NATIONAL INSTITUTE OF TECHNOLOGY

THEORY OF MACHINES

Module 3

2. GRASHOFF'S LAW

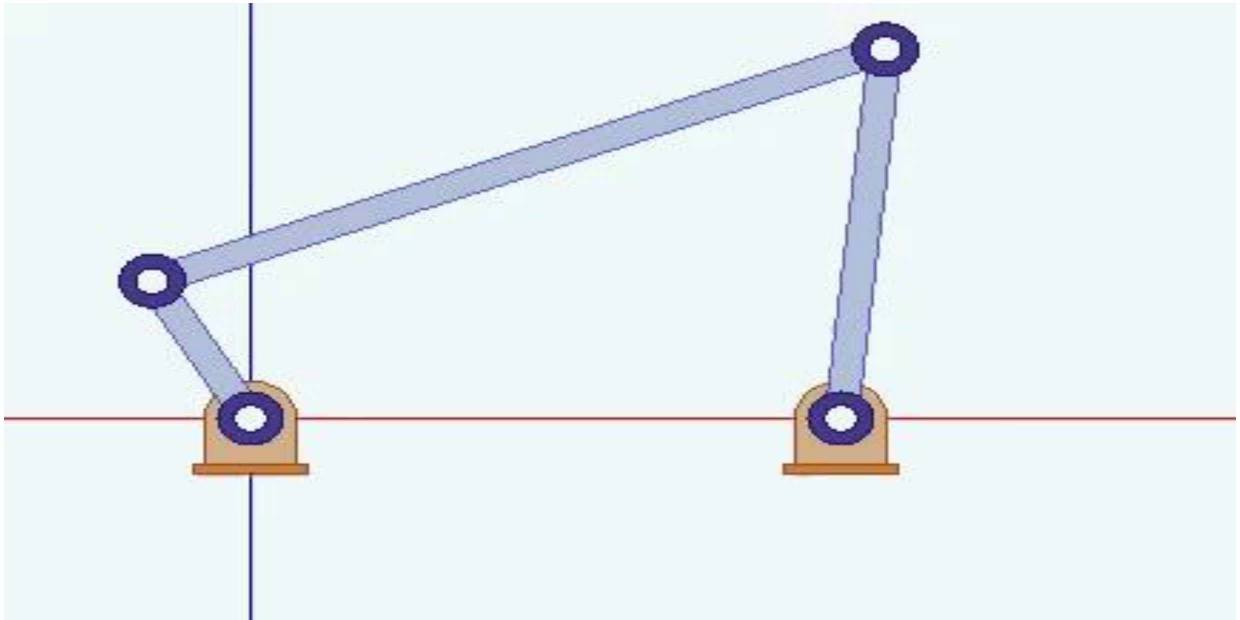
According to Grashoff's law for a four bar mechanism, the sum of the shortest and longest links cannot be greater than the sum of the remaining two links lengths, if there is to be continuous relative motion between two members or it will have at least one revolving link.

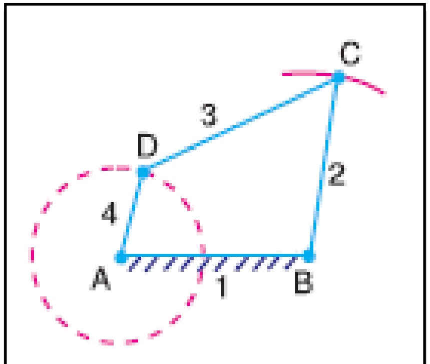
3+4 < 1+2

According to him, mechanism is not useful if no link makes complete revolution.

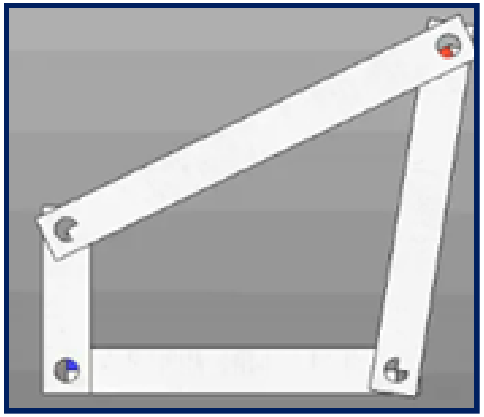
3 and 4 be the length of longest and shortest link.

1 and 2 be the length of the other two links.

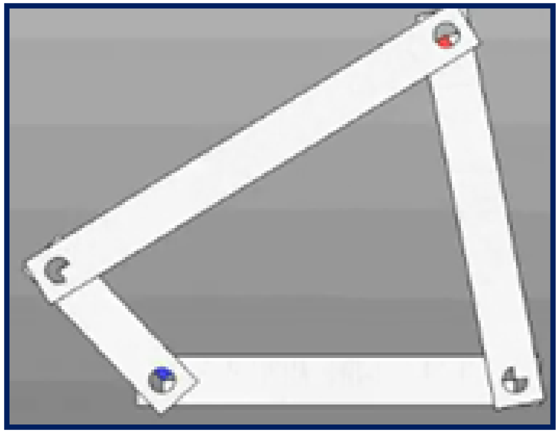




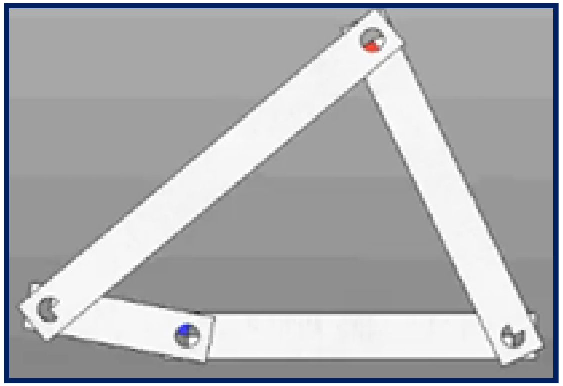
Four bar chain.



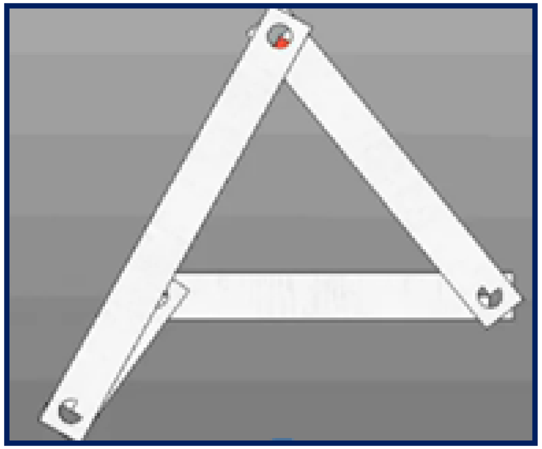
Position 1



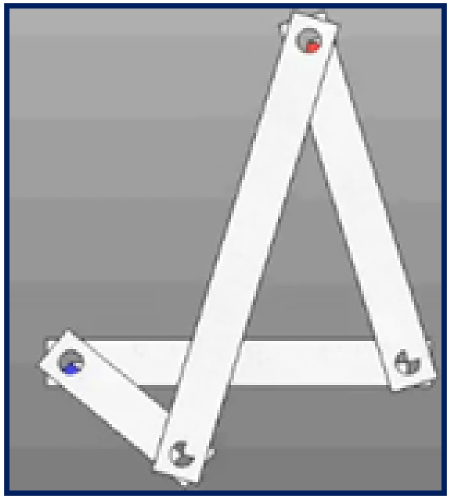
Position 2



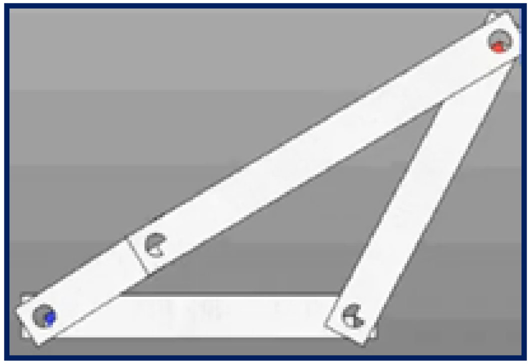
Position 3



Position 4



Position 5



Position 6

3. KINEMATIC INVERSIONS OF MECHANISM

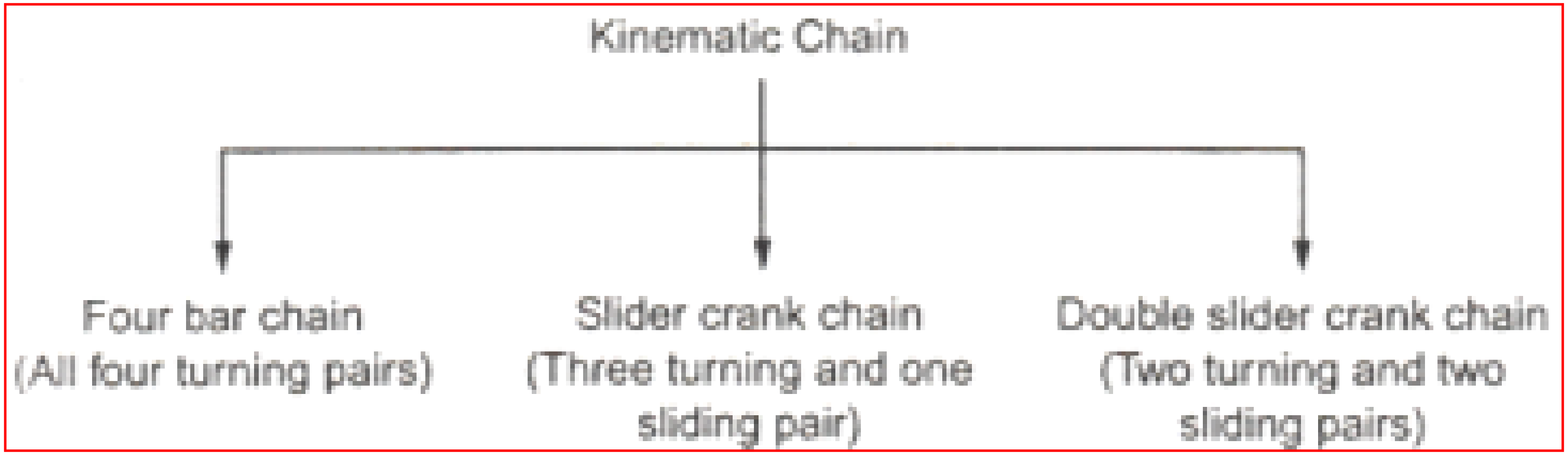
INVERSION OF MECHANISM:

Method of obtaining different mechanisms by fixing different links in a kinematic chain is known as inversion of the mechanism.

For Example, a kinematic chain having 4 link makes four different mechanisms by fixing its different links one at a time. Thus we get many mechanisms as the number of links in a kinematic chain by fixing its different links in turn.

TYPES OF KINEMATIC CHAINS:

The kinematic chains with four lower pairs are more important kinematic chains (from the subject point of view).



Four Bar Chain or Quadric Cycle Chain

In four bar kinematic chain, if one link is fixed, it is known as Mechanism.

A four-link mechanism or linkage is the most fundamental and the simplest kinematic chain. It consists of four links and four turning pairs as shown in figure.

Link that makes complete rotation is known as crank.

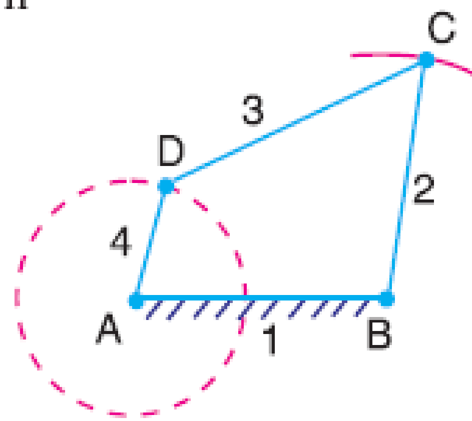
The link opposite to fixed link is called coupler. Fourth link is called a lever or rocker, if it oscillates, or crank if it rotates.

AD (link 4) is a crank.

The link BC (link 2) which makes a partial rotation or oscillates is known as *lever* or *rocker* or *follower*

the link CD (link 3) which connects the crank and lever is called *connecting rod* or *coupler.*

The fixed link AB (link 1) is known as *frame* of the mechanism.



Four bar chain.

When the crank (link 4) is the driver, the mechanism is transforming rotary motion into oscillating motion.

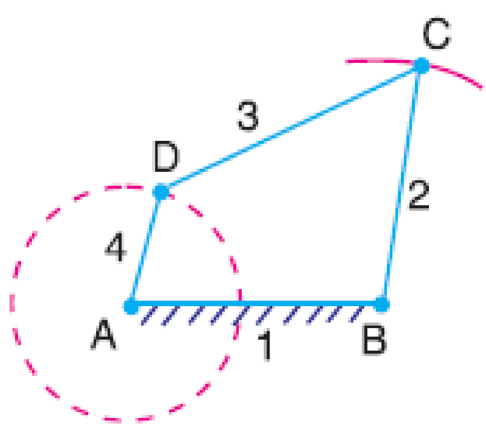
Four Bar Chain or Quadric Cycle Chain

If both link 4 and 2 rotates through full circle. . The link 3 also makes full circle with link 1 (AB).
The mechansim thus obtained is known as crank-crank or double – crank or drag-crank mechanism or rotary-.
Here rotation is observed relative to the fixed link 1.

Now consider the movement of link 3
With respect to 4.
Complete rotation of 3 relative to 4 is possible only if angle ∠ADC is > 180°

Similarly complete rotation of 3 relative to 2 is possible only if ∠BCD is > 180°.

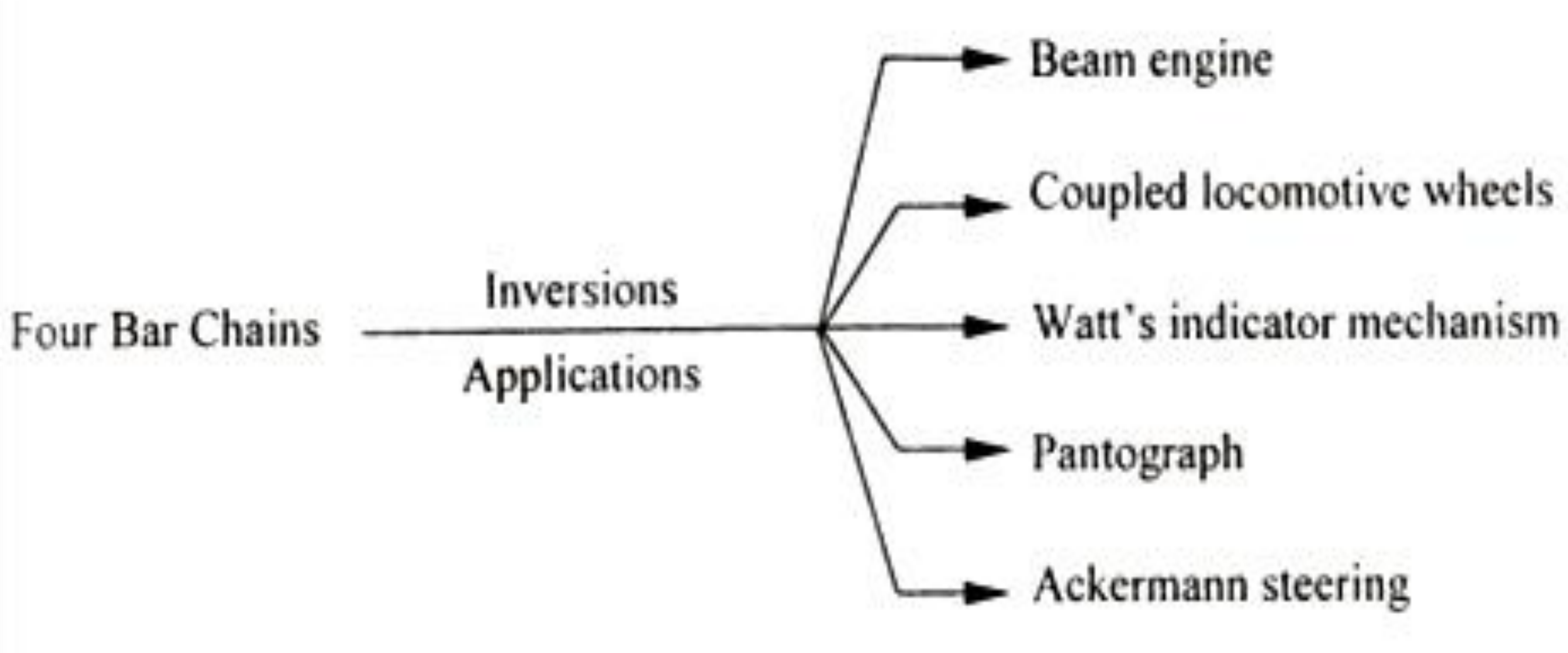
Four bar chain.



FOUR BAR CHAIN

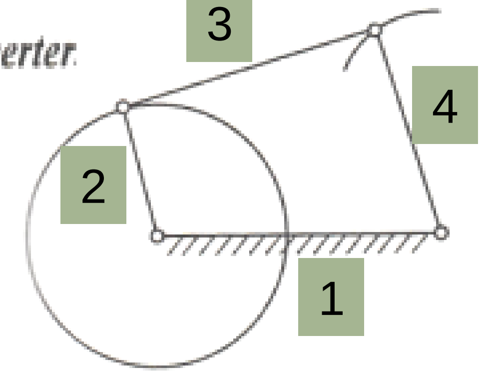
Important inversions of four bar chain are

1.Beam engine (Crank and lever mechanism)
2.Coupling rod of locomotive (Double crank mechanism)
3.Watt's indicator mechanism (Double lever mechanism)



1. First Inversion

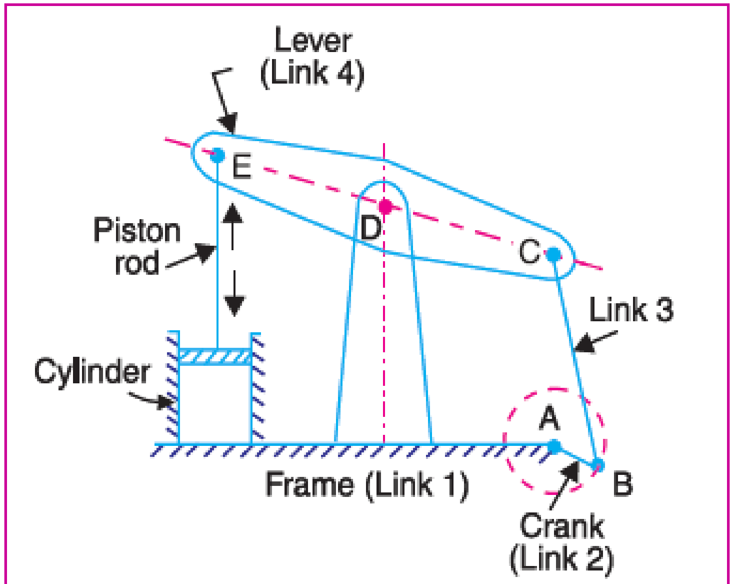
rotary-oscillating converter



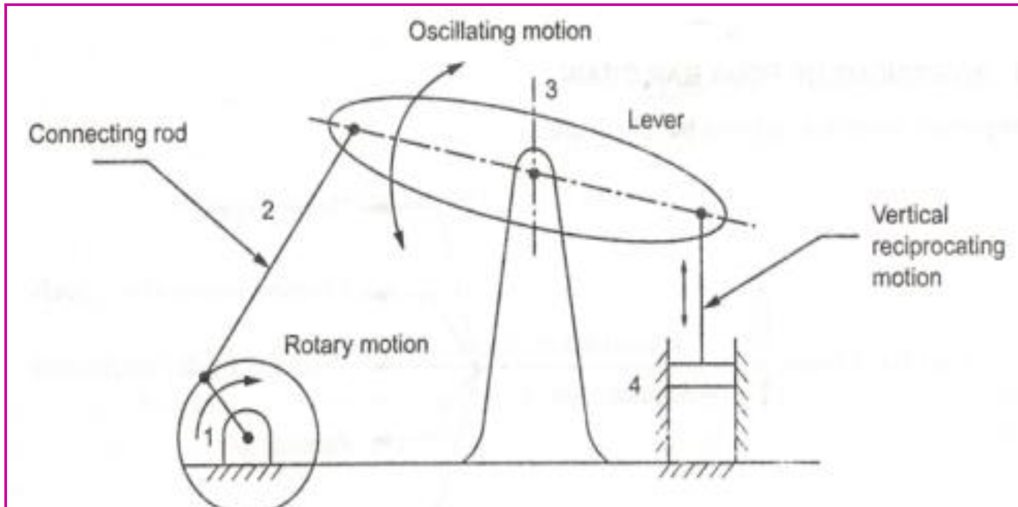
1.BEAM ENGINE (CRANK AND LEVER MECHANISM)

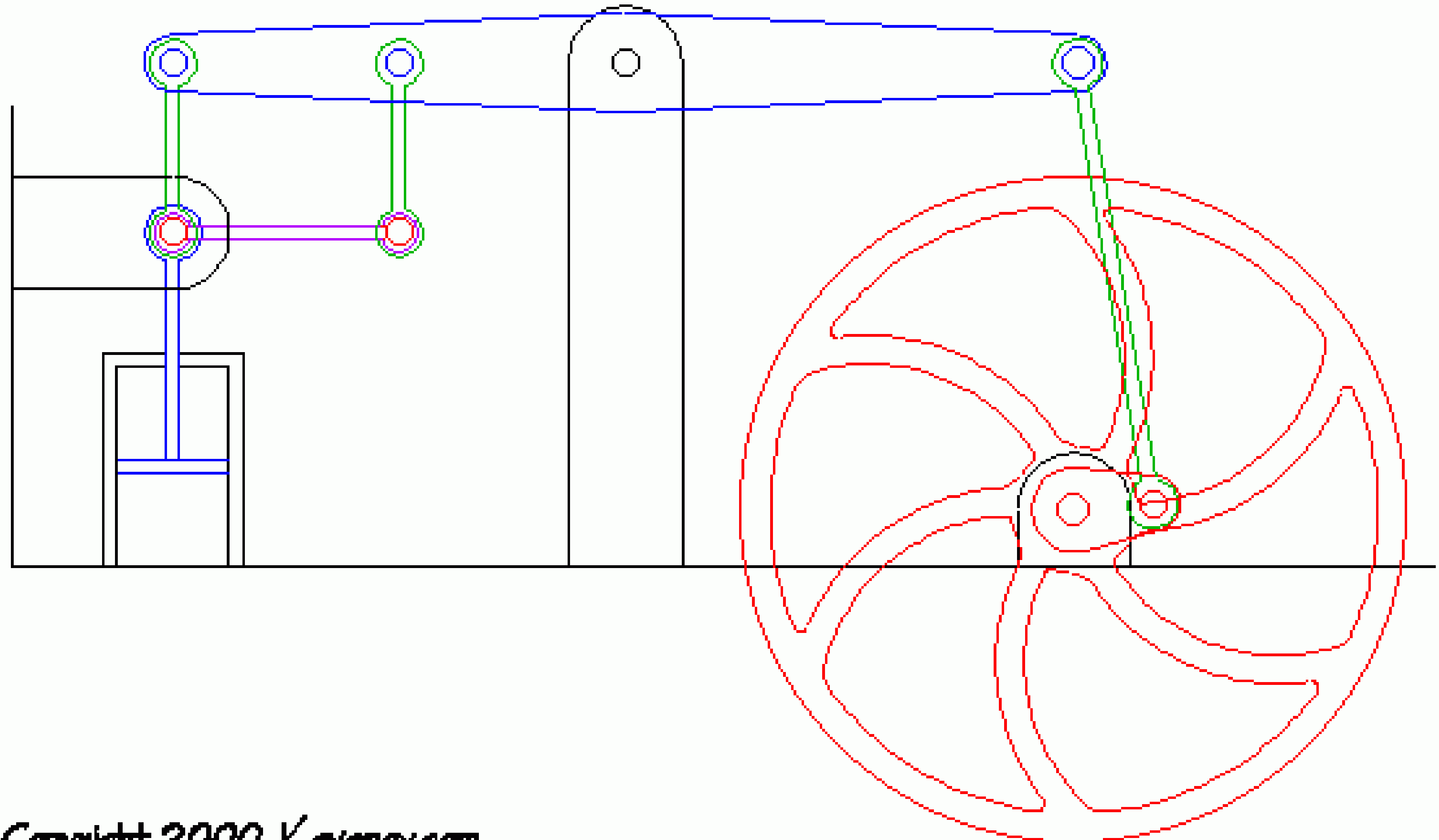
Main purpose of this mechanism is to convert rotary motion in to reciprocating motion.

- It consists of four links.(Frame, Crank, Connecting rod, Lever)
- In this mechanism, when the crank rotates about the fixed center A, the lever oscillates about a fixed centre D.
- The end E of the lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank.

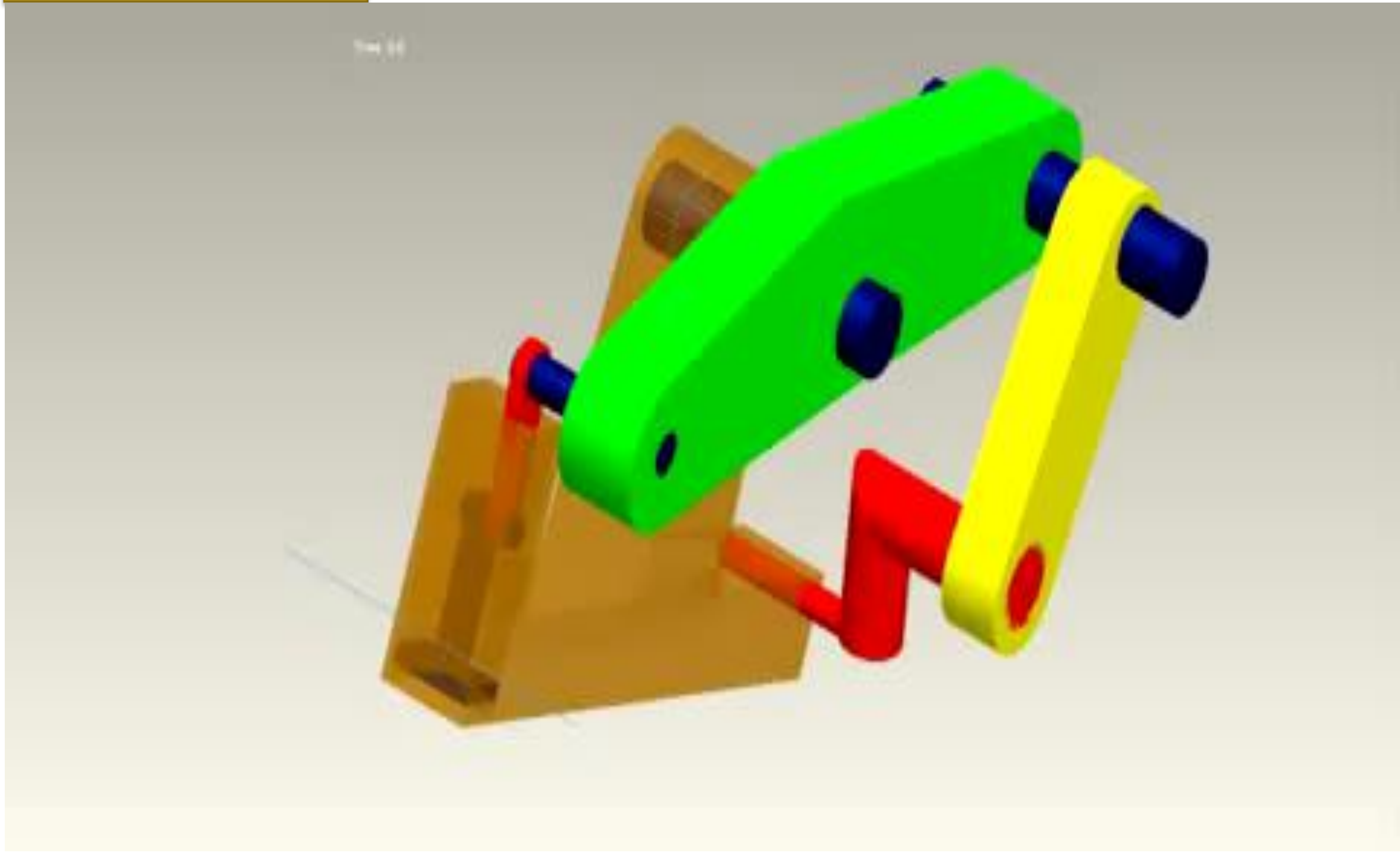


Beam engine.

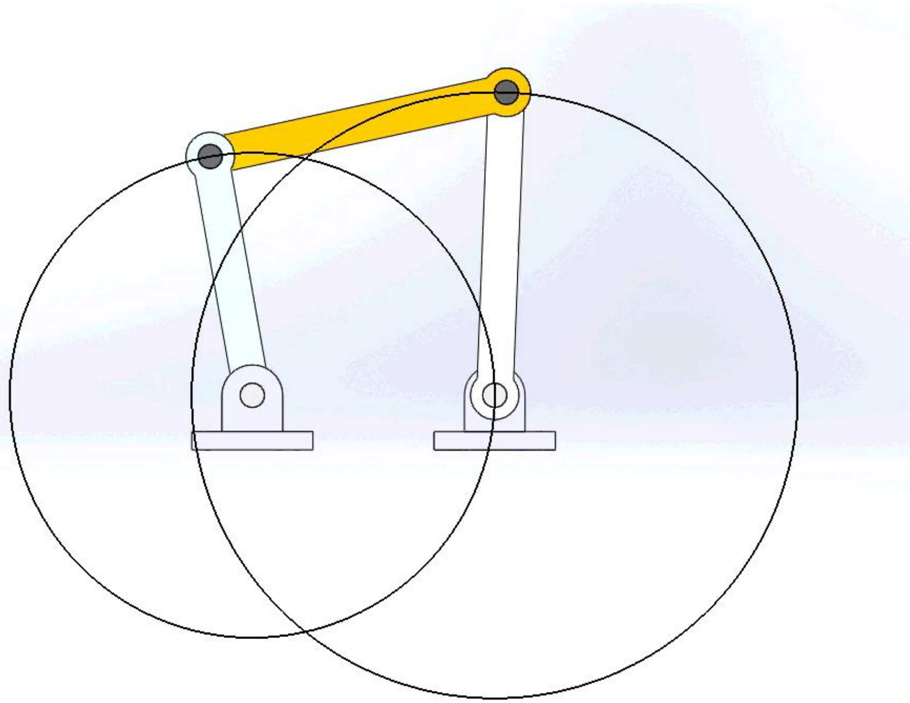


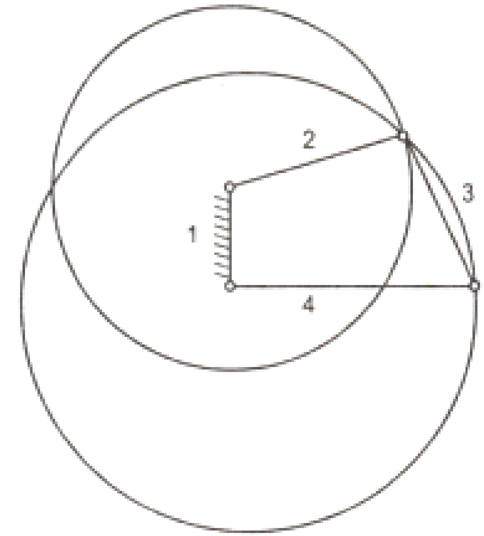


1.BEAM ENGINE



Second Inversion (Double Crank Mechanism) :





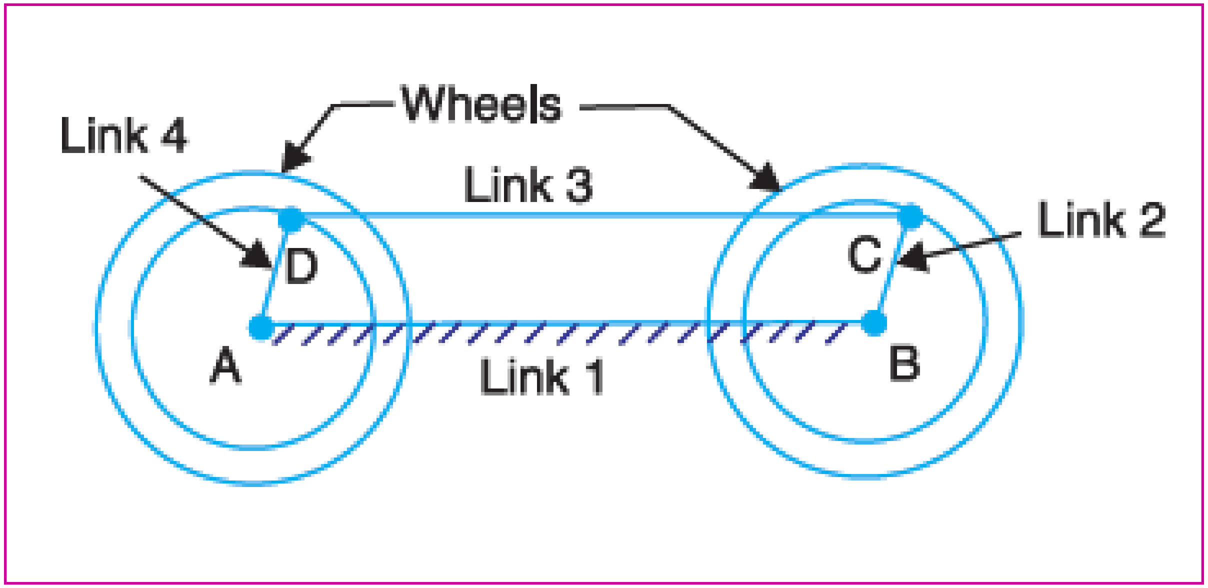
If the shortest link, *i.e.*, link 1 (crank) is fixed, the adjacent links 2 and 4 would make complete revolutions, as shown in Fig. . The mechanism thus obtained is known as ***crank-crank*** or ***double crank mechanism*** or ***rotary-rotary converter.***

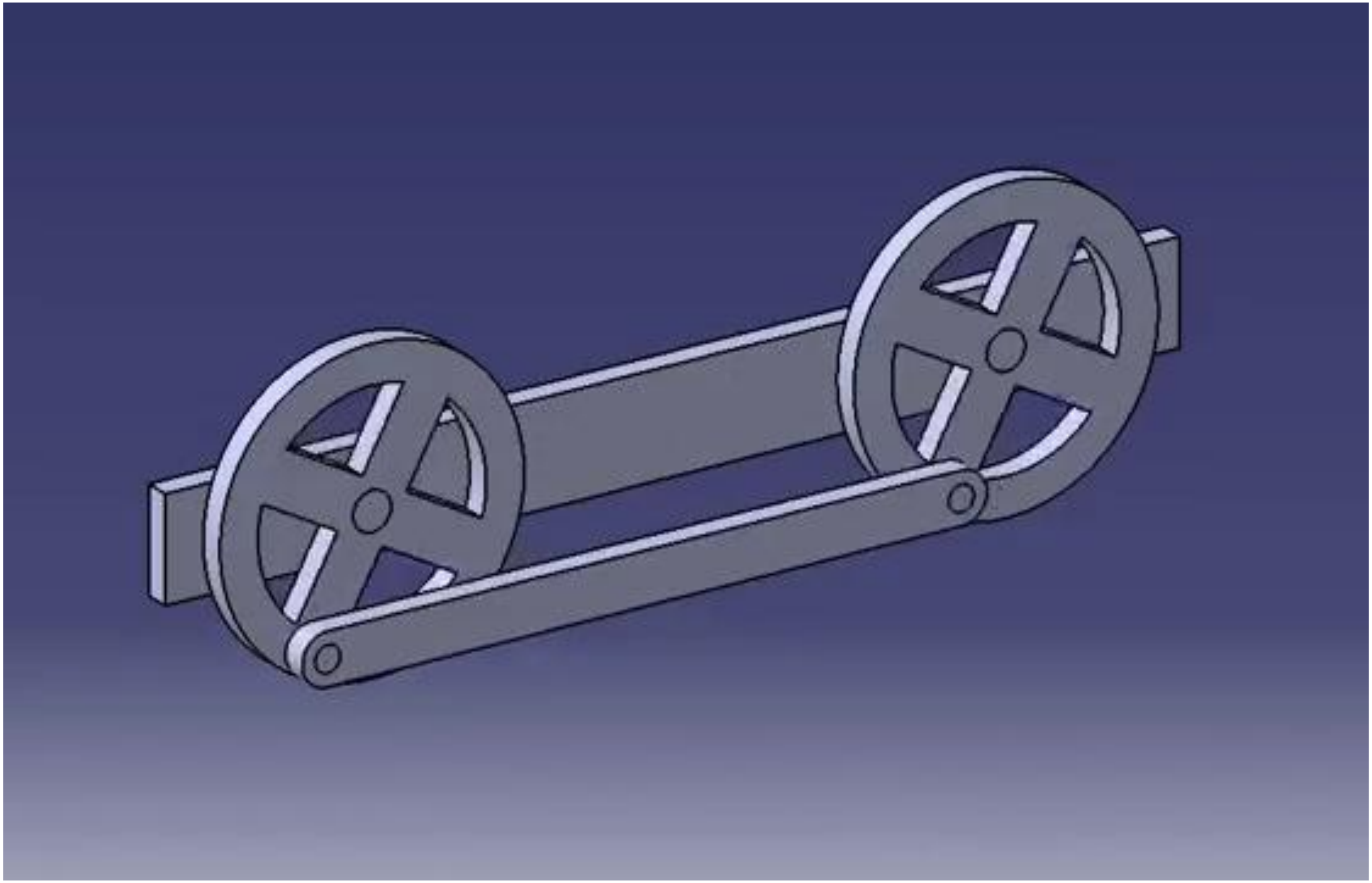
Application : Coupling rod of a locomotive.

2.COUPLING ROD OF LOCOMOTIVE (DOUBLE CRANK MECHANISM)

This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

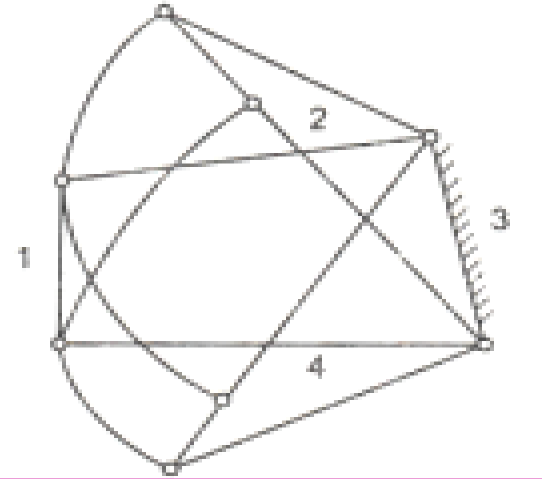
- It consists of four links.
- In this mechanism, the link AD and BC having equal length act as cranks and are connected to the respective wheels.
- The link CD act as a coupling rod and link AB is fixed inorder to maintain a constant center to center distance between them.
- Link 1 is fixed, adjacent links 2 and 4 would make complete revolutions.





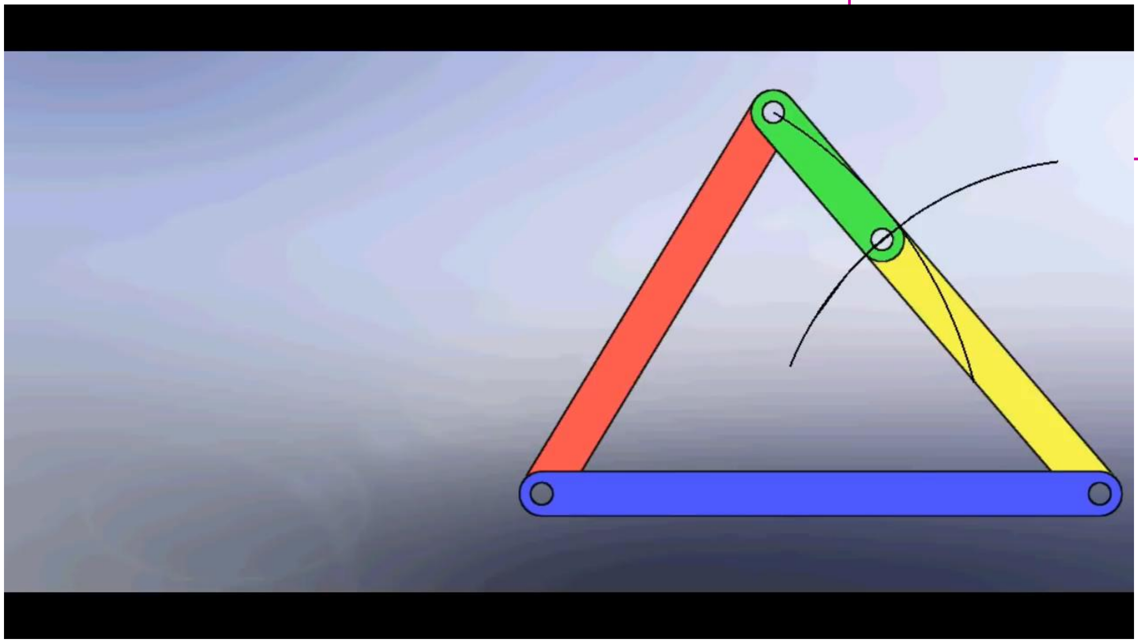
Third Inversion :

- Link opposite to the crank is fixed – Link 1 (shortest link) is made coupler and the other two links oscillates.
- Double rocker mechanism or double lever mechanism



Applications :

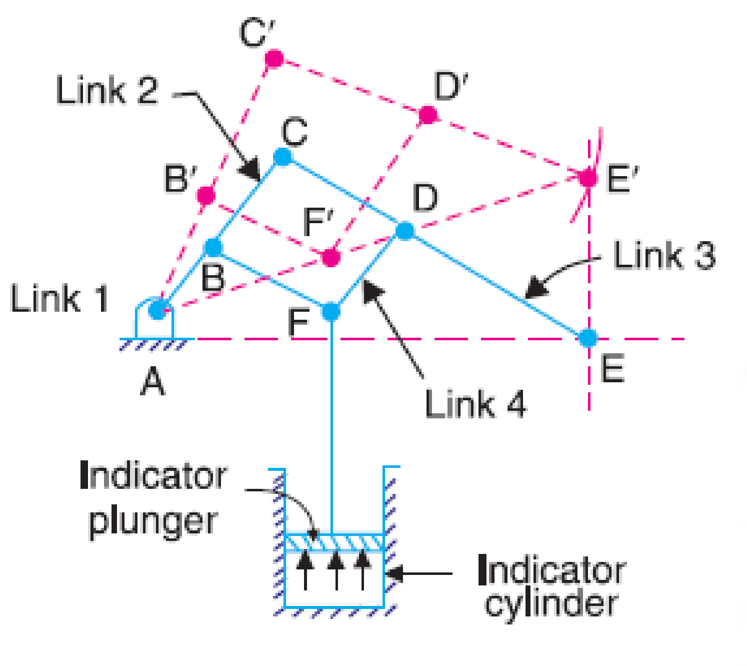
1. Watt's indicator mechanism
2. Pantograph
3. Ackermann steering



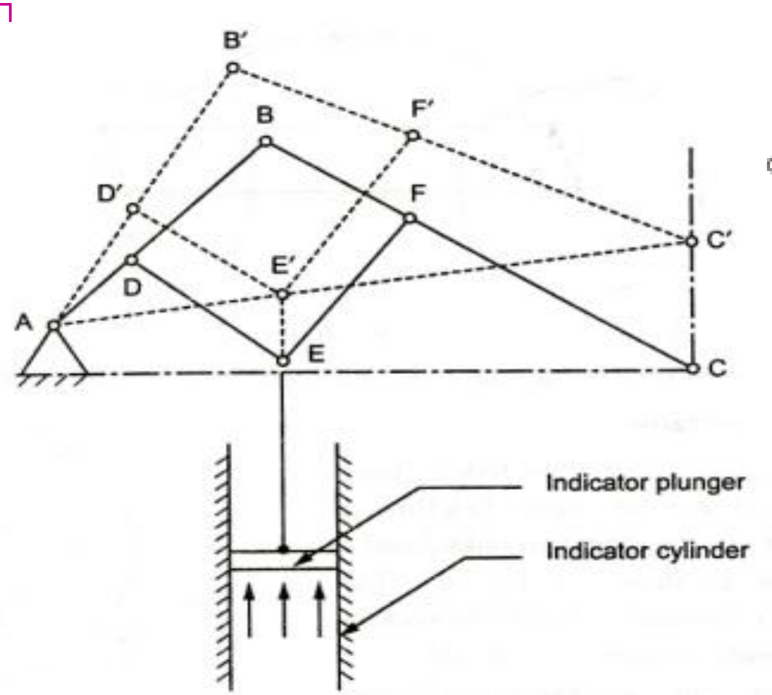
3.WATT'S INDICATOR MECHANISM (DOUBLE LEVER MECHANISM)

- It consists of four links. (fixed link at A, Link AC, Link CE and Link BFD)
- It may be noted that BF and FD forms one link because these two point have no relative motion between them.
- Link CE and BFD act as levers.

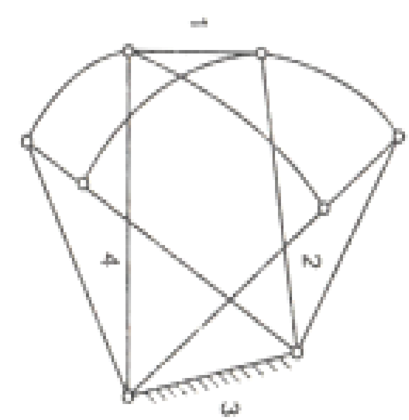
•The displacement of the link BFD is directional to the pressure of gas which acts on the indicator plunger. On any small displacement of the mechanism, the tracing point E at the end of the link CE traces out approximately a straight line.



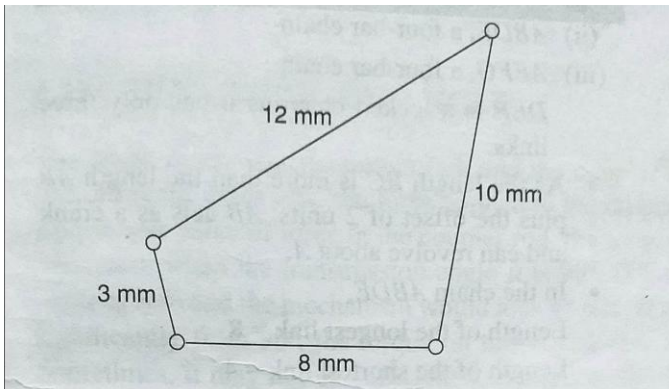
Watt's indicator mechanism.



Watt's engine indicator (double lever mechanism)



Find all the inversions of the chain given in Figure



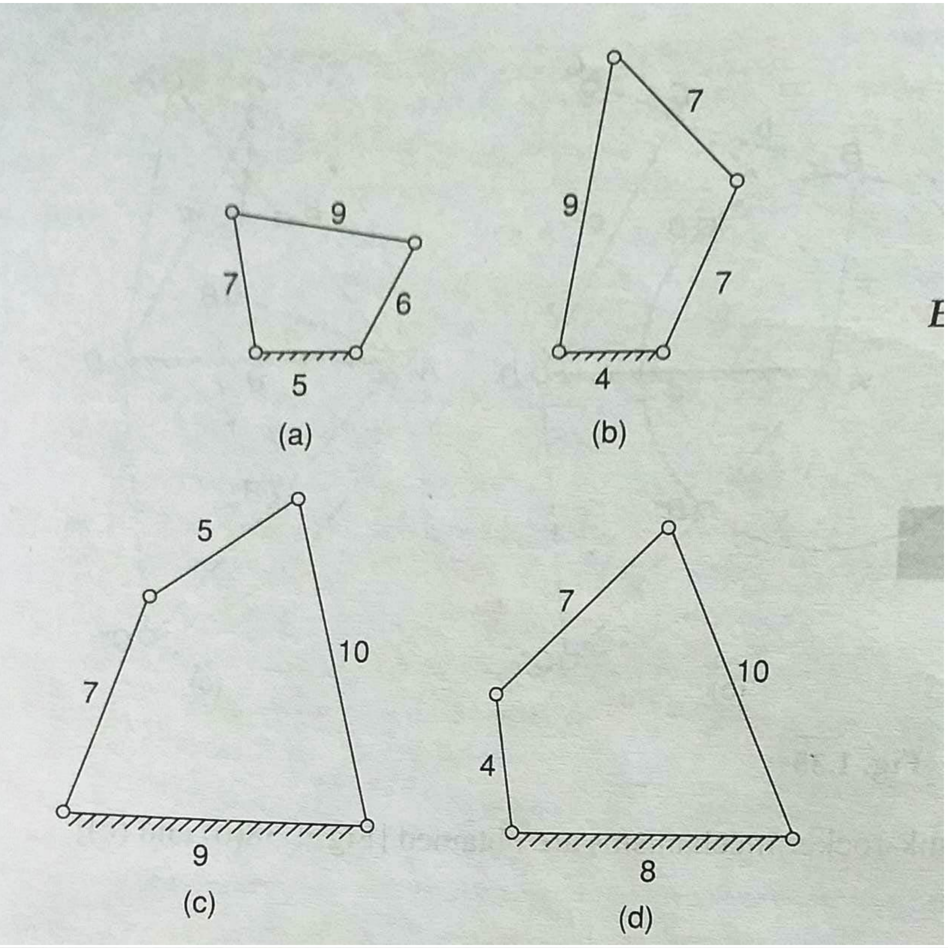
Length of the longest link = 12 mm
Length of the shortest link = 3 mm
Length of other links = 10 mm and 8 mm
Since $12 + 3 < 10 + 8$, it belongs to the class-I mechanism and according to Grashoff's law, three distinct inversions are possible.

Shortest link fixed, i.e., when the link with 3-mm length is fixed, the chain will act as double-crank mechanism in which links with lengths of 12 mm and 8 mm will have complete revolutions.

Link opposite to the shortest link fixed, i.e., when the link with 10-mm length is fixed, the chain will act as double-rocker mechanism in which links with lengths of 12 mm and 8 mm will oscillate.

Link adjacent to the shortest link fixed, i.e., when any of the links adjacent to the shortest link, i.e., link with a length of 12-mm or 8 mm is fixed, the chain will act as crank-rocker mechanism in which the shortest link of 3-mm length will revolve and the link with 10-mm length will oscillate.

Figure shows some four link mechanisms in which the dimensions in standard unit of length is mentioned. Indicate the type of each mechanism whether it is crank rocker or double crank or double rocker mechanism.



(a) Length of the longest link = 9
Length of the shortest link = 5
Length of other links = 7 and 6
Since $9 + 5 > 7 + 6$, it does not belong to the class-I mechanism. Therefore, it is a double-rocker mechanism.

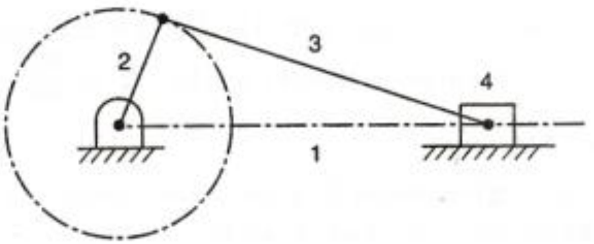
(b) Length of the longest link = 9
Length of the shortest link = 4
Length of other links = 7 and 7
Since $9 + 4 < 7 + 7$, it belongs to the class-I mechanism. In this case as the shortest link is fixed, it is a double-crank mechanism.

(c) Length of the longest link = 10
Length of the shortest link = 5
Length of other links = 9 and 7
Since $10 + 5 < 9 + 7$, it belongs to the class-I mechanism. In this case as the link opposite to the shortest link is fixed, it is a double-rocker mechanism.

(d) Length of the longest link = 10
Length of the shortest link = 4
Length of other links = 8 and 7
Since $10 + 4 < 8 + 7$, it belongs to the class-I mechanism. In this case as the link adjacent to the shortest link is fixed, it is a crank-rocker mechanism.

SINGLE SLIDER CRANK CHAIN

A single slider crank chain is a modification of basic four bar chain. It consists of one sliding pair and three turning pair. This is used to convert reciprocating motion to rotary motion and vice versa. The most familiar form of a slider crank chain is that of the reciprocating steam engine mechanism as shown in Fig.



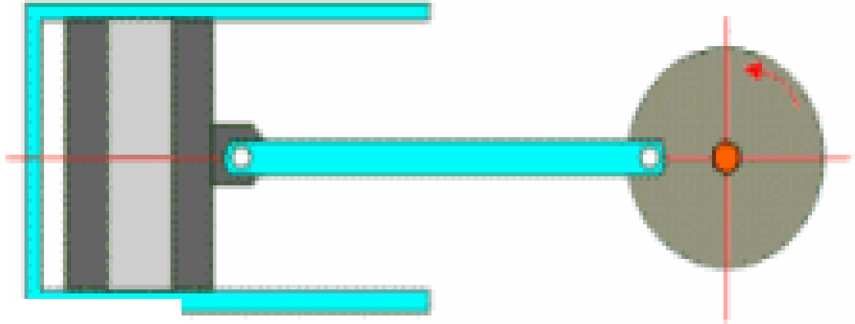
Four links of a single slider crank chain are :

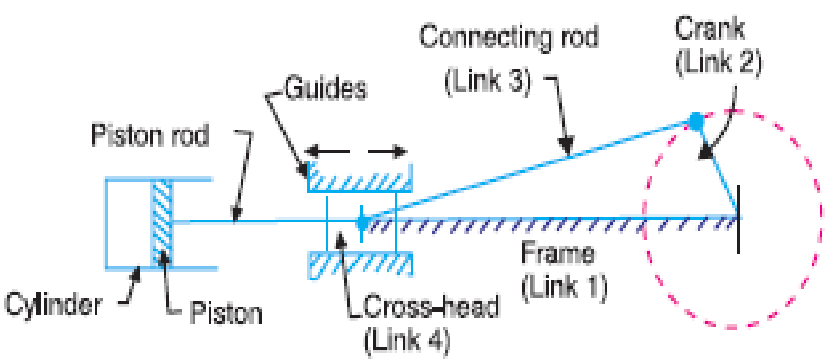
Link 1 : Frame : The fixed link is known as frame.

Link 2 : Crank : A link that makes complete revolutions is called as crank.

Link 3 : Connecting rod : The link opposite to the fixed link is known as connecting rod.

Link 4 : Cross-head or slider.

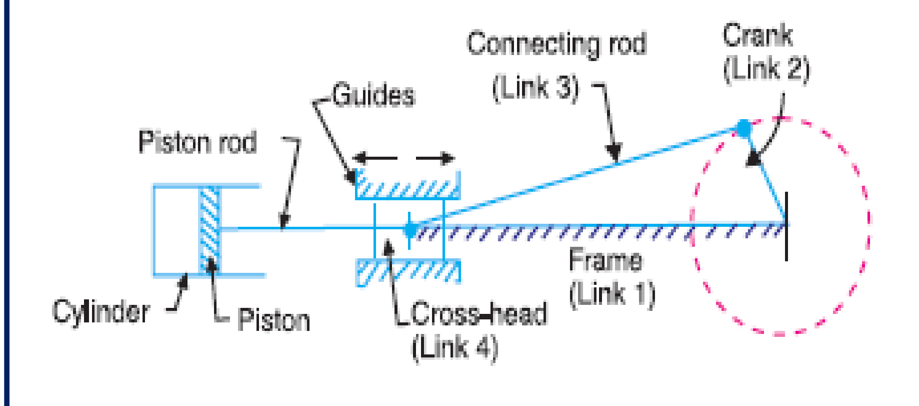




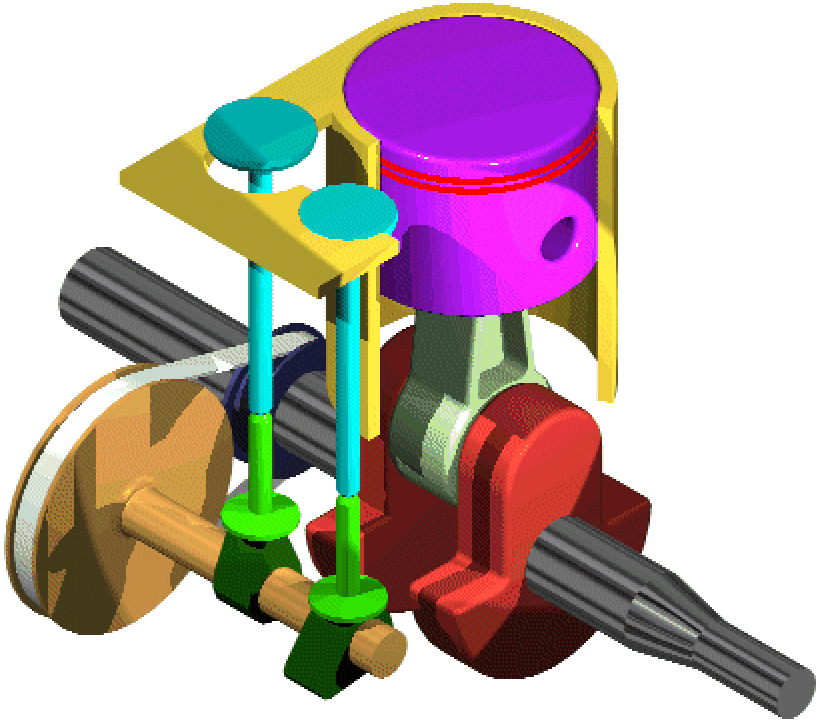
Single slider crank chain.

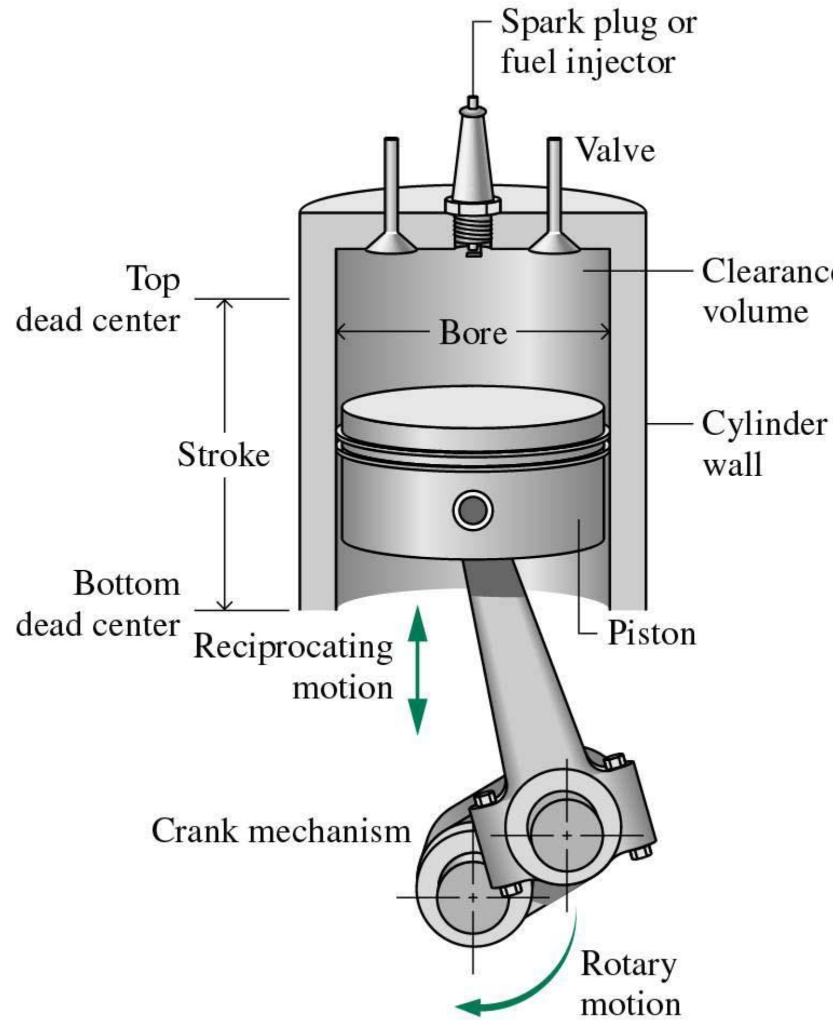
SINGLE SLIDER CRANK CHAIN

When one turning pair is replaced by one sliding pair, it is called single slider crank chain or simply a slider crank chain.
The straight line of the slder may be passing through the fixed pivot or it may be displaced.
The distance between the fixed pivot and the straight line path of the slider is called the offset.
The chain thus formed is called offset slider crank chain.
Different mechanism thus obtained by fixing the different links of the kinematic chain is called inversion.

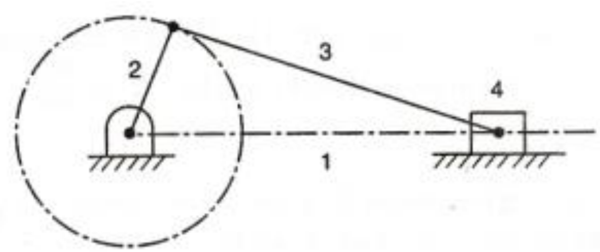


Single slider crank chain.



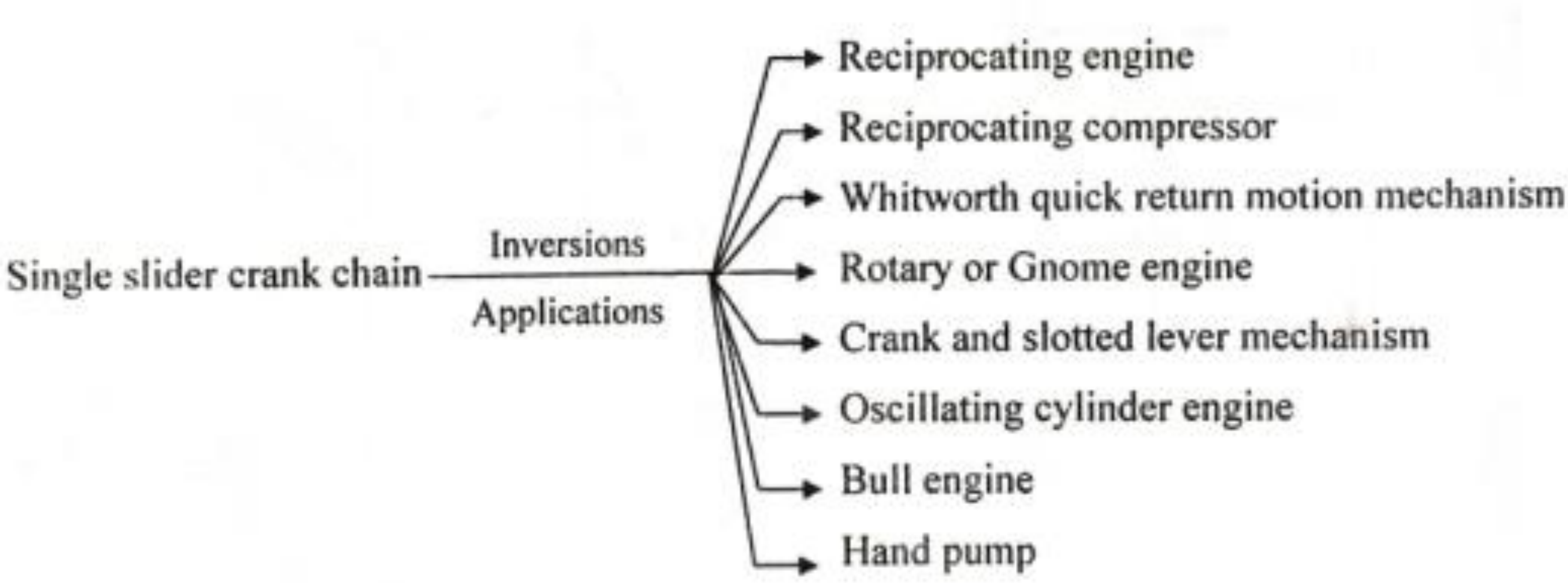


Spark plug or
fuel injector
Valve
Clearance
volume
Top
dead center
Bore
Cylinder
wall
Stroke
Bottom
dead center
Piston
Reciprocating
motion
Crank mechanism
Rotary
motion



2
3
4
1

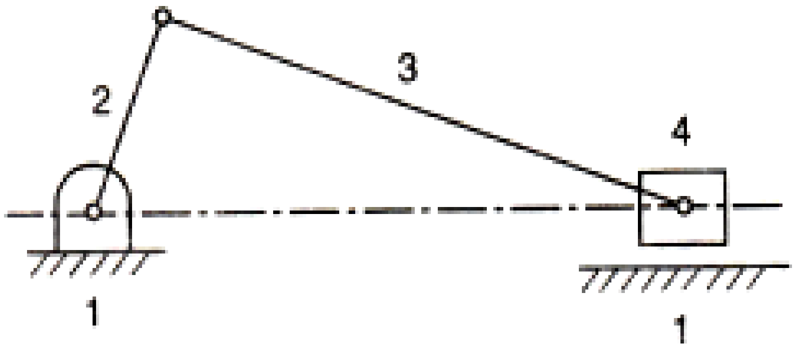
Important inversions of Slider crank chain are

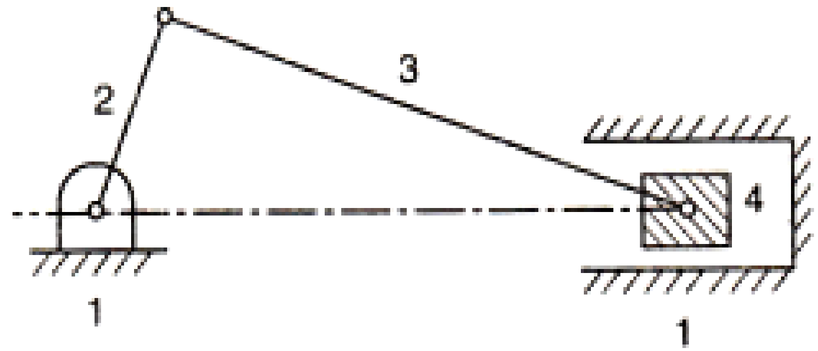


1. Pendulum pump or Bull engine
2. Oscillating cylinder engine
3. Rotary internal combustion engine.
4. Crank and slotted lever quick return mechanism
5. Whitworth quick return mechanism

.1. First Inversion :

First inversion is obtained when the link 1 (*i.e.*, frame) is fixed and links 2 and 4 are made the crank and the slider respectively, as shown in Fig. (a).





(b)

Applications : 1. Reciprocating engine

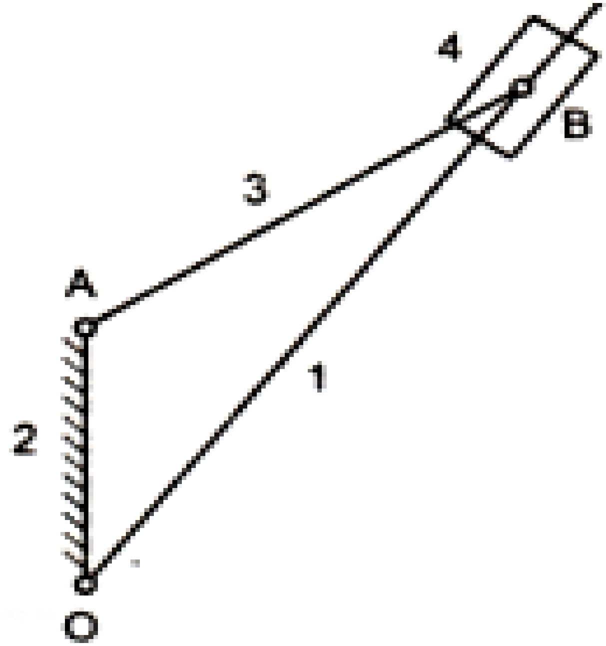
2. Reciprocating compressor

Reciprocating Engine and Reciprocating Compressor :

In both reciprocating type engine and compressor, the link 1 (*i.e.*, frame) is fixed. If it is a reciprocating engine, link 4 (*i.e.*, piston) is the driver [Refer Fig. a)] and if it is a compressor, link 2 (*i.e.*, crank) is the driver [Refer Fig. b)].

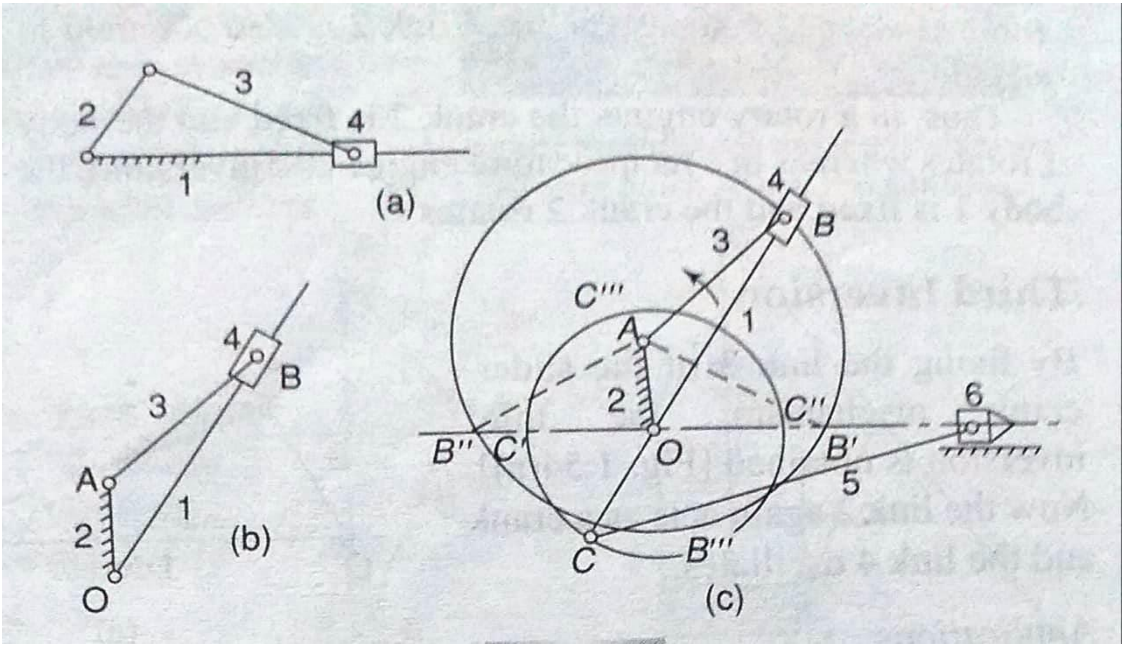
Second Inversion :

Second inversion is obtained by fixing the link 2 (*i.e.*, crank) of a slider crank chain. As shown in Fig , when the link 2 is fixed, then the link 3 along with the slider at its end B becomes a crank. This makes link 1 to rotate about O along with the slider which also reciprocates on it.



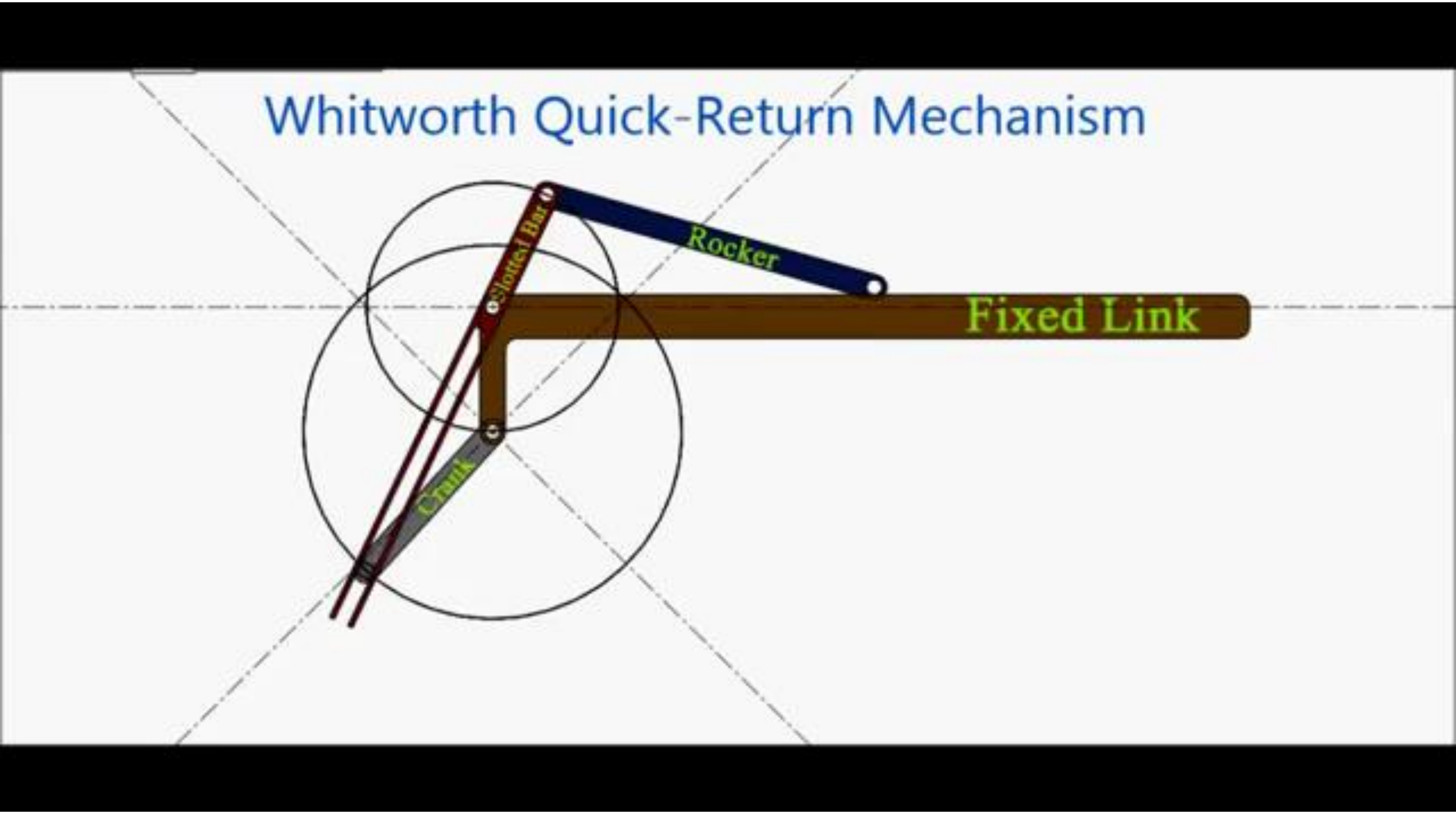
Applications : 1. Whitworth quick-return mechanism

2. Rotary engine



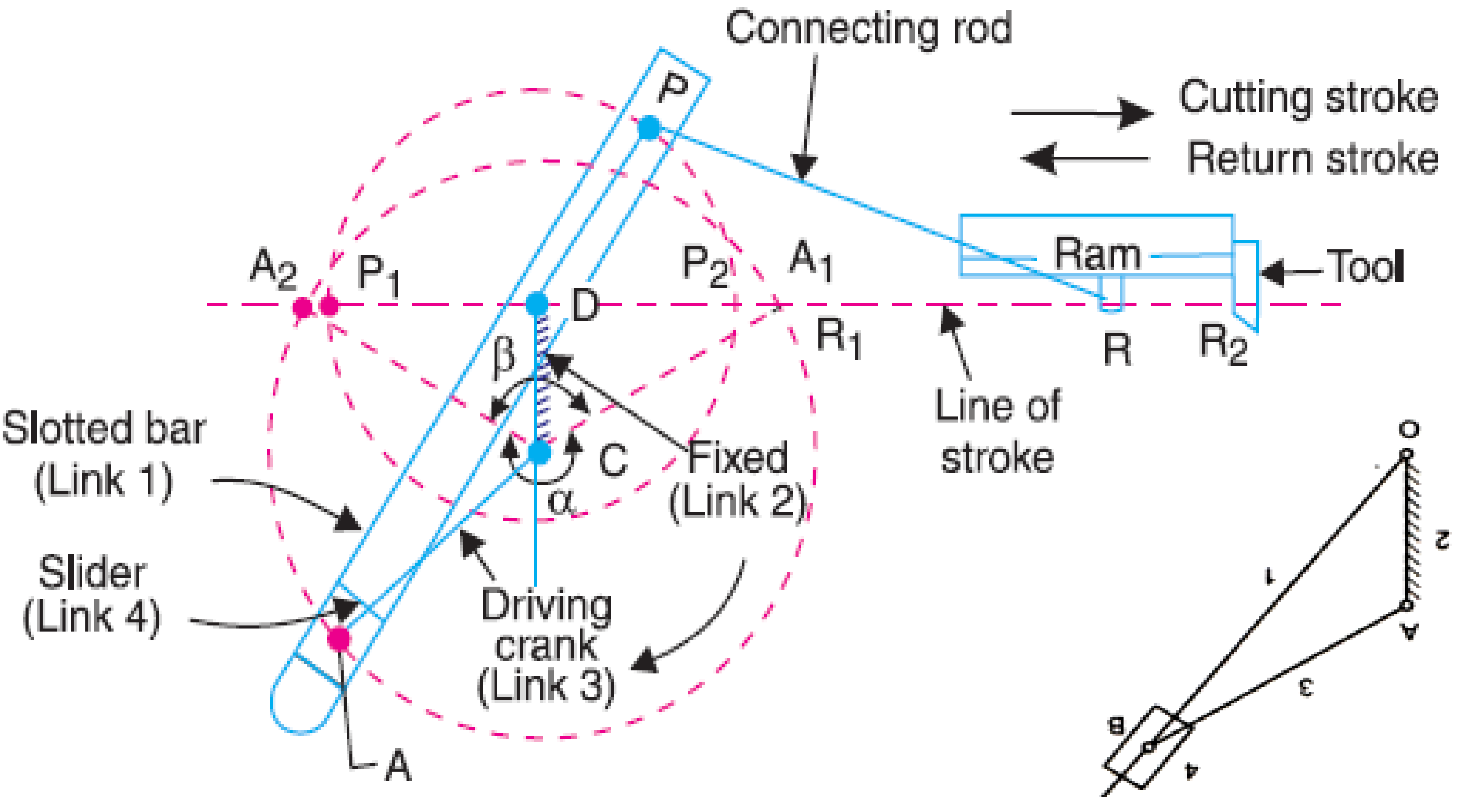
Whitworth Quick-Return Mechanism :

It is a mechanism used in workshops *i.e.*, in shaping and slotting machines to cut metals. In this mechanism, link 2 (*i.e.*, crank) is fixed, link 3 rotates, link 4 reciprocates and link 1 oscillates as shown in Fig.



Whitworth Quick-Return Mechanism :

It is a mechanism used in workshops *i.e.*, in shaping and slotting machines to cut metals. In this mechanism, link 2 (*i.e.*, crank) is fixed, link 3 rotates, link 4 reciprocates and link 1 oscillates as shown in Fig.



The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank CA (link 3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at A slides along the slotted bar PA (link 1) which oscillates at a pivoted point D. The connecting rod PR carries the ram at R to which a cutting tool is fixed. The motion of the tool is constrained along the line RD produced, *i.e.* along a line passing through D and perpendicular to CD.

When the driving crank CA moves from the position CA_1 to CA_2 (or the link DP from the position DP_1 to DP_2) through an angle α in the clockwise direction, the tool moves from the left hand end of its stroke to the right hand end through a distance 2 PD.

Now when the driving crank moves from the position CA_2 to CA_1 (or the link DP from DP_2 to DP_1) through an angle β in the clockwise direction, the tool moves back from right hand end of its stroke to the left hand end.

A little consideration will show that the time taken during the left to right movement of the ram (*i.e.* during forward or cutting stroke) will be equal to the time taken by the driving crank to move from CA_1 to CA_2. Similarly, the time taken during the right to left movement of the ram (or during the idle or return stroke) will be equal to the time taken by the driving crank to move from CA_2 to CA_1.

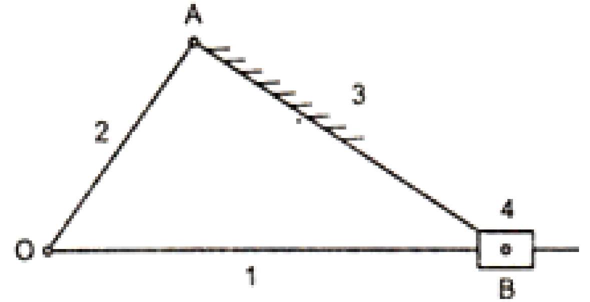
Since the crank link CA rotates at uniform angular velocity therefore time taken during the cutting stroke (or forward stroke) is more than the time taken during the return stroke. In other words, the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke. The ratio between the time taken during the cutting and return strokes is given by

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360° - \alpha} \quad \text{or} \quad \frac{360° - \beta}{\beta}$$

Note. In order to find the length of effective stroke $R_1 R_2$, mark $P_1 R_1 = P_2 R_2 = PR$. The length of effective stroke is also equal to 2 PD.

Third Inversion

Third inversion is obtained by fixing the link 3 of the slider crank mechanism, as shown in Fig. . In this, link 2 again acts as a crank and link 4 oscillates.



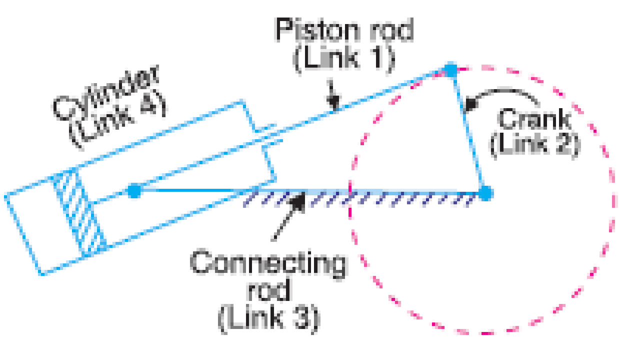
Applications : 1. Oscillating cylinder engine, and

2. Crank and slotted lever mechanism.

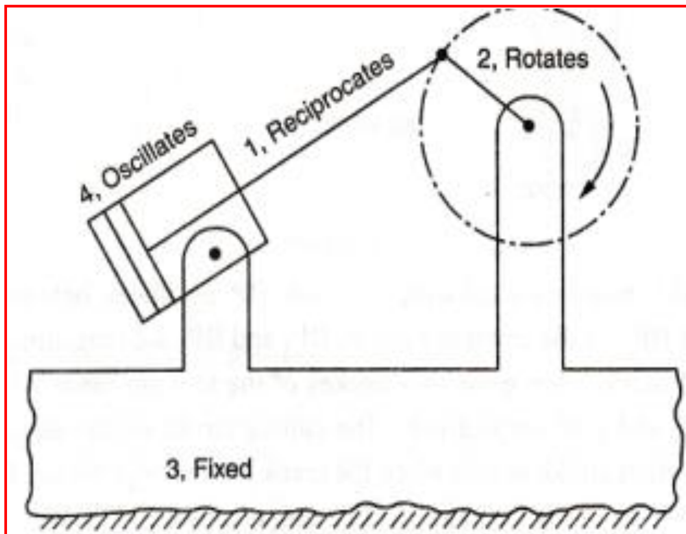
Oscillating Cylinder Engine :

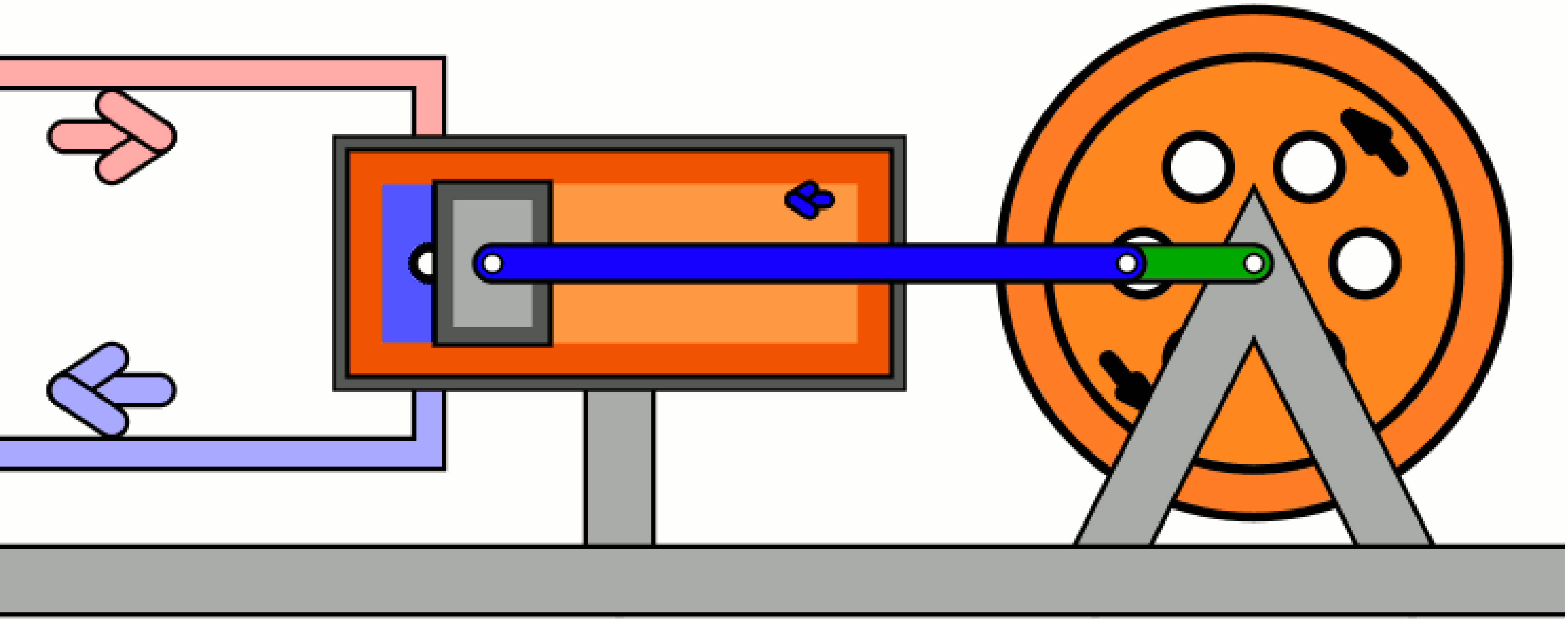
The oscillating cylinder engine mechanism is used to convert reciprocating motion into rotary motion, as shown in Fig.

In this mechanism, link 3 is fixed. When the link 2 (crank) rotates, the piston attached to link 1 (piston rod) reciprocates and the link 4 (cylinder) oscillates about a pin pivoted to the fixed link at A.



Oscillating cylinder engine.





CRANK AND SLOTTED LEVER QUICK RETURN MECHANISM

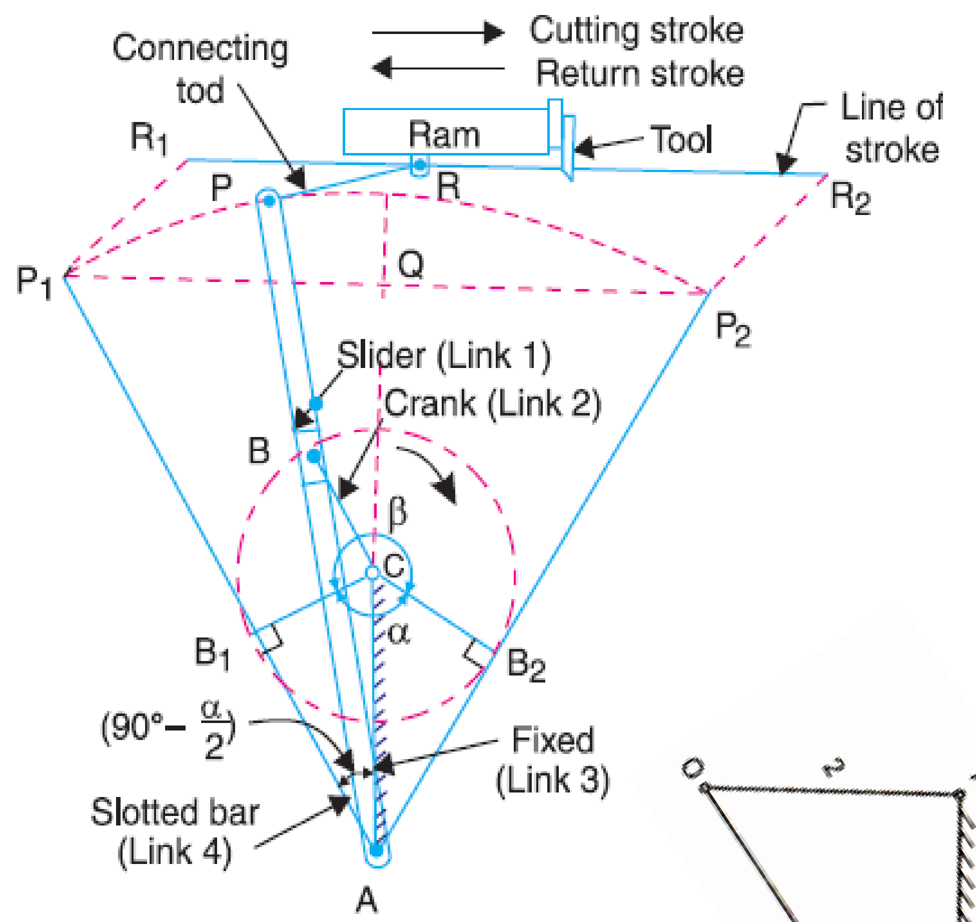
Mostly used in shaping machine.

In this mechanism ,the link AC is fixed.

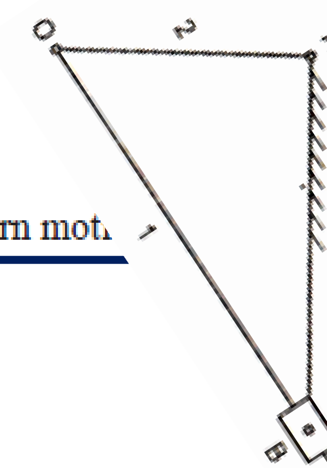
The driving crank CB revolves with uniform angular speed about the fixed centre C.

A short link PR transmits the motion from AP to the ram which carries the tool.

Sliding block attached to the crank pin at B slides along the slotter bar AP and thus cause AP to oscillates about the pivoted point A.



Crank and slotted lever quick return moti

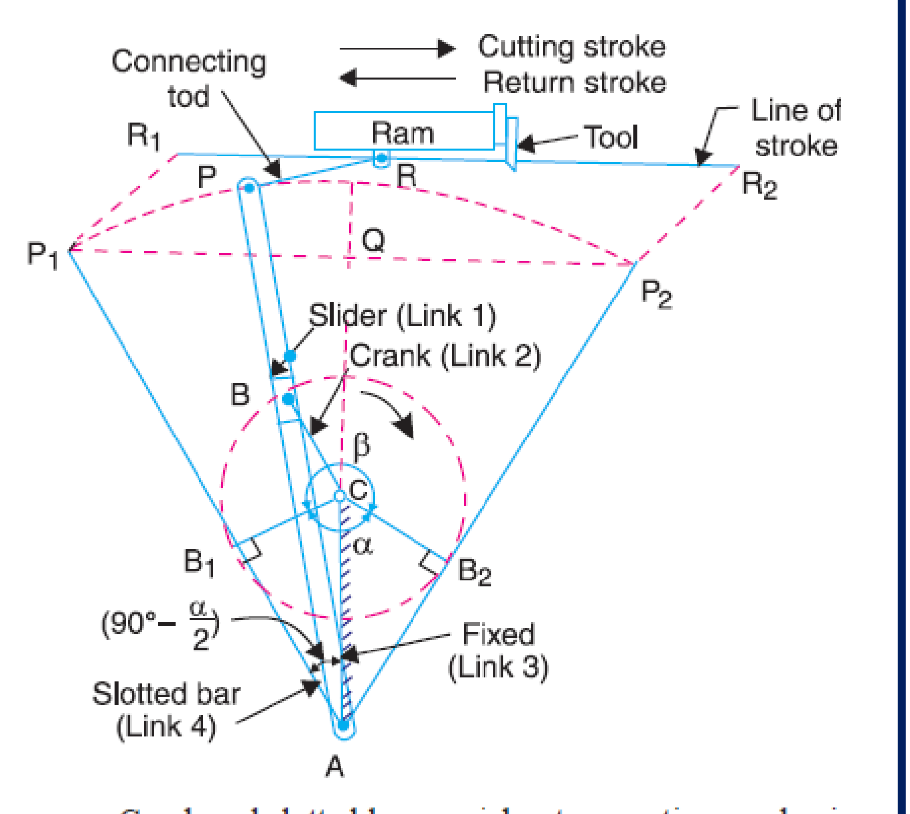


• Tool reciprocates along the line of stroke R_1R_2.

• In the extreme positions,AP1 and AP2 are tangential to the circle and the cutting tool is at the end of the stroke.

• Forward cutting stroke occurs when the crank rotates from the position CB_1 to CB_2 in the clockwise direction.

•Return stroke occures when the crank rotates from the position CB_2 to CB_1 in the clockwise direction.



Crank and slotted lever quick return motion mechanism.

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360° - \beta} \text{ or } \frac{360° - \alpha}{\alpha}$$

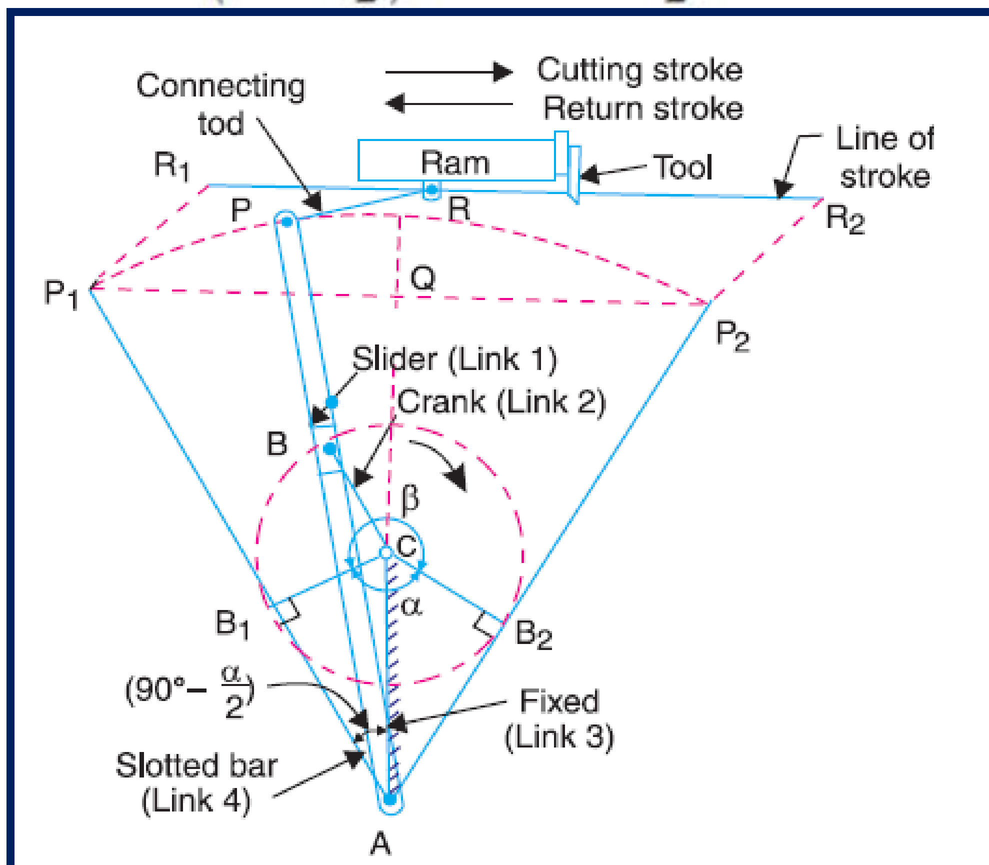
Since the tool travels a distance of $R_1 R_2$ during cutting and return stroke, therefore travel of the tool or length of stroke

$$= R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 \sin \angle P_1AQ$$

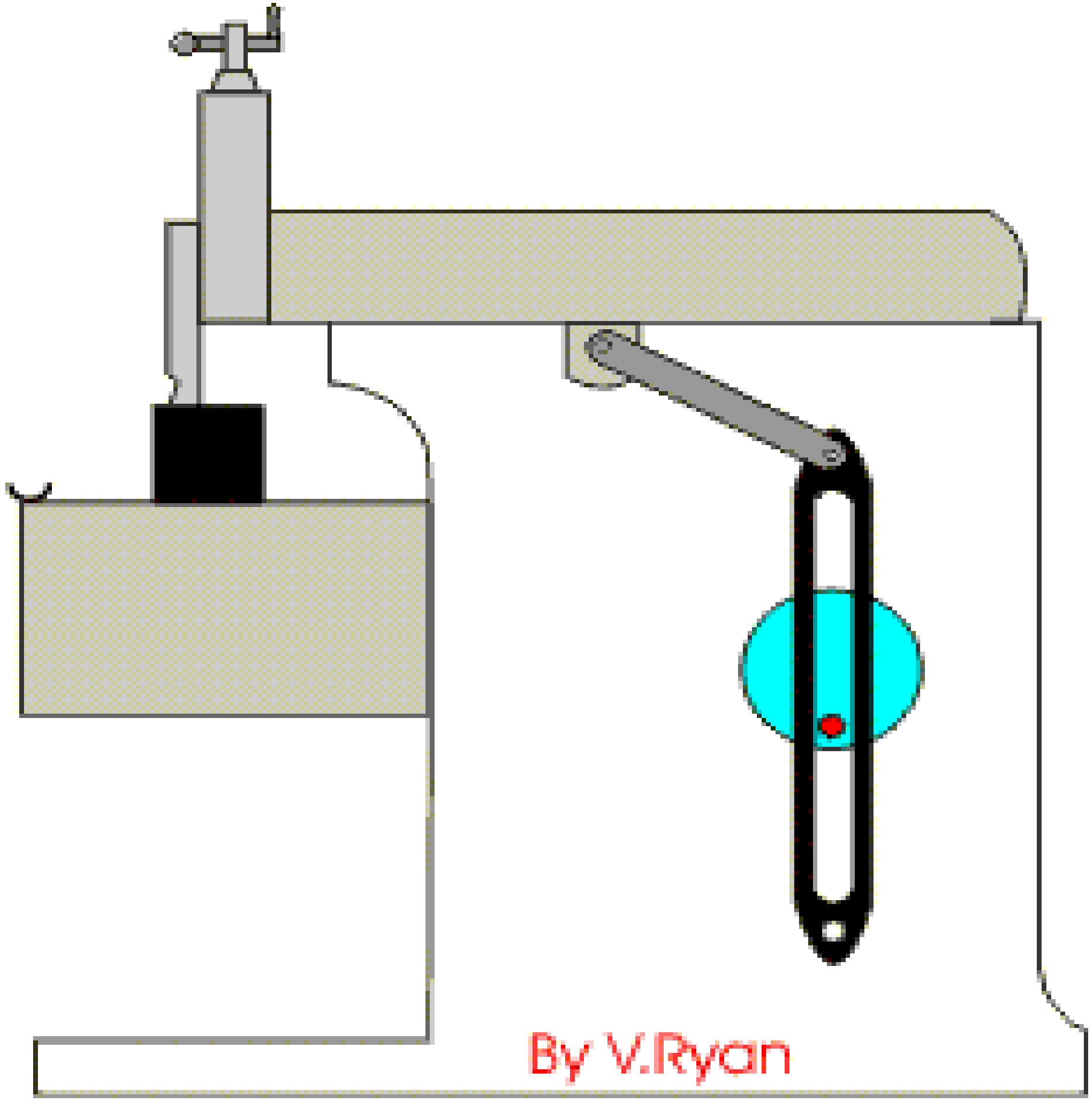
$$= 2AP_1 \sin\left(90° - \frac{\alpha}{2}\right) = 2AP \cos\frac{\alpha}{2} \qquad \text{..... } (\because AP_1 = AP)$$

$$= 2AP \times \frac{CB_1}{AC} \cdots \left(\because \cos\frac{\alpha}{2} = \frac{CB_1}{AC}\right)$$

$$= 2AP \times \frac{CB}{AC} \qquad \text{... } (\because CB_1 = CB)$$



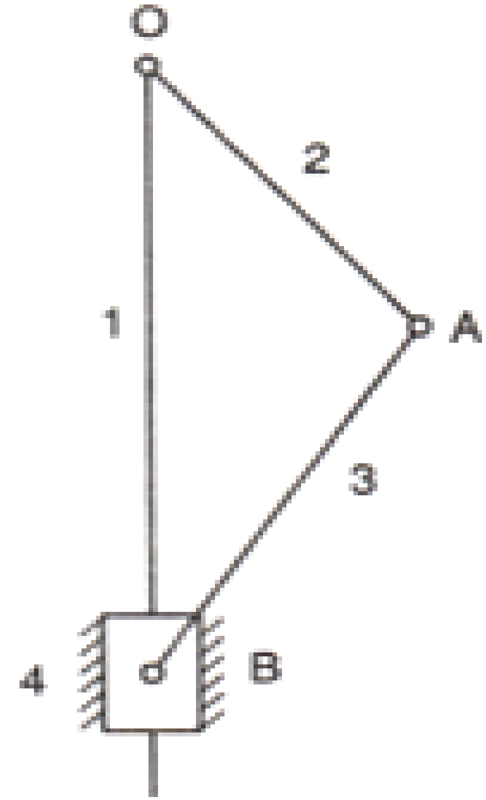
Crank and slotted lever quick return motion mechanism.



THE SHAPING MACHINE

Fourth Inversion :

Fourth inversion is obtained by fixing the link 4 of the slider crank mechanism, as shown in Fig. In this inversion, link 3 can oscillate about the fixed pivot B on link 4. This makes end A of link 2 to oscillate about B and end O to reciprocate along the axis of the fixed link 4.

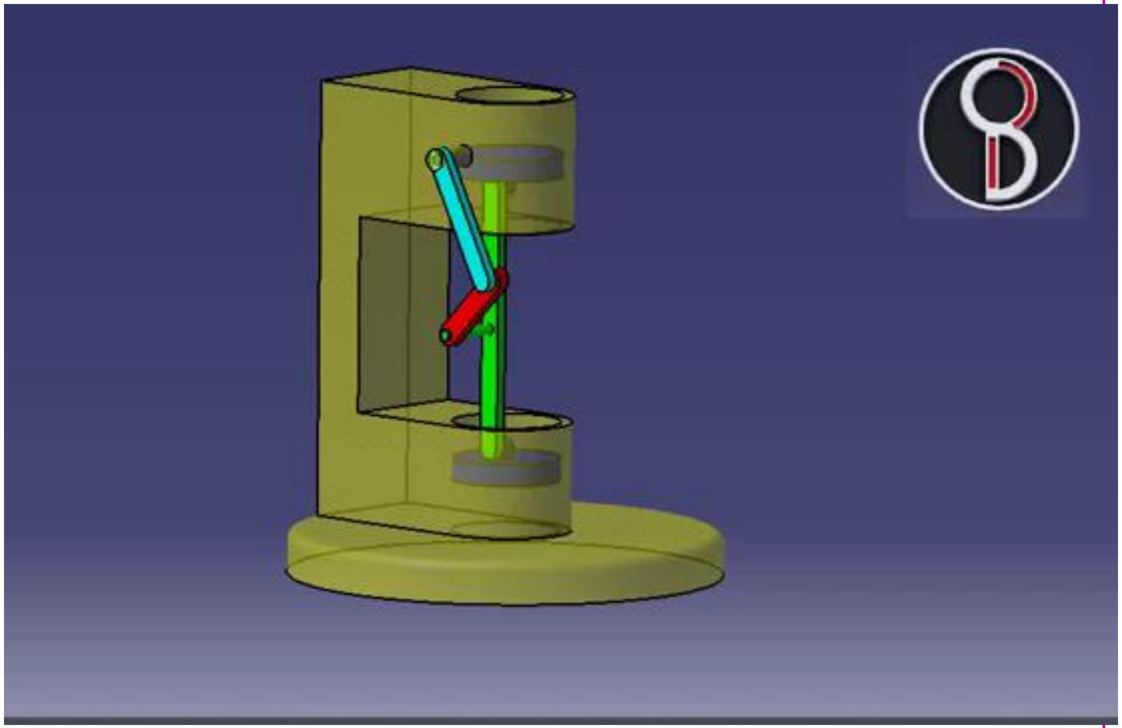


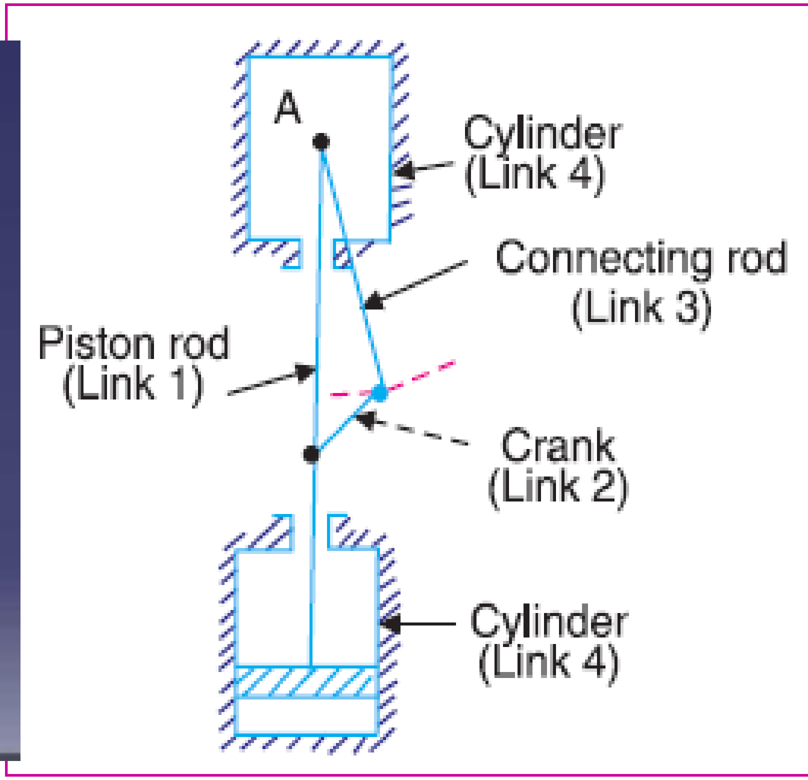
Applications : 1. Pendulum pump or Bull engine, and

2. Hand pump.

Pendulum pump or Bull engine

- It is used to supply feed water to boilers.
- Inversions is obtained by fixing the cylinder or link 4.
- When the crank rotates, connecting rod oscillates about a pin pivoted to the fixed link 4 at A and the piston attached to the piston rod(link1)reciprocates.



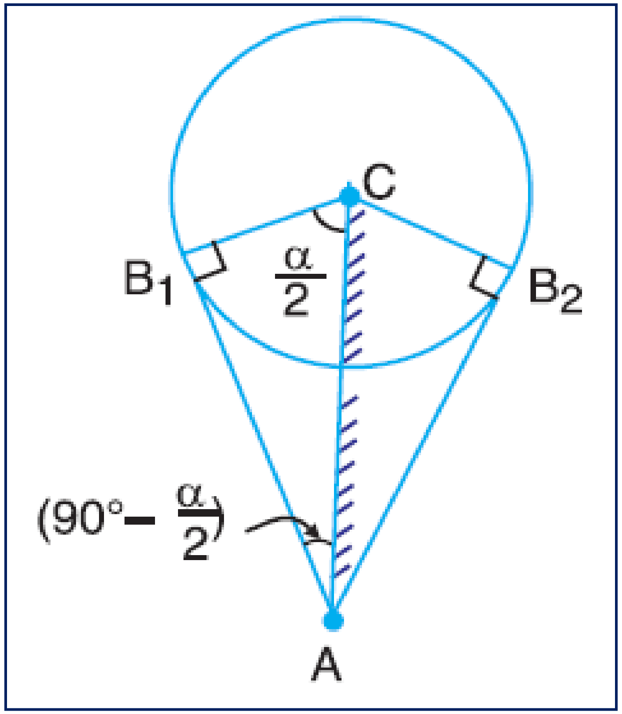


Problems :

Example .1. *A crank and slotted lever mechanism used in a shaper has a centre distance of 300 mm between the centre of oscillation of the slotted lever and the centre of rotation of the crank. The radius of the crank is 120 mm. Find the ratio of the time of cutting to the time of return stroke.*

Solution. Given : $AC = 300$ mm ; $CB_1 = 120$ mm

The extreme positions of the crank are shown in Fig. We know that



$$\sin \angle CAB_1 = \sin(90° - \alpha/2)$$

$$= \frac{CB_1}{AC} = \frac{120}{300} = 0.4$$

$$\sin \angle CAB_1 = 90° - \alpha/2$$

$$\angle CAB_1 = \sin^{-1} 0.4 = 23.6°$$

$$90° - \alpha/2 = 23.6°$$

$$\alpha/2 = 90° - 23.6° = 66.4°$$

$$\alpha = 2 \times 66.4 = 132.8°$$

We know that

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360° - \alpha}{\alpha} = \frac{360° - 132.8°}{132.8°} = 1.72$$ **Ans.**

Example .2. *In a crank and slotted lever quick return motion mechanism, the distance between the fixed centres is 240 mm and the length of the driving crank is 120 mm. Find the inclination of the slotted bar with the vertical in the extreme position and the time ratio of cutting stroke to the return stroke.*

If the length of the slotted bar is 450 mm, find the length of the stroke if the line of stroke passes through the extreme positions of the free end of the lever.

Given : $AC = 240$ mm ; $CB_1 = 120$ mm ; $AP_1 = 450$ mm

Inclination of the slotted bar with the vertical

Let $\angle CAB_1$ = Inclination of the slotted bar with the vertical.

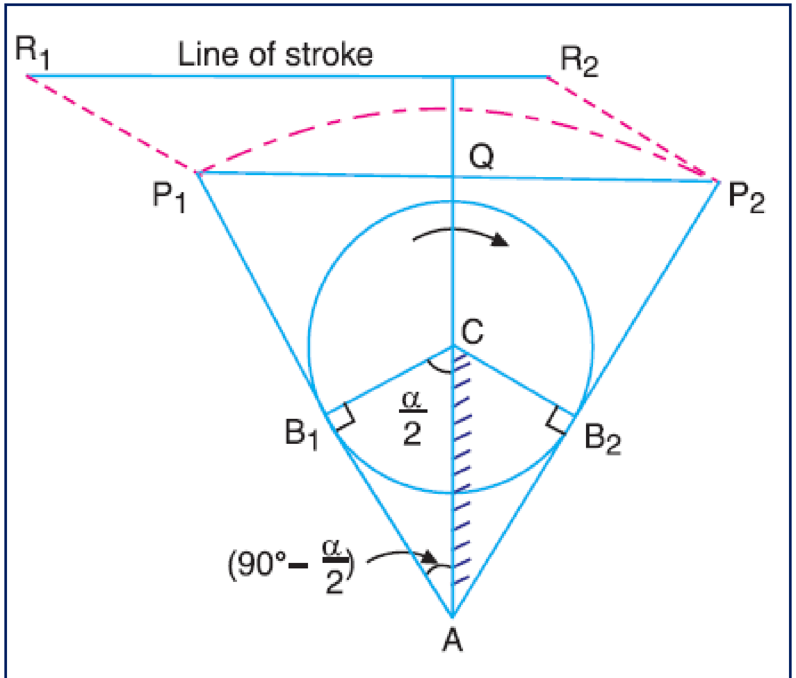
The extreme positions of the crank are

$$\sin \angle CAB_1 = \sin\left(90^\circ - \frac{\alpha}{2}\right)$$

$$= \frac{B_1C}{AC} = \frac{120}{240} = 0.5$$

$$\therefore \angle CAB_1 = 90^\circ - \frac{\alpha}{2}$$

$$= \sin^{-1} 0.5 = 30^\circ \quad \textbf{Ans.}$$



Time ratio of cutting stroke to the return stroke

We know that

$$90° - \alpha / 2 = 30°$$

$$\therefore \quad \alpha / 2 = 90° - 30° = 60°$$

or

$$\alpha = 2 \times 60° = 120°$$

$$\therefore \quad \frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360° - \alpha}{\alpha} = \frac{360° - 120°}{120°} = 2$$ **Ans.**

Length of the stroke

We know that length of the stroke,

$$R_1R_2 = P_1P_2 = 2\,P_1Q = 2\,AP_1 \sin(90° - \alpha / 2)$$

$$= 2 \times 450 \sin(90° - 60°) = 900 \times 0.5 = 450 \text{ mm}$$ **Ans.**

Example 1.14 ***In a crank and slotted lever quick return motion mechanism, the length of the fixed link is 300 mm and that of the driving crank is 150 mm. Determine the maximum angle the slotted lever will make with the fixed link. Also determine the ratio of the time of cutting and the return strokes.***

If the length of the slotted lever is 700 mm, what would be the length of the stroke, assuming that the line of the stroke passes through the extreme positions of the free end of the slotted lever ?

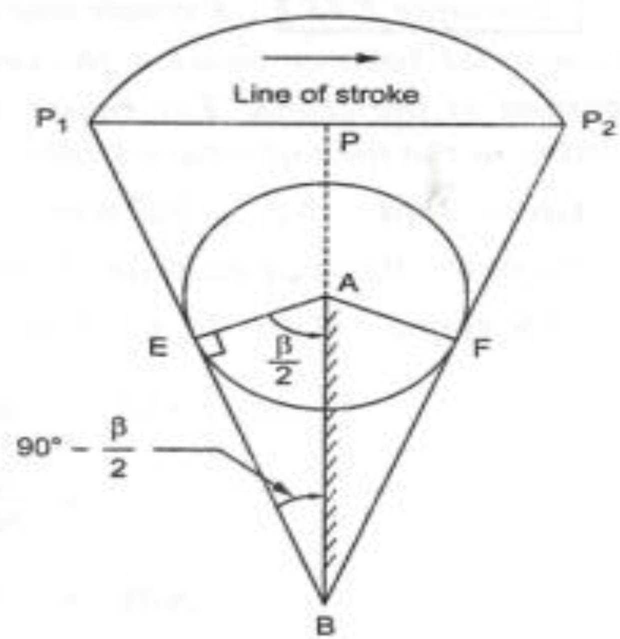
Given Data :

$$AB = 300 \text{ mm} = 0.3 \text{ m};$$
$$AE = 150 \text{ mm} = 0.15 \text{ m};$$
$$BP_1 = 700 \text{ mm} = 0.7 \text{ m}.$$

☺ ***Solution : (i) Inclination of the slotted bar with the fixed link :***

Let $\angle ABE$ = inclination of the slotted bar with the vertical

Fig. shows the extreme positions of the crank.



We know that, $\sin \angle ABE = \sin\left(90° - \frac{\beta}{2}\right)$

$$= \frac{AE}{AB}$$

$$= \frac{0.15}{0.3} = 0.5$$

$$\angle ABE = 90° - \frac{\beta}{2}$$

$$= \sin^{-1}(0.5)$$

$$\beta = 30° \textbf{ Ans.}$$

(ii) Time ratio of cutting stroke to the return stroke :

We know that
$$90° - \frac{\beta}{2} = 30°$$
$$\text{or} \quad \beta = 120°$$
$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{360° - \beta}{\beta}$$
$$= \frac{360° - 120°}{120°} = \mathbf{2}$$
Ans. ☜

(iii) Length of the stroke :

$$\text{Length of the stroke} = P_1P_2 = 2\,(P_1P)$$
$$= 2\,(BP_1)\ \sin\left(90° - \frac{\beta}{2}\right)$$
$$= 2 \times 0.7\ \sin\ (90° - 60°)$$
$$L = \mathbf{450\ mm}$$
Ans. ☜

THANK YOU