Theory of Machines

Unit – II
Module 1

Velocity of Mechanisms

Velocity Analysis

Analysis of Mechanism

study of motions and forces concerning their different parts.

velocity analysis.

- linear velocity of various points on different links of a mechanism.
- angular velocity of the links.

velocity analysis – prerequisite of acceleration analysis
– useful to force analysis

For doing analysis

machine (or) mechanism is represented by a skeleton (or) a line diagram commonly known as ~~base~~ configuration diagram.

velocity } — analytically — computer & calculator
acceleration } (or) graphically.

But graphical analysis is more direct and accurate. to an acceptable degree ~~and~~

Two methods of graphical approach.

- relative velocity method
- instantaneous. centre method

Instantaneous Centre Method

It is convenient and easy to apply in simple mechanism -

we have to ~~be~~ know abt ~~a) Motion~~ the follows

motion

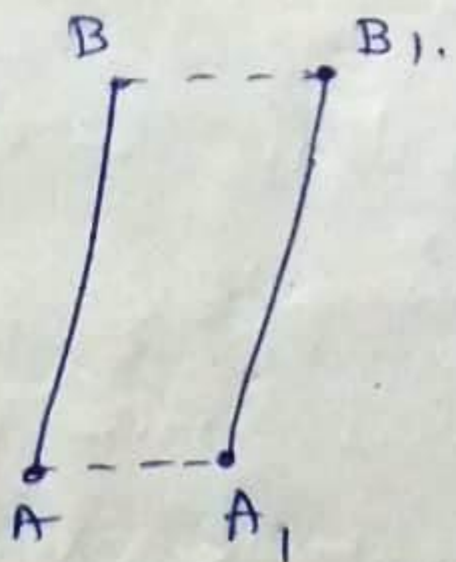
(i) Motion of translation.

2) " " rotation.

3) combination of Motion of translation. and motion of rotation.

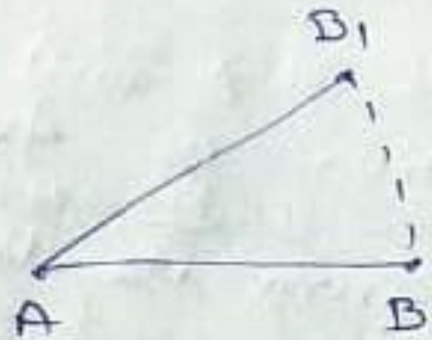
Motion of translation

If a body movez in such a way that all particle movez in parallel plane and travelz same distance. ~~the~~

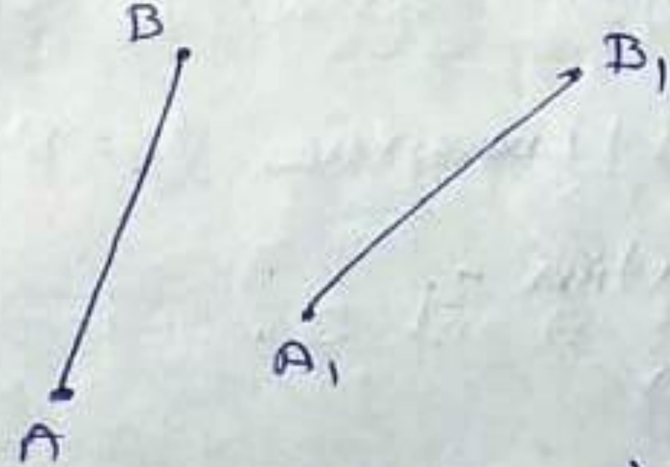


motion of Rotation

If a body rotates abt fixed pt, such that, all its particle moves in a circular path.. Then body is said to have motion of rotation.



Motion of Translation and rotation (combination)

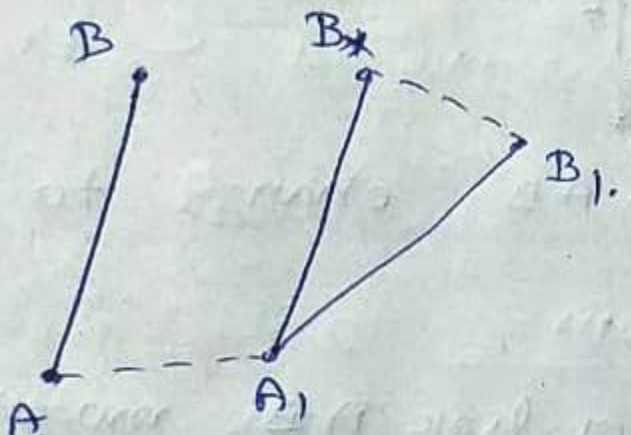


In the link shown above.

The motion is neither entirely linear (translation), nor entirely rotation. But it is combination of the two.

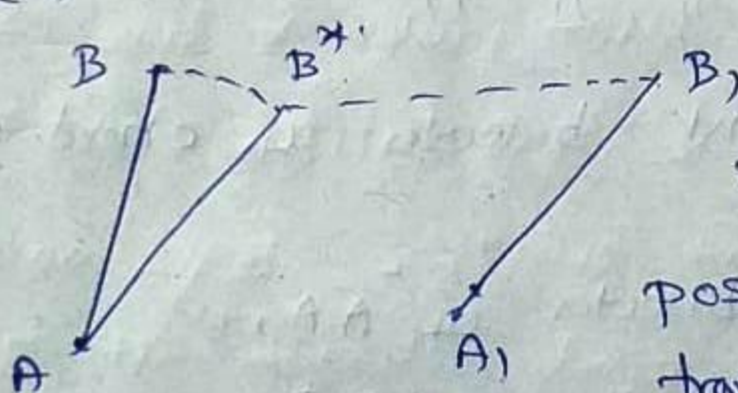
It can be represented as.

case (i) Entirely translation from AB to AB^*. such that AB^* is ||el to A_1B^* then moves to position A_1B_1.



case (ii)

1) Entirely rotates abt A from B to B^*



2) Then moves from position AB^* to A_1B_1 in translation motion.

case (iii)

combined motion of translation and rotation of link from its initial position AB to A_1B_1.

It may be assumed as motion of entirely rotation about a certain point.

This point is known as Instantaneous centre of rotation.

Let link AB changes to Link A_1B_1 in short time.

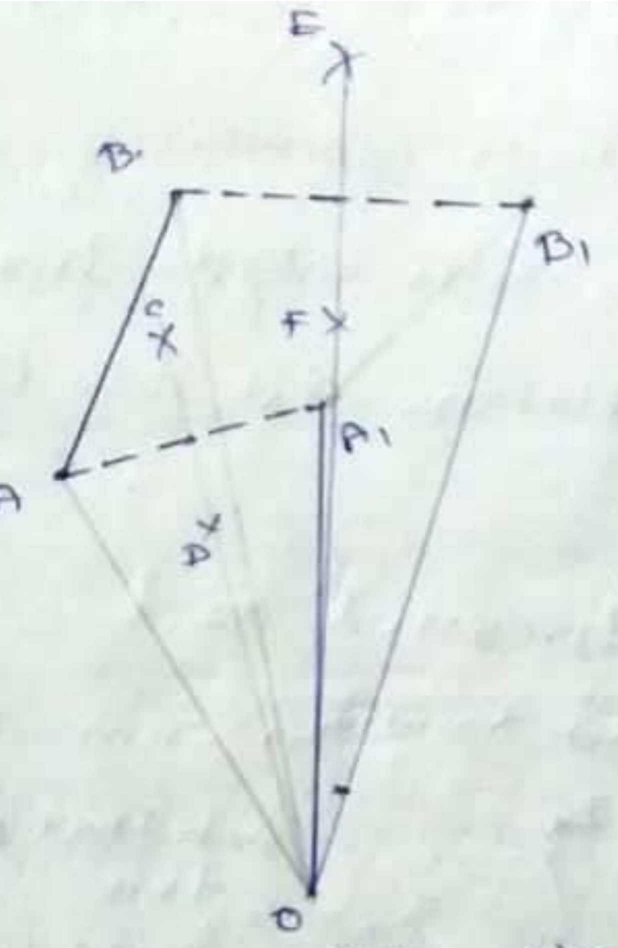
(ie) pt A of link AB moved to pt A_1

pt B " " " " " pt B_1

Draw the right bisecter of chord. AA_1 and BB_1.

CD rt bisecter of AA_1

EF " " " BB_1.



Extend this line. It meets at pt O. called instantaneous centre.

link AB as a whole rotated. about O.

V_A = linear velocity of pt A.

V_B = " " " pt B.

ω = angular velocity. of link AB abt. O.

(ie) angular velocity of link at pt A and point B abt O is ω..

we know linear velocity $V = r\omega$

$$\therefore \quad V_A = AO \times \omega$$

$$\omega = \frac{V_A}{AO} \quad -(1)$$

similarly. linear velocity at pt B.

$$V_B = \cancel{AB} \times \omega \times BO$$

$$\omega = \frac{V_B}{\cancel{OB}\, BO} \quad -(2)$$

Equating (1) and (2)

$$\frac{V_A}{AO} = \frac{V_B}{\cancel{B}BO} = \omega$$

direction of velocity at A will be rt angle to AO

11ly " " " B " " " to BO

∴ If directions of velocities at A and B are known,

Then instantaneous centre ~~wil~~ of AB is obtained by drawing ⊥r to the direction of the velocities at A and B.

The ~~directions~~ point where these two ⊥rs meet is known as instantaneous centre

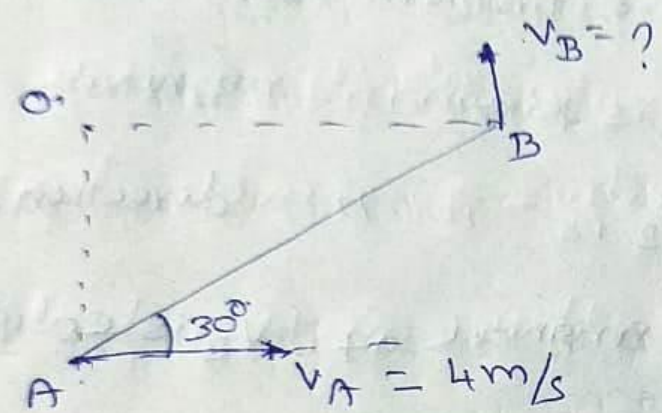
If C is any other point on the link AB

$$\frac{V_A}{AO} = \frac{V_B}{BO} = \frac{V_C}{CO} = \omega.$$

where CO distance of C from instantaneous centre O.

Simple problem

1) link AB is moving in a vertical plane. At a certain instant, when the link is inclined at 30° to the horizontal, pt A is moving horizontally at 4 m/s. B is moving vertically upwards. Find velocity of B.



since direction of velocities are known, Instantaneous centre can be obtained by drawing line ⊥r to direction of velocity at A & B (ie) AO $\perp$r to V_A

BO $\perp$r to V_B.

$$\frac{V_B}{BO} = \frac{V_A}{AO}.$$

$$\tan 30 = \frac{AO}{BO}$$

$$\therefore \frac{V_A}{V_B} = \frac{AO}{BO} = \tan 30^\circ.$$

$$V_B = \frac{\cancel{\tan 30}}{} \frac{V_A}{\tan 30} = \frac{4}{\tan 30}$$

$$= 6.928 \text{ m/s}$$

From. above problem it is noted that.

(i) If V_A is known in magnitude and direction

and V_B is " " direction.

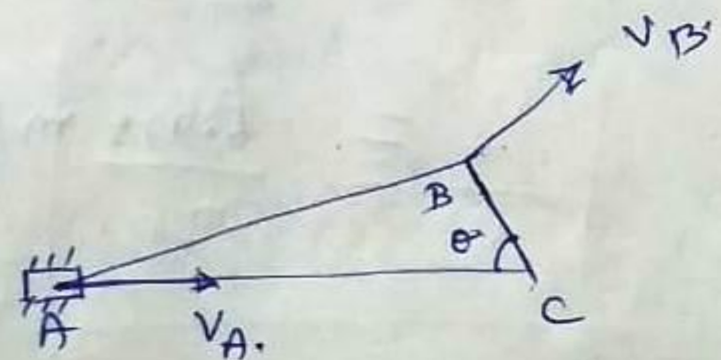
Then magnitude of. V_B is calculated.

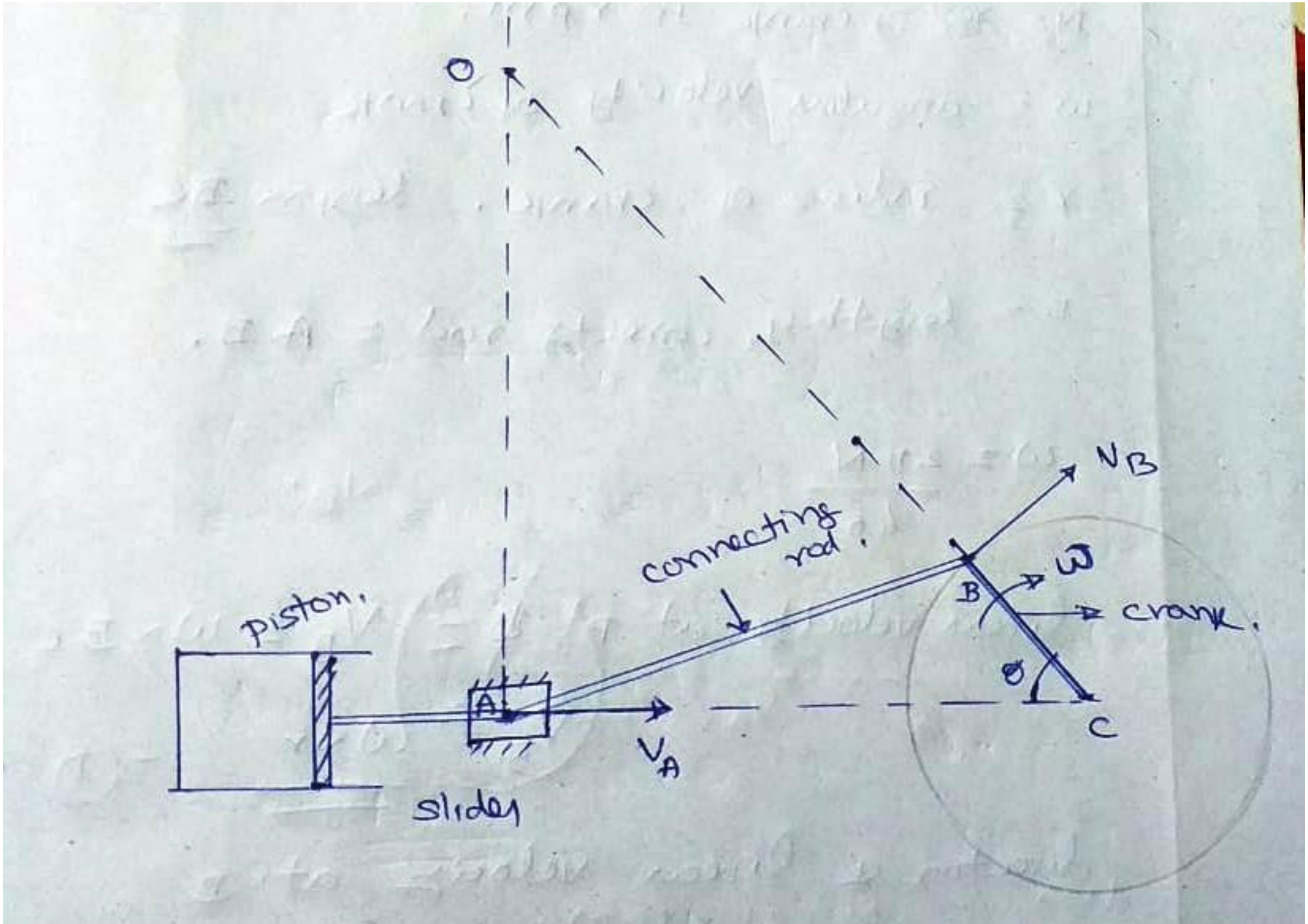
~~(i)~~ 2) velocity of any other pt c lying in AB

can be determined. in magnitude and

direction.

Analysis of Reciprocating Engine Mechanism by Instantaneous Centre method.




O
V_B
connecting rod.
ω
B
crank.
piston.
θ
A
C
V_A
slider

AB – connecting rod

BC – crank.

BC rotates at a uniform angular velocity ω.
in c.w. direction. abt pt. C.

pt. A connected to piston rod and connecting rod
having to and fro motion. in horizontal
plane.

C.R having combined motion of translation
and rotation.

A – translation motion.

B – rotary motion. w.r.t. C.

N = rev of crank in rpm.

ω = angular velocity of crank

r = radius of crank. length $\underline{\underline{BC}}$

L = length of connecting rod = AB.

$$\omega = \frac{2\pi N}{60}$$

linear velocity at pt B = $V_B = \omega \times BC$.

$$\underline{\underline{V_B = \omega \times r}} \quad -(1)$$

direction of linear velocity at B

is along tangent at B to crank circle.

V_B is at $\perp$ to BC.

V_A is horizontal, to AC.

Instantaneous centre is obtained by.

extending. the line BC $\perp$r to V_B.

drawing // from A $\perp$r to V_O.

It meets at pt. O (Instantaneous centre).

Here C·R can be considered as having entirely rotation about pt O

ω_{AB} = angular velocity of C·R AB abt pt. O

$$V_A = \omega_{AB} \times AO. \quad -(2)$$

$$V_B = \omega_{AB} \times BO. \quad -(3)$$

equaty (1) and (3).

$$V_B = \omega \times r. = \omega_{AB} \times BO.$$

$$\boxed{\omega_{AB} = \frac{\omega \times r}{BO.}} \quad -(4)$$

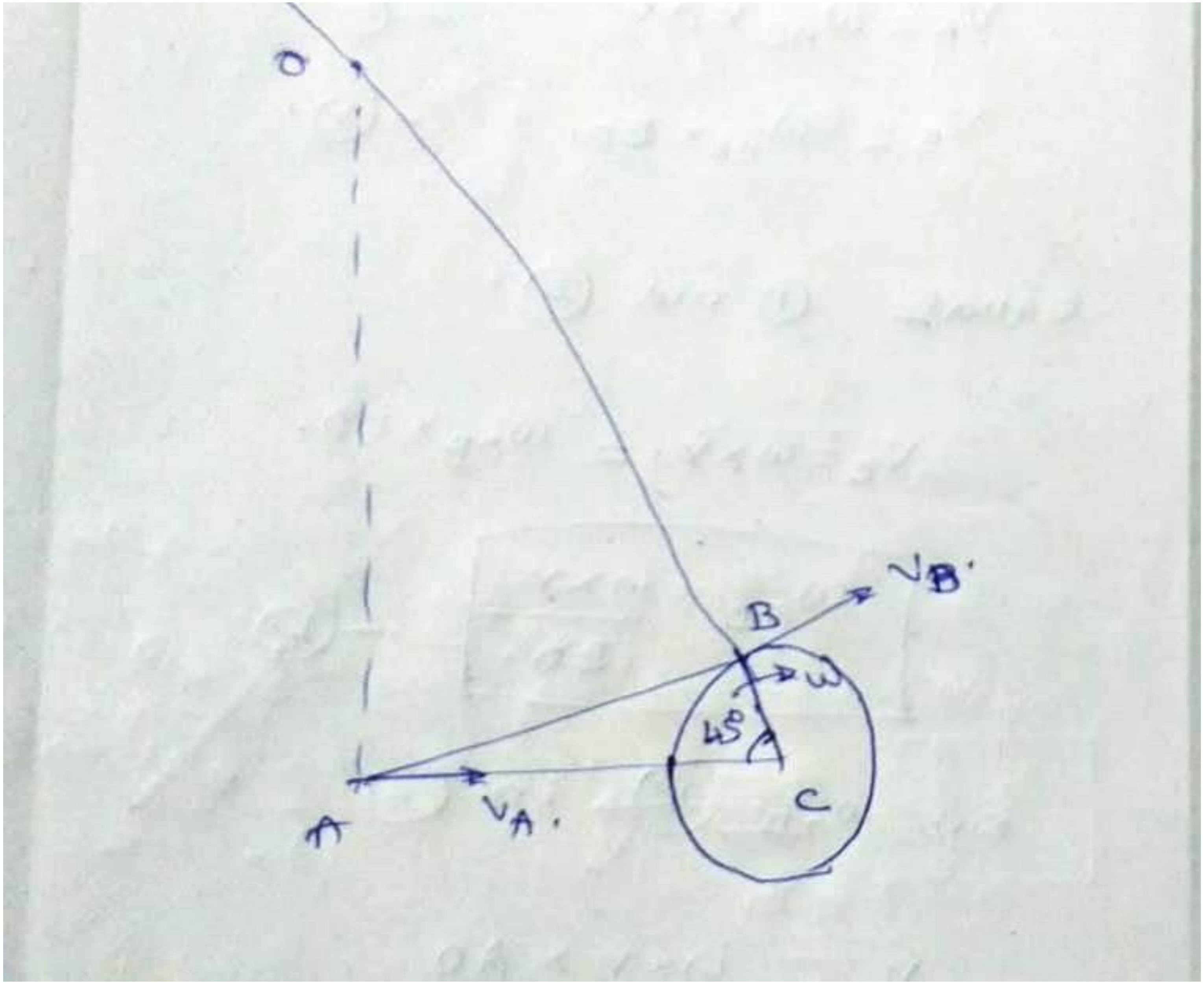
sub ω_{AB} (4) in. (2)

$$V_A = \frac{\omega \times r \times AO}{BO} \quad -(5)$$

Hence meauy scale of length AO and BO. velocy at A can be determned from equation. (5)

problem - 2

The crank of a reciprocating engine is rotating at 210 rpm. The length of the crank and C.R are 20 cm and 100 cm respectively. Find the velocities of the point A (ie velocity of piston) when crank rotates through an angle of 45^0.


O
B
V_B
ω
$45°$
C
A
V_A

data

$N = 210$ rpm.

$$\omega = \frac{2\pi N}{60} = 22 \text{ rad/s}$$

$r = BC = 20\,cm = \cancel{0.020\,m}$ $0.20\,m$

$l = AB = 100\,cm = \cancel{0.100\,m}$ $1\,m$

$\theta = 45^\circ$.

$V_B = \omega \times BC = \omega \times r$.

$$\frac{V_B}{BO} = \frac{V_A}{AO}$$

$$\frac{\omega \times r}{BO} = \frac{V_A}{AO} \qquad V_A = \frac{\omega \times r \times AO}{BO}$$

$$V_A = \frac{22 \times 0.200 \times 1.15}{1.41} = 3.58 \text{ m/s}$$