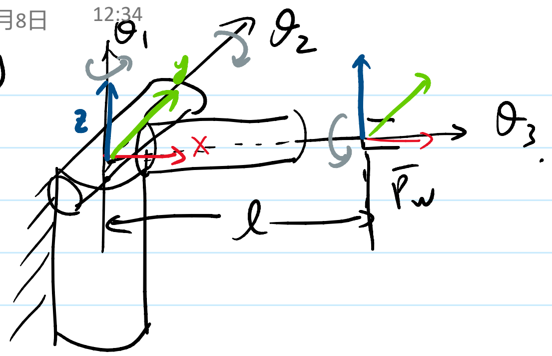
Hw3

2020年2月8日 12:34

Q1 (a)



$$g_{st}(\bar{0}) = \begin{bmatrix} 1 & 0 & 0 & l \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\xi_1 = (0,0,0,0\ 0\ 1)^T$$
$$\xi_2 = (0,0,0,0,1,0)^T$$
$$\xi_3 = (0,0,0,1,0,0)^T$$

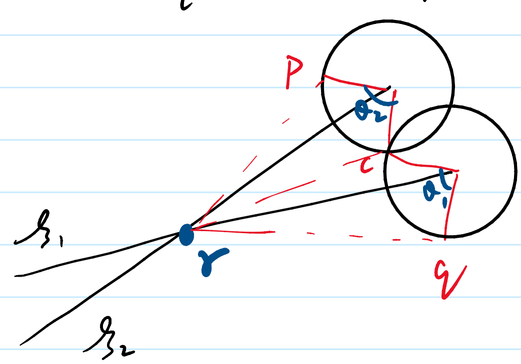
$$g_{st}(\bar{\theta}) = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} g(\bar{0})$$

(b) assume the end-effector is g_{st}^d

$$e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} = g_{st}^d\, g(\bar{0})^{-1}$$
$$e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} \bar{p}_w = g_{st}^d\, g(\bar{0})^{-1} \bar{p}_w$$

This is subproblem of Rotation about two subsequent axes.

$$e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} p = gp = q$$
$$e^{\hat{\xi}_2\theta_2} p = e^{-\hat{\xi}_1\theta_1} q = c$$



$$e^{\hat{\xi}_2\theta_2}(p-r) = c-r = e^{-\hat{\xi}_1\theta_1}(q-r)$$
$$u = p-r$$
$$v = q-r$$
$$z = c-r$$

$$e^{\hat{\xi}_2\theta_2} u = z = e^{-\hat{\xi}_1\theta_1} v$$

implies

$$w_2^T u = w_2^T z \quad , \quad w_1^T v = w_1^T z$$

$$\|u\|^2 = \|v\|^2 = \|z\|^2$$

$$z = \alpha w_1 + \beta w_2 + \gamma (w_1 \times w_2)$$

$$\alpha = \frac{(w_1^T w_2) w_2^T u - w_1^T v}{(w_1^T w_2)^2 - 1}$$

$$\beta = \frac{(w_1^T w_2) w_1^T v - w_2^T u}{(w_1^T w_2)^2 - 1}$$

$$\gamma^2 = \frac{\|u\|^2 - \alpha^2 - \beta^2 - 2\alpha\beta w_1^T w_2}{\|w_1 \times w_2\|^2}$$

if two circle intersect 2 points, there are two solutions.
if two circles are tangential, there is one solution.
if they fail to intersect, no solution.

Then we can seperate to two subproblem

$$e^{\hat{\xi}_2\theta_2} u = z \quad \text{and} \quad e^{\hat{\xi}_1\theta_1} z = v$$

$$u' = u - w_2 w_2^T u \qquad \theta_2 = \operatorname{atan2}(w_2^T (u' \times z''), u'^T z'')$$
$$z'' = z - w_2 w_2^T z$$

$$v' = v - w_1 w_1^T v \qquad \theta_1 = \operatorname{atan2}(w_1^T (z' \times v_1'), z'^T v')$$
$$z' = z - w_1 w_1^T z$$

(c)

$$J_{st}^s = (\bar{\xi}_1, \bar{\xi}_2', \bar{\xi}_3')$$
$$= (\bar{\xi}_1, Ad_{e^{\hat{\xi}_1\theta_1}} \bar{\xi}_2, Ad_{e^{\hat{\xi}_1\theta_1}} Ad_{e^{\hat{\xi}_2\theta_2}} \bar{\xi}_3)$$

$$e^{\hat{\xi}_1\theta_1} = \begin{bmatrix} e^{\hat{w}_1\theta_1} & (I - e^{\hat{w}_1\theta_1})(w_1 \times v_1) + (w_1 w_1^T v_1)\theta_1 \\ \bar{0} & 1 \end{bmatrix} = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix}$$

$$Ad_{e^{\hat{\xi}_1\theta_1}} = \begin{bmatrix} R & \hat{P}R \\ \bar{0} & R \end{bmatrix}$$

With same method we can compute $e^{\hat{\xi}_2\theta_2}$ and $Ad_{e^{\hat{\xi}_2\theta_2}}$

$J_{st}^b = (Ad \ldots$

$$\begin{bmatrix} \cdot & \cdot \\ \bar{0} & R \end{bmatrix}$$ we can compute $e^{\bar{\xi}_2\theta_2}$ and $Ad_{e^{\bar{\xi}_2\theta_2}}$

$$J_{st}^{b} = \left(Ad_{-\bar{\xi}_3\theta_3} \cdot Ad_{-\bar{\xi}_2\theta_2} \bar{\xi}_1 ,\ Ad_{-\bar{\xi}_3\theta_3} \bar{\xi}_2 ,\ \bar{\xi}_3 \right)$$

$$e^{-\bar{\xi}_3\theta_3} = \begin{bmatrix} e^{-\hat{w}_3\theta_3} & (I - e^{-\hat{w}_3\theta_3})(w_3 \times v_3) + (w_3 w_3^T v_3)\theta_3 \\ \bar{0} & 1 \end{bmatrix} = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix}$$

$$Ad_{e^{-\bar{\xi}_3\theta_3}} = \begin{bmatrix} R & \hat{P}R \\ \bar{0} & R \end{bmatrix}$$ With same method we can compute $e^{-\bar{\xi}_2\theta_2}$ and $Ad_{e^{-\bar{\xi}_2\theta_2}}$

Q2. $W = \int V_{st}^{b\,T} F_t \, dt = \int \dot{\theta} \tau \, dt$

$\Rightarrow \quad \dot{\theta}\tau = V_{st}^{b\,T} dt = (J_{st}^{b} \dot{\theta})^T F_t = \dot{\theta}^T J_{st}^{b\,T} F_t$

$\Rightarrow \quad \tau = J_{st}^{b\,T} F_t$

$$J_{st}^{b\,T} = \begin{bmatrix} \xi_1'^{T} \\ \xi_2'^{T} \\ \vdots \\ \xi_n'^{T} \end{bmatrix}$$ which has dimension $n \times 6$

if Manipulator is at singular configuration,
We can apply SVD for $J_{st}^{b\,T} = u \Sigma V^T$
Σ has dimension $n \times 6$ if $n < 6$ There must exist $F = VF'$ which the last $6-n$ entries can be freely assigned without generating any torque The other entries must be 0

if $n > 6$, Because at singular configuration. one of the sigular value must be 0.
Then exist $F = VF'$ which the entry in F' w.r.t

one of the singular value must be 0.
Then exist $F = VF'$ which the entry in F' w.r.t
the 0 singlar value can be freely assigned
The other entries must be 0

Problem3

part 1

The Jacobian matrix is shown below.

0.7200	0.1344	0.4396	0.7708	-0.3219	1.8178	-1.1043
-0.6100	-1.3393	-0.6245	-1.7636	-0.8168	-1.5531	-1.4406
0	0.6555	0.1349	0.5781	0.4599	0.2569	0.9476
0	-0.9950	-0.0198	-0.9216	-0.1692	-0.6414	-0.5622
0	-0.0998	0.1977	-0.3836	0.5334	-0.6981	0.7100
1.0000	0	0.9801	0.0587	0.8288	0.3183	0.4242

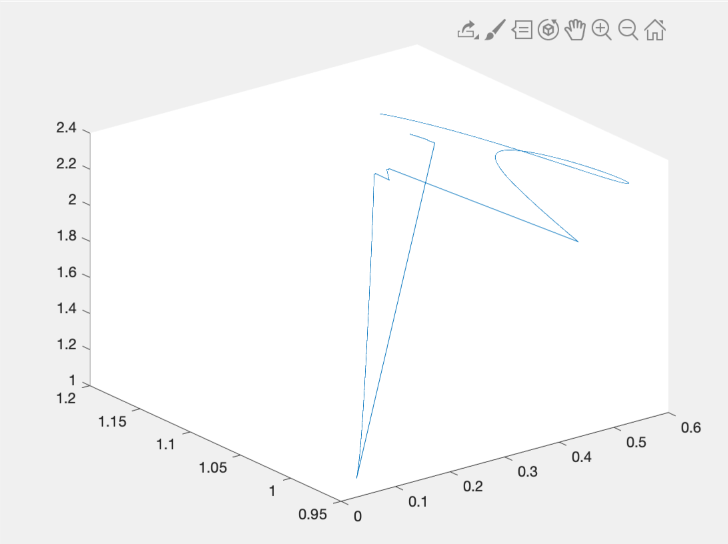
part 2

To compute the velocity, we should first calculate the difference between the desire point and current point.
The err of position if [0.0178, 0.0408, -0.0059]
The err of orientation in quaternion is [-0.98061 - 0.084062i + 0.077586j + 0.15908k]
Thus the velocity can be represent as [0.0889, 0.2041, -0.0297, 0.4122, -0.3804, -0.7800]

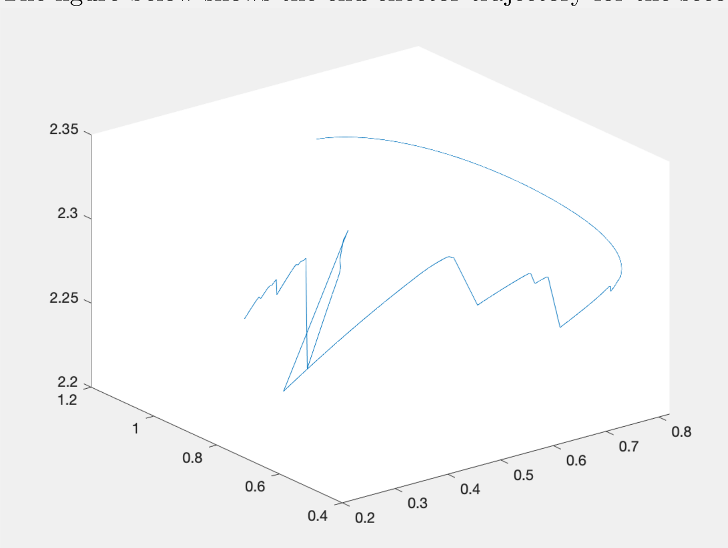
part 3

step sizes = 0.005
stopping conditions is the l2 norm or error is less than 1e-3
gain for the position error is 1
gain for the quaternion error is 1
For the first desire point, θ converge at step 4276, and the value is [-2.893657, -0.361559, -0.046428, 6.242551, 0.348168, -0.684040, -2.464989]
For the second desire point, θ converge at step 3236, and the value is [6.785208, 0.371810, -2.838329, 0.273743, -3.139266, 0.178172, -0.401902]

The figure below shows the end effector trajectory for the first desire point.



The figure below shows the end effector trajectory for the second desire point.

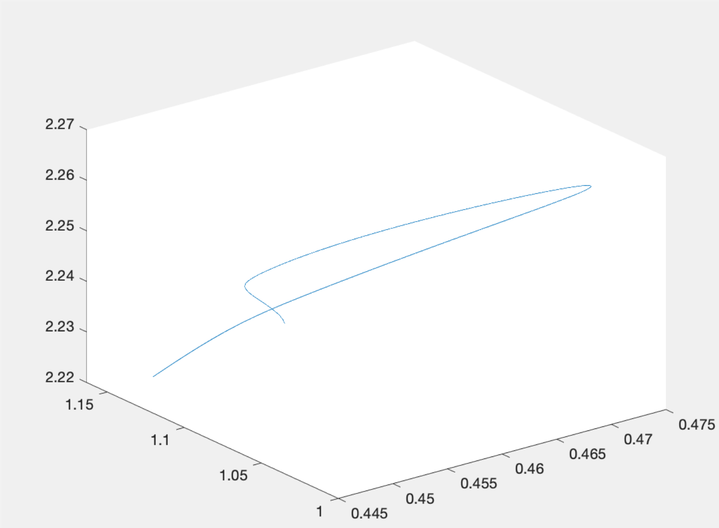


From the figures, we can see the algorithm is not very stable. Both of the trajectories are very jumpy at the beginning, but it will become continuous and converge to the desire point.

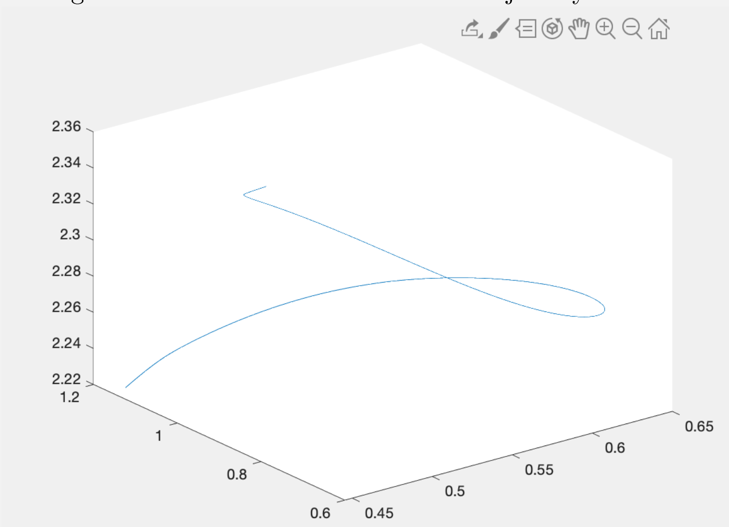
part 4

step sizes = 0.005
stopping conditions is the l2 norm or error is less than 1e-3
lambda for the least square damping is 0.01
gain for the position error is 1
gain for the quaternion error is 1
For the first desire point, θ converge at step 2615, and the value is [0.252718, 0.387967, 0.118049, -0.035701, 0.162026, 0.732013 ,0.698144]
For the second desire point, θ converge at step 5041, and the value is [-2.704026, -0.361894, 0.113953 ,0.228825 ,3.127296, 0.147645 ,-0.135513]

The figure below shows the end effector trajectory for the first desire point.



The figure below shows the end effector trajectory for the second desire point.

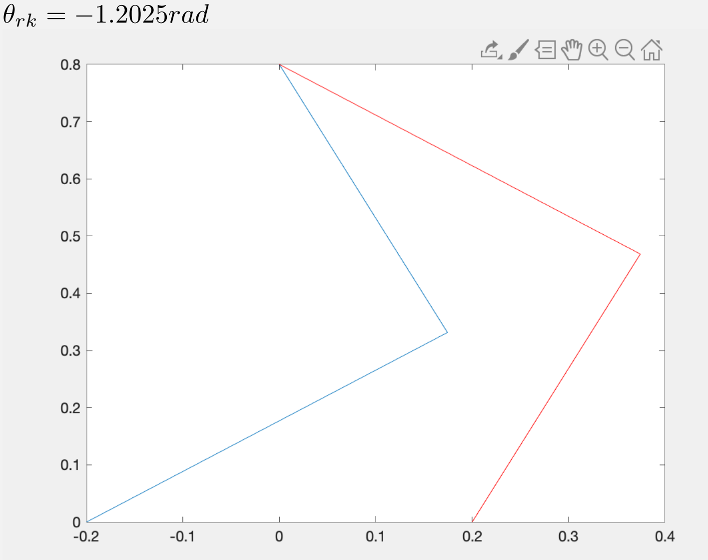


From the figures, we can see the algorithm is very stable. Both of the trajectories are very smooth. It will not affected by the singularity point

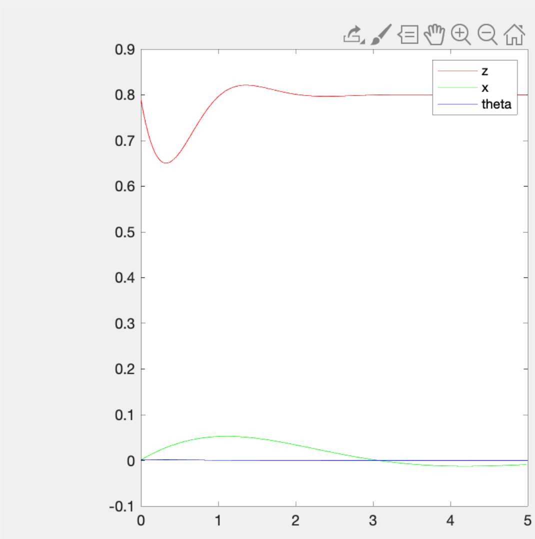
Problem4

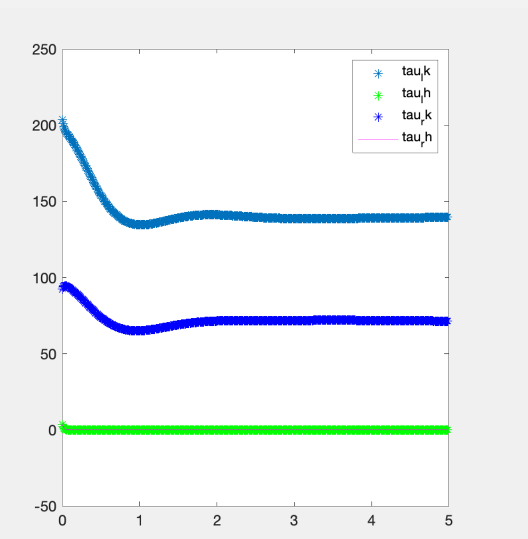
part 1

$\theta_{la} = 0.8462rad$
$\theta_{lk} = -1.2025rad$
$\theta_{ra} = 0.3562rad$
$\theta_{rk} = -1.2025rad$



part 2 and 3





The state will eventually converge to the desired state.
The motor on the left ankle will be saturated.

part 5

Imagine I am driving a car and I suddenly saw my friend is at the cross waiting the green light. I look at him and because my car is turning I have to turn my head. Thus in the frame of car, my head is turning with a certain twist. But because the car is moving and my friend is looking at me and follow my view, this twist under my friend perspective is different. With adjoint matrics, we can transfer the twist under the car coordinate to my friend view coordinate.