Laguerre functions can be found by using the following relation:

$$\Gamma_N(z)=\Gamma_{N-1}(z)\left[\frac{z^{-1}-a}{1-az^{-1}}\right] \tag{24}$$

Let the inverse z-transforms in vector form be, L(k). Using the realization (24), the discrete-time Laguerree functions can be written in the form of difference equation with initial condition L(0):

$$L(k+1)=A_l L(k) \tag{25}$$

where,

$$\beta=\left(1-a^2\right)$$

$$L(0)^T=\sqrt{\beta}\left[1\ \ -a\ \ a^2\ \ -a^3 \ldots\ (-1)^{N-1}a^{N-1}\right.$$

$$L(k)=\left[l_1(k)\ l_2(k) \ldots l_N(k)\right]$$

$$A_l=\begin{bmatrix} a & 0 & \ldots & 0 \\ \beta & a & \ldots & 0 \\ -a\beta & \beta & \ldots & 0 \\ a^2\beta & -a\beta & \ldots & 0 \\ : & & & \\ (-1)^{N-2}a^{N-2}\beta & (-1)^{N-3}a^{N-3}\beta & \ldots & a \end{bmatrix}$$

Thus at time k discrete Laguerre functions can be used to capture the control trajectory:

$$\Delta u\left(k_i+m\right)=\sum_{i=1}^{N} C_i\left(k_i\right) l_i(m) \quad =L(m)^T\eta \tag{26}$$

where, m=0,1,2…N_p , $\eta^T=\left[c_1\, c_2 \,..\, c_N\right]$

The value of $x(k_i+m)$ for single-input is found as:

$$x\left(k_i+m|k_i\right) \quad =A^m x\left(k_i\right)+\sum_{i=0}^{m-1} A^{m-i-1} BL(i)^T\eta \tag{27}$$

If the system is extended from single-input to multi-input input. The control variable will be formulated as:

$$\Delta u(k)=\left[\Delta u_1(k)\ \Delta u_2(k) \ .... \ \Delta u_p(k)\right]^T \tag{28}$$

where, p is the number of inputs and $\Delta u_i(k)=L_i(k)\eta_i$

Now the prediction of future state at time m :

$$x\left(k_i+m|k_i\right)=A^m x\left(k_i\right)+\phi(m)^T\eta \tag{29}$$

where,

$$\phi(m)=\sum_{j=0}^{m-1} A^{m-j-1}\left[B_1 L_1(j)^T B_2 L_2(j)^T .. B_m L_m(j)^T\right] \tag{30}$$

The cost function in terms of Laguerre parameter η is given by,

$$J=\eta^T\Omega\eta+2\eta^T\Psi x\left(k_i\right)+\sum_{m=1}^{N_p} x\left(k_i\right)^T\left(A^T\right)^m QA^m x\left(k_i\right) \tag{31}$$

where,

$$\Omega=\sum_{m=1}^{N_p}\phi(m)\,Q\,\phi(m)^T+R_L$$

$$\Psi=\sum_{m=1}^{N_p}\phi(m)\,QA^m$$

After optimal value of η is found, the receding horizon control is realized.

Constrained solution can be found in terms of η by solving the quadratic programming problem:

$$M\eta\leq\gamma \tag{32}$$

V. Simulation Studies

Simulation results show the performance comparison between the basic MPC approach based on GPC and MPC approach based on Laguerre functions when applied to SISO nonlinear Coupled Tanks System. MPC approach based on Laguerre functions is also applied to MIMO nonlinear CTS. Prediction horizon for all simulations is set to 40.

A. SISO nonlinear CTS results

The reference level H_2 used to evaluate the response of MPC controllers applied to CTS is a pulse train whose amplitude varies between 0 and 0.12 meter.

1) MPC approach results (Fig.2), (Fig.3)and (Fig.4) show that in order to improve performance a long control horizon is required and not much improvement is seen after the control horizon exceeds the value 10. There are lots of undershoots and overshoots in the plant response which are undesirable.

Irrespective of the value of the control horizon the overshoots and undershoots are not eliminated.

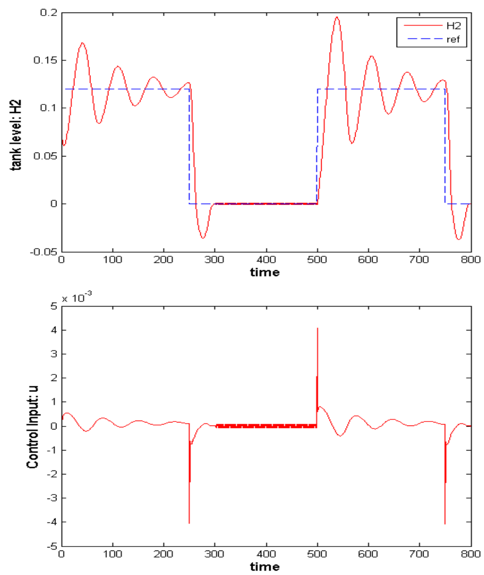

Figure 2. The basic MPC approach with a control horizon of 2

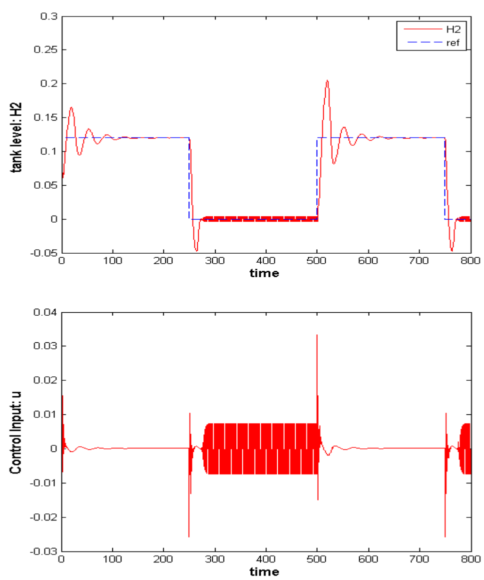

Figure 3. The basic MPC approach with a control horizon of 10

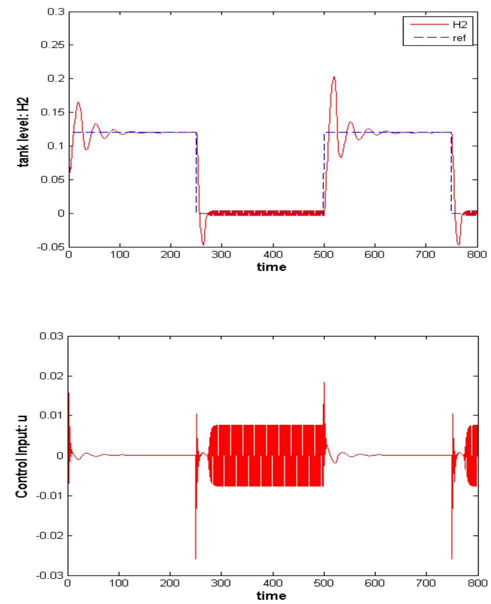

Figure 4. The basic MPC algorithm with a control horizon of 30

2) Simulation result (Fig.5) indicates improvement in performance when MPC with Laguerre functions is employed for control of SISO CTS as compared to the performance of basic MPC (Fig.4). The size of the undershoot and overshoot are reduced considerably. This approach also reduces the computational cost since there are only 4 parameters required to capture the control trajectory. Parameter values for this simulation are a=0.4 and N=4.

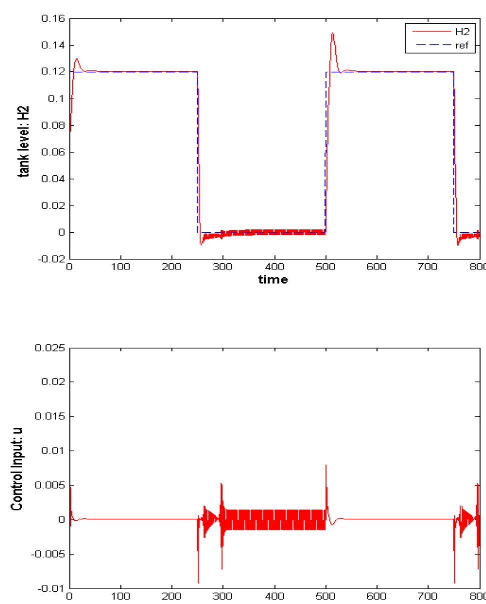

Figure 5. MPC based on Laguerre functions with scaling factor of 0.4

B. *MIMO nonlinear CTS results*

The reference levels used for H_1 and H_2 are pulse train with amplitude variations of 0 to 0.12m and 0 to 0.15m respectively. Parameter values are a=0.4 and N=4. The performance of the MPC with Laguerre functions with MIMO plants outperforms the basic MPC approach. Tighter control is observed throughout the simulation.

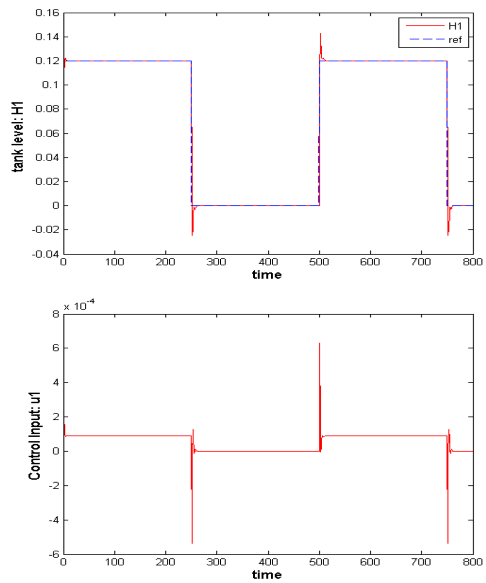

Figure 6. Tank 1 level control using MPC based on Laguerre functions.

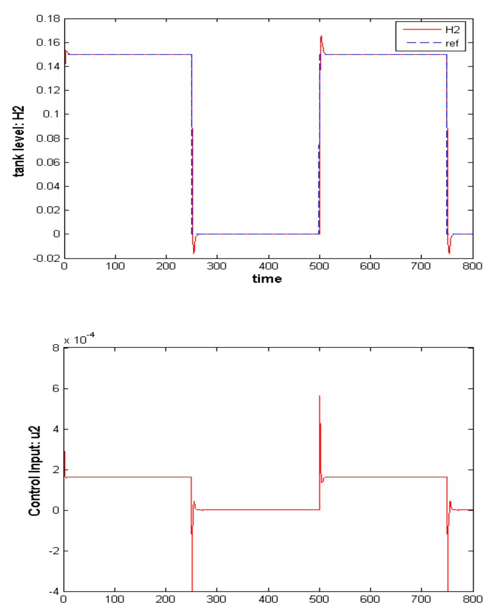

Figure 7. Tank 2 level control using MPC based on Laguerre functions.

CONCLUSION

In this paper two MPC schemes are employed to control nonlinear CTS having parameters given in Table 1. The simulation results presented in this paper indicate that the MPC using Laguerre functions reduces computational cost and performs better than the basic MPC approach using GPC. Further aspects of the Discrete-time Model Predictive Controller concerning the reduction of noise in steady state without degradation of system performance and formulation of control problem with constraints on the control variable incremental variation and constraints on the amplitude of the control variable, are under research.

REFERENCES

[1] D. W. Clarke, C. Mohtadi, and P. S. Tuffs, "Generalized predictive control-part 1: The basic algorithm," Automatica, 137-148 vol.23, 1987.

[2] Tan, K.K., Lee, T.H., Hunag, S.N. & Leu, F.M., "Pid control design based on a gpc approach," Industrial & Engineering Chemistry Research 41(8), 2002.

[3] D.Y. Valeryl, Predictive pid control of non-minimum phase systems, Advances in pid control, InTech, Croatia, 2011.

[4] L. Wang, "Discrete time model predictive control design using laguerre functions,"Proceedings of the American Control Conference,2430-2435 Vol.3, 2001.

[5] B. Khan and J. A. Rossiter, "Triple mode or laguerre mpc: a comparison,"American Control Conference, 2011.

[6] A.Boubakir, F.Boudjema, C.Boubakir, and S.Labiod, "A fuzzy sliding mode controller using nonlinear sliding surface applied to the coupled tanks system," International Journal of Fuzzy Systems, Vol. 10, 2008.

[7] J. Huang, L. Wang and Y. Huang , "Continuous time model predictive control for a magnetic bearing system,"PIERS ONLINE, vol.3, No.2, 2007.