The Statistical Analysis of U.S. Quarterly GDP Growth Rate

Xiye Zhong

2021/5/9

Abstract

In this project, we analyze the quarterly GDP growth rate of U.S. and there are two candidate models proposed. After testing their significance and diagnostics, an ARIMA(0,1,2) model is selected to predict the future 10 quarters with 95% confidence interval. Through spectrum analysis, the first three predominant periods and the upper and lower bounds are also calculated.

Introduction

U.S. as the world's largest economy plays a significant role in worldwide economic activity. Investigating GDP of U.S. can appeal the potential model of its growth rate and help people to predict the future economy. In this project, the analysis of quarterly U.S. GDP would be considered. The data starts at 1947(1) and ends at 2018(3); it has n = 287 observations. Figure 1 shows a plot of data,

$$y_t$$

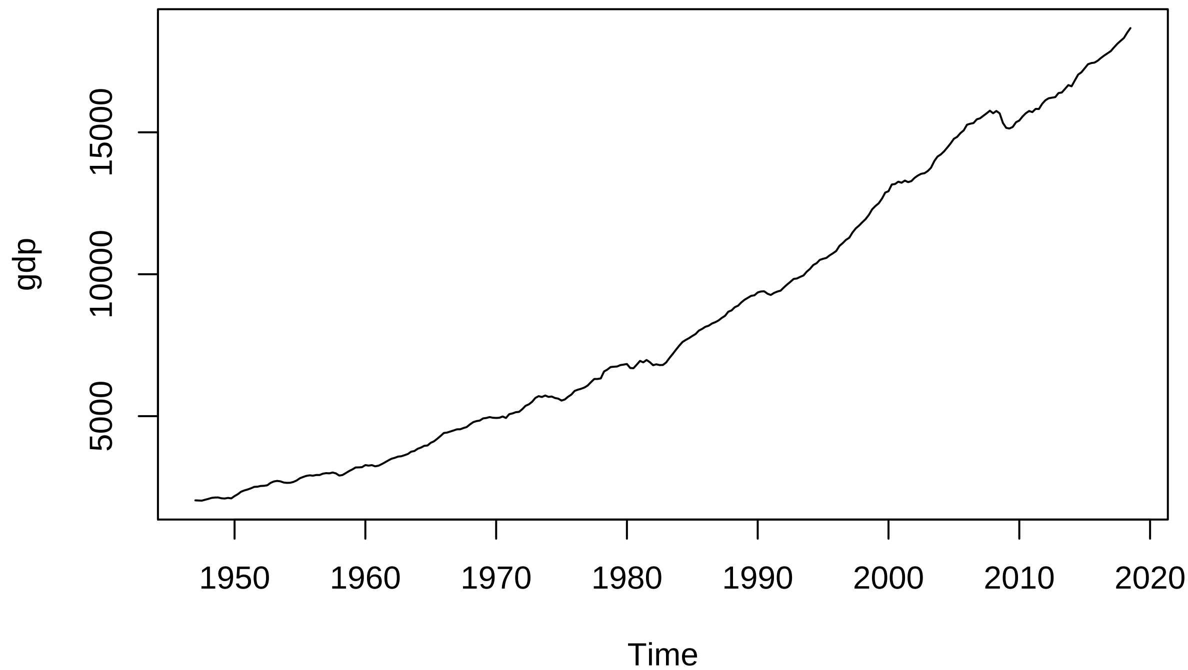
, but it has a strong trend and the sample ACF plot also shows a slow decay which indicates that log transformation and differencing may be needed.

```r
data("gdp")
gdp
```

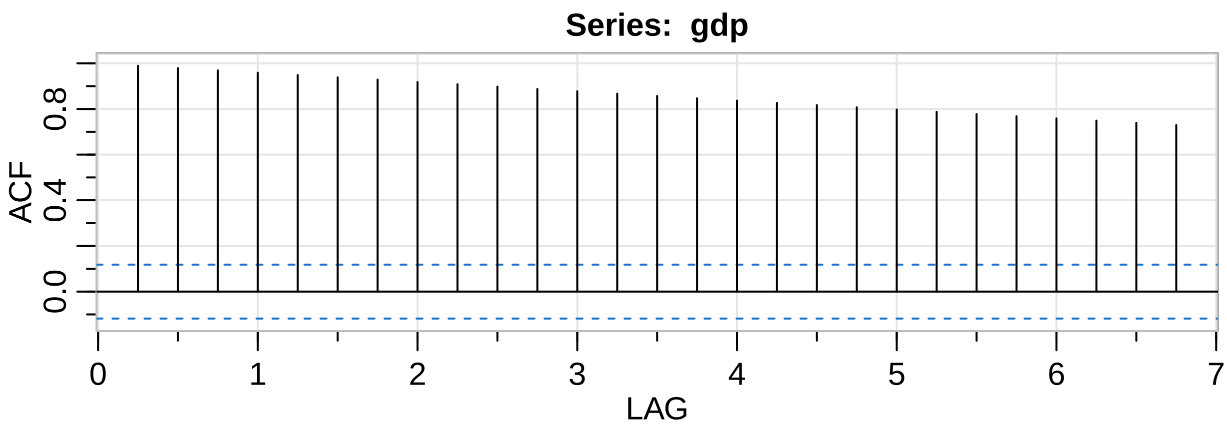
```
##          Qtr1     Qtr2     Qtr3     Qtr4
## 1947 2033.061 2027.639 2023.452 2055.103
## 1948 2086.017 2120.450 2132.598 2134.981
## 1949 2105.562 2098.380 2120.044 2102.251
## 1950 2184.872 2251.507 2338.514 2383.291
## 1951 2415.660 2457.517 2508.166 2513.690
## 1952 2540.550 2546.022 2564.401 2648.621
## 1953 2697.855 2718.709 2703.411 2662.482
## 1954 2649.755 2652.643 2682.601 2735.091
## 1955 2813.212 2858.988 2897.598 2914.993
## 1956 2903.671 2927.665 2925.035 2973.179
## 1957 2992.219 2985.663 3014.919 2983.727
## 1958 2906.274 2925.379 2993.068 3063.085
## 1959 3121.936 3192.380 3194.653 3203.759
## 1960 3275.757 3258.088 3274.029 3232.009
## 1961 3253.826 3309.059 3372.581 3438.721
## 1962 3500.054 3531.683 3575.070 3586.827
## 1963 3625.981 3666.669 3747.278 3771.845
## 1964 3851.366 3893.296 3954.121 3966.335
## 1965 4062.311 4113.629 4205.086 4301.973
## 1966 4406.693 4421.747 4459.195 4495.777
```

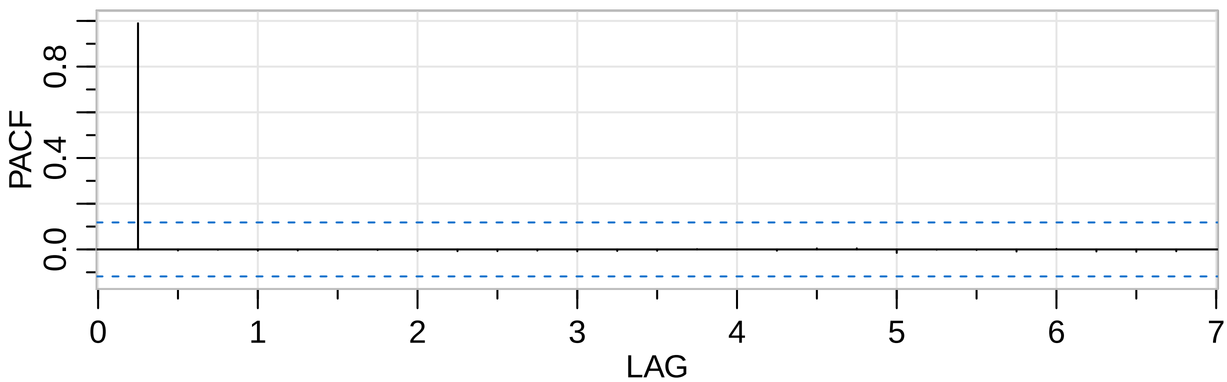
```
## 1967  4535.591  4538.370  4581.309  4615.853
## 1968  4709.993  4788.688  4825.799  4844.779
## 1969  4920.605  4935.564  4968.164  4943.935
## 1970  4936.594  4943.600  4989.159  4935.693
## 1971  5069.746  5097.179  5139.128  5151.245
## 1972  5245.974  5365.045  5415.712  5506.396
## 1973  5642.669  5704.098  5674.100  5727.960
## 1974  5678.713  5692.210  5638.411  5616.526
## 1975  5548.156  5587.800  5683.444  5759.972
## 1976  5889.500  5932.711  5965.265  6008.504
## 1977  6079.494  6197.686  6309.514  6309.652
## 1978  6329.791  6574.390  6640.497  6729.755
## 1979  6741.854  6749.063  6799.200  6816.203
## 1980  6837.641  6696.753  6688.794  6813.535
## 1981  6947.042  6895.559  6978.135  6902.105
## 1982  6794.878  6825.876  6799.781  6802.497
## 1983  6892.144  7048.982  7189.896  7339.893
## 1984  7483.371  7612.668  7686.059  7749.151
## 1985  7824.247  7893.136  8013.674  8073.239
## 1986  8148.603  8185.303  8263.639  8308.021
## 1987  8369.930  8460.233  8533.635  8680.162
## 1988  8725.006  8839.641  8891.435  9009.913
## 1989  9101.508  9170.977  9238.923  9257.128
## 1990  9358.289  9392.251  9398.499  9312.937
## 1991  9269.367  9341.642  9388.845  9421.565
## 1992  9534.346  9637.732  9732.979  9834.510
## 1993  9850.973  9908.347  9955.641 10091.049
## 1994 10188.954 10327.019 10387.382 10506.372
## 1995 10543.644 10575.100 10665.060 10737.478
## 1996 10817.896 10998.322 11096.976 11212.205
## 1997 11284.587 11472.137 11615.636 11715.393
## 1998 11832.486 11942.032 12091.614 12287.000
## 1999 12403.293 12498.694 12662.385 12877.593
## 2000 12924.179 13160.842 13178.419 13260.506
## 2001 13222.690 13299.984 13244.784 13280.859
## 2002 13397.002 13478.152 13538.072 13559.032
## 2003 13634.253 13751.543 13985.073 14145.645
## 2004 14221.147 14329.523 14464.984 14609.876
## 2005 14771.602 14839.782 14972.054 15066.597
## 2006 15267.026 15302.705 15326.368 15456.928
## 2007 15493.328 15582.085 15666.738 15761.967
## 2008 15671.383 15752.308 15667.032 15328.027
## 2009 15155.940 15134.117 15189.222 15356.058
## 2010 15415.145 15557.277 15671.967 15750.625
## 2011 15712.754 15825.096 15820.700 16004.107
## 2012 16129.418 16198.807 16220.667 16239.138
## 2013 16382.964 16403.180 16531.685 16663.649
## 2014 16621.696 16830.111 17033.572 17113.945
## 2015 17254.744 17397.029 17438.802 17456.225
## 2016 17523.374 17622.486 17706.705 17784.185
## 2017 17863.023 17995.150 18120.843 18223.758
## 2018 18323.963 18511.576 18671.497
```

```
plot.ts(gdp)
```



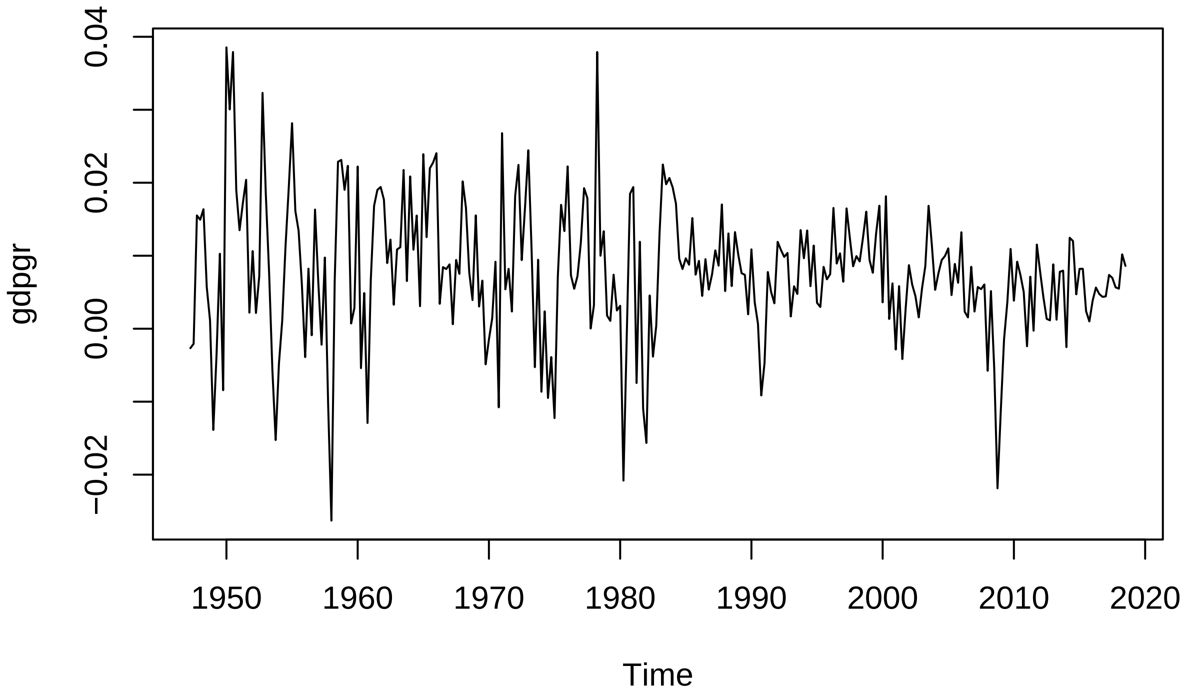

```
acf2(gdp)
```





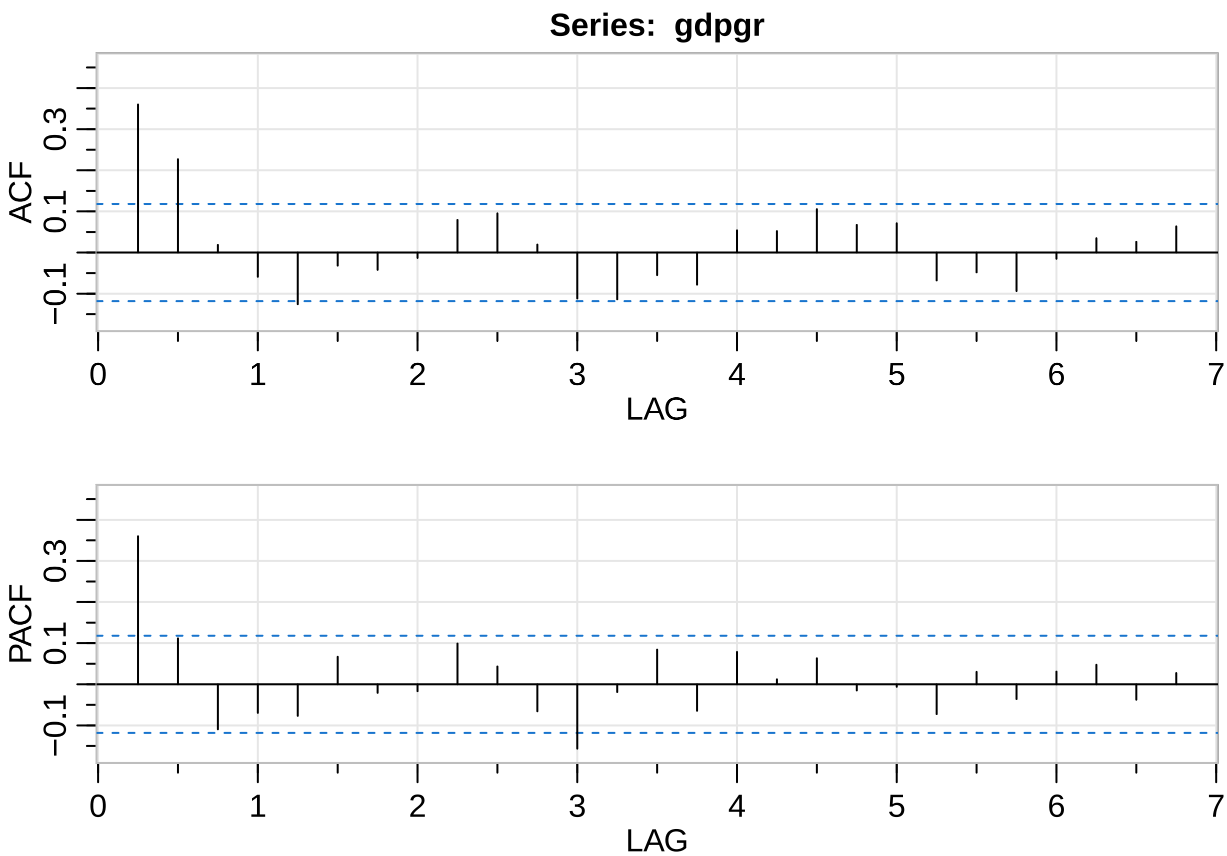

```
##      [,1] [,2] [,3]  [,4] [,5] [,6] [,7]  [,8]  [,9] [,10] [,11] [,12] [,13]
## ACF  0.99 0.98 0.97  0.96 0.95 0.94 0.93  0.92  0.91  0.90  0.89  0.88  0.87
## PACF 0.99 0.00 0.00 -0.01 0.00 0.00 0.00 -0.01 -0.01 -0.01 -0.01 -0.01 -0.01
##      [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25]
## ACF   0.86  0.85  0.84  0.83  0.82  0.81  0.80  0.79  0.78  0.77  0.76  0.75
## PACF -0.01  0.00  0.00 -0.01  0.00  0.00 -0.02  0.00  0.00 -0.01  0.00 -0.01
##      [,26] [,27]
## ACF   0.74  0.73
## PACF -0.01 -0.01
```

```
gdpgr = diff(log(gdp))
plot.ts(gdpgr)
```



gdpgr
0.04
0.02
0.00
−0.02
1950
1960
1970
1980
1990
2000
2010
2020
Time


```
acf2(gdpgr)
```




```
##      [,1] [,2]  [,3]  [,4]  [,5]  [,6]  [,7]  [,8] [,9] [,10] [,11] [,12] [,13]
## ACF  0.36 0.23  0.02 -0.06 -0.13 -0.03 -0.04 -0.01 0.08  0.10  0.02 -0.11 -0.11
## PACF 0.36 0.11 -0.11 -0.07 -0.08  0.07 -0.02 -0.02 0.10  0.04 -0.07 -0.16 -0.02
##      [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25]
## ACF  -0.05 -0.08  0.05  0.05  0.11  0.07  0.07 -0.07 -0.05 -0.09 -0.02  0.03
## PACF  0.08 -0.06  0.08  0.01  0.06 -0.02 -0.01 -0.07  0.03 -0.04  0.03  0.05
##      [,26] [,27]
## ACF   0.03  0.06
## PACF -0.04  0.03
```

Hence, in Figure 2, we plotted
$$x_t = \nabla\left(log\left(y_t\right)\right)$$
. It means the growth rate of U.S. GDP. The plot shows a more stable process. The sample ACF and PACF plots are also quickly decayed.

Since a stable process has been found. In this project, we are going to inspect the sample ACF and PACF plot to propose some candidate models for U.S. GDP growth rate. Then, those candidate models would be fitted and tested. Finally, we would find a better model from candidates to predict the future and do a spectral analysis of it.

Statistical Methods

From the sample ACF and PACF plots in Figure 2, we may feel the ACF cuts off at lag 2 and PACF tails off. It would suggest log GDP follows an ARIMA(0, 1, 2) model. Also, it appears that the ACF tails off at lag 1 and PACF cuts off at lag 1. An ARIMA(1, 1, 0) model would also be considered for log GDP. We would fit those two models in RStudio. We would use MLE to estimate the model for growth rate. Then diagnostics would be considered. The first one is standardized residuals, if the model fits well the residual would behave as an iid sequence with mean zero and variance one. The second one is checking the normality by normal Q-Q plot. If there is a departure from normality then the data does not meet the normal assumption. The third one is the ACF of residuals. The sample autocorrelations are almost independently and normally distributed with mean zero and variance 1/n. If the

$$\hat{\rho}\left(h\right)$$

is along with the bounds of

$$\pm 2/\sqrt{(n)}$$

, then the model fits well. The last one is Ljung-Box plot. It is to check for any H > 1, whether

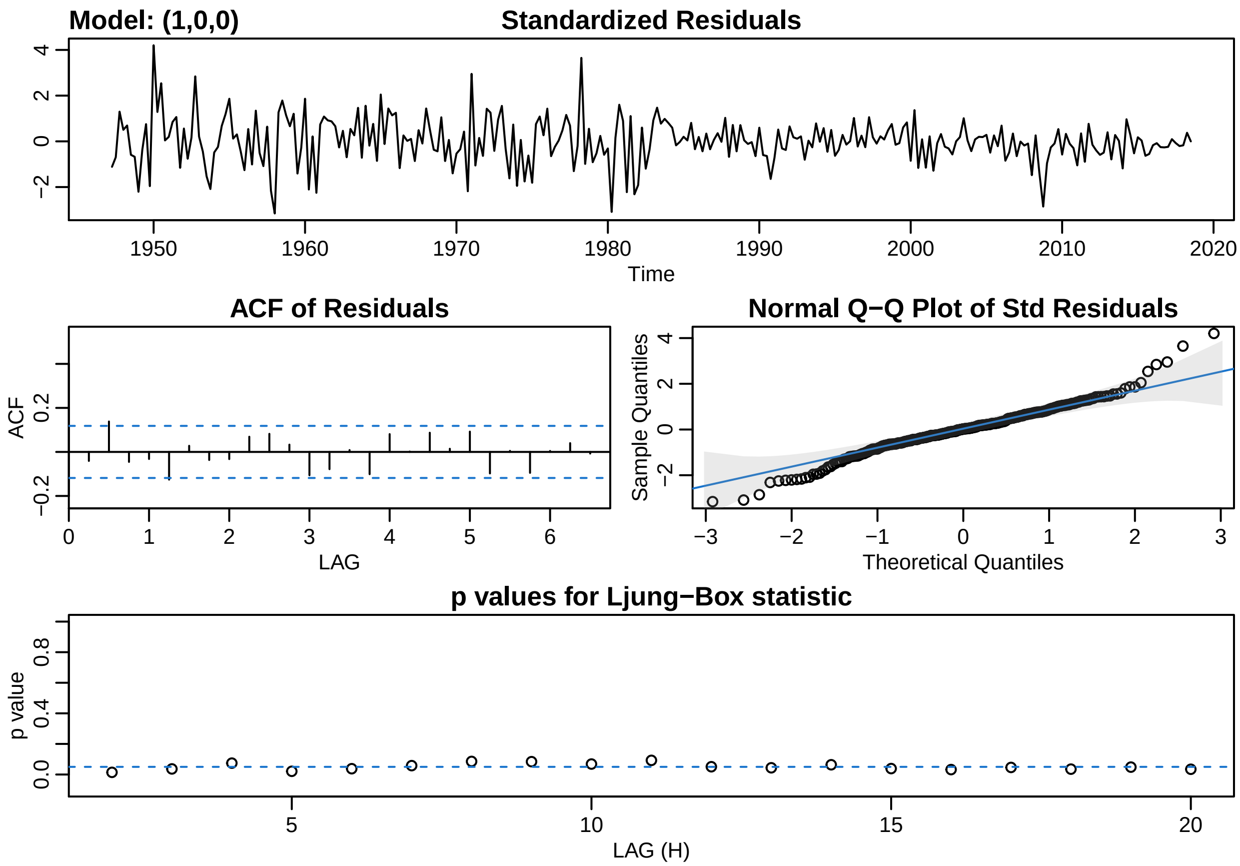
$$\rho\left(1\right) = \ldots = \rho\left(H\right) = 0$$

. If the plots are departure from 0.05, the model fits well. Moreover, we would compare the AIC, the AICc and the BIC. The smaller they are, the better the model fits.

Results

```
#fit the ARIMA(1,1,0) model
sarima(diff(log(gdp)),1,0,0)
```

```
## initial  value -4.673186
## iter   2 value -4.742918
## iter   3 value -4.742921
## iter   4 value -4.742923
## iter   5 value -4.742925
## iter   6 value -4.742925
## iter   6 value -4.742925
## final  value -4.742925
## converged
## initial  value -4.742229
## iter   2 value -4.742234
## iter   3 value -4.742245
## iter   3 value -4.742245
## iter   3 value -4.742245
## final  value -4.742245
## converged
```




```
## $fit
##
## Call:
## stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
##     Q), period = S), xreg = xmean, include.mean = FALSE, transform.pars = trans,
##     fixed = fixed, optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1   xmean
##       0.3603  0.0077
## s.e.  0.0551  0.0008
##
## sigma^2 estimated as 7.598e-05:  log likelihood = 950.47,  aic = -1894.93
##
## $degrees_of_freedom
## [1] 284
##
## $ttable
##       Estimate     SE t.value p.value
## ar1     0.3603 0.0551  6.5366       0
## xmean   0.0077 0.0008  9.5915       0
##
## $AIC
## [1] -6.625634
##
## $AICc
```

```
## [1] -6.625485
##
## $BIC
## [1] -6.587284
```

The estimated ARIMA(1,1,0) model is:

$$x_t = 0.0077_{(0.0008)}(1 - 0.3603) + 0.3603_{(0.0551)}x_{t-1} + w_t$$

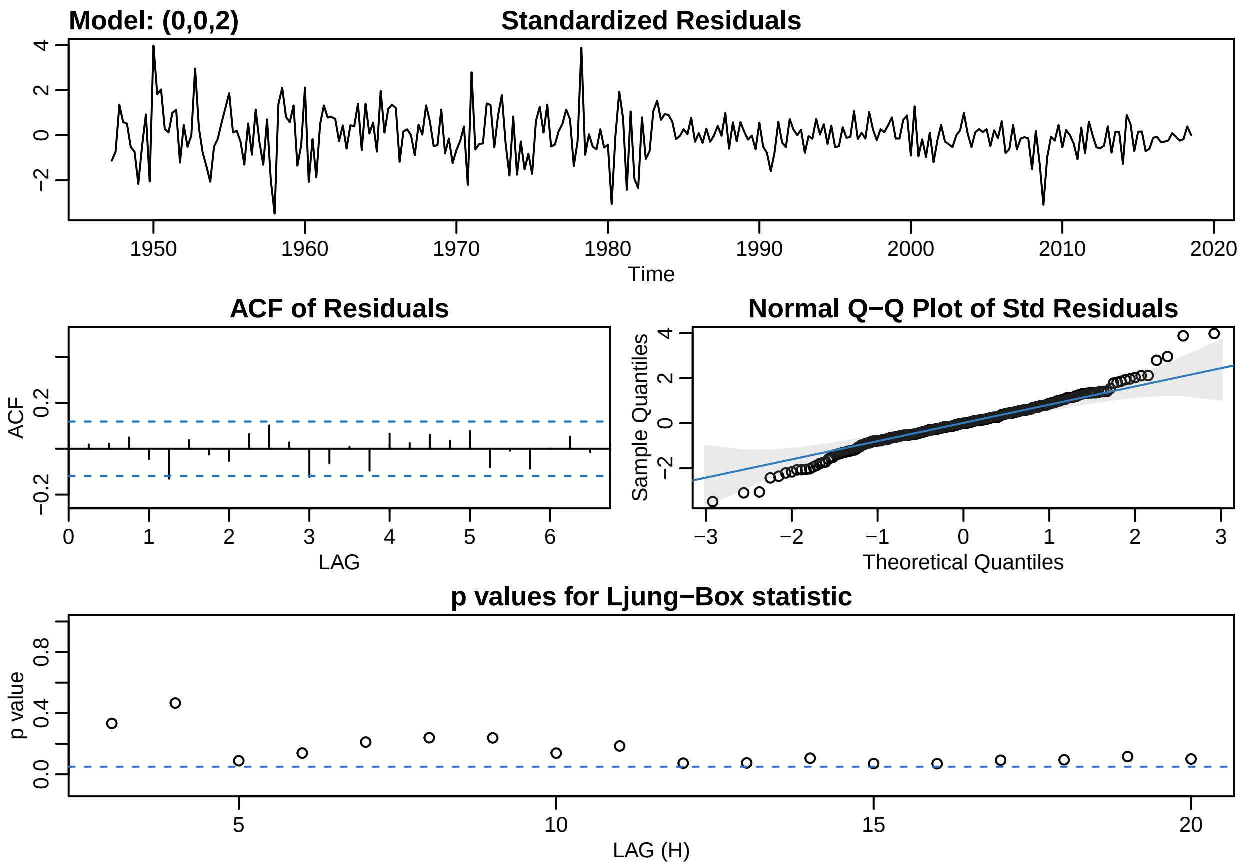
, Where

$$\sigma_w^2 = 7.599e - 05$$

on 284 degrees of freedom; the constant is 0.0049. All coefficients' p-value are less than 0.05 which indicates that both ar1 and constant are significant.

```
#fit the ARIMA(0,0,2) model
sarima(diff(log(gdp)),0,0,2)
```

```
## initial  value -4.672758
## iter   2 value -4.749239
## iter   3 value -4.750696
## iter   4 value -4.750723
## iter   5 value -4.750724
## iter   6 value -4.750725
## iter   7 value -4.750725
## iter   7 value -4.750725
## iter   7 value -4.750725
## final  value -4.750725
## converged
## initial  value -4.751078
## iter   2 value -4.751080
## iter   3 value -4.751080
## iter   4 value -4.751081
## iter   5 value -4.751081
## iter   5 value -4.751081
## iter   5 value -4.751081
## final  value -4.751081
## converged
```




```
## $fit
##
## Call:
## stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
##     Q), period = S), xreg = xmean, include.mean = FALSE, transform.pars = trans,
##     fixed = fixed, optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ma1     ma2   xmean
##       0.3070  0.2258  0.0077
## s.e.  0.0579  0.0547  0.0008
##
## sigma^2 estimated as 7.465e-05:  log likelihood = 952.99,  aic = -1897.99
##
## $degrees_of_freedom
## [1] 283
##
## $ttable
##       Estimate     SE t.value p.value
## ma1     0.3070 0.0579  5.2988       0
## ma2     0.2258 0.0547  4.1271       0
## xmean   0.0077 0.0008  9.8631       0
##
## $AIC
## [1] -6.636312
##
```

```
## $AICc
## [1] -6.636015
##
## $BIC
## [1] -6.58518
```

The estimated ARIMA(0,1,2) model is:

$$x_t = 0.0077_{(0.0008)} + w_t + 0.307_{(0.0579)} w_{t-1} + 0.2258_{(0.0547)} w_{t-2}$$

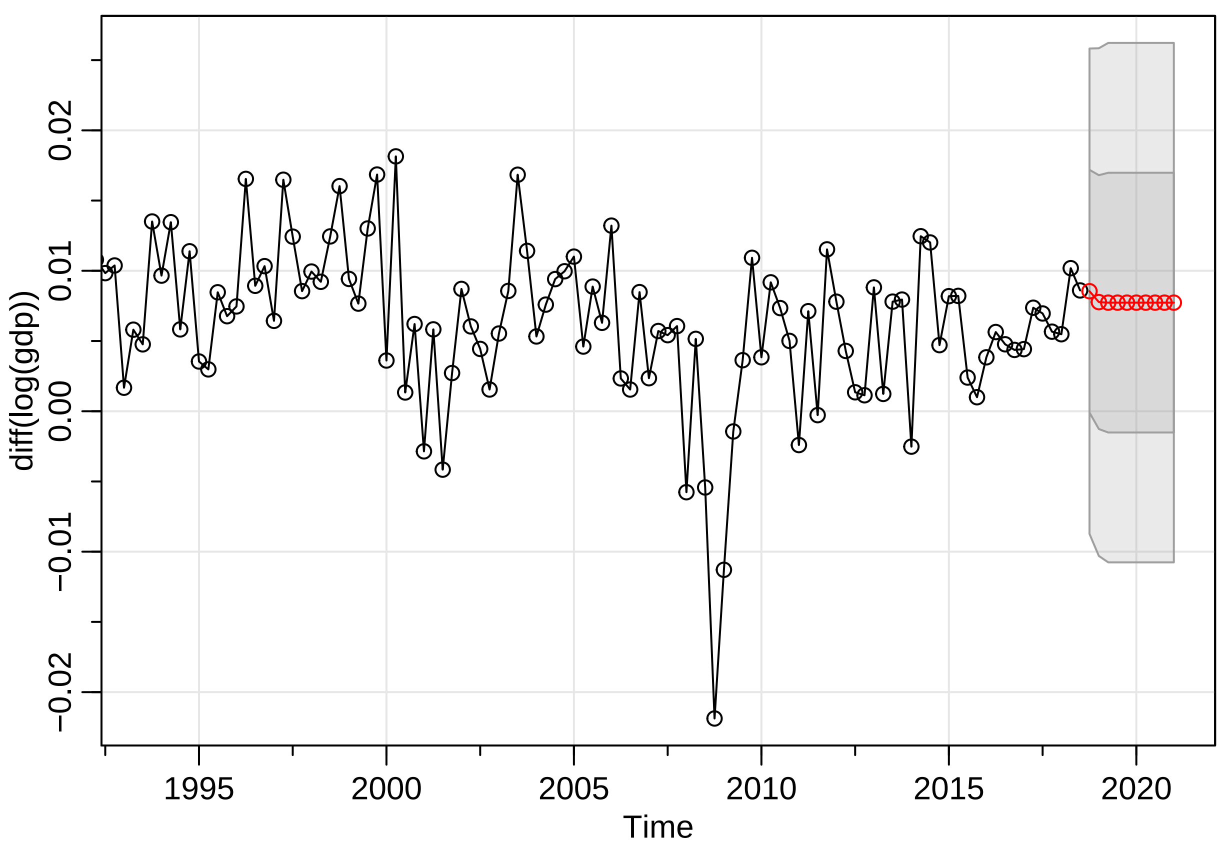
, Where

$$\sigma_w^2 = 7.465e - 05$$

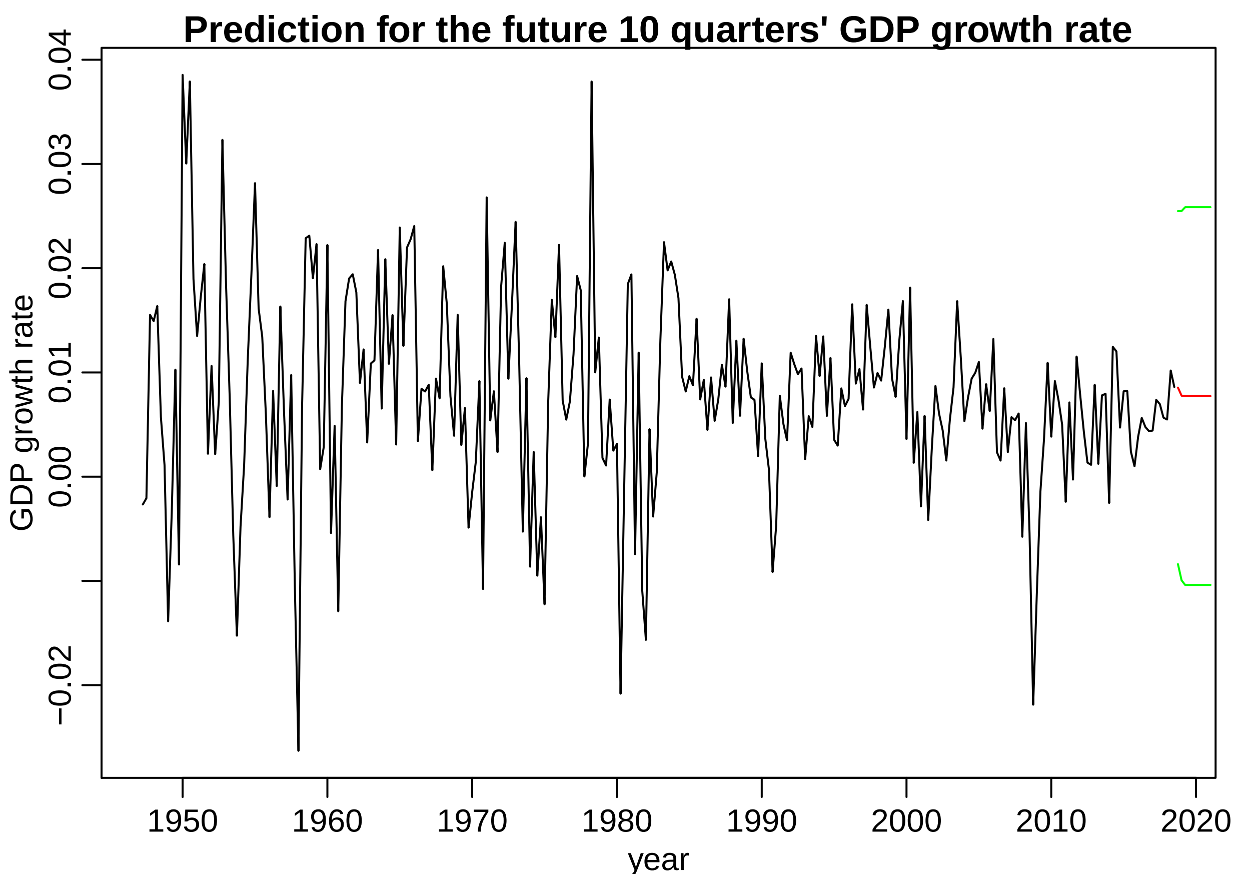
on 283 degrees of freedom. All coefficients' p-value are less than 0.05 which indicates that ma1, ma2 and constant are significant. Then we would focus on the diagnostics.

For the diagnostic plots for ARIMA(1, 1, 0) and ARIMA(0, 1, 2). Both the standardized residuals have no obvious patterns. There are few outliers. The ACF Residuals plots show a significant spike in both cases. The residuals' normal Q-Q plots show that the assumption of normality is reasonable enough, except for some possible outliers. The p-values for Ljung-Box statistics of ARIMA(1,1,0) are all closed to or below the reasonable significant level but the initial several p-values for Ljung-Box of ARIMA(0,1,2) are above 0.05. Hence, we are proposing the ARIMA(0,1,2) model for prediction. ARIMA(1,1,0): $AIC: -6.625634 $AICc: -6.625485 $BIC: -6.587284 ARIMA(0,1,2): $AIC: -6.636309 $AICc: -6.636001 $BIC: -6.585176 Meanwhile, the ARIMA(0, 1, 2) model has smaller AIC, AICc, but a larger BIC. Considering the ARIMA(0, 1, 2) model performs better with diagnostics, we select the ARIMA(0, 1, 2) model to predict the future 10 quarters.

```
forecast <- sarima.for(diff(log(gdp)),10,0,0,2)
```

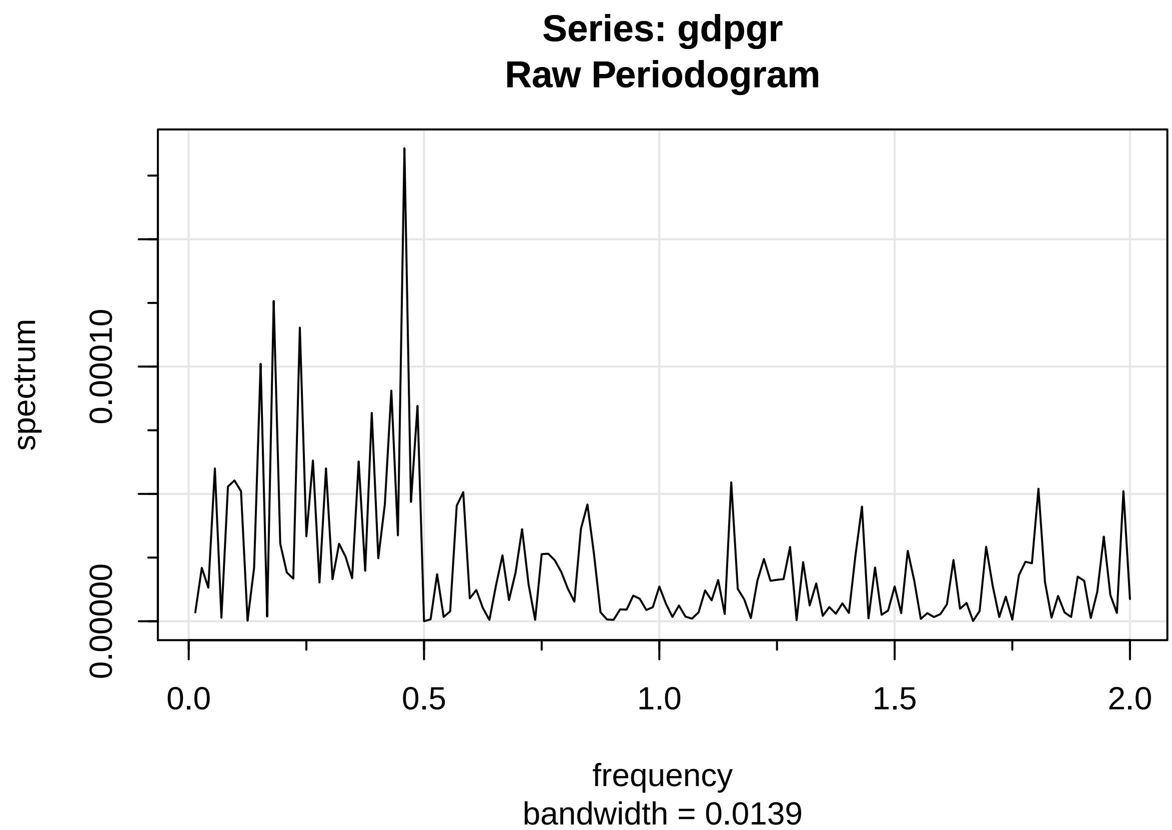


```
plot(gdpgr,xlab="year",ylab="GDP growth rate", main="Prediction for the future 10 quarters' GDP growth r
lines(forecast$pred,col="red")
lines(forecast$pred-1.96*forecast$se,col="green")
lines(forecast$pred+1.96*forecast$se,col="green")
```



From Figure 5, the future GDP growth rate is around 1% and the 95% confidence interval is around (-1%, 3%). But from the U.S. Department of Commerce, in 2020, U.S. GDP decreased by 3.5% which is out of our 95% confidence interval. (https://www.bea.gov/news/2021/gross-domestic-product-4th-quarter-and-year-2020-advance-estimate) Finally, we would do a spectrum analysis.

```
gdp.per = mvspec(gdpgr,log = "no")
```




```
gdp.per$details
```

```
##         frequency  period spectrum
##    [1,]    0.0139 72.0000    0e+00
##    [2,]    0.0278 36.0000    0e+00
##    [3,]    0.0417 24.0000    0e+00
##    [4,]    0.0556 18.0000    1e-04
##    [5,]    0.0694 14.4000    0e+00
##    [6,]    0.0833 12.0000    1e-04
##    [7,]    0.0972 10.2857    1e-04
##    [8,]    0.1111  9.0000    1e-04
##    [9,]    0.1250  8.0000    0e+00
##   [10,]    0.1389  7.2000    0e+00
##   [11,]    0.1528  6.5455    1e-04
##   [12,]    0.1667  6.0000    0e+00
##   [13,]    0.1806  5.5385    1e-04
##   [14,]    0.1944  5.1429    0e+00
##   [15,]    0.2083  4.8000    0e+00
##   [16,]    0.2222  4.5000    0e+00
##   [17,]    0.2361  4.2353    1e-04
##   [18,]    0.2500  4.0000    0e+00
##   [19,]    0.2639  3.7895    1e-04
##   [20,]    0.2778  3.6000    0e+00
##   [21,]    0.2917  3.4286    1e-04
##   [22,]    0.3056  3.2727    0e+00
```

```
## [23,] 0.3194 3.1304 0e+00
## [24,] 0.3333 3.0000 0e+00
## [25,] 0.3472 2.8800 0e+00
## [26,] 0.3611 2.7692 1e-04
## [27,] 0.3750 2.6667 0e+00
## [28,] 0.3889 2.5714 1e-04
## [29,] 0.4028 2.4828 0e+00
## [30,] 0.4167 2.4000 0e+00
## [31,] 0.4306 2.3226 1e-04
## [32,] 0.4444 2.2500 0e+00
## [33,] 0.4583 2.1818 2e-04
## [34,] 0.4722 2.1176 0e+00
## [35,] 0.4861 2.0571 1e-04
## [36,] 0.5000 2.0000 0e+00
## [37,] 0.5139 1.9459 0e+00
## [38,] 0.5278 1.8947 0e+00
## [39,] 0.5417 1.8462 0e+00
## [40,] 0.5556 1.8000 0e+00
## [41,] 0.5694 1.7561 0e+00
## [42,] 0.5833 1.7143 1e-04
## [43,] 0.5972 1.6744 0e+00
## [44,] 0.6111 1.6364 0e+00
## [45,] 0.6250 1.6000 0e+00
## [46,] 0.6389 1.5652 0e+00
## [47,] 0.6528 1.5319 0e+00
## [48,] 0.6667 1.5000 0e+00
## [49,] 0.6806 1.4694 0e+00
## [50,] 0.6944 1.4400 0e+00
## [51,] 0.7083 1.4118 0e+00
## [52,] 0.7222 1.3846 0e+00
## [53,] 0.7361 1.3585 0e+00
## [54,] 0.7500 1.3333 0e+00
## [55,] 0.7639 1.3091 0e+00
## [56,] 0.7778 1.2857 0e+00
## [57,] 0.7917 1.2632 0e+00
## [58,] 0.8056 1.2414 0e+00
## [59,] 0.8194 1.2203 0e+00
## [60,] 0.8333 1.2000 0e+00
## [61,] 0.8472 1.1803 0e+00
## [62,] 0.8611 1.1613 0e+00
## [63,] 0.8750 1.1429 0e+00
## [64,] 0.8889 1.1250 0e+00
## [65,] 0.9028 1.1077 0e+00
## [66,] 0.9167 1.0909 0e+00
## [67,] 0.9306 1.0746 0e+00
## [68,] 0.9444 1.0588 0e+00
## [69,] 0.9583 1.0435 0e+00
## [70,] 0.9722 1.0286 0e+00
## [71,] 0.9861 1.0141 0e+00
## [72,] 1.0000 1.0000 0e+00
## [73,] 1.0139 0.9863 0e+00
## [74,] 1.0278 0.9730 0e+00
## [75,] 1.0417 0.9600 0e+00
## [76,] 1.0556 0.9474 0e+00
```

```
##  [77,]    1.0694  0.9351    0e+00
##  [78,]    1.0833  0.9231    0e+00
##  [79,]    1.0972  0.9114    0e+00
##  [80,]    1.1111  0.9000    0e+00
##  [81,]    1.1250  0.8889    0e+00
##  [82,]    1.1389  0.8780    0e+00
##  [83,]    1.1528  0.8675    1e-04
##  [84,]    1.1667  0.8571    0e+00
##  [85,]    1.1806  0.8471    0e+00
##  [86,]    1.1944  0.8372    0e+00
##  [87,]    1.2083  0.8276    0e+00
##  [88,]    1.2222  0.8182    0e+00
##  [89,]    1.2361  0.8090    0e+00
##  [90,]    1.2500  0.8000    0e+00
##  [91,]    1.2639  0.7912    0e+00
##  [92,]    1.2778  0.7826    0e+00
##  [93,]    1.2917  0.7742    0e+00
##  [94,]    1.3056  0.7660    0e+00
##  [95,]    1.3194  0.7579    0e+00
##  [96,]    1.3333  0.7500    0e+00
##  [97,]    1.3472  0.7423    0e+00
##  [98,]    1.3611  0.7347    0e+00
##  [99,]    1.3750  0.7273    0e+00
## [100,]    1.3889  0.7200    0e+00
## [101,]    1.4028  0.7129    0e+00
## [102,]    1.4167  0.7059    0e+00
## [103,]    1.4306  0.6990    0e+00
## [104,]    1.4444  0.6923    0e+00
## [105,]    1.4583  0.6857    0e+00
## [106,]    1.4722  0.6792    0e+00
## [107,]    1.4861  0.6729    0e+00
## [108,]    1.5000  0.6667    0e+00
## [109,]    1.5139  0.6606    0e+00
## [110,]    1.5278  0.6545    0e+00
## [111,]    1.5417  0.6486    0e+00
## [112,]    1.5556  0.6429    0e+00
## [113,]    1.5694  0.6372    0e+00
## [114,]    1.5833  0.6316    0e+00
## [115,]    1.5972  0.6261    0e+00
## [116,]    1.6111  0.6207    0e+00
## [117,]    1.6250  0.6154    0e+00
## [118,]    1.6389  0.6102    0e+00
## [119,]    1.6528  0.6050    0e+00
## [120,]    1.6667  0.6000    0e+00
## [121,]    1.6806  0.5950    0e+00
## [122,]    1.6944  0.5902    0e+00
## [123,]    1.7083  0.5854    0e+00
## [124,]    1.7222  0.5806    0e+00
## [125,]    1.7361  0.5760    0e+00
## [126,]    1.7500  0.5714    0e+00
## [127,]    1.7639  0.5669    0e+00
## [128,]    1.7778  0.5625    0e+00
## [129,]    1.7917  0.5581    0e+00
## [130,]    1.8056  0.5538    1e-04
```

```
## [131,]    1.8194  0.5496    0e+00
## [132,]    1.8333  0.5455    0e+00
## [133,]    1.8472  0.5414    0e+00
## [134,]    1.8611  0.5373    0e+00
## [135,]    1.8750  0.5333    0e+00
## [136,]    1.8889  0.5294    0e+00
## [137,]    1.9028  0.5255    0e+00
## [138,]    1.9167  0.5217    0e+00
## [139,]    1.9306  0.5180    0e+00
## [140,]    1.9444  0.5143    0e+00
## [141,]    1.9583  0.5106    0e+00
## [142,]    1.9722  0.5070    0e+00
## [143,]    1.9861  0.5035    1e-04
## [144,]    2.0000  0.5000    0e+00
```

```r
df <- as.data.frame(gdp.per$details)
df
```

```
##     frequency  period spectrum
## 1      0.0139 72.0000    0e+00
## 2      0.0278 36.0000    0e+00
## 3      0.0417 24.0000    0e+00
## 4      0.0556 18.0000    1e-04
## 5      0.0694 14.4000    0e+00
## 6      0.0833 12.0000    1e-04
## 7      0.0972 10.2857    1e-04
## 8      0.1111  9.0000    1e-04
## 9      0.1250  8.0000    0e+00
## 10     0.1389  7.2000    0e+00
## 11     0.1528  6.5455    1e-04
## 12     0.1667  6.0000    0e+00
## 13     0.1806  5.5385    1e-04
## 14     0.1944  5.1429    0e+00
## 15     0.2083  4.8000    0e+00
## 16     0.2222  4.5000    0e+00
## 17     0.2361  4.2353    1e-04
## 18     0.2500  4.0000    0e+00
## 19     0.2639  3.7895    1e-04
## 20     0.2778  3.6000    0e+00
## 21     0.2917  3.4286    1e-04
## 22     0.3056  3.2727    0e+00
## 23     0.3194  3.1304    0e+00
## 24     0.3333  3.0000    0e+00
## 25     0.3472  2.8800    0e+00
## 26     0.3611  2.7692    1e-04
## 27     0.3750  2.6667    0e+00
## 28     0.3889  2.5714    1e-04
## 29     0.4028  2.4828    0e+00
## 30     0.4167  2.4000    0e+00
## 31     0.4306  2.3226    1e-04
## 32     0.4444  2.2500    0e+00
## 33     0.4583  2.1818    2e-04
## 34     0.4722  2.1176    0e+00
## 35     0.4861  2.0571    1e-04
```

```
## 36      0.5000  2.0000    0e+00
## 37      0.5139  1.9459    0e+00
## 38      0.5278  1.8947    0e+00
## 39      0.5417  1.8462    0e+00
## 40      0.5556  1.8000    0e+00
## 41      0.5694  1.7561    0e+00
## 42      0.5833  1.7143    1e-04
## 43      0.5972  1.6744    0e+00
## 44      0.6111  1.6364    0e+00
## 45      0.6250  1.6000    0e+00
## 46      0.6389  1.5652    0e+00
## 47      0.6528  1.5319    0e+00
## 48      0.6667  1.5000    0e+00
## 49      0.6806  1.4694    0e+00
## 50      0.6944  1.4400    0e+00
## 51      0.7083  1.4118    0e+00
## 52      0.7222  1.3846    0e+00
## 53      0.7361  1.3585    0e+00
## 54      0.7500  1.3333    0e+00
## 55      0.7639  1.3091    0e+00
## 56      0.7778  1.2857    0e+00
## 57      0.7917  1.2632    0e+00
## 58      0.8056  1.2414    0e+00
## 59      0.8194  1.2203    0e+00
## 60      0.8333  1.2000    0e+00
## 61      0.8472  1.1803    0e+00
## 62      0.8611  1.1613    0e+00
## 63      0.8750  1.1429    0e+00
## 64      0.8889  1.1250    0e+00
## 65      0.9028  1.1077    0e+00
## 66      0.9167  1.0909    0e+00
## 67      0.9306  1.0746    0e+00
## 68      0.9444  1.0588    0e+00
## 69      0.9583  1.0435    0e+00
## 70      0.9722  1.0286    0e+00
## 71      0.9861  1.0141    0e+00
## 72      1.0000  1.0000    0e+00
## 73      1.0139  0.9863    0e+00
## 74      1.0278  0.9730    0e+00
## 75      1.0417  0.9600    0e+00
## 76      1.0556  0.9474    0e+00
## 77      1.0694  0.9351    0e+00
## 78      1.0833  0.9231    0e+00
## 79      1.0972  0.9114    0e+00
## 80      1.1111  0.9000    0e+00
## 81      1.1250  0.8889    0e+00
## 82      1.1389  0.8780    0e+00
## 83      1.1528  0.8675    1e-04
## 84      1.1667  0.8571    0e+00
## 85      1.1806  0.8471    0e+00
## 86      1.1944  0.8372    0e+00
## 87      1.2083  0.8276    0e+00
## 88      1.2222  0.8182    0e+00
## 89      1.2361  0.8090    0e+00
```

```
## 90     1.2500  0.8000    0e+00
## 91     1.2639  0.7912    0e+00
## 92     1.2778  0.7826    0e+00
## 93     1.2917  0.7742    0e+00
## 94     1.3056  0.7660    0e+00
## 95     1.3194  0.7579    0e+00
## 96     1.3333  0.7500    0e+00
## 97     1.3472  0.7423    0e+00
## 98     1.3611  0.7347    0e+00
## 99     1.3750  0.7273    0e+00
## 100    1.3889  0.7200    0e+00
## 101    1.4028  0.7129    0e+00
## 102    1.4167  0.7059    0e+00
## 103    1.4306  0.6990    0e+00
## 104    1.4444  0.6923    0e+00
## 105    1.4583  0.6857    0e+00
## 106    1.4722  0.6792    0e+00
## 107    1.4861  0.6729    0e+00
## 108    1.5000  0.6667    0e+00
## 109    1.5139  0.6606    0e+00
## 110    1.5278  0.6545    0e+00
## 111    1.5417  0.6486    0e+00
## 112    1.5556  0.6429    0e+00
## 113    1.5694  0.6372    0e+00
## 114    1.5833  0.6316    0e+00
## 115    1.5972  0.6261    0e+00
## 116    1.6111  0.6207    0e+00
## 117    1.6250  0.6154    0e+00
## 118    1.6389  0.6102    0e+00
## 119    1.6528  0.6050    0e+00
## 120    1.6667  0.6000    0e+00
## 121    1.6806  0.5950    0e+00
## 122    1.6944  0.5902    0e+00
## 123    1.7083  0.5854    0e+00
## 124    1.7222  0.5806    0e+00
## 125    1.7361  0.5760    0e+00
## 126    1.7500  0.5714    0e+00
## 127    1.7639  0.5669    0e+00
## 128    1.7778  0.5625    0e+00
## 129    1.7917  0.5581    0e+00
## 130    1.8056  0.5538    1e-04
## 131    1.8194  0.5496    0e+00
## 132    1.8333  0.5455    0e+00
## 133    1.8472  0.5414    0e+00
## 134    1.8611  0.5373    0e+00
## 135    1.8750  0.5333    0e+00
## 136    1.8889  0.5294    0e+00
## 137    1.9028  0.5255    0e+00
## 138    1.9167  0.5217    0e+00
## 139    1.9306  0.5180    0e+00
## 140    1.9444  0.5143    0e+00
## 141    1.9583  0.5106    0e+00
## 142    1.9722  0.5070    0e+00
## 143    1.9861  0.5035    1e-04
```

```
## 144    2.0000  0.5000    0e+00
```

```
U = qchisq(.025,2)
L = qchisq(.975,2)
spec <- df %>% arrange(desc(spectrum)) %>% slice(1:3)
spec
```

```
##   frequency  period spectrum
## 1    0.4583  2.1818    2e-04
## 2    0.0556 18.0000    1e-04
## 3    0.0833 12.0000    1e-04
```

After we sort and subset the data, we get the first three predominant periods. Then we calculate the upper and lower bounds, and those two columns would be combined in the dataset.

```
spec%>%
  mutate(Upper= 2*spec$spectrum/U)%>%mutate(Lower = 2*spec$spectrum/L)
```

```
##   frequency  period spectrum       Upper        Lower
## 1    0.4583  2.1818    2e-04 0.007899578 5.421701e-05
## 2    0.0556 18.0000    1e-04 0.003949789 2.710850e-05
## 3    0.0833 12.0000    1e-04 0.003949789 2.710850e-05
```

From Figure 7, the dominant periods are 2.1818, 18 and 12. Those 95% confidence intervals are too wide to establish the significance of the peak.

Discussion

Since this is a quarterly time series, there seems to be some seasonal trend. So, an ARIMA model may not be the best fitted model and SARIMA models may have been a better one. Meanwhile, there are some outliers at the tails of the Q-Q plot which limiting the model prediction. So, SARIMA models may have been a better fit for this data. Moreover, in the prediction of the future 10 quarters, the estimated 95% interval of 2020 GDP growth rate is (-0.01, 0.03) but the GDP growth rate of 2020 in real world is -0.035. It is due to the current COVID-19 pandemic which is one of the most significant black swan events in history. There is no model can predict that.