

# Accelerations in Mechanism

## Accelerations in Mechanism

If velocity at various pts known.

acceleration of " " can be found -

If Acc is known, external force  $F = m \times a$  can be calculated.

If force is known - stresses developed at various pts can be calculated.

$\therefore$  stress due to accelerations will be sometimes  $>$  stresses developed by working load.

$$a = \omega^2 r. \quad \underline{\underline{a \propto \omega^2}}$$

$$a = \omega^2 r. \quad \underline{\underline{a \propto \omega^2}}$$

If speed increases two times.

Centripetal force will become four times

$$(ie) F_c = \frac{mv^2}{r}$$

Acceleration diagrams are therefore fundamental to stress analysis of mechanism.

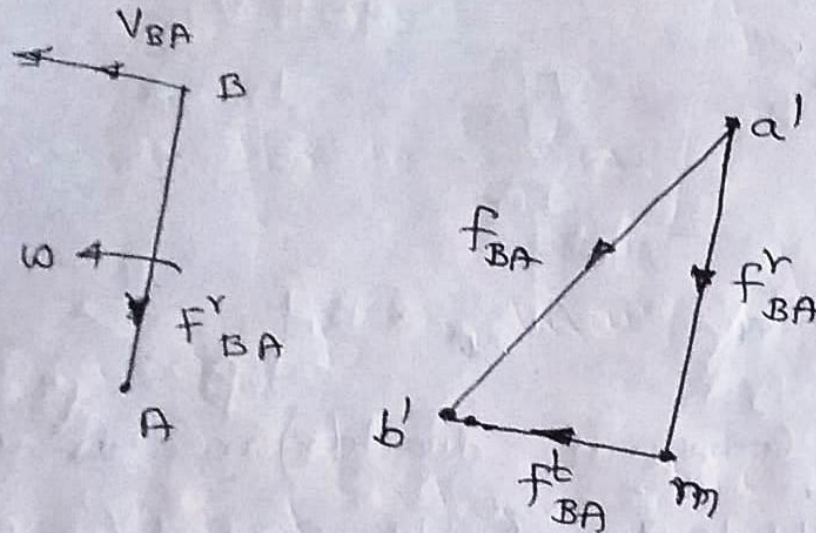
### Acceleration

rate of change of velocity.

### Acceleration diagram for a link

- 1) Analytical method.
- 2) Graphical method.

## Acceleration diagram method



AB is rigid link.

Pt-B moving w.r.t Pt-A.

To find

Acceleration of B w.r.t. A.

Find acceleration of B assuming Pt-A to be fixed.

If A is fixed, possible motion of B will be rotation about A.



Let  $\omega =$  angular velocity of link AB.

$\alpha =$  angular acceleration of " .

velocity of B changing in magnitude and direction.

acceleration of pt B will have 2 components.

1) radial component (centripetal component).

It is due to angular velocity.

acting along BA (|| to BA),

directed from B towards A.

$$\text{radial component} = \frac{V^2}{r} = \omega^2 \times r.$$

$$F_{BA}^r = \frac{V_{BA}^2}{BA}$$

$$F_{BA}^r = \omega^2 \times BA$$

2) Tangential Component due to ( $\alpha$ ) angular acceleration  
It acts  $\parallel$  to the velocity (or)  $\perp$  to AB.  
magnitude of this Component.

$$f_{BA}^t = \alpha \times BA$$

The total acceleration of B w.r.t A

$$\text{is } f_{BA} = f_{BA}^r + f_{BA}^t$$

$$= \frac{v_{BA}^2}{BA} + \alpha \times BA.$$



## Acceleration Diagram for the link A-B

- 1) Take any point  $a'$  (Here fixed pt should be taken).

radial Component of acceleration is acting along

B-A -

tangential Component  $\perp$  to B-A in direction of velocity.



From  $a'$  draw  $a'm$   $\parallel$  to  $BA$  to represent radial component of acceleration of  $B$  w.r.t.  $A$ .

$$a'm = f_{BA}^r = a_{BA}^r = \frac{V_{BA}^2}{BA}$$

2) From pt  $m$  draw  $mb'$   $\perp$  to  $AB$  to represent tangential component of acceleration of  $B$  w.r.t.  $A$ .

$$mb' = f_{BA}^t = a_{BA}^t = \alpha BA$$

3) Join  $b|a|$

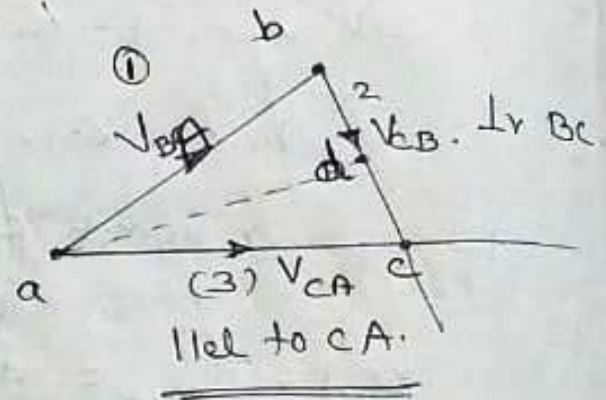
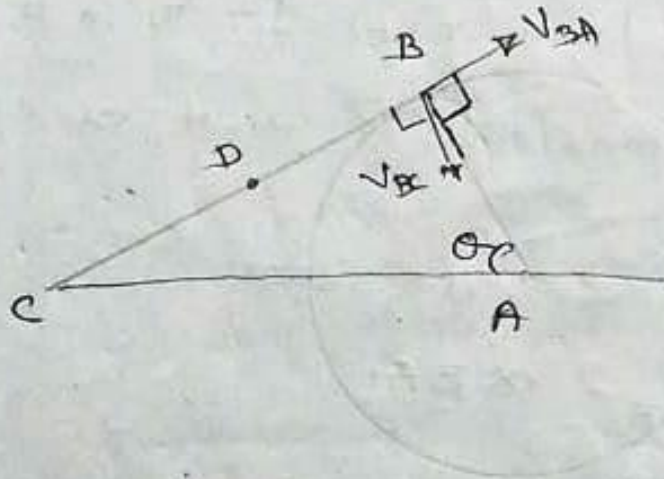
vector ~~the~~  $a|b|$  represents total acceleration  
of B w.r.t - A.

Note

- \* small letters with a prime is used for acceleration diagrams
- \* Lines from the common fixed points in acceleration diagram represents absolute accelerations.
- \* Abs acceleration  $\rightarrow$  sum of  $a_r$  and  $a_t$ .

# Acceleration diagram for a slider crank mechanism

~~ab~~  $\parallel V_{BA}$   
 $\perp r_{BA}$



$$\frac{bd}{bc} =$$

$$V_{BA} = V_B = \text{velocity of B w.r.t. A. } V_{BA} = \overline{ab}$$

$$V_{CA} = V_C = \text{ " " C w.r.t. A } V_{CA} = V_C = \overline{ac}$$

$$V_{CB} = \overline{bc} = \text{velocity of C w.r.t. B.}$$



first draw velocity diagram

take pt a as fixed point. ~~for~~

draw  $\overline{ab}$   $\parallel$  to velocity  $V_{BA}$ . to a

suitable scale  $\nabla$

$$\boxed{V_{BA} = \omega \times BA}$$

$V_{BC} \perp$  line BC.

draw vector  $\overline{bc}$   $\parallel$  to  $V_{BC}$  (Infinite line)

draw line from a such that it intersects  $\overline{bc}$  at a pt c



To find ~~any~~ velocity at any pt D,

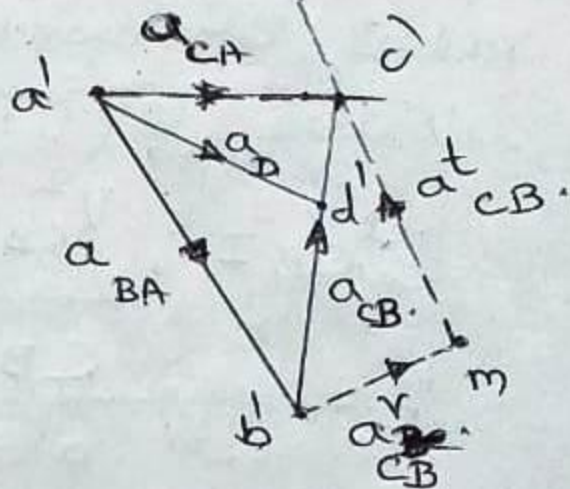
divide vector bc such that,

$$\frac{bd}{bc} = \frac{BD}{BC}$$

Join. ad  
pts a, d

measure ad convert it to suitable scale,

to find velocity at any point.



To find acceleration diagram

Crank BA has radial component.

$$f_{BA}^r = a_{BA}^r = \frac{V_{BA}^2}{BA}$$

Since it is rotating at constant angular velocity  $\omega$   
angular acceleration is  $\alpha_{BA} = 0$ .

$$\therefore f_{BA}^t = a_{BA}^t = 0.$$

$$\begin{aligned} \text{resultant acceleration } a_{BA} &= f_{BA} = a_{BA}^r + a_{BA}^t \\ &= a_{BA}^r = \frac{V_{BA}^2}{BA} \end{aligned}$$

$a_{BA}$  lies ~~in~~ acting along B.A.

[ draw  $a'b'$  lkl to B.A. equal in magnitude  
of  $a_{BA}$  to some suitable scale

For link CB (connecting rod)

radial component of acceleration of C w.r.t B.

$$a_{CB}^r = \frac{V_{CB}^2}{CB}$$

It acts along CB.

tangential component of C w.r.t B.  $\perp$  CB.

$$a_{CB}^t = \alpha_{CB} CB$$

used to find angular acceleration of ~~CB~~.

C with respect to B.

$$a_{CB} = a_{CB}^r + a_{CB}^t$$

$$f_{CB} = f_{CB}^r + f_{CB}^t$$



$V_{CB}$  obtained from velocity diagram.

from this

$a_{CB}^r$  can be found.

Now for acceleration diagram.



for  $a_{CB}^r$  with suitable scale draw

vector  $b'm$  || to  $CB$



$a_{CB}^r$  and  $a_{CB}^t$  are ⊥ r.

From m draw a line ⊥ r  $a_{CB}^r$  (or)  $b'm$ .

In order to find resultant acceleration  $a_{CB}$ . (or)

$b'c'$

for line CA

At C moving in straight line w.r.t. line CA.

∴ radial component is zero.

But tangential component exists. It is in direction of C w.r.t. A.

But C moves ~~along~~ CA w.r.t. A.

∴ from  $a'$  draw a line  $\parallel$  to  $CA$ .

It intersects  $a_{CB}^t$  at  $c'$ .

Join.  $a'c'$   
 $b'c'$

from  $a_{CB}^t$  } angular acceleration.  $\alpha_{CB}$  can  
from diagram } be calculated

$$a_{CB}^t = \alpha_{CB} \times C_B$$

acceleration  $a_{CB}$  can be measured from scale.

acceleration  $a_{CA}$  " " " "

To find acceleration at any pt D.

$$\frac{b'd'}{b'c'} = \frac{BD}{BC}$$

mark  $d'$

connect  $a'd'$ .

means to line  $a'd'$  to get the acceleration at pt D on connecting rod.



### Problem

The crank of a slider crank mechanism is 15 cm and the connecting rod is 60 cm long. The crank makes 300 rpm in the clockwise direction; when it has turned  $45^\circ$  from the inner dead centre position determine

(i) Acceleration of the mid point of the connecting rod -

2) angular acceleration of C.R.

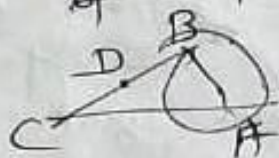
data

$$r = AB = 15 \text{ cm}$$

$$l = BC = 60 \text{ cm.}$$

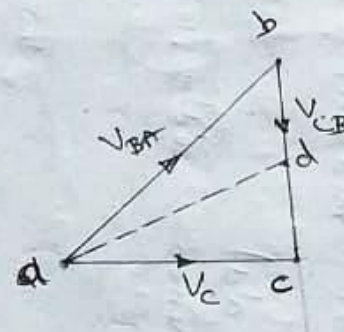
$$N = 300 \text{ rpm.}$$

$$\theta = 45^\circ \quad \text{Find.}$$

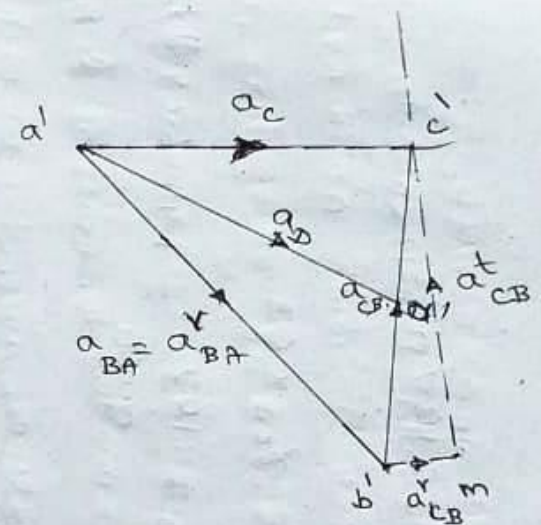


acceleration at mid pt D  
angular acceleration of BC





velocity diagram



$$\omega_{BA} = \frac{2\pi N}{60} = 31.4 \text{ rad/sec}$$

$$V_{BA} = \omega_{BA} \times BA$$

$$= 0.4712 \text{ m/sec.}$$

Take scale  $1 \text{ cm} = 0.1 \text{ m/sec}$

from velocity diagram.

$$V_C = \overline{ac} = 0.4 \text{ m/s}$$

$$V_{CB} = \overline{bc} = 0.34 \text{ m/s}$$

For acceleration diagram

crank No acceleration  $a_{BA}^t = 0$

$$a_{BA}^r = \frac{V_B^2}{BA} = \frac{0.4712^2}{0.015} = 14.8 \text{ m/s}^2$$

For c.R.

$$a_{CB}^r = \frac{V_{CB}^2}{CB} = \frac{0.34^2}{0.06} = 1.93 \text{ m/s}^2$$

we need to find  $a_{CB}^t$   $a_{CB}^t \perp a_{CB}^r$

By measurement  $a_{CB}^t = \frac{10.3}{11.7} \text{ m/s}^2$

$$\therefore a_{CB}^t = \alpha_{CB} \times CB$$

angular acceleration,  $\alpha_{CB} = \frac{10.3}{0.060} = \underline{\underline{171.67 \text{ rad/s}^2}}$

To find acceleration at midpoint D of C-R.

draw vector  $a'd'$

by measurement  $a'd' = \underline{\underline{11.7 \text{ m/s}^2}}$

# Assignment and Self Practice

## Assignment

The crank of a slider crank Engine 200 mm long and connecting rod length to crank radius is 4. The crank has turned through  $45^\circ$  from inner dead centre position. The instantaneous speed of rotation of the crank is 240 rpm - clockwise and it is increasing at the rate of  $100 \text{ rad/s}^2$ .

Determine.

- 1) acceleration of the midpoint of C.R.
- 2) angular acceleration of connecting rod,
- 3) acceleration of the slider.



Ans

Here crank will have tangential component and radial component both.

Here it doesn't rotate speed.

~~crank~~ crank accelerates Hence  $a^t \neq 0$ .

$$\therefore a_{\text{crank}} = a^t_{\text{crank}} + a^r_{\text{crank}}$$

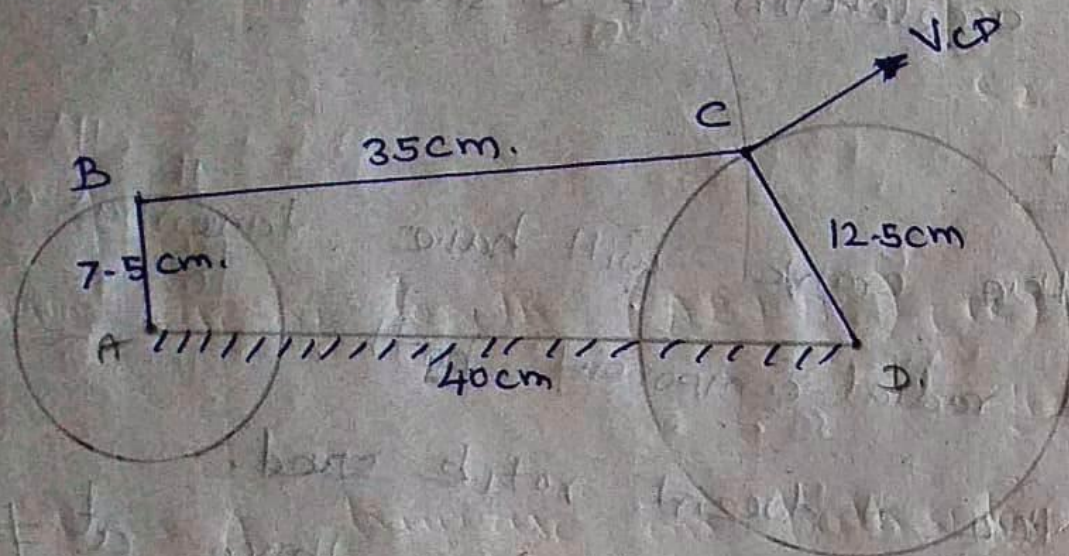
$$a^t = 100 \text{ rad/s}^2 \quad a^r = \frac{V_{\text{crank}}}{\text{crank length. (or) radius}}$$

$\therefore$  extra tang & radial component comes.

A four bar kinematic chain is represented by a quadrilateral ABCD in which AD is fixed is 400 mm long. The crank AB 75 mm long rotates in clockwise direction at 120 rpm and drives the link CD 125 mm long by means of link BC 350 mm long. 1) Determine angle through which C.D oscillates  
2) find angular velocities of the link B.C and C.D in one of the position when B.C is AB.

Scale 1 cm = 50 mm.





find 1) angle through which C.D oscillates.

2)  $\omega_{BC}$ ,  $\omega_{CD}$  when  $BC \perp AB$ .

soln

data

$N_{AB} =$

120 rpm.

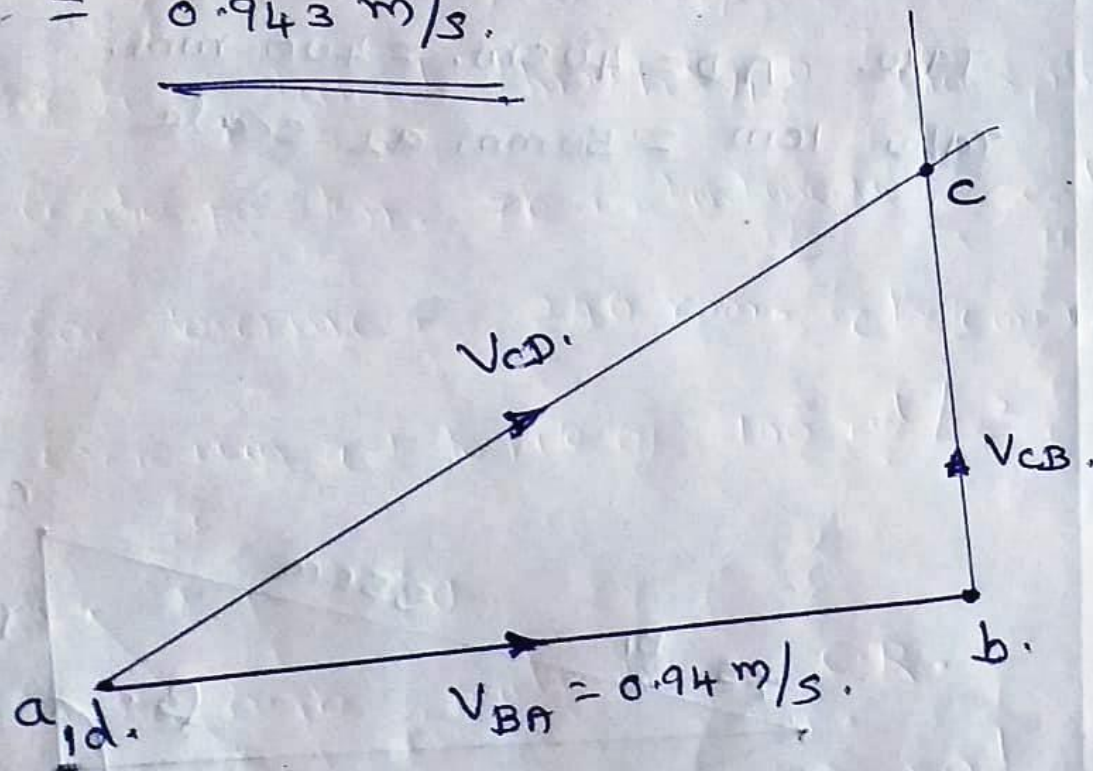
$$\omega_{AB} = \frac{2\pi N_{AB}}{60} = 12.57 \text{ rad/s}$$



$$V_{BA} = \omega_{BA} \times BA$$

$$= 12.57 \times 7.5 \text{ cm}$$

$$= 0.943 \text{ m/s.}$$



IF  $BC \perp AB$ .

$V_{BA}$  pass through line  $BC$ .

$V_{CB} \perp$  to line  $CB$ .

$\parallel$  to  $AB$ .

By measurement  $V_{CB} = 0.3675 \text{ m/s}$ .

$V_{CD} = 1.008 \text{ m/s}$ .

$$\omega_{CB} = \frac{V_{CB}}{CB} = \frac{0.3675}{0.35} = 1.05 \text{ rad/s.}$$

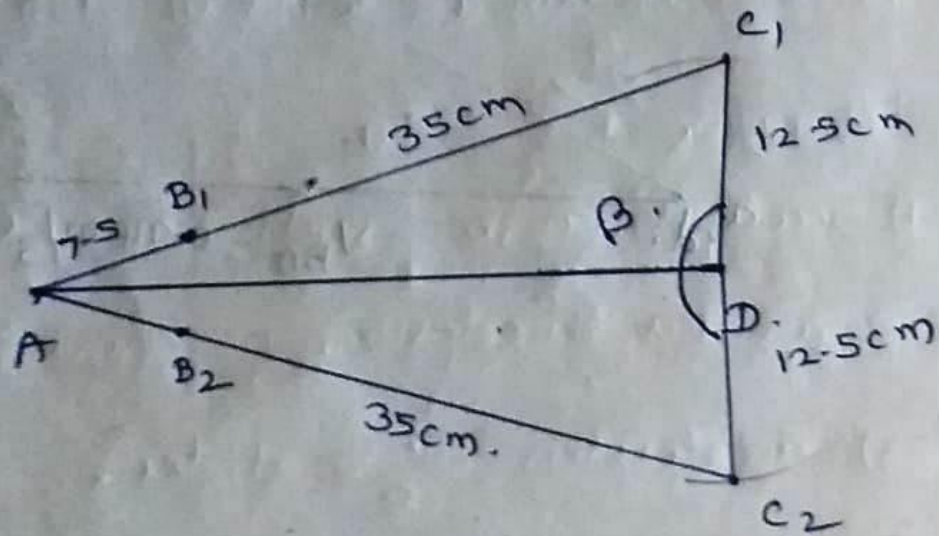
$$\omega_{CD} = \frac{V_{CD}}{CD} = \frac{1.008}{0.125} = 8.06 \text{ rad/s.}$$

(ii) Angle through which C.D oscillates.

Let  $\beta$  angle through which C.D oscillates.

Take  $AD = 40 \text{ cm.} = 400 \text{ mm.}$

Take  $1 \text{ cm} = 50 \text{ mm}$  as scale





Take arc of length  $(AB+BC)$ . (i.e.)  $(7.5+35)$   
42.5 cm.

From A draw arc on both sides

From D draw arc having radius 12.5 cm

measure angle  $\beta$   $\angle C_1DC_2$

by measurement ~~186~~  $\beta = 186^\circ$