

CameraML

Inpainting and Outpainting with Diffusion Models

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Outline

- Motivation
- Diffusion Models for unconditional image generation
- Unsupervised Inpainting and Outpainting:
 - Denoising Diffusion Probabilistic Models (DDPM) with optional constraints
 - Diffusion GAN and Wavelet Diffusion Models
- Supervised Inpainting and Outpainting
- Free-Size Inpainting and Outpainting
 - Denoising Diffusion Null-Space Models (DDNM) with Mask-Shift Restoration and Hierarchical Restoration
 - Future work: Diffusion models + GAN

Motivation

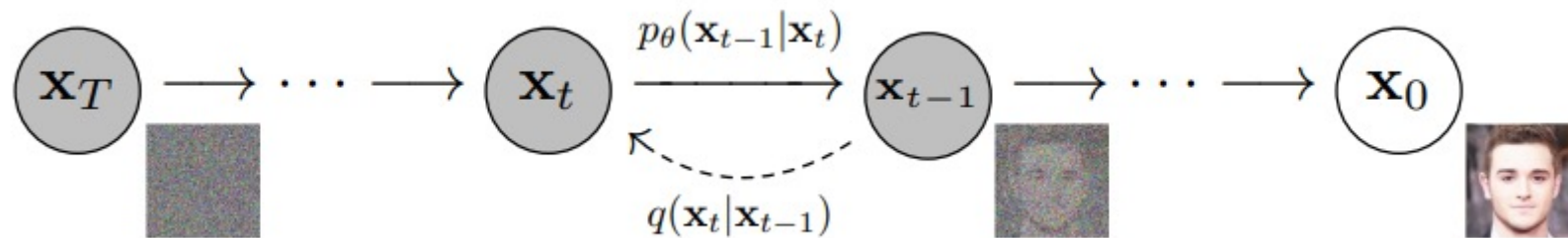
- Inpainting and Outpainting are essential to computational photography.
- The goal is to fill the unknown target region of an image.



Diffusion Models

[\[2006.11239\] Denoising Diffusion Probabilistic Models \(arxiv.org\)](#)

By learning the reverse process of degrading clean image, diffusion models can generate high-quality and diverse images from random noise.

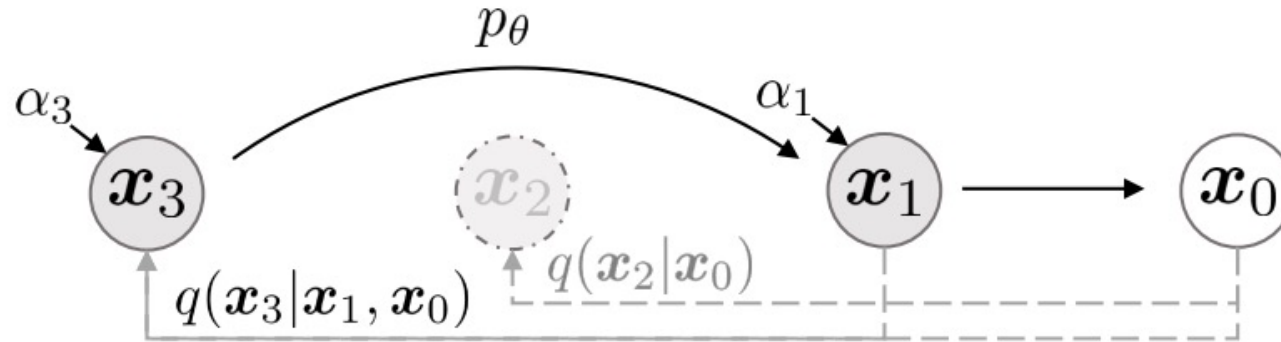


Denoising Diffusion Probabilistic Model (DDPM)

Denoising Diffusion Implicit Models (DDIM)

[2010.02502.pdf \(arxiv.org\)](#)

- A well-trained DDPM with T steps can generalize to sampling with τ steps (a subsequence of T steps), since we only consider the mapping from x_T to x_0 .
- Thus, we can skip-sampling by rescaling the denoising strength.
- It also provides a deterministic sampling approach.



Evaluation Metrics for Unconditional Generation

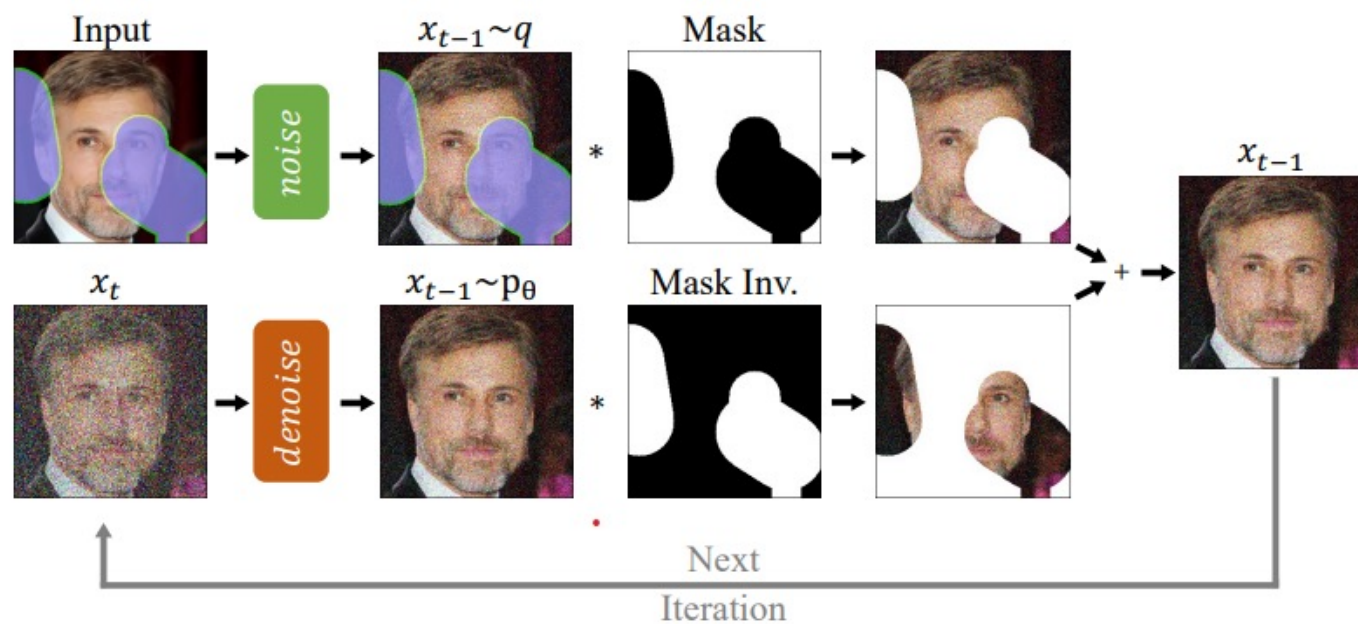
- LPIPS: a learned distance metric based on the deep feature space.
- FID/sFID/KID: 2-Wasserstein distance.
- Precision and Recall: Classification results of K-NN.
- Inception score: Inception-V3 classification statistics.
- CA: classification accuracy of a pretrained model.
- VOTES: human

Unsupervised Inpainting and Outpainting

Repainting

[\[2201.09865\] RePaint: Inpainting using Denoising Diffusion Probabilistic Models \(arxiv.org\)](#)

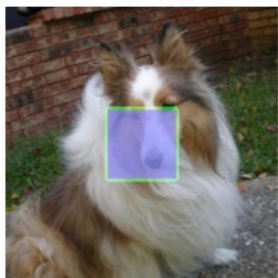
- Training a diffusion model could takes weeks and require multiple powerful GPUs, e.g., A100.
- By modifying the inference process, we can utilize a pretrained unconditional image generation model (e.g., DDPM) for inpainting and outpainting without further training.



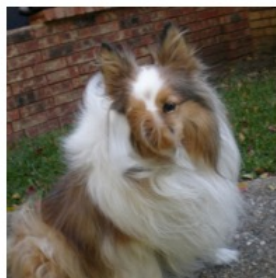
Repainting

[\[2201.09865\] RePaint: Inpainting using Denoising Diffusion Probabilistic Models \(arxiv.org\)](#)

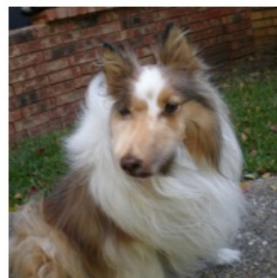
- The generated results will be correct considering the texture but wrong considering the content with single reverse pass.
- By travel back to the previous time step n times during sampling, the content will be also correct.



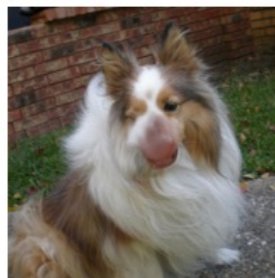
Input



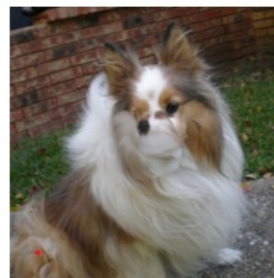
$n = 1$



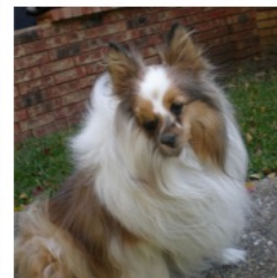
$n = 2$



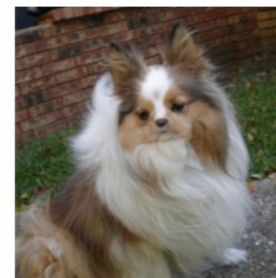
$n = 3$



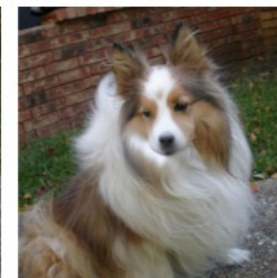
$n = 4$



$n = 5$



$n = 10$

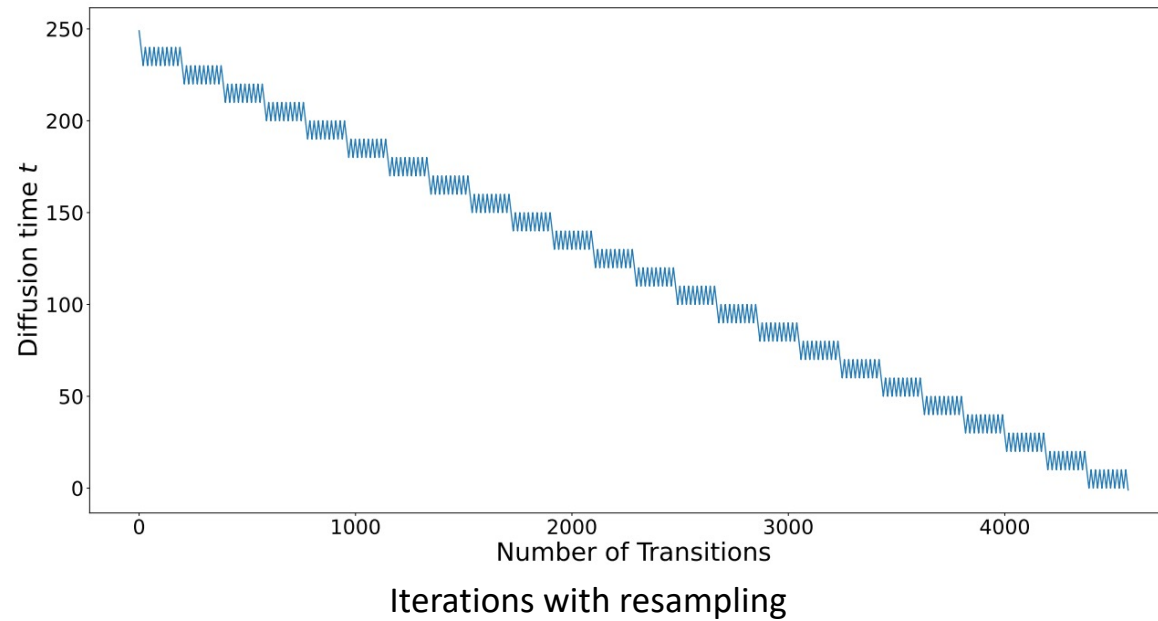


$n = 20$

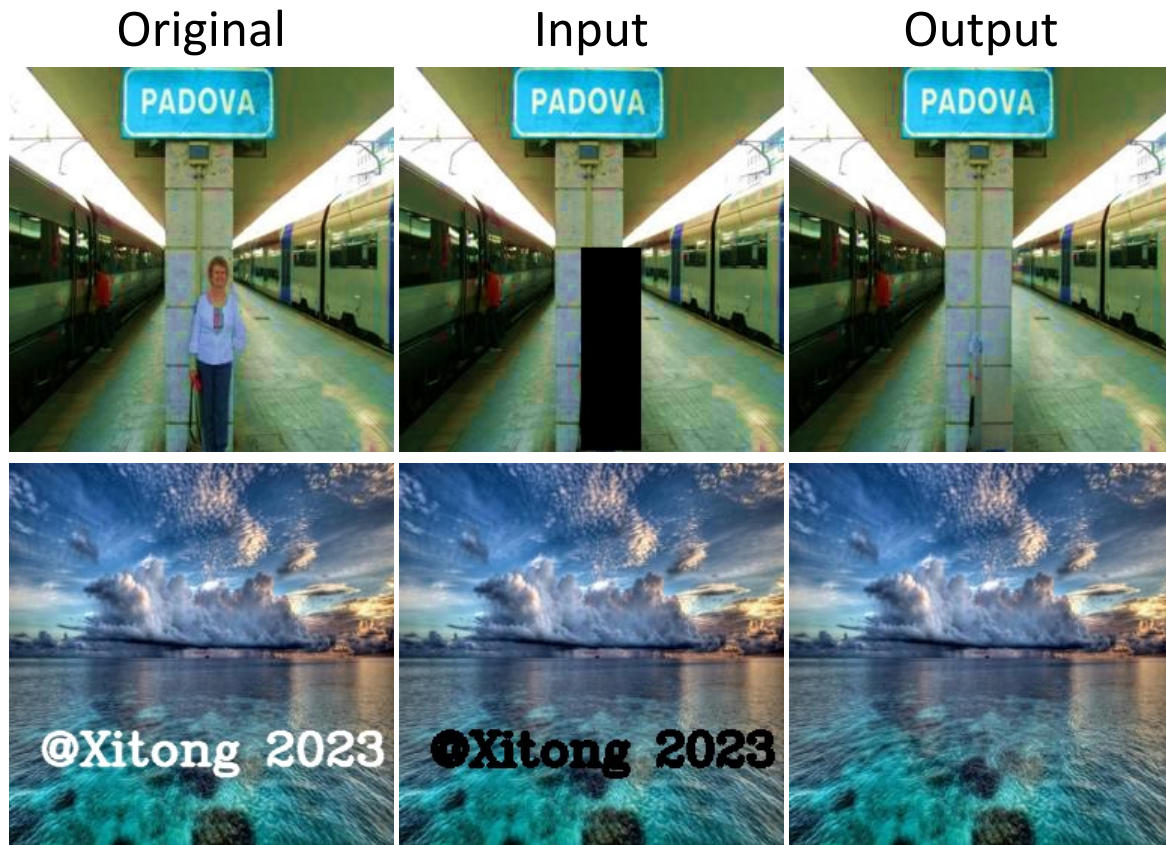
Repainting

[\[2201.09865\] RePaint: Inpainting using Denoising Diffusion Probabilistic Models \(arxiv.org\)](#)

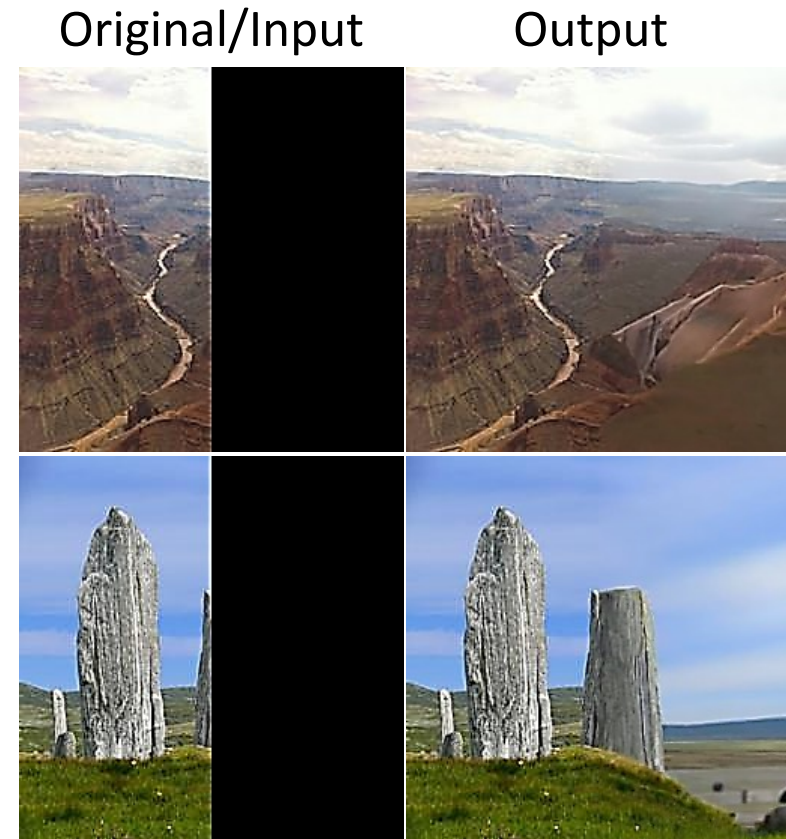
- Resampling (traveling forward and backward) is time consuming.
- It takes over 4000 steps for one 256*256 image.



Repainting Results



inpainting



outpainting

Classifier Guidance sampling

[2105.05233.pdf \(arxiv.org\)](#)

- Perturb the output towards including more information of the target class.
- p_ϕ is trained on the same noising distribution as the corresponding diffusion model.
- The classifier guidance can also be replaced by the gradient of other functions, e.g., total variation (TV).

Algorithm 1 Classifier guided diffusion sampling, given a diffusion model $(\mu_\theta(x_t), \Sigma_\theta(x_t))$, classifier $p_\phi(y|x_t)$, and gradient scale s .

Input: class label y , gradient scale s
 $x_T \leftarrow$ sample from $\mathcal{N}(0, \mathbf{I})$
for all t from T to 1 **do**
 $\mu, \Sigma \leftarrow \mu_\theta(x_t), \Sigma_\theta(x_t)$
 $x_{t-1} \leftarrow$ sample from $\mathcal{N}(\mu + s \underbrace{\Sigma \nabla_{x_t} \log p_\phi(y|x_t)}_{\text{Classifier guidance}}, \Sigma)$
end for
return x_0 Guidance strength Classifier guidance

Variants of Repaint

Improving Diffusion Models for Inverse Problems

[\[2206.00941\] Improving Diffusion Models for Inverse Problems using Manifold Constraints \(arxiv.org\)](#)

- We can avoid the resampling process by the gradient guidance of MCG.
- However, computing the gradient is expensive and time consuming.

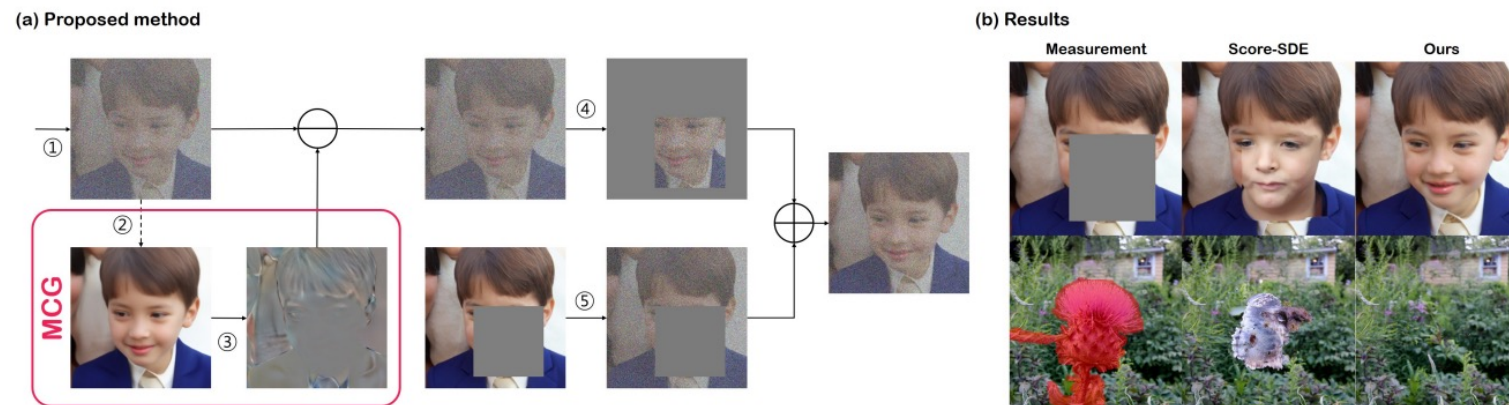


Figure 1: Visual schematic of the MCG correction step. (a) ① Unconditional reverse diffusion generates x_i ; ② Q_i maps the noisy x_i to generate \hat{x}_0 ; ③ Manifold Constrained Gradient (MCG) $\frac{\partial}{\partial x_i} \|\mathbf{W}(\mathbf{y} - \mathbf{H}\hat{x}_0)\|_2^2$ is applied to fix the iteration on manifold; ④ Takes the orthogonal complement; ⑤ Samples from $p(\mathbf{y}_i|\mathbf{y})$, then combines $\mathbf{A}x'_{i-1}$ and \mathbf{y}_i . (b) Representative results of inpainting, compared with score-SDE [41]. Reconstructions with score-SDE produce incoherent results, while our method produces high fidelity solutions.

Pseudoinverse-Guided Diffusion

[pdf \(openreview.net\)](#)

The problem-specific score can be decomposed via Bayes' rule:

$$\nabla_{\mathbf{x}_t} \log p_t(\mathbf{x}_t|\mathbf{y}) = \nabla_{\mathbf{x}_t} \log p_t(\mathbf{x}_t) + \nabla_{\mathbf{x}_t} \log p_t(\mathbf{y}|\mathbf{x}_t),$$

where the first term can be approximated with the score network $S_\theta(\mathbf{x}_t; \sigma_t)$ (Vincent, 2011), and the second term is a *guidance* term which is the score of $p_t(\mathbf{y}|\mathbf{x}_t)$.

$$y = Hx_0 + n, n \sim N(0, \sigma_y)$$

$$p_t(\mathbf{x}_0|\mathbf{x}_t) \approx \mathcal{N}(\hat{\mathbf{x}}_t, r_t^2 \mathbf{I}) \quad \hat{\mathbf{x}}_t \text{ the estimated } x_0 \text{ (DDIM)} \quad r_t \text{ depends on } \sigma_t$$

Pseudoinverse-Guided Diffusion

[pdf \(openreview.net\)](#)

Our next step is to approximate the score of $p_t(\mathbf{y}|\mathbf{x}_t)$. Since the measurement model obtains \mathbf{y} by performing a linear transform on \mathbf{x}_0 and adding independent Gaussian noise (Eq. 2), and $p_t(\mathbf{x}_0|\mathbf{x}_t)$ is Gaussian under our approximation (Eq. 4), the distribution of \mathbf{y} conditioned on \mathbf{x}_t is also Gaussian under our approximation, as follows:

$$p_t(\mathbf{y}|\mathbf{x}_t) \approx \mathcal{N}(\mathbf{H}\hat{\mathbf{x}}_t, r_t^2 \mathbf{H}\mathbf{H}^\top + \sigma_y^2 \mathbf{I}). \quad (6)$$

Thus, we have the following approximation to the score²:

$$\nabla_{\mathbf{x}_t} \log p_t(\mathbf{y}|\mathbf{x}_t) \approx \left(\underbrace{(\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}_t)^\top (r_t^2 \mathbf{H}\mathbf{H}^\top + \sigma_y^2 \mathbf{I})^{-1} \mathbf{H}}_{\text{vector}} \underbrace{\frac{\partial \hat{\mathbf{x}}_t}{\partial \mathbf{x}_t}}_{\text{Jacobian}} \right)^\top. \quad (7)$$

This is a vector-Jacobian product and can be computed with backpropagation.

Pseudoinverse-Guided Diffusion

[pdf \(openreview.net\)](#)

In many cases, we have that $\sigma_{\mathbf{y}} = 0$, and thus, Eq. 7 can be simplified to:

$$\nabla_{\mathbf{x}_t} \log p_t(\mathbf{y}|\mathbf{x}_t) \approx r_t^{-2} ((\mathbf{H}^\dagger \mathbf{y} - \mathbf{H}^\dagger \mathbf{H} \hat{\mathbf{x}}_t)^\top \frac{\partial \hat{\mathbf{x}}_t}{\partial \mathbf{x}_t})^\top; \quad (8)$$

$$\nabla_{\mathbf{x}_t} \log p_t(\mathbf{y}|\mathbf{x}_t) \approx r_t^{-2} ((h^\dagger(\mathbf{y}) - h^\dagger(h(\hat{\mathbf{x}}_t)))^\top \frac{\partial \hat{\mathbf{x}}_t}{\partial \mathbf{x}_t})^\top, \quad (9)$$

which generalizes the linear case (Eq. 8) when $h(\mathbf{x}) = \mathbf{H}\mathbf{x}$ and $h^\dagger(\mathbf{x}) = \mathbf{H}^\dagger \mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^n$.

Table 1: Comparison of different guidance methods.

Guidance	Expression	$\mathbf{x}_t \rightarrow \mathbf{y}$ differentiable	Train on $(\mathbf{x}_t, \mathbf{y})$	Noisy \mathbf{y}
Classifier	$\nabla_{\mathbf{x}_t} \log q(\mathbf{y} \mathbf{x}_t)$	Required	Yes	-
Reconstruction	$\nabla_{\mathbf{x}_t} \ \mathbf{y} - \mathbf{H} \hat{\mathbf{x}}_t\ _2^2$	Required	No	No
Pseudoinverse	Eqs. 7 to 9	Not required	No	Yes

Diffusion Null-space Model

[Unlimited-Size Diffusion Restoration \(thecvf.com\)](https://thecvf.com)

- Assuming $y = Ax + n, n = 0$ or $n \sim N(0, \sigma_y)$, A is a known transformation and y is the observation, then the reverse process solves an inversion task.
- DDNM = Repaint + correction

Algorithm 1 Sampling of DDNM

```

1:  $\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ 
2: for  $t = T, \dots, 1$  do

3:    $\mathbf{x}_{0|t} = \frac{1}{\sqrt{\bar{\alpha}_t}} (\mathbf{x}_t - \mathcal{Z}_\theta(\mathbf{x}_t, t) \sqrt{1 - \bar{\alpha}_t})$ 
4:    $\hat{\mathbf{x}}_{0|t} = \mathbf{A}^\dagger \mathbf{y} + (\mathbf{I} - \mathbf{A}^\dagger \mathbf{A}) \mathbf{x}_{0|t}$ 
5:    $\mathbf{x}_{t-1} \sim p(\mathbf{x}_{t-1} | \mathbf{x}_t, \hat{\mathbf{x}}_{0|t})$ 
6: return  $\mathbf{x}_0$ 
  
```

Algorithm 2 Sampling of DDNM⁺

```

1:  $\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ 
2: for  $t = T, \dots, 1$  do
3:    $L = \min\{T - t, l\}$ 
4:    $\mathbf{x}_{t+L} \sim q(\mathbf{x}_{t+L} | \mathbf{x}_t)$ 
5:   for  $j = L, \dots, 0$  do
6:      $\mathbf{x}_{0|t+j} = \frac{1}{\sqrt{\bar{\alpha}_{t+j}}} (\mathbf{x}_{t+j} - \mathcal{Z}_\theta(\mathbf{x}_{t+j}, t+j) \sqrt{1 - \bar{\alpha}_{t+j}})$ 
7:      $\hat{\mathbf{x}}_{0|t+j} = \mathbf{x}_{0|t+j} - \Sigma_{t+j} \mathbf{A}^\dagger (\mathbf{A} \mathbf{x}_{0|t+j} - \mathbf{y})$ 
8:      $\mathbf{x}_{t+j-1} \sim \hat{p}(\mathbf{x}_{t+j-1} | \mathbf{x}_{t+j}, \hat{\mathbf{x}}_{0|t+j})$ 
9: return  $\mathbf{x}_0$ 
  
```

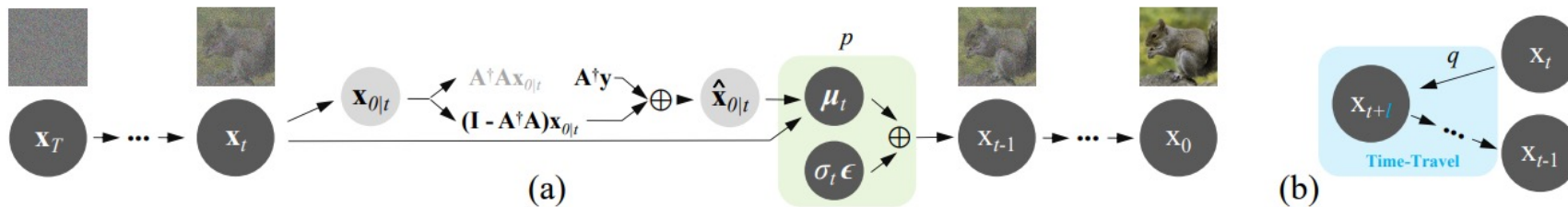


Figure 2: Illustration of (a) DDNM and (b) the time-travel trick.

Diffusion Null-space Model

[Unlimited-Size Diffusion Restoration \(thevcv.com\)](https://thevcv.com)

The pseudo-inverse should satisfy $AA^\dagger = I$, it could be solved by SVD, or

- Inpainting: $AAA = A$, so A can be the mask.
- Colorization: $A = \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]$, $A^\dagger = [1, 1, 1]^T$
- Super resolution: $A = \left[\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right]$, $A^\dagger = [1, 1, 1, 1]^T$

Plug-and-Play Image Restoration

[Denoising Diffusion Models for Plug-and-Play Image Restoration \(the cvf.com\)](https://the cvf.com)

- We want to solve this optimization problem:

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \underbrace{\frac{1}{2\sigma_n^2} \|\mathbf{y} - \mathcal{H}(\mathbf{x})\|^2 + \lambda \mathcal{P}(\mathbf{x})}_{\text{x appears both terms, unstable during optimization}} \quad \mathbf{y} = \mathcal{H}(\mathbf{x}_0) + \mathbf{n}$$

x appears both terms, unstable during optimization

↓ HQS algorithm

$$\begin{cases} \mathbf{z}_k = \arg \min_{\mathbf{z}} \frac{1}{2(\sqrt{\lambda/\mu})^2} \|\mathbf{z} - \mathbf{x}_k\|^2 + \mathcal{P}(\mathbf{z}) & (10a) \\ \mathbf{x}_{k-1} = \arg \min_{\mathbf{x}} \underbrace{\|\mathbf{y} - \mathcal{H}(\mathbf{x})\|^2 + \mu\sigma_n^2 \|\mathbf{x} - \mathbf{z}_k\|^2}_{\text{x has closed form here}} & (10b) \end{cases}$$



$\mathbf{z}_k = \mathbf{x}_k + n, n \sim N(0, \frac{\lambda}{\mu})$
 \mathbf{z}_k is a denoised version of \mathbf{x}_k



\mathbf{x}_k is a noised version of \mathbf{x}_0

Let \mathbf{z}_k be $\widehat{\mathbf{x}}_0$

Plug-and-Play Image Restoration

[Denoising Diffusion Models for Plug-and-Play Image Restoration \(the cvf.com\)](https://the cvf.com)

Algorithm 1 DiffPIR

Require: $\mathbf{s}_\theta, T, \mathbf{y}, \sigma_n, \{\bar{\sigma}_t\}_{t=1}^T, \zeta, \lambda$

1: Initialize $\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$, pre-calculate $\rho_t \triangleq \lambda \sigma_n^2 / \bar{\sigma}_t^2$.

2: **for** $t = T$ **to** 1 **do**

Solve optimization at t { 3: $\mathbf{x}_0^{(t)} = \frac{1}{\sqrt{\bar{\alpha}_t}}(\mathbf{x}_t + (1 - \bar{\alpha}_t)\mathbf{s}_\theta(\mathbf{x}_t, t))$ // Predict $\hat{\mathbf{z}}_0$ with score model as denoisor

4: $\hat{\mathbf{x}}_0^{(t)} = \arg \min_{\mathbf{x}} \|\mathbf{y} - \mathcal{H}(\mathbf{x})\|^2 + \rho_t \|\mathbf{x} - \mathbf{x}_0^{(t)}\|^2$ // Solving data proximal subproblem

x_t depends on both ϵ and x_0 ,
 ϵ should be updated when \hat{x}_0 changes { 5: $\hat{\epsilon} = \frac{1}{\sqrt{1 - \bar{\alpha}_t}}(\mathbf{x}_t - \sqrt{\bar{\alpha}_t}\hat{\mathbf{x}}_0^{(t)})$ // Calculate effective $\hat{\epsilon}(\mathbf{x}_t, \mathbf{y})$

6: $\epsilon_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$

Sampling based on DDIM { 7: $\mathbf{x}_{t-1} = \sqrt{\bar{\alpha}_{t-1}}\hat{\mathbf{x}}_0^{(t)} + \sqrt{1 - \bar{\alpha}_{t-1}}(\sqrt{1 - \zeta}\hat{\epsilon} + \sqrt{\zeta}\epsilon_t)$ // Finish one step reverse diffusion sampling

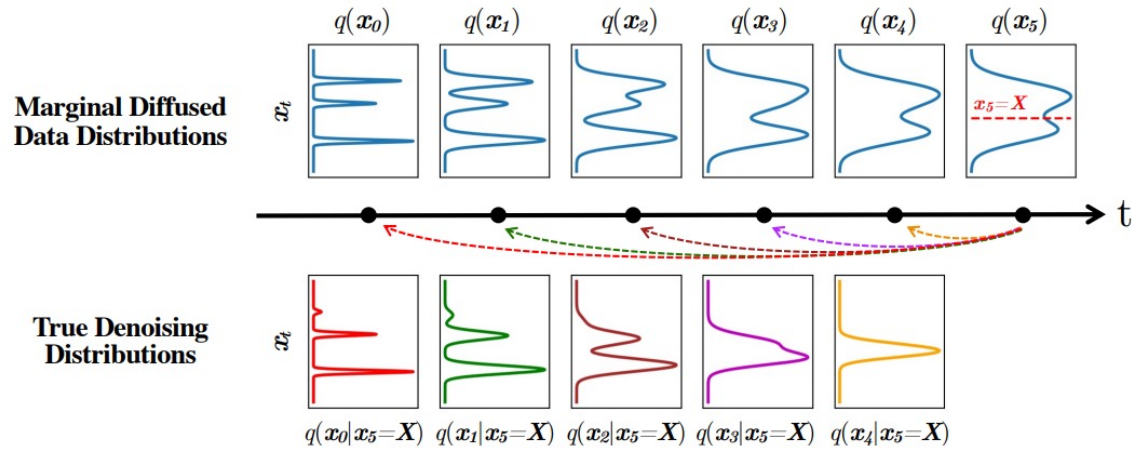
8: **end for**

9: **return** \mathbf{x}_0

Diffusion-GAN and Wavelet Diffusion

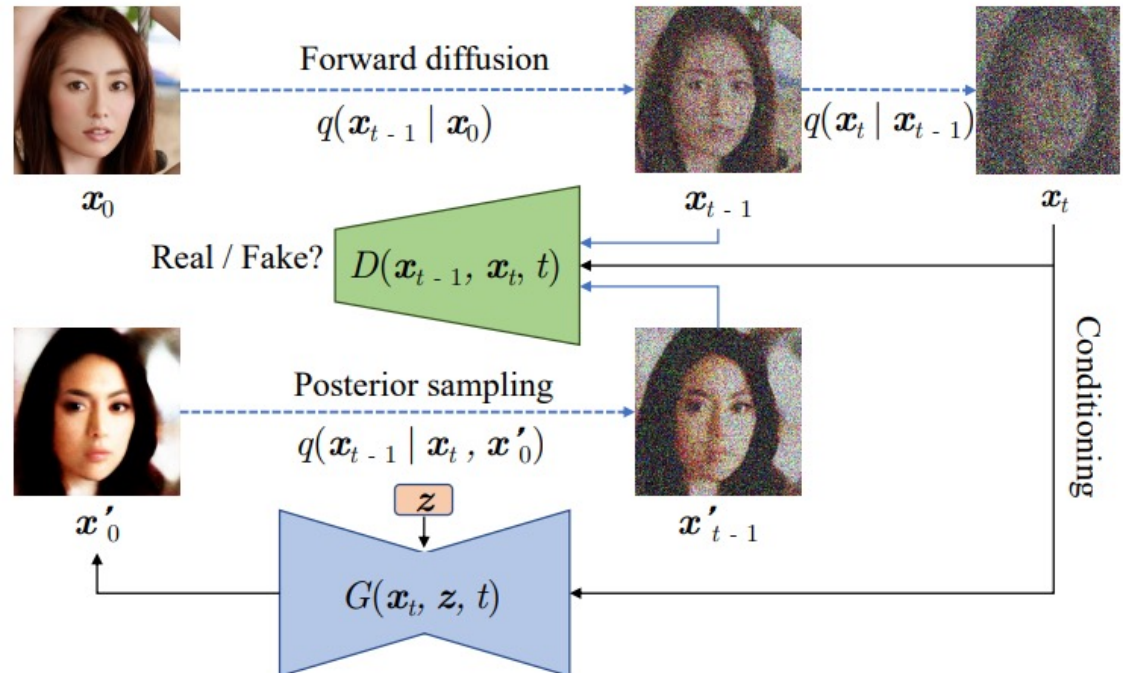
Diffusion GAN

[2112.07804.pdf \(arxiv.org\)](#)



Advantages:

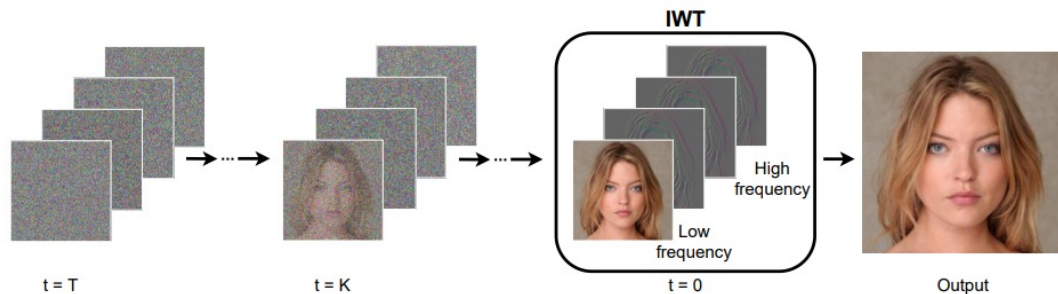
- Consistent with DDPM
- Using a single network to predict x_{t-1} directly at different t may be difficult.



Wavelet Diffusion Model

[Wavelet Diffusion Models Are Fast and Scalable Image Generators \(thecvf.com\)](https://thecvf.com)

The wavelet diffusion model is faster and smaller than the Diffusion GAN.



Adversarial objective Following [49], we optimize the generator and the discriminator through the adversarial loss:

$$\begin{aligned}\mathcal{L}_{adv}^D &= -\log(D(y_{t-1}, y_t, t)) + \log(D(y'_{t-1}, y_t, t)), \\ \mathcal{L}_{adv}^G &= -\log(D(y'_{t-1}, y_t, t)).\end{aligned}\quad (4)$$

Reconstruction term In addition to the adversarial objective in Eq. (4), we add a reconstruction term to not only impede the loss of frequency information but also preserve the consistency of wavelet subbands. It is formulated as an L1 loss between a generated image and its ground-truth:

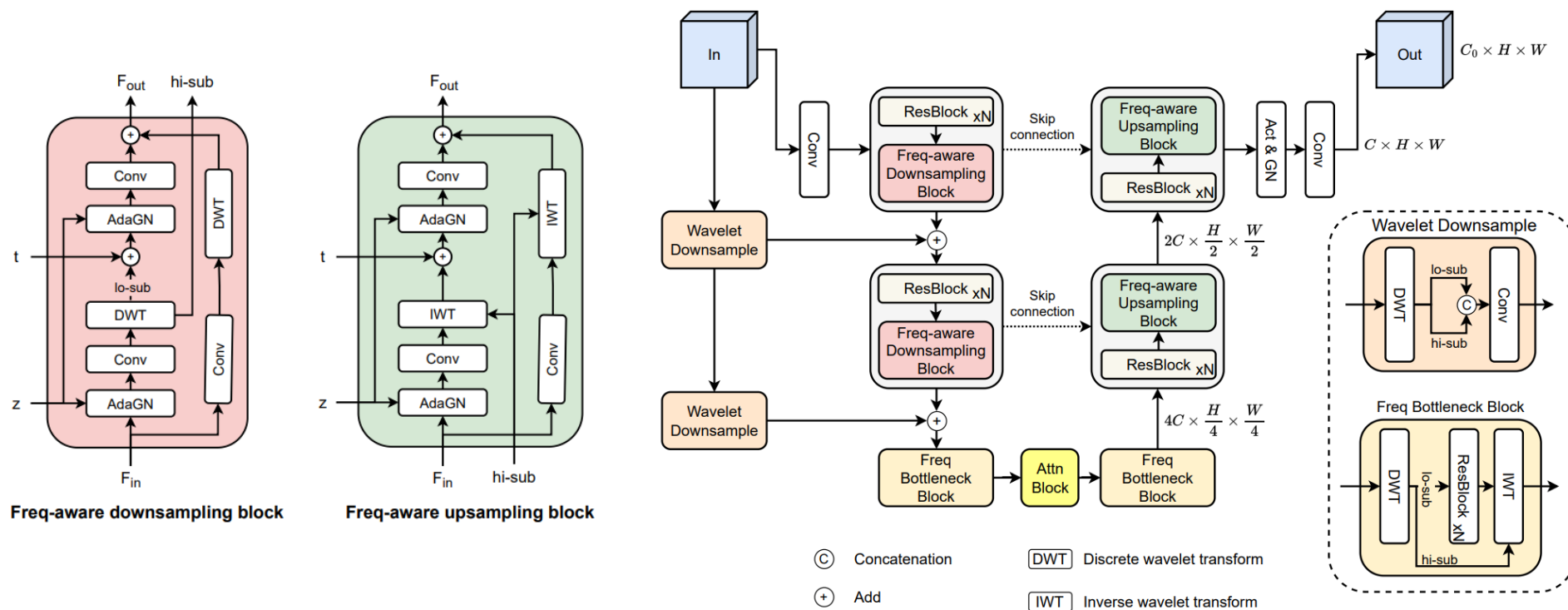
$$\mathcal{L}_{rec} = \|y'_0 - y_0\|. \quad (5)$$

The overall objective of the generator is a linear combination of adversarial loss and reconstruction loss:

$$\mathcal{L}^G = \mathcal{L}_{adv}^G + \lambda \mathcal{L}_{rec}, \quad (6)$$

Wavelet Diffusion Model

Wavelet Diffusion Models Are Fast and Scalable Image Generators (thecvf.com)



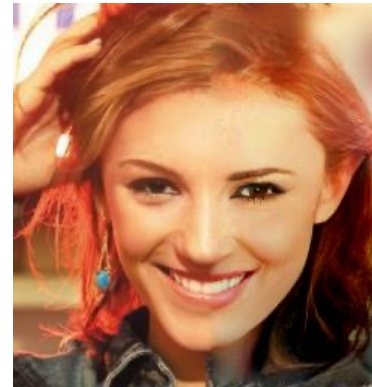
Blending

[Gradient Domain Fusion Using Poisson Blending \(brown.edu\)](http://brown.edu)

- Applying repainting on the wavelet diffusion model will generate results with the distribution shift.
- The Poisson Blending algorithm is introduced as a postprocessing step to solve the issue.



Repaint

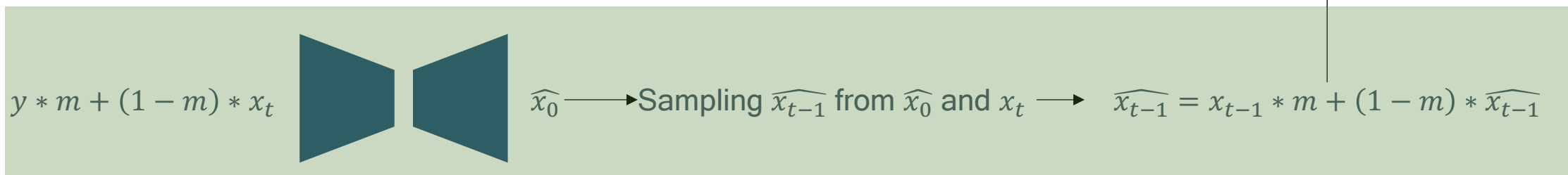
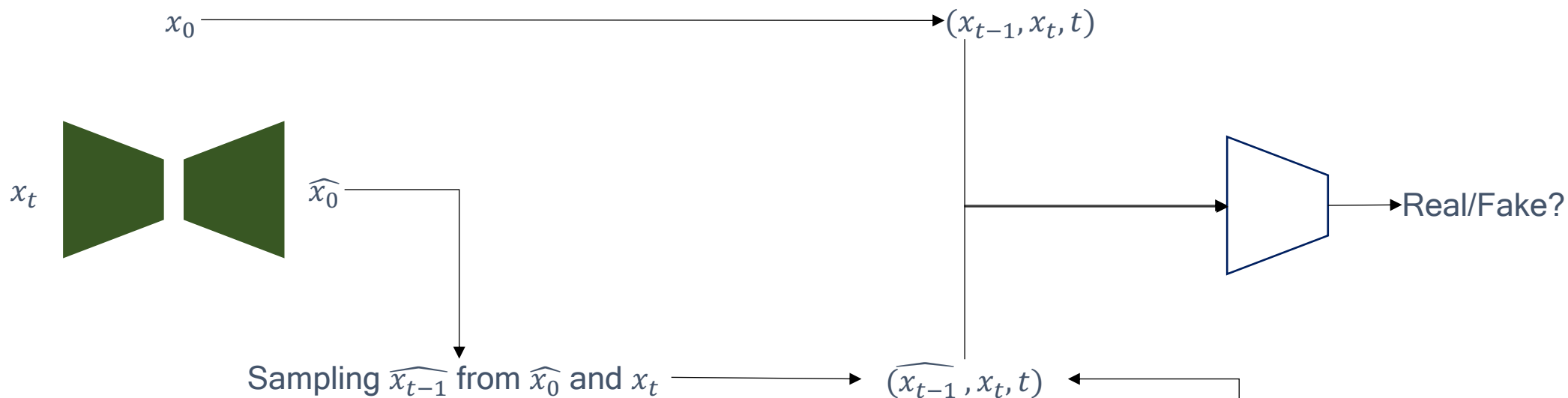


Poisson blend

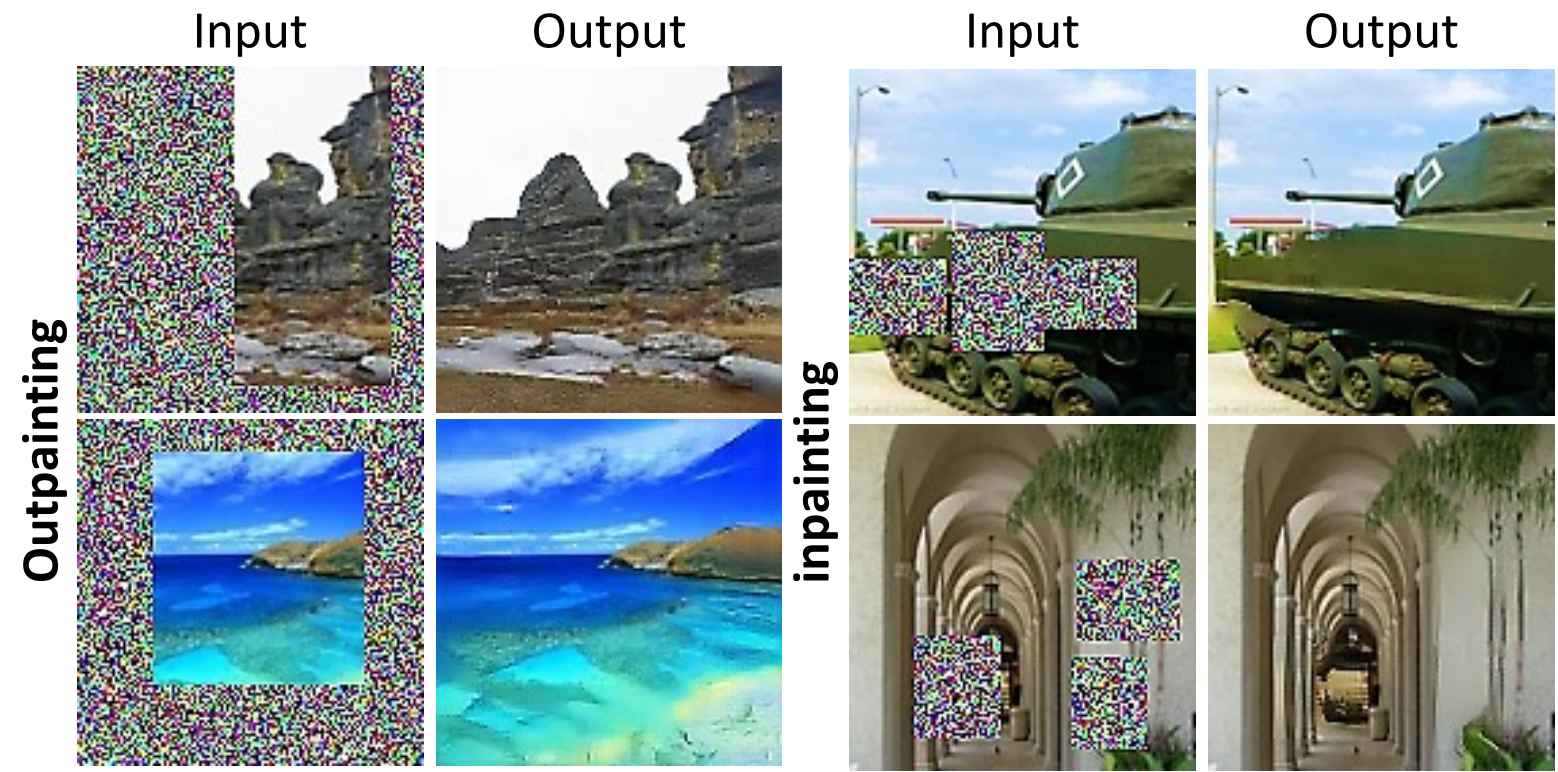
Outpainting the right half of the face.

Supervised Inpainting and Outpainting

Supervised Inpainting and Outpainting



Results



Takeaways

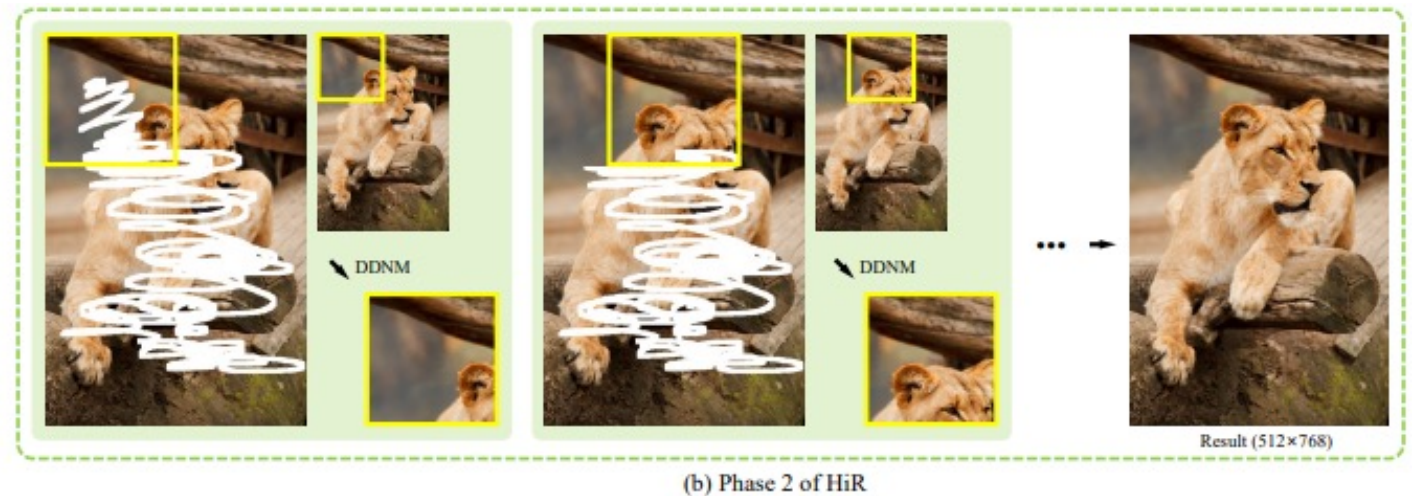
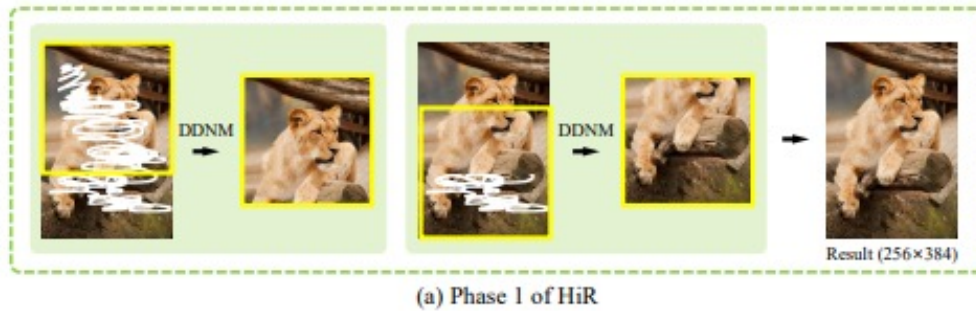
- The mask should be given to both discriminator and the generator.
- The inference results depend more on the design of the discriminators.
- Without the Haar wavelet transformation, the model fails to generate high frequency details.
- If the input of the discriminator is the Haar wavelet features, the results include more high-frequency details, but also include checker-box artifacts.
- The wavelet generator architecture does not provide high-frequency details if the input of the discriminator is the image but not the Haar wavelet feature.
- The results include high frequency details w/o the artifacts if using two discriminators, one works in the image domain and the other works in the wavelet domain.
- Reconstruction loss is important to make the training faster and stable.
- More discriminators could be incorporated, e.g., the patch-wised discriminator.
- More reconstruction loss could be considered, e.g., perceptual loss.

Free-Size Inpainting and Outpainting

Free-Size Inpainting and Outpainting

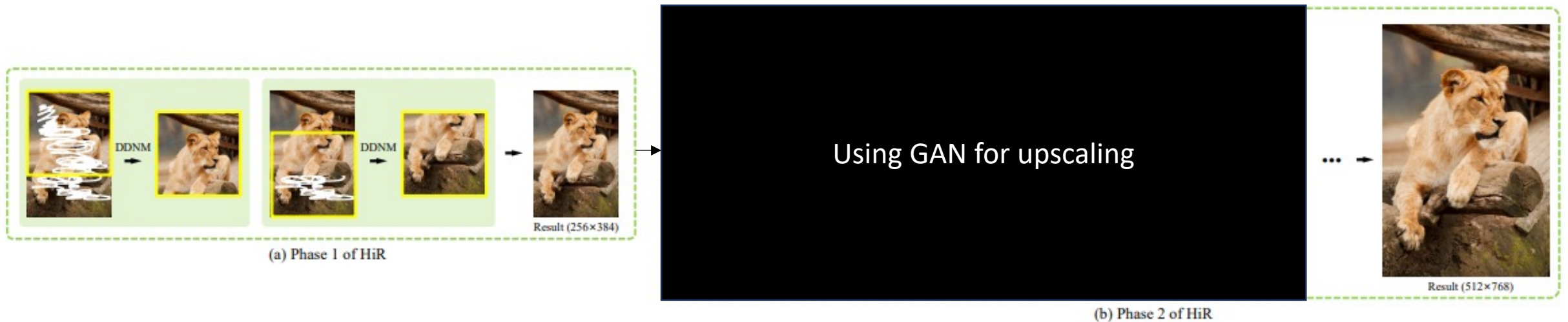
[Unlimited-Size Diffusion Restoration \(thecvf.com\)](https://thecvf.com)

To reuse a low resolution inpainting/outpainting model, we can first inpaint the low-resolution image patch by patch and then inpaint the high-resolution image condition on the low-resolution results.



Future work: Diffusion + GAN

- Using diffusion models for patch-wised inpainting/outpainting is time-consuming.
- The other option is first using the diffusion model to inpaint the low-resolution image and then upscaling the image using a GAN to the target resolution.



Data Sources

- Places365 [MIT Places Database for Scene Recognition](#)
- CelebA HQ [CelebA-HQ resized \(256x256\) | Kaggle](#)

Conclusion

- The diffusion model is a proven to generating realistic results for inpainting and outpainting.
- The results of unsupervised methods are realistic, but they are time consuming and are biased to the pretrained dataset.
- The supervised method is 1000 times faster than the unsupervised methods, but it requires a large dataset and is time consuming during training.
- The supervised method is potential to be applied on mobile devices.