# Problem 1

## Part 1

From the system dynamic equation, we can derive the following state space:

$$\begin{bmatrix} \dot{\varphi_1} \\ \varphi_2 \\ \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-k}{J_1} & \frac{k}{J_1} & \frac{-c}{J_1} & \frac{c}{J_1} \\ \frac{k}{J_2} & \frac{-k}{J_2} & \frac{c}{J_2} & \frac{-c}{J_2} \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ k_I I \\ \tau_d \end{bmatrix}$$

$$\begin{bmatrix} \dot{\varphi_1} \\ \varphi_2 \\ \frac{\dot{\varphi_1}}{\omega_0} \\ \frac{\dot{\varphi_2}}{\omega_0} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-k}{J_1\omega_0} & \frac{k}{J_1\omega_0} & \frac{-c}{J_1} & \frac{c}{J_1} \\ \frac{k}{J_2\omega_0} & \frac{-k}{J_2\omega_0} & \frac{c}{J_2} & \frac{-c}{J_2} \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \frac{\dot{\varphi_1}}{\omega_0} \\ \frac{\dot{\varphi_2}}{\omega_0} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{k_I}{\omega_0} \\ 0 \end{bmatrix} I + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{\tau_d}{\omega_0} \end{bmatrix}$$

$$where\ \omega_0 = \sqrt{\frac{k(J_1 + J_2)}{J_1 J_2}}$$

## Part2

After substitute the symbolic variable with the actual number, (eg. $J1 = 10/9, J2 = 10, c = 0, k = 1, k_I = 1$) we can use matlab to derive the eigenvalues for this open loop system is:

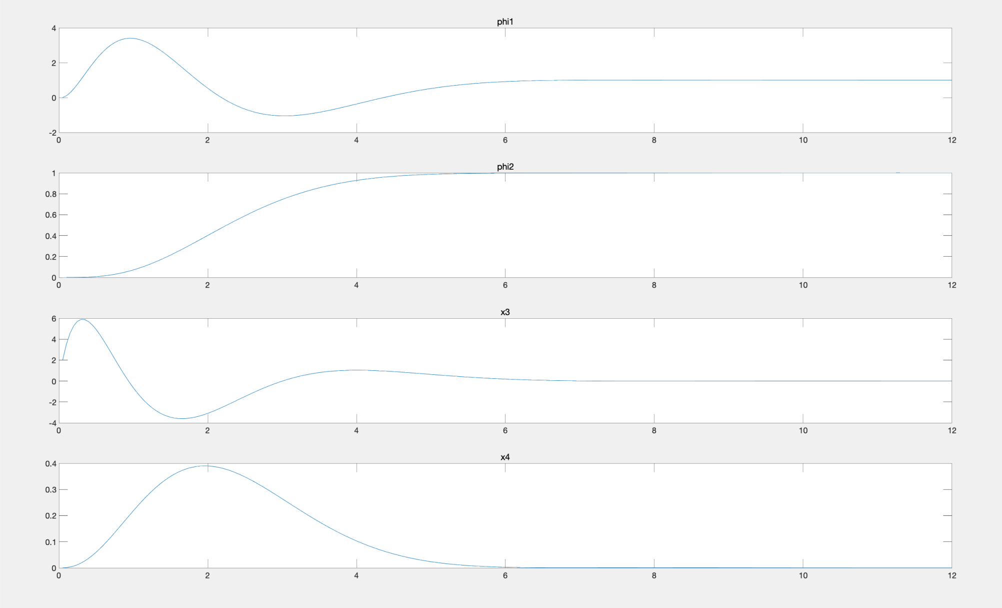
```
e =

  -0.0500 + 0.9987i
  -0.0500 - 0.9987i
  -0.0000 + 0.0000i
   0.0000 + 0.0000i
```

## Part3

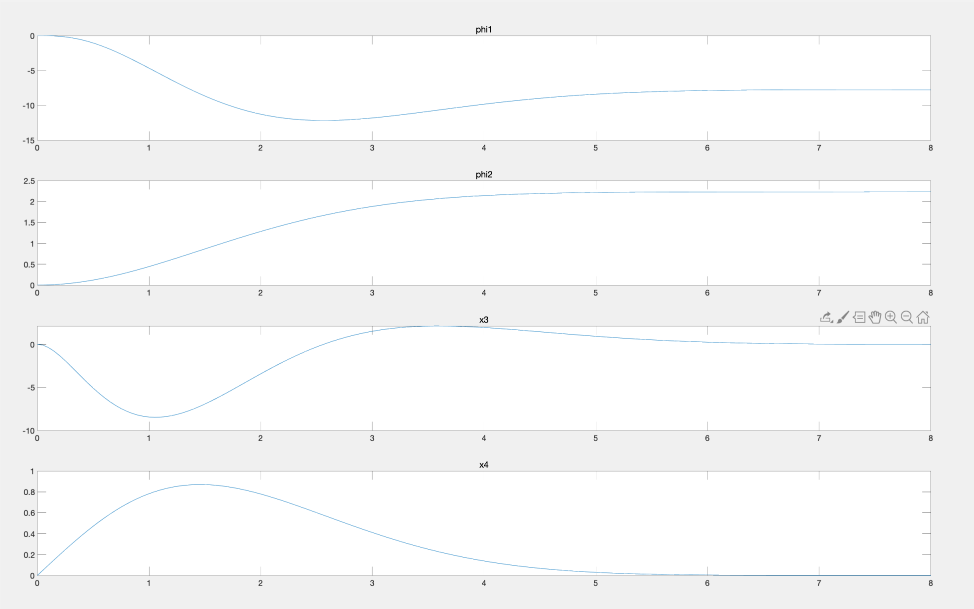
After using the pole placement method in matlab, the state feedback for system can be derived as:
$I = K * x$
x is the state. K is feedback matrix, which has value:
$\begin{bmatrix} 8.04 & 31.96 & 4.90 & 91.10 \end{bmatrix}$

**Part4**

The response for the step changes in the reference signal for $\varphi_2$is:




When a new reference $\varphi_2$ is set, motor need to generate torque to drive the J2. Thus $\varphi_1$ first increase intensively to generate torque on the other side of spring. Then to decelerate motor need to turn the opposite way. As we can see because the effect of spring, the motor need to turn a bigger angle to generate enough torque and also can lean to a lag.
The response for the step changes on disturbance signal is:




In the figure, we can see that in order to compensate the effection of the disturbance torque, motor is turning to an opposite direction to mitigate the effection

# Problem 2

## Part1

If the discrete model is $x[k] = Ax[k-1] + B(u + v[k-1])$, then the expected real position and velocity is $E(x[k]) = A\mu_{k-1} + Bu$
Because E(x[0]) = 0, we can calculate $E(x[k]) = (\sum_{i=0}^{k-1} A^i B)u$
The covariance matrix of x[k] is $E(x[k]x[k]^T) = A^k P_0 A^{Tk} + \sum_{i=0}^{k-1} A^i B R_v B^T A^{Ti}$

## Part2

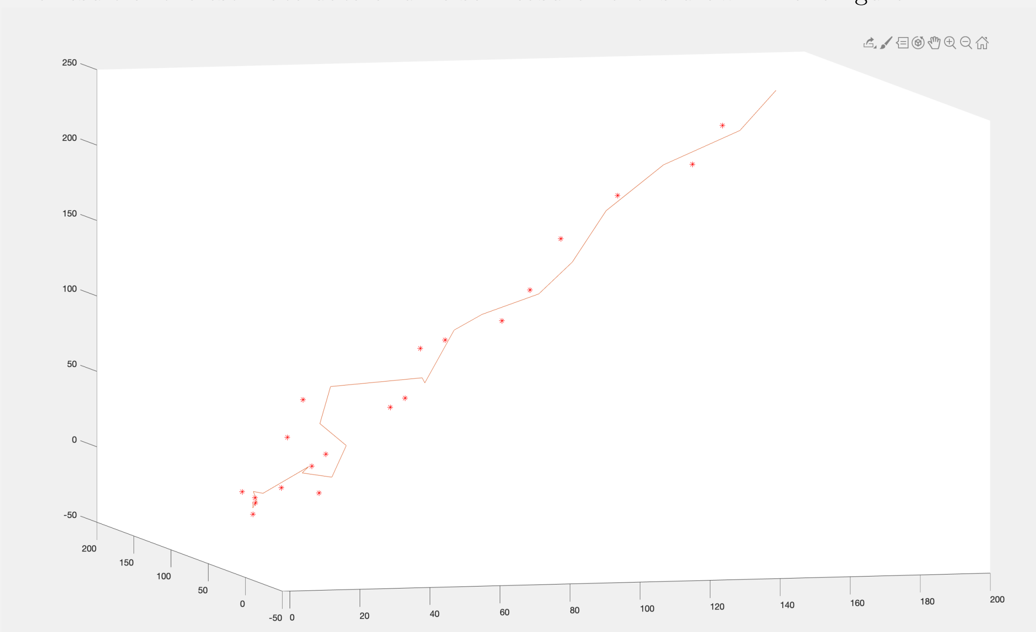
First we can derive the continuous-time LTI model for the insect as:

$$\dot{\begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{M} & 0 & 0 \\ 0 & \frac{1}{M} & 0 \\ 0 & 0 & \frac{1}{M} \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

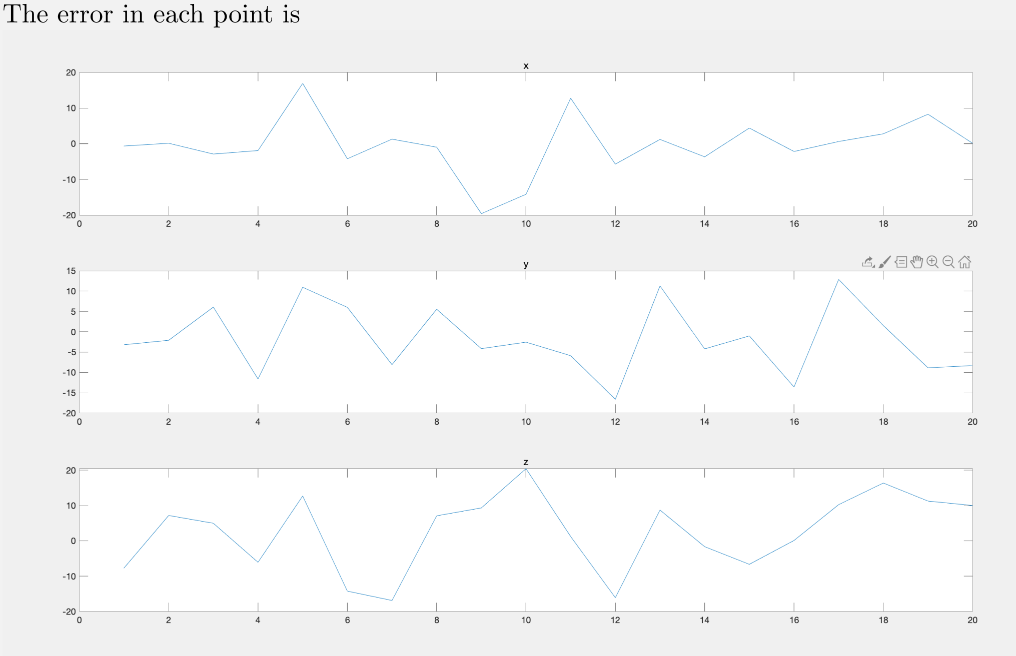
$$\begin{aligned} x[k] &= (I + A_c T)x[k-1] + B_c T(u + v[k-1]) \\ &= A_d x[k-1] + B_d(u + v[k-1]) \\ &= \begin{bmatrix} 1 & 0 & 0 & T & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & 0 & T \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} x[k-1] + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{T}{M} & 0 & 0 \\ 0 & \frac{T}{M} & 0 \\ 0 & 0 & \frac{T}{M} \end{bmatrix} (u + v[k-1]) \end{aligned}$$

## Part3

The result of the estimate data and noise measurement is drawn in one figure:




The error in each point is




the steady-state expected value and error covariance of the last point is : $\begin{bmatrix} 185.256799002814 \\ 176.403634361333 \\ 226.151263135416 \\ 19.8678244445396 \\ 18.8895310658419 \\ 22.8351390174236 \end{bmatrix}$

$$\begin{bmatrix} 17.52 & 0 & 0 & 2.59 & 0 & 0 \\ 0 & 17.52 & 0 & 0 & 2.598 & 0 \\ 0 & 0 & 17.52 & 0 & 0 & 2.59 \\ 2.59 & 0 & 0 & 0.77 & 0 & 0 \\ 0 & 2.59 & 0 & 0 & 0.77 & 0 \\ 0 & 0 & 2.59 & 0 & 0 & 0.77 \end{bmatrix}$$

I implement the method in Karl Johan Aström Richard M. Murray's book and also tried the method in wiki and find they are different.