

Empirical IO

2. Consumer Demand

Utility Function → Demand Systems

Demand Systems in Product Space

- standard approach in EIO until 1990s.
- consumer preference are defined over products themselves.

Demand System

The Linear Expenditure Demand System

Stone-Geary Utility Function

$$U = (q_0 - \gamma_0)^{\theta_0} (q_1 - \gamma_1)^{\theta_1} \dots (q_J - \gamma_J)^{\theta_J} \quad (2.5)$$

Demand System

$$q_j = \gamma_j + \alpha_j \left[ \frac{p_j - \beta_j}{P} \right] \quad (2.6)$$

where  $P$  is the aggregate price index  $\sum_{j=0}^J \alpha_j P_j$ .

$$q_j = \gamma_j + \alpha_j \frac{p_j - \beta_j}{P} + \beta_j \frac{p_j}{P} + \xi_j \quad (2.7)$$

with  $\beta_j = -\alpha_j \gamma_j$ . Variable  $\xi_j$  is an error term that can come, for instance, from measurement error in purchased quantity  $q_j$  or from time variation in the coefficient  $\gamma_j$ . The intercept and slope parameters in these linear regression models can be estimated using instrumental variable methods.

Properties

All the products are complements in consumption.

$$\frac{\partial q_j}{\partial p_j} = -\alpha_j \beta_j / P_j < 0$$

Constant Elasticity of Substitution Demand System

CES Utility Function

$$v = \left( \sum_{j=0}^J \alpha_j^{\frac{1}{\sigma}} \right)^{\sigma} \quad (2.8)$$

Demand System

$$q_j = \frac{\gamma_j}{P_j} \left[ \frac{p_j}{P} \right]^{1-(1-\sigma)} \quad (2.10)$$

where  $P_j$  is the following aggregate price index:

$$P_j = \left( \sum_{k=0}^J \alpha_k^{1-\sigma} \right)^{1/(1-\sigma)} \quad (2.11)$$

$$\ln \left( \frac{q_j}{\alpha_j} \right) = \beta_k + \beta_j \ln(p_j) + \beta_k \ln(P_{jk}) + \xi_j \quad (2.12)$$

Properties

Same degree of substitution for all pairs of products.

"Almost Ideal" Demand System

Expenditure Function

$$E(p, u) = \min_{q_0, \dots, q_J} \sum_{j=0}^J p_j q_j \quad \text{subject to } U(q_0, q_1, \dots, q_J) = u \quad (2.14)$$

Demand System

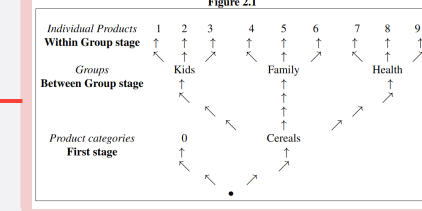
$$q_j = \alpha_j + \beta_j [\ln(u) - \ln(P_{jk})] + \sum_{k=0}^J \gamma_{jk} \ln p_k \quad (2.20)$$

where  $\gamma_{jk} = (\gamma_{kj}^* + \gamma_{kj}^{**})/2$  such that the model implies the symmetry condition  $\gamma_{jk} = \gamma_{kj}$ ; and  $P_{jk}$  is a price index with the following form:

$$\ln(P_{jk}) = \sum_{k=0}^J \alpha_k \ln p_k + \frac{1}{2} \sum_{k=0}^J \sum_{l=0}^J \gamma_{kl} \ln p_k \ln p_l \quad (2.21)$$

Properties

Estimation of this demand system requires that  $T \gg J$  (as shown in equation 2.22), which is hard to be satisfied.



$$q_j = \alpha_j^{(j)} + \beta_j^{(j)} \ln \left( \frac{p_j}{P_j} \right) + \sum_{k=0}^J \gamma_{jk}^{(j)} \ln p_k \quad (2.24)$$

Group Stage

$$q_j^{(g)} = \alpha_j^{(g)} + \beta_j^{(g)} \ln \left( \frac{p_j}{P_j} \right) + \sum_{k=0}^J \gamma_{jk}^{(g)} \ln p_k \quad (2.25)$$

Top Stage

$$q_j = \alpha_j^{(t)} + \beta_j^{(t)} [\ln(q_j) - \ln(p_j)] \quad (2.26)$$

Deal with high dimensionality of parameters

Multi-stage Budgeting Demand System

Demand Systems in Characteristics Space

- predominant approach in EIO over the past two decades.
- a product is a bundle of characteristics.
- consumers have preference over characteristics of products.

Demand System

Logit Model

$$U_{ij} = -\alpha p_j + X_j \beta + \xi_j + \varepsilon_{ij} \quad (2.41)$$

Market Share

$$s_j = \frac{e^{U_j}}{\sum_{j=0}^J e^{U_j}} \quad (2.42)$$

where  $\delta_j \equiv -\alpha p_j + X_j \beta + \xi_j$  represents the mean utility of buying product  $j$ .

Nested Logit Model

$$U_{ij} = \lambda U_j^{(g)} + \varepsilon_{ij}^{(g)} \quad (2.43)$$

Market Share

$$s_j = \frac{\exp\{\lambda U_j^{(g)}\}}{\sum_{j \in g} \exp\{\lambda U_j^{(g)}\}} \frac{\exp\{\delta_j\}}{\sum_{j=0}^J \exp\{\delta_j\}} \quad (2.44)$$

Random Coefficients Logit Model

$$U_{ij} = -\alpha p_j + X_j \beta + \xi_j + \varepsilon_{ij} \quad (2.46)$$

where  $\varepsilon_{ij} = -\alpha_j^{(g)} p_j + \beta_j^{(g)} X_j + \dots + \beta_j^{(g)} X_{K_j}$  has a heteroskedastic normal distribution.

Market Share

$$s_j = \frac{e^{U_j}}{J} = \frac{\exp\{\delta_j + \xi_j\}}{1 + \sum_{j=0}^J \exp\{\delta_j + \xi_j\}} \quad (2.47)$$

with  $\delta_j = -\alpha p_j + X_j \beta + \xi_j$  still representing the mean utility of buying product  $j$ .

BLP(1995)

Market share can be derived using a mapping from mean utility (conditioned on price, exogenous variables, etc.)

In general, for any distribution of consumer heterogeneity  $u_0$ , the model implies a mapping between the  $J+1$  vector of mean utilities  $\delta = (\delta_j, j=1,2,\dots,J)$  and the  $J+1$  vector of market shares  $s = (s_j, j=1,2,\dots,J)$ :

$$s_j = \sigma_j(\delta) \quad \text{for } j=1,2,\dots,J \quad (2.48)$$

or in vector form  $s = \sigma(\delta) \quad (p, X, Z)$ .

**Berry's Inversion Property**

Berry (1984) shows that, under some regularity conditions (more later), the demand system  $s = \sigma(\delta) \quad (p, X, Z)$  is invertible in  $\delta$  such that there is an inverse function  $\sigma^{-1}$ , and:

$$\delta = \sigma^{-1}(s) \quad (p, X, Z) \quad (2.51)$$

or for a product  $j$ ,  $\delta_j = \sigma_j^{-1}(s) \quad (p, X, Z)$ . The form of the inverse mapping  $\sigma^{-1}$  depends on the PDF  $f_{\varepsilon}$ .

This inversion property has important implications for the estimation of the demand system. Under this inversion, the unobserved product characteristics  $\xi_j$  enter additively in the equation  $\delta_j = \sigma_j^{-1}(s) \quad (p, X, Z)$ . Under this additivity, and the mean independence of the unobservable  $\xi_j$  conditional on the response product characteristics  $X_j$ , we can construct moment conditions and obtain GMM estimates of the structural parameters that deal with the endogeneity of prices.

consumer's optimization problem

$$\max_{(q_0, \dots, q_J)} U(q_0, \dots, q_J) \quad (2.1)$$

subject to:  $q_0 + p_1 q_1 + \dots + p_J q_J \leq y$

consumer's optimization problem

$$\max_{(d_0, d_1, \dots, d_J)} u(C; u_0) = \sum_{j=0}^J d_j u_j(X_j, \xi_j; u_0) \quad (2.37)$$

subject to:  $C = \sum_{j=0}^J d_j p_j \leq y$   
 $d_{0j} \in [0, 1]$  and  $\sum_{j=0}^J d_{0j} \in [0, 1]$