Compositional Reasoning about Connectors Using Coq and Z3

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Abstract

Reo is a channel-based exogenous coordination language in which complex coordinators, called connectors, are compositionally built out of simpler ones. In this paper, we present an approach to model and reason about connectors using Coq and Z3. The model reflects the original structure of connectors simply and clearly. In our framework, basic connectors (channels) are interpreted as axioms and composition operations are specified as inference rules. Furthermore, connectors are interpreted as logical predicates which describe the relations between inputs and outputs. With such definitions provided, connector properties, as well as equivalence and refinement relations between different connectors, can be naturally formalized as *goals* in Coq and easily proved using pre-defined *tactics*. Particularly, to fulfill the proof of refinement relation, Z3 is used as an assistant tool for solving logical formulas.

Keywords: Coordination language, Reo, Coq, Z3, Reasoning

1. Introduction

Modern software systems are typically distributed over large networks of computing devices, and usually the components that comprise a system do not exactly fit together as pieces of a jigsaw puzzle, but leave significant interfacing gaps that must somehow be filled with additional code. Compositional coordination models and languages provide a formalization of the "glue code" that interconnects the constituent components and organizes the mutual interactions among them in a distributed processing environment, and played a crucial role for the success of component-based systems in the past decades.

As an example, Reo [3], which is a channel-based exogenous coordination language, offers a powerful framework for implementation of coordinating component connectors. Connectors provide the protocols that control and organize the communication, synchronization and cooperation among the components that they interconnect. Primitive connectors, called *channels* in Reo, can be composed to build complex connectors. Reo has been successfully applied in different application domains, such as service-oriented computing and bioinformatics [7, 19]. In recent years, verifying the correctness of connectors is becoming a critical challenge, especially due to the advent of Cloud computing technologies. The rapid growth of size and complexity of the computing infrastructures has made it more difficult to model and verify connector properties, and thus leads to less confidence on the correctness of connectors.

Several works have been done for formal modeling and verifying connectors. An operational semantics for Reo using Constraint Automata (CA) was provided by Baier et al. [6], and later the symbolic model checker Vereofy [5] was developed, which can be used to check CTL-like properties. Besides, one attractive approach is to translate from Reo to other formal models such as Alloy [13], mCRL2 [15], UTP [2, 20], etc., which makes it possible to take advantage of existing verification tools. A comparison of existing semantic models for Reo can be found in [12].

In a previous paper [21] we showed how to formally model and reason about connectors' properties in Coq. The basic idea of our approach is to model the behavior of a connector by representing it as a logical

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predicate which describes the relation among the timed data streams on the input and output nodes, and to reason about connectors' properties, as well as the equivalence and refinement relations between connectors, by using proof principles and tactics in Coq. Compared with existing approaches for verifying connectors' properties [5, 14, 15], reasoning about connectors using theorem provers like Coq is especially helpful when we take infinite behavior into consideration. The coinductive proof principle makes it possible to prove connectors' properties easily while it is difficult (sometimes impossible) for other approaches (like model checking) because of the huge (or maybe infinite) number of states.

In the current paper, we aim to provide a more complete account of the approach in [21] to formally model and reason about connectors. The main drawback of failing to check the complementary refinement relation in Coq is made up by using the constraint solver Z3. While Coq provides a platform to prove properties with a series of tactics and strategies to be considered, Z3 can be used as a component in the context that requires solving logical formulas. The logical formulas involved in the model of Reo should reflect the constraints of connectors and be consistent with their behavior.

The approach is not a brand new idea, as we have already provided a solution for modeling Reo in Coq in [16], where connectors are represented in a constructive way, and verification is essentially based on simulations. We do believe that the approach in this paper is reasonably different from its predecessor [16] where Coq seldom shows its real power. To be more specific, our new work has its certain advantages comparing with [16] in the following aspects:

- **Modeling Method:** We use axioms to describe basic channels and their composition operations, which is more natural on a proof-assistant platform than the simulation-based approach in [16].
- **Expression Power:** Any valid Coq expression can be used to depict properties, which is obviously more powerful than just using LTL formulas in [16]. Furthermore, support for continuous time behavior is also possible in our approach in this paper.
- **Refinement and Equivalence Checking:** In our framework, equivalence and refinement relations can be proved among different connectors, while the approach in [16] is not capable of either equivalence or refinement checking. Furthermore, the constraint solver Z3 is adopted to prove non-refinement relations between connectors, while in [21] we can only derive proofs for refinement relations.

The paper is organized as follows: After this general introduction, we briefly summarize Reo, Coq and Z3 in Section 2. Section 3 shows the notion of timed data streams and some pre-defined auxiliary functions and predicates. Section 4 presents the formal modeling of basic channels and operators, as well as complex connectors. Section 5 shows how to reason about connector properties and equivalence (or refinement) relations in our framework. Section 6 concentrates on the proof of refinement relation between two connectors. In Section 7, we conclude with some further research directions. Full source codes can be found at [**?**] for further reference.

2. Preliminaries

In this section, we provide a brief introduction to the coordination language Reo, and the proof assistant tools Coq and Z3.

2.1. The Coordination Model Reo

Reo is a channel-based exogenous coordination language wherein complex coordinators, called connectors, are compositionally built out of simpler ones [3]. Further details about Reo and its semantics can be found in [3, 4, 6]. The simplest connectors are channels with well-defined behavior such as synchronous channels, FIFO channels, etc. Each channel in Reo has exactly two directed ends, with their own identities. There are two types of channel ends: *source* ends and *sink* ends. A source channel end accepts data into the channel. A sink channel end dispenses data out of the channel.

The graphical notations of some basic channels are presented in Fig. 1, and their behavior can be interpreted as follows:

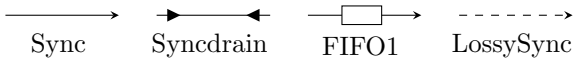


Figure 1: Four types of basic channels.

- **Sync:** a synchronous channel with one source end and one sink end. The pair of I/O operations on its two ends can succeed only simultaneously.
- **SyncDrain:** a synchronous channel which has two source ends. The pair of input operations on its two ends can succeed only simultaneously. All data items written to this channel are lost.
- **FIFOn:** an asynchronous channel with one source end and one sink end, and a bounded buffer with capacity n. It can accept data items from its source end. The accepted data items are kept in the internal buffer, and dispensed to the sink end in FIFO order. Especially, the FIFO1 channel is an instace of FIFOn where the buffer capacity is 1.
- **LossySync:** a synchronous channel with one source end and one sink end. The source end always accepts all data items. If there is no matching output operation on the sink end of the channel at the time that a data item is accepted, then the data item is lost; otherwise, the channel transfers the data item exactly the same as a Sync channel, and the output operation at the sink end succeeds.

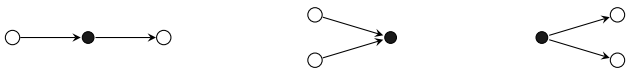


Figure 2: Operations of channel composition.

Complex connectors are constructed by composing simpler ones via the join and hiding operations. Channels are joined together in nodes. The set of channel ends coincident on a node is disjointly partitioned into the sets of source and sink channel ends that coincide on the node, respectively. Nodes are categorized into source, sink and mixed nodes, depending on whether all channel ends that coincide on a node are source ends, sink ends or a combination of the two. The hiding operation is used to hide the internal topology of a connector. The hidden nodes can no longer be accessed or observed from outside. There are three types of operations for channel composition: *flow-through*, *merge* and *replicate*. Fig. 2 provides the graphical representation of these operations.

2.2. The Proof Assistant tools

Coq[10] is a widely-used proof assistant tool, where denotational formalizations (e.g. theorem and hypothesis) and operational formalizations (e.g. functions and algorithms) are naturally integrated. Moreover, it allows the interactive construction of formal proofs. The formal language used in Coq, *Gallina*, provides a convenient way to define both programming statements and mathematical propositions, for example:

```
1   (* a variable definition *)
2   Variables a b: nat.
3   (* a simple non-recursive function *)
4   Definition inc(a:nat) := a + 1.
5   (* axioms don't have to be proved *)
6   Axiom inc_ax: forall c:nat, inc(c) > c.
7   (* theorems rely on proving *)
8   Theorem inc_eq: forall c:nat, inc(c) = c + 1.
9   Proof.
10    (* interactive proving based on tactics *)
11    auto.
12  Qed.
```

As shown in this example, there are two rather different mode in Coq's interactive shell. When we start Coq, we can write declarations and definitions in a functional-programming mode. Then, when we start a *Theorem*, or *Lemma*, Coq jumps into the proving mode. We need to write different *tactics* to reduce the proving goal and finally finish the formal proof.

Furthermore, Coq is equipped with a set of well-written standard libraries. For example, as used in this paper, *List* describes the widely-used finite list structure, *Stream* provides a co-inductive definition of infinite lists, and *Reals* defines various operations and theorems on real numbers. Usually, quite a few lemmas and theorems are pre-defined in such libraries, making it substantially easier to prove our goals.

Z3 [8] is an efficient SMT (Satisfiability Modulo Theories) Solver and a high performance theorem prover freely available from Microsoft. It can be used in various software verification and analysis applications. Acting as a SMT solver, Z3 expands to deciding the satisfiability (or dually the validity) of first order formulas with respect to combinations of theories such as: arithmetic, bit-vectors, arrays, and uninterpreted functions. Given the data and time constraints of connectors, it allows us to decide the satisfiability against properties or refinement relations. Z3 provides bindings for several programming languages. In this paper, we use the Z3 API in Python to construct the models and carry out experiments. The following example provides an intuitive understanding of the Z3 solver.

```
1  x = Int('x')
2  y = Int('y')
3
4  s = Solver()
5  s.add(x > 10, Or(x + y > 3, x - y < 2))
6  #Solving constraints in the solver s
7  print s.check()
8
9  #Create a new scope
10 s.push()
11 s.add(y < 11)
12 #Solving updated set of constraints
13 print s.check()
14
15 #Restoring state
16 s.pop()
17 #Solving restored set of constraints
18 print s.check()
```

This example contains two constraints at first. From these two constraints, we can see that Z3 supports Boolean operators, such as *And*, *Or*, *Not* and *Implies (implication)*, apart from the operators like $<$, $>$ for comparison. The command *Solver()* creates a general purpose solver. Constraints can be added using the method *add*. Once they are added to the solver, we can say that the constraints have been asserted in the solver. The method *check()* solves the asserted constraints. Finally, the result is satisfiable (unsatisfiable) if a solution is (not) found, respectively. A solver may fail to solve a system of constraints and *unknown* is returned. Each solver maintains a stack of assertions. The command *push* creates a new scope by saving the current stack size. The command *pop* removes any assertion performed between it and the matching *push*.

3. Basic Definitions

In this section, we briefly introduce the notion of timed data streams and some pre-defined auxiliary functions and predicates in Coq, which are used in the following sections for modeling connectors.

The behavior of a connector can be formalized by means of data-flows at its sink and source nodes which are essentially infinite sequences. With the help of the stream library in Coq, such infinite data-flows can

be defined as *timed data streams*:

```
1     Definition Time := R.
2     Definition Data := nat.
3     (*Inductive Data : Set :=
4         |Natdata : nat-> Data
5         |Empty : Data.*)
6     Definition TD := Time * Data.
7     Variable Input : Stream TD.
8     Variable Output : Stream TD.
```

In our framework, time is represented by real numbers. The continuity of the set of real numbers is sufficiently enough for our modeling approach. Also the continuous time model is more appropriate than the discrete model since it is very expressive and closer to the nature of time in the real world. Thus, the time sequence consists of increasing and diverging time moments. For simplicity, here we take the natural numbers as the definition of data, which can be easily expanded according to different application domains. The Cartesian product of time and data defines a TD object. We use the stream module in Coq to produce streams of TD objects.

Some auxiliary functions and predicates are defined to facilitate the representation of axioms for basic channels in Reo. This part can be extended for further use in different problems. The terms "PrL" and "PrR" take a pair of values (a, b) that has Cartesian product type A $\times$ B as the argument and return the first or second value of the pair, respectively. The following functions provide some judgment of time, which can make the description of axioms and theorems for connectors more concise and clear: "Teq" means that time of two streams are equal and "Tneq" has the opposite meaning. "Tle" ("Tgt") represents that time of the first stream is strictly less (greater) than the second stream. The judgement about equality of data is analogous to the judgement of time. The complete definition of these functions can be found at [1].

4. Formal Modeling of Basic Channels and Operators

In this section, we show how primitive connectors, i.e., channels, and operators for connector composition are specified in Coq and used for modeling of complex connectors. Then we can apply the tactics provided in Coq to reason about connector properties. Basic channels, which can be regarded as axioms of the whole framework, are specified as logical predicates illustrating the relation between the timed data streams of input and output. When we need to construct a more complex connector, appropriate composition operators are applied depending on the topological structure of the connector.

4.1. Formal Modeling of Basic Channels

We use a pair of predicates to describe the constraints on time and data, respectively, and their intersection to provide the complete specification of basic channels. This model offers convenience for the analysis and proof of connector properties. In the following, we present a few examples of the formal model of basic channels.

The simplest form of a synchronous channel is denoted by the Sync channel type. For a channel of the Sync type, a read operation on its sink end succeeds only if there is a write operation pending on its source end. Thus, the time and data of a stream flowing into the channel are exactly the same as the stream that flows out of the channel [1]. The Sync channel can be defined as follows in the Coq system:

[1] If we use α, β to denote the data streams that flow through the channel ends of a channel and a, b to denote the time stream corresponding to the data streams, i.e., the i-th element $a(i)$ in a denotes exactly the time moment of the occurrence of $\alpha(i)$, then we can easily obtain the specifications for different channels, as discussed in [18, 20]. For example, a synchronous channel can be expressed as $\alpha = \beta \wedge a = b$.

```
1     Definition Sync (Input Output:Stream TD) : Prop :=
2     Teq Input Output /\ Deq Input Output.
```

The channel of type SyncDrain is a synchronous channel that allows pairs of write operations pending on its two ends to succeed simultaneously. All written data items are lost. Thus, the SyncDrain channel is used for synchronising two timed data streams on its two source ends. This channel type is an important basic synchronization building block for the construction of more complex connectors. The SyncDrain channel can be defined as follows:

```
1     Definition SyncDrain (Input Output:Stream TD) : Prop :=
2     Teq Input Output.
```

The channel types FIFO and FIFOn where n is an integer greater than 0 represent the typical unbounded and bounded asynchronous FIFO channels. A write to a FIFO channel always succeeds, and a write to a FIFOn channel succeeds only if the number of data items in its buffer is less than its bounded capacity n. A read or take from a FIFO or FIFOn channel suspends until the first data item in the channel buffer can be obtained and then the operation succeeds. For simplicity, we take the FIFO1 channel as an example. This channel type requires that the time when it consumes a data item through its source end is earlier than the time when the data item is delivered through its sink end. Besides, as the buffer has the capacity 1, time of the next data item that flows in should be later than the time when the data in the buffer is delivered. We use intersection of predicates in its definition as follows:

```
1     Definition FIFO1(Input Output:Stream TD) : Prop :=
2     Tle Input Output /\ Tle Output (tl Input) /\ Deq Input Output.
```

For a FIFO1 channel whose buffer already contains a data element e, the communication can be initiated only if the data element e can be taken via the sink end. In this case, the data stream that flows out of the channel should get an extra element e settled at the beginning of the stream. And time of the stream that flows into the channel should be earlier than time of the tail of the stream that flows out. But as the buffer contains the data element e, new data can be written into the channel only after the element e has been taken. Therefore, time of the stream that flows out is earlier than time of the stream that flows in. The channel can be represented as the

```
1     Definition FIFO1e(Input Output:Stream TD)(e:Data) : Prop :=
2     Tgt Input Output /\ Tle Input (tl Output)
3     /\ PrR (hd Output) = e /\ Deq Input (tl Output).
```

In the following we choose Axiom to define LossySync and AsyncDrain because it is easier to use the coinductive expression to specify their behavior.

A LossySync channel behaves the same as a Sync channel, except that a write operation on its source end always succeeds immediately. If a compatible read or take operation is already pending on its sink end, the written data item is transferred to the pending operation and both succeed. Otherwise, the write operation succeeds and the data item is lost. The LossySync channel can be defined as follows:

```
1    Parameter LossySync: Stream TD -> Stream TD -> Prop.
2    Axiom LossySync_coind:
3      forall Input Output: Stream TD,
4      LossySync Input Output ->
5        (
6         (hd Output = hd Input /\ LossySync (tl Input)(tl Output)) \/
7         LossySync(tl Input) Output
8        ).
```

AsyncDrain is analogous to SyncDrain except that it guarantees that the pairs of write operations on the two channel ends never succeed simultaneously. Similarly it only has requirements on the time of the two streams on its opposite ends, but it requires that the times of the two streams are always different. The AsyncDrain channel can be defined as follows:

```
1     Parameter AsyncDrain: Stream TD -> Stream TD -> Prop.
2     Axiom AsyncDrain_coind:
3       forall Input1 Input2: Stream TD,
4       AsyncDrain Input1 Input2 ->
5         (~ PrL(hd Input1) = PrL (hd Input2) ) /\
6         (
7           ( (PrL(hd Input1) < PrL (hd Input2)) /\ AsyncDrain (tl Input1) Input2) /\
8           ( (PrL(hd Input1) > PrL (hd Input2)) /\ AsyncDrain Input1 (tl Input2) )
9         ).
```

Defining basic channels by intersection of predicates provides the following benefits:

- Firstly, this makes the model intuitive and concise as each predicate describes a simple order relation on time or data.
- Secondly, we can easily split predicates for proofs of different properties which can make the proving process simpler.

4.2. Formal modeling of Operators

We have just described the way to define channel types, by means of definitions in Coq. Now we start defining the composition operators for connector construction. There are three types of composition operators for connector construction, which are *flow-through*, *replicate* and *merge*, respectively.

The flow-through operator simply allows data items to flow through the junction node, from one channel to the other. We need not to give the flow-through operator a specific definition in the Coq system. For example, while we illustrate two channels *Sync(A,B)* and *FIFO1(B,C)*, a flow-through operator that acts on node B for these two channels has been achieved implicitly.

The replicate operator puts the source ends of different channels together into one common node, and a write operation on this node succeeds only if all the channels are capable of consuming a copy of the written data. Similar to the flow-through operator, it can be implicitly represented by the structure of connectors. For example, for two channels *Sync(A,B)* and *FIFO1(C,D)*, we can illustrate *Sync(A,B)* and *FIFO1(A,D)* in Coq instead of defining a function like *rep(Sync(A,B),FIFO1(C,D))* and the replicate operator is achieved directly by renaming C with A for the FIFO1 channel.

The merge operator is more complicated. We consider merging two channels AB and CD. When the merge operator acts on these two channels, it leads to a choice of taking from the common node that delivers a data item out of AB or CD. Similar to the definition of basic channels, we define merge as the intersection of two predicates and use recursive definition here:

```
1     Parameter merge:
2     Stream TD -> Stream TD ->Stream TD -> Prop.
3     Axiom merge_coind:
4       forall s1 s2 s3:Stream TD,
5       merge s1 s2 s3-> (
6         ~ (PrL(hd s1) = PrL(hd s2)) /\
7         ( (PrL(hd s1) < PrL(hd s2)) -> ((hd s3 = hd s1) /\ merge (tl s1) s2 (tl s3)) )
              /\
8         ( (PrL(hd s1) > PrL(hd s2)) -> ((hd s3 = hd s2) /\ merge s1 (tl s2) (tl s3)) )
9       ).
```

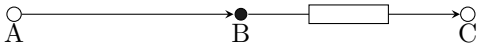


Figure 3: A connector consisting of a Sync channel and a FIFO1 channel.

Based on the definition of basic channels and operators, more complex connectors can be constructed structurally. To show how a composite connector is constructed, we consider a simple example as shown in Fig. 3, where a FIFO1 channel is attached to the sink end of a Sync channel. Assume AB is of type Sync and BC is of type FIFO1, then we can construct the required connector by illustrating *Sync(A,B)* and *FIFO1(B,C)*. The configuration and the functionality of the required connector can be specified using this concise method. Note that the composition operations can be easily generalized to the case of multiple nodes, where the modeling of connectors is similar. More examples can be found in Section 5.

5. Reasoning about Connectors

After modeling a connector in Coq, we can analyse and prove important properties of the connector. In this section, we give some examples to elucidate how to reason about connector properties and prove refinement/equivalence relations between different connectors, with the help of Coq.

5.1. Derivation of Connector Properties

The proof process of a property is as follows: the user states the proposition that needs to be proved, called a *goal*, then he/she applies commands called *tactics* to decompose this goal into simpler subgoals or solve it directly. This decomposition process ends when all subgoals are completely solved. In the following, we use some examples to illustrate our approach instead of giving all the complex technical details.

Example 5.1. *We first consider the connector given in Fig. 3, which consists of two channels* AB *and* BC *with types Sync and FIFO1, respectively. We use a and b to denote the time streams when the corresponding data streams flow into and out of the Sync channel* AB*, and c to denote the time stream for the data stream that flows out of the FIFO1 channel* BC*. Here we can see that a flow-through operation has acted on the mixed node* B*. The time when the stream flows into the FIFO1 channel* BC *is equal to the time when the stream flows out of the Sync channel* AB*. The following theorem states the property $a < c$ for this connector. The connector is based on the axioms Sync and FIFO1, which can be used as hypotheses for the proof of the following theorem.*

Theorem 5.1. $\forall A, B, C.\, Sync(A, B) \wedge FIFO1(B, C) \rightarrow Tle(A, C)$.

In Coq, the theorem can be proved as follows:

```
1    Theorem test1: forall A B C,
2      Sync A B /\ FIFO1 B C -> Tle A C.
3    Proof.
4       intros. destruct H. destruct H0.
5       intro n. rewrite H. apply H0.
6    Qed.
```

First we give the Coq system a proposition `test1` which needs to be proved. The proposition is represented by a logical expression. Table 1 shows the detailed proving steps and the feedback that the Coq system provides during the proof.

The advantages of using intersection of logical predicates to describe basic channels have emerged while proving this example. After constructing the new connector, we use "intros" to split conditions and conclusions. Then we can use "destruct" to obtain the conditions for time and data separately, and make the proving procedure much more convenient. Once the concrete conditions are obtained, using "intro" contributes to comparing each time point in a sequence element by element. Then by using "rewrite" H, we can

Step	Feedback
Theorem test1: *forall A B C,* *Sync A B → FIFO1 B C* *→ Tle A C.*	1 subgoal: *forall A B C,* *Sync A B → FIFO1 B C → Tle A C*
intros	1 subgoal: *Tle A C* *H : Sync A B; H0 : FIFO1 B C*
destruct *H*	1 subgoal: *Tle A C* *H : Teq A B; H1 : Deq A B;* *H0 : FIFO1 B C*
destruct *H0*	1 subgoal: *Tle A C* *H : Teq A B; H1 : Deq A B;* *H0 : Tle B C; H2 : Tle C (tl B) ∧ Deq B C*
intro *n*	1 subgoal: *PrL (Str_nth n A) < PrL (Str_nth n C)* *H : Teq A B; H1 : Deq A B; H0 : Tle B C;* *H2 : Tle C (tl B)∧ Deq B C*
rewrite *H*	1 subgoal: *PrL (Str_nth n B) < PrL (Str_nth n C)* *H : Teq A B; H1 : Deq A B; H0 : Tle B C;* *H2 : Tle C (tl B)∧Deq B C; n : nat*
apply *H0*	No more subgoals

Table 1: Steps and feedbacks for proving Theorem 5.1

(a) Before merge (b) After merge

Figure 4: Alternator

make the proof a step forward with known conditions of the comparison of time a and b, and finally by "apply" *H0* we can prove the goal. This is the implementation for reasoning about the constructed connector. Note that proper selection of strategies and tactics is essential for the proof of connector properties.

Example 5.2. *In this example, we show a more interesting connector named* alternator *which consists of three channels* AB, AC *and* BC *of type Syncdrain, FIFO1 and Sync, respectively. With the help of this connector, we can get data from node* B *and* A *alternatively at node* C. *By using the axioms for the basic channels and operators of composition, we can get the connector as shown in Figure 4(b). The two channels* AC *and* BC *are merged together at node* C. *Before the merge operation, the connector's structure is as shown in Figure 4(a), which is useful in the reasoning about the alternator.*

Here the replicate operation has been applied twice for the alternator: node A *becomes the common source node of* Syncdrain (A,B) *and* FIFO1(A,C1), *and node* B *becomes the common source node of* Syncdrain(A,B) *and* Sync(B,C2). *Let the time streams when the data streams flow into the two source nodes* A *and* B *be denoted by a and b, and the time streams when the data streams flow out of the channels* FIFO1(A,C1) *and* Sync(B,C2) *be denoted by* $c1$ *and* $c2$, *respectively. Theorem 5.2 specifies the property* $c2 < c1 \wedge c1 < tl(c2)$ *of the connector in Fig. 4(a). The connector is based on the axioms Sync, Syncdrain and FIFO1. These three corresponding axioms are used as hypotheses for the proof of this theorem.*

Theorem 5.2 (subtest)**.**

$$\forall A, B, C1, C2\,.\,SyncDrain(A, B) \wedge FIFO1(A, C1) \wedge Sync(B, C2) \rightarrow Tle(C2, C1) \wedge Tle(C1, tl(C2))$$

We first introduce some lemmas to facilitate the proof.

```
1    Lemma transfer_eq : forall s1 s2 s3 : Stream TD,
2    ((Teq s1 s2) /\ (Teq s2 s3)) -> (Teq s1 s3).
3    Lemma transfer_eqtl : forall s1 s2 : Stream TD,
4    (Teq s1 s2) -> (Teq tl s1)) (tl s2)).
5    Lemma transfer_leeq : forall s1 s2 s3 : Stream TD,
6    ((Tle s1 s2) /\ (Teq s2 s3)) -> (Tle s1 s3).
7    Lemma transfer_hdle : forall s1 s2 : Stream TD,
8    (Tle s2 s1) -> (PrL (hd s1) > PrL (hd s2)).
```

In Coq, the theorem can be proved as follows. Note that the formalism is slightly different from the previous one. By the *section* environment, Coq is able to encapsulate hypothesizes as assumptions of the theorem. So the two definitions are exactly equivalent.

```
1    Section Alt.
2    Hypothesis D1: SyncDrain A B.
3    Hypothesis D2: FIFO1 A C1.
4    Hypothesis D3: Sync B C2.
5    Theorem subtest:
6       (Tle C2 C1) /\ (Tle C1 (tl C2)).
```

After constructing the connector in Fig. 4(a), we use "destruct" to obtain the conditions for time and data, respectively. Since the goal we are going to prove is an intersection of logical predicates, we use "split" to obtain the single subgoals represented by logical predicates. Besides, "intros" contributes to comparing each data in a sequence element by element. Then "rewrite" and "apply" are used similarly for multiple times until the goal is proved finally. Concrete proof steps and feedbacks are specified in Table 2.

An additional hypothesis is needed for the proof of alternator which merges *C1* and *C2* into a common node *C*. Based on the three hypotheses for channels and the additional hypothesis, the theorem of alternator is presented as the following proposition which needs to be proved:

```
1    Hypothesis D4: merge C1 C2 C.
2    Theorem test:
3    hd(C) = hd(C2) /\ merge C1 (tl C2) (tl C).
4    Proof.
5      destruct subtest. (* ... *)
```

Here we only present the first step which shows how a proven theorem can be applied in another proof and omit the full details. And it greatly simplifies the process of proving the property of alternator.

5.2. Refinement and Equivalence

A refinement relation between connectors which allows us to systematically develop connectors in a stepwise fashion, may help to bridge the gap between requirements and the final implementations. The notion of refinement has been widely used in different system descriptions. For example, in data refinement[9], the 'concrete' model is required to have *enough redundancy* to represent all the elements of the 'abstract' one. This is captured by the definition of a surjection from the former into the latter (the *retrieve map*). If models are specified in terms of pre and post-conditions, the former are weakened and the latter strengthened under refinement [11]. In process algebra, refinement is usually discussed in terms of several 'observation' preorders, and most of them justify transformations entailing *reduction of nondeterminism* (see, for example,

Step	Feedback
Theorem subtest	1 subgoal: *Tle C2 C1 ∧ Tle C1 (tl C2)*
destruct *D2*	1 subgoal: *Tle C2 C1 ∧ Tle C1 (tl C2)* *H : Tle A C1; H0 : Tle C1 (tl A) ∧ Deq A C1*
destruct *D3*	1 subgoal: *Tle C2 C1 ∧ Tle C1 (tl C2)* *H : Tle A C1; H0 : Tle C1 (tl A) ∧ Deq A C1;* *H1 : Teq B C2; H2 : Deq B C2*
destruct *H0*	1 subgoal: *Tle C2 C1 ∧ Tle C1 (tl C2)* *H : Tle A C1; H0 : Tle C1 (tl A);* *H3 : Deq A C1; H1 : Teq B C2; H2 : Deq B C2*
split	2 subgoals: *Tle C2 C1; Tle C1 (tl C2)* *H : Tle A C1; H0 : Tle C1 (tl A);* *H3 : Deq A C1; H1 : Teq B C2; H2 : Deq B C2*
intros *n*	2 subgoals: *PrL (Str_nth n C2) < PrL (Str_nth n C1); Tle C1 (tl C2)* *H : Tle A C1; H0 : Tle C1 (tl A); H3 : Deq A C1;* *H1 : Teq B C2; H2 : Deq B C2; n : nat*
rewrite ← *H1*	2 subgoals: *PrL (Str_nth n B) < PrL (Str_nth n C1); Tle C1 (tl C2)* *H : Tle A C1; H0 : Tle C1 (tl A); H3 : Deq A C1;* *H1 : Teq B C2; H2 : Deq B C2; n : nat*
rewrite ← *D1*	2 subgoals: *PrL (Str_nth n A) < PrL (Str_nth n C1); Tle C1 (tl C2)* *H : Tle A C1; H0 : Tle C1 (tl A); H3 : Deq A C1;* *H1 : Teq B C2; H2 : Deq B C2; n : nat*
apply *H*	1 subgoal: *Tle C1 (tl C2)* *H : Tle A C1; H0 : Tle C1 (tl A); H3 : Deq A C1;* *H1 : Teq B C2; H2 : Deq B C2*
intros *n*	1 subgoal: *Tle C1 (tl C2)* *H : Tle A C1; H0 : Tle C1 (tl A); H3 : Deq A C1;* *H1 : Teq B C2; H2 : Deq B C2*
rewrite ← *D4*	2 subgoals: *PrL (Str_nth n C1) < PrL (Str_nth n (tl B)); Teq B C2* *H : Tle A C1; H0 : Tle C1 (tl A); H3 : Deq A C1;* *H1 : Teq B C2; H2 : Deq B C2; n : nat*
rewrite ← *D5*	3 subgoals: *PrL (Str_nth n C1) < PrL (Str_nth n (tl A)); Teq A B; Teq B C2* *H : Tle A C1; H0 : Tle C1 (tl A); H3 : Deq A C1;* *H1 : Teq B C2; H2 : Deq B C2; n : nat*
apply *H0*	2 subgoals: *Teq A B; Teq B C2* *H : Tle A C1; H0 : Tle C1 (tl A); H3 : Deq A C1;* *H1 : Teq B C2; H2 : Deq B C2; n : nat*
apply *D1*	1 subgoal: *Teq B C2* *H : Tle A C1; H0 : Tle C1 (tl A); H3 : Deq A C1;* *H1 : Teq B C2; H2 : Deq B C2; n : nat*
apply *D3*	No more subgoals.

Table 2: Steps and feedback

[17]). For connectors, the refinement relation can be defined as in [20], where a proper refinement order over connectors has been established based on the implication relation on predicates.

Here we adopt the definition of refinement in [20]. Two connectors are equivalent if each one of them is a refinement of the other. In the following, we show two examples of such refinement and equivalence relations for connectors.

Example 5.3 (Refinement)**.** *Taking the two connectors in Fig. 5 into consideration, connector* Q *is a refinement of connector* P *(denoted by* $\mathrm{P} \sqsubseteq \mathrm{Q}$*).*

We have mentioned that newly constructed connectors can be specified as theorems. Given arbitrary input timed data stream at node A *and output timed data streams at nodes* C, D*, essentially connector* Q *is a refinement of another connector* P *only if the behavior property of* P *can be derived from* theorem Q*, i.e., the property of connector* Q*. Intuitively, connector* P *enables the data written to the source node* A *to be asynchronously taken out via the two sink nodes* C *and* D*, but it has no constraints on the relationship between the time of the two output events. On the other hand, connector* Q *refines this behavior by synchronizing the two sink nodes, which means that the two output events must happen simultaneously. To be more precise, we use* c, d *to denote the time streams of the two outputs and* a *to denote the time stream of*

Figure 5: Example of connector refinement

the input. Connector P *satisfies condition* $a < c \wedge a < d$ *and connector* Q *satisfies* $a < c \wedge a < d \wedge c = d$.

The refinement relation can be formally defined in Coq as:

```
1    Theorem refinement : forall A C D,
2    (exists B, (FIFO1 A B) /\ (Sync B C) /\ (Sync B D)) ->
3    (exists E, (Sync A E) /\ (FIFO1 E C) /\ (FIFO1 E D)).
```

To prove this refinement relation, we first introduce a lemma which is frequently used in the proof.

Lemma 5.1 (Eq)**.** $\forall$*A,B: Stream TD. Sync (A,B)* $\Leftrightarrow$ *A=B.*

The lemma means that *Sync(A,B)* and *A=B* can be derived from each other. Although this lemma seems to make the presence of Sync channels in connectors redundant, it is not the case for most connectors. For example, if we consider the alternator in Example 2, it can not accept any input data if we remove the synchronous channel BC and use one node for it.

By using the axioms for the basic channels and the operators of composition, we can obtain the two connectors easily. In the process of constructing the connectors, the flow-through and replicate operations act once for each connector, respectively.

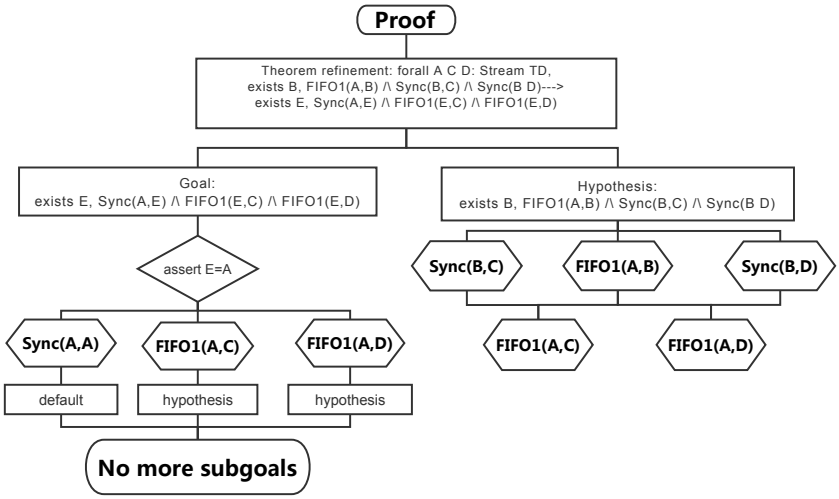


Figure 6: Proof Steps Flow Chart

Fig. 6 shows the flow chart for the proof steps of connector refinement in this example.

We now show the specific tactics used in the proof of refinement $P \sqsubseteq Q$ for connectors P and Q in this example. We need to find a timed data stream which specifies the data-flow through node E of connector P, i.e., an appropriate E that satisfies $Sync(A, E) \wedge FIFO1(E, C) \wedge FIFO1(E, D)$.

First we employ 'intros' to acquire a simpler subgoal $\exists E_0.Sync(A, E_0) \wedge FIFO1(E_0, C) \wedge FIFO1(E_0, D)$. Then we assert that $E = A$.After using 'split', we split the goal into two subgoals *Sync (A, E)* and *FIFO1 (E, C)* $\wedge$ *FIFO1 (E, D)*. And by 'rewrite' H_0 *(H_0: $E = A$)*, we replace the two subgoals with *Sync (A, A)* and *FIFO1 (E, C)* $\wedge$ *FIFO1 (E, D)*, respectively.

Through 'apply' Lemma 5.1 (*Eq*), we have $A = A$ in place of *Sync (A, A)*. Next the tactic *reflexivity* makes the subgoal $A = A$ proved directly. Up to now, the initial subgoal *Sync (A, E)* has been achieved.

Using 'split' again, the remaining unproven subgoal is split into two subgoals *FIFO1 (E, C)* and *FIFO1 (E, D)*. After destructing the precondition three times, we succeed in obtaining three hypotheses: *H: FIFO1 (A, x); H1: Sync (x, C); H2: Sync (x, D)*. Assume x = C and then using tactics *apply Eq* and *assumption*, *assertion* $x = C$ is proved easily. Meanwhile, we get hypothesis *H3:* $x = C$. Via *Rewrite* $\leftarrow$*H3*, we bring left in place of the right side of the equation *H3:* $x = C$ into *FIFO1 (E, C)* and have *FIFO1 (E, x)*. Similarly, rewrite *H0* and further we get the result *FIFO1 (A, x)* which is exactly hypothesis *H*. By using 'assumption', the second subgoal is proved already. Using substantially the same tactic steps, *FIFO1 (E, D)* can be proved. Finally, we have no more subgoals. Note that there is a new tactic 'reflexivity' used in the proof, which is actually synonymous with 'apply refl equal'. We can use it to prove that two statements are equal.

Example 5.4 (Equivalence)**.** *For the connector* P *in Example 5.3, we can add three more basic channels to build a new connector* R *which is equivalent to* Q. R *can be interpreted similarly based on basic channels and operators. We will omit the details for its construction here and prove the equivalence between the two connectors* R *and* Q *directly.*

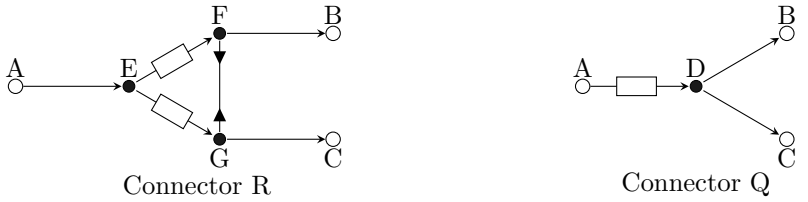


Figure 7: Example of connector equivalence

Equivalence relationship between the two connectors can be formalized as:

```
1     Theorem equivalence: forall A B C,
2       (exists E F G,
3         (Sync A E) /\ (FIFO1 E F) /\ (Sync F B) /\
4         (FIFO1 E G) /\ (Sync G C) /\ (SyncDrain F G)
5       ) <->
6       (exists D,
7         (FIFO1 A D) /\ (Sync D B) /\ (Sync D C)
8       ).
```

The proof of this theorem has two steps. Firstly, we prove that the new connector R is a refinement of connector Q. We hope to find an appropriate D that satisfies

$$FIFO1(A, D) \wedge Sync(D, B) \wedge Sync(D, C)$$

Similar to Example 5.3, we first assert $D = F$, which leads to

$$FIFO1(A, F) \wedge Sync(F, B) \wedge Sync(F, C)$$

From Lemma 5.1, we have $Sync(A, E)$, or $A = E$. Therefore, *FIFO1(E,F)* can be replaced by *FIFO1(A,F)*. By adopting *FIFO1(E,F)* and *FIFO1(E,G)*, we can prove that the data sequences at F and G are equal. Similarly, data sequences at C, G and F are also equal, wrt. $Sync(G, C)$.

Further according to *Sync(G,C)* and *Syncdrain(F,G)*, the time sequences at F and C are proved equal. With the combination of relations on time and data between F and C, we can draw the conclusion *Sync(F,C)*.

Up to now, we present a proof for *Sync(F,C)* and *FIFO1(A,F)* by the derivation. Besides, *Sync(F,B)* is already declared in the assumptions. Consequently, the refinement relation has been proved.

Secondly, we prove that connector Q is a refinement of connector R. We hope to find appropriate timed data streams at E,F,G which satisfy

$$\begin{aligned} \mathit{Sync(A,E)} \quad &\wedge \quad \mathit{Sync(G,C)} \wedge \mathit{FIFO1(E,G)} \wedge \mathit{Sync(F,B)} \\ &\wedge \quad \mathit{FIFO1(E,F)} \wedge \mathit{Syncdrain(F,G)}. \end{aligned}$$

We can directly assume $E = A$, $F = D$ and $G = D$. Now we only need to prove *Sync(A,A)* $\wedge$ *Sync(D,C)* $\wedge$ *FIFO1(A,D)* $\wedge$ *Sync(D,B)* $\wedge$ *FIFO1(A,D)* $\wedge$ *Syncdrain(D,D)*, which can be easily derived from the assumptions.

6. Refinement Checking in Z3

Modelling and reasoning of Reo connectors in Coq can only provide a proof of that one connector is indeed a refinement of the other connector when coming to refinement checking, although it allows us to reason about a relatively wide range of properties. Moreover, the refinement relation is a fairly important part of properties, which has been investigated in many applications. Therefore, in order to fulfill the proof of the refinement relation, Z3 solver is needed as an assistant tool, particularly applied to refinement checking.

6.1. Basic Channels

Before using Z3 to fulfill the proof of refinement relation, we need to implement the construction of connectors and also define the refinement checking function in a way that fits in with Z3 solver. Typically, connector construction starts with the definition of basic channels and composition operators. The formalization of basic framework is important and primary, so that we can further formalize our reasoning within this model.

Coinciding with the definition of channels in Coq, we dealt with both time and data relation constraints in the basic definition. An auxiliary function *Conjunction* is used to take the conjunction of every constraint in the constraint lists parameterized by *constraints*.

By means of function *Conjunction*, basic channels are defined concisely and neatly. For example, the specific behavior constraints for *Sync* channel are classified into two types: data constraints and time constraints. The description and the definition of *Sync* in Coq have already clearly expressed the behavior pattern it needs to follow: each item in data and time streams of output specified as "node[1]" is equivalent to the timed data items of input specified as "node[0]", which are exactly data and time constraints. As they are both requirements that we need to satisfy no matter data or time is related, each constraint in the list will all be merged finally.

```
1  def Sync(nodes, bound):
2     assert len(nodes) == 2
3     constraints = []
4     for i in range(bound):
5       constraints +=
6          [ nodes[0]['data'][i] == nodes[1]['data'][i] ]
7       constraints +=
8          [ nodes[0]['time'][i] == nodes[1]['time'][i] ]
9     return Conjunction(constraints)
```

Syncdrain channel are defined in a similar way. On the basis of the behavior of *Syncdrain* channel, there are only time related constraints to be needed, i.e., equivalence of each item in two time streams of the two inputs.

```
def SyncDrain(nodes, bound):
   assert len(nodes) == 2
   constraints = []
   for i in range(bound):
      constraints +=
          [nodes[0]['time'][i] == nodes[1]['time'][i]]
   return Conjunction(constraints)
```

Regarding *FIFO1* channels, constraints related data is the same as *Sync* channels. But there are more time related constraints. As the buffer capacity is one, the relations of each item in the input time stream and output time stream need to be carefully dealt with, especially that the next input should be later than the present output. If the buffer contains a data item at first, i.e. the variant version *FIFO1e* channel, then the constraints related to data and time are just a little different. Note that the data item "*e*" in the buffer should be the first one for transit and all the differences made to the constraints result from it. At last, their corresponding constraint lists consist of constraint statements similar to the statements above. In the programming, the data item "e" is specifically replaced by the integer 1.

```
def Fifo1(nodes, bound):
   assert len(nodes) == 2
   constraints = []
   for i in range(bound):
      constraints +=
                [ nodes[0]['data'][i] == nodes[1]['data'][i] ]
      constraints +=
                [ nodes[0]['time'][i] < nodes[1]['time'][i] ]
      if i:
         constraints +=
                [ nodes[0]['time'][i] > nodes[1]['time'][i-1] ]
   return Conjunction(constraints)
```

```
def Fifo1e(nodes, bound):
   assert len(nodes) == 2
   constraints = []
   constraints += [nodes[1]['data'][0] == 1]
   for i in range(bound-1):
      constraints +=
                [nodes[0]['data'][i] == nodes[1]['data'][i + 1]]
      constraints +=
                [nodes[0]['time'][i] < nodes[1]['time'][i + 1]]
   for i in range(bound):
      constraints +=
                [nodes[0]['time'][i] > nodes[1]['time'][i]]
   return Conjunction(constraints)
```

In accordance with the behavior of *Lossy* channel, each item of the input stream may or may not be lost. If a timed data item is lost, then the corresponding output gets nothing. If not, it behaves exactly like *Sync* channel, so in this case the data and time related constraints are identical with those in the definition of *Sync* channel. The biggest difference is that *Lossy* channel should be defined in a recursive way according to its own distinctive behavior. Note that the constraints are added recursively which is in great coincidence with the original definition.

```
def Lossy(nodes, bound, idx = 0, num = 0):
  assert len(nodes) == 2
  if bound == num:
    return True
  if bound == idx:
    return True
  constraints_0 = []
  constraints_1 = []
  constraints_0 +=
        [ nodes[0]['data'][idx] != nodes[1]['data'][num]]
  constraints_0 +=
        [ nodes[0]['time'][idx] != nodes[1]['time'][num]]
  constraints_1 +=
        [ nodes[0]['data'][idx] == nodes[1]['data'][num]]
  constraints_1 +=
        [ nodes[0]['time'][idx] == nodes[1]['time'][num]]
  return Or(And(Conjunction(constraints_0),
           Channel.Lossy(nodes, bound, idx + 1, num)),
           And(Conjunction(constraints_1),
           Channel.Lossy(nodes, bound, idx + 1, num + 1))
           )
```

6.2. Composition Operators

While the basic channels are already well defined as the basis of the whole construction framework, the composition operators also need to be specified for the large-scale connector construction just like cement for bricks.

Among the three types of composition operators, the most important and difficult one is *Merge*, which is specially dealt with as a channel *Merger* in our model. *Replicate* and *flow-through* operators are both implicitly achieved when we compose channels to construct connectors using the same node names. A simple example for the implementation of *replicate* is *c1.connect('Sync', 'A', 'E'), c1.connect('Fifo1', 'E', 'F'), c1.connect('Fifo1', 'E', 'G')* where messages from node 'E' are copied to send to nodes 'F' and 'G' simultaneously. As for *flow-through* operator, a simple implemented example is *c2.connect('Fifo1', 'A', 'D'), c2.connect('Sync', 'D', 'B')* where the timed data stream flows from node 'A' to node 'B' through the mixed node 'D'. In these two examples, both *c1* and *c2* belong to the *connector* class which will be elaborated in detail in the next module.

Although the composition operator *Merge* is treated as a channel *Merger*, it also plays the same part in the construction of connectors. Plus the two composition operators above: *replicate* and *flow-through*, they are sufficiently enough for the whole construction of any connector. For each item of the output stream, there is a nondeterministic choice between the two input nodes. Hence, the *Merger* channel also needs to be defined in a recursive way like *Lossy* channel. Note that the constraints also conform well to the original definition of *Merge*. From the definition and implementation of the three composition operators, they are ensured to serve as same as the original composition operations.

Concluded from the two recursive definitions, an additional advantage of defining *Lossy* and *Merger* channels in this way is that it can preserve the behavioral pattern to a great extent without any assumption, such as priorities of reading or taking data from the two input streams.

```
def Merger(nodes, bound, idx_1 = 0, idx_2 = 0):
  assert len(nodes) == 3
  if bound == idx_1 + idx_2:
    return True
```

```
    constraints_1 = []
    constraints_2 = []
    constraints_1 +=
     [ nodes[0]['data'][idx_1] == nodes[2]['data'][idx_1 + idx_2]]
    constraints_1 +=
     [ nodes[0]['time'][idx_1] == nodes[2]['time'][idx_1 + idx_2]]
    constraints_1 +=
     [ nodes[0]['time'][idx_1] < nodes[1]['time'][idx_2]]
    constraints_2 +=
     [ nodes[1]['data'][idx_2] == nodes[2]['data'][idx_1 + idx_2]]
    constraints_2 +=
     [ nodes[1]['time'][idx_2] == nodes[2]['time'][idx_1 + idx_2]]
    constraints_2 +=
     [ nodes[1]['time'][idx_2] < nodes[0]['time'][idx_1]]
    return Or(And(Conjunction(constraints_1),
            Channel.Merger(nodes, bound, idx_1 + 1, idx_2)),
            And(Conjunction(constraints_2),
            Channel.Merger(nodes, bound, idx_1, idx_2 + 1))
            )
```

6.3. Reasoning about Refinement Relation

With basic channels and composition operators well formulated above, the next step is to construct Reo connectors in this framework. In our model, the *connector* is defined as a new class, which provides a method of constructing complex connectors out of the basic channels and composition operators. The statements inside a class definition are function definitions. We have seen the function 'connect' used in the two simple examples of *replicate* and *flow-through*.

```
class Connector:
    def __init__(self):
        self.channels = []
    def connect(self, channel, *nodes):
        self.channels += [(channel, nodes)]
        return self
```

Another function ***isRefinementOf*** is the most crucial one in refinement checking. The result of the function is Boolean, i.e., a connector either is or not a refinement of another connector. According to the definition of refinement relation in section 5, we have a formula to represent it properly: $Q \rightarrow P$ if and only if the behavior property of connector P can be derived from the property of connector Q, i.e., $P \sqsubseteq Q$. The background theory of the definition of function ***isRefinementOf*** is based on first order logic. Initially, if Q is indeed a refinement of P, i.e., $Q \rightarrow P$, it can be rephrased as $\neg Q \vee P$. If Z3 solver presents a model satisfying the negation of this formula, then the refinement relation is falsified by a counterexample. On the contrary, if there doesn't exist a counterexample, we say that the refinement relation, $P \subseteq Q$, holds within the given bound.

The soundness of the function ***isRefinementOf*** is presented as a theorem as follows.

Theorem 6.1 (Soundness Theorem)**.** *Fake counterexamples won't be produced by the algorithm.*

Proof. Assume that the bound we set is b. For any channel, the constraints added to Z3 solver are bounded, i.e., only part of the constraints of the timed data streams are fed to Z3. Let the set of all constraints of connectors being considered be Σ and the set of bounded ones be σ. We have $\sigma \sqsubset \Sigma$. If there exists a counterexample π and the weaker constraints can't be satisfied: $\pi \nvDash \sigma$, then we can deduce that $\pi \nvDash \Sigma$. Let Π be the full timed data stream where π is the prefix of it and $\Pi \setminus \pi$ satisfies all the left constraints $\Sigma \setminus \sigma$.

A valid prefix leads to a valid timed data stream, hence we have $\Pi \not\models \Sigma$. Thus, the counterexamples are always sound and genuine. □

The formal definition of ***isRefinementOf*** is presented in the Algorithm. 1, with the details left out. The complete definition of the function can be found at [1].

Example 6.1. *To get some intuition for this, consider a simple case where the two connectors are constructed as follows:* sync = Connector(), sync.connect ('Sync', 'A', 'B'); fifo1 = Connector(), fifo1.connect('Fifo1', 'A', 'B'). *The two simple connectors are actually* Sync(A,B) *and* Fifo1(A,B). *Note that a* FIFO1 *channel is not a refinement of a* Sync *channel. After invoking the function* isRefinementOf, *we can obtain the result of the refinement relation checking:* fifo1.isRefinementOf(sync). *The result and the counterexample Z3 solver yields are presented as follows.*

```
1 False
2 A_d_0 = 0, A_d_1 = 0, A_d_2 = 0, A_d_3 = 0, A_d_4 = 0,
3 A_d_5 = 0, A_d_6 = 0, A_d_7 = 0, A_d_8 = 0, A_d_9 = 0,
4 A_t_0 = 0, A_t_1 = 2, A_t_2 = 4, A_t_3 = 6, A_t_4 = 8,
5 A_t_5 = 10,A_t_6 = 12,A_t_7 = 14,A_t_8 = 16,A_t_9 = 18,
6 B_d_0 = 0, B_d_1 = 0, B_d_2 = 0, B_d_3 = 0, B_d_4 = 0,
7 B_d_5 = 0, B_d_6 = 0, B_d_7 = 0, B_d_8 = 0, B_d_9 = 0,
8 B_t_0 = 1, B_t_1 = 3, B_t_2 = 5, B_t_3 = 7, B_t_4 = 9,
9 B_t_5 = 11,B_t_6 = 13,B_t_7 = 15,B_t_8 = 17,B_t_9 = 19.
```

Note that the counterexample is easy to understand. There exist two corresponding timed data streams whose time satisfies the constraints of FIFO1 *channel, but doesn't satisfy the time constraints of* Sync *channel.*

Example 6.2. *Another simple example is also to seek the refinement relation between two basic channels. The construction process is similar to the above example:* sync = Connector(), sync.connect('Sync', 'A', 'B'); lossy = Connector(), lossy.connect('Lossy', 'A', 'B'). *We know that* Sync *channel is a refinement of* Lossy *channel. When* Lossy *channel behaves well enough to lose no data items, it behaves exactly like* Sync *channel. On the contrary,* Lossy *channel is not a refinement of* Sync *channel, which is consistent with the result* False *given by the Z3 solver. The counterexample is also very intelligible. Note that the first four timed data items were lost with the last timed data item transferred successfully, which is consistent with the behavior constraints of* Lossy *channel and obviously not in concordance with the constraints of* Sync *channel.*

```
1 False
2 A_d_0 = 2, A_d_1 = 0, A_d_2 = 2, A_d_3 = 3, A_d_4 = 1,
3 A_t_0 = 0, A_t_1 = 1, A_t_2 = 2, A_t_3 = 3, A_t_4 = 4,
4 B_d_0 = 1,
5 B_t_0 = 4.
```

Before demonstrating complementary refinement checking between large-scale connectors, we will present some experiment results of bounded refinement and equivalence relation checking in the next two examples.

Example 6.3. *In this example, we consider about the two connectors in Fig. 5 again. In Example 5.3, we have seen the refinement relation checking between the two connectors in Coq. However, tactics and strategies can be too professional to comprehend or to tackle with. Although Z3 can only provide a bounded refinement relation checking for connectors, it will be much easier to check the refinement relation as we don't need to master concrete theorem proving tactics. All we need to do is to construct the connectors in the way accepted by Z3 solver and use the well-defined function* isRefinementOf.

(a) Router (b) Variant Router

Figure 8: Example of connector complementary refinement

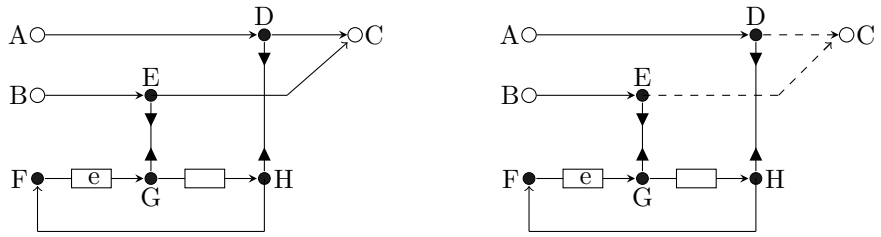
```
1 c1 = Connector()
2 c1.connect('Sync', 'A', 'M')
3 c1.connect('Fifo1', 'M', 'B')
4 c1.connect('Fifo1', 'M', 'C')
5 c2 = Connector()
6 c2.connect('Fifo1', 'A', 'N')
7 c2.connect('Sync', 'N', 'B')
8 c2.connect('Sync', 'N', 'C')
```

Notice that the replicate *operator is applied twice in the construction process. Assume the bound is ten and then carry out the function* c2.isRefinementOf(c1, 10). *Finally, we can get the result* True *which means that* Connector Q *in Fig. 5 is indeed a refinement of* Connector P. *We can further extend the bound and the time complexity increases linearly as no recursively defined channels are involved.*

Example 6.4. *This example checks the equivalence relation between the connectors in Fig. 7. Construct the two connectors first in the way similar to the example 6.3 and invoke the function* isRefinementOf *in two directions. When giving the bound ten, the results of refinement checking for the two directions are both* True. *With the yielded results, we can conclude that the two connectors are in equivalence relation within the bound ten.*

Example 6.5. *In this example, we will take the well-known Reo connector* Exclusive Router *into consideration. This connector and its variant are shown in Fig. 8. The* variant router *is defined like* router *with one major difference: one of the two* Lossy *channels is replaced with a* Sync *channel. Therefore, the behavior of the variant router is actually not exclusive and all data items will be transported to node* E *rather than being transmitted to node* E *and* F *exclusively in the original router. Notice that the variant router is a refinement of the original router and conversely not. Construct these two connectors first and set the bound to ten. Invoke the function* isRefinementOf *in two directions, i.e.,* c2.isRefinementOf(c1, 10), c1.isRefinementOf(c2, 10) *where* c2 *is the variant router. We can find the results Z3 solver yields are* True *and* False *respectively, which are consistent with our intuition. The counterexample is omitted here and the complete results including the counterexamples can be found at [1].*

Example 6.6. *Fig. 9 shows an example of the application of a two-node sequencer, with the addition of a pair of* Sync *channels and a* SyncDrain *channel connecting each of the nodes of the sequencer to the nodes* A *and* C, *and* B *and* C, *respectively. The leftmost channel of the sequencer is initialized to have a data item in its buffer, as indicated by the presence of the symbol* e *in the box, which is actually a* FIFO1e *channel. As the sequencer ensures that the take operations on nodes* G *and* H *can succeed only in the strict left-to-right order, the behavior of the connector* P *can be seen as imposing an order on the flow of the data items written to* A *and* B. *The timed data stream obtained by successive take operations on* C *consists of the first data item written to* B, *followed by the first data item written to* A, *followed by the second data item written to* B, *and so on. The variant connector* Q *is obtained by replacing the pair of* Sync *channels with a pair of* Sync *channel and* Lossy *channel. On account of the behavior of* Lossy *channel, the first data item written*



(a) Connector P (b)Variant Connector Q

Figure 9: Example of connector complementary refinement

to B *can be lost, which leads to the failure of the sequence order. Thus, the variant connector* Q *is not a refinement of the original connector* P*, but the original connector is indeed a refinement of the variant connector. The details of the concrete construction and the counterexample are left out. The results Z3 solver presents are* False *for* c2.isRefinementOf(c1) *and* True *for* c1.isRefinementOf(c2)*, which are consistent with the theoretical deduction.*

6.4. Z3 vs Coq

Having seen the examples of property proving or relation checking provided by Coq and Z3 solver, we can summarize their features and make a comparison between them.

In the preceding proof of the refinement relation or other properties in Coq, the main feature is that users need to master the tactics and strategies well enough to give a proof. But there is no need to worry about the time consumed in the proof. It will be efficient no matter how large the connectors are as long as users have chosen proper tactics. Behavior properties or relations of infinite timed data streams can also be proven, which makes Coq sufficiently powerful. Besides, the connectors whose properties have been proved can be regarded as theorems or lemmas to assist in proving properties of much larger ones. Recall that the main reason of introducing Z3 solver is that Coq can't provide the proof of complementary refinement checking, i.e., one connector is not a refinement of the other one, let alone provide counterexamples.

As we have seen, Z3 solver has shone light on complementary refinement checking. Besides, Z3 solver is much easier to make use of after the definition of the function *isRefinementOf* and the construction framework of Reo connectors, compared with using specific tactics and strategies in Coq. Moreover, counterexamples can be provided as the additional diagnostic feedback while checking the refinement relation between two connectors. Soundness and completeness of proofs in Z3 solver need a careful thought. In fact, the biggest drawback of Z3 solver is the failure of ensuring completeness. In simple terms, we didn't provide refinement checking for ideally infinite timed data streams. A finite data stream is not enough to ensure the refinement relation between two Reo connectors for certain. As a result, the equivalence relation is also not ensured completely. Nevertheless, it has no influence on the complementary refinement checking. As long as Z3 presents a counterexample of finite timed data streams, the complementary refinement checking is already done, which is ensured by the soundness theorem. As for time complexity, the efficiency of Z3 solver is well-known. Proofs can be time-consuming when the bound increases gradually, especially for the connectors consisting of one of the two recursively defined channels: *Lossy* and *Merger*. However, when checking complementary refinement relation, the experiments show that a bound limit from 5 to 10 can guarantee a proper counterexample to be given in a reasonable time.

7. Conclusion and Future Work

In this paper, we present a new approach to model and reason about connectors in the Coq system and Z3 API in Python. The model naturally preserves the original structure of connectors. This also makes the connector description reasonably readable. We implement the proof of properties for connectors using

identified techniques and tactics provided by Coq. Properties are defined in terms of predicates which provide an appropriate description of the relation among different timed data streams on the nodes of a connector. All the analysis and verification work are based on the logical framework where basic channels are viewed as axioms and composition operations are viewed as operators. As we can address the relation among different timed data streams, we can easily reason about temporal properties as well as equivalence and refinement relations for connectors. In particular, the refinement relation is fulfilled by Z3 solver.

As some of the benefits of this approach are inherited from Coq, our approach has also got some of its drawbacks as well. The main limitation is that the analysis needs much more tactics and techniques when the constructor becomes large. In the future work, we plan to enhance our framework by two different approaches. Firstly, we may try to encapsulate frequently-used proof patterns as new tactics, which may reduce lots of repetitive work. After that, automation methods may also help us to avoid tons of hand-written proof. For example, Coq provides several auto tactics to solve proof goals. With proper configuration, perhaps such tactics will work well in our framework. More attention is needed to precisely evaluate how expressive this way is for modeling temporal properties.

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Algorithm 1 Q.isRefinementOf (P, bound)

Require: Both P and Q are connectors.
Ensure: A boolean result: *True* or *False with a counterexample*

for channel $t \leftarrow Q$ **do**
  for node $n \leftarrow t$ **do**
    if n is a new node **then**
      i.generate the corresponding variable like
      {'time':[n_t_0, n_t_1,...,n_t_(bound-1)], 'date':[n_d_0, n_d_1,...,n_d_(bound-1)]}

      ii.add time constraints like "n_t_0 $\geq$ 0 and n_t_i < n_t_i+1" into *Solver()*
    end if
  end for
  Add the constraints between different nodes of the same channel into *Solver()* according to the property of channels.
end for

foralls = []
absGlobalConstr = None
absTimeConstr = None
for channel $t \leftarrow P$ **do**
  for node $n \leftarrow t$ **do**
    if n is a new node **then**
      i.generate the corresponding variable like
      {'time':[n_t_0, n_t_1,...,n_t_(bound-1)], 'date':[n_d_0, n_d_1,...,n_d_(bound-1)]}

      ii.add n into forall

      iii.add time constraints like "n_t_0 $\geq$ 0 and n_t_i < n_t_i+1" into absTimeConstr
    end if
    add the constraints between different nodes of the same channel into absGlobalConstr according to the property of channels.
  end for
end for

if absTimeConstr is not None **then**
  $absGlobalConstr = \neg\ (absTimeConstr \cap absGlobalConstr)$
else
  $absGlobalConstr = \neg\ absGlobalConstr$
end if
if forall is not empty **then**
  apply *ForAll* to absGlobalConstr and add those constraints into *Solver()*
else
  add absGlobalConstr into *Solver()*
end if
Solver.check().