

# The Statistical Analysis of U.S. Quarterly GDP Growth Rate

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## Abstract

In this project, we analyze the quarterly GDP growth rate of U.S. and there are two candidate models proposed. After testing their significance and diagnostics, an ARIMA(0,1,2) model is selected to predict the future 10 quarters with 95% confidence interval. Through spectrum analysis, the first three predominant periods and the upper and lower bounds are also calculated.

## Introduction

U.S. as the world's largest economy plays a significant role in worldwide economic activity. Investigating GDP of U.S. can appeal the potential model of its growth rate and help people to predict the future economy. In this project, the analysis of quarterly U.S. GDP would be considered. The data starts at 1947(1) and ends at 2018(3); it has  $n = 287$  observations. Figure 1 shows a plot of data,

$y_t$

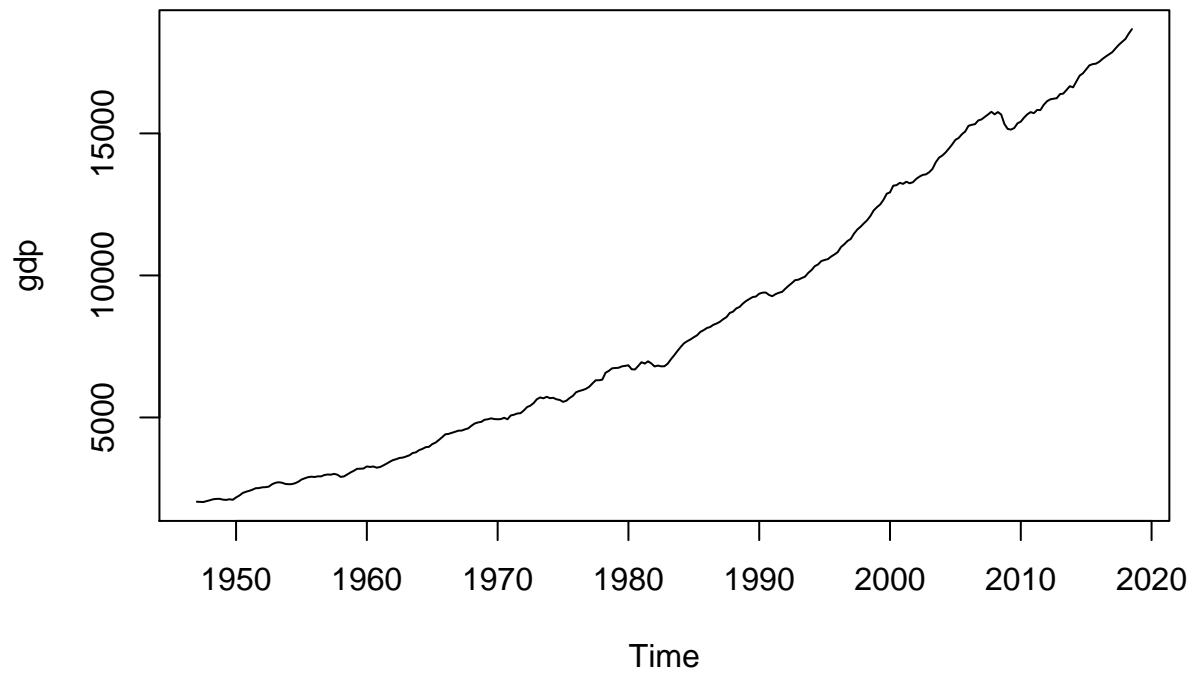
, but it has a strong trend and the sample ACF plot also shows a slow decay which indicates that log transformation and differencing may be needed.

```
data("gdp")
gdp
```

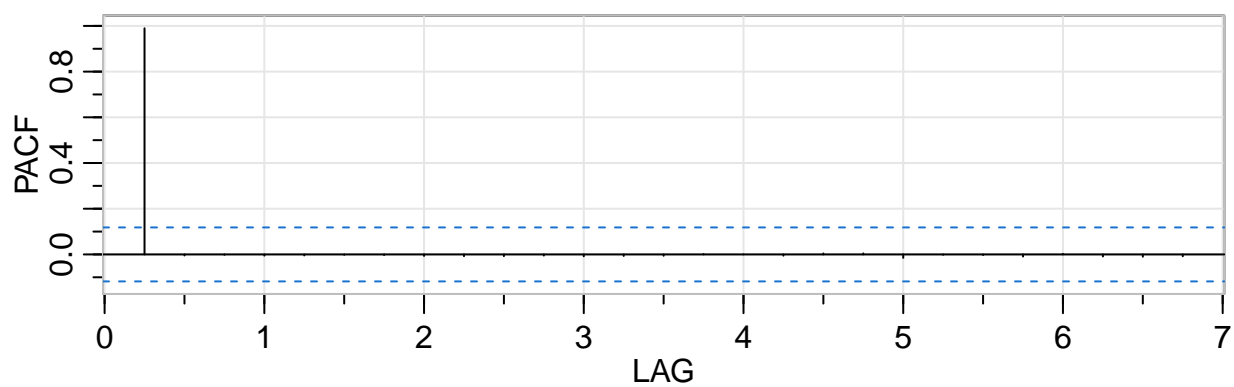
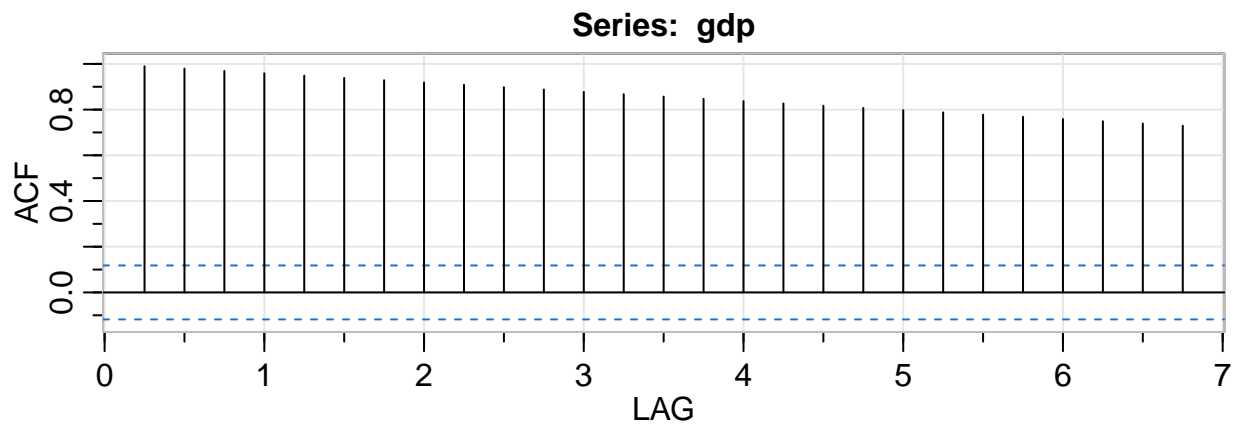
##		Qtr1	Qtr2	Qtr3	Qtr4
##	1947	2033.061	2027.639	2023.452	2055.103
##	1948	2086.017	2120.450	2132.598	2134.981
##	1949	2105.562	2098.380	2120.044	2102.251
##	1950	2184.872	2251.507	2338.514	2383.291
##	1951	2415.660	2457.517	2508.166	2513.690
##	1952	2540.550	2546.022	2564.401	2648.621
##	1953	2697.855	2718.709	2703.411	2662.482
##	1954	2649.755	2652.643	2682.601	2735.091
##	1955	2813.212	2858.988	2897.598	2914.993
##	1956	2903.671	2927.665	2925.035	2973.179
##	1957	2992.219	2985.663	3014.919	2983.727
##	1958	2906.274	2925.379	2993.068	3063.085
##	1959	3121.936	3192.380	3194.653	3203.759
##	1960	3275.757	3258.088	3274.029	3232.009
##	1961	3253.826	3309.059	3372.581	3438.721
##	1962	3500.054	3531.683	3575.070	3586.827
##	1963	3625.981	3666.669	3747.278	3771.845
##	1964	3851.366	3893.296	3954.121	3966.335
##	1965	4062.311	4113.629	4205.086	4301.973
##	1966	4406.693	4421.747	4459.195	4495.777

##	1967	4535.591	4538.370	4581.309	4615.853
##	1968	4709.993	4788.688	4825.799	4844.779
##	1969	4920.605	4935.564	4968.164	4943.935
##	1970	4936.594	4943.600	4989.159	4935.693
##	1971	5069.746	5097.179	5139.128	5151.245
##	1972	5245.974	5365.045	5415.712	5506.396
##	1973	5642.669	5704.098	5674.100	5727.960
##	1974	5678.713	5692.210	5638.411	5616.526
##	1975	5548.156	5587.800	5683.444	5759.972
##	1976	5889.500	5932.711	5965.265	6008.504
##	1977	6079.494	6197.686	6309.514	6309.652
##	1978	6329.791	6574.390	6640.497	6729.755
##	1979	6741.854	6749.063	6799.200	6816.203
##	1980	6837.641	6696.753	6688.794	6813.535
##	1981	6947.042	6895.559	6978.135	6902.105
##	1982	6794.878	6825.876	6799.781	6802.497
##	1983	6892.144	7048.982	7189.896	7339.893
##	1984	7483.371	7612.668	7686.059	7749.151
##	1985	7824.247	7893.136	8013.674	8073.239
##	1986	8148.603	8185.303	8263.639	8308.021
##	1987	8369.930	8460.233	8533.635	8680.162
##	1988	8725.006	8839.641	8891.435	9009.913
##	1989	9101.508	9170.977	9238.923	9257.128
##	1990	9358.289	9392.251	9398.499	9312.937
##	1991	9269.367	9341.642	9388.845	9421.565
##	1992	9534.346	9637.732	9732.979	9834.510
##	1993	9850.973	9908.347	9955.641	10091.049
##	1994	10188.954	10327.019	10387.382	10506.372
##	1995	10543.644	10575.100	10665.060	10737.478
##	1996	10817.896	10998.322	11096.976	11212.205
##	1997	11284.587	11472.137	11615.636	11715.393
##	1998	11832.486	11942.032	12091.614	12287.000
##	1999	12403.293	12498.694	12662.385	12877.593
##	2000	12924.179	13160.842	13178.419	13260.506
##	2001	13222.690	13299.984	13244.784	13280.859
##	2002	13397.002	13478.152	13538.072	13559.032
##	2003	13634.253	13751.543	13985.073	14145.645
##	2004	14221.147	14329.523	14464.984	14609.876
##	2005	14771.602	14839.782	14972.054	15066.597
##	2006	15267.026	15302.705	15326.368	15456.928
##	2007	15493.328	15582.085	15666.738	15761.967
##	2008	15671.383	15752.308	15667.032	15328.027
##	2009	15155.940	15134.117	15189.222	15356.058
##	2010	15415.145	15557.277	15671.967	15750.625
##	2011	15712.754	15825.096	15820.700	16004.107
##	2012	16129.418	16198.807	16220.667	16239.138
##	2013	16382.964	16403.180	16531.685	16663.649
##	2014	16621.696	16830.111	17033.572	17113.945
##	2015	17254.744	17397.029	17438.802	17456.225
##	2016	17523.374	17622.486	17706.705	17784.185
##	2017	17863.023	17995.150	18120.843	18223.758
##	2018	18323.963	18511.576	18671.497	

```
plot.ts(gdp)
```

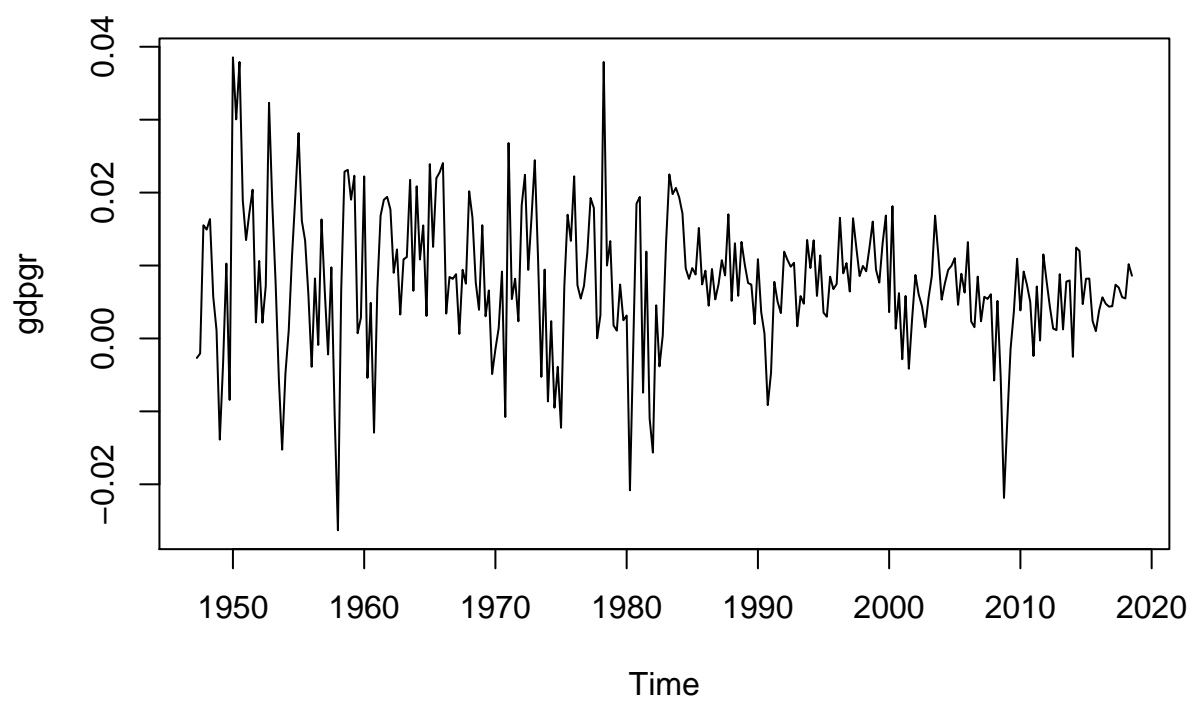


```
acf2(gdp)
```

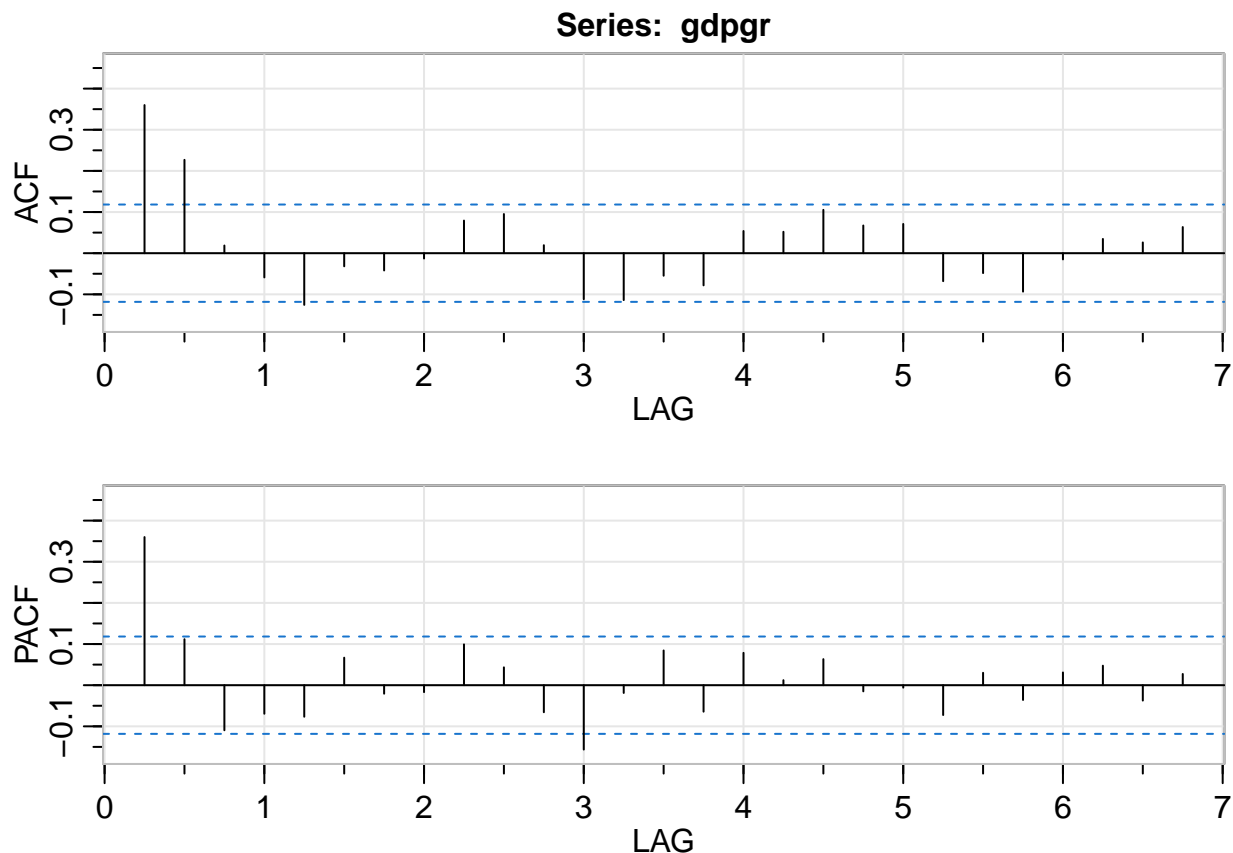


```
##      [,1] [,2] [,3]  [,4] [,5] [,6] [,7]  [,8]  [,9] [,10] [,11] [,12] [,13]
## ACF  0.99 0.98 0.97  0.96 0.95 0.94 0.93  0.92 0.91 0.90 0.89 0.88 0.87
## PACF 0.99 0.00 0.00 -0.01 0.00 0.00 0.00 -0.01 -0.01 -0.01 -0.01 -0.01 -0.01
##      [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25]
## ACF  0.86 0.85 0.84 0.83 0.82 0.81 0.80 0.79 0.78 0.77 0.76 0.75
## PACF -0.01 0.00 0.00 -0.01 0.00 0.00 -0.02 0.00 0.00 -0.01 0.00 -0.01
##      [,26] [,27]
## ACF  0.74 0.73
## PACF -0.01 -0.01
```

```
gdpgr = diff(log(gdp))
plot.ts(gdpgr)
```



```
acf2(gdpgr)
```



```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13]
## ACF  0.36 0.23  0.02 -0.06 -0.13 -0.03 -0.04 -0.01 0.08  0.10  0.02 -0.11 -0.11
## PACF 0.36 0.11 -0.11 -0.07 -0.08  0.07 -0.02 -0.02 0.10  0.04 -0.07 -0.16 -0.02
##      [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25]
## ACF  -0.05 -0.08  0.05  0.05  0.11  0.07  0.07 -0.07 -0.05 -0.09 -0.02  0.03
## PACF  0.08 -0.06  0.08  0.01  0.06 -0.02 -0.01 -0.07  0.03 -0.04  0.03  0.05
##      [,26] [,27]
## ACF   0.03  0.06
## PACF -0.04  0.03
```

Hence, in Figure 2, we plotted

$$x_t = \nabla (\log(y_t))$$

. It means the growth rate of U.S. GDP. The plot shows a more stable process. The sample ACF and PACF plots are also quickly decayed.

Since a stable process has been found. In this project, we are going to inspect the sample ACF and PACF plot to propose some candidate models for U.S. GDP growth rate. Then, those candidate models would be fitted and tested. Finally, we would find a better model from candidates to predict the future and do a spectral analysis of it.

#### Statistical Methods

From the sample ACF and PACF plots in Figure 2, we may feel the ACF cuts off at lag 2 and PACF tails off. It would suggest log GDP follows an ARIMA(0, 1, 2) model. Also, it appears that the ACF tails off at lag 1 and PACF cuts off at lag 1. An ARIMA(1, 1, 0) model would also be considered for log GDP. We would fit those two models in RStudio. We would use MLE to estimate the model for growth rate. Then diagnostics

would be considered. The first one is standardized residuals, if the model fits well the residual would behave as an iid sequence with mean zero and variance one. The second one is checking the normality by normal Q-Q plot. If there is a departure from normality then the data does not meet the normal assumption. The third one is the ACF of residuals. The sample autocorrelations are almost independently and normally distributed with mean zero and variance  $1/n$ . If the

$$\hat{\rho}(h)$$

is along with the bounds of

$$\pm 2/\sqrt{n}$$

, then the model fits well. The last one is Ljung-Box plot. It is to check for any  $H > 1$ , whether

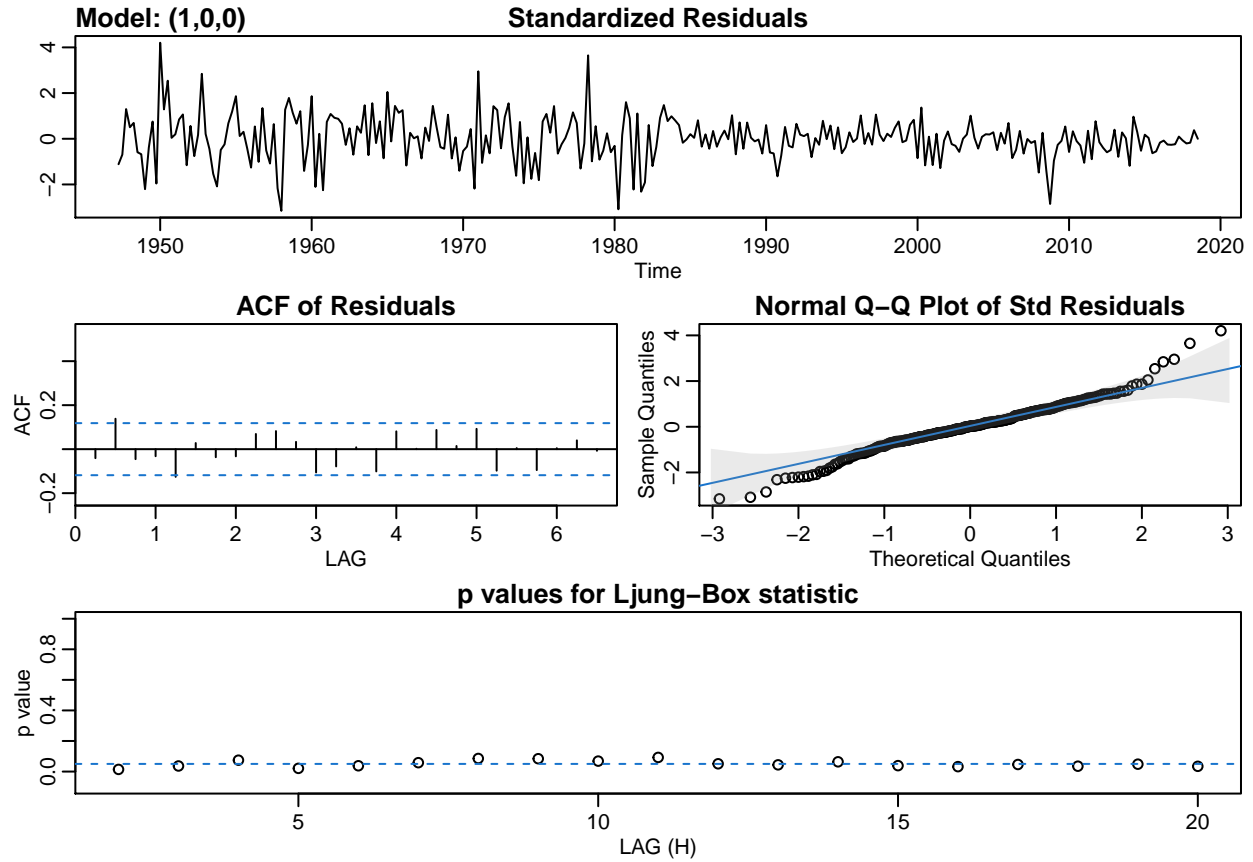
$$\rho(1) = \dots = \rho(H) = 0$$

. If the plots are departure from 0.05, the model fits well. Moreover, we would compare the AIC, the AICc and the BIC. The smaller they are, the better the model fits.

Results

```
#fit the ARIMA(1,1,0) model
sarima(diff(log(gdp)),1,0,0)
```

```
## initial  value -4.673186
## iter    2 value -4.742918
## iter    3 value -4.742921
## iter    4 value -4.742923
## iter    5 value -4.742925
## iter    6 value -4.742925
## iter    6 value -4.742925
## final   value -4.742925
## converged
## initial  value -4.742229
## iter    2 value -4.742234
## iter    3 value -4.742245
## iter    3 value -4.742245
## iter    3 value -4.742245
## final   value -4.742245
## converged
```



```
## $fit
##
## Call:
## stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
##     Q), period = S), xreg = xmean, include.mean = FALSE, transform.pars = trans,
##     fixed = fixed, optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1    xmean
##       0.3603  0.0077
## s.e.  0.0551  0.0008
##
## sigma^2 estimated as 7.598e-05:  log likelihood = 950.47,  aic = -1894.93
##
## $degrees_of_freedom
## [1] 284
##
## $ttable
##      Estimate      SE t.value p.value
## ar1    0.3603 0.0551  6.5366      0
## xmean   0.0077 0.0008  9.5915      0
##
## $AIC
## [1] -6.625634
##
## $AICc
```



```
## [1] -6.625485
##
## $BIC
## [1] -6.587284
```

The estimated ARIMA(1,1,0) model is:

$$x_t = 0.0077_{(0.0008)}(1 - 0.3603) + 0.3603_{(0.0551)}x_{t-1} + w_t$$

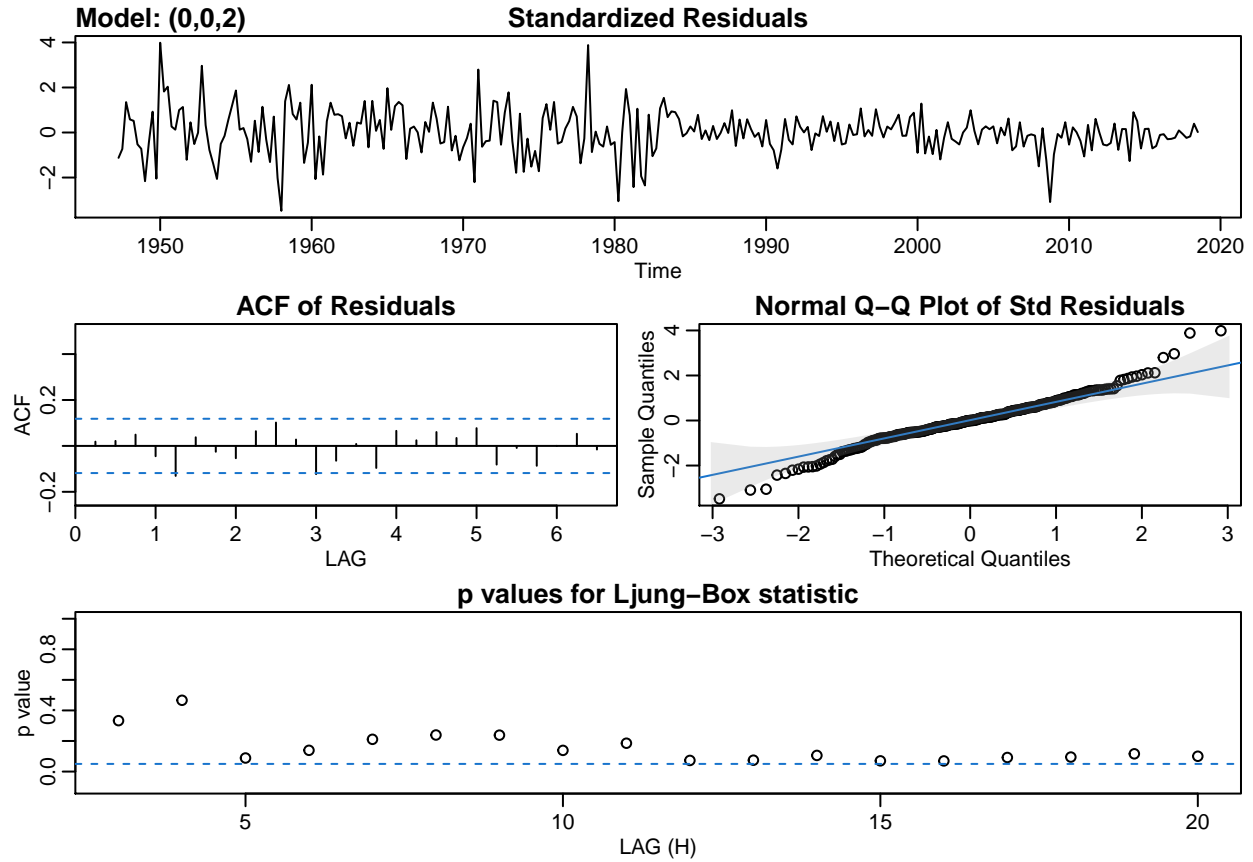
, Where

$$\sigma_w^2 = 7.599e - 05$$

on 284 degrees of freedom; the constant is 0.0049. All coefficients' p-value are less than 0.05 which indicates that both ar1 and constant are significant.

```
#fit the ARIMA(0,0,2) model
sarima(diff(log(gdp)),0,0,2)
```

```
## initial  value -4.672758
## iter    2 value -4.749239
## iter    3 value -4.750696
## iter    4 value -4.750723
## iter    5 value -4.750724
## iter    6 value -4.750725
## iter    7 value -4.750725
## iter    7 value -4.750725
## iter    7 value -4.750725
## final   value -4.750725
## converged
## initial  value -4.751078
## iter    2 value -4.751080
## iter    3 value -4.751080
## iter    4 value -4.751081
## iter    5 value -4.751081
## iter    5 value -4.751081
## iter    5 value -4.751081
## final   value -4.751081
## converged
```



```
## $fit
##
## Call:
## stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
##     Q), period = S), xreg = xmean, include.mean = FALSE, transform.pars = trans,
##     fixed = fixed, optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ma1      ma2    xmean
##          0.3070  0.2258  0.0077
## s.e.  0.0579  0.0547  0.0008
##
## sigma^2 estimated as 7.465e-05:  log likelihood = 952.99,  aic = -1897.99
##
## $degrees_of_freedom
## [1] 283
##
## $ttable
##      Estimate      SE t.value p.value
## ma1      0.3070 0.0579  5.2988      0
## ma2      0.2258 0.0547  4.1271      0
## xmean     0.0077 0.0008  9.8631      0
##
## $AIC
## [1] -6.636312
##
```

```
## $AICc
## [1] -6.636015
##
## $BIC
## [1] -6.58518
```

The estimated ARIMA(0,1,2) model is:

$$x_t = 0.0077_{(0.0008)} + w_t + 0.307_{(0.0579)}w_{t-1} + 0.2258_{(0.0547)}w_{t-2}$$

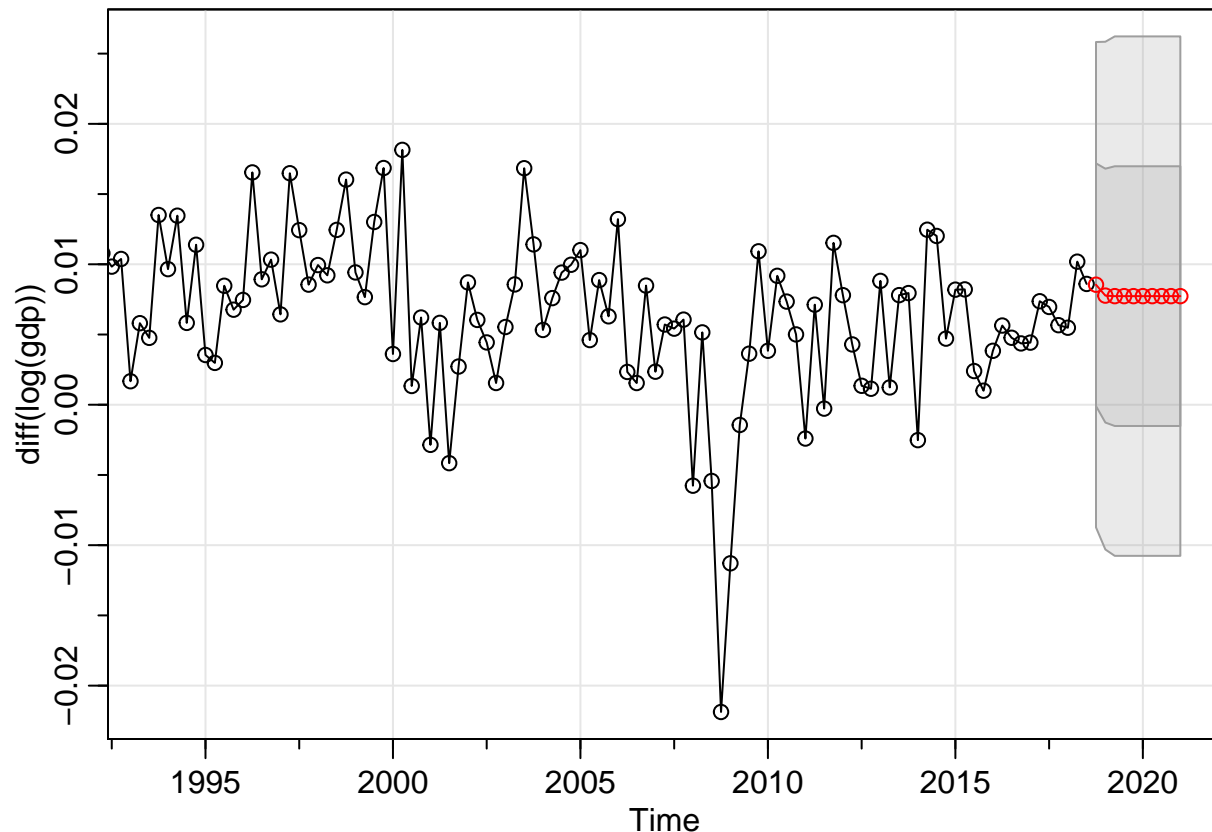
, Where

$$\sigma_w^2 = 7.465e - 05$$

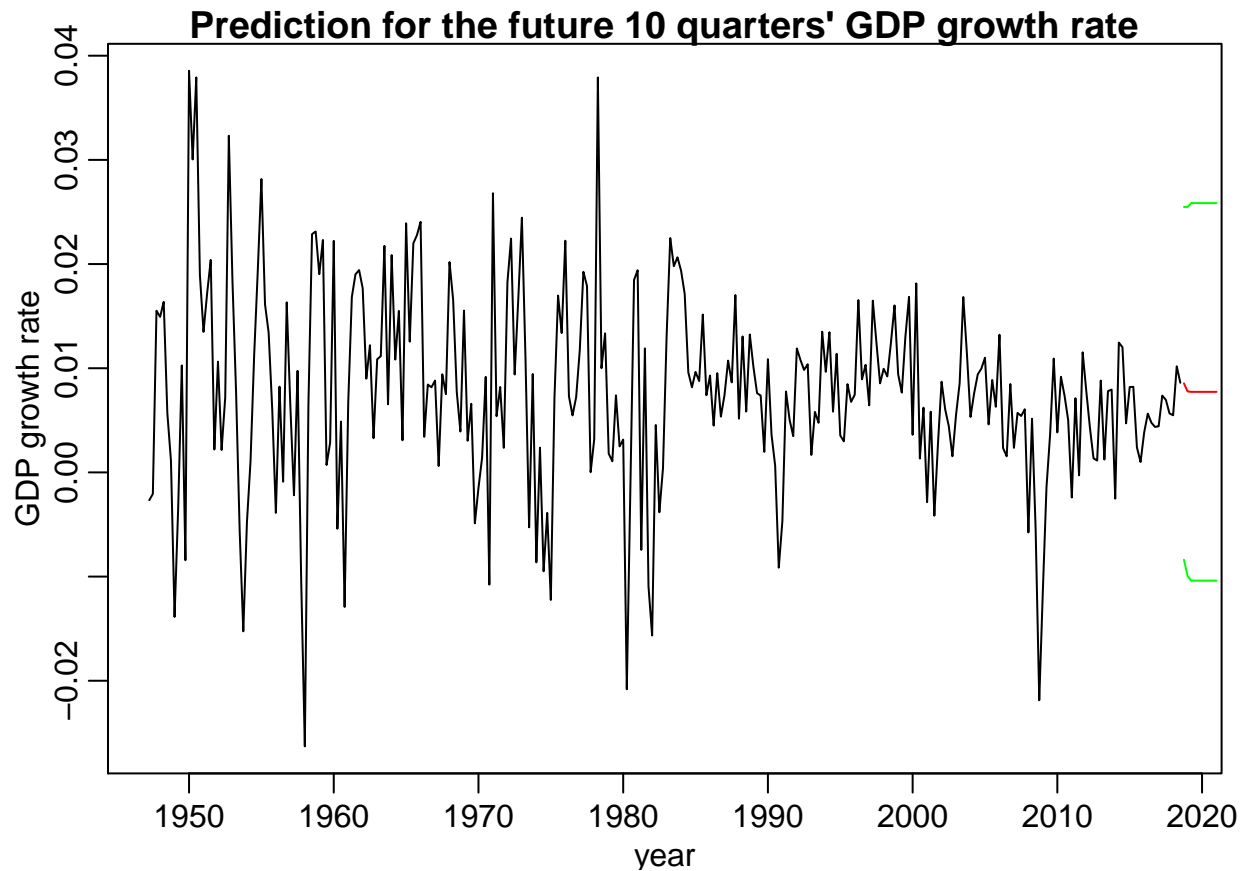
on 283 degrees of freedom. All coefficients' p-value are less than 0.05 which indicates that ma1, ma2 and constant are significant. Then we would focus on the diagnostics.

For the diagnostic plots for ARIMA(1, 1, 0) and ARIMA(0, 1, 2). Both the standardized residuals have no obvious patterns. There are few outliers. The ACF Residuals plots show a significant spike in both cases. The residuals' normal Q-Q plots show that the assumption of normality is reasonable enough, except for some possible outliers. The p-values for Ljung-Box statistics of ARIMA(1,1,0) are all closed to or below the reasonable significant level but the initial several p-values for Ljung-Box of ARIMA(0,1,2) are above 0.05. Hence, we are proposing the ARIMA(0,1,2) model for prediction. ARIMA(1,1,0): \$AIC: -6.625634 \$AICc: -6.625485 \$BIC: -6.587284 ARIMA(0,1,2): \$AIC: -6.636309 \$AICc: -6.636001 \$BIC: -6.585176 Meanwhile, the ARIMA(0, 1, 2) model has smaller AIC, AICc, but a larger BIC. Considering the ARIMA(0, 1, 2) model performs better with diagnostics, we select the ARIMA(0, 1, 2) model to predict the future 10 quarters.

```
forecast <- sarima.for(diff(log(gdp)),10,0,0,2)
```



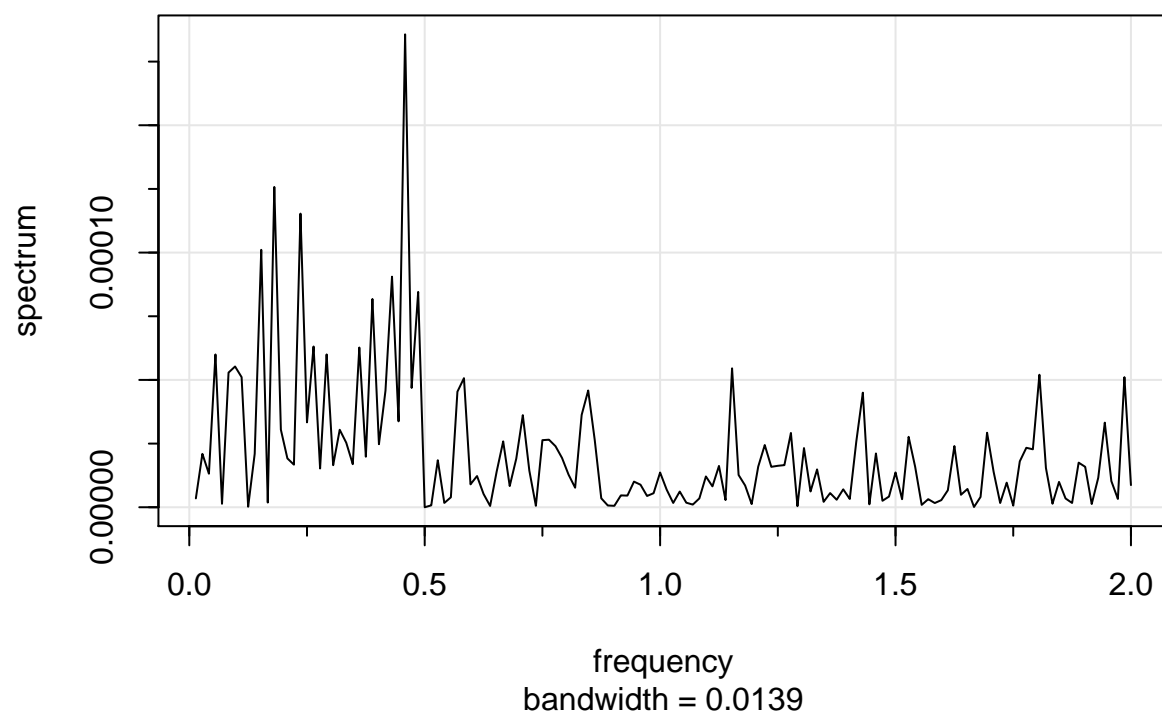
```
plot(gdpgr,xlab="year",ylab="GDP growth rate", main="Prediction for the future 10 quarters' GDP growth .")
lines(forecast$pred,col="red")
lines(forecast$pred-1.96*forecast$se,col="green")
lines(forecast$pred+1.96*forecast$se,col="green")
```



From Figure 5, the future GDP growth rate is around 1% and the 95% confidence interval is around (-1%, 3%). But from the U.S. Department of Commerce, in 2020, U.S. GDP decreased by 3.5% which is out of our 95% confidence interval. (<https://www.bea.gov/news/2021/gross-domestic-product-4th-quarter-and-year-2020-advance-estimate>) Finally, we would do a spectrum analysis.

```
gdp.per = mvspec(gdpgr,log = "no")
```

# Series: gdpgr Raw Periodogram



gdp.per\$details

##		frequency	period	spectrum
##	[1,]	0.0139	72.0000	0e+00
##	[2,]	0.0278	36.0000	0e+00
##	[3,]	0.0417	24.0000	0e+00
##	[4,]	0.0556	18.0000	1e-04
##	[5,]	0.0694	14.4000	0e+00
##	[6,]	0.0833	12.0000	1e-04
##	[7,]	0.0972	10.2857	1e-04
##	[8,]	0.1111	9.0000	1e-04
##	[9,]	0.1250	8.0000	0e+00
##	[10,]	0.1389	7.2000	0e+00
##	[11,]	0.1528	6.5455	1e-04
##	[12,]	0.1667	6.0000	0e+00
##	[13,]	0.1806	5.5385	1e-04
##	[14,]	0.1944	5.1429	0e+00
##	[15,]	0.2083	4.8000	0e+00
##	[16,]	0.2222	4.5000	0e+00
##	[17,]	0.2361	4.2353	1e-04
##	[18,]	0.2500	4.0000	0e+00
##	[19,]	0.2639	3.7895	1e-04
##	[20,]	0.2778	3.6000	0e+00
##	[21,]	0.2917	3.4286	1e-04
##	[22,]	0.3056	3.2727	0e+00

##	[23,]	0.3194	3.1304	0e+00
##	[24,]	0.3333	3.0000	0e+00
##	[25,]	0.3472	2.8800	0e+00
##	[26,]	0.3611	2.7692	1e-04
##	[27,]	0.3750	2.6667	0e+00
##	[28,]	0.3889	2.5714	1e-04
##	[29,]	0.4028	2.4828	0e+00
##	[30,]	0.4167	2.4000	0e+00
##	[31,]	0.4306	2.3226	1e-04
##	[32,]	0.4444	2.2500	0e+00
##	[33,]	0.4583	2.1818	2e-04
##	[34,]	0.4722	2.1176	0e+00
##	[35,]	0.4861	2.0571	1e-04
##	[36,]	0.5000	2.0000	0e+00
##	[37,]	0.5139	1.9459	0e+00
##	[38,]	0.5278	1.8947	0e+00
##	[39,]	0.5417	1.8462	0e+00
##	[40,]	0.5556	1.8000	0e+00
##	[41,]	0.5694	1.7561	0e+00
##	[42,]	0.5833	1.7143	1e-04
##	[43,]	0.5972	1.6744	0e+00
##	[44,]	0.6111	1.6364	0e+00
##	[45,]	0.6250	1.6000	0e+00
##	[46,]	0.6389	1.5652	0e+00
##	[47,]	0.6528	1.5319	0e+00
##	[48,]	0.6667	1.5000	0e+00
##	[49,]	0.6806	1.4694	0e+00
##	[50,]	0.6944	1.4400	0e+00
##	[51,]	0.7083	1.4118	0e+00
##	[52,]	0.7222	1.3846	0e+00
##	[53,]	0.7361	1.3585	0e+00
##	[54,]	0.7500	1.3333	0e+00
##	[55,]	0.7639	1.3091	0e+00
##	[56,]	0.7778	1.2857	0e+00
##	[57,]	0.7917	1.2632	0e+00
##	[58,]	0.8056	1.2414	0e+00
##	[59,]	0.8194	1.2203	0e+00
##	[60,]	0.8333	1.2000	0e+00
##	[61,]	0.8472	1.1803	0e+00
##	[62,]	0.8611	1.1613	0e+00
##	[63,]	0.8750	1.1429	0e+00
##	[64,]	0.8889	1.1250	0e+00
##	[65,]	0.9028	1.1077	0e+00
##	[66,]	0.9167	1.0909	0e+00
##	[67,]	0.9306	1.0746	0e+00
##	[68,]	0.9444	1.0588	0e+00
##	[69,]	0.9583	1.0435	0e+00
##	[70,]	0.9722	1.0286	0e+00
##	[71,]	0.9861	1.0141	0e+00
##	[72,]	1.0000	1.0000	0e+00
##	[73,]	1.0139	0.9863	0e+00
##	[74,]	1.0278	0.9730	0e+00
##	[75,]	1.0417	0.9600	0e+00
##	[76,]	1.0556	0.9474	0e+00

##	[77,]	1.0694	0.9351	0e+00
##	[78,]	1.0833	0.9231	0e+00
##	[79,]	1.0972	0.9114	0e+00
##	[80,]	1.1111	0.9000	0e+00
##	[81,]	1.1250	0.8889	0e+00
##	[82,]	1.1389	0.8780	0e+00
##	[83,]	1.1528	0.8675	1e-04
##	[84,]	1.1667	0.8571	0e+00
##	[85,]	1.1806	0.8471	0e+00
##	[86,]	1.1944	0.8372	0e+00
##	[87,]	1.2083	0.8276	0e+00
##	[88,]	1.2222	0.8182	0e+00
##	[89,]	1.2361	0.8090	0e+00
##	[90,]	1.2500	0.8000	0e+00
##	[91,]	1.2639	0.7912	0e+00
##	[92,]	1.2778	0.7826	0e+00
##	[93,]	1.2917	0.7742	0e+00
##	[94,]	1.3056	0.7660	0e+00
##	[95,]	1.3194	0.7579	0e+00
##	[96,]	1.3333	0.7500	0e+00
##	[97,]	1.3472	0.7423	0e+00
##	[98,]	1.3611	0.7347	0e+00
##	[99,]	1.3750	0.7273	0e+00
##	[100,]	1.3889	0.7200	0e+00
##	[101,]	1.4028	0.7129	0e+00
##	[102,]	1.4167	0.7059	0e+00
##	[103,]	1.4306	0.6990	0e+00
##	[104,]	1.4444	0.6923	0e+00
##	[105,]	1.4583	0.6857	0e+00
##	[106,]	1.4722	0.6792	0e+00
##	[107,]	1.4861	0.6729	0e+00
##	[108,]	1.5000	0.6667	0e+00
##	[109,]	1.5139	0.6606	0e+00
##	[110,]	1.5278	0.6545	0e+00
##	[111,]	1.5417	0.6486	0e+00
##	[112,]	1.5556	0.6429	0e+00
##	[113,]	1.5694	0.6372	0e+00
##	[114,]	1.5833	0.6316	0e+00
##	[115,]	1.5972	0.6261	0e+00
##	[116,]	1.6111	0.6207	0e+00
##	[117,]	1.6250	0.6154	0e+00
##	[118,]	1.6389	0.6102	0e+00
##	[119,]	1.6528	0.6050	0e+00
##	[120,]	1.6667	0.6000	0e+00
##	[121,]	1.6806	0.5950	0e+00
##	[122,]	1.6944	0.5902	0e+00
##	[123,]	1.7083	0.5854	0e+00
##	[124,]	1.7222	0.5806	0e+00
##	[125,]	1.7361	0.5760	0e+00
##	[126,]	1.7500	0.5714	0e+00
##	[127,]	1.7639	0.5669	0e+00
##	[128,]	1.7778	0.5625	0e+00
##	[129,]	1.7917	0.5581	0e+00
##	[130,]	1.8056	0.5538	1e-04

```
## [131,] 1.8194 0.5496 0e+00
## [132,] 1.8333 0.5455 0e+00
## [133,] 1.8472 0.5414 0e+00
## [134,] 1.8611 0.5373 0e+00
## [135,] 1.8750 0.5333 0e+00
## [136,] 1.8889 0.5294 0e+00
## [137,] 1.9028 0.5255 0e+00
## [138,] 1.9167 0.5217 0e+00
## [139,] 1.9306 0.5180 0e+00
## [140,] 1.9444 0.5143 0e+00
## [141,] 1.9583 0.5106 0e+00
## [142,] 1.9722 0.5070 0e+00
## [143,] 1.9861 0.5035 1e-04
## [144,] 2.0000 0.5000 0e+00
```

```
df <- as.data.frame(gdp.per$details)
df
```

```
##      frequency  period spectrum
## 1      0.0139 72.0000 0e+00
## 2      0.0278 36.0000 0e+00
## 3      0.0417 24.0000 0e+00
## 4      0.0556 18.0000 1e-04
## 5      0.0694 14.4000 0e+00
## 6      0.0833 12.0000 1e-04
## 7      0.0972 10.2857 1e-04
## 8      0.1111 9.0000 1e-04
## 9      0.1250 8.0000 0e+00
## 10     0.1389 7.2000 0e+00
## 11     0.1528 6.5455 1e-04
## 12     0.1667 6.0000 0e+00
## 13     0.1806 5.5385 1e-04
## 14     0.1944 5.1429 0e+00
## 15     0.2083 4.8000 0e+00
## 16     0.2222 4.5000 0e+00
## 17     0.2361 4.2353 1e-04
## 18     0.2500 4.0000 0e+00
## 19     0.2639 3.7895 1e-04
## 20     0.2778 3.6000 0e+00
## 21     0.2917 3.4286 1e-04
## 22     0.3056 3.2727 0e+00
## 23     0.3194 3.1304 0e+00
## 24     0.3333 3.0000 0e+00
## 25     0.3472 2.8800 0e+00
## 26     0.3611 2.7692 1e-04
## 27     0.3750 2.6667 0e+00
## 28     0.3889 2.5714 1e-04
## 29     0.4028 2.4828 0e+00
## 30     0.4167 2.4000 0e+00
## 31     0.4306 2.3226 1e-04
## 32     0.4444 2.2500 0e+00
## 33     0.4583 2.1818 2e-04
## 34     0.4722 2.1176 0e+00
## 35     0.4861 2.0571 1e-04
```



## 36	0.5000	2.0000	0e+00
## 37	0.5139	1.9459	0e+00
## 38	0.5278	1.8947	0e+00
## 39	0.5417	1.8462	0e+00
## 40	0.5556	1.8000	0e+00
## 41	0.5694	1.7561	0e+00
## 42	0.5833	1.7143	1e-04
## 43	0.5972	1.6744	0e+00
## 44	0.6111	1.6364	0e+00
## 45	0.6250	1.6000	0e+00
## 46	0.6389	1.5652	0e+00
## 47	0.6528	1.5319	0e+00
## 48	0.6667	1.5000	0e+00
## 49	0.6806	1.4694	0e+00
## 50	0.6944	1.4400	0e+00
## 51	0.7083	1.4118	0e+00
## 52	0.7222	1.3846	0e+00
## 53	0.7361	1.3585	0e+00
## 54	0.7500	1.3333	0e+00
## 55	0.7639	1.3091	0e+00
## 56	0.7778	1.2857	0e+00
## 57	0.7917	1.2632	0e+00
## 58	0.8056	1.2414	0e+00
## 59	0.8194	1.2203	0e+00
## 60	0.8333	1.2000	0e+00
## 61	0.8472	1.1803	0e+00
## 62	0.8611	1.1613	0e+00
## 63	0.8750	1.1429	0e+00
## 64	0.8889	1.1250	0e+00
## 65	0.9028	1.1077	0e+00
## 66	0.9167	1.0909	0e+00
## 67	0.9306	1.0746	0e+00
## 68	0.9444	1.0588	0e+00
## 69	0.9583	1.0435	0e+00
## 70	0.9722	1.0286	0e+00
## 71	0.9861	1.0141	0e+00
## 72	1.0000	1.0000	0e+00
## 73	1.0139	0.9863	0e+00
## 74	1.0278	0.9730	0e+00
## 75	1.0417	0.9600	0e+00
## 76	1.0556	0.9474	0e+00
## 77	1.0694	0.9351	0e+00
## 78	1.0833	0.9231	0e+00
## 79	1.0972	0.9114	0e+00
## 80	1.1111	0.9000	0e+00
## 81	1.1250	0.8889	0e+00
## 82	1.1389	0.8780	0e+00
## 83	1.1528	0.8675	1e-04
## 84	1.1667	0.8571	0e+00
## 85	1.1806	0.8471	0e+00
## 86	1.1944	0.8372	0e+00
## 87	1.2083	0.8276	0e+00
## 88	1.2222	0.8182	0e+00
## 89	1.2361	0.8090	0e+00

## 90	1.2500	0.8000	0e+00
## 91	1.2639	0.7912	0e+00
## 92	1.2778	0.7826	0e+00
## 93	1.2917	0.7742	0e+00
## 94	1.3056	0.7660	0e+00
## 95	1.3194	0.7579	0e+00
## 96	1.3333	0.7500	0e+00
## 97	1.3472	0.7423	0e+00
## 98	1.3611	0.7347	0e+00
## 99	1.3750	0.7273	0e+00
## 100	1.3889	0.7200	0e+00
## 101	1.4028	0.7129	0e+00
## 102	1.4167	0.7059	0e+00
## 103	1.4306	0.6990	0e+00
## 104	1.4444	0.6923	0e+00
## 105	1.4583	0.6857	0e+00
## 106	1.4722	0.6792	0e+00
## 107	1.4861	0.6729	0e+00
## 108	1.5000	0.6667	0e+00
## 109	1.5139	0.6606	0e+00
## 110	1.5278	0.6545	0e+00
## 111	1.5417	0.6486	0e+00
## 112	1.5556	0.6429	0e+00
## 113	1.5694	0.6372	0e+00
## 114	1.5833	0.6316	0e+00
## 115	1.5972	0.6261	0e+00
## 116	1.6111	0.6207	0e+00
## 117	1.6250	0.6154	0e+00
## 118	1.6389	0.6102	0e+00
## 119	1.6528	0.6050	0e+00
## 120	1.6667	0.6000	0e+00
## 121	1.6806	0.5950	0e+00
## 122	1.6944	0.5902	0e+00
## 123	1.7083	0.5854	0e+00
## 124	1.7222	0.5806	0e+00
## 125	1.7361	0.5760	0e+00
## 126	1.7500	0.5714	0e+00
## 127	1.7639	0.5669	0e+00
## 128	1.7778	0.5625	0e+00
## 129	1.7917	0.5581	0e+00
## 130	1.8056	0.5538	1e-04
## 131	1.8194	0.5496	0e+00
## 132	1.8333	0.5455	0e+00
## 133	1.8472	0.5414	0e+00
## 134	1.8611	0.5373	0e+00
## 135	1.8750	0.5333	0e+00
## 136	1.8889	0.5294	0e+00
## 137	1.9028	0.5255	0e+00
## 138	1.9167	0.5217	0e+00
## 139	1.9306	0.5180	0e+00
## 140	1.9444	0.5143	0e+00
## 141	1.9583	0.5106	0e+00
## 142	1.9722	0.5070	0e+00
## 143	1.9861	0.5035	1e-04

```
## 144      2.0000  0.5000      0e+00
```

```
U = qchisq(.025,2)
L = qchisq(.975,2)
spec <- df %>% arrange(desc(spectrum)) %>% slice(1:3)
spec
```

```
##   frequency  period spectrum
## 1    0.4583   2.1818    2e-04
## 2    0.0556  18.0000    1e-04
## 3    0.0833  12.0000    1e-04
```

After we sort and subset the data, we get the first three predominant periods. Then we calculate the upper and lower bounds, and those two columns would be combined in the dataset.

```
spec%>%
  mutate(Upper= 2*spec$spectrum/U)%>%mutate(Lower = 2*spec$spectrum/L)
```

```
##   frequency  period spectrum      Upper      Lower
## 1    0.4583   2.1818    2e-04 0.007899578 5.421701e-05
## 2    0.0556  18.0000    1e-04 0.003949789 2.710850e-05
## 3    0.0833  12.0000    1e-04 0.003949789 2.710850e-05
```

From Figure 7, the dominant periods are 2.1818, 18 and 12. Those 95% confidence intervals are too wide to establish the significance of the peak.

#### Discussion

Since this is a quarterly time series, there seems to be some seasonal trend. So, an ARIMA model may not be the best fitted model and SARIMA models may have been a better one. Meanwhile, there are some outliers at the tails of the Q-Q plot which limiting the model prediction. So, SARIMA models may have been a better fit for this data. Moreover, in the prediction of the future 10 quarters, the estimated 95% interval of 2020 GDP growth rate is (-0.01, 0.03) but the GDP growth rate of 2020 in real world is -0.035. It is due to the current COVID-19 pandemic which is one of the most significant black swan events in history. There is no model can predict that.