EECS240 – Spring 2010

Lecture 5: Electronic Noise

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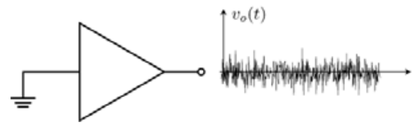
Electronic Noise

- Why is noise important?
  - Sets minimum signals we can deal with – often sets lower limit on power
- Signal-to-noise ratio
  - Signal Power $P_{sig} \sim (V_{DD})^2$
  - Noise Power $P_{noise} \sim k_B T/C$
  - SNR = P_{sig} / P_{noise}
- Technology Scaling
  - V_{DD} goes down → lower signal
  - Increase C to compensate → increases power

Types of "Noise"

- Interference
  - Not "fundamental" – deterministic
  - Signal coupling
    - Capacitive, inductive, subtrate, etc.
  - Supply noise
- <u>Device noise</u>
  - Caused by discreteness of charge
  - "fundamental" – thermal noise
  - "manufacturing process related" – flicker noise

Noise in Amplifiers

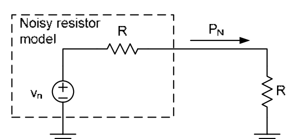


- All amplifiers generate noise
  - Comes from carrier random thermal motion and discreteness of charge
- Noise is <u>random</u>
  - Has to be treated statistically – can't predict actual value
  - Deal with mean (average), variance, spectrum

Thermal Noise of a Resistor

- Origin: Brownian Motion
  - Thermally agitated particles
  - E.g. ink in water, electrons in a conductor
- Available noise power: $P_N = k_B T \Delta f$
  - Noise power in bandwidth Δf delivered to a matched load
  - Example: Δf = 1Hz → P_N = 4 x 10^{-21}W = -174 dBm
  - Reference: J.B. Johnson, "Thermal Agitation of Electricity in conductors," Phys. Rev., pp. 97-109, July 1928.

Resistor Noise Model



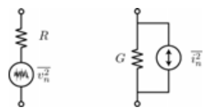
$$P_N = k_B T \Delta f = \frac{\overline{v_n^2}}{4R}$$

Mean square noise voltage:

$$\overline{v_n^2} = 4 k_B T R \Delta f$$

Thermal Noise

- Present in all dissipative elements
  - I.e., resistors
- Independent of DC current flow
- Random fluctuations of *v*(*t*) or *i*(*t*)
  - Mean is 0
  - Distribution (pdf) is Gaussian
  - Power spectral density is “white”
    - Up to ~THz frequencies
  - $k_BT = 4 \times 10^{-21}$ J (T = 290K = 16.9°C)
- Example:
  R = 1kΩ → <u>4nV/rt-Hz</u>
  1MHz bandwidth → σ = <u>4uV</u>



$$\overline{v_n^2} = 4k_BTRB \qquad \overline{i_n^2} = \frac{4k_BTB}{R}$$

Thermal Noise in Capacitors?

Noise Calculations

- Noise calculations
  - Instantaneous voltages add
  - Power spectral densities add
  - RMS voltages do <u>NOT</u> add
- Example: R_1+R_2 in series

Calculating Noise in Passive Networks

- Capacitors and inductors only shape spectrum:

$$\overline{v_{on,T}^2}(f) = \sum_x \left|H_x(s)\right|^2_{s=2\pi jf} \overline{v_x^2}(f)$$

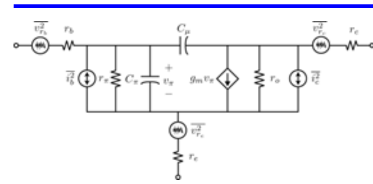
Noise in Diodes

- Shot noise
  - Zero mean, Gaussian pdf, white
  - Proportional to current
  - Independent of temperature

$$\overline{i_n^2} = 2qI_D\Delta f$$

- Example:
  I_D = 1mA → <u>17.9pA/rt-Hz</u>
  1MHz bandwidth → σ = <u>17.9nA</u>
- Shot noise versus thermal noise
  - $g_{diode} = I_d/(k_bT/q)$
  - Thermal noise density: $4k_bTg_{diode} = 4qI_d$
  - Shot noise half of this (current flow in 1 direction)

BJT Noise



$$\overline{v_b^2} = 4k_BTr_b\Delta f$$

$$\overline{i_b^2} = 2qI_B\Delta f + K_1 I_B^{\alpha}\frac{\Delta f}{f}$$

$$\overline{i_c^2} = 2qI_C\Delta f$$

- Just like diodes: shot noise
  - Collector and base noise partially correlated
- Extrinsic resistors contribute noise
  - Small signal resistors (e.g., r_o) don’t
  - These aren’t physical resistors

Triode MOSFET Noise

- **Channel resistance contributes thermal noise**
- **Channel conductance when $V_{ds} = 0$:**

$$g_{ds0} = \mu C_{ox} \frac{W}{L} (V_{GS} - V_{th})$$

- **Device is truly a resistor when $V_{ds} = 0$, so:**

$$\overline{i_d^2} = 4kTg_{ds0}\Delta f$$

Saturation Noise

- **Noise distributed along the channel:**

$$\overline{i_d^2} = 4kT\frac{2}{3}\frac{W}{L}\mu C_{ox}(V_{GS} - V_T)\Delta f$$

- **For long channel model, can substitute γg_m for g_{ds0}**

$$\overline{i_d^2} = 4kT\gamma g_m \Delta f$$

- **More correct formulation uses inversion charge in the channel [Tsividis]**
  - **This is what SPICE/BSIM use**

Thermal Noise for Short Channels

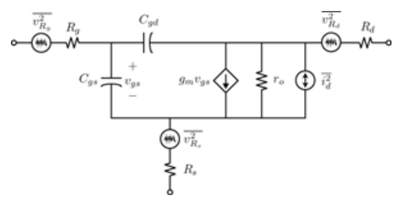
- **Strong inversion → thermal noise**
  - **Drain current: g_{ds0} is what you really care about**

$$\overline{i_d^2} = 4kT\gamma g_{ds0}\Delta f \; = 4kT\frac{\gamma}{\alpha}g_m\Delta f$$

- **g_m more convenient for input-referred noise**
  - **For low field (long L), $\gamma = 2/3$ relates g_m to g_{ds}**
  - **For high field, use α to capture increase in noise**
  - **High-field noise can be 2-3 times larger than low field**
- **MOS actually has intrinsic gate induced noise (142/242 topic)**
- **Gate leakage → shot noise**

Weak Inversion Noise

FET Noise Model



- **Model neglects intrinsic gate noise**
- **BSIM3 does not directly include α**

1/f Noise

- **Flicker noise**
  - **$K_{f,NMOS} = 2.0 \times 10^{-29}$ AF**
    $K_{f,PMOS} = 3.5 \times 10^{-30}$ AF
  - **Strongly process dependent**

$$\overline{i_{1/f}^2} = \frac{K_f I_D}{L^2 C_{ox}} \frac{\Delta f}{f}$$

- **<u>Example</u>: $I_D = 10\mu A$, $L = 1\mu m$,**
  - **$C_{ox} = 5.3fF/\mu m^2$, $f_{hi} = 1MHz$**

f_{lo} = 1Hz → σ = 722pA
f_{lo} = 1/year → σ = 1083pA

$$\overline{i_{1/f,total}^2} = \int_{f_{lo}}^{f_{hi}} \frac{K_f I_D}{L^2 C_{ox}} \frac{df}{f} = \frac{K_f I_D}{L^2 C_{ox}} \ln \frac{f_{hi}}{f_{lo}}$$

1/f Noise Corner Frequency

- **Definition (MOS)**

$$\frac{K_f I_D}{L^2 C_{ox}} \frac{\Delta f}{f_{co}} = 4 k_B T_r \gamma g_m \Delta f \qquad f_{co} = \frac{K_f I_D}{L^2 C_{ox}} \frac{1}{4 k_B T_r \gamma g_m}$$

$$= \frac{K_f}{4 k_B T_r \gamma C_{ox}} \frac{1}{L^2} \frac{1}{g_m/I_D}$$

$$= \frac{K_f}{8 k_B T_r \gamma C_{ox}} \frac{V^*}{L^2}$$

- **<u>Example</u>:**
  - **V* = 200mV, γ = 1**

		NMOS	***PMOS***
L = 0.35μm	→	**192kHz**	**34kHz**
L = 1.00μm	→	**24kHz**	**4kHz**

Noise Calculations with Actives

- **Method:**
  1) **Create small-signal model**
  2) **All inputs = 0 (linear superposition)**
  3) **Pick output v_o or i_o**
  4) **For each noise source v_x, i_x**
     Calculate $H_x(s) = v_o(s) / v_x(s)$ (… i_o, i_x)
  5) **Total noise at output is:**

$$\overline{v_{on,T}^2(f)} = \sum_x \left|H_x(s)\right|^2_{s=2\pi jf} \overline{v_x^2(f)}$$

$$\text{simpler notation:} \quad \overline{v_{on,T}^2(f)} = S_n(f)$$

Tedious but simple …

SPICE Noise Analysis