EECS240 – Spring 2010

Lecture 13: Settling

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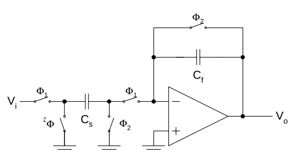
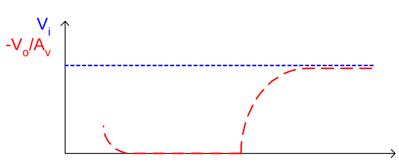
Settling

- Why interested in settling?
  - Oscilloscope: track input waveform without ringing
  - ADC (switched-cap amplifier): gain a signal up by a precise amount within T_{sample}

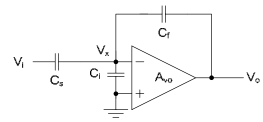


Step Response

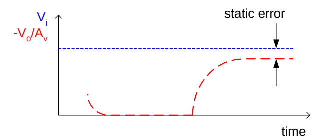
- Two types of settling "errors":
  - Static
    - Finite gain, capacitor mismatch
  - Dynamic
    - Takes time to reach final value



Static Error



$$\frac{V_o}{V_i} = -\frac{c}{1+\frac{1}{\underbrace{FA_{vo}}_{T_o}}}$$



$$F = \frac{C_f}{C_f + C_s + C_i}$$

$$c = \frac{C_s}{C_f}$$

Static Error (cont.)

$$\frac{V_o}{V_i} = -\frac{c}{1+\frac{1}{FA_{vo}}} \approx -c\left(1 - \underbrace{\frac{1}{FA_{vo}}}_{\text{relative error}}\right)$$

- <u>Example:</u>
  - Closed loop gain: c = -4, C_f = 1pF, C_s = 4pF, C_i = 1pF
  - F = 1/6 (C_i hurts!)

- Error specification: <0.1%
  - FA_{vo} > 1000
  - <u>A_{vo} > 6000</u> over output range

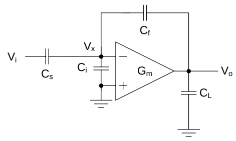
Dynamic Errors

- Many possible dynamic effects that impact settling error:
  - Finite bandwidth
  - Feedforward zero
  - Non-dominant poles
  - Doublets
  - Slewing

- Approximate analysis approach:
  - Decompose each error source, isolate interactions
  - Add all errors together

Single Time Constant Linear Settling

- For dynamic settling (and for T_0 >> 1), can generally ignore r_o



$$\frac{V_o}{V_i} = -c\frac{1 - s\dfrac{C_f}{G_m}}{1 + s\dfrac{C_L + (1-F)C_f}{FG_m}}$$

Time Domain Step Response

Frequency domain:

Time domain:

$$V_{o,step} = -c\frac{1+s/z}{1+s/p}\frac{V_{step}}{s}$$

Time Domain Step Response

Frequency domain:

input step:

$$V_{i,step} = \frac{V_{step}}{s}$$

output step:

$$V_{o,step} = -c\frac{1+s/z}{1+s/p}V_{i,step}$$
$$= -c\frac{1+s/z}{1+s/p}\frac{V_{step}}{s}$$

Time domain:
(inverse Laplace transform)

$$v_{o,step}(t) = \underbrace{-V_{step}c}_{\text{ideal response}}\left[1 - \underbrace{\left(\underbrace{1-\frac{p}{z}}_{\substack{\text{initial error}\\\text{(feedforward)}}}\right)e^{-pt}}_{\text{exponentially decaying error}}\right]$$

Case 1: |p/z| << 1

$$v_{o,step}(t) \cong \underbrace{-V_{step}c}_{\text{ideal response}}\left[1 - e^{-t/\tau}\right]$$

Relative settling error:

$$\varepsilon = \frac{v_o(t\to\infty) - v_o(t=t_s)}{v_o(t\to\infty)} = e^{-t/\tau}$$

$$\frac{t_s}{\tau} = -\ln\varepsilon$$

- Easiest number to remember: 2.3τ per decade
- Example: 1% settling, 4.6ns clock cycle: τ = 1ns
- $C_{L,eff}$ usually set by noise – use settling to determine required g_m

Case 2: |p/z| not negligible

$$v_{o,step}(t) \cong \underbrace{-V_{step}c}_{\text{ideal response}}\left[1 - \left(1-\frac{p}{z}\right)e^{-t/\tau}\right]$$

Relative settling error:

$$\varepsilon = \frac{v_o(t\to\infty) - v_o(t=t_s)}{v_o(t\to\infty)} = \left(1-\frac{p}{z}\right)e^{-t/\tau}$$

$$\frac{t_s}{\tau} = -\ln\left(\frac{\varepsilon}{1+F\,C_f/C_{Leff}}\right)$$

- **Example:**
  - c = 0.25, C_f = 1pF, C_s = 250fF, C_i = 250fF, C_L = 1pF
  - F = 0.67, $C_{L,eff}$ = 1.33pF
- ε = 0.1%:
  - t_s (no feedforward) = 6.9τ
  - t_s (with feedforward) = -ln[1e-3/(1+0.67*0.75)]=7.3τ

Non-Dominant Pole

- Ignore feed-forward zero for simplicity
  - (Just increases final swing by $1+FC_f/C_{L,eff}$)

$$H(s) = \frac{V_o}{V_{in}} = -c\frac{1}{1+s\dfrac{C_{Leff}}{FG_m}}$$

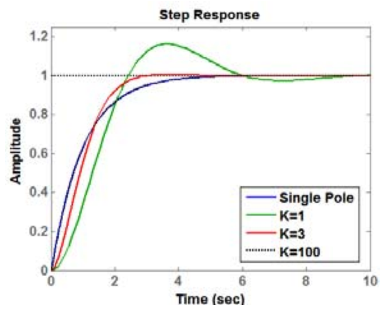
- Model for non-dominant pole:

$$G_m(s) = \frac{G_{m0}}{1+s/p_2}$$

$$p_2 = K\omega_u$$

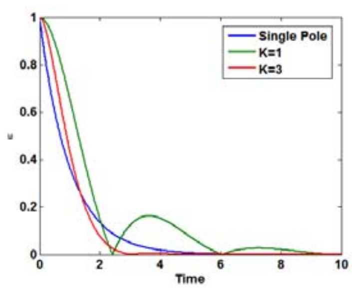
ω_u is unity gain bandwidth of $T(s)$

Step Response



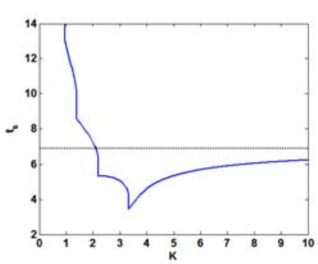
Non-Dominant Pole (cont.)

Relative error: $\varepsilon = |1 - v_s(t, K)|$



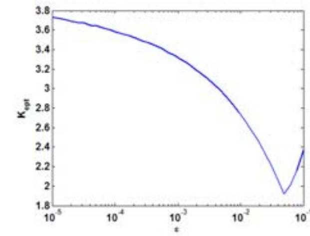
Settling Time

Settling time: $t_s(\mathrm{K})$ for $\varepsilon = 10^{-3}, \ \tau = 1$



- Optimum at K=3.3

Non-Dominant Pole vs. ε



- Optimum K actually depends on required accuracy
- Still, always want to avoid K<~2

Doublets

- **Amplifier model:**

$$G_m(s) = G_{mo}\frac{1+s/\omega_z}{1+s/\omega_p} \quad \textbf{with} \quad \begin{array}{l} \omega_p = \beta\omega_{-3dB}, \quad \omega_{-3dB} \text{ is bandwidth of } T(s) \\ \omega_z = \dfrac{\omega_p}{\alpha} \\ \alpha = 1+\varepsilon \quad \text{with} \quad |\varepsilon| << 1 \end{array}$$

- **Closed-loop gain (ignore feedforward zero):**

$$\frac{V_o}{V_{in}} = -c\frac{1}{1+s\dfrac{C_{Leff}}{FG_m(s)}} \approx -\frac{c}{1+s/\omega_{-3dB}}\left(\frac{1+s/\omega_z}{1+s/\omega_{pp}}\right) \qquad \omega_{-3dB} \cong \frac{FG_{mo}}{C_{Leff}}, \quad \omega_{pp} \cong \omega_p$$

Doublet Analysis

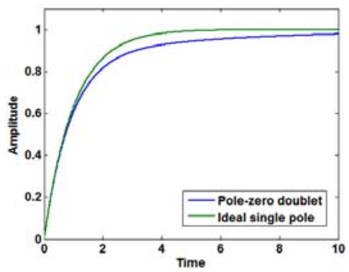
- **Step response**

$$v_{o,step}(t) = -cV_{step}\left(1 - Ae^{-t\omega_{-3dB}} - Be^{-t\omega_{pp}}\right)$$

$$B \approx \varepsilon\frac{\beta}{(1-\beta)^2}$$

$$A \approx 1 - B \approx 1$$

Doublet Example



α=1.5
β=0.3

Doublet Conclusions

- **Case A:** $\tau_2 \le \tau_1$ i.e. $\beta \ge 1$
  - **Doublet settles faster than amplifier**
  - **Has no impact on overall settling time**
- **Case B:** $\tau_2 > \tau_1$
  - **Doublet settles more slowly than amplifier**
  - **Determines overall settling time (unless ε within settling accuracy requirements)**
- **→ Avoid "slow" doublets!**

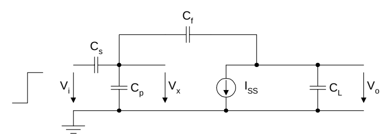
Final Note on Doublets

Slewing

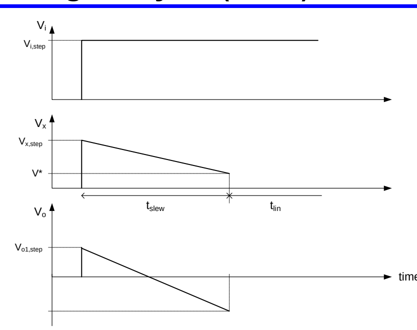
- **Transconductor ΔI vs. ΔV:**
- **Model for (nonlinear) slewing amplifier**
  - **Piecewise linear approximation:**
    - **Slewing with constant current, followed by**
    - **Linear settling exponential**
    - **$t_s = t_{slew} + t_{s,lin}$**

Slewing Analysis

- **Circuit model during slewing:**



Slewing Analysis (cont.)



Slewing Analysis (cont.)

- **Slewing period:**

$$V_{x,step} = V_{i,step} \frac{C_s}{C_s + C_2} \quad \text{with} \quad C_2 = C_i + \frac{C_f C_L}{C_f + C_L}$$

$$\Delta V_x = V_{x,step} - V^* \rightarrow \Delta V_o = \frac{\Delta V_x}{F}$$

$$t_{slew} = \frac{\Delta V_o}{SR} = \frac{\Delta V_x C_{Leff}}{F I_{SS}}$$

- **Linear settling during final V* of swing at V_x:**
  - **Step during linear settling:** V^*/F
  - **Linear settling time:** $\frac{t_{s,lin}}{\tau} = -\ln\left(\varepsilon \frac{cV_{i,step} F}{V^*}\right)$