

## EECS240 – Spring 2010

### Lecture 3: MOS Models for Design



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### Why Not Square Law?

- Square law model most widely known:

$$I_{D,sat} = \frac{1}{2} \cdot \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{GS} - V_{th})^2$$

- But, totally inadequate for “short-channel” behavior
- Also doesn’t capture *moderate* inversion
  - (i.e., in between sub-threshold and strong inversion)

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### Why Modeling?

- Analog circuits more sensitive to detailed transistor behavior
  - Precise currents, voltages, etc. matter
  - Digital circuits have much larger “margin of error”
- Models allow us to reason about circuits
  - Provide window into the physical device and process
  - “Experiments” with SPICE much easier to do

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### Square Law Model Assumptions

- Charge density determined only by vertical field
- Drift velocity set only by lateral field
- Neglect diffusion currents (“magic”  $V_{th}$ )
- Constant mobility
- And many more...

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### Levels of Abstraction

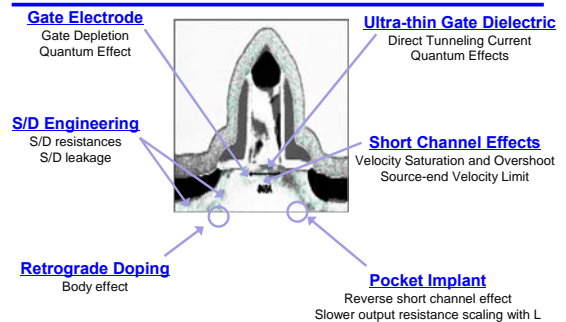
- Best abstraction depends on questions you want to answer
- Digital functionality:
  - MOSFET is a switch
- Digital performance:
  - MOSFET is a current source and a switch
- Analog characteristics:
  - MOSFET described by BSIM with 100’s of parameters?
  - MOSFET described by measurement results?

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### A Real Transistor



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## To Make Matters Worse...

- Run-to-run parameter variations:
  - E.g. implant doses, layer thickness, dimensions
  - Affect  $V_{TH}$ ,  $\mu$ ,  $C_{ox}$ ,  $R_{sq}$  ...
- In SPICE use device “corners”: nominal / slow / fast parameters (tt, ss, ff)
  - E.g. fast: low  $V_{TH}$ , high  $\mu$ , high  $C_{ox}$ , low  $R_{sq}$
  - Combine with supply & temperature extremes
  - Pessimistic but numerically tractable
    - improves chances for working Silicon

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## $V_{TH}$ : Halo Doping



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## Corner example: $V_{TH}$



- Corners just shift  $V_{th}$ 
  - Probably not real
  - (PMOS doesn't look real anyways)
- Variations probably bigger than reality too
  - Fab wants you to buy everything they make

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## $V_{TH}$ : Reverse Short-Channel Effect

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## Now What?

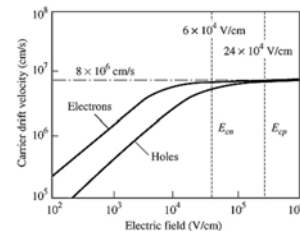
- Rely purely on simulator to tell us how devices behave?
  - Models not always based on real measurements
  - Model extraction is hard
  - Models inherently compromise accuracy for speed
- Need to know about important effects
  - So that know what to look for
  - Model might be wrong, or doesn't automatically include some effects
    - E.g., gate leakage

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## $I_D$ : Velocity Saturation



- Drift velocity initially increases linearly with field
- Eventually carriers hit a “speed limit”
- In the limit,  $I_D \propto (V_{GS} - V_{th})$

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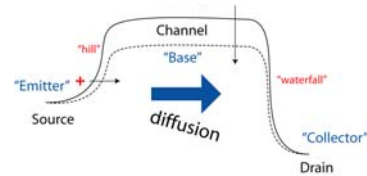
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### I<sub>D</sub>: Vertical Field Mobility Reduction

- Mobility actually depends on gate field
  - “Hard to run when there is wind blowing you sideways (into a wall)”
- More technical explanation:
  - E-field pushes carriers close to the surface
  - Enhanced scattering lowers mobility

$$\mu = \frac{\mu_0}{1 + \theta(V_{GS} - V_T) + \theta_B V_{SB}}$$

### I<sub>D</sub>: Weak Inversion Current

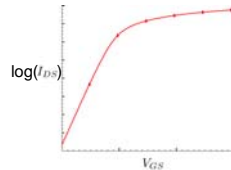


- Current set by diffusion – borrow BJT equation:

$$I_{ds} = \frac{W}{L} I_{ds,0} e^{\frac{q(V_{GS} - V_T)}{n k T}} \left( 1 - e^{\frac{-q V_{DS}}{k T}} \right)$$

### I<sub>D</sub>: Sub-Threshold Region

- Current doesn't really go to 0 at  $V_{GS} = V_{th}$



- Lateral BJT:

### I<sub>D</sub>: Operating in Weak Inversion

- Usually considered “slow”:
  - “large”  $C_{GS}$  for “little” current drive (see later)
- But, weak (or moderate) inversion becoming more common:
  - Low power
  - Submicron L means “high speed” even in weak inversion
- Not well modeled, matching poor:
  - $V_{TH}$  mismatch amplified exponentially
  - Avoid in mirrors

### I<sub>D</sub>: Weak Inversion Channel Potential

- “Base” controlled through capacitive divider

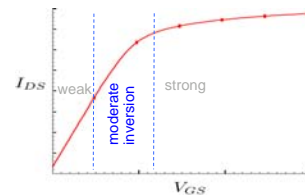
$$\delta V_{ch} \approx \frac{C_{ox}}{C_{dep} + C_{ox}} \delta V_g = \frac{\delta V_g}{n}$$

- Non-ideality factor of channel control  $n > 1$ :

$$n = 1 + \frac{C_{dep}}{C_{ox}} = 1 + \frac{c_{dep} t_{ox}}{\epsilon_{ox} t_{dep}}$$

- ( $n$  varies somewhat with bias – const. approx. usually OK)

### I<sub>D</sub>: Moderate Inversion



- Moderate inversion: both *drift* and *diffusion* contribute to the current.
- Closed form equations for this region don't really exist.

## $I_D$ : Patching Models?

- Have “good” models for weak inversion and strong inversion.
  - Why not just interpolate in between?
- Example (EKV):

$$I_{DS} = \frac{W}{L} \mu C_{ox} (2n) \left( \frac{kT}{q} \right)^2 \left( \left( \ln \left( 1 + e^{\frac{q(V_{GS} - V_{th0} - \eta V_{DS})}{2kT}} \right) \right)^2 - \left( \ln \left( 1 + e^{\frac{q(V_{GS} - V_{th0} - \eta V_{DS})}{2kT}} \right) \right)^2 \right)$$

## Output Resistance: SCBE

- “Substrate Current Body Effect”
- At high electric fields, get “hot” electrons
  - Have enough energy to knock electrons off Si lattice (impact ionization)

- Extra  $e^-$  -  $h^+$  pairs – extra (substrate) current
  - Models usually empirical

$$I_{sub} = \frac{A_i}{B_i} I_{ds} (V_{ds} - V_{d sat}) \exp \left( - \frac{B_i l}{V_{ds} - V_{d sat}} \right)$$

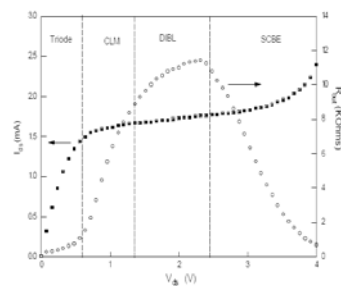
## Output Resistance: CLM

- “Channel Length Modulation”
  - Depletion region varies with  $V_{DS}$
  - Changes effective channel length

- If perturbation is small:

$$I \propto \frac{1}{L - \delta L(V_{ds})} \approx \frac{1}{L} \left( 1 + \frac{\delta L(V_{ds})}{L} \right) \longrightarrow \frac{I_{ds}}{I_{ds0}} = (1 + \lambda V_{ds})$$

## Output Resistance Mechanisms



- All effects active simultaneously
- CLM at relatively low fields
- DIBL dominates for high fields
- SCBE at very high fields

## Output Resistance: DIBL

- “Drain Induced Barrier Lowering”

- Drain controls the channel too
  - Charge gets imaged – lowers effective  $V_{th}$
  - Model with  $V_{th} = V_{th0} - \eta V_{DS}$

## Comprehensive Model: BSIM

- **Berkeley Short-channel IGFET Model (BSIM)**
  - Industry standard model for modern devices
  - BSIM3v3 is model for this course
- Typically 40-100+ parameters
  - Advanced software and expertise needed to perform extraction

## BSIM “Hand Calculation” Model

- Requires many, many, many... assumptions

- Vertical mobility degradation:

Define:  $u_d = \frac{UA}{t_{ox}}$  mobility degradation coefficient  
 $u_d = 0.5V^{-1}$  for  $t_{ox}=10nm$

- Velocity saturation:

Define:  $E_c = \frac{2v_{sat}}{U_0}$  critical E-field for velocity saturation  
 $E_c = 2 \times 10^4 V/cm$  (typical value)

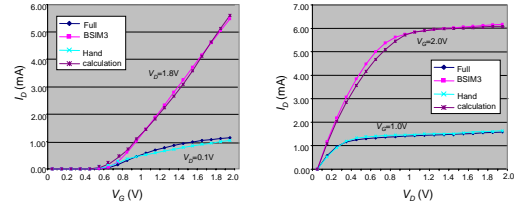
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## Fitting Results

### Comparison between full and simplified model



Parameter detail: TSMC 0.18um process  
 $t_{ox} = 4.1nm$ ,  $W=10mm$ ,  $V_{FB}=-0.39V$

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## Strong Inversion Current

$$V_{Dsat} = (V_G - V_T) \left[ \frac{1 + u_d (V_G - V_T)}{1 + \left( u_d + \frac{1}{E_c L} \right) (V_G - V_T)} \right]$$

$$I_{Dlin} = \mu_0 C_{ox} \frac{W}{L} \left( V_G - V_T - \frac{V_D}{2} \right) \left[ \frac{1}{1 + u_d (V_G - V_T) + \left( \frac{V_D}{E_c L} \right)} \right] = I_{Dsat(long)} \left[ \frac{1}{1 + u_d (V_G - V_T) + \left( \frac{V_D}{E_c L} \right)} \right]$$

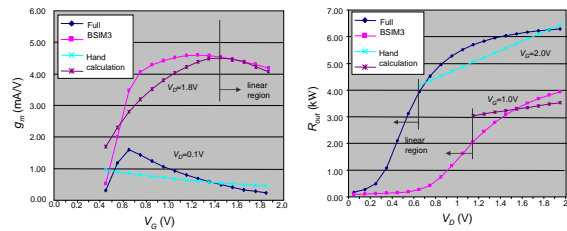
$$I_{Dsat} = \mu_0 C_{ox} \frac{W}{2L} \left[ \frac{(V_G - V_T)^2}{1 + \left( u_d + \frac{1}{E_c L} \right) (V_G - V_T)} \right] = I_{Dsat(long)} \left[ \frac{1}{1 + \left( u_d + \frac{1}{E_c L} \right) (V_G - V_T)} \right]$$

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## Weakness of Model First Derivatives



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## Equations of Derivatives

$$g_{mout} = \frac{I_{Dsat}}{(V_G - V_T)} \left[ 1 + \frac{I_{Dsat}}{I_{Dsat(long)}} \right] = \frac{I_{Dsat}}{(V_G - V_T)} \left[ 1 + \frac{1}{1 + \left( u_d + \frac{1}{E_c L} \right) (V_G - V_T)} \right]$$

$$r_{out} = \frac{2[(V_D - V_{Dsat}) + [1 + u_d (V_G - V_T)](V_G - V_T)]L^2}{\mu_0 C_{ox} W I_{PCLM} [1 + u_d (V_G - V_T)](V_G - V_T)^2}$$

$$= \frac{[(V_D - V_{Dsat}) + [1 + u_d (V_G - V_T)](V_G - V_T)]L}{I_{Dsat(long)} I_{PCLM} [1 + u_d (V_G - V_T)]}$$

with  $l = \sqrt{3\mu_0 X_j}$

- Required parameters  $W, L, TOX, U_0, UA, VSAT, VTH0, PCLM, XJ$

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## “Hand Model” Conclusion

- Even “simple” model is not convenient
  - $r_o$  is key for gain, but really hard to model
  - Missing important regions such as moderate inversion
- Hand models really best to build intuition
- But for design (i.e., how to choose  $W, L$ , etc.):
  - Will learn how to use the simulator as a “calculator”

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