UBC SAUDER
SCHOOL OF BUSINESS

CHAPTER 14

INTEREST RATES AND MORTGAGES

Learning Objectives

After studying this chapter, a student should be able to:

- ✓ Discuss the concepts of equivalent rates and effective annual rates of interest
- ✓ Calculate equivalent interest rates, payments for constant payment mortgages, outstanding balances, and other related mortgage finance calculations
- ✓ Calculate the portion of a mortgage payment allocated to principal and interest
- ✓ Discuss the concept of final payments
- ✓ Calculate and discuss the benefits of accelerated payments

EQUIVALENT INTEREST RATES

equivalent interest rates
different interest rates that result in the same effective annual interest rate, i.e., they accumulate the same amount of interest for the same loan amount over the same period of time

The previous chapter introduced the concepts of compound interest and compounding periods. To expand upon these concepts, the analyst must be able to calculate a nominal rate of interest for a particular compounding frequency that is *equivalent* to a stated nominal rate of interest for a given compounding frequency. For example, an analyst may be asked to calculate a nominal rate of interest with monthly compounding that is equivalent to a particular nominal rate of interest expressed with semi-annual compounding. Other commonly encountered examples include converting a semi-annually or monthly compounded nominal rate to an equivalent nominal rate of interest with annual compounding. The purpose of the first section of this chapter is to introduce the techniques necessary for interest rate standardization and conversion.

The basis upon which interest rate calculations are performed is stated as follows:

> Two interest rates are said to be equivalent if, for the same amount borrowed, over the same period of time, the same amount is owed at the end of the period of time.

The effective annual rate (j_1), introduced earlier, is very helpful when considering equivalency of interest rates. Because the effective rate is the annual rate with annual compounding, it does not hide any impacts from compounding within the year – in effect, this serves as the clearest way to express the true rate of interest on an annual basis. As such, the effective annual rate is often used to standardize interest rates to allow borrowers and lenders to compare different rates on a common basis.

A variation of the above statement with reference to the effective annual rate:

> If two interest rates accumulate the same amount of interest for the same loan amount over the same period of time, they have the same effective annual interest rate. Therefore, two interest rates are said to be equivalent if they result in the same effective annual interest rate.

The HP 10bII+ calculator uses the effective annual interest rate directly in its functions to convert between equivalent nominal interest rates. This relationship is demonstrated in Illustration 14.1.

Illustration 14.1

A borrower is considering interest accruing loans from two different lenders. Either loan would be for $100. Loan A accrues interest at j_{12} = 11.234%, and Loan B accrues interest at j_2 = 11.5%. Both loans have a one-year term. How much interest accrues on each loan over the one-year term?

Solution:

First, calculate how much interest will accrue with Loan A. This can be done either by solving for FV in the formula, $FV = PV \times (1 + j_m/m)^n$, or by using the financial keys on the calculator. Only the method using the financial keys will be shown, as it is quicker and is good practice for later operations. The calculation for Loan A follows:

Calculation

Press	Display	Comments
11.234 I/YR	11.234	Enter stated nominal rate
12 ■ P/YR	12	Enter stated compounding frequency
12 N	12	Enter number of compounding periods
100 PV	100	Enter present value
0 PMT	0	No payment during the term
FV	–111.830865	Total amount owed at the end of the term
+/– –100 =	11.830865	Subtract original loan amount to find amount of interest accrued

So in Loan A, $11.83 of interest accrues on a $100 loan over one year. Now perform the same calculation for Loan B.

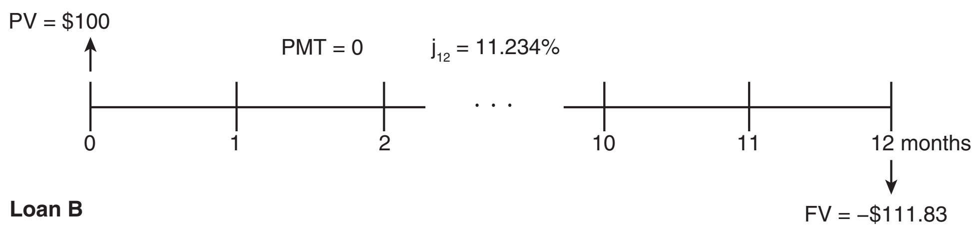
Calculation

Press	Display	Comments
11.5 I/YR	11.5	Enter stated nominal rate
2 ■ P/YR	2	Enter stated compounding frequency
2 N	2	Enter number of compounding periods
100 PV	100	Enter present value
0 PMT	0	No payments during the term
FV	–111.830625	Total amount owed at end of term
+/– –100 =	11.830625	Subtract original loan amount to find amount of interest accrued

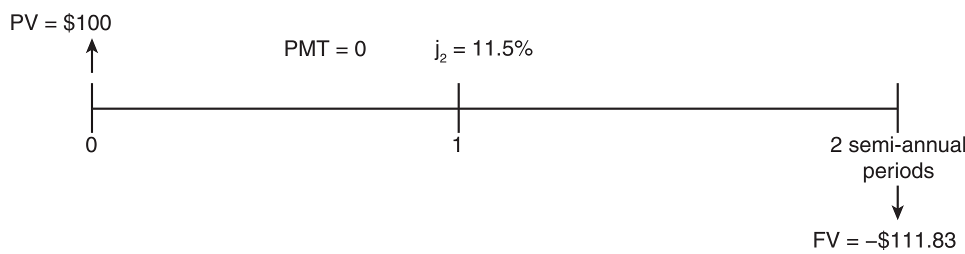
This calculation shows that $11.83 in interest also accrues on the $100 of principal over one year in Loan B. Recall the statement at the beginning of the chapter that *two interest rates are said to be equivalent if, for the same amount borrowed, over the same period of time, the same amount is owed at the end of the period of time.* From this example, it is clear that j_2 = 11.5% and j_{12} = 11.234% are equivalent interest rates, because for the same amount borrowed ($100), over the same period of time (1 year), the same amount is owed at the end of the period of time ($111.83).[1]

Consider the time diagrams for these two loans:

Loan A



Loan B

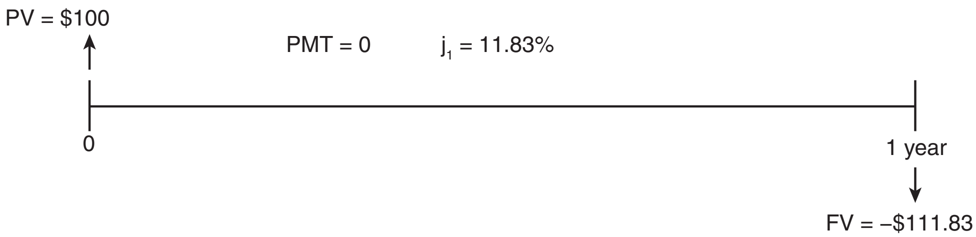


Now consider the second statement at the beginning of the chapter that *if two interest rates accumulate the same amount of interest for the same loan amount over the same time period, they have the same effective annual interest rate*. From the example, one can calculate the effective annual interest rate (j_1) on both Loan A and Loan B. The effective annual interest rate is the interest rate per annum with annual compounding. The calculations have shown us that both loans accrue $11.83 in interest over the one-year term. This amount comes due only once, at the end of the one-year term, and is therefore compounded annually. The effective annual interest rate on both loans is 11.83%.

Calculation

Press	Display	Comments
1 ■ P/YR	1	Enter compounding frequency
100 PV	100	Enter present value
111.83 +/– FV	–111.83	Amount owed at the end of the term
1 N	1	Enter number of compounding periods
0 PMT	0	No payments during the term
I/YR	11.83	Effective annual interest rate

[1] If your calculator is set to display more than six decimals, you may have noticed that these two interest rates do not accrue *exactly* the same amount of interest and thus are not *exactly* equivalent. More precise methods of finding equivalent interest rates will be shown later in the chapter.



These two loans, which accrue the same amount of interest for the same loan amount over the same time period, have the same effective annual interest rate. This proves the final statement that *two interest rates are said to be equivalent if they result in the same effective annual interest rate*. These two interest rates, $j_2 = 11.5\%$ and $j_{12} = 11.234\%$ are equivalent, and they do result in the same effective annual interest rate. The fact that equivalent interest rates have the same effective annual interest rate will be used in computing equivalent rates with the financial calculator.

There are several reasons to convert interest rates from one compounding frequency to another:

- The *Interest Act* requires that the rate of interest quoted in a mortgage contract be quoted as a nominal rate with either semi-annual or annual compounding, so interest rates quoted with any other compounding frequency must be converted to their equivalent nominal rates with semi-annual or annual compounding.
- When comparing interest rates to one another, it is necessary to compare rates that have the same frequency of compounding to accurately assess the cost of borrowing.
- When using the financial calculator to compute periodic payments under a mortgage contract, conversion is required to make the frequency of compounding for the interest rate match the frequency of the periodic payments.

There are two methods of calculating equivalent interest rates. The first involves the use of mathematical formulas, but this involves complex and time-consuming algebra. The alternative, and equally valid, approach to calculating equivalent interest rates is to use the financial keys of a business calculator.

There are two financial keys on the HP 10bII+ that have not yet been introduced but are needed for *interest rate conversion* problems. These are:

interest rate conversion
a mathematical process that changes a stated interest rate (and its associated compounding frequency) into an equivalent rate with a different compounding frequency

- ■ NOM% Nominal interest rate per year (j_m)
- ■ EFF% Effective interest rate (j_1), which is calculated based on the nominal rate (j) entered in ■ NOM% and the compounding frequency (m) entered in ■ P/YR

The interest rate conversion process involves entering both the stated nominal interest rate and its compounding frequency, and then converting the nominal interest rate into its effective annual equivalent. Then, the desired compounding frequency (which is usually the number of payment periods per year) is entered. The final step is to solve for the equivalent nominal rate with the desired compounding frequency. The interest rate conversion is usually a five-step process; an interest rate conversion template is provided for you to follow as a general guide (see Helpful Hint!).

It is important to note that the HP 10bII+ financial calculations require nominal interest rates to be entered into the financial keys. Periodic interest rates must first be expressed as nominal interest rates before they are entered into the financial keys.

HELPFUL HINT!

The following could be used as an interest rate conversion template:

? ■ NOM%	[Enter stated nominal rate for ?]
? ■ P/YR	[Enter stated compounding frequency for ?]
■ EFF%	(Compute effective annual interest rate)
? ■ P/YR	[Enter desired compounding frequency for ?]
■ NOM%	(Compute equivalent nominal rate with desired compounding frequency)

In the interest rate conversions illustrated in this manual, the first step shown is to enter the stated nominal rate using ■ NOM%. Students may notice that similar results can also be achieved by pressing I/YR alone.

Illustration 14.2

The *Interest Act* requires that a mortgage contract contain a statement indicating both the principal amount of the loan and the rate of interest charged on the principal, expressed as an annual rate of interest, with either semi-annual or annual compounding. Assume that a bank agrees to give a loan at an interest rate of 6% per annum, compounded monthly (j_{12}). To determine the rate that the bank must disclose under the *Interest Act*, calculate the nominal rate per annum, with semi-annual compounding (j_2) that is equivalent to 6% per annum, compounded monthly (j_{12} = 6%).

Solution:

Enter the given nominal rate and the stated number of compounding periods per year (12, in this case). Solve for the effective annual rate, i.e., the nominal rate with annual compounding. Then, enter the desired compounding periods (2, in this case). Solve for the equivalent nominal rate. Using the interest rate conversion template shown in the Helpful Hint, the calculator steps are as follows:[2]

Calculation

Press	Display	Comments
6 ■ NOM%	6	Enter stated nominal rate
12 ■ P/YR	12	Enter stated compounding frequency
■ EFF%	6.167781	Compute effective annual interest rate
2 ■ P/YR	2	Enter desired compounding frequency
■ NOM%	6.075502	Compute equivalent nominal rate with desired compounding frequency

The nominal rate per annum with semi-annual compounding equivalent to j_{12} = 6% is j_2 = 6.075502%. If it was necessary to calculate the periodic rate per semi-annual period, this could be done by dividing the nominal rate (j_2 = 6.075502%) by the number of compounding periods per year (2) to determine the periodic rate (i_{sa} = 3.037751%).

Illustration 14.3

Assume that a bank agrees to give a loan at an interest rate of 4% per annum, compounded semi-annually (j_2 = 4%). Calculate the equivalent nominal rate per annum with monthly compounding (j_{12}).

Solution:

Enter the given nominal rate and the stated number of compounding periods per year (2, in this case). Solve for the effective annual rate, i.e., the nominal rate with annual compounding. Then, enter the desired number of compounding periods per year (12, in this case). Solve for the equivalent nominal rate. The calculator steps are as follows:

Calculation

Press	Display	Comments
4 ■ NOM%	4	Enter stated nominal rate
2 ■ P/YR	2	Enter stated compounding frequency
■ EFF%	4.04	Compute effective annual interest rate
12 ■ P/YR	12	Enter desired compounding frequency
■ NOM%	3.967068	Compute equivalent nominal rate with desired compounding frequency

The nominal rate per annum with monthly compounding equivalent to j_2 = 4% is j_{12} = 3.967068%. If it was necessary to calculate the monthly periodic rate, this could be done by dividing the nominal rate (j_{12} = 3.967068%) by the number of compounding periods per year (12) to determine the periodic rate (i_{mo} = 0.330589%).

[2] This chapter shows the calculator steps for finding equivalent interest rates using the HP 10bII+ calculator. If you elect to use a different calculator, it is your responsibility to ensure that the alternative calculator will perform all necessary functions.

Illustration 14.4

A borrower is considering a loan that will charge interest at a nominal rate of 5% per annum, compounded monthly (j_{12} = 5%). The borrower is more familiar with interest rates that are quoted as nominal rates with semi-annual compounding (j_2). What nominal rate with semi-annual compounding is equivalent to 5% per annum, compounded monthly?

Solution:

The borrower wants to convert the interest rate of j_{12} = 5% to its j_2 equivalent. This is done by using the calculator's financial keys.

Calculation

Press	Display	Comments
5 ■ NOM%	5	Enter nominal rate with monthly compounding
12 ■ P/YR	12	Enter stated compounding frequency
■ EFF%	5.11619	Compute effective annual rate
2 ■ P/YR	2	Enter desired compounding frequency
■ NOM%	5.052374	Compute nominal rate with semi-annual compounding

This calculation shows that the nominal rate of j_2 = 5.052374% is equivalent to j_{12} = 5%.

Illustration 14.5

A borrower is considering mortgage loans from two different lenders. Lender A will loan funds at a rate of j_2 = 9.5%. Lender B will loan funds at a rate of j_{12} = 9.4%. Which of these two interest rates represents the lowest cost of borrowing?

Solution:

When comparing interest rates, it is necessary that both interest rates being compared have the same compounding frequency. Both interest rates should be converted to their equivalent effective annual interest rates so they can be compared. The calculation to convert the rate from Lender A to its equivalent effective annual rate using the financial keys on the calculator follows:

Calculation

Press	Display	Comments
9.5 ■ NOM%	9.5	Enter nominal rate with semi-annual compounding
2 ■ P/YR	2	Enter stated compounding frequency
■ EFF%	9.725625	Compute equivalent effective annual interest rate

This calculation shows that Lender A is charging an effective annual rate of j_1 = 9.725625% on funds loaned. Now, compute the effective annual interest rate charged by Lender B.

Calculation

Press	Display	Comments
9.4 ■ NOM%	9.4	Enter nominal rate with monthly compounding
12 ■ P/YR	12	Enter stated compounding frequency
■ EFF%	9.815747	Compute equivalent effective annual interest rate

This calculation shows that Lender B charges an effective annual rate of j_1 = 9.815747% on loans. By comparing the two effective annual rates, it is evident that Lender A, who is charging a nominal rate of j_2 = 9.5%, has a lower cost of borrowing than Lender B, who is charging a nominal rate of j_{12} = 9.4%. This shows the importance of converting both rates to their equivalent effective annual rates before comparing them. The rate that, at first glance, appears lower, j_{12} = 9.4%, represents a higher effective annual rate, and thus a higher cost of borrowing, than the rate that appears to be higher, j_2 = 9.5%.

The following illustration converts a monthly periodic rate to an effective annual interest rate.

Illustration 14.6

A borrower has arranged a loan to fund the construction of a new house. This loan calls for interest to be calculated at the rate of 0.5% per month, compounded monthly (i_{mo}). Since the borrower is more familiar with interest expressed as an effective annual rate (j_1), the borrower asks you to calculate the effective annual interest rate that is equivalent to 0.5% per month, compounded monthly.

Solution:

In the above illustration, the borrower is considering a contract in which interest is charged at the rate of 0.5% per month (i_{mo} = 0.5%). The borrower wants to calculate the equivalent effective annual interest rate (j_1). This will be done in two steps. First, convert the monthly periodic interest rate to a nominal interest rate with monthly compounding, and then use the financial keys on the calculator to convert this to the equivalent effective annual interest rate. Remember from the previous chapter the relationship, $j_m = i \times m$. In this case, the relationship will be used to calculate j_{12}.

$j_{12} = i_{mo} \times 12$

$j_{12} = 0.5\% \times 12$

$j_{12} = 6\%$

Now, convert the j_{12} = 6% rate to its equivalent effective annual rate using the financial keys on the calculator.

Calculation

Press	Display	Comments
6 ■ NOM%	6	Enter nominal rate with monthly compounding
12 ■ P/YR	12	Enter given compounding frequency
■ EFF%	6.167781	Compute effective annual rate

The above analysis has demonstrated that an effective annual interest rate of j_1 = 6.167781% is equivalent to a periodic rate per month of i_{mo} = 0.5%.

Exercise 14.1

Consider the following table:

1. The first column specifies a nominal rate of interest with a given compounding frequency.
2. The second column provides the desired compounding frequency.
3. The third column presents an equivalent nominal interest rate with the desired frequency of compounding.

Given the nominal rate, use the desired frequency of compounding to calculate an equivalent nominal interest rate. Confirm your calculation matches the answer shown in the third column.

Nominal interest rate	Desired number of compounding periods per annum	Equivalent nominal interest rate with desired compounding frequency
j_{12} = 5.5%	1	j_1 = 5.640786%
j_2 = 4%	12	j_{12} = 3.967068%
j_4 = 8%	2	j_2 = 8.08%
j_1 = 9%	365	j_{365} = 8.618787%
j_4 = 7.5%	12	j_{12} = 7.453607%
j_1 = 6%	12	j_{12} = 5.841061%

APPLICATION OF EQUIVALENT RATES TO PERIODIC MORTGAGE PAYMENTS

Interest rate conversions are often required when calculating the periodic payments to be made on a loan.

Typically, interest rates are quoted as an annual rate with semi-annual compounding, but borrowers prefer to make their payments monthly. The nominal rate with semi-annual compounding must be converted to its equivalent rate with monthly compounding so the amount of interest charged each month can be calculated.

Illustration 14.7 calculates the equivalent nominal interest rates for a loan using the same method as outlined in the previous illustrations.

Illustration 14.7

A borrower has arranged a loan with interest at a rate of 7% per annum, compounded semi-annually (j_2 = 7%). The loan requires monthly payments to repay the $50,000 loan amount over 25 years. To calculate the payments on this loan, the lender needs to know the nominal interest rate with monthly compounding that is equivalent to 7% per annum, compounded semi-annually. Calculate the j_{12} rate that is equivalent to j_2 = 7%.

Solution:

Calculation

Press	Display	Comments
7 ■ NOM%	7	Enter stated nominal interest rate
2 ■ P/YR	2	Enter stated compounding frequency
■ EFF%	7.1225	Compute equivalent effective annual rate
12 ■ P/YR	12	Enter desired compounding frequency
■ NOM%	6.900047	Compute equivalent nominal rate

The nominal rate with monthly compounding that is equivalent to 7% per annum, compounded semi-annually is j_{12} = 6.900047%.

As a Licensee...

As a Licensee, it is crucial that you understand the mechanics of different types of loans and the impact of compounding frequency. Failing to properly detail the specifics of a loan could result in litigation, where a court could assign a different meaning to the terms of the loan than was otherwise intended. The following is an example where such was the case.

In *Haptom v Pahl*, 2013 BCSC 71, the lender/petitioner was seeking to foreclose on a mortgage granted by the borrowers/respondents. The loan agreement provided for “the advance of a $150,000 non-revolving term facility with interest at 24% per annum”. Furthermore, the repayment clause stated that the payments would be “[i]nterest only, $3,000 per month.” It appears that the monthly payments were calculated by taking the annual total interest of 24% on $150,000 ($36,000) and simply dividing it into 12 equal payments of $3,000. Doing this ignores the fact that monthly interest payments of $3,000 does not equate to a 24% per annum interest rate, since the $3,000 payments could be invested before the year’s end to generate a return (the lender could earn interest on the interest).

The Court was faced with an ambiguous loan agreement. It had to decide which term it should enforce: either that the monthly interest payments should be at 24% per annum (at which case, the monthly payments should have only been $2,713.14), or whether the $3,000 monthly interest payments should account for interest only (at which case, the respondent borrower would be paying more than 24% per annum of interest). The Court sided with the borrower, stating:

> As I am satisfied that the reinvestment principle applies the payments made under the loan agreements by the respondents to the petitioner of $3,000 per month must be recalculated in accordance with it. That is, on the basis that the annual rate of interest agreed to was 24% per annum compounded annually with monthly payments for interest only. This will also address the second issue of recalculation of the $3,000 per month interest only payment. Any overpayment of interest is to be credited to the [respondents].

ANALYSIS OF CONSTANT PAYMENT MORTGAGES

Introduction

Real estate financing mainly involves the use of *constant payment mortgages*. Real estate licensees and mortgage brokers are often asked to determine the size of payments on a particular loan, the size of a loan that a given payment will support, or the balance owing on an existing loan. The purpose of the rest of this chapter is to review the financial calculations necessary to determine loan amounts, periodic payments (monthly or otherwise), amortization periods, interest rates, outstanding balances, and final payments. In addition, these techniques will serve as the basis for the analysis of discounted and bonused mortgage loans, which are presented in a later chapter.

constant payment mortgages
a mortgage loan that is repaid by equal and consecutive instalments that include principal and interest

There are four basic financial components in all constant payment mortgage loans:

1. **Loan Amount:** The loan amount (or face value of the mortgage) is the amount the borrower agrees to repay at the interest rate stated in the mortgage contract. In financial terms, the loan amount is the present value of the required payments.
2. **Nominal Rate of Interest:** The frequency of compounding of the nominal interest rate must match the frequency of the payments. For example, if a loan calls for interest at 10% per annum, compounded semi-annually with monthly payments, the equivalent nominal rate of interest with monthly compounding needs to be calculated.
3. **Amortization Period:** The amortization period is used to calculate the size of the required payments. The amortization period must be specified in terms of the number of payment periods, so a loan calling for monthly payments over 25 years has 300 (25 × 12) payment periods.
4. **Payment:** The constant payment required to repay the loan amount over the amortization period is calculated such that, if payments are made regularly, the last payment will repay all remaining principal as well as interest due at the end of the final payment period.

The calculator also uses a fifth piece of information, the future value. However, the future value is equal to zero when doing basic calculations for constant payment mortgages because these mortgages are always completely paid off (have a future value of zero) at the end of the amortization period.

Calculations for Constant Payment Mortgages

The financial calculator used in this course is pre-programmed to calculate loan amounts (PV), future values (FV), payments (PMT), amortization periods (N), and interest rates (I/YR). By entering any four of these variables (PV, FV, PMT, N, and I/YR), the calculator can then determine the fifth variable.

The following conditions must occur to use the calculator to analyze a constant payment mortgage:

1. The present value must occur at the *beginning* of the first payment/compounding period.
2. The payments must be equal in amount, occur at regular intervals, and be made at the *end* of each payment period.
3. The rate of interest must be stated as, or *converted to*, a nominal rate with compounding frequency matching the payment frequency.

! **ALERT**

Cash Flow

Most of the calculations in the remainder of this course are for mortgage loans. In these problems, the borrower receives loan funds at the beginning of the loan term (cash in, so a positive amount) and makes periodic payments during the loan term and an outstanding balance payment at the end of the loan term (cash out, so negative amounts). In these examples, PV will be shown as positive, while PMT and FV will be shown as negatives.

Calculation of Loan Amount

Illustration 14.8(a)

An individual is considering buying a residential condominium but wants to limit mortgage payments to \$700 per month. If mortgage rates are 5.5% per annum, compounded monthly, and the lender will permit monthly payments to be made over a 25-year amortization period, determine the maximum allowable loan.

Solution:

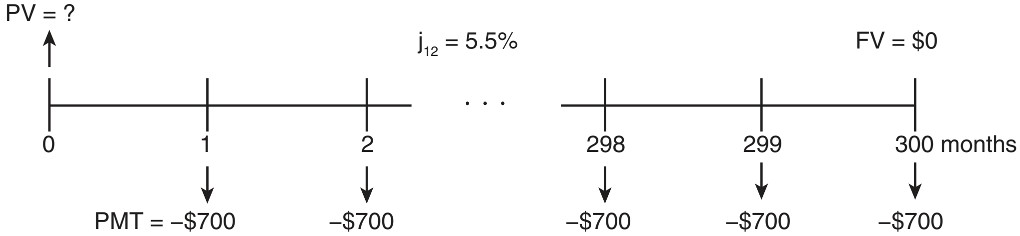
The financial terms of the proposed loan may be summarized as follows:

j_{12} = 5.5%

PMT = \$700 per month

N = 300 months (25 × 12)

PV = ?



As the frequencies of payment and compounding correspond (both are monthly), the problem may be solved directly as follows; an interest rate conversion is not required.

Calculation

Press	Display	Comments
5.5 I/YR	5.5	Nominal rate with monthly compounding
12 ■ P/YR	12	Number of payments per year
700 +/– PMT	−700	Monthly payment
25 × 12 = N	300	Months in amortization period
0 FV	0	No future value*
PV	113,990.271549	Present value (or loan amount)

* A future value amount is not used in this problem because at the end of 300 months the entire principal amount (or outstanding balance) has been repaid, making the future value of the loan zero.

The maximum loan based on the interest rate, payments, and amortization period specified, is \$113,990.27.

Illustration 14.8(b)

If the previous loan called for interest at a rate of 7% per annum, compounded monthly, determine the maximum loan amount.

Solution:

N = 300 months; j_{12} = 7%; PMT = \$700; PV = ?

Because PMT, N, ■ P/YR, and FV are already stored and do not require revision, the calculation is straightforward. Simply change the interest rate and then solve for the present value:

Calculation (continued)

Press	Display	Comments
7 I/YR	7	Nominal rate with monthly compounding
PV	99,040.83237	Loan amount at j_{12} = 7%

Therefore, increasing the interest rate from j_{12} = 5.5% to j_{12} = 7% has the effect of decreasing the maximum allowable loan by almost \$15,000 (from \$113,990.27 to \$99,040.83).

Illustration 14.8(c)

Recalculate the maximum loan, given the details from Illustration 14.8(a) and assume interest rates increased to j_{12} = 10%.

Solution:

N = 300 months; j_{12} = 10%; PMT = $700; PV = ?

Calculation (continued)

Press	Display	Comments
10 I/YR	10	Nominal rate with monthly compounding
PV	77,033.061042	Loan amount at j_{12} = 10%

The maximum loan at an interest rate of j_{12} = 10% reduces to $77,033.06.

From the preceding illustration, the rate of interest charged on a loan has a large impact on the size of the loan that a *fixed* series of payments will support. With constant payment mortgage loans, a large portion of each of the early payments is allocated to the payment of interest. Increased interest rates reduce the amount of each payment available for principal repayment, making a significant impact on an individual's ability to borrow a given amount. This topic is addressed further in Chapter 17, "Mortgage Underwriting and Borrower Qualification".

Many borrowers and lenders, particularly in the residential real estate market, repay loans using monthly payments. It is easier for borrowers to budget for 12 smaller payments, rather than 1 or 2 large payments during the year. However, as noted earlier, the *Interest Act* requires that all blended payment mortgages specify the rate of interest as compounded annually or semi-annually. To conform to this provision of the *Interest Act*, interest rates are typically quoted with semi-annual compounding, but most mortgage loans specify that payments are to be made monthly. This means that the techniques of mortgage analysis presented in Illustration 14.9 must include an extra step to convert the nominal rate with semi-annual compounding to an equivalent nominal rate with monthly compounding.

Illustration 14.9

A local trust company has been approached by a real estate investor seeking a mortgage loan. The investor can pay $4,000 per month over a 15-year period. What size of loan will the trust company advance if it desires a yield (or interest rate) of j_2 = 5%?

Solution:

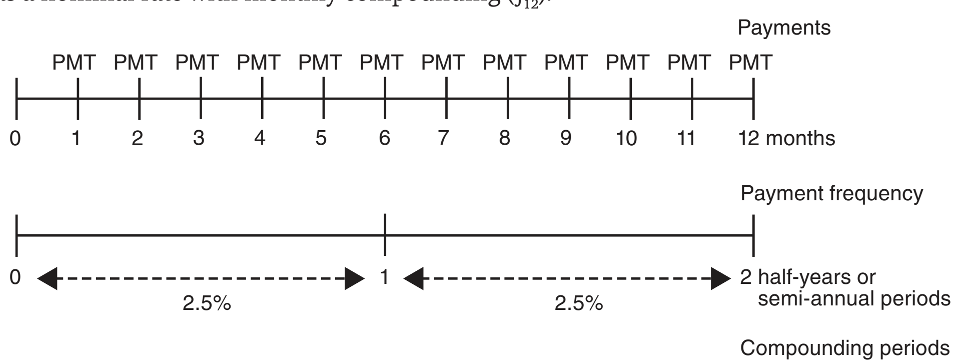
j_2 = 5%

PMT = $4,000 per month

N = 15 years (180 payment periods and 30 compounding periods)

PV = ?

The payments are monthly, but the stated interest rate is compounded semi-annually. Because these do not match, an interest rate conversion is needed. The nominal rate with semi-annual compounding (j_2) must be restated as a nominal rate with monthly compounding (j_{12}).



Calculation

Press	Display	Comments
5 ■ NOM%	5	Enter stated nominal rate
2 ■ P/YR	2	Enter stated compounding frequency
■ EFF%	5.0625	Compute equivalent effective annual rate
12 ■ P/YR	12	Enter desired compounding frequency
■ NOM%	4.948699	Compute equivalent nominal rate with desired compounding frequency

The borrower will make 180 monthly payments (15 years × 12 payments per year) of $4,000, and the rate of interest is 4.948699% per annum, compounded monthly. Since the rate of j_{12} = 4.948699% is already entered as the nominal interest rate with monthly compounding, it does not have to be entered again. **Equivalent interest rates should not be "keyed" into the calculator. Instead, they should be calculated and used directly to avoid errors in re-entering the number, and to retain full accuracy of calculations.**

Having determined the equivalent j_{12} interest rate, the maximum loan amount is calculated as follows:

Calculation (continued)

Press	Display	Comments
	4.948699	j_{12} rate displayed from previous calculation
4000 +/– PMT	–4,000	Payment per month
15 × 12 = N	180	Number of monthly payments
0 FV	0	Indicates that FV is not to be used (because all of the loan is totally repaid at the end of 180 months)
PV	507,534.472267	Present value or loan amount

The lender, desiring to earn 5% per annum, compounded semi-annually, would be willing to advance $507,534.47 in exchange for the borrower's promise to pay $4,000 per month for 180 months.

Exercise 14.2

A mortgage loan officer is reviewing several loan applications. The following table provides a summary of the loan terms under consideration. Calculate the maximum loan amount the borrowers should be offered for each loan.

Loan	Size of Payment	Frequency of Payment	Amortization Period	Nominal Rate
1	$500.00	monthly	20 years	j_2 = 5%
2	$1,712.15	monthly	25 years	j_2 = 17%
3	$6,000.00	annually	25 years	j_1 = 4%
4	$17,250.00	quarterly	15 years	j_2 = 12%
5	$623.00	monthly	25 years	j_2 = 4.5%

Abbreviated Solution:

1.	j_{12} = 4.948699%	PMT = $500.00	N = 240	PV = $76,089.02
2.	j_{12} = 16.427423%	PMT = $1,712.15	N = 300	PV = $122,953.40
3.	j_1 = 4%	PMT = $6,000.00	N = 25	PV = $93,732.48
4.	j_4 = 11.825206%	PMT = $17,250.00	N = 60	PV = $481,906.22
5.	j_{12} = 4.458383%	PMT = $623.00	N = 300	PV = $112,561.96

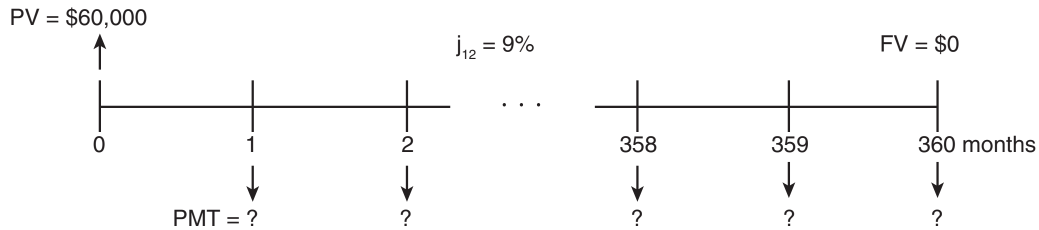
Calculation of Payments to Amortize a Loan

Illustration 14.10

A mortgage loan for $60,000 is to be repaid by equal monthly payments over a 30-year period at an interest rate of 9% per annum, compounded monthly. Calculate the size of the required monthly payments.

Solution:

j_{12} = 9%; N = 360 months (12 × 30); PV = $60,000; PMT = ?



Calculation

Press	Display	Comments
9 I/YR	9	Enter nominal rate compounded monthly (same as payment frequency, so no conversion needed)
12 ■ P/YR	12	Enter payment frequency
60000 PV	60,000	Enter present value
30 × 12 = N	360	Enter amortization period in months
0 FV	0	Indicates that FV will not be used (loan is fully repaid at the end of 360 months)
PMT	–482.77357	Size of required monthly payments

The calculated monthly payments are $482.77357. Since borrowers cannot make payments that involve fractions of cents, the payments must be rounded to at least the nearest cent. **Regular rounding rules apply,** *unless the facts indicate differently* (e.g., a problem may indicate that payments be rounded up to the next higher $10 or $100 to obtain a round number). Therefore, the payments on this loan would be $482.77 when rounded.

Exercise 14.3

A prospective borrower has contacted four lenders and has collected the information summarized below. For each loan alternative, calculate the required payment, rounded to the nearest cent.

Loan	Loan Amount	Nominal Rate	Amortization Period	Frequency of Payment
1	$75,000	j_2 = 4%	20 years	monthly
2	$100,000	j_1 = 7%	25 years	quarterly
3	$51,125	j_4 = 5%	240 months	monthly
4	$60,000	j_2 = 6.25%	25 years	monthly

Abbreviated Solution:

1.	N = 240	j_{12} = 3.967068%	PV = $75,000	PMT = $453.18
2.	N = 100	j_4 = 6.82341%	PV = $100,000	PMT = $2,091.14
3.	N = 240	j_{12} = 4.97931%	PV = $51,125	PMT = $336.82
4.	N = 300	j_{12} = 6.17014%	PV = $60,000	PMT = $392.84

Although payments are typically made monthly, most lenders offer other repayment options for constant payment mortgage loans (as mentioned previously). For example, some borrowers are choosing to "accelerate" their payments to save thousands of dollars in interest. A detailed example of accelerated payments is provided later in the chapter.

! ALERT

Loan payments can be rounded up to any amount specified in a loan contract, for example to the next higher dollar, ten dollars, or even hundred dollars. However, without specified payment rounding, assume that loan payments are rounded to the nearest cent.

Calculation of Interest Rates

Illustration 14.11

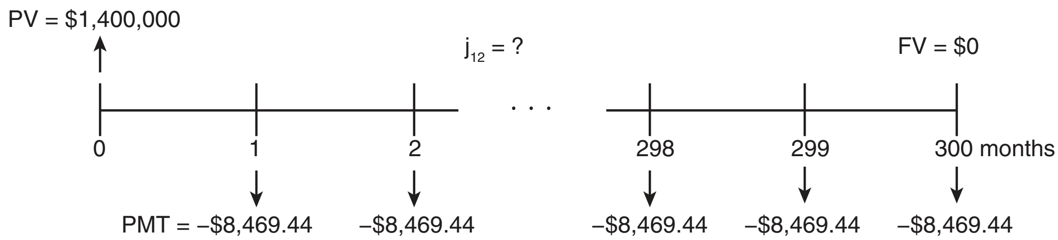
A $1,400,000 mortgage calls for monthly payments of $8,469.44 over 25 years. Calculate the annual rate of interest with semi-annual compounding (j_2) for this mortgage.

Solution:

Since the loan calls for monthly payments, first determine the nominal rate of interest with monthly compounding, and then convert this to an equivalent nominal rate with semi-annual compounding.

a. Calculate the nominal rate with monthly compounding:

PV = $1,400,000; PMT = $8,469.44; N = 300 months; j_{12} = ?



Calculation

Press	Display	Comments
12 ■ P/YR	12	Enter payment frequency
1400000 PV	1,400,000	Enter present value
8469.44 +/– PMT	–8,469.44	Enter monthly payment
25 × 12 = N	300	Number of monthly payments
0 FV	0	Loan is fully paid off over 300 months
I/YR	5.346594	j_{12} rate

b. Calculate the equivalent nominal rate with semi-annual compounding:

j_{12} = 5.346594% → j_2 = ?

Calculation (continued)

Press	Display	Comments
	5.346594	j_{12} rate displayed from previous calculation
■ EFF%	5.479579	Equivalent effective annual rate
2 ■ P/YR	2	Enter desired compounding frequency
■ NOM%	5.406503	Equivalent j_2 rate

The interest rate on this $1,400,000 loan with monthly payments of $8,469.44 over 25 years is j_2 = 5.406503%.

Exercise 14.4

A private investor is considering three alternative mortgage investments, each of which involves a $60,000 loan amount:

1. Under the first alternative, the investor will receive 60 monthly payments of $1,104.93.
2. Under the second alternative, the investor will receive 66 monthly payments of $1,025.05.
3. Under the third alternative, the investor will receive 54 monthly payments of $1,207.48.

Assist the investor in choosing among these alternatives by calculating the yield, as a nominal rate with semi-annual compounding, in each case.

Abbreviated Solution:

1. j_{12} = 3.997735%, j_2 = 4.031179%
2. j_{12} = 4.395223%, j_2 = 4.435666%
3. j_{12} = 3.684914%, j_2 = 3.713319%

The investor should choose the second alternative since it provides the highest return (rate of interest).

Calculation of Amortization Periods

The amortization period is the time that it takes to fully pay off a loan, given the required periodic payments. Most lenders will state the amortization period they require. The stated amortization period is then used to calculate the size of the required payments, i.e., to fully pay off the loan's principal plus all interest owing over this specified time period. Due to government rule changes, the maximum amortization period for insured mortgages is 25 years, and 30 to 35 years for uninsured mortgages with a 20% or more down payment.

While mortgage payments may be based on a relatively long amortization period, it does not mean that the payments will continue throughout the entire period. Most mortgages written in Canada have a much shorter contractual term. The periodic payments are made throughout the contractual term but do not fully repay the loan over this time. At the end of the contractual term, the principal balance remaining (outstanding) at that time becomes due and payable.

Illustration 14.12

To facilitate the sale of his property, a vendor has agreed to provide partial financing to a purchaser. Under this mortgage, the vendor will loan the purchaser $50,000, at a rate of 8% per annum, compounded semi-annually. The required payments are $684.51 per month. What is the amortization period of this loan (in months)?

Solution:

PV = $50,000

PMT = $684.51 per month

j_2 = 8%

N = ?

Since the contract rate is compounded semi-annually and the payments are made monthly, the interest rate given must be converted and expressed as an equivalent nominal rate with monthly compounding. Then calculate the amortization period.

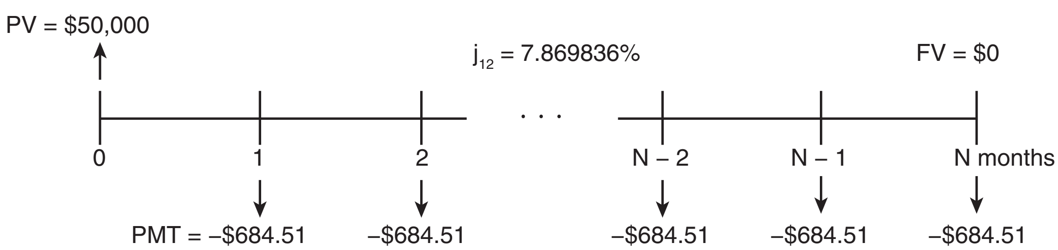
a. Calculate the equivalent nominal rate with monthly compounding:

$j_2 = 8\% \rightarrow j_{12} = ?$

Calculation

Press	Display	Comments
8 ■ NOM%	8	Enter stated nominal rate
2 ■ P/YR	2	Enter stated compounding frequency
■ EFF%	8.16	Compute equivalent effective annual rate
12 ■ P/YR	12	Enter desired compounding frequency
■ NOM%	7.869836	Compute equivalent nominal rate with monthly compounding

b. Calculate the number of months:



Calculation (continued)

Press	Display	Comments
	7.869836	Displayed from previous calculation
50000 PV	50,000	Enter loan amount
684.51 +/– PMT	–684.51	Enter monthly payment
0 FV	0	Loan is fully paid off over N months
N	99.756695	Number of monthly payments

The amortization period is 99.756695, or 100 months.

When computing amortization periods, it is usually the case that the amortization period will not be a round number but will have several decimal places, as in the previous illustration. This indicates, in the case of the previous illustration, that the borrower must make 99 full monthly payments of $684.51 and one final payment of an amount less than $684.51.

Note that if the message, “no Solution,” appears on the calculator’s display when computing an amortization period, it indicates that the given payments are not large enough to ever repay the loan amount at this interest rate.

Exercise 14.5

For each set of the following loan terms, calculate the amortization period.

Loan	Loan Amount	Nominal Rate	Payment
1	$100,000	j_{12} = 5%	$659.96 per month
2	$100,000	j_2 = 6%	$839.89 per month
3	$50,000	j_1 = 10%	$6,000 per year
4	$62,500	j_2 = 11.5%	$623.40 per month

Abbreviated Solution:

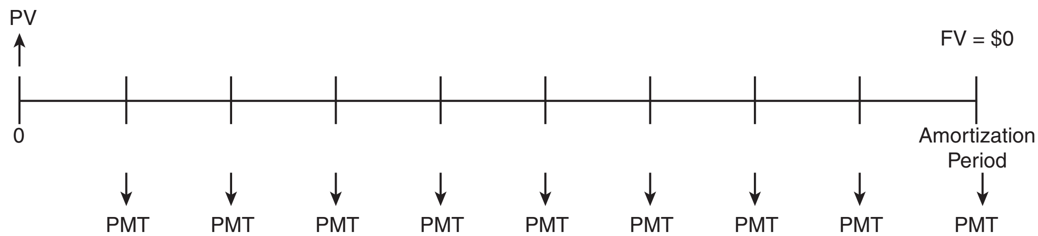
1. j_{12} = 5% N = 239.997341 months
2. j_{12} = 5.926346% N = 179.997514 months
3. j_1 = 10% N = 18.799246 years
4. j_{12} = 11.233783% N = 299.374382 months

Calculation of Outstanding Balances

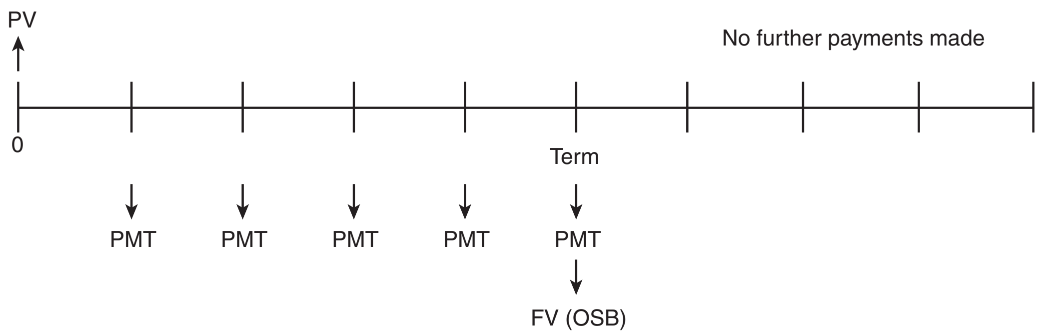
It is important to know how to calculate the *outstanding balance (OSB)* of a loan, or the amount of principal owing at a specific point in time, for several reasons. Most vendors want to know how much they will receive from the sale of their property after they have repaid the outstanding balance on their mortgage. While mortgage payments are calculated using the *amortization period*, the actual length of the mortgage contract may be different than the amortization period. The length of the mortgage contract is called the *term*. If the mortgage term and amortization period are the same length of time, the mortgage is said to be *fully amortized*. If the mortgage term is shorter than the amortization period, the mortgage is said to be *partially amortized*. Since mortgages are typically partially amortized with one to five-year contractual terms, the amount of money that the borrower owes the lender when the contract expires must be calculated.

outstanding balance (OSB)
the amount of principal owing on a loan at a specific point in time

To calculate the outstanding balance on an amortized loan, the payments are first calculated based on the full amortization period:



The outstanding balance is then calculated at the end of the loan term:



As shown below, the outstanding balance can be calculated quickly on your calculator.

Illustration 14.13

A borrower requires a $60,000 mortgage loan that is written at j_{12} = 6%, has a 20-year amortization period, a 3-year term, and monthly payments, rounded up to the next higher dollar. What is the outstanding balance of the mortgage at the end of its term? In other words, what is the outstanding balance just after the 36th payment (OSB_{36}) has been made?

Solution:

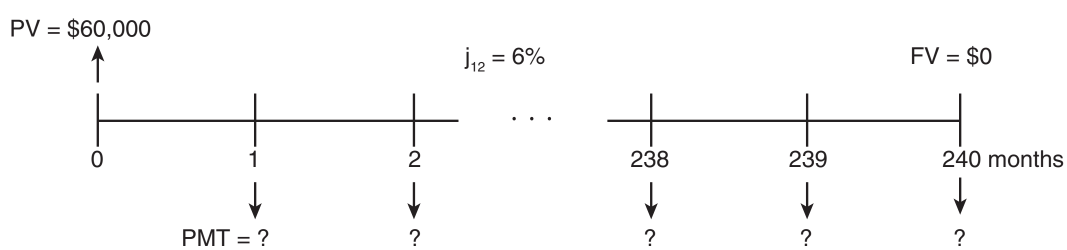
a. Calculate the required monthly payments:

N = 240 months (20 × 12)

PV = $60,000

j_{12} = 6%

PMT = ?

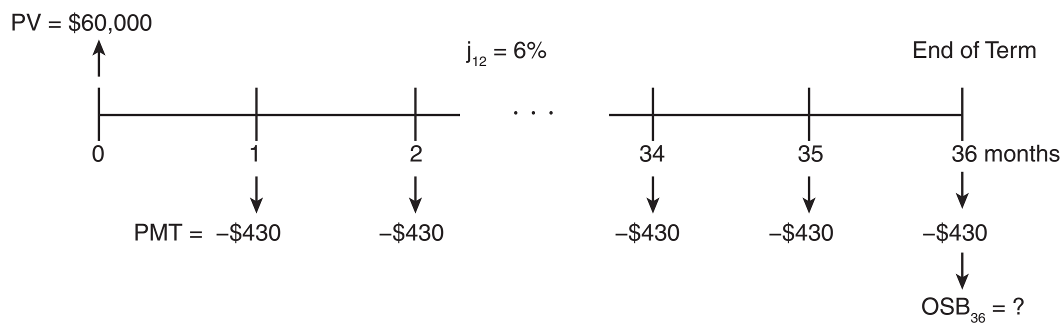


Calculation

Press	Display	Comments
6 I/YR	6	Enter nominal rate with monthly compounding (no conversion needed)
12 ■ P/YR	12	Enter compounding frequency
60000 PV	60,000	Enter loan amount
20 × 12 = N	240	Enter number of payments
0 FV	0	240 monthly payments would fully amortize loan
PMT	–429.858635	Calculate payment

The monthly payment is $430, rounded up to the next higher dollar.

b. Calculate the outstanding balance due immediately after (with) the 36th monthly payment:



Before the outstanding balance can be calculated, the rounded payment must be entered into the calculator. The $430 payment is slightly larger than the $429.858635 calculated minimum payment required to repay the loan. By rounding this up to $430, the borrower is overpaying by nearly $0.15 with each payment. This causes more principal to be repaid in each month than is required to amortize the loan. The rounding of the payment results in a faster repayment of the loan amount and, consequently, reduces the number of full payments needed to amortize the loan. When the new (larger) payment is entered, the pre-programmed function of your calculator revises the amortization period as part of the outstanding balance calculation:

Calculation (continued)

Press	Display	Comments
	–429.858635	Payment from previous calculation (not yet rounded up)
430 +/– PMT	–430	Enter rounded payment
N	239.84778	Recompute amortization period given higher payment (see ALERT)
36 INPUT ■ AMORT	PER 36 – 36	
= = =	54,886.306355	Outstanding balance after 36th monthly payment

ALERT

Recalculating N

This step is included for illustrative purposes and is not required in outstanding balance calculations. Recomputing the amortization period will not affect further calculations such as the outstanding balance, as long as the rounded payment has been re-entered.

If this loan was fully amortized, it would take 239.84778 months to pay off the loan: 239 payments of $430 plus a 240th payment, which is less than $430 (this will be explained further in the "Final Payments on Fully Amortized Loans" section of this chapter). However, since this loan is partially amortized, the borrower will make 36 monthly payments of $430 and pay the remaining balance of $54,886.31 at the end of the 36th month (or renegotiate for a new loan).

Readers may have noticed that while pressing the "=" sign three times, three different numbers appeared on the screen. The first number that appears is the principal paid in the 36th payment, the second number is the interest paid in the 36th payment, and the final number is the outstanding balance owing immediately after the 36th payment. These functions will be explained further in the next section.

HELPFUL HINT!

(1) Checklist for Outstanding Balance Calculations

The following steps should be followed when calculating the outstanding balance of a mortgage loan:

1. Interest rate conversion, if required; i.e., convert the contract interest rate so that the compounding frequency matches the payment frequency
2. Calculate the payment based on the contract rate, loan amount, and amortization
3. Re-enter the rounded payment
4. Calculate the outstanding balance using the INPUT and AMORT keys

(2) The AMORT Key

This helpful function calculates an AMORTization schedule for the loan, showing the outstanding balance at any point in the loan term, as well as interest and principal paid for a single payment or over multiple payments. Assuming monthly payments over a 3-year term, the following calculator steps can be used:

- Interest/Principal paid in Month 36: 36 INPUT ■ AMORT OR 36 INPUT 36 ■ AMORT
- Interest/Principal paid over a 3-year loan term: 1 INPUT 36 ■ AMORT
- Outstanding Balance at Month 36: 36 INPUT ■ AMORT OR 36 INPUT 36 ■ AMORT OR 1 INPUT 36 ■ AMORT

All three calculations show the same outstanding balance at Month 36. It is only the interest and principal amounts that will differ – as the first two calculations show one month's principal and interest amounts; the third shows the total principal repaid and interest paid for 36 payments over a 3-year term.

Exercise 14.6

For each of the loans in Exercise 14.5, assume each loan has a two-year term and calculate the outstanding balance owing at the end of the term.

Abbreviated Solution:

1. OSB = $93,872.43
2. OSB = $91,206.14
3. OSB = $47,900.00
4. OSB = $61,474.51

Calculation of Principal and Interest Components of Payments

In addition to the outstanding balance, it is often necessary to calculate the principal and interest components of payments on constant payment mortgages. These calculations are important because interest on payments can sometimes be deducted as an expense for income tax purposes. As well, borrowers like to know how much principal they have paid off in a single payment or over a series of payments. The calculation of principal and interest components of payments is done using the same keys on the calculator as the outstanding balance calculation shown above. This function will be explained using the following illustration.

Illustration 14.14

Three years ago, Tom and Nancy bought a house with a mortgage loan of $175,000, written at j_2 = 9.5%, with a 25-year amortization, monthly payments rounded up to the next higher dollar, and a 3-year term. Tom and

Nancy are about to make their 36th monthly payment, the last one in the loan's term, and want to know the following information:

a. How much interest will they be paying with their 36th payment?
b. How much principal will they be paying off with their 36th payment?
c. What will the outstanding balance be immediately following the 36th payment? In other words, what will be the amount they will have to refinance after the 36th payment?
d. How much interest did they pay over the entire 3-year term?
e. How much principal did they pay off during the 3-year term?
f. What is the total amount of interest paid during the second year of the loan?

Solution:

To answer any of these questions, first it is necessary to find the j_{12} interest rate and the monthly payments under the mortgage.

Calculation

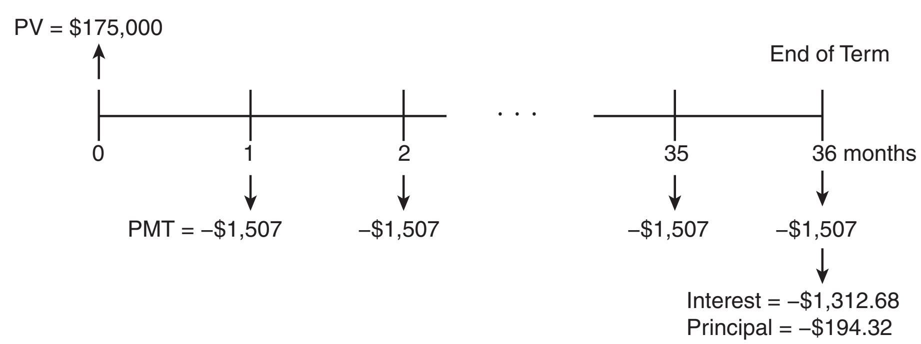
Press	Display	Comments
9.5 ■ NOM%	9.5	Enter stated nominal rate
2 ■ P/YR	2	Enter stated compounding frequency
■ EFF%	9.725625	Compute equivalent effective annual rate
12 ■ P/YR	12	Enter desired compounding frequency
■ NOM%	9.31726	Compute nominal rate with monthly compounding
175000 PV	175,000	Enter present value
25 × 12 = N	300	Enter amortization period in months
0 FV	0	Payment is calculated to fully amortize loan
PMT	–1,506.798355	Compute the monthly payment
1507 +/– PMT	–1,507	Enter rounded payment (to the next higher dollar)

The monthly payment on the loan is $1,507, rounded up to the next higher dollar. The answers to questions a, b, and c can all be found using the calculator's pre-programmed amortization function as shown in the following calculation.

Calculation (continued)

Press	Display	Comments
	–1,507	Payment displayed from previous calculation
36 INPUT ■ AMORT	PER 36 – 36	
=	–194.316609	Principal portion of 36th payment
=	–1,312.683391	Interest portion of 36th payment
=	168,870.419441	Outstanding balance immediately following 36th payment (OSB_{36})

The amount of interest paid in the 36th payment is $1,312.68. The amount of principal paid off in the 36th payment is $194.32. The outstanding balance immediately following the 36th payment (OSB_{36}) is $168,870.42. As expected, the principal paid and interest paid in the 36th payment total $1,507, which is the amount of the monthly payment.

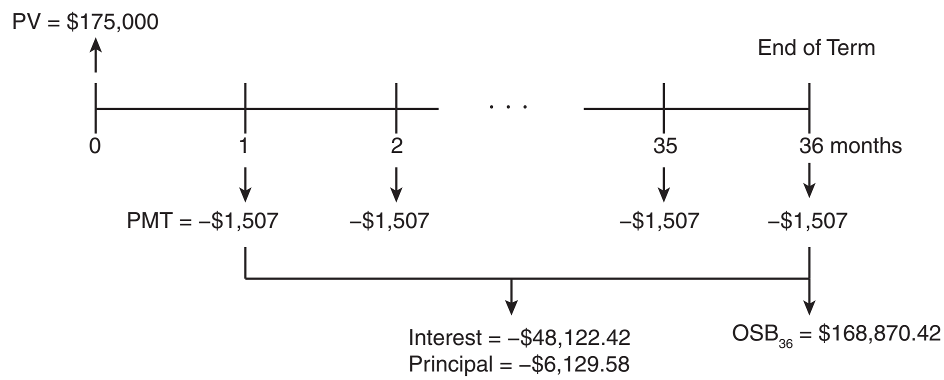


The same pre-programmed amortization function can be used to find the amounts of principal and interest paid over a series of payments, as in questions d and e. In this case, the series of payments will be the entire 36-month loan term, although the amortization function can be used over any series of payments. Questions d and e can be answered using the following calculator steps:

Calculation (continued)

Press	Display	Comments
1 INPUT 36	36	Enter desired series of payments
■ AMORT	PER 1 – 36	Series of payments being amortized
=	–6,129.580559	Total principal paid off in payments 1 – 36
=	–48,122.419441	Total interest paid in payments 1 – 36
=	168,870.419441	OSB_{36}

The total amount of interest paid over the term of the loan was $48,122.42. The total amount of principal paid off during the loan term was $6,129.58.



Calculation (continued)

Press	Display	Comments
13 INPUT 24	24	Enter desired series of payments
■ AMORT	PER 13 – 24	Series of payments being amortized
=	–2,037.339299	Total principal paid off during second year
=	–16,046.660701	Total interest paid during second year
=	171,105.90272	OSB_{24}

The answer to question f is shown by the above calculation. The total amount of interest paid during the second year of the loan was $16,046.66. It is important to note that the second year of the loan is comprised of monthly payments 13 through 24.

Final Payments on Fully Amortized Loans

As discussed earlier in the outstanding balance section of this chapter, if payments are rounded up, more principal will be repaid with each payment than is required to amortize the loan, resulting in faster repayment of the loan amount. In other words, you are overpaying slightly with each payment. When the payments are rounded up to the nearest cent, the final payment necessary to repay the loan amount will be *smaller* than the regular payments. If the payments are rounded up to the next higher dollar, or the next higher ten dollars, etc., *the number of payments* required may decline in addition to reducing the size of the final payment.

final payment
the last instalment that is made on a fully amortized loan

Alternatively, if payments are rounded down to the nearest cent, you may in fact be slightly underpaying by a portion of a cent with each payment, meaning an interest adjustment will be owing as part of the final payment – the final payment will be larger than a regular payment. However, this is not a common issue in practice, since fully amortized loans in Canada are very rare.

For the purposes of this course, final payment calculations are not required; however, students are required to understand the concept of final payments.

Calculation of Accelerated Payments

Constant payment mortgage loans have historically required monthly payments. In recent years, lenders have become more flexible in offering a variety of different payment frequencies, such as biweekly or weekly payments. *Biweekly payments* are particularly popular in that they match most people's earning frequency (paid every two weeks). The more frequent payments can create substantial interest savings and reduce the loan's amortization period. The following illustration demonstrates three options for a borrower: constant monthly payments, biweekly payments, and accelerated biweekly payments.

biweekly payments
constant payments that are paid every two weeks

Accelerating payments is a highly effective way to pay off a mortgage loan faster and to reduce interest costs. An accelerated payment simply means paying more with each payment than the bare minimum required to fully amortize the loan. If the borrower can afford to pay even just a little more with each payment, this extra goes directly to reducing loan principal and can have a dramatic impact on interest paid over the loan term as well as the time needed to pay off the loan.

One of the most common forms of accelerated payments is the accelerated biweekly mortgage plan. Since many people are paid every two weeks, making biweekly mortgage payments is a convenient and efficient way to pay off a mortgage loan. The *accelerated biweekly payment* is calculated as a monthly payment divided in half – instead of paying once a month, one-half of the monthly payment is paid every two weeks. The effect of biweekly accelerated payments is that the borrower makes the equivalent of one extra monthly payment per year, thus paying down the principal faster and paying less interest (it should be noted that a borrower could make 26 or 27 biweekly payments in a given year, depending on the payment date).

accelerated biweekly payments
constant payments that are equal to ½ of the regular monthly payment and paid every two weeks

Illustration 14.15

A borrower is considering buying a home and has arranged a mortgage loan with a face value of $200,000, an interest rate of j_2 = 5.5%, an amortization period of 20 years, and a term of 5 years. This borrower is considering three repayment plans with different payment frequencies. Option 1 requires constant monthly payments. Option 2 involves increasing the payment frequency so that payments are made biweekly instead of monthly. Option 3 allows the borrower to make accelerated biweekly payments. All options require the mortgage payments to be rounded up to the next highest dollar.

In deciding between these options, the borrower wants to know the amount of interest that will be paid over the term of the mortgage under each option, as well as principal paid off over the term, and the time it will take to fully amortize the loan.

Solution:

Option 1 – Constant Monthly Payments

Calculation

Press	Display	Comments
5.5 ■ NOM%	5.5	j_2
2 ■ P/YR	2	
■ EFF%	5.575625	j_1
12 ■ P/YR	12	
■ NOM%	5.438018	j_{12}
200000 PV	200,000	
20 × 12 = N	240	20-year amortization
0 FV	0	
PMT	–1,368.782577	
1369 +/– PMT	–1,369	
N	239.931169	Months to fully amortize loan
÷ 12 =	19.994264	Years to fully amortize loan
1 INPUT 60 ■ AMORT	PER 1 – 60	5-year term
=	–31,818.122986	Principal paid during term
=	–50,321.877014	Interest paid during term
=	168,181.877014	OSB_{60}

The minimum monthly payment to fully pay this loan over 20 years is $1,369.

Option 2 – Biweekly Payments

Calculation

Press	Display	Comments
5.5 ■ NOM%	5.5	j_2
2 ■ P/YR	2	
■ EFF%	5.575625	j_1
26 ■ P/YR	26	
■ NOM%	5.431399	j_{26}
200000 PV	200,000	
20 × 26 = N	520	20-year amortization
0 FV	0	
PMT	–630.976818	
631 +/– PMT	–631	
N	519.965498	Biweekly periods to fully amortize loan
÷ 26 =	19.998673	Years to fully amortize loan
1 INPUT 130 ■ AMORT	PER 1 – 130	5-year term
=	–31,806.628914	Principal paid during term
=	–50,223.371086	Interest paid during term
=	168,193.371086	OSB_{130}

Since many people are paid every two weeks, making biweekly mortgage payments conveniently matches mortgage payments to income. As well, by paying interest earlier in the month, the borrower saves some interest over the loan term. By accelerating the biweekly payment, as we will see in Option 3, the borrower can save more interest and substantially reduce the time required to pay off this loan.

Option 3 – Accelerated Biweekly Payments

Under Option 3, the biweekly payment will be set as one-half of the monthly payment. First, calculate the monthly payment, shown in Option 1 as −1,368.782577. Then, the accelerated biweekly payment is the monthly payment of $1,369 divided by two, or $685 (rounded up to next higher dollar).

Calculation

Press	Display	Comments
5.5 ■ NOM%	5.5	j_2
2 ■ P/YR	2	
■ EFF%	5.575625	j_1
12 ■ P/YR	12	
■ NOM%	5.438018	j_{12}
200000 PV	200,000	
20 × 12 = N	240	20-year amortization
0 FV	0	
PMT	–1,368.782577	
÷ 2 =	–684.391289	Half of one monthly payment
685 +/– PMT	–685	
5.5 ■ NOM%	5.5	j_2
2 ■ P/YR	2	
■ EFF	5.575625	j_1
26 ■ P/YR	26	
■ NOM%	5.431399	j_{26}
N	451.126959	Biweekly periods to fully amortize loan
÷ 26 =	17.351037	Years to fully amortize loan
1 INPUT 130 ■ AMORT	PER 1 – 130	5-year term
=	–39,862.713082	Principal paid during term
=	–49,187.286918	Interest paid during term
=	160,137.286918	OSB_{130}

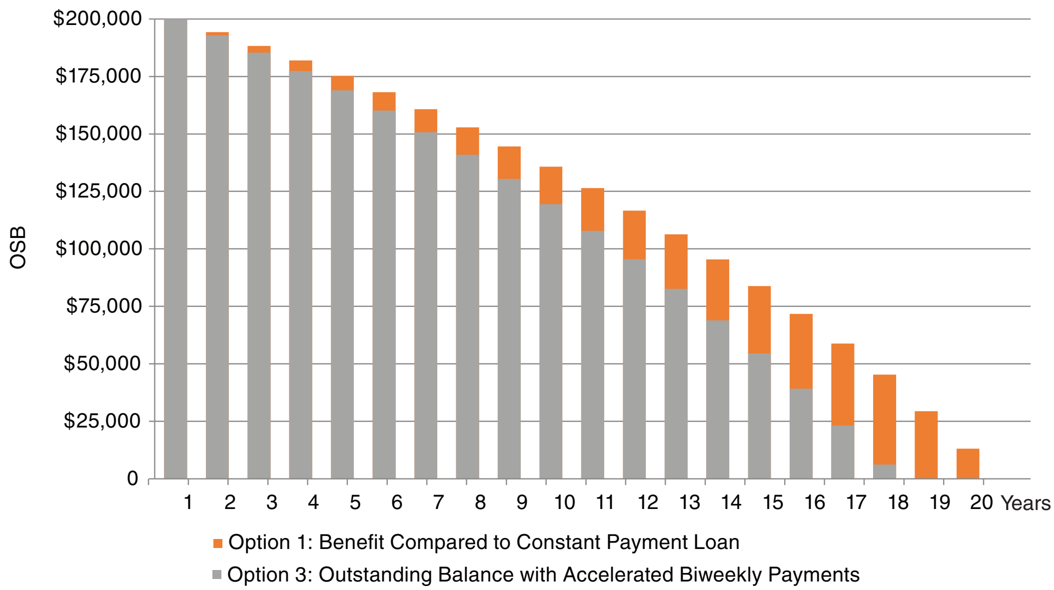
The following table summarizes the impact of this accelerated biweekly payment on the interest paid and amortization period for the loan.

Option	Repayment Plan	Principal Paid Over Term	Interest Paid Over Term	OSB @ End of Term	Years to Fully Amortize Loan	Interest Paid Over Amortization Period
1	Constant Monthly Payments	$31,818.12	$50,321.88	$168,181.88	19.994264	$128,465.97
2	Constant Biweekly Payments	$31,806.63	$50,223.37	$168,193.37	19.998673	$128,098.25
3	Accelerated Biweekly Payments	$39,862.71	$49,187.29	$160,137.29	17.351037	$109,022.05

If the borrower makes accelerated biweekly payments over the 5-year term, the borrower will have paid off substantially more principal. This results in a smaller outstanding balance owing at the end of the term. The interest paid is slightly less than both the monthly repayment plan and the constant biweekly plan, but the real impact is in the borrower paying off the loan faster. The accelerated biweekly plan helps the borrower save nearly three years of payments.

Another significant impact is in the interest paid over the full amortization period. This loan only has a five-year term, but if we assumed that it would be renewed with the same terms until fully paid off, we can calculate the total interest paid over the amortization period – shown in the right column in the table above. The accelerated biweekly option saves more than $19,000 in interest over the amortization period compared to Option 1. Figure 14.1 illustrates the benefit in interest savings over the amortization period for Illustration 14.15.

FIGURE 14.1: Benefit of Accelerated Biweekly Payments



Accelerated biweekly payments have become popular because of the interest savings and reduced time required to repay a loan. Since many employees are paid biweekly, the impact of paying every two weeks versus monthly is probably negligible for most borrowers from a financial planning perspective. The biweekly payment matches the frequency of income receipt.

In this example, if the borrower is paid biweekly, the borrower may wish to consider either of the biweekly payment plans. If the borrower can afford the extra $54 payment every two weeks, this offers clear benefits. One small drawback to consider with accelerated biweekly payments: if the borrower is paid semi-monthly rather than biweekly, then the payment and income frequencies do not match perfectly. The borrower would be paid 24 times per year but must make 26 mortgage payments – meaning twice in the year, the borrower must make two mortgage payments with only one paycheque.

CONCLUSION

Having a solid foundation of mortgage finance is beneficial for those in the real estate industry. Given that you will be dealing with clients who are making financial commitments to purchase real estate, it is imperative that you be able to communicate with them using the correct terminology. As well, being able to perform financial calculations will give your client a sense of their potential financial obligations and assist in determining the type of property to show a potential client.