Design of Shaft

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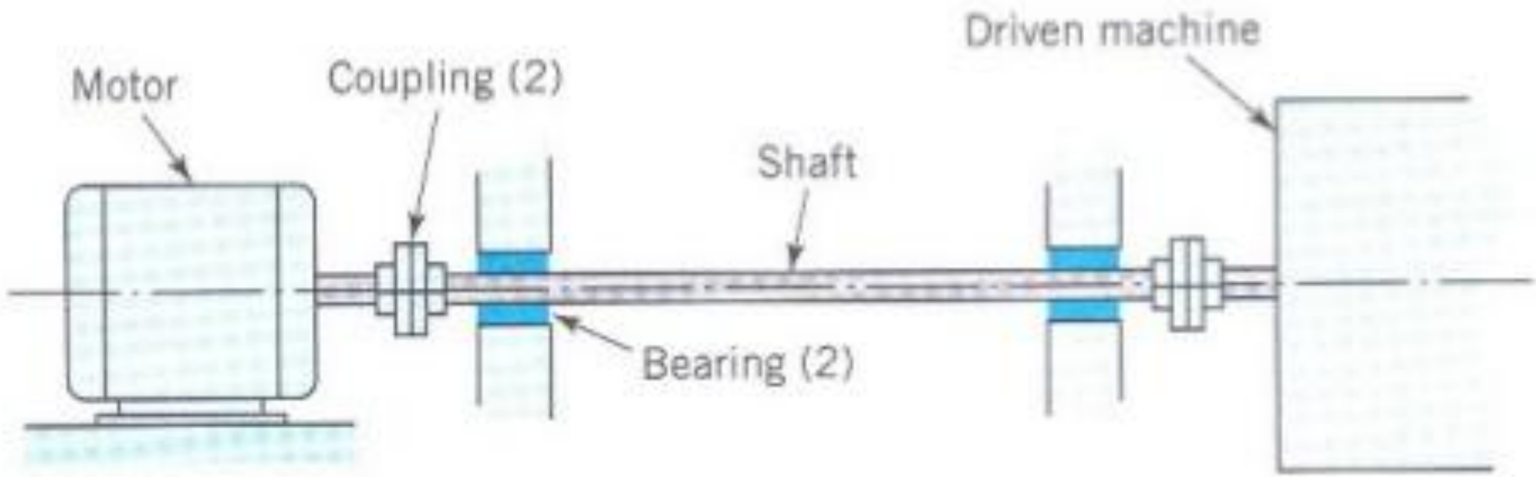
- A shaft is a rotating member usually of circular cross-section (solid or hollow), which transmits power and rotational motion.
- Machine elements such as gears, pulleys (sheaves), flywheels, clutches, and sprockets are mounted on the shaft and are used to transmit power from the driving device (motor or engine) through a machine.
- Press fit, keys, dowel, pins and splines are used to attach these machine elements on the shaft.
- The shaft rotates on rolling contact bearings or bush bearings.
- Various types of retaining rings, thrust bearings, grooves and steps in the shaft are used to take up axial loads and locate the rotating elements.
- Couplings are used to transmit power from drive shaft (e.g., motor) to the driven shaft (e.g. gearbox, wheels).

Talking Points

- Shaft?
- Shaft Design
- ASME Shaft Equations
- Design of Shaft for Torsional Rigidity
- Standard Sizes of Shafts
- Bending and Torsional Moments

The connecting shaft is loaded primarily in torsion.



(*a*) Connecting shaft

Introduction

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft.

In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending.

In other words, we may say that ***a shaft is used for the transmission of torque and bending moment***. The various members are mounted on the shaft by means of keys or splines.

Notes:

1. The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used.

2. An *axle*, though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave.

3. A *spindle* is a short shaft that imparts motion either to a cutting tool (e.g. drill press spindles) or to a work piece (*e.g. lathe spindles)*.

Material Used for Shafts

The material used for shafts should have the following properties :

1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12.

The mechanical properties of these grades of carbon steel are given in the following table.

Table 14.1. Mechanical properties of steels used for shafts.

Indian standard designation	*Ultimate tensile strength, MPa*	*Yield strength, MPa*
40 C 8	560 - 670	320
45 C 8	610 - 700	350
50 C 4	640 - 760	370
50 C 12	700 Min.	390

When a shaft of high strength is required, then an alloy steel such as nickel, nickel-chromium or

Manufacturing of Shafts

Shafts are generally manufactured by hot rolling and finished to size by cold drawing or turning and grinding. The cold rolled shafts are stronger than hot rolled shafts but with higher residual stresses.

The residual stresses may cause distortion of the shaft when it is machined, especially when slots or keyways are cut. Shafts of larger diameter are usually forged and turned to size in a lathe.

Types of Shafts

The following two types of shafts are important from the subject point of view :

1. ***Transmission shafts.*** These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2. ***Machine shafts.*** These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

Standard Sizes of Transmission Shafts

The standard sizes of transmission shafts are :
25 mm to 60 mm with 5 mm steps; 60 mm to 110 mm with 10 mm steps ; 110 mm to 140 mm with 15 mm steps ; and 140 mm to 500 mm with 20 mm steps.
The standard length of the shafts are 5 m, 6 m and 7 m.

Stresses in Shafts

The following stresses are induced in the shafts :

1. Shear stresses due to the transmission of torque (*i.e. due to torsional load*).
2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.
3. Stresses due to combined torsional and bending loads.

Maximum Permissible Working Stresses for Transmission Shafts

According to American Society of Mechanical Engineers (ASME) code for the design of transmission shafts, the maximum permissible working stresses in tension or compression may be taken as

(a) 112 MPa for shafts without allowance for keyways.

(b) 84 MPa for shafts with allowance for keyways.

For shafts purchased under definite physical specifications, the permissible tensile stress (σt) may be taken as 60 percent of the elastic limit in tension (σel), *but not more than 36 per cent of the* ultimate tensile strength (σu). *In other words, the permissible tensile stress,*

$$\sigma t = 0.6\ \sigma el \text{ or } 0.36\ \sigma u, \text{ whichever is less.}$$

The maximum permissible shear stress may be taken as

(a) 56 MPa for shafts without allowance for key ways.

(b) 42 MPa for shafts with allowance for keyways.

For shafts purchased under definite physical specifications, the permissible shear stress (τ) may be taken as 30 per cent of the elastic limit in tension (σel) *but not more than 18 percent of the ultimate* tensile strength (σu). *In other words, the permissible shear stress,*

$$\tau = 0.3\ \sigma el \text{ or } 0.18\ \sigma u, \text{ whichever is less.}$$

Design of Shafts

The shafts may be designed on the basis of

1. Strength, and **2. Rigidity and stiffness.**

In designing shafts on the basis of strength, the following cases may be considered :

(*a*) ***Shafts subjected to twisting moment or torque only,***
(*b*) ***Shafts subjected to bending moment only,***
(*c*) ***Shafts subjected to combined twisting and bending moments, and***
(*d*) ***Shafts subjected to axial loads in addition to combined torsional and bending loads.***

We shall now discuss the above cases, in detail, in the following pages.

Shafts Subjected to Twisting Moment Only

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that

$$\frac{T}{J} = \frac{\tau}{r} \qquad \ldots(i)$$

where

T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation,

τ = Torsional shear stress, and

r = Distance from neutral axis to the outer most fibre

= d / 2; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation *(i) may now be written as*

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3 \qquad \ldots(ii)$$

From this equation, we may determine the diameter of round solid shaft (*d*).
We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

where *do and di = Outside and inside diameter of the shaft, and r = do / 2.*
Substituting these values in equation *(i), we have*

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau\left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] \qquad \ldots(iii)$$

Let *k = Ratio of inside diameter and outside diameter of the shaft*
= di / do

Now the equation ***(iii) may be written as***

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau \, (d_o)^3 \, (1 - k^4) \qquad \ldots(iv)$$

From the equations ***(iii)* or *(iv)*, the outside and inside diameter of a hollow shaft may be** determined.

It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous thanwould be possible for a solid shaft.

When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \quad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3 \, (1 - k^4) = d^3$$

2. The twisting moment (*T*) *may be obtained by using the following relation :*
We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

where T = *Twisting moment in N-m, and*
N = *Speed of the shaft in r.p.m.*

3. In case of belt drives, the twisting moment (*T*) *is given by*

$$T = (T1 - T2)\, R$$

where
T1 and T2 = Tensions in the tight side and slack side of the belt respectively, and
R = Radius of the pulley.

Example 1

A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.

Solution.

Given : N = 200 r.p.m. ; P = 20 kW = 20 × 103 W; τ = 42 MPa = 42 N/mm2

Let d = *Diameter of the shaft.*

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

$$\therefore \quad d^3 = 955 \times 10^3 / 8.25 = 115\,733 \text{ or } d = 48.7 \text{ say } 50 \text{ mm}$$ **Ans.**

Example 2.

A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the *shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximu allowable shear stress as 60 MPa.*

Solution.

Given : *P = 1 MW = 1 × 106 W ; N = 240 r.p.m. ; Tmax = 1.2 Tmean ;*

τ = 60 MPa = 60 N/mm²

Let $\quad d$ = *Diameter of the shaft.*

We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{1 \times 10^6 \times 60}{2\pi \times 240} = 39\,784 \text{ N-m} = 39\,784 \times 10^3 \text{ N-mm}$$

Maximum torque transmitted,

$T_{max} = 1.2\ T_{mean} = 1.2 \times 39\,784 \times 10^3 = 47\,741 \times 10^3$ N-mm

We know that maximum torque transmitted (*Tmax*),

$$47\,741 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\, d^3$$

$$d^3 = 47\,741 \times 10^3 / 11.78 = 4053 \times 10^3$$

$$d = 159.4 \text{ say } 160 \text{ mm}$$ Ans.

Example 3.

Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8. If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

Solution. Given : $P = 20\ kW = 20 \times 10^3\ W$; $N = 200$ r.p.m. ; $\tau u = 360\ MPa = 360\ N/mm^2$; $F.S. = 8$; $k = di / do = 0.5$

We know that the allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{360}{8} = 45 \text{ N/mm}^2$$

Diameter of the solid shaft

Let d = *Diameter of the solid shaft.*

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\, d^3$$

$$d^3 = 955 \times 10^3 / 8.84 = 108\,032 \quad \text{or} \quad d = 47.6 \text{ say } 50 \text{ mm } \textbf{Ans.}$$

Diameter of hollow shaft

Let *di = Inside diameter, and*

do = Outside diameter.

We know that the torque transmitted by the hollow shaft (*T*),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau\, (d_o)^3\ (1 - k^4)$$

$$= \frac{\pi}{16} \times 45\, (d_o)^3\, [1 - (0.5)^4] = 8.3\, (d_o)^3$$

$$(d_o)^3 = 955 \times 10^3 / 8.3 = 115\,060 \quad \text{or} \quad d_o = 48.6 \text{ say } 50 \text{ mm} \textbf{ Ans.}$$

$$d_i = 0.5\, d_o = 0.5 \times 50 = 25 \text{ mm} \textbf{ Ans.}$$

Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{M}{I} = \frac{\sigma_b}{y} \qquad \text{...}(i)$$

where *M = Bending moment,*
I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,
σb = Bending stress, and
y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation (i), we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.
We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4\,(1 - k^4) \qquad \text{...(where } k = d_i / d_o\text{)}$$

$$y = d_o / 2$$

Again substituting these values in equation ***(i), we have***

$$\frac{M}{\frac{\pi}{64}(d_o)^4\,(1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \qquad \text{or} \qquad M = \frac{\pi}{32} \times \sigma_b\,(d_o)^3\,(1 - k^4)$$

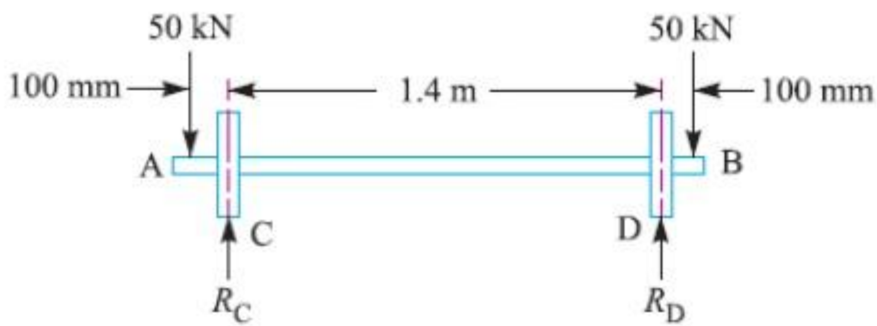
From this equation, the outside diameter of the shaft (do) may be obtained.

Example 4.

A pair of wheels of a railway wagon carries a load of 50 kN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.

Solution.

Given : $W = 50\ kN = 50 \times 10^3\ N$; $L = 100\ mm$; $x = 1.4\ m$; $\sigma b = 100\ MPa = 100\ N/mm^2$



The axle with wheels is shown in Fig. 1.

A little consideration will show that the maximum bending moment acts on the wheels at C and D. Therefore maximum bending moment,

*$M = W.L = 50 \times 10^3 \times 100 = 5 \times 10^6$ N-mm

The maximum B.M. may be obtained as follows :

$RC = RD = 50\ kN = 50 \times 10^3\ N$

B.M. at A, $MA = 0$

B.M. at C, $MC = 50 \times 10^3 \times 100 = 5 \times 10^6$ *N-mm*

B.M. at D, $MD = 50 \times 10^3 \times 1500 - 50 \times 10^3 \times 1400 = 5 \times 10^6$ *N-mm*

B.M. at B, $MB = 0$

Let d = Diameter of the axle.
We know that the maximum bending moment (M),

$$5 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\, d^3$$

$$d^3 = 5 \times 10^6 / 9.82 = 0.51 \times 10^6 \text{ or } d = 79.8 \text{ say } 80 \text{ mm}$$ **Ans.**

Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view :

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.

2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σb = *Bending stress (tensile or compressive) induced due to bending* moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Substituting the values of τ and σb *from Art. 14.9 and Art. 14.10, we have*

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi\,d^3}\right)^2 + 4\left(\frac{16\,T}{\pi\,d^3}\right)^2} = \frac{16}{\pi\,d^3}\left[\sqrt{M^2 + T^2}\right]$$

$$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2}$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation (*i*) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \qquad \text{...}(ii)$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\,\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} \qquad \text{...}(iii)$$

$$= \frac{1}{2} \times \frac{32\,M}{\pi\,d^3} + \frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi\,d^3}\right)^2 + 4\left(\frac{16\,T}{\pi\,d^3}\right)^2}$$

$$= \frac{32}{\pi\,d^3}\left[\frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)\right]$$

or $$\frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right. \qquad \text{...}(iv)$$

The expression $\frac{1}{2}\left[(M + \sqrt{M^2 + T^2})\right]$ is known as ***equivalent bending moment*** and is denoted by M_e. The equivalent bending moment may be defined as **that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment**. By limiting the maximum normal stress $[\sigma_{b(max)}]$ equal to the allowable bending stress (σ_b), then the equation (*iv*) may be written as

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3 \qquad \ldots(v)$$

From this expression, diameter of the shaft (d) may be evaluated.

Notes: 1. In case of a hollow shaft, the equations (*ii*) and (*v*) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

and

$$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{\pi}{32} \times \sigma_b\,(d_o)^3\,(1 - k^4)$$

2. It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of the two values is adopted.

Example 5.

A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.

Solution. Given : $M = 3000$ N-m $= 3 \times 10^6$ N-mm ; $T = 10\ 000$ N-m $= 10 \times 10^6$ N-mm ; $\sigma_{tu} = 700$ MPa $= 700$ N/mm^2 ; $\tau_u = 500$ MPa $= 500$ N/mm^2

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let d = Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36\ d^3$$

$\therefore$ $d^3 = 10.44 \times 10^6 / 16.36 = 0.636 \times 10^6$ or $d = 86$ mm

According to maximum normal stress theory, equivalent bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\, d^3$$

$$\therefore \quad d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6 \quad \text{or} \quad d = 83.7 \text{ mm}$$

Taking the larger of the two values, we have

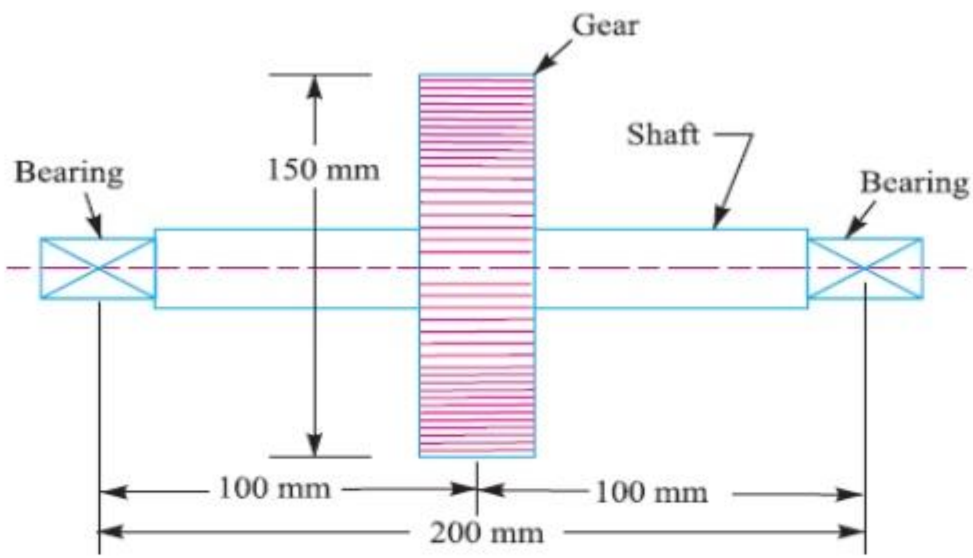
$$d = 86 \text{ say } 90 \text{ mm}$$ **Ans.**

Example 6.

A shaft supported at the ends in ball bearings carries a straight tooth spur gear at its mid span and is to transmit 7.5 kW at 300 r.p.m. The pitch circle diameter of the gear is 150 mm. The distances between the centre line of bearings and gear are 100 mm each. If the shaft is made of steel and the allowable shear stress is 45 MPa, determine the diameter of the shaft. Show in a sketch how the gear will be mounted on the shaft; also indicate the ends where the bearings will be mounted?
The pressure angle of the gear may be taken as 20°.

Solution. Given : P = 7.5 kW = 7500 W ; N = 300 r.p.m. ; D = 150 mm = 0.15 m ; L = 200 mm = 0.2 m ; τ = 45 MPa = 45 N/mm^2 ; α = 20°

Fig. 14.2 shows a shaft with a gear mounted on the bearings.



We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{7500 \times 60}{2\pi \times 300} = 238.7 \text{ N-m}$$

$\therefore$ Tangential force on the gear,

$$F_t = \frac{2T}{D} = \frac{2 \times 238.7}{0.15} = 3182.7 \text{ N}$$

and the normal load acting on the tooth of the gear,

$$W = \frac{F_t}{\cos\alpha} = \frac{3182.7}{\cos 20^\circ} = \frac{3182.7}{0.9397} = 3387 \text{ N}$$

Since the gear is mounted at the middle of the shaft, therefore maximum bending moment at the centre of the gear,

$$M = \frac{W.L}{4} = \frac{3387 \times 0.2}{4} = 169.4 \text{ N-m}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(169.4)^2 + (238.7)^2} = 292.7 \text{ N-m}$$

$$= 292.7 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$292.7 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\ d^3$$

$\therefore$ $d^3 = 292.7 \times 10^3 / 8.84 = 33 \times 10^3$ or $d = 32$ say 35 mm **Ans.**

Example 14.7. *A shaft made of mild steel is required to transmit 100 kW at 300 r.p.m. The supported length of the shaft is 3 metres. It carries two pulleys each weighing 1500 N supported at a distance of 1 metre from the ends respectively. Assuming the safe value of stress, determine the diameter of the shaft.*

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $N = 300$ r.p.m. ; $L = 3$ m ; $W = 1500$ N

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{100 \times 10^3 \times 60}{2\pi \times 300} = 3183 \text{ N-m}$$

The shaft carrying the two pulleys is like a simply supported beam as shown in Fig. 14.3. The reaction at each support will be 1500 N, *i.e.*

$$R_A = R_B = 1500 \text{ N}$$

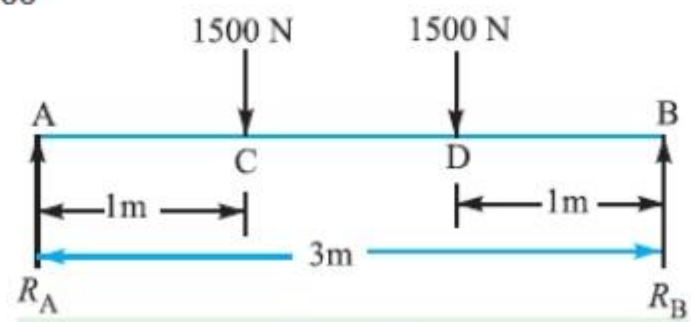


Fig. 14.3

A little consideration will show that the maximum bending moment lies at each pulley *i.e.* at *C* and *D*.

∴ Maximum bending moment,

$$M = 1500 \times 1 = 1500 \text{ N-m}$$

Let d = Diameter of the shaft in mm.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(1500)^2 + (3183)^2} = 3519 \text{ N-m}$$

$$= 3519 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$3519 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.8\, d^3 \quad \text{...(Assuming } \tau = 60 \text{ N/mm}^2\text{)}$$

$$\therefore \quad d^3 = 3519 \times 10^3 / 11.8 = 298 \times 10^3 \text{ or } d = 66.8 \text{ say } 70 \text{ mm } \textbf{Ans.}$$

Example 8. *A line shaft is driven by means of a motor placed vertically below it. The pulley on the line shaft is 1.5 metre in diameter and has belt tensions 5.4 kN and 1.8 kN on the tight side and slack side of the belt respectively. Both these tensions may be assumed to be vertica If the pulley be overhang from the shaft, the distance of the centre line of the pulley from the centre line of the bearing being 400 mm, find the diameter of the shaft. Assuming maximum allowable shear stress of 42 MPa.*

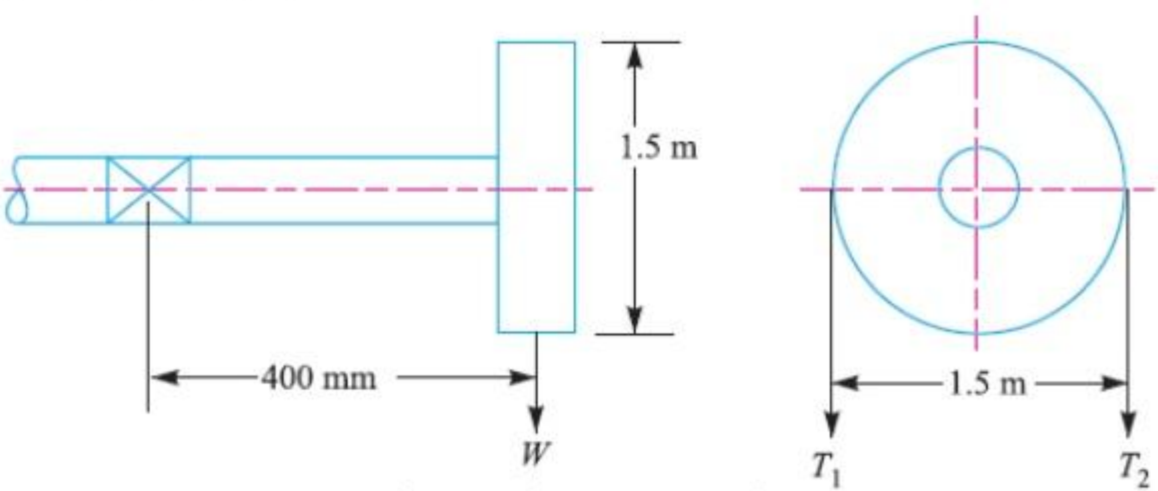
Solution .

Given : $D = 1.5$ m or $R = 0.75$ m; $T1 = 5.4$ kN $= 5400$ N ; $T2 = 1.8$ kN $= 1800$ N ;
$L = 400$ mm ; $\tau = 42$ MPa $= 42$ N/mm2

A line shaft with a pulley is shown in Fig 14.4.

We know that torque transmitted by the shaft,

$$T = (T1 - T2)\, R = (5400 - 1800)\, 0.75 = 2700 \text{ N-m}$$
$$= 2700 \times 103 \text{ N-mm}$$



Neglecting the weight of shaft, total vertical load acting on the pulley,

$$W = T_1 + T_2 = 5400 + 1800 = 7200 \text{ N}$$

$\therefore$ Bending moment, $M = W \times L = 7200 \times 400 = 2880 \times 10^3$ N-mm

Let d = Diameter of the shaft in mm.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(2880 \times 10^3)^2 + (2700 \times 10^3)^2}$$

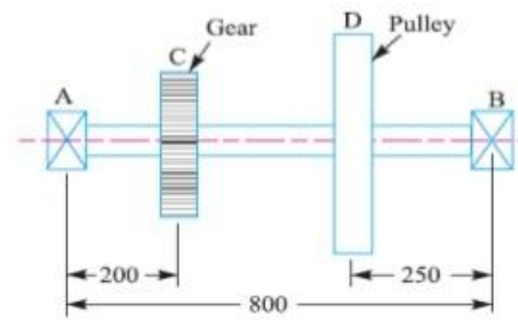
$$= 3950 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$3950 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

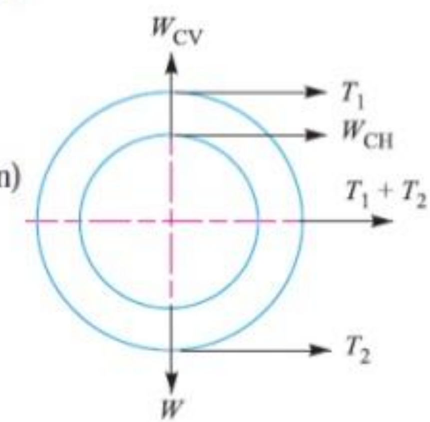
$\therefore$ $d^3 = 3950 \times 10^3 / 8.25 = 479 \times 10^3$ or $d = 78$ say 80 mm **Ans.**

Example 14.9. *A shaft is supported by two bearings placed 1 m apart. A 600 mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25 kN. Another pulley 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180° and μ = 0.24. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in tension and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley.*



Solution. Given : $AB = 1$ m ; $D_C = 600$ mm or $R_C = 300$ mm $= 0.3$ m ; $AC = 300$ mm $= 0.3$ m ; $T_1 = 2.25$ kN $= 2250$ N ; $D_D = 400$ mm or $R_D = 200$ mm $= 0.2$ m ; $BD = 200$ mm $= 0.2$ m ; $\theta = 180° = \pi$ rad ; $\mu = 0.24$; $\sigma_b = 63$ MPa $= 63$ N/mm² ; $\tau = 42$ MPa $= 42$ N/mm²

The space diagram of the shaft is shown in Fig. 14.5 (*a*).



Let T_1 = Tension in the tight side of the belt on pulley $C = 2250$ N ...(Given)

T_2 = Tension in the slack side of the belt on pulley C.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.24 \times \pi = 0.754$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.754}{2.3} = 0.3278 \text{ or } \frac{T_1}{T_2} = 2.127 \quad \text{...(Taking antilog of 0.3278)}$$

and

$$T_2 = \frac{T_1}{2.127} = \frac{2250}{2.127} = 1058 \text{ N}$$

∴ Vertical load acting on the shaft at C,

$$W_C = T_1 + T_2 = 2250 + 1058 = 3308 \text{ N}$$

and vertical load on the shaft at D

$$= 0$$

The vertical load diagram is shown in Fig. 14.5 (*c*).

We know that torque acting on the pulley C,

$$T = (T_1 - T_2)\, R_C = (2250 - 1058)\, 0.3 = 357.6 \text{ N-m}$$

The torque diagram is shown in Fig. 14.5 (*b*).

Let T_3 = Tension in the tight side of the belt on pulley D, and

T_4 = Tension in the slack side of the belt on pulley D.

Since the torque on both the pulleys (*i.e.* C and D) is same, therefore

$$(T_3 - T_4)\, R_D = T = 357.6 \text{ N-m} \quad \text{or} \quad T_3 - T_4 = \frac{357.6}{R_D} = \frac{357.6}{0.2} = 1788 \text{ N} \qquad \text{...}(i)$$

We know that

$$= \frac{T_3}{T_4} = \frac{T_1}{T_2} = 2.127 \quad \text{or} \quad T_3 = 2.127\, T_4 \qquad \text{...}(ii)$$

From equations (*i*) and (*ii*), we find that

$$T_3 = 3376 \text{ N, and } T_4 = 1588 \text{ N}$$

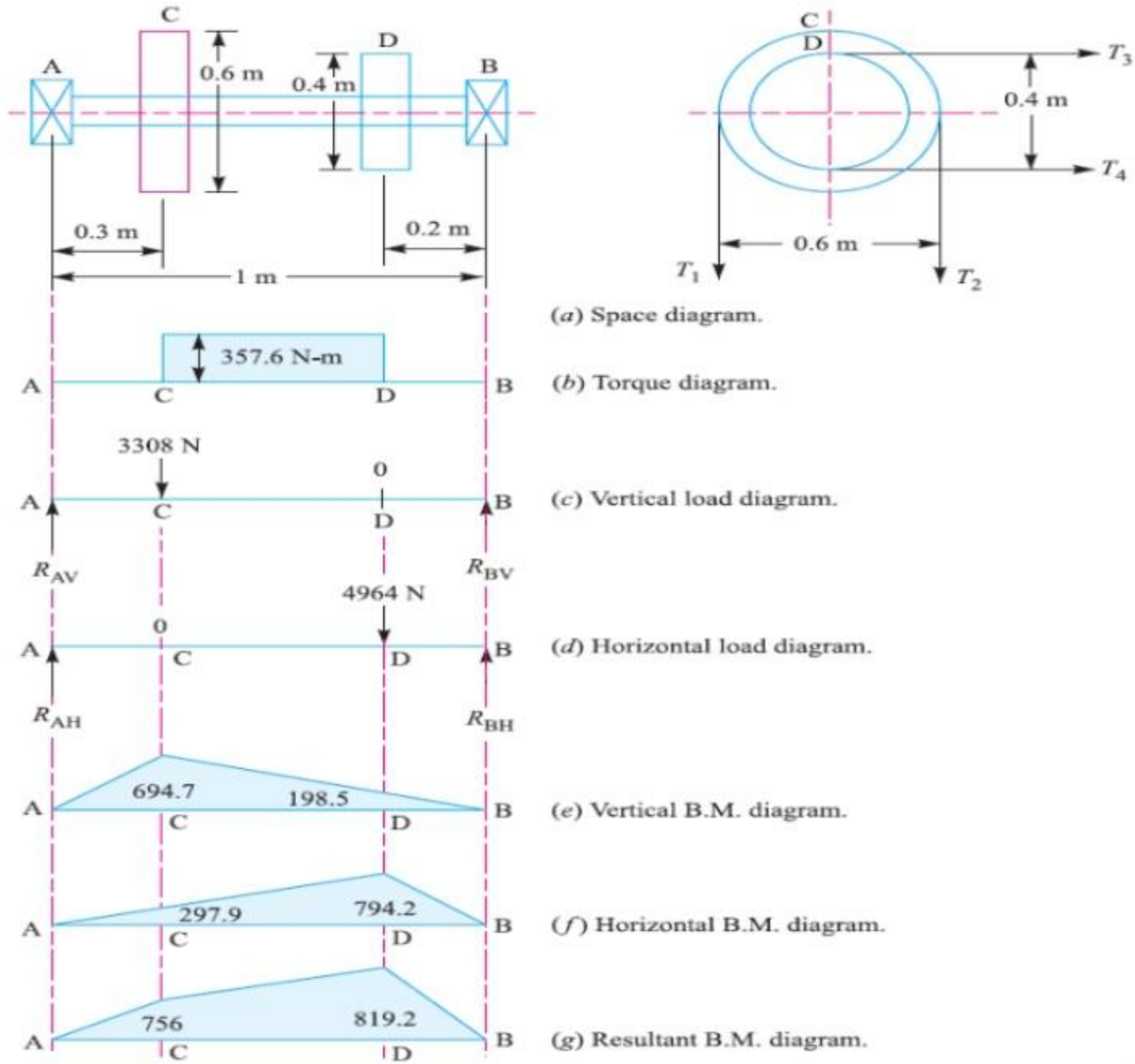
∴ Horizontal load acting on the shaft at D,

$$W_D = T_3 + T_4 = 3376 + 1588 = 4964 \text{ N}$$

and horizontal load on the shaft at $C = 0$

The horizontal load diagram is shown in Fig. 14.5 (*d*).

Now let us find the maximum bending moment for vertical and horizontal loading.



(a) Space diagram.

(b) Torque diagram.

(c) Vertical load diagram.

(d) Horizontal load diagram.

(e) Vertical B.M. diagram.

(f) Horizontal B.M. diagram.

(g) Resultant B.M. diagram.

First of all, considering the vertical loading at C. Let R_{AV} and R_{BV} be the reactions at the bearings A and B respectively. We know that

$$R_{AV} + R_{BV} = 3308 \text{ N}$$

Taking moments about A,

$$R_{BV} \times 1 = 3308 \times 0.3 \text{ or } R_{BV} = 992.4 \text{ N}$$

and $$R_{AV} = 3308 - 992.4 = 2315.6 \text{ N}$$

We know that B.M. at A and B,

$$M_{AV} = M_{BV} = 0$$

B.M. at C, $$M_{CV} = R_{AV} \times 0.3 = 2315.6 \times 0.3 = 694.7 \text{ N-m}$$

B.M. at D, $$M_{DV} = R_{BV} \times 0.2 = 992.4 \times 0.2 = 198.5 \text{ N-m}$$

The bending moment diagram for vertical loading in shown in Fig. 14.5 (*e*).

Now considering horizontal loading at D. Let R_{AH} and R_{BH} be the reactions at the bearings A and B respectively. We know that

$$R_{AH} + R_{BH} = 4964 \text{ N}$$

Taking moments about A,

$$R_{BH} \times 1 = 4964 \times 0.8 \text{ or } R_{BH} = 3971 \text{ N}$$

and $$R_{AH} = 4964 - 3971 = 993 \text{ N}$$

We know that B.M. at A and B,

$$M_{AH} = M_{BH} = 0$$

B.M. at C, $$M_{CH} = R_{AH} \times 0.3 = 993 \times 0.3 = 297.9 \text{ N-m}$$

B.M. at D, $$M_{DH} = R_{BH} \times 0.2 = 3971 \times 0.2 = 794.2 \text{ N-m}$$

The bending moment diagram for horizontal loading is shown in Fig. 14.5 (*f*).

Resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(694.7)^2 + (297.9)^2} = 756 \text{ N-m}$$

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(198.5)^2 + (794.2)^2} = 819.2 \text{ N-m}$$

The resultant bending moment diagram is shown in Fig. 14.5 (g).

We see that bending moment is maximum at D.

$\therefore$ Maximum bending moment,

$$M = M_D = 819.2 \text{ N-m}$$

Let $\quad d$ = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(819.2)^2 + (357.6)^2} = 894 \text{ N-m}$$

$$= 894 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$894 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$$\therefore \quad d^3 = 894 \times 10^3 / 8.25 = 108 \times 10^3 \text{ or } d = 47.6 \text{ mm}$$

Again we know that equivalent bending moment,

$$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \tfrac{1}{2}\,(M + T_e)$$

$$= \tfrac{1}{2}\,(819.2 + 894) = 856.6 \text{ N-m} = 856.6 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$856.6 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 63 \times d^3 = 6.2\, d^3$$

$$\therefore \qquad d^3 = 856.6 \times 10^3/6.2 = 138.2 \times 10^3 \text{ or } d = 51.7 \text{ mm}$$

Taking larger of the two values, we have

$$d = 51.7 \text{ say } 55 \text{ mm}$$ **Ans.**

Shafts Subjected to Fluctuating Loads

In the previous articles we have assumed that the shaft is subjected to constant torque and bending moment. But in actual practice, the shafts are subjected to fluctuating torque and bending moments. In order to design such shafts like line shafts and counter shafts, the combined shock and fatigue factors must be taken into account for the computed *twisting moment* (T) *and bending moment* (M). Thus for a shaftsubjected to combined bending and torsion, the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t + T)^2}$$

and equivalent bending moment,

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right]$$

where *Km = Combined shock and fatigue factor for bending, and*
Kt = Combined shock and fatigue factor for torsion.

The following table shows the recommended values for *Km and Kt.*

Table 14.2. Recommended values for K_m and K_t

Nature of load	K_m	K_t
1. **Stationary shafts**		
(*a*) Gradually applied load	1.0	1.0
(*b*) Suddenly applied load	1.5 to 2.0	1.5 to 2.0
2. **Rotating shafts**		
(*a*) Gradually applied or steady load	1.5	1.0
(*b*) Suddenly applied load with minor shocks only	1.5 to 2.0	1.5 to 2.0
(*c*) Suddenly applied load with heavy shocks	2.0 to 3.0	1.5 to 3.0

Example 14.12. *A mild steel shaft transmits 20 kW at 200 r.p.m. It carries a central load of 900 N and is simply supported between the bearings 2.5 metres apart. Determine the size of the shaft, if the allowable shear stress is 42 MPa and the maximum tensile or compressive stress is not to exceed 56 MPa. What size of the shaft will be required, if it is subjected to gradually applied loads?*

Solution. Given : $P = 20$ kW $= 20 \times 10^3$ W ; $N = 200$ r.p.m. ; $W = 900$ N ; $L = 2.5$ m ; $\tau = 42$ MPa $= 42$ N/mm^2 ; $\sigma_b = 56$ MPa $= 56$ N/mm^2

Size of the shaft

Let d = Diameter of the shaft, in mm.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

and maximum bending moment of a simply supported shaft carrying a central load,

$$M = \frac{W \times L}{4} = \frac{900 \times 2.5}{4} = 562.5 \text{ N-m} = 562.5 \times 10^3 \text{ N-mm}$$

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(562.5 \times 10^3)^2 + (955 \times 10^3)^2}$$

$$= 1108 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1108 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25 \, d^3$$

$$\therefore \quad d^3 = 1108 \times 10^3 / 8.25 = 134.3 \times 10^3 \text{ or } d = 51.2 \text{ mm}$$

We know that the equivalent bending moment,

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(562.5 \times 10^3 + 1108 \times 10^3) = 835.25 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$835.25 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 56 \times d^3 = 5.5\, d^3$$

$$\therefore \quad d^3 = 835.25 \times 10^3 / 5.5 = 152 \times 10^3 \text{ or } d = 53.4 \text{ mm}$$

Taking the larger of the two values, we have

$$d = 53.4 \text{ say } 55 \text{ mm}$$ **Ans.**

Size of the shaft when subjected to gradually applied load

Let d = Diameter of the shaft.

From Table 14.2, for rotating shafts with gradually applied loads,

$$K_m = 1.5 \text{ and } K_t = 1$$

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(1.5 \times 562.5 \times 10^3)^2 + (1 \times 955 \times 10^3)^2} = 1274 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1274 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$$\therefore \quad d^3 = 1274 \times 10^3 / 8.25 = 154.6 \times 10^3 \text{ or } d = 53.6 \text{ mm}$$

We know that the equivalent bending moment,

$$M_e = \tfrac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \tfrac{1}{2}\left[K_m \times M + T_e\right]$$

$$= \tfrac{1}{2}\left[1.5 \times 562.5 \times 10^3 + 1274 \times 10^3\right] = 1059 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$1059 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 56 \times d^3 = 5.5\, d^3$$

$$\therefore \quad d^3 = 1059 \times 10^3 / 5.5 = 192.5 \times 10^3 = 57.7 \text{ mm}$$

Taking the larger of the two values, we have

$$d = 57.7 \text{ say } 60 \text{ mm} \quad \textbf{Ans.}$$

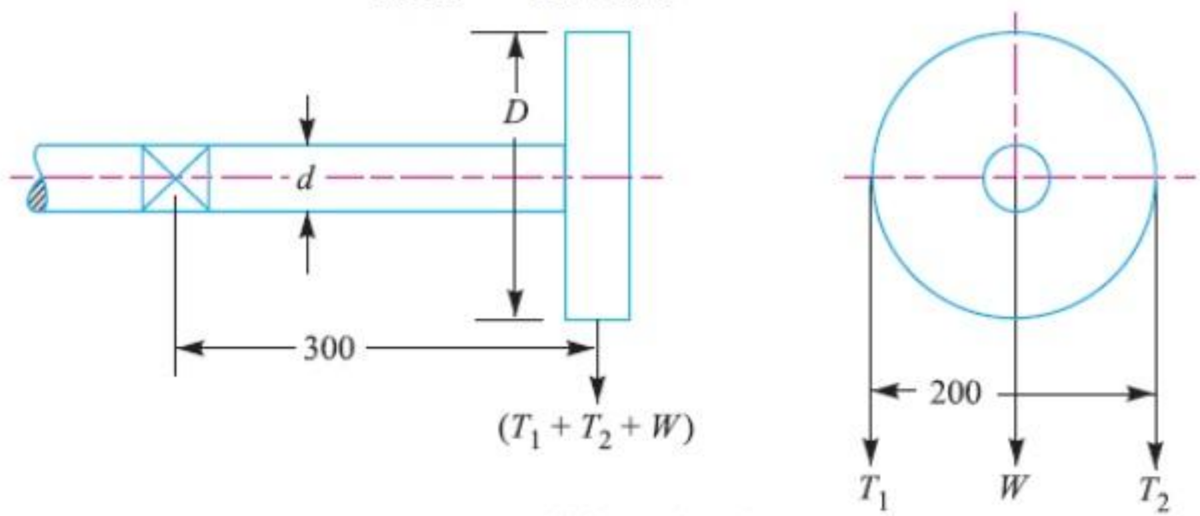
Example 14.13. *Design a shaft to transmit power from an electric motor to a lathe head stock through a pulley by means of a belt drive. The pulley weighs 200 N and is located at 300 mm from the centre of the bearing. The diameter of the pulley is 200 mm and the maximum power transmitted is 1 kW at 120 r.p.m. The angle of lap of the belt is 180° and coefficient of friction between the belt and the pulley is 0.3. The shock and fatigue factors for bending and twisting are 1.5 and 2.0 respectively. The allowable shear stress in the shaft may be taken as 35 MPa.*

Solution. Given : W = 200 N ; L = 300 mm ; D = 200 mm or R = 100 mm ; P = 1 kW = 1000 W ; N = 120 r.p.m. ; $\theta = 180° = \pi$ rad ; $\mu = 0.3$; $K_m = 1.5$; $K_t = 2$; $\tau = 35$ MPa = 35 N/mm²

The shaft with pulley is shown in Fig. 14.9.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{1000 \times 60}{2\pi \times 120} = 79.6 \text{ N-m} = 79.6 \times 10^3 \text{ N-mm}$$



All dimensions in mm.

Let T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively in newtons.

$\therefore$ Torque transmitted (T),

$$79.6 \times 10^3 = (T_1 - T_2)\, R = (T_1 - T_2)\, 100$$

$$\therefore \quad T_1 - T_2 = 79.6 \times 10^3 / 100 = 796 \text{ N} \qquad \text{...}(i)$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3\,\pi = 0.9426$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.9426}{2.3} = 0.4098 \text{ or } \frac{T_1}{T_2} = 2.57 \qquad \text{...}(ii)$$

...(Taking antilog of 0.4098)

From equations (*i*) and (*ii*), we get,

$$T_1 = 1303 \text{ N, and } T_2 = 507 \text{ N}$$

We know that the total vertical load acting on the pulley,

$$W_{\mathrm{T}} = T_1 + T_2 + W = 1303 + 507 + 200 = 2010 \text{ N}$$

$\therefore$ Bending moment acting on the shaft,

$$M = W_{\mathrm{T}} \times L = 2010 \times 300 = 603 \times 10^3 \text{ N-mm}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t + T)^2}$$

$$= \sqrt{(1.5 \times 603 \times 10^3)^2 + (2 \times 79.6 \times 10^3)^2} = 918 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$918 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 35 \times d^3 = 6.87\, d^3$$

$$\therefore \quad d^3 = 918 \times 10^3 / 6.87 = 133.6 \times 10^3 \text{ or } d = 51.1 \text{ say } 55 \text{ mm}$$ **Ans.**

Design of Shafts on the basis of Rigidity

1. ***Torsional rigidity.*** The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be effected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3 degree per metre length may be used as limiting value. The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft.

The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G \cdot \theta}{L} \quad \text{or} \quad \theta = \frac{T \cdot L}{J \cdot G}$$

where

θ = Torsional deflection or angle of twist in radians,

T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$$= \frac{\pi}{32} \times d^4 \quad \text{...(For solid shaft)}$$

$$= \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \quad \text{...(For hollow shaft)}$$

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

2. ***Lateral rigidity.*** It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam, *i.e.*

$$\frac{d^2 y}{dx^2} = \frac{M}{EI}$$

Example 14.21. *A steel spindle transmits 4 kW at 800 r.p.m. The angular deflection should not exceed 0.25° per metre of the spindle. If the modulus of rigidity for the material of the spindle is 84 GPa, find the diameter of the spindle and the shear stress induced in the spindle.*

Solution. Given : $P = 4$ kW = 4000 W ; $N = 800$ r.p.m. ; $\theta = 0.25° = 0.25 \times \frac{\pi}{180} = 0.0044$ rad ; $L = 1$ m = 1000 mm ; $G = 84$ GPa $= 84 \times 10^9$ N/m² $= 84 \times 10^3$ N/mm²

Diameter of the spindle

Let d = Diameter of the spindle in mm.

We know that the torque transmitted by the spindle,

$$T = \frac{P \times 60}{2\pi N} = \frac{4000 \times 60}{2\pi \times 800} = 47.74 \text{ N-m} = 47\,740 \text{ N-mm}$$

We also know that $\frac{T}{J} = \frac{G \times \theta}{L}$ or $J = \frac{T \times l}{G \times \theta}$

or $$\frac{\pi}{32} \times d^4 = \frac{47\,740 \times 1000}{84 \times 10^3 \times 0.0044} = 129\,167$$

$\therefore$ $d^4 = 129\,167 \times 32 / \pi = 1.3 \times 10^6$ or $d = 33.87$ say 35 mm **Ans.**

Shear stress induced in the spindle

Let τ = Shear stress induced in the spindle.

We know that the torque transmitted by the spindle (T),

$$47\,740 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times \tau\,(35)^3 = 8420\,\tau$$

$\therefore$ $\tau = 47\,740 / 8420 = 5.67$ N/mm² $= 5.67$ MPa **Ans.**

Example 14.22. *Compare the weight, strength and stiffness of a hollow shaft of the same external diameter as that of solid shaft. The inside diameter of the hollow shaft being half the external diameter. Both the shafts have the same material and length.*

Solution. Given : $d_o = d$; $d_i = d_o / 2$ or $k = d_i / d_o = 1/2 = 0.5$

Comparison of weight

We know that weight of a hollow shaft,

$$W_H = \text{Cross-sectional area} \times \text{Length} \times \text{Density}$$

$$= \frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right] \times \text{Length} \times \text{Density} \qquad ...(i)$$

and weight of the solid shaft,

$$W_S = \frac{\pi}{4} \times d^2 \times \text{Length} \times \text{Density} \qquad ...(ii)$$

Since both the shafts have the same material and length, therefore by dividing equation (*i*) by equation (*ii*), we get

$$\frac{W_H}{W_S} = \frac{(d_o)^2 - (d_i)^2}{d^2} = \frac{(d_o)^2 - (d_i)^2}{(d_o)^2} \qquad ...(\because d = d_o)$$

$$= 1 - \frac{(d_i)^2}{(d_o)_2} = 1 - k^2 = 1 - (0.5)^2 = 0.75 \textbf{ Ans.}$$

Comparison of strength

We know that strength of the hollow shaft,

$$T_H = \frac{\pi}{16} \times \tau \, (d_o)^3 \; (1 - k^4) \qquad ...(iii)$$

and strength of the solid shaft,

$$T_S = \frac{\pi}{16} \times \tau \times d^3 \qquad ...(iv)$$

Dividing equation (*iii*) by equation (*iv*), we get

$$\frac{T_H}{T_S} = \frac{(d_o)^3 \; (1 - k^4)}{d^3} = \frac{(d_o)^3 \; (1 - k^4)}{(d_o)^3} = 1 - k^4 \qquad ...(\because d = d_o)$$

$$= 1 - (0.5)^4 = 0.9375 \text{ Ans.}$$

Comparison of stiffness

We know that stiffness

$$= \frac{T}{\theta} = \frac{G \times J}{L}$$

∴ Stiffness of a hollow shaft,

$$S_H = \frac{G}{L} \times \frac{\pi}{32} \left[(d_o)^4 - (d_i)^4 \right] \qquad ...(v)$$

and stiffness of a solid shaft,

$$S_S = \frac{G}{L} \times \frac{\pi}{32} \times d^4 \qquad ...(vi)$$

Dividing equation (*v*) by equation (*vi*), we get

$$\frac{S_H}{S_S} = \frac{(d_o)^4 - (d_i)^4}{d^4} = \frac{(d_o)^4 - (d_i)^4}{(d_o)^4} = 1 - \frac{(d_i)^4}{(d_o)^4} \qquad ...(\because d = d_o)$$

$$= 1 - K^4 = 1 - (0.5)^4 = 0.9375 \quad \text{Ans.}$$

Assignment Problem

1. A transmission shaft, supporting two pulleys *A and B and mounted between two bearings C1 and C2 is shown in Fig. 1. Power is* transmitted from the pulley *A to B. The shaft* is made of plain carbon steel 45C8 (S_{ut} = *600* and S_{yt} = *380 N/mm²). The pulleys are keyed* to the shaft.

   Assume $k_b = 1.5$ *and* $k_t = 1.0$

a) Determine the shaft diameter

b) Also, determine the shaft diameter on the basis of torsional rigidity, if the permissible angle of twist between the two pulleys is 0.5° and the modulus of rigidity is 79 300 N/mm²