4

Acceleration in Mechanism

4.1. Introduction

If the velocities of various points in a mechanism are known, then the acceleration of these points can be easily determined. After knowing acceleration the external force, which is the product of mass and acceleration, can be obtained. From the forces, the stresses (which are equal to force divided by area) at the various points of the mechanism will be known. These stresses are in addition to the stresses caused by the working loads. With increasing speeds, higher and higher accelerations are being called for. Hence the forces and stresses due to higher accelerations are sometimes more than the stresses caused by the working loads. As acceleration is proportional to the square of the speed, *i.e.*, acceleration = $\omega^2 \times r$, hence if the speed of the machines becomes two times, the centripetal force will become four times. Thus the acceleration diagrams are, therefore, fundamental to stress analysis of mechanism.

4.2. Acceleration of a Body Moving along a Circular Path

Acceleration is defined as the rate of change of velocity. Acceleration and velocity are vector quantities.

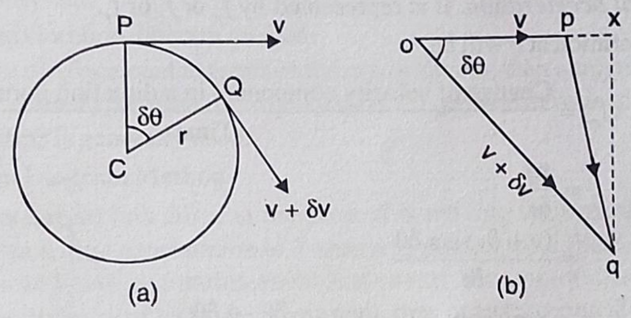


Fig. 4.1

Let a body is moving along a circular path as shown in Fig. 4.1. The body is initially at point P and in time δt moves to the point Q. Let us assume that the velocity of the body goes on changing in magnitude as the body is moving along the circular path.

Let v = Velocity of the body at point P

$v + \delta v$ = Velocity of the body at point Q

r = Radius of the circle

δt = Time taken by the body in moving from P to Q

$\delta\theta$ = Angle covered by the body in moving from P to Q.

The change of velocity, as the body moves from P to Q can be determined by drawing the vector triangle opq, as shown in Fig. 4.1 (b), in which op represents the velocity v and oq represents the velocity $v + \delta v$. In this triangle pq represents the change of velocity in time δt. The vector pq is resolved in two components px and xq parallel to op and perpendicular to op respectively.

Now, $px = ox - op = oq \cos \delta\theta - v$

$= (v + \delta v) \cos \delta\theta - v$ $(\because \; oq = v + \delta v)$

and $xq = oq \sin \delta\theta.$

$= (v + \delta v) \sin \delta\theta.$

The change of velocity (*i.e.*, vector pq) has two components (*i.e.*, px and xq) which are mutually perpendicular. Hence the rate of change of velocity will also have two components which will be mutually perpendicular. But rate of change of velocity is acceleration. Hence a body moving in a circular path has two components of acceleration which are perpendicular to each other. These two components are tangential component and normal component of acceleration.

(*i*) *Tangential component of the acceleration* (f_t). The component of acceleration in the tangential direction is known as tangential acceleration and is denoted by 'f_t'.

$\therefore$ Tangential acceleration at P will be

$$f_t = \frac{\text{Change of velocity component in tangential direction}}{\text{Time}} = \frac{px}{\delta t} = \frac{(v + \delta v) \cos \delta\theta - v}{\delta t}$$

In the limit, when δt approaches to zero, then $\cos \delta\theta \to 1$.

$$\therefore \quad f_t = \frac{(v + \delta v) - v}{\delta t} = \frac{\delta v}{\delta t} = \frac{dv}{dt} = \frac{d(\omega \times r)}{dt} = \frac{r d\omega}{dt} = r \times \alpha \quad \text{...(4.1)}$$

(*ii*) *Normal component of the acceleration* (f_n). The component of the acceleration in a direction normal to the tangent at that instant is known as normal component of the acceleration. This component is directed towards the centre of the circular path (*i.e.*, in the direction from P to C). It is also called the *radial acceleration* or *centripetal acceleration*. It is represented by f_n or f_c or f_r.

$\therefore$ Normal acceleration at P will be,

$$f_n \,(\text{or} f_r \text{ or} f_c) = \frac{\text{Change of velocity component in a direction normal to tangent}}{\text{Time}}$$

$$= \frac{xq}{\delta t}$$

$$= \frac{(v + \delta v) \sin \delta\theta}{\delta t}$$

In the limit, when δt approaches to zero, then $\sin \delta\theta \to \delta\theta$

$$\therefore \quad f_n = \frac{(v + \delta v)\, \delta\theta}{\delta t} = \frac{v \delta\theta}{\delta t} + \frac{\delta v \delta\theta}{\delta t}$$

$$= v \frac{\delta\theta}{\delta t} \quad \text{(Neglecting product of } \delta v \text{ and } \delta\theta)$$

$$= v \frac{d\theta}{dt} \quad \left(\text{But } \frac{d\theta}{dt} = \omega\right)$$

$$= v \times \omega = (\omega \times r) \times \omega \quad (\because \; v = \omega \times r)$$

$$= \omega^2 \times r \quad \text{or} \quad \frac{v^2}{r} \quad \text{...(4.2)}$$

Total acceleration of the body will be the vector sum of the tangential component and normal component as shown in Fig. 4.2.

$\because$ Total acceleration or resultant acceleration is given by,

$$f = \sqrt{f_t^2 + f_c^2}.$$

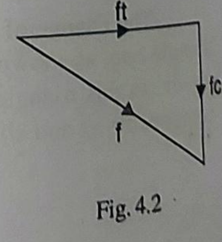


Fig. 4.2

Important Points :

1. If the body is moving with **uniform velocity,** then $\frac{dv}{dt}$ will be zero. Hence from equation (4.1), f_t *i.e.,* tangential acceleration will become zero. Hence the body will have only normal acceleration or radial acceleration or centripetal acceleration. This value is given by,

$$f_c = f_n = f_r = \frac{v^2}{r} = \omega^2 \times r$$

Also the total acceleration will be equal to radial acceleration or centripetal acceleration.

2. If the body is moving on a straight path, the radius r will be infinity and $\frac{v^2}{r}$ will be equal to zero. Hence there will be no normal (or radial) acceleration. Only tangential acceleration will exist. The value of tangential acceleration is given by,

$$f_t = \frac{dv}{dt} = \alpha \times r.$$

Also the total acceleration will be equal to tangential acceleration.

Hence for a body moving on a straight path, total acceleration is equal to $\alpha \times r$ whereas for a body moving along a circular path with uniform velocity, the total acceleration will be equal to radial acceleration *i.e.,* equal to $\omega^2 \times r$.

4.3. Acceleration Diagram for a Link

The acceleration of any point in a mechanism can be determined by two important methods, which are :

(*i*) Analytical method and

(*ii*) Graphical or acceleration diagram method.

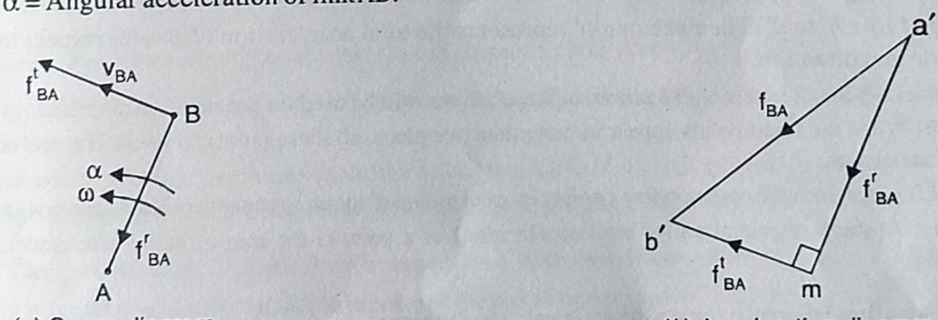
If an expression for displacement in terms of time (t) is known, then analytical method can be applied. But for most of mechanism, the expression for displacement cannot be determined easily. Hence graphical or acceleration diagram method is generally used.

4.3.1. Acceleration Diagram Method

Fig. 4.3 (*a*) shows a rigid link AB. Let the point B is moving with respect to point A. To find the acceleration of B relative to A, find acceleration of B assuming point A to be fixed. And if A is assumed fixed, then only possible motion of B will be rotation about A as centre. For a particular instant,

Let ω = Angular velocity of link AB

α = Angular acceleration of link AB.



(*a*) Space diagram (*b*) Acceleration diagram

Fig. 4.3

As point B is rotating about A, the velocity of B is changing in magnitude* and direction. Hence the acceleration of the point B will have two components :

*The change in magnitude of velocity is due to angular acceleration. If point B is rotating with uniform angular velocity, then angular acceleration will be zero.

1. Radial component (or centripetal component) which is due to angular velocity. It will be acting along BA (or parallel to BA) and will be directed from B towards A. The magnitude of this component will be $\frac{v^2}{r}$ or $\omega^2 \times r$. Hence radial component of B with respect to A,

$$f^r_{BA} = \frac{V_{BA}^2}{BA} \quad \text{or} \quad \omega^2 \times BA \qquad (\because \quad r = BA) \qquad ...(4.3)$$

2. Tangential component which is due to angular acceleration (α). This acts parallel to the velocity or it is perpendicular to AB. The magnitude of this component is $\alpha \times BA$. Hence tangential component of B with respect to A, is given by

$$f^t_{BA} = \alpha \times BA \qquad ...(4.4)$$

The total acceleration of B with respect to A is the vector sum of the above two components. This is represented by f_{BA}. Hence total acceleration of B with respect to A, is given by

$$f_{BA} = \text{vector sum of } f^r_{BA} + f^t_{BA} \qquad ...(4.5)$$

$$= \text{vector sum of } \frac{v^2_{BA}}{BA} + \alpha \, . \, BA \qquad ...(4.6)$$

Note. If point B is rotating with uniform angular velocity with respect to A, then angular acceleration (α) will be zero. And hence the tangential component (which is equal to α . BA) will be zero. Then total acceleration of B with respect to A will be given by, $f_{BA} = \frac{v^2_{BA}}{BA}$.

Acceleration Diagram for the link AB

The acceleration diagram for the link AB is shown in Fig. 4.3 (*b*). This is drawn as given below :

(*i*) Take any point a' (Here fixed point should be taken). The radial component of acceleration is acting along BA whereas tangential component is acting perpendicular to BA in the direction of velocity. From a', draw vector $a'm$ parallel to BA to represent the radial component of acceleration of B with respect to A *i.e.*, f^r_{BA} such that vector

$$a'm = f^r_{BA} = \frac{v^2_{BA}}{BA}.$$

(*ii*) From point m, draw vector mb' perpendicular to AB (or at right angles to vector mb') to represent the tangential component of acceleration of B with respect to A *i.e.*, f^t_{BA} such that vector $mb' = f^t_{BA}$.

The tangential component of acceleration acts in the direction of velocity.

(*iii*) Join b' to a'. Then vector $a'b'$ represents the total acceleration of B with respect to A (*i.e.*, f_{BA}) in magnitude and direction.

Note. (*i*) Small letters with a prime such as a', b' etc. will be used for acceleration diagram.

(*ii*) When the fixed points appear in more than one place, all these points are located at the same place in the acceleration diagram.

(*iii*) Lines from the common fixed points in acceleration diagram represent absolute accelerations.

(*iv*) Absolute acceleration (or total acceleration) of a point is the sum of radial acceleration and tangential acceleration.

4.4. Acceleration Diagram for a Slider Crank Mechanism

Fig. 4.4 (*a*) shows a slider crank mechanism in which the crank AB is rotating clockwise with a *uniform* angular velocity ω about fixed point A. Let the crank makes an angle θ with the inner deed centre (I.D.C.). The connecting rod is represented by CB.

Now the velocity of B with respect to A (or only the velocity of B as point A is fixed) is given by,

$$v_{BA} \quad \text{or} \quad v_B = \omega \times AB$$

This velocity will be perpendicular to AB as shown in Fig. 4.4 (a). Now the velocity diagram can be drawn as shown in Fig. 4.4 (b). The procedure of drawing velocity diagram is given in Art. 3.11.

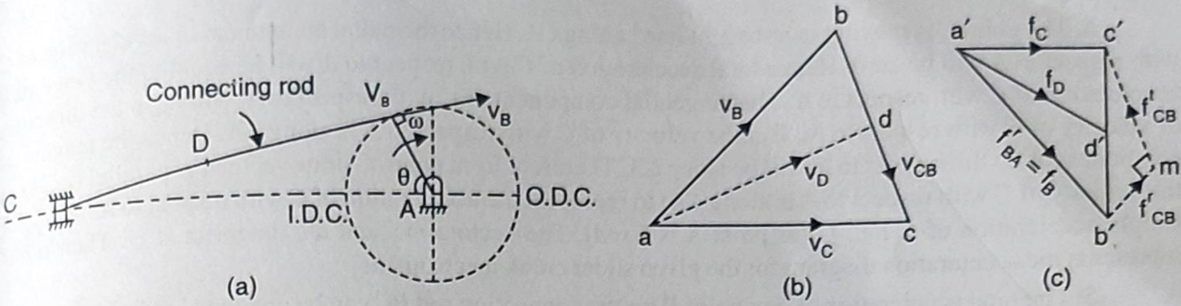


Fig. 4.4. (a) Space diagram (b) Velocity diagram (c) Acceleration diagram.

For drawing the acceleration diagram, first calculate the radial components of acceleration of different links.

Now radial acceleration of B with respect to A (or simply radial acceleration of B as point A is fixed) is given by,

$$f^r_{BA} \quad \text{or} \quad f^r_B = \frac{v^2_{BA} \text{ or } v^2_B}{AB} = \frac{v^2_B}{AB}$$

As the point B is rotating with uniform angular velocity with respect to A, hence angular acceleration of B is zero. Therefore the tangential acceleration of B (which is equal to angular acceleration × BA) with respect to A, will be zero. Hence total acceleration of B with respect to A [which is given by equation (4.3)] will be,

$$f_{BA} \quad \text{or} \quad f_B = f^r_{BA} \qquad (\text{as } f^t_{BA} = \text{Tangential acceleration is zero})$$

$$= \frac{v^2_{BA}}{AB} \quad \text{or} \quad \frac{v^2_B}{AB}$$

$$\therefore \qquad f_B = \frac{v^2_B}{AB}$$

This acceleration is acting along BA. Hence the magnitude and direction of total acceleration of B with respect to A (or simply total acceleration of B) is known.

The radial component of acceleration of C with respect to B is given by, $f^r_{CB} = \dfrac{v^2_{CB}}{BC}$.

The magnitude of v_{CB} ($i.e.$, velocity of C with respect to B) will be obtained from velocity diagram by vector bc as shown in Fig. 4.4 (b).

Now the acceleration diagram can be drawn as shown in Fig. 4.4 (c) as per method given below :

1. Choose a suitable scale for drawing the acceleration diagram. Take any point a' corresponding to fixed point A. Total acceleration of point B with respect to A is known. It is equal to $\dfrac{v^2_B}{AB}$ and is acting along BA. From a' draw vector $a'b'$ parallel to BA to represent the radial component of acceleration of B with respect to A (which is in this case is equal to f_B $i.e.$, total acceleration of B with respect to A) such that vector $a'b' = f_B$.

2. The point C is having radial component of acceleration with respect to B and also tangential component of acceleration with respect to B. Radial component is acting along CB whereas tangentical component is acting perpendicular to BC. Hence from b', draw vector $b'm$ parallel to CB to represent the radial component of acceleration of C with respect to B ($i.e.$, f^r_{CB}). As the magnitude of f^r_{CB} is known, therefore vector $b'm = f^r_{CB} = \dfrac{v^2_{CB}}{BC}$.

3. From point m, draw vector mc' perpendicular to BC to represent the tangential component of the acceleration of C with respect to B (*i.e.*, f^t_{CB}). The magnitude of f^t_{CB} is not yet known. The line mc' will also be at right angles to $b'm$.

4. The point C is moving in a straight line* along CA. Hence the radial component of acceleration of C with respect to A will be zero. Hence total acceleration of C with respect to A will be equal to the tangential acceleration of C with respect to A. The tangential component of C with respect to A, will be in the direction of velocity of C with respect to A. But the velocity of C with respect to A is along CA. Hence the tangential component of C with respect to A will be along CA. Therefore from point a', draw vector $a'c'$ parallel to CA (as the velocity of C with respect to A is along CA) to represent the acceleration of C with respect to A *i.e.*, f_{CA} or simply acceleration of C *i.e.*, f_C (as point A is fixed). The vectors mc' and $a'c'$ intersect at c'. Then $a'b'c'$ represents the acceleration diagram for the given slider crank mechanism.

5. The total acceleration of any point D on the connecting rod BC can be obtained by dividing vector $b'c'$ at d' in the same ratio as D divides BC. This means that

$$\frac{b'd'}{b'c'} = \frac{BD}{BC}.$$

$$\therefore \quad b'd' = \frac{BD}{BC} \times b'c'.$$

6. The angular acceleration of connecting rod CB is given by,

$$f^t_{CB} = \alpha_{CB} \times CB$$

$$\therefore \quad \alpha_{CB} = \frac{f^t_{CB}}{CB} \qquad \text{where } \alpha_{CB} = \text{Angular acceleration of } CB.$$

Problem 4.1. *The crank of a slider crank mechanism is 15 cm and the connecting rod is 60 cm long. The crank makes 300 r.p.m. in the clockwise direction. When it has turned 45° from the inner dead centre position, determine :*

(*i*) *Acceleration of the mid-point of the connecting rod and*

(*ii*) *Angular acceleration of the connecting rod.*

Sol. Given :

Crank length, $AB = 15$ cm

Connecting rod length, $BC = 60$ cm

Crank speed, $N = 300$ r.p.m.

Angle turned by crank from I.D.C. $= 45°$

Angular velocity of crank, $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 300}{60} = 31.42$ rad/s

$\therefore$ Velocity of B with respect to A (or velocity of B only as point A is fixed) is given by,

$$v_{BA} = v_B = \omega \times AB = 31.42 \times 15 = 471.3 \text{ cm/s}$$

The direction of v_B will be perpendicular to AB.

First draw the space diagram to a suitable scale as shown in Fig. 4.5 (*a*). Now draw the velocity diagram as shown in Fig. 4.5 (*b*) as per method given below :

1. Take any point a (fixed point). Choose a suitable velocity scale and draw vector ab perpendicular to AB to represent the velocity of B with respect to A or simply the velocity of B such that vector $ab = v_{BA}$ or v_B $= 471.3$ cm/s.

2. As the point C is moving with respect to B and also with respect to A. The velocity of C with respect to B is perpendicular to BC whereas the velocity of C with respect to A is along CA (*i.e.*, horizontally). Hence

*When a point moves along a straight line, then its radial acceleration will be zero.

from point b, draw vector bc perpendicular to BC to represent the velocity of C with respect to B and from point a draw vector ac horizontally to represent the velocity of C with respect to A. The vectors bc and ac intersect at point c. Then abc is the velocity diagram.

3. As the point D is the middle point of CB, hence corresponding point d will be the middle point of vector bc. Join ad. Then vector ad represents the velocity of point D.

From velocity diagram, we have :

(*i*) Velocity of D, v_D = vector $ad = 410$ cm/s

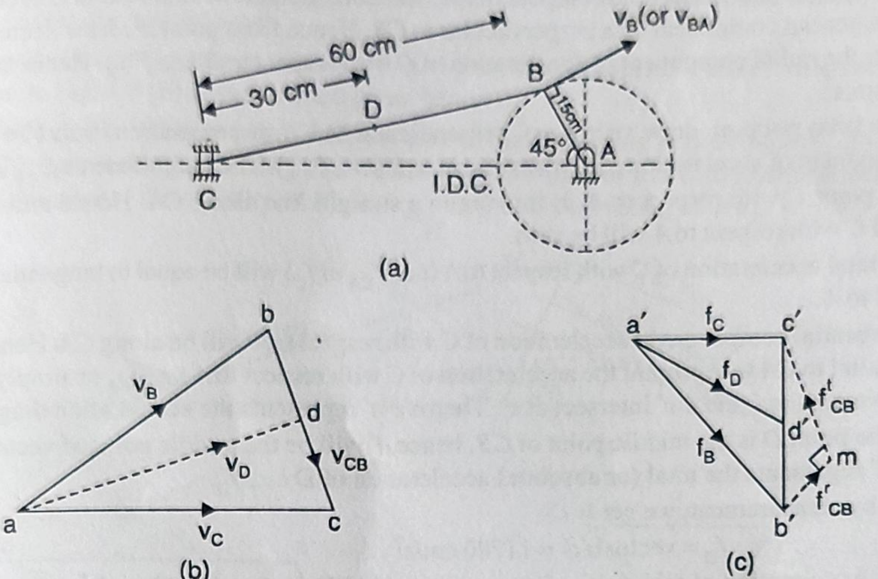


Fig. 4.5. (*a*) Space diagram (*b*) velocity diagram (*c*) Acceleration diagram.

(*ii*) Velocity of C, v_C = vector $ac = 400$ cm/s

(*iii*) Velocity of C with respect to B, v_{CB} = vector $bc = 340$ cm/s.

Acceleration diagram

Before drawing acceleration diagram, we must calculate the radial acceleration $\left(i.e., \dfrac{v^2}{r}\right)$ of each link.

$\therefore$ Radial acceleration of B with respect to A (or simply radial acceleration of B as point A is fixed) is given by,

$$f^r_{BA} \text{ or } f^r_B = \frac{v^2_B}{AB} = \frac{471.3^2}{15} = 14808 \text{ cm/s}^2$$

Radial acceleration of C with respect to B is given by,

$$f^r_{CB} = \frac{v^2_{CB}}{CB} = \frac{340^2}{60} \qquad (\because\ v_{CB} = 340 \text{ and } CB = 60 \text{ cm})$$
$$= 1926.7 \text{ cm/s}^2$$

As the point B is rotating with uniform angular velocity with respect to A, hence angular acceleration of B with respect to A is zero. Therefore the tangential acceleration of B with respect to A will be zero. Hence total acceleration of B with respect to A will be equal to radial acceleration of B with respect to A and will be acting along BA.

$$\therefore \quad f_{BA} \text{ or } f_B = f^r_{BA} \text{ or } f^r_B = 14808 \text{ cm/s.}$$

Now the acceleration diagram can be drawn as shown in Fig. 4.5 (c), as per method given below :

1. Choose a suitable scale for drawing acceleration diagram. Take any point a' corresponding to fixed point A. Total acceleration of B with respect to A is known. It is equal to radial acceleration of B and is acting along BA. From a' draw vector $a'b'$ parallel to BA to represent the radial acceleration of B with respect to A (which is in this case is equal to f_{BA} or f_B *i.e.*, total acceleration of B). Hence cut vector $a'b' = f_B$ or $f^r_{BA} = 14808$ cm/s.

2. The point C with respect to B is having radial component of acceleration and also tangential component of acceleration. These two components are mutually perpendicular. Radial component acts along CB whereas tangential component acts perpendicular to CB. Hence from point b', draw vector $b'm$ parallel to CB to represent the radial component of acceleration of C with respect to B *i.e.*, f^r_{CB}. Hence take vector $b'm = f^r_{CB} = 1926.7$ cm/s^2.

3. Now from point m, draw vector mc' perpendicular to CB or perpendicular to $b'm$ to represent the tangential component of acceleration of C with respect to B (*i.e.*, f^t_{CB}). The magnitude to f^t_{CB} is not yet known.

4. The point C with respect to A, is moving in a straight line along CA. Hence radial component of acceleration of C with respect to A will be zero.

Hence total acceleration of C with respect to A (*i.e.*, f_{CA} or f_C) will be equal to tangential acceleration of C with respect to A.

The tangential component of acceleration of C with respect to A will be along CA. Hence from a', draw vector $a'c'$ parallel to CA to represent the acceleration of C with respect to A *i.e.*, f_{CA} or simply acceleration of C *i.e.*, f_C. The vectors mc' and $a'c'$ intersect at c'. Then $a'b'c'$ represents the acceleration diagram.

5. As the point D is the middle point of CB, hence d' will be the middle point of vector $b'c'$. Join $a'd'$. The vector $a'd'$ represents the total (or absolute) acceleration of D *i.e.*, f_D.

Hence by measurement, we get

$$f_D = \text{vector } a'd' = 11700 \text{ cm/s}^2$$

(*i*) $\therefore$ Acceleration of mid-point of connecting rod (CB) is $f_D = \mathbf{11700 \ cm/s^2}$. **Ans.**

(*ii*) *Angular acceleration of connecting rod (CB).* We know that

$$f^t_{CB} = \alpha_{CB} \times CB$$

where f^t_{CB} = Tangential acceleration of C with respect to B (or tangential acceleration of connecting rod CB).

$= \text{Vector } mc' = 10300$ cm/s^2 (by measurement)

α_{CB} = Angular acceleration of connecting rod

$$= \frac{f^t_{CB}}{\text{Length } CB}$$

$$= \frac{10300}{60} = \mathbf{171.67 \ rad/s^2}. \quad \textbf{Ans.}$$

Problem 4.2. *A reciprocating engine mechanism is shown in Fig. 4.6 (a). The crank CB = 10 cm and connecting rod BA = 30 cm with the centre of gravity G, 10 cm from B. In the position shown, the crank has a velocity of 75 rad/s and an angular acceleration of 1200 rad/s². Find : (a) the velocity and acceleration of G and (b) the angular velocity and angular acceleration of AB.* (AMIE, Summer 1981)

Sol. Given :

Crank length, $CB = 10$ cm, connecting rod, $BA = 30$ cm

Distance, $BG = 10$ cm

Angular velocity of crank, $\omega_{BC} = 75$ rad/s

Angular acceleration of crank, $\alpha_{BC} = 1200$ rad/s^2.

$\therefore$ Linear velocity of crank, $v_{BC} = \omega_{BC} \times \text{Length } BC$

$$= 75 \times 10 = 750 \text{ cm/s} = \frac{750}{100} = 7.5 \text{ m/s}$$

This velocity will be perpendicular to BC.

Now tangential acceleration of crank = Angular acceleration of crank × Crank length

$$\therefore \quad f^t_{BC} = \alpha_{BC} \times \text{Length } BC$$
$$= 1200 \times 10 = 12000 \text{ cm/s}^2$$
$$= \frac{1200}{100} = 120 \text{ m/s}^2$$

Please note that in this problem, the crank is having tangential acceleration whereas in problem 4.1 the tangential acceleration of crank was zero.

Now draw the space diagram as shown in Fig. 4.6 (*a*) to some suitable scale. Now draw the velocity diagram as shown in Fig. 4.6 (*b*) as per method given below :

1. Choose a suitable velocity scale. Take any point c corresponding to fixed point C. The velocity of B with respect to C is perpendicular to CB. Draw vector cb perpendicular to BC to represent the velocity of B with respect to C (or simply the velocity of B) *i.e.*, v_{BC} or v_B such that vector $cb = v_{BC} = v_B = 7.5$ m/s.

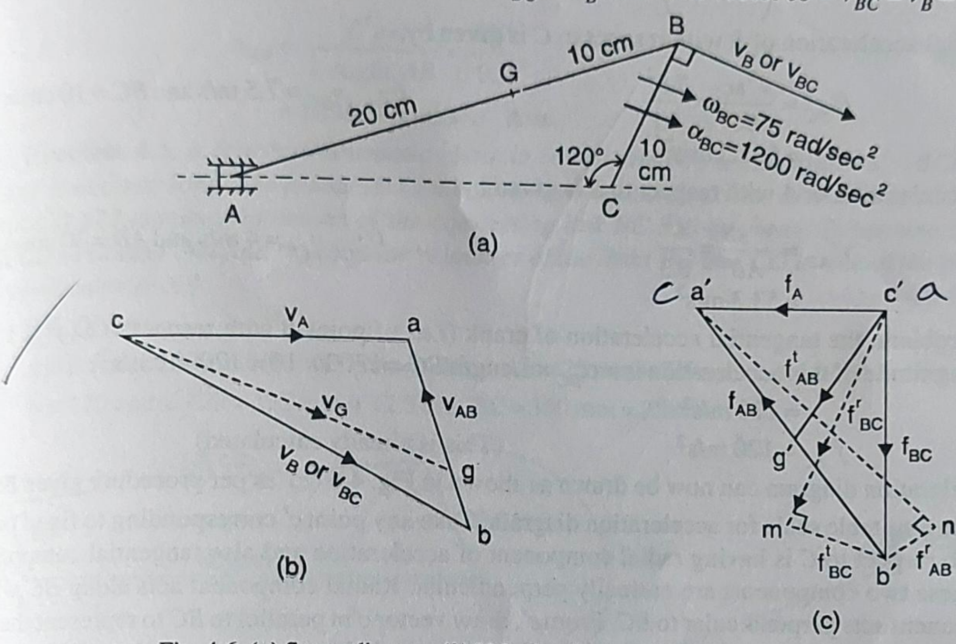


Fig. 4.6. (*a*) Space diagram (*b*) Velocity diagram (*c*) Acceleration diagram.

2. The point A is moving with respect to B and also with respect to C. The velocity of A with respect to B is perpendicular to AB whereas the velocity of A with respect to C is along AC. Hence from the point b, draw vector ba perpendicular to AB to represent the velocity of A with respect to B *i.e.*, v_{AB} and also from point c, draw a vector ca along the path of motion of A (which is horizontal) to represent the velocity of A with respect to C. The vectors ba and ca intersect at a.

3. The point G lies in AB and hence corresponding point g will lie in vector ab of velocity diagram. To find the location of g, divide vector ab at g in the same ratio as G divides AB. This means

$$\frac{bg}{ab} = \frac{BG}{AB}$$

$$\therefore \quad bg = \frac{BG}{AB} \times ab = \frac{10}{30} \times ab = \frac{ab}{3}$$

The vector cg represents the velocity of G *i.e.*, v_G.

By measurement, vector $cg \doteq 6.8$ m/s.

$\therefore$ Velocity of G, v_G = vector cg = **6.8 m/s. Ans.**

The angular velocity of connecting rod is given by,

$$\omega_{AB} = \frac{v_{AB}}{\text{Length } AB}$$

But v_{AB} (*i.e.*, the velocity of A with respect to B) from velocity diagram is given by vector ba. By measurement vector $ba = 4$ m/s.

$$\therefore \quad v_{AB} = \text{vector } ba = 4 \text{ m/s.}$$

$$\therefore \quad \omega_{AB} = \frac{4}{0.3} \qquad (\because \; AB = 30 \text{ cm} = 0.3 \text{ m})$$

$$= \mathbf{13.3 \text{ rad/s.}} \quad \textbf{Ans.}$$

Acceleration of G and angular acceleration of AB

These will be obtained from acceleration diagram. Before drawing acceleration diagram, we must calculate the radial acceleration $\left(i.e., \dfrac{v^2}{r}\right)$ of each link.

$\therefore$ Radial acceleration of B with respect to C is given by,

$$f^r_{BC} = \frac{v^2_{BC}}{BC} = \frac{7.5^2}{0.1} \qquad (\because \; v_{BC} = 7.5 \text{ m/s and } BC = 10 \text{ cm} = 0.1 \text{ m})$$

$$= 562.5 \text{ m/s}^2$$

Radial acceleration of A with respect to B is given by,

$$f^r_{AB} = \frac{v^2_{AB}}{AB} = \frac{4^2}{0.3} \qquad (\because \; v_{AB} = 4 \text{ m/s and } AB = 30 \text{ cm} = 0.3 \text{ m})$$

$$= 53.3 \text{ m/s}^2$$

In this problem, the tangential acceleration of crank (*i.e.*, of point B with respect to C, f^t_{BC}) is also existing. The magnitude of this acceleration is $= \alpha_{BC} \times \text{Length } BC = 1200 \times 10 = 12000 \text{ cm/s}^2$

$$= 120 \text{ m/s}^2$$

$$\therefore \quad f^t_{BC} = 120 \text{ m/s}^2 \qquad \text{(This is already calculated)}$$

The acceleration diagram can now be drawn as shown in Fig. 4.6 (c) as per procedure given below :

1. Choose a suitable scale for acceleration diagram. Take any point c' corresponding to fixed point C. The point B with respect to C is having radial component of acceleration and also tangential component of acceleration. These two components are mutually perpendicular. Radial component acts along BC whereas tangential component acts perpendicular to BC. From c', draw vector $c'm$ parallel to BC to represent the radial component of acceleration of point B with respect to C *i.e.*, f^r_{BC} such that vector $c'm = f^r_{BC} = 562.5 \text{ m/s}^2$.

2. From m, draw vector mb' perpendicular to $c'm$ (or perpendicular to BC) to represent the tangential component of acceleration of B with respect to C *i.e.*, f^t_{BC} such that vector $mb' = f^t_{BC} = 120 \text{ m/s}^2$. [$mb'$ is in the direction of velocity of B]

3. Join $c'b'$. Then vector $c'b'$ represents the total acceleration of B with respect to C *i.e.*, f_{BC}.

4. The point A with respect to B is having radial component of acceleration and also tangential component of acceleration. These two components are mutually perpendicular. Radial component acts along AB whereas tangential component acts perpendicular to AB. From point b', draw vector $b'n$ parallel to AB to represent radial component of acceleration of A with respect to B *i.e.*, f^r_{BA} such that vector $b'n = f^r_{AB} = 53.3$ m/s^2. And from point n, draw vector na' perpendicular to $b'n$ (or perpendicular to AB) to represent tangential component of acceleration of A with respect to B *i.e.*, f^t_{AB}. But the magnitude of f^t_{AB} is not known.

5. The point A with respect to C is moving in a straight line along AC. Hence total acceleration of A will be along AC. Now from point c', draw vector $c'a'$ parallel to AC to represent the acceleration of A with respect to C *i.e.*, f_{AC} or simply the acceleration of A *i.e.*, f_A. The vectors na' and $c'a'$ intersect at a'. Join $b'a'$. The vector $b'a'$ represent the total acceleration of A with respect to B *i.e.*, f_{AB}.

6. The point G lies in AB and hence corresponding point g' in acceleration diagram will lie in vector $a'b'$. To find the location of g', divide vector $a'b'$ at g' in the same ratio as G divides AB. This means

$$\frac{b'g'}{a'b'} = \frac{BG}{AB}$$

$$\therefore \quad b'g' = \frac{BG}{AB} \times a'b' = \frac{10}{30} \times a'b' = \frac{a'b'}{3}$$

Join c' to g'. Then vector $c'g'$ represents the total acceleration of G *i.e.*, f_G.

By measurement, vector $c'g' = 415 \text{ m/s}^2$.

$\therefore$ Acceleration of $G, f_G =$ vector $c'g' = \mathbf{415 \text{ m/s}^2}$. **Ans.**

The angular acceleration of connecting rod AB is given by,

$$f^t_{AB} = \alpha_{AB} \times \text{Length } AB$$

where f^t_{AB} = Tangential acceleration of A with respect to B

= Vector $na' = 550 \text{ m/s}^2$ (By measurement)

$$\therefore \quad \alpha_{AB} = \frac{f^t_{AB}}{\text{Length } AB} = \frac{550}{0.3} \qquad (\because \ AB = 30 \text{ cm} = 0.3 \text{ m})$$

$$= \mathbf{1833.34 \text{ rad/s}^2}. \quad \textbf{Ans.}$$

Problem 4.3. *A four bar kinematic chain is represented by a quadrilateral ABCD in which AD is fixed and is 400 mm long. The crank AB 75 mm long rotates in a clockwise direction at 120 r.p.m. and drives the link CD 125 mm long by means of the connecting link BC 350 mm long. Determine the angle through which CD oscillates and find the angular velocities of the links BC and CD in one of the positions when BC is perpendicular to AB.* (AMIE, Summer 1987)

Sol. Given :

AD is fixed. Length $AD = 400$ mm $= 40$ cm, $AB = 75$ mm $= 7.5$ cm,

$N = 120$ r.p.m., $CD = 125$ mm $= 12.5$ cm, $BC = 350$ mm $= 35$ cm

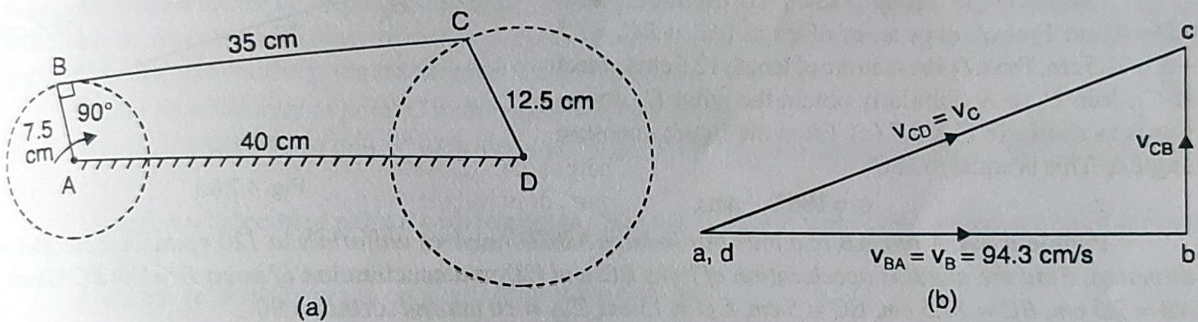


Fig. 4.7. (*a*) Space diagram (*b*) Velocity diagram.

Now $\quad N_{BA} = 120$ r.p.m.

$$\therefore \quad \omega_{BA} = \frac{2\pi \times 120}{60} = 12.57 \text{ rad/s}$$

Now velocity of B with respect to A,

$$v_{BA} = \omega_{BA} \times \text{Length } BA$$
$$= 12.57 \times 7.5 = 94.3 \text{ cm/s}$$

As point A is fixed, hence velocity of B (or velocity of B with respect to A) will be 94.3 cm/s.

$$\therefore \quad v_{BA} = v_B = 94.3 \text{ cm/s.}$$

First draw the space diagram to a suitable scale as shown in Fig. 4.7 (*a*). Now draw the velocity diagram as shown in Fig. 4.7 (*b*) as per method given below :

1. Since *A* and *D* are fixed points, therefore these points (*a*, *d*) lie at one place in velocity diagram. Choose a suitable velocity scale and from point *a* draw vector *ab* perpendicular to *AB* to represent the velocity of *B* with respect to *A* (*i.e.*, v_{BA}). Take $ab = 94.3$ cm/s.

2. The point *C* moves with respect to point *B* and also with respect to point *D*. The velocity of *C* with respect to *B* is perpendicular to *BC* whereas the velocity of *C* with respect to *D* is perpendicular to *DC*. Hence from point *b*, draw vector *bc* perpendicular to *BC*. Since link *AD* is fixed hence point *a* and *d* will coincide in the velocity diagram. From point *d* (or point *a*), draw vector *dc* (or *ac*) perpendicular to *DC* so as to intersect vector *bc* in *c*. Then *abc* is the velocity diagram in which :

Vector *bc* = Velocity of *C* with respect to *B* (*i.e.*, v_{CB})

Vector *dc* = Velocity of *C* with respect to *D* (*i.e.*, v_{CD}). As point *D* is fixed, this is also the velocity of point *C* (*i.e.*, v_C).

By measurement, from velocity diagram, we get

$$v_{CB} = \text{vector } bc = 36.75 \text{ cm/s}$$

and $$v_{CD} \text{ or } v_C = \text{Vector } dc = 100.8 \text{ cm/s}$$

(*i*) *Angular velocities of the links BC and CD*

Let ω_{CB} = Angular velocity of link *CB*

ω_{CD} = Angular velocity of link *CD*

Then $$\omega_{CB} = \frac{v_{CB}}{CB} = \frac{36.75}{35} = \mathbf{1.05 \text{ rad/s.}} \quad \textbf{Ans.}$$

and $$\omega_{CD} = \frac{v_{CD}}{CD} = \frac{100.8}{12.5} = \mathbf{8.06 \text{ rad/s.}} \quad \textbf{Ans.}$$

(*ii*) *Angle through which CD oscillates*

Let α = angle through which *CD* oscillates. Take $AD = 40$ cm. From *A*, draw an arc of length ($AB_1 + BC_1 = 7.5 + 35$) 42.5 cm. From *D*, draw an arc of length 12.5 cm to meet at C_1. Join C_1 to *A*. Similarly obtain the point C_2 downwards as shown in Fig. 4.7 (*c*). From the figure, measure angle α. This is equal to 186°.

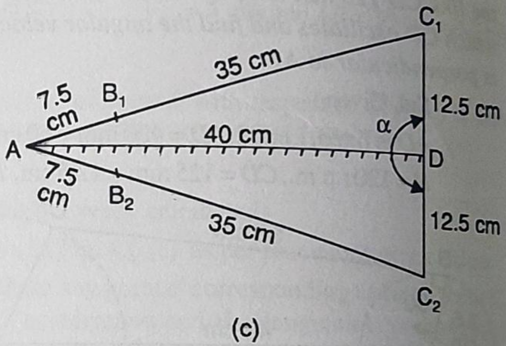


Fig. 4.7 (*c*)

$\therefore$ $$\alpha = \mathbf{186°.} \quad \textbf{Ans.}$$

Problem 4.4. *A link AB of a four-bar linkage ABCD revolves uniformly at 120 r.p.m. in a clockwise direction. Find the angular acceleration of links BC and CD and acceleration of point E in link BC. Given : AB = 7.5 cm, BC = 17.5 cm, EC = 5 cm, CD = 15 cm, DA = 10 cm and ∠BAD = 90°.*

(AMIE, Winter 1979)

Sol. Given :

Speed of link *AB*, $N_{BA} = 120$ r.p.m.

$\therefore$ Angular velocity of link *AB*,

$$\omega_{BA} = \frac{2\pi \times N_{BA}}{60} = \frac{2\pi \times 120}{60} = 12.57 \text{ rad/s}$$

Length $AB = 7.5$ cm

$\therefore$ Velocity of *B* with respect to *A* (or velocity of *B* as point *A* is fixed),

$$v_{BA} = v_B = \omega_{BA} \times \text{Length } BA = 12.57 \times 7.5 = 94.3 \text{ cm/s}$$

First draw the space diagram of the four-bar linkage *ABCD* with the given dimensions as shown in Fig. 4.8 (*a*) to same suitable scale. Now draw the velocity diagram as shown in Fig. 4.8 (*b*) as per method given below :

1. Since *A* and *D* are fixed points, therefore, these points lie at one place in velocity diagram. Choose a suitable velocity scale and from point *a* draw vector *ab* perpendicular to *AB* and cut off this length equal to 94.3 cm/s. Then this vector *ab* represents the velocity of *B* with respect to *A* (*i.e.*, v_{BA}). As the point *A* is fixed, hence the vector *ab* also represents the velocity of *B* (*i.e.*, v_B).

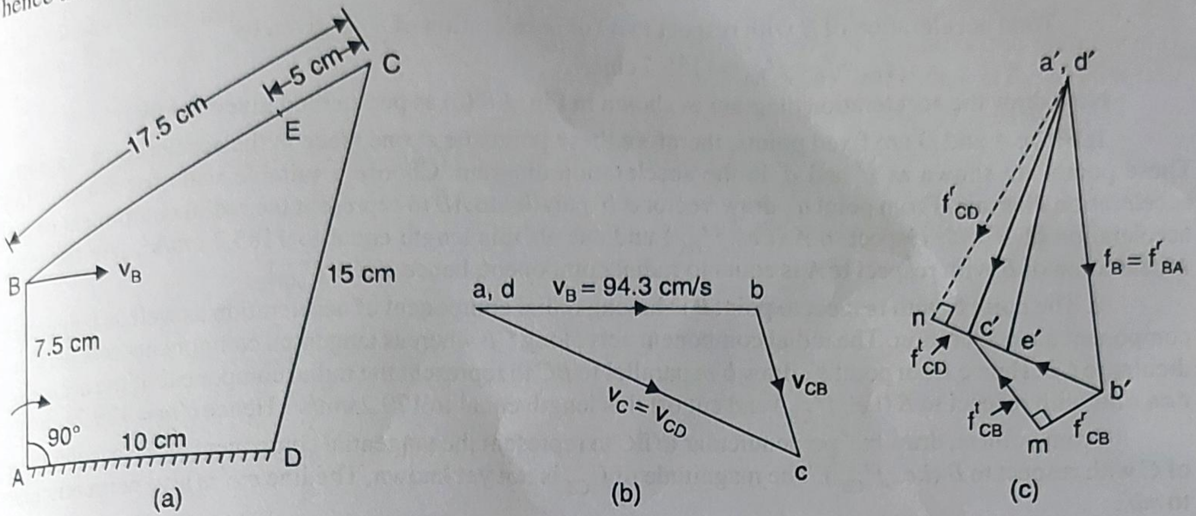


Fig. 4.8. (*a*) Space diagram (*b*) Velocity diagram (*c*) Acceleration diagram.

2. The point *C* moves with respect to point *B* and also with respect to point *D*. The velocity of *C* with respect to *B* is perpendicular to *BC* whereas the velocity of *C* with respect to *D* is perpendicular to *DC*. Hence from point *b*, draw vector *bc* perpendicular to *BC*. Since *AD* is fixed hence point *a* and *d* in the velocity diagram will coincide. From point *d*, draw vector *dc* perpendicular to *DC* so as to intersect vector *bc* in *c*. Then *abc* is the velocity diagram for the given mechanism. In this diagram :

Vector *bc* = Velocity of point *C* with respect to point *B* or v_{CB}

Vector *dc* = Velocity of point *C* with respect to point *D* (*i.e.*, v_{CD}) or velocity of *C* as point *D* is fixed (*i.e.*, v_C).

Vector *ab* = Velocity of point *B* with respect to point *A* (*i.e.*, v_{BA}) or velocity of point *B* as point *A* is fixed *i.e.*, v_{BA} or v_B.

From the velocity diagram, on measurement we find that

$$v_{CB} = \text{Vector } bc = 56 \text{ cm/s},$$

$$v_C = v_{CD} = \text{Vector } dc = 124 \text{ cm/s}.$$

Acceleration Diagram

Before drawing the acceleration diagram, we must calculate the radial components of the acceleration $\left(i.e. \dfrac{v^2}{r}\right)$ of each link. Here *v* is the velocity and *r* is the length of the link.

∴ The radial component of the acceleration of *B* with respect to *A* is given by,

$$f^r_{BA} = \frac{v^2_{BA}}{BA} = \frac{94.3^2}{7.5} = 1185.7 \text{ cm/s}^2$$

Similarly the radial component of the acceleration of *C* with respect to *B*,

$$f^r_{CB} = \frac{v^2_{CB}}{CB} = \frac{56^2}{17.5} = 179.2 \text{ cm/s}^2$$

and radial component of the acceleration of C with respect to D,

$$f^r_{CD} = \frac{v^2_{CD}}{CD} = \frac{124^2}{15} = 102.5 \text{ cm/s}^2$$

As the link AB revolves uniformly, the angular acceleration of AB is zero, therefore there will be no tangential component of acceleration of B with respect to A.

∴ Total acceleration of B with respect to A (or acceleration of B) is given by

$$f_{BA} = f_B = f^r_{BA} = 1185.7 \text{ cm/s}^2$$

Now draw the acceleration diagram as shown in Fig. 4.8 (c) as per method given below :

1. Since A and D are fixed points, therefore these points lie at one place in the acceleration diagram. These points are shown as a' and d' in the acceleration diagram. Choose a suitable scale for drawing the acceleration diagram. From point a', draw vector $a'b'$ parallel to AB to represent the radial component of the acceleration of B with respect to A (*i.e.*, f^r_{BA}) and cut off this length equal to 1185.7 cm/s^2. [As the total acceleration of B with respect to A is equal to radial component, hence $a'b' = f^r_{AB}$]

2. The point C with respect to point B is having radial component of acceleration as well as tangential component of acceleration. The radial component acts along CB whereas tangential components acts perpendicular to CB. Hence from point b', draw $b'm$ parallel to BC to represent the radial component of the acceleration of C with respect to B (*i.e.*, f^r_{CB}) and cut off this length equal to 179.2 cm/s^2. Hence $b'm = 179.2$ cm/s^2.

From point m, draw mc' perpendicular to BC to represent the tangential component of the acceleration of C with respect to B (*i.e.*, f^t_{CB}). The magnitude of f^t_{CB} is not yet known. The line mc' is also perpendicular to mb'.

3. The point C with respect to point D is having radial component of acceleration as well as tangential component of acceleration. The radial component acts along CD whereas tangential component acts perpendicular to CD. Hence from point d', which coincides with point a', draw $d'n$ parallel to CD to represent the radial component of acceleration of C with respect to D (*i.e.*, f^r_{CD}) and cut off this length = 1025 cm/s^2. Hence $d'n = 1025$ cm/s^2.

4. From point n, draw nc' perpendicular to CD (or perpendicular to nd') to represent the tangential component of acceleration of C with respect to D (*i.e.*, f^t_{CD}) intersecting mc' at c'. Then $a'b'c'$ represents the acceleration diagram.

By measurement, we find that

$$f^t_{CB} = \text{Vector } mc' = 250 \text{ cm/s}^2 \text{ and}$$
$$f^t_{CD} = \text{Vector } nc' = 185 \text{ cm/s}^2.$$

(i) Angular accelerations of links BC and CD

$$\text{Angular acceleration of } BC = \frac{\text{Tangential acceleration of } BC}{\text{Length of link } BC}$$

$$= \frac{f^t_{CB}}{CB} = \frac{250}{17.5} = \mathbf{14.3\ rad/s^2.} \quad \textbf{Ans.}$$

$$\text{Angular acceleration of } CD = \frac{\text{Tangential acceleration of } CD}{\text{Length of } CD}$$

$$= \frac{f^t_{CD}}{CD} = \frac{185}{15} = \mathbf{12.3\ rad/s^2.} \quad \textbf{Ans.}$$

(ii) Acceleration of point E

In order to find the acceleration of the point E, first locate the corresponding point e' in the acceleration diagram such that

$$\frac{b'e'}{b'c'} = \frac{BE}{BC}.$$

Now join $a'e'$ as shown. Then $a'e'$ is the acceleration of the point E.

By measurement, we find that $a'e' = 1075 \text{ cm/s}^2$.

$\therefore$ Acceleration of $E = \mathbf{1075 \text{ cm/s}^2}$. **Ans.**

Problem 4.5. *In the steam engine mechanism, shown in Fig. 4.9, the crank AB rotates at 200 r.p.m. Find the velocities of C, D, E, F and P. Also find the acceleration of the slider at C. The dimensions of the various links are : AB = 12 cm, BC = 48 cm, CD = 18 cm, DE = 36 cm and EF = 12 cm and FP = 36 cm.*

(AMIE, Winter 1976)

Sol. Given :

$$N_{BA} = 200 \text{ r.p.m.}$$

$$\therefore \quad \omega_{BA} = \frac{2\pi \times 200}{60} = 20.94 \text{ rad/s}$$

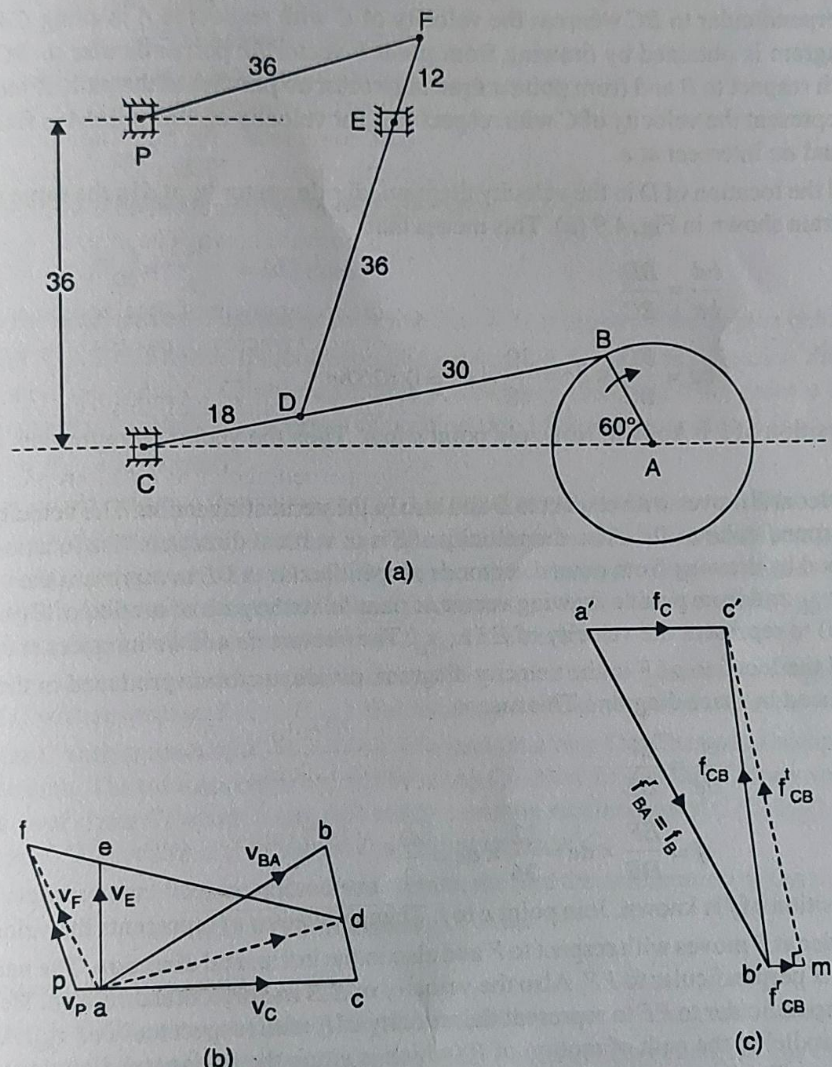


Fig. 4.9. (*a*) Space diagram (*b*) Velocity diagram (*c*) Acceleration diagram.

Now the velocity of point B with respect to A is given by,

$$v_{BA} = \omega_{BA} \times \text{Length } AB$$
$$= 20.94 \times 12 \text{ cm/s} \qquad (\because \; AB = 12 \text{ cm})$$
$$= 251.3 \text{ cm/s}$$

The direction of this velocity will be perpendicular to AB. As point A is fixed, hence this is also the velocity of point B.

$$\therefore \qquad v_{BA} = v_B = 251.3 \text{ cm/s.}$$

First draw the space diagram of the given mechanism to some suitable scale as shown in Fig. 4.9 (*a*). Now draw the velocity diagram as shown in Fig. 4.9 (*b*) as per method given below :

1. The point A is fixed hence the corresponding point will be a in velocity diagram. Take point a any where choose a suitable velocity scale and from point a draw vector ab perpendicular to AB to represent the velocity of B with respect to A or velocity of B (*i.e.*, v_{BA} or v_B), such that vector $ab = v_{BA} = v_B = 251.3$ cm/s.

2. Now the point C is moving with respect to B and also with respect to A. The velocity of C with respect to B is perpendicular to BC whereas the velocity of C with respect to A is along CA. The location of C in velocity diagram is obtained by drawing from point b vector bc perpendicular to BC to represent the velocity of C with respect to B and from point a drawing vector ac parallel to the path of motion of C (which is along CA) to represent the velocity of C with respect to A (or velocity of C as point A is fixed) *i.e.*, v_{CA} or v_C. The vectors bc and ac intersect at c.

3. To find the location of D in the velocity diagram, divide vector bc at d in the same ratio as D divides BC in space diagram shown in Fig. 4.9 (*a*). This means that

$$\frac{bd}{bc} = \frac{BD}{BC}$$

$$\therefore \qquad bd = \frac{BD}{BC} \times bc = \frac{30}{48} \times bc = 0.625\, bc$$

Hence position of d is known. Now join point a to d. Then the vector ad represents the velocity of D *i.e.*, v_D.

4. The slider at E moves with respect to D and also in the vertical direction. The velocity of slider E with respect to D is perpendicular to DE. Also the velocity of E is in vertical direction. The location of E in velocity diagram is obtained by drawing from point d vector de perpendicular to DE to represent the velocity of E with respect to D *i.e.*, v_{ED} and from point a drawing vector ae parallel to the path of motion of E (which is along the vertical direction) to represent the velocity of E *i.e.*, v_E. The vectors de and ae intersect at e.

5. To find the location of F in the velocity diagram, divide vector de produced in the same ratio as F divides DE produced in space diagram. This means

$$\frac{ef}{de} = \frac{EF}{DE}$$

or
$$ef = \frac{EF}{DE} \times de = \frac{12}{36} \times de = \frac{de}{3}$$

Hence position of f is known. Join point a to f. Then the vector af represents the velocity of F *i.e.*, v_F.

6. The slider at P moves with respect to F and also in the horizontal direction. The velocity of slider P with respect to F is perpendicular to PF. Also the velocity of P is in horizontal direction. Hence from point f draw vector fp perpendicular to PF to represent the velocity of P with respect to F *i.e.*, v_{PF}. Also from point a draw vector ap parallel to the path of motion of P (which is along the horizontal direction) to represent the velocity of P *i.e.*, v_P. The vectors fp and ap intersect at p.

Now by measurements from velocity diagram, we get :

(i) Velocity of C, v_C = Vector ac = **242.5 cm/s. Ans.**

(ii) Velocity of D, v_D = Vector ad = **235 cm/s. Ans.**

(iii) Velocity of E, v_E = Vector ae = **100 cm/s. Ans.**

(iv) Velocity of F, v_F = Vector af = **140 cm/s. Ans.**

(v) Velocity of P, v_P = Vector ap = **30 cm/s. Ans.**

Acceleration of the slider at C

This is obtained by drawing acceleration diagram. Before drawing acceleration diagram, let us calculate the radial components of acceleration $\left(i.e., \frac{v^2}{r}\right)$ of points B and C.

The radial component of acceleration of B with respect to A,

$$f^r_{BA} = \frac{v^2_{BA}}{AB} = \frac{251.3^2}{12} \qquad (\because \; v_{BA} = 251.3 \text{ cm/s})$$

$$= 5240 \text{ cm/s}^2$$

The radial component of acceleration of C with respect to B,

$$f^r_{CB} = \frac{v^2_{CB}}{BC}$$

But from velocity diagram v_{CB} = Vector bc = 130 cm/s.

$$\therefore \qquad f^r_{CB} = \frac{130^2}{48} = 352 \text{ cm/s}^2$$

Now draw the acceleration diagram as shown in Fig. 4.9 (c) as per method given below :

1. The point A is fixed hence the corresponding point will be a' in acceleration diagram. Start the acceleration diagram from point a'. Choose a suitable acceleration scale and from point a draw vector $a'b'$ parallel to AB to represent the radial component of acceleration of B with respect to A ($i.e.$, f^r_{BA}) such that

Vector $a'b' = f^r_{BA} = 5240 \text{ cm/s}^2$.

As the total acceleration of B with respect to A is equal to radial component, hence $a'b' = f^r_{AB}$.

2. The point C with respect to point B is having radial component of acceleration as well as tangential component. The radial component acts along CB whereas the tangential component acts perpendicular to CB. Now from point b', draw vector $b'm$ parallel to BC to represent the radial component of acceleration of C with respect to B ($i.e.$, f^r_{CB}) such that, vector $b'm = f^r_{CB} = 352 \text{ cm/s}^2$.

From point m, draw vector mc' perpendicular to the vector $b'm$ to represent the tangential component of acceleration of C with respect to B ($i.e.$, f^t_{CB}). But the magnitude of f^r_{CB} is not known.

3. The point C with respect to A, is moving in a straight along CA. The radial component of C with respect to A will be zero. The total acceleration will be along CA. Now from point a', draw vector $a'c'$ parallel to the path of motion of slider C (which along AC) to represent the acceleration of C ($i.e.$, f_C). The vectors mc' and $a'c'$ intersect at c'. The vector $a'c'$ represents the acceleration of C.

Hence by measurement from acceleration diagram, we find the acceleration of the slider at C,

f_C = vector $a'c'$ = **1900 cm/s². Ans.**

4.5. Coriolis Acceleration Component

When a slider is sliding along a rotating link, Coriolis component of acceleration comes into existence.

Consider a link OP rotating clockwise. There is a slider (link 3), sliding along the link OP. Hence the link 3 is moving along a straight line. At any instant there is a point A on the link OP and the corresponding point