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THRESHOLD MODELS OF DIFFUSION AND COLLECTIVE BEHAVIOR

MARK GRANOVETTER
State University of New York at Stony Brook

and

ROLAND SOONG
Arbitron Ratings Company
New York, New York

INTRODUCTION: DIFFUSION MODELS AND HOMOGENEITY ASSUMPTIONS

Historically, diffusion models in both social and biological science have made two crucial homogeneity assumptions, of which only one has been explicitly recognized—that of homogeneous mixing in the population. That is, it is usually assumed that each member of the group is in contact with each other member. This is universally acknowledged to be a gross simplification adopted only to facilitate mathematical analysis. A number of researchers, notably A. Rapoport, have attempted to take explicit account of the complexities introduced when the assumption of homogeneous mixing is dropped.

A second homogeneity assumption, equally important, is rarely given equal attention: this is the assumption that the populations in question are made up of homogeneous or identical *individuals*—at least with respect to those characteristics which bear on the likelihood of adoption or contagion. In practice, of course, we know perfectly well that people vary significantly from one another in their likelihood of adoption. Rather than attempt to model this heterogeneity explicitly, it has usually been treated instead as an unfortunate complication which leads to the need for models which take chance factors into accout. Bailey, for example, in the standard treatise on models of epidemics, remarks at the outset that whether or not

> ...an infective actually communicates his disease to susceptibles in his vicinity is plainly a matter of chance. The magnitude of this chance may depend on the virulence of the organisms, the extent to which they are discharged, the natural resistance of the susceptibles, the degree of the latter's proximity to the infective, and so on. (1975, p. 22)

Is this actually "plainly" a matter of chance? In this paper, we will argue that such situations can be modelled deterministically if the proper assumptions are made. Consider the four factors cited by Bailey as reflecting the operation of chance in the transmission of disease. Two of these—the extent of discharge of infectious organisms and the degree of proximity to the infective—can be eliminated by assuming homogeneous mixing. The variation in virulence of a disease organism points to the need for a parameter in the model indexing such virulence. This leaves us with the "natural resistance of the susceptibles"—that is, with population heterogeneity. If such heterogeneity can be explicitly built into the model, the need for stochastic treatment will vanish. We believe that it is worth making such an attempt, since stochastic models of diffusion and epidemics are notoriously intractable; strategically, our position here is that deterministic models ought not to be given up until absolutely necessary.

Threshold models of collective behavior, then, may be viewed as one attempt to construct models of diffusion or contagion that explicitly builds in population heterogeneity thus avoiding the necessity of stochastic models. Because such heterogeneity in propensity to adopt or succumb is empirically obvious, these models have the further advantage of resting closer to observed facts than stochastic models which must leave as a mystery why this rather than that person adopted or succumbed. We make no claim that threshold models are the only or best way to deal with population heterogeneity; they are most apt in situation where people's decisions or actions depend on other people's previous ones; but most cases of diffusion fit this criterion. Note also that, at least in the simple models treated in this paper, we do retain the assumption of homogeneous mixing. Though this can be relaxed, ensuing complications cannot be dealt with here. (See Granovetter, 1978, pp. 1428-1430.)

GENERAL DESCRIPTION OF THRESHOLD MODELS[1]

Many binary decision situations have the following characteristics: i) at any time, t, one's choice between the two available decisions depends, in part, on the choices of some relevant group of others in the preceding time period, t - 1; ii) each unit (person) is distinct, and will react differently from any other to the immediately previous distribution of choices; iii) because each decision depends on the set of decisions in the previous time period, there is an evolution of the "state vector" over time—a trajectory —which may or may not lead to an equilibrium position.

To illustrate these three characteristics, take the decision whether or not to "join a riot." [This particular example has no special conceptual status, and is merely a colorful illustration. For other examples, such as diffusion of innovations, rumors or diseases, decisions to engage in political activity (striking, voting, joining a party), migration, and various types of conformity, see Granovetter, 1978, pp. 1423-1424.] i) At any point in the progress of a riot, whether one decides to join depends in part on how many others are currently involved, as it is riskier to join when few others have—chances of being apprehended are greater. ii) Some individuals are more daring than others (a personality factor); some are more committed to radical causes (ideological influences); some have more to lose by being arrested (rational economic motives: e.g., the employed vs. the unemployed). Combinations of such influences give the result that while one individual might be prepared to join very early—in the risky stage—others might not be willing to join until the behavior was quite safe, nearly universal. Call these, as in common parlance, low versus high threshold individuals. iii) The state vector—telling us by 1's and 0's, for example, who is and is not rioting at time t—changes over time, and results, at any time, in part from the configuration in the previous time period.

These characteristics can be represented by the following formal model[2]: assign to each person a "threshold" (the number or proportion of the group he must see choose one of the two decisions (e.g., to riot) before he will. It is irrelevant to the formal model, though not, of course, to the substantive analysis, whether such thresholds result from calculations of interest, propensity to imitate (conformism) or some mixture of rational and irrational motives. If no members of the group have threshold zero, the resulting equilibrium—here defined as the total number ultimately making one of the decisions—will be zero. If, on the other hand, some k individuals have threshold zero, this number will choose the relevant decision, to riot, say, and at $t = 1$ we have k "rioters." If there are some m individuals with threshold less than or equal to k (or k/N where thresholds are expressed as proportions), then they will be "activated" and at $t = 2$ we will have $k + m$ rioters, etc. For a finite number of individuals, N, this forward recursion must always yield some equilibrium value $\leq N$ (100%).

DYNAMIC ANALYSIS IN THRESHOLD MODELS

The main interest of threshold models lies in the complex relation between individual preferences and aggregate (equilibrium) outcomes. A particular outcome can have been generated by any number of very different threshold distributions; correspondingly, very small changes in such distributions may result in very large changes of outcome. Consider as an example the case of one hundred individuals with thresholds distributed uniformly across the integers 0 to 99. The individual with threshold zero (the "instigator") would precipitate the "riot"; the presence of one rioter now activates the individual with threshold one, bring the number of rioters to two, etc. It is clear that the equilibrium is for all to riot. But if we replaced an individual of threshold one by one of threshold two, the instigator's action would activate no one, and the equilibrium would be one rioter. Our usual habit of inferring individual preferences from aggregate outcomes is confounded by situations such as this, since the individual preferences are nearly indistinguishable, but the outcomes entirely different. This habit of mind carries over into our use of multiple regression on individual traits, to explain behavior. In the present case, this procedure would represent riot behavior by a dummy variable, 1 or 0, and aggregate the individuals in the two groups described, using individual traits to predict whether or not one rioted. Such a regression would then show these traits to be uncorrelated with riot behavior But this is misleading since whatever traits are measured probably do help determine thresholds; the missing link, which washes out the causal relationship, is the mechanism of forward recursion which relates individual preferences to final outcomes. Regression, used in this way, implements an atomized conception of social structure, in which only individual traits, not the relationships among individuals, determine behavior.

Call distributions such as the uniform one reported above "unstable," in the sense that their equilibrium undergoes large changes when slightly altered. This statement is a matter of comparative statics. To go beyond such qualitative statements to more systematic and dynamic analysis requires a more analytical formulation of the underlying model.

Suppose r% of the individuals have joined a riot by time t. What % will have joined by time t + 1? By definition of threshold, it must be all those whose thresholds are less than or equal to r%. But since the usual cumulative distribution function gives that proportion, we have the relation:

$$r(t + 1) = F(r(t)) \tag{1}$$

a first-order difference equation, where F is the c.d.f. of thresholds.

For known F, we may find the equilibria by setting $r(t + 1) = r(t) = F(r(t))$.

Of more general use is the picture in Figure 1. Since we have equilibrium where $F(r) = r$, any intersection of F with the 45° line will be an equilibrium. In this particular picture, there are three. Where $F(0)$ is greater than zero and the process begins with no individuals active, then the lowest equilibrium, here labelled r_e, will be attained. But what of the other two equilibria? Can we ever expect the system to be found at or near them?

This question is closely related to whether these equilibria are stable or unstable, since a system near a stable equilibrium will move closer to it and one near an unstable one is repelled from it. Suppose that $r(t) = (r_e + h(t)) = r^*$: the system is some small distance away from an equilibrium point. We would like to know whether $h(t)$ will grow or decline. The usual technique for examining this question is called "linearization," or more impressively, Liapunov's indirect method (see, e.g., Luenberger, 1979, Ch. 9). This consists of treating the nonlinear system as linear in the

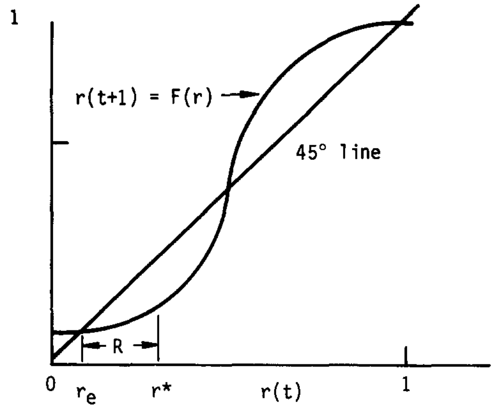

Figure 1

neighborhood of the equilibrium point, thus making available simpler results from the theory of linear systems. From equation (1) we now have $r(t + 1) = r_e + h(t + 1) = F(r_e + h(t))$. The linear approximation consists of replacing $F(r_e + h(t))$ by its first order Taylor series, $F(r_e) + F'(r_e)(h(t))$. This gives us the approximate equality: $r_e + h(t + 1) \doteq F(r_e) + F'(r_e)(h(t))$. But since, by definition, $r_e = F(r_e)$, this reduces to:

$$h(t + 1) \doteq F'(r_e)(h(t)) \qquad (2)$$

from which it is clear by inspection that the necessary and sufficient condition for h to decline over time is that abs $(F'(r_e))$ be less than unity; if greater, h increases and the equilibrium is unstable; at unity, we have neutral stability—stable if h is negative, unstable if negative.

In general, large classes of empirical threshold distributions could be reasonably approximated by polynomial functions. Consideration of such functions therefore will give some insight into the general issues of stability and dynamics. For the polynomial density, $f(x) = ax^n + bx^{n-1} + \ldots k$, we have $F(x) = (a/(n + 1))x^{n+1} + (b/n)x^n + \ldots C$. Equilibria can then be found by setting $F(r) = r$. This equation has n + 1 possible roots, of which the real ones such that $F(0) \leq r_e \leq 1$ are equilibria. Unity is always an equilibrium, as can be seen either from the equation or from the logic of the model; but as will be indicated, it need not be stable.

The possibility of as many as n + 1 different available equilibria means that the dynamic behavior of these systems may be quite complex and counterintuitive. The simple case of n = 2 will serve to illustrate this. Let $f = ax^2 + bx + c$, so $F = (a/3)x^3 + (b/2)x^2 + cx + d$. Since F is a c.d.f., we have $F(1) = 1$, and by inspection, $F(0) = d$. This gives $1 = (a/3) + (b/2) + c + F(0)$. To reduce the number of parameters we confine attention to the special case where f is symmetric about $x = \frac{1}{2}$—i.e., $f'(\frac{1}{2}) = 0$, hence $c = (a/6) - F(0) + 1$. We then have $F(x) = (a/3)x^3 - (a/2)x^2 +$

$+ ((a/6) - F(0) + 1)x + F(0)$. Substituting into the equation $0 = F(x) - x$ yields:

$$0 = (a/3)x^3 - (a/2)x^2 + ((a/6) - F(0))x + F(0). \quad (3)$$

This factors into $(x-1)$, $x-\frac{1}{4}-\frac{1}{4}(1 + (48F(0)/a))^{\frac{1}{2}}$, and $x-\frac{1}{4}+\frac{1}{4}(1 + 48F(0)/a)^{\frac{1}{2}}$. For any F, then, the roots of the homogeneous equation depend only on F(0) and a, and are given by

$$1,\ \tfrac{1}{4} - \tfrac{1}{4}(1 + (48F(0)/a))^{\frac{1}{2}},\ \text{and}$$

$$\tfrac{1}{4} + \tfrac{1}{4}(1 + (48F(0)/a))^{\frac{1}{2}}. \quad (4)$$

Sketches indicate that the curves, f, are sections of parabolas, and the parameter a controls the steepness and direction of the hump. For the "riot" interpretation, we may think of the value of a as indicating the level of polarization of the crowd.

Consider the case where $F(0) = .05$ and $a = -3$. Pursuing the above analysis, the three equilibrium points given by (4) are 1, .1382 and .3618; all fall within the permissible range to be equilibria. To gauge their stability, we take $F'(1) = f(1) = .45$; $f(.1382) = .8073$; $f(.3618) = 1.143$. Figures 2a and 2b show f and F for these values. The lower

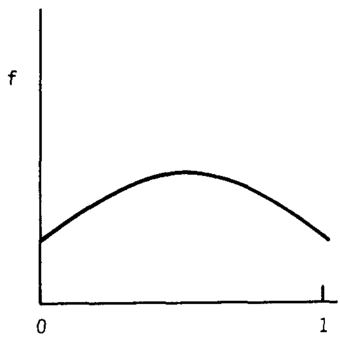

Figure 2a

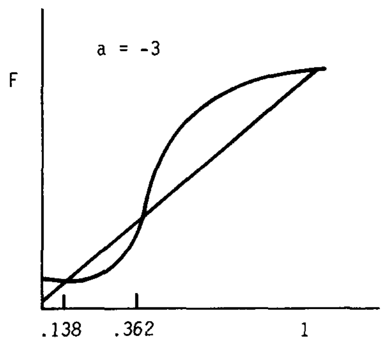

Figure 2b

and upper equilibria are stable and the middle one unstable. In general, since F is monotonic, an equilibrium occurring where F cuts the 45° line from below will be unstable, and if from above, stable.

Suppose we increase a a bit—corresponding to a slight polarization in the crowd. From (4), or the picture in Figure 3, it will be seen that this corresponds to bringing the two lower equilibria closer together. At $a = -2.4$, the square root term in (4) vanishes, and we have a double root of $\frac{1}{4}$; Figure 4 shows that this corresponds to tangency of F to the 45° line. After this point, the slightest increase in a makes the two non-unity solutions imaginary; Figure 5 shows that F now misses the 45° line in the vicinity of .25,

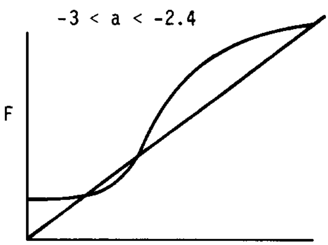

Figure 3

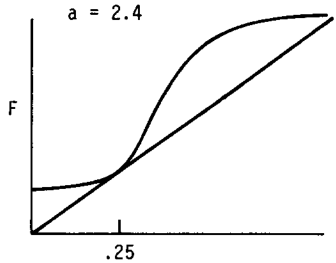

Figure 4

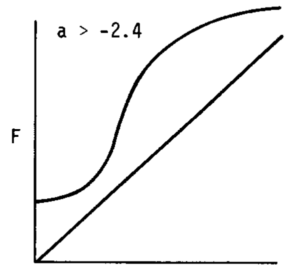

Figure 5

touching it only at 1. So this process is unstable in the vicinity of a = -2.4; a slight change in crowd composition would change the equilibrium from .25 to 1—a discontinuous jump.[3]

More generally, Figure 6 gives the various possible equilibria for this equation as a function of a. Stable

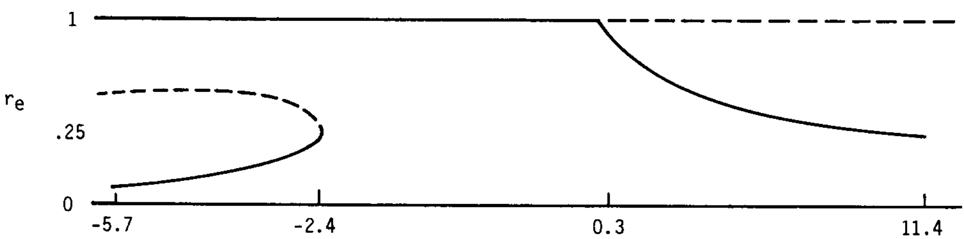

Figure 6

equilibria have their locus given by full lines, unstable ones dotted lines. The picture makes it clear not only that there is a discontinuous jump from .25 to 1, but also, and perhaps more interesting, that such a jump is irreversible. That is, if the crowd polarizes a bit, from a = -2.4 to 1 = -2.35, so that the process jumps to an equilibrium of 1, and then the crowd changes back to -2.4 or even much lower levels of polarization, the process will not return to the lower equilibrium since the higher one, unity, is stable, attracting all trajectories near it over the entire negative range of a. So the outcome of the process can be seen to depend not merely on the distribution of f, but also on the previous history of the system.

Such effects will occur where discontinuities lie in the neighborhood of multiple stable equilibria. For polynomials of general order this should occur more frequently the higher the order. Since real threshold distributions are not especially likely to be unimodal, but rather to have a series of hills and valleys, they might in practice be better approximated by polynomials of order higher than two, so that the effect indicated here could be quite typical, rather than a peculiarity.

THE DROPOUT PROBLEM: BANDWAGON VS. SNOB EFFECTS

So far we have neglected that some people drop out of any process: fatigue is often the reason—people get tired of rioting or bored with passing on a rumor. In the theory of epidemics, recovery from a disease is the equivalent state-change. In cases where the dropping out is more-or-less random, and occurs some time after equilibrium has been reached, the stability analysis outlined above should be sufficient to treat it: the dropouts can be seen as a perturbation.

There are, however, systematic reasons for dropping out. Diffusion models for innovations rarely consider one obvious and important possibility: that one adopts but later reverts to non-adopter status because of bad experience with the innovation. The usual models, moreover, blithely assume that contact of a non-adopter with one who has previously adopted always increases the chance of adoption. But where the previous adopter has reverted with (what seems to him or her) good cause, such an assumption is foolish. Women who remove IUD's as the result of bleeding will not spur their contacts to new and higher levels of adoption. When new products fail, it is often on account of word-of-mouth warnings from dissatisfied users. Thus, a certain potato chip, heavily advertised for its virtue of stacking neatly in a can, was also widely reported by its buyers to taste uncannily like cardboard. Market failure promptly followed.

This type of non-adoption could be modelled in the threshold framework, but we do not attempt to do so here. Consider instead a second type of systematic dropping out: the result of a second threshold having been reached. Some behaviors are such that people will perform them once they see some minimum number of others do so, but stop once some maximum number is passed. Such effects can be imagined for riots, for the decision about whether to remain at some social occasion, or for the wearing of clothing styles. Before wearing wide lapels, for example, most men will want to see at least a few others do so; but when even the suits in mass distribution catalogues sport this style, some may decide that things have gone too far, and return to the narrow. The economist Leibenstein (1976) calls the former "bandwagon" effects, and the latter "snob" effects; neither is common in economic analysis as they require the assumption of interdependent utilities between consumers, a possibility which, along with nonconvex preferences, is generally banished to the never-never land of "sociological influences."[4]

The formal analysis of such lower and upper thresholds goes as follows: for the lower, or bandwagon thresholds, we have the same model as above. Call the c.d.f. $F_1(x)$. In addition we have a second c.d.f. for the upper, or snob thresholds: call it $F_2(x)$. Now suppose at time = 2 we have 45% wearing wide lapels. What will we have at t = 3? Let 60% have lower thresholds less than or equal to 45%, but 25% have upper thresholds less than or equal to 45%. These

25% must be included in the 60%, since, by the obvious definition, an upper threshold cannot be smaller than a lower one. These 25% would therefore have been deactivated by $t = 3$, leaving only the balance, 35%, to wear wide lapels. In other words, those wearing the style in the next period will be people whose lower threshold, but not yet their upper has been exceeded—simply the difference between the two c.d.f.'s: $r(t + 1) = F_1(r(t)) - F_2(r(t))$. Call $F_1 - F_2 = G$. This model is then very similar to that for lower thresholds alone: $r(t + 1) = G(r(t))$. But where the F of the previous model was monotone increasing, since it was a c.d.f., G is not, which has important consequences for dynamic analysis.

Take as an example the bivariate density $24(1-x_2)(x_2-x_1)$ *where* x_1 *and* x_2 *denote lower and upper* thresholds, respectively. The marginal densities give the lower and upper threshold distributions, and by integrating them we get F_1 and F_2. Their difference, G, is $2x(x-1)^2(x+2)$—sketched in Figure 7. As in the previous model, equilibrium is given by

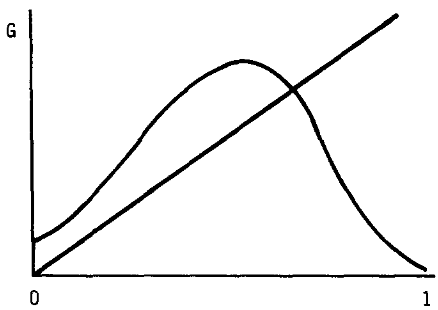

Figure 7

$G(x) = x$—the intersection of G with the 45° line, and again, the same analysis shows that it is stable if abs $(G'(r_e))$ less than unity. For this particular equation, equilibrium occurs at about .56, and $G'(.56) = -1.3$

Thus, the only available equilibrium is unstable, repelling all trajectories in its vicinity. What, then, is the dynamic behavior of such a system? Computing the difference equation through six or eight time periods shows that the process settles into a stable oscillation of period two, going back and forth between about .348 and .695. A graph of $r(t + 2)$ against $r(t)$ would show that these two values cross the 45° line with slopes less than one, indicating the stability of the oscillation. In general, May (1976) has shown that for a wide class of difference equations, namely those with a hump whose steepness is controlled by one parameter—in the present case, the 2 can be seen as one value of such a parameter—the dynamic behavior of the system is subject to successive bifurcations as the parameter is increased. That is, for some lower range of the parameter, there is one stable equilibrium. As it is increased, the hump becomes steeper, and the curve crosses the 45° line with slope greater than one, and the single solution bifurcates into two, with stable oscillation. Further increases lead again to each of the two solutions bifurcating, resulting in four solutions, with stable oscillation among these. after n such occasions, the system will be oscillating among 2^n values. But with each successive bifurcation, the "window" of parameter values within which the current oscillations occur becomes narrower, until a "critical value" of the parameters is reached, beyond which the system no longer oscillates among 2^n values, but rather shows dynamic behavior which, while some mathematical statements can be made about it, is for all intents and purposes indistinguishable from random noise. The sketch of Figure 8, while it does not

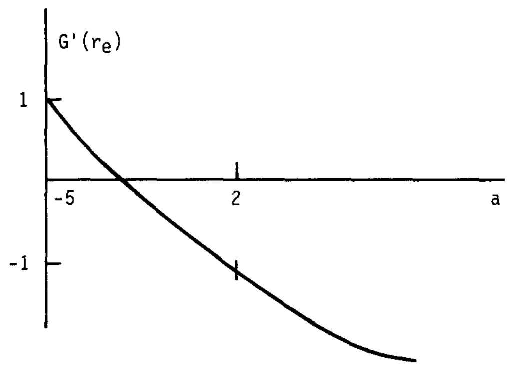

Figure 8

pinpoint this critical value, does indicate that if the initial coefficient of 2 for G is considered as a special value of a parameter a, the slope of $G'(r_e)$ does behave, as a function of a, in exactly the way required to yield this sequence of events.

The interest of this conclusion lies in the qualitative statements which may be made as a result of it. Consider, for example, an innovation which catches on quickly and has a great deal of staying power, but ultimately falls prey to snob effects. What will be the dynamic history of this item? Figures 9a, b and c indicate the situation. G will be steep, which, from the above analysis, means that we could expect wildly unpredictable oscillations in the level of adoption of this innovation. By contrast, one which catches on slowly and suffers snob effects almost immediately (Figures 10a, b, c) will reach a single stable equilibrium, since G will be flat. These conclusions seem to us not so much counter-intuitive as non-intuitive: without the formal model it is unclear what one might have expected "intuitively" for these two cases. It should be noted that the G used as an example here is a special case, as it is unimodal; there is no special reason to expect in general that the difference of two c.d.f.'s should be such, and where G has hills and valleys, dynamic analysis would be correspondingly more complex.

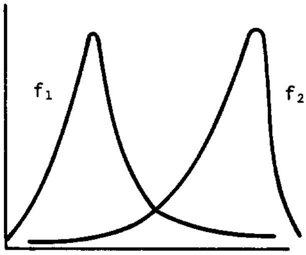

Figure 9a

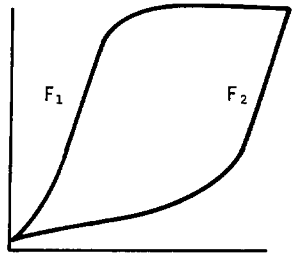

Figure 9b

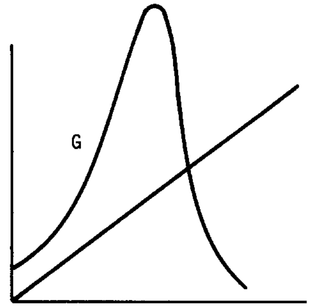

Figure 9c

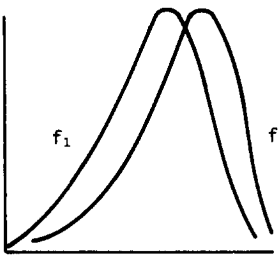

Figure 10a

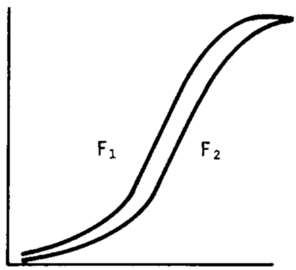

Figure 10b

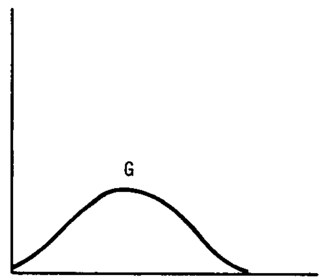

Figure 10c

FURTHER COMPLEXITIES AND ANALOGIES

As presented thus far, threshold models are structurally innocent, due to the major simplifying assumption that each individual reacts to all the others without regard to who they are. Another way of saying this is that one reacts in effect to an *average* of what others are doing. This can only be an approximation, though such approximations can at times give good results, as in mean field theories in physics for such phenomena as ferromagnetism. In practice, it is clear that except in unusual cases, one is influenced more by some individuals than by others. This can be the result of differential leadership, or "charisma," or the existence of structures of friendship. It may also be related to the structure of the situation; for example, the economist Stiglitz argues that investment in education is undertaken because without such a "screen" one would be assumed by employers to possess only the average productivity of the unscreened. Thus, when another individual purchases the screen and thereby removes himself from among the unscreened, he changes that average. A very high productivity person does so by more than a lower productivity person would, and his influence on whether you then do so would be correspondingly different (Stiglitz, 1975).

Whatever the source of differential influence, it can always be handled by brute force: construct an nxn sociomatrix, and for each entry, ij, indicate how much i's behavior is to be weighted in determining j's. Any number of assumptions are possible: e.g., those unknown to j may be assigned influence 0 and those known, 1. Or those not known can be assigned 1 and those known some number greater than or equal to 1, depending on the relationship. Using such a matrix, one can proceed by forward recursion to determine an equilibrium. (For details, see Granovetter, 1978, p. 1429.) While this method works, it is not illuminating in general. A better strategy would be to think of the sociomatrix as being characterized by a few summary parameters, and think of such social structures as perturbations acting to modify the equilibrium one would have in the underlying threshold distribution with no such structure impinging on it. One can then begin with various distributions and ask how different social structures affect their equilibria; this makes the sociometric question into a sub-case of stability analysis. Some results are reported in Granovetter (1978, p. 1430), but much of this work is still in progress.

An alternate approach to this complexity would be to use a spatial model: to assume that the direct influence of others on each individual varies with the distance of others from him. Distance here could be merely actual Euclidean distance: then such a model might be useful in analyzing large riot situations such as that occurring in the Watts section of Los Angeles in 1965 (see Stark et al., 1974). The "distance" could also be indexed by more complex metrics, indicating some sort of network distance or distance as measured by some scaling procedure. For such an approach, analogies to models in physics are surprisingly apt. Where, for example, distances are arranged in two dimensions, one has a situation quite close to that depicted by the two-dimensional Ising model of ferromagnetism. This model abstracts away from complex atomic structure and simply postulates a lattice with spin vectors occupying each lattice-point. The vectors may be oriented either up or down, and the overall magnetization is determined by the excess of up spins over down. Furthermore, whether any particular spin is up or down is in part determined by the orientation of nearby spins, as the total energy of adjacent spins is less when they are parallel. The Ising model simplifies the situation by suggesting that each spin is affected only by its four nearest neighbors. Nevertheless, the situation is remarkably difficult to analyze as a disturbance in any part of the lattice will, because of the uniform connectivity of the structure, propagate over wide areas. Ising models, which can also be used for phase transitions if one substitutes for up and down spins the presence or absence of a molecule at lattice-points, have recently become of increasing interest in physics, as new methods have been developed which promise to solve some of the previously intractable problems involved in finding or explaining the empirically observed equilibria and sharp discontinuities typical of magnetic and phase transition research. (See, e.g., Wilson, 1979; Nelson, 1977). It remains to be seen how easily these methods, such as those of "renormalization theory" (Wilson, 1979) will be adaptable to the complexities of threshold models.

Particularly intriguing is the finding of renormalization theory that as the number of dimensions in these models increases, the results of mean-field theory become better approximations, and in four or more dimensions, those results become exact (Wilson, 1979, pp. 173-177). This suggests that for sociometric structures where only two dimensions are required to portray reasonably the "distances" among individuals, threshold models will not make good predictions of outcomes; but where as many as four are required, these models, which might seem especially to be crude approximations in so complex a structure, will paradoxically give excellent results. In the physical case, this occurs, roughly speaking, because the number of nearby neighbors in four or more dimensions is so great that it is approximately the same as the average in the whole structure. But as Wilson remarks, it "remains a mystery...why four...should mark a sharp boundary above which the mean-field exponents are exact." (1979, p. 177). Considerably more work will be needed to give the above statements analytical precision. But it is not surprising that models in biology, physics and sociology should find certain common formal ground. All deal with units which may often be in binary "choice" situations, and where the choice of one unit depends in part on that of some others in some relationship to it. In biology, the work especially of May has sparked a new interest in discontinuities and critical points; in physics, these have always been important matters, but recent analytical breakthroughs have led to a substantial revival of interest. In sociology, linear models have dominated not only because they are tractable in execution, but more importantly, because they correspond to a basic sociological intuition that the size of effects must invariably be proportional to the size of causes. If threshold models do no more than shake sociology out of this unreasonable assumption, which rules out the possibility of surprises in social life—even though surprises are always confounding our expectations—they will have served a useful purpose.

PROBLEMS OF DATA ANALYSIS IN THRESHOLD MODELS

Is there any relation between threshold models and available real-world data? At a fairly descriptive level, some interesting observations can be made. For any set of data where binary behavior *and* the exact time-sequence of decisions is recorded (easier said than found) we have immediately, by definition, a distribution of "observed thresholds." If, for example, we had month-by-month data from various Korean villages, over a ten-year period, indicating for each woman at risk of pregnancy whether she was using contraceptive techniques, we could take the number of adopters before any particular woman as an estimate of her threshold of adoption. From the shape of the resulting c.d.f., we could judge whether the distribution was a stable one, and thus "overdetermined," or whether it was one in which a slight change in thresholds would have resulted in a very different outcome. Comparing distributions from village to village would allow us to judge whether inter-village differences in adoption were due to differences at the level of individual characteristics—as would be the case if high adoption villages had different sorts of women than did low-adoption ones—or whether the variation is related to instability of threshold distributions. Ten villages, for example, might have nearly identical but unstable distributions; one could expect then that the ten would vary widely in adoption levels, even if the women in each village were quite similar to those in each other one.

All this is in the way of post-diction. Is prediction possible? It would be if we had some notion of what the determinants of thresholds were, a question not so far addressed here (although see Granovetter, 1978, pp. 1420-1421 for justification of this inattention). Given a data set with observed thresholds, and substantial other data about the individuals, one could imagine constructing regression procedures to estimate determinants of thresholds. One could then take the estimated coefficients to some similar setting, not yet examined, and see whether the predicted threshold distribution for that setting, and the corresponding predicted adoption-level, were more accurate than that obtained by methods focussing more on individual characteristics. In the case of twenty-five Korean villages (a real data-set, collected by the Seoul National University School of Public Health in 1973, and which we are now analyzing) one could use coefficients estimated in certain villages to predict the situation in others. A jackknife technique would be to estimate coefficients in all twenty-five subsets of twenty-four villages, using these coefficients to estimate the situation in the twenty-fifth. Various issues arise which are not trivial to resolve: e.g., the data-set is "censored, in the sense that women who, at the time of the study had not adopted, might have ultimately done so. Their apparent threshold is 100%, but this is in most cases not an accurate reflection of their "real" threshold. The presence of sociometric data (quite rich in this Korean data set) also introduces complex problems.

In general, data sets concerning diffusion of innovations are promising sources for analysis with these models. Other types of data may also be tractable: movies of riots could be analyzed frame-by-frame; conformity experiments, and studies of "risky-shift," from experimental social psychology, could bear detailed examination in these terms. There may be ways to adapt stock-market data, or statistics on epidemics to this purpose. We are presently seeking out a variety of data sets, and welcome suggestions about which would be relevant, and how they can be gotten. There is much more to say about empirical analysis in such models, but we defer this to future papers.

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NOTES

1. The present paper, though self-contained, is an extension and report of further progress on the models reported in the May, 1978 *American Journal of Sociology* article, "Threshold Models of Collective Behavior."

2. For a comparison of threshold models with other traditions in sociological theory, and with game theory, see Granovetter, 1978, pp. 1420-1421, 1433-1437.

3. This jump is similar to those discussed in "catastrophe theory," but since the functions here are non-negative, it does not appear possible to fit the "cusp" catastrophe to this model.

4. For a systematic analysis of the impact of lower and upper thresholds on consumer demand, see Granovetter (1983).

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