1) $x = \sqrt{R} \Rightarrow x^2 = R \Rightarrow x^2 - R \Rightarrow f(x) = x^2 - R$ Formula 1) $f(x) = x^2 - R$

Find Derivative, $f'(x) = 2x$ Formula 2) $f'(x) = 2x$

$$P_{n+1} = P_n - \frac{f(P_n)}{f'(P_n)} \Rightarrow x - \frac{x^2 - R}{2x}$$ is Equation for Newton's Method given $x = \sqrt{R}$.

Newton's Method's approximation depends heavily on the initial guess X_0. The reason is due to the error terms growth in Newton's Method which is displayed by the formula $|r - P_{n+1}| < C(f)\,|r - P_n|^2$. Firstly we can see that picking a bad X_0 can lead to a $|r - P_n| > 1$ which mean the error grows drastically in comparison to the bisection method $|r - P_{n+1}| < \frac{1}{2}|r - P_n|$. Picking a bad X_0 can also lead to a high $C(\delta)$ term which grows the error term more. Inversly a good X_0 can lead to an exponentially decreasing error term with $|r - P_n| < 1$ and a $C(f) < 1$ which can give you extremly precise results faster then the bisection method or false position methods. (Bad X_0 can result in divergence).

2) $f(x) = \frac{x^2}{1+x^2}$ $\qquad P_{n+1} = x - \left(\frac{x^2}{1+x^2} \cdot \frac{(1+x^2)^2}{2x}\right) = x - \frac{x(1+x^2)}{2} = x - \frac{x+x^3}{2}$

$f'(x) = \frac{2x}{(1+x^2)^2}$ $\qquad \Rightarrow \frac{x-x^3}{2} \Rightarrow \frac{x(1-x^2)}{2}$

Find Roots of P_{n+1}, get $x = 0, 1, -1$. (cannot Divide by 0)

Values which are not 0, 1, or -1 should converge to the root that also satisfy $-1 < x_0 < 1$. This range is pretty safe, and should converge well given the $|r - P_n|^2 < 1$ property of this problem. Conditions where $x_0 > 2$ or $x_0 < -2$ will most likely diverge.

3) $f(x) = 2x^3 - 9x^2 + 12x + 15$ $\qquad P_{n+1} = P_n - \frac{f(P_n)}{f'(P_n)}$

$f'(x) = 6x^2 - 18x + 12$

$P_{n+1} = x - \frac{2x^3-9x^2+12x+15}{6x^2-18x+12}$

$\Rightarrow \frac{2x^3-9x^2+12x+15}{6(x^2-3x+2)} \Rightarrow \frac{2x^3-9x^2+12x+15}{6(x-2)(x-1)}$

$X_0 = 3 \quad 3 - \frac{2(3)^3-9(3)^2+12(3)+15}{6(3-2)(3-1)} = 3 - \frac{2\cdot 27 - 9\cdot 9 + 36 + 15}{6\cdot 1\cdot 2} = 3 - \frac{54-81+51}{12}$

$\Rightarrow 3 - \frac{105-81}{12} \Rightarrow 3 - \frac{24}{12} \Rightarrow 3-2 \Rightarrow 1 \qquad X_1 = 1$

$1 - \frac{2(1)^3-9(1)^2+12(1)+15}{6(1)^2-18(1)+12} = 1 - \frac{\text{stuff}}{0}$ $\qquad X_0 = 3$ doesn't lead to convergence.

$X_0 = 4 \quad (x>3) \qquad 4 - \frac{2(4)^3-9(4)^2+12(4)+15}{6(4-2)(4-1)} \Rightarrow 4 - \frac{128-144+48+15}{96-72+12}$

$\Rightarrow 4 - \frac{47}{36} = \frac{97}{36} \Rightarrow$ ugly number, division by 0 is unlikely, leading to covergence.

$X_0 = 2 \quad (x<3) \qquad 2 - \frac{\text{stuff}}{6(2-2)(2-1)} = 2 - \frac{\text{stuff}}{0}$ $\qquad X_0 = 2$ doesn't lead to convergence.

$X_0 = 3$ dosen't converge

$X_0 > 3$ Probably converges

$X_0 < 3$ Probably Convergences

$X_0 = 1$ or $X_0 = 2$ doesn't converge. $\rightarrow$

$\boxed{\text{(if } x \text{ becomes 1 or 2 ever, then no convergence, this goes for any } X_0 \text{ the main goal is to hop over these values.)}}$

4) $f(x) = x^m - R$ vs $f(x) = 1 - \frac{R}{x^m}$

$f'(x) = m x^{(m-1)}$ vs $f'(x) = m R x^{(-m-1)}$

$P_{n+1} = X - \frac{x^m - R}{m x^{(m-1)}}$ v.s $P_{n+1} = X - \frac{1 - \frac{R}{x^m}}{m R x^{(-m-1)}}$

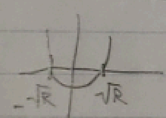
$R > 0, m \geq 2$

~~so R = 1, m = 2~~

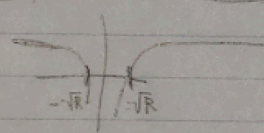
$\cancel{P_{n+1} = X - \frac{x^2 - 1}{2x}}$ ~~vs~~ $\cancel{P_{n+1} = X - \frac{1 - \frac{1}{x^2}}{2x^{(-3)}}}$ ~~or~~ $\cancel{X - 2x^3 + 2x}$

The first formula $(x^m - R)$ is better in this case because it's not prone to divergence. When $m \geq 2$.

Formula 1



Formula 2



In the first case, badly picked X_0's can still converge. But badly picked X_0's will result in divergence in the second formula.