EECS240 – Spring 2010

Lecture 6: Noise Analysis Techniques

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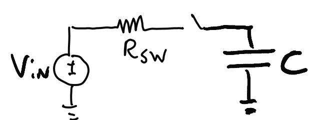
Noise Variance in a Real Circuit: Sample and Hold



- **Noise on the capacitor:**

$$\overline{v_{on}^2(f)} = 4k_BTR\left|\frac{1}{1+sRC}\right|^2$$

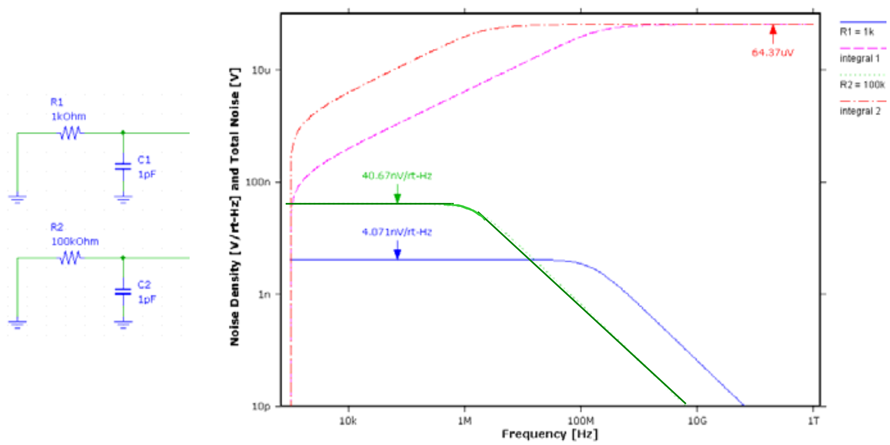
$$\rightarrow \overline{v_{oT}^2} = \int_0^\infty \overline{v_{on}^2(f)}df = \frac{k_BT}{C}$$

- **So effective bandwidth is:**

$$4k_BTR\Delta f = \frac{k_BT}{C}$$

$$\rightarrow \Delta f = \frac{1}{4RC} = \frac{\pi}{2}f_o$$

SPICE Verification



Energy-Based Analysis

Sampled Signal Signal-To-Noise Ratio

- **SNR:** $$SNR = \frac{P_{sig}}{P_{noise}}$$
- **Signal Power (sinusoidal source):**

$$P_{sig} = \tfrac{1}{2} V_{zero-peak}^2$$

- **Noise Power (assuming thermal noise dominates):**

$$P_{noise} = \frac{k_B T}{C} n_f$$

- **So:**

$$\boxed{SNR = \frac{\tfrac{1}{2} C V_{zero-peak}^2}{n_f k_B T}}$$

$$SNR \quad \uparrow +6dB$$
$$\downarrow$$
$$C \quad \uparrow \times 4$$

dB versus Bits

- **Quantization "noise"**
  - **Quantizer step size:** Δ
  - **Box-car pdf variance:** $S_Q = \frac{\Delta^2}{12}$

N	dB
8	50
16	98
24	146

- **SNR of N-Bit sinusoidal signal**
  - **Signal power** $$P_{sig} = \frac{1}{2}\left(2^N \frac{\Delta}{2}\right)^2$$
  - **SNR** $$SNR = \frac{P_{sig}}{S_Q} = 1.5 \times 2^{2N}$$
  - **6.02 dB per Bit** $$= [1.76 + 6.02N] \text{ dB}$$

SNR versus Power

- **1 Bit → 6dB → 4x SNR**
- **4x SNR → 4x C**
- **Circuit bandwidth ~g_m/C → 4x g_m**
- **Keeping V* constant → 4x I_D, 4x W**

- **Thermal noise limited circuit:**
  - **Each bit QUADRUPLES power!**

- **Overdesign is <u>expensive</u>**
  - **Better do the analysis right!**
  - **Need to know how to get analytical expressions for more general circuits**

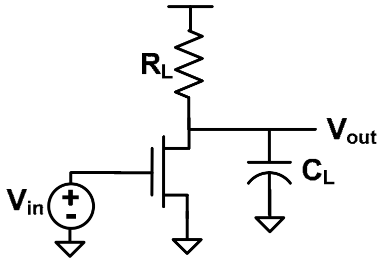
Important Integrals

$$\int_0^\infty \left| \frac{1}{1+\frac{s}{\omega_o}} \right|^2 df = \frac{\omega_o}{4}$$

$$\int_0^\infty \left| \frac{1}{1+\frac{s}{\omega_o Q}+\frac{s^2}{\omega_o^2}} \right|^2 df = \int_0^\infty \left| \frac{\frac{s}{\omega_o}}{1+\frac{s}{\omega_o Q}+\frac{s^2}{\omega_o^2}} \right|^2 df = \frac{\omega_o Q}{4}$$

$$\int_0^\infty \left| \frac{\frac{s}{\omega_z}+1}{1+\frac{s}{\omega_o Q}+\frac{s^2}{\omega_o^2}} \right|^2 df = \frac{\omega_o Q}{4}\left(\frac{\omega_o^{\ 2}}{\omega_z^{\ 2}}+1 \right)$$

CS Amplifier



$$\overline{v_{on}^2(f)} = 4k_BT\left(\frac{1}{R_L}+\frac{2}{3}g_m\right)\left|\frac{R_L}{1+sR_LC_L}\right|^2$$

$$\overline{v_{oT}^2} = 4k_BT\left(\frac{1}{R_L}+\frac{2}{3}g_m\right)R_L^2\int_0^\infty\left|\frac{1}{1+sR_LC_L}\right|^2 df$$

$$= 4k_BT\left(\frac{1}{R_L}+\frac{2}{3}g_m\right)R_L^2\frac{1}{4R_LC_L}$$

$$= \frac{k_BT}{C_L}\left(1+\frac{2}{3}g_mR_L\right)$$

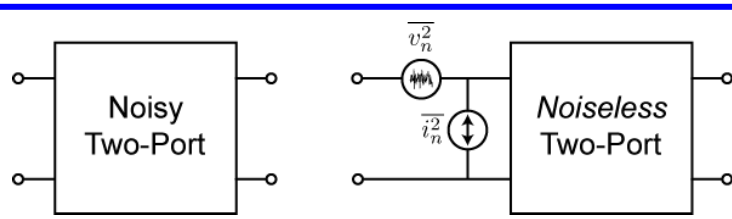
$$= \frac{k_BT}{C_L}\left(1+\frac{2}{3}|A_{vo}|\right)$$

$$= \frac{k_BT}{C_L}n_F$$

Two-Stage Amplifier

Input Equivalent Noise

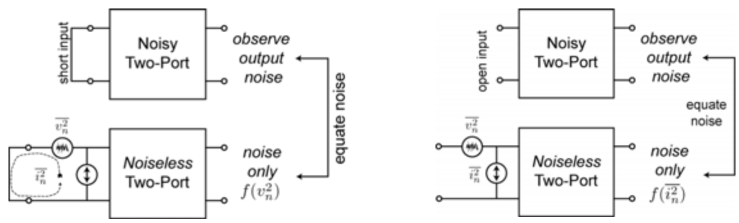
Equivalent Noise Generators



- **Model for noisy two-port:**
  - ***Noiseless* two-port**
  - **Plus equivalent input noise sources**

- **In general, v_n and i_n are correlated.**
  - **Ignore that for now**

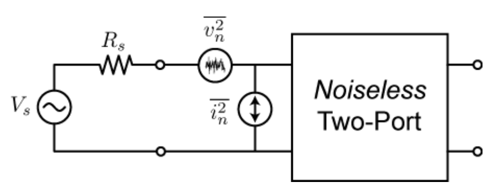
Finding the Equivalent Generators



- Find v_n and i_n by opening and shorting the input

- Shorted input:
  - Output noise due only to v_n
- Open input:
  - Output noise due only to i_n

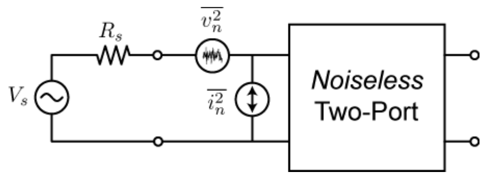
Role of Source Resistance



- If R_s is large:
  - Design amplifier with low i_n (MOS)
- If R_s is low:
  - Design amplifier with low v_n (BJT)

- For a given R_s, there is an optimal v_n/i_n ratio
  - Alternatively, for a given amp, there is an optimal R_s

Total Output Noise



$$\overline{v_o^2} = (\overline{v_n^2}A_v^2 + \overline{v_{R_s}^2}A_v^2)\left(\frac{R_{in}}{R_{in}+R_s}\right)^2 + \left(\frac{R_{in}}{R_{in}+R_s}\right)^2 R_s^2\overline{i_n^2}A_v^2$$

$$= (\overline{v_n^2} + \overline{i_n^2}R_s^2 + \overline{v_{R_s}^2})\left(\frac{R_{in}}{R_{in}+R_s}\right)^2 A_v^2$$

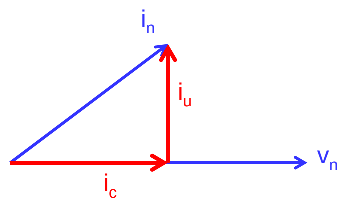
Optimum Source Impedance

- **Can use this to optimize source impedance for minimum added noise from two-port (noise figure):**

$$R_n \equiv \frac{v_n^2}{4kT\Delta f} \qquad G_n \equiv \frac{i_n^2}{4kT\Delta f}$$

$$R_{opt} = \sqrt{\frac{R_n}{G_n}} = \sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}}$$

Correlated Noise Sources



$$i_n = i_c + i_u$$

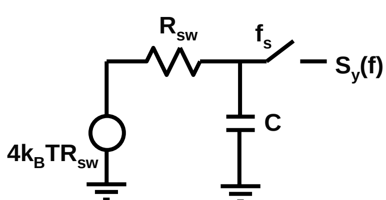
$$< i_u, v_n >= 0$$

$$i_c = Y_C v_n$$

- **Partition i_n into two components:**
  - **Correlated ("parallel") to v_n**
  - **Uncorrelated ("perpendicular") to v_n**

- **Can use this to re-derive optimum source Z**

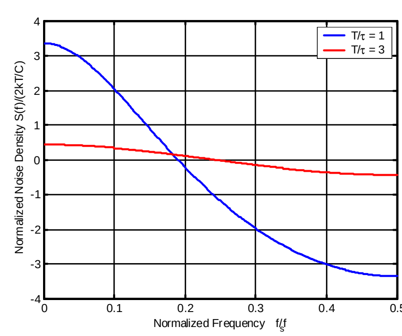
Loose Ends: Sampled Noise Spectrum



$$S_y(f) = \frac{k_B T_r}{C} \frac{2}{f_s} \frac{1 - e^{-2a}}{1 + e^{-2a}(1 - \cos 2\pi f T)}$$

$$a = \frac{T}{R_{sw}C} = \frac{T}{\tau} \qquad \text{and} \qquad T = \frac{1}{f_s}$$

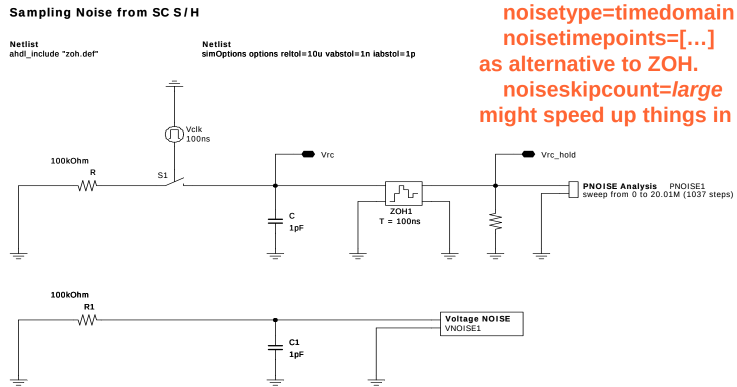
$$\int_0^{f_s/2} S_y(f)\,df = \frac{k_B T_r}{C}$$



- What if RC doesn't completely settle every cycle?
  - Noise between samples correlated → spectrum not white
- If T/τ > 3, correlation small
  - Sampled spectrum white
  - In practice usually the case

Loose Ends: Periodic Noise Analysis

SpectreRF PNOISE: check
noisetype=timedomain
noisetimepoints=[…]
as alternative to ZOH.
noiseskipcount=*large*
might speed up things in this case.




```
PSS pss period=100n maxacfreq=1.5G errpreset=conservative
PNOISE ( Vrc_hold 0 ) pnoise start=0 stop=20M lin=500 maxsideband=10
```