NATIONAL INSTITUTE OF TECHNOLOGY

THEORY OF MACHINES – I

Module 2

Topics Discussed So Far

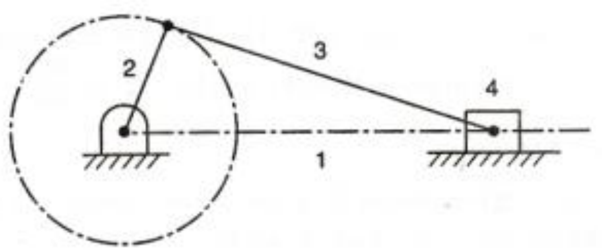
- TOM: Introduction
- Kinematics, Dynamics
- Kinematic Link
- Rigid Body
- Resistant Body
- Types of Link: Rigid, Flexible, Fluid
- Machine
- Structure
- Mechanism
- Kinematic Pair
- Constrained Motion
- Types of Constrained Motion: Completely, incompletely and Successfully Constrained Motion.
- Classification of Kinematic Pair

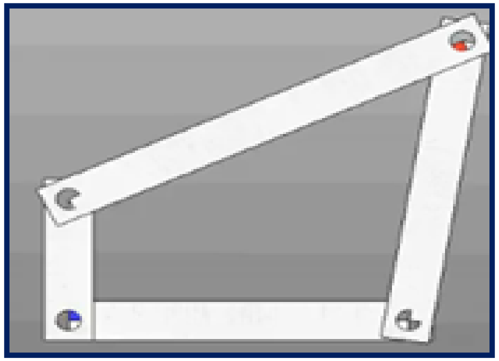
Topics of Discussion

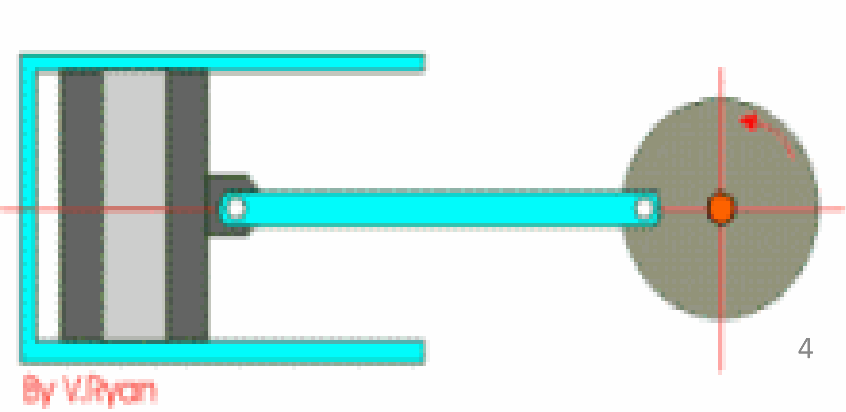
- Kinematic Chain
- Arrangement of Links
- Types of Joints
- Plane and Spatial Mechanism
- Degree of Freedom

1.3.KINEMATIC CHAIN:

When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion (*i.e.* completely or successfully constrained motion), it is called a ***kinematic chain.***







To determine the given assemblage of links forms the kinematic chain or not:

Two equations for lower pairs are available to determine the assemblage of links and pairs forms the chain or not.

The two equations are;

$$\mathbf{l = 2p-4}$$

$$\mathbf{j = (3/2)\, l-2}$$

Where, l= Number of links.

p= Number of pairs.

j= Number of joints.

If the above equations and are satisfied, then the assemblage of links forms a kinematic chain.

Three possible cases are:

i) If L.H.S = R.H.S, then the given chain is called constrained kinematic chain.

ii) If L.H.S > R.H.S, then the given chain is called locked chain or structure.

iii) If L.H.S < R.H.S, then the given chain is called unconstrained kinematic chain.

1. Arrangement of three links:

Number of links, $l = 3$

Number of pairs, $p = 3$

number of joints, $j = 3$

From equation $l = 2p - 4$

$$3 = 2 \times 3 - 4 = 2$$

L.H.S. > R.H.S.

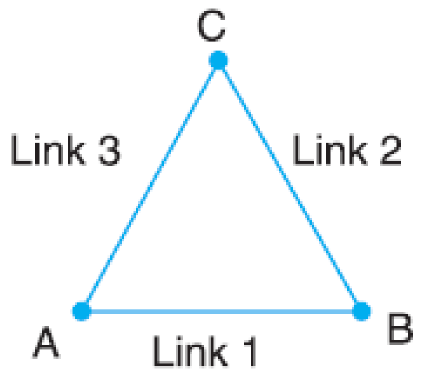


Fig. Arrangement of three links.

Now from equation $j = \frac{3}{2} l - 2$ or $3 = \frac{3}{2} \times 3 - 2 = 2.5$ L.H.S. > R.H.S.

This arrangement is not a kinematic chain LHS>RHS it is a Locked Chain or Structure

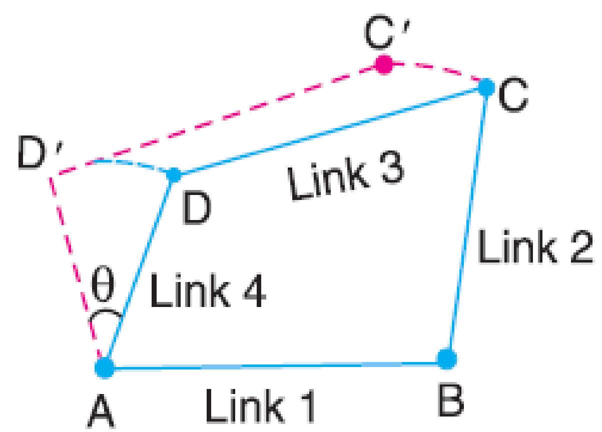
2. Arrangement of four links:

$l = 4$, $p = 4$, and $j = 4$

From equation (*i*), $l = 2p - 4$

$$4 = 2 \times 4 - 4 = 4$$

L.H.S. = R.H.S.



From equation (*ii*), $j = \frac{3}{2}l - 2$

$$4 = \frac{3}{2} \times 4 - 2 = 4$$

L.H.S. = R.H.S.

Since the arrangement of four links, as shown in Fig. satisfy the equations (*i*) and (*ii*), therefore it is a ***kinematic chain of one degree of freedom***.

Arrangement of five links.

$l = 5$, $p = 5$, and $j = 5$

From equation (*i*),

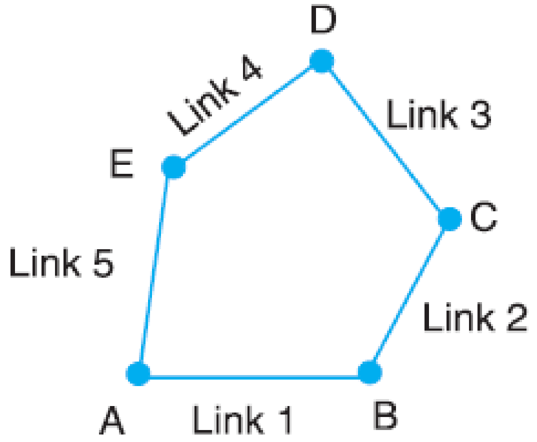
$l = 2p - 4$ or $5 = 2 \times 5 - 4 = 6$

i.e. L.H.S. < R.H.S.

From equation (*ii*),

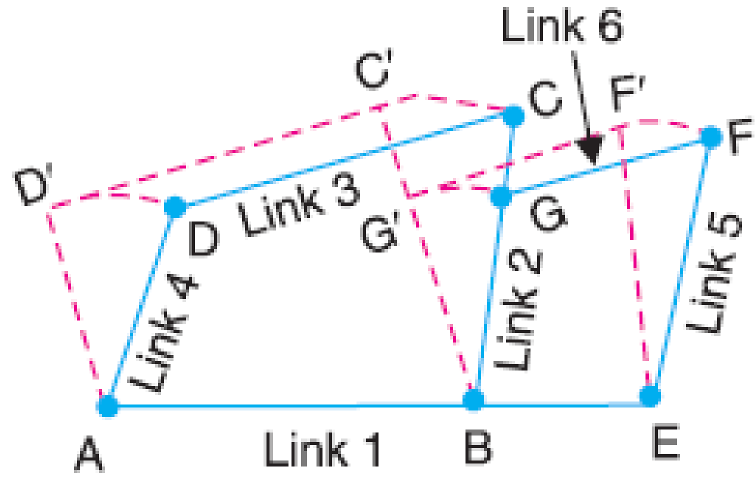
$j = \frac{3}{2}l - 2$ or $5 = \frac{3}{2} \times 5 - 2 = 5.5$

i.e. L.H.S. < R.H.S.



Since the arrangement of five links, as shown in Fig. does not satisfy the equations and left hand side is less than right hand side, therefore it is not a kinematic chain. Such a type of chain is called ***unconstrained chain*** *i.e.* the relative motion is not completely constrained. This type of chain is of little practical importance.

ARRANGEMENT OF SIX LINKS



$$l = 6,\ p = 5, \text{ and } \quad j = 7$$

From equation (*i*),

$$l = 2p - 4 \quad \text{or} \quad 6 = 2 \times 5 - 4 = 6$$

L.H.S. = R.H.S.

From equation (*ii*),

$$j = \frac{3}{2} l - 2 \quad \text{or} \quad 7 = \frac{3}{2} \times 6 - 2 = 7$$

L.H.S. = R.H.S.

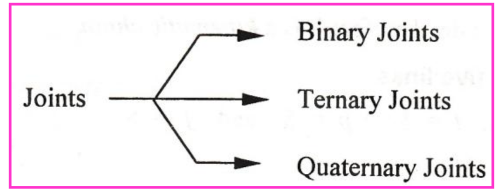
Since the arrangement of six links, as shown in Fig. satisfies the equations (*i.e.* left hand side is equal to right hand side), therefore it is a kinematic chain.

A chain having more than four links is known as **compound kinematic chain.**

Types of Joints in a Chain

1. *Binary joint*. When two links are joined at the same connection, the joint is known as binary joint. For example, a chain as shown in Fig. has four links and four binary joins at A, B, C and D.

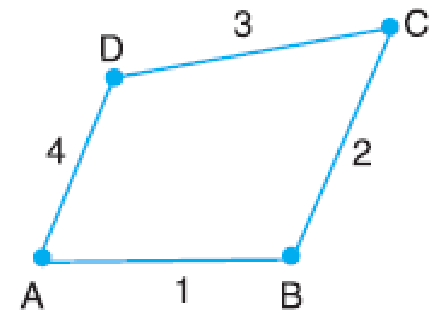
$$j + \frac{h}{2} = \frac{3}{2} l - 2$$



j = Number of binary joints,

h = Number of higher pairs, and

l = Number of links.



Kinematic chain with all binary joints.

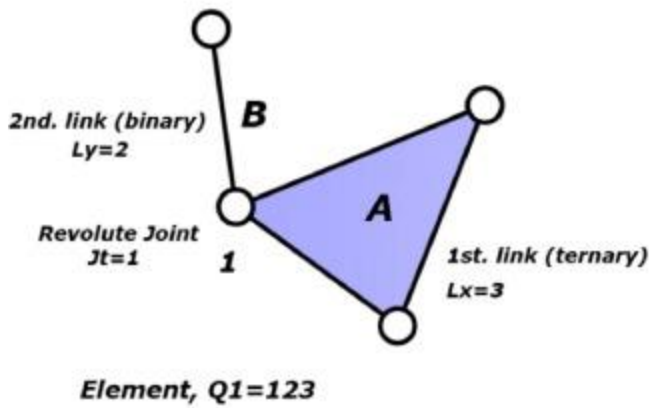
When $h = 0$, the equation *(i)*, may be written as

$$j = \frac{3}{2} l - 2 \quad \ldots (ii)$$

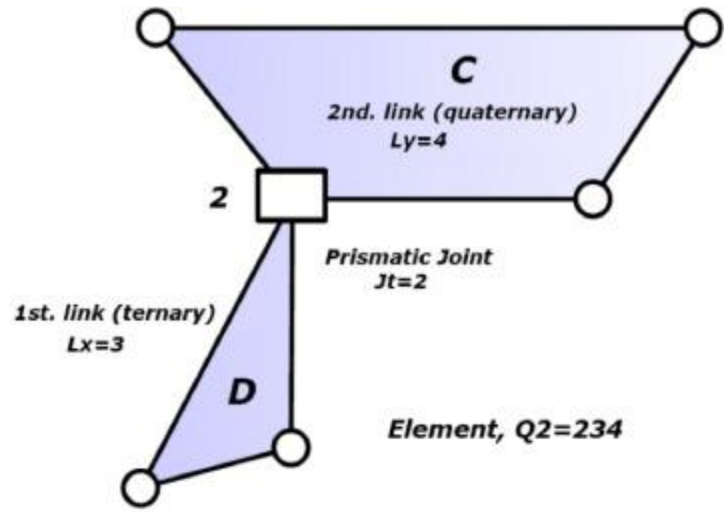
Applying this equation to a chain, as shown in Fig. 5.10, where $l = 4$ and $j = 4$, we have

$$4 = \frac{3}{2} \times 4 - 2 = 4$$

Since the left hand side is equal to the right hand side, therefore the chain is a kinematic chain or constrained chain.


B
2nd. link (binary)
Ly=2
A
Revolute Joint
Jt=1
1
1st. link (ternary)
Lx=3
Element, Q1=123

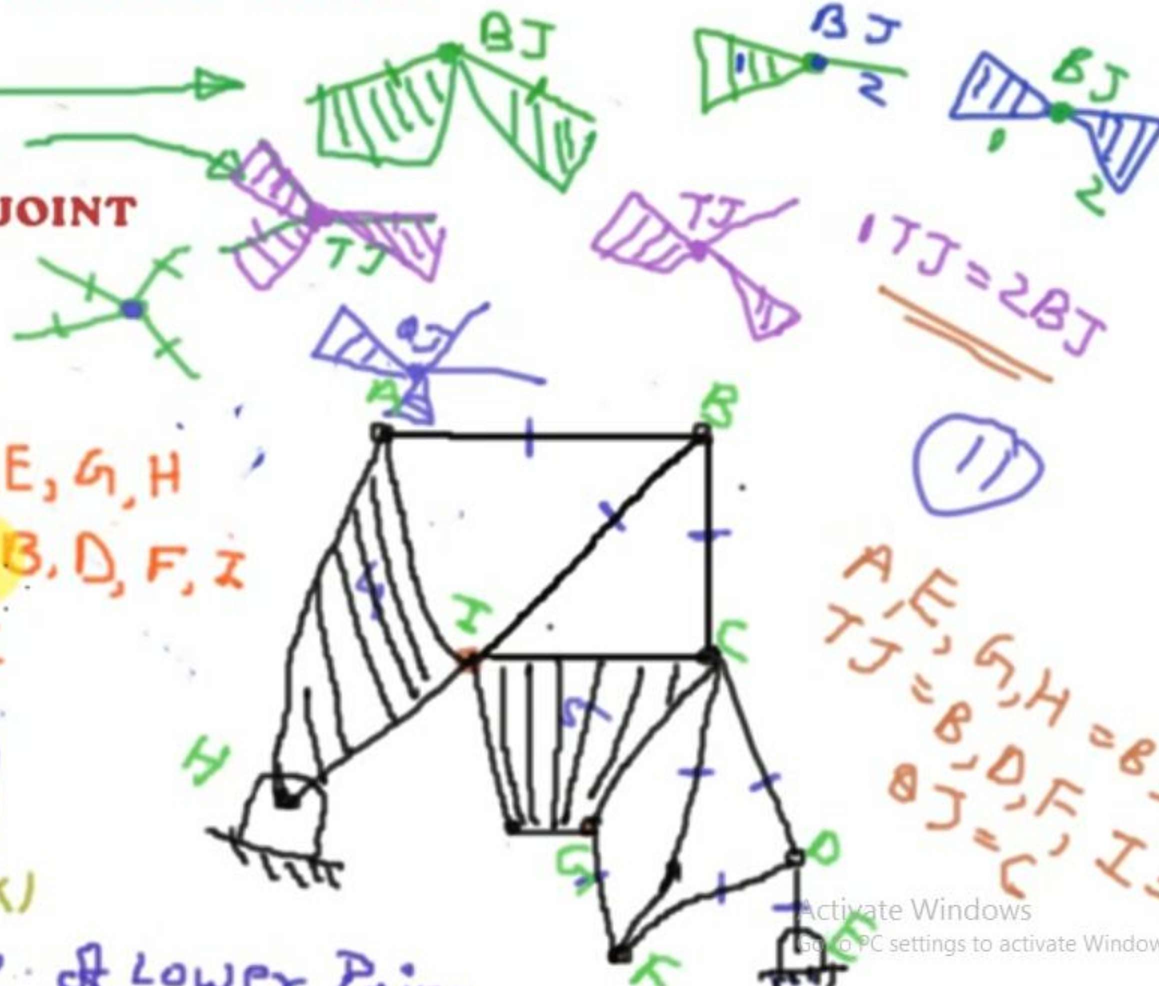
(a)


C
2nd. link (quaternary)
Ly=4
2
Prismatic Joint
Jt=2
1st. link (ternary)
Lx=3
D
Element, Q2=234

(b)

TYPES OF JOINTS

1) BINARY JOINT
2) TERNARY JOINT
3) QUARTERNARY JOINT

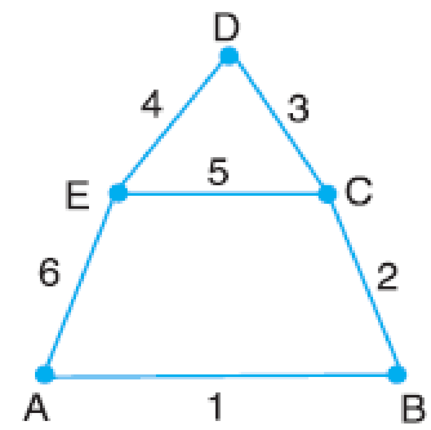


Here, 1 QJ = 3BJ
No. of links = 11
Binary Joints = A, E, G, H
Ternary Joints = B, D, F, I
Quarternary = C
Total No. of Binary Joints =
$4 + (2 \times 4) + 3 \times 1$
$= 15$ = No. of Lower Pairs

2. *Ternary joint*.

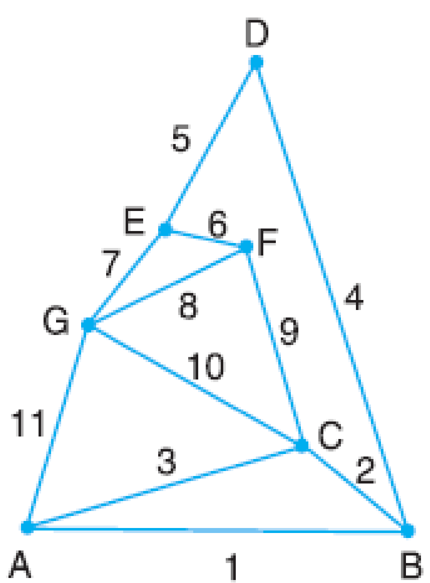
$$j = \frac{3}{2} l - 2$$

$$7 = \frac{3}{2} \times 6 - 2 = 7$$



Kinematic chain having binary and ternary joints.

3. *Quaternary joint*.



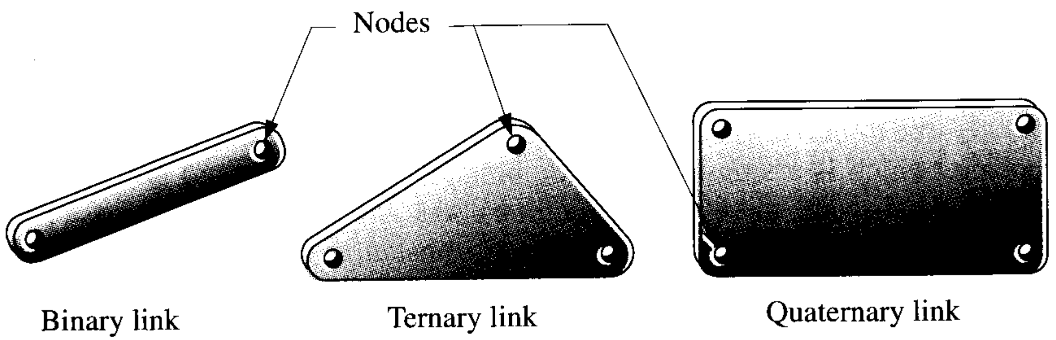
(a) Looked chain having binary, ternary and quaternary joints.

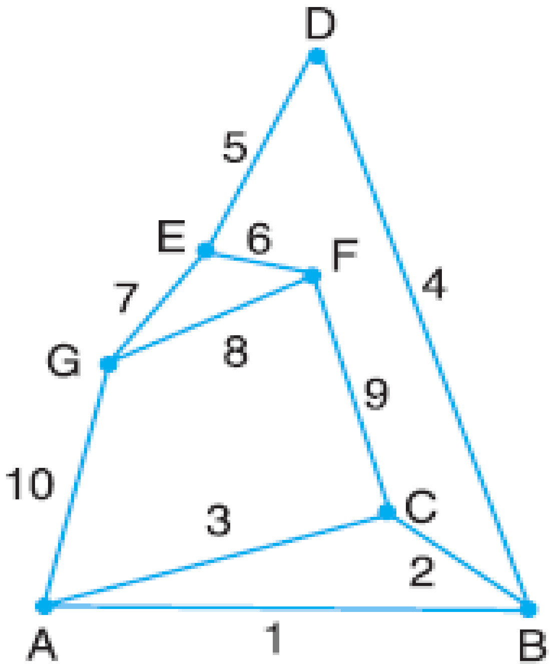
$1 + 4 \times 2 + 2 \times 3 = 15$

We know that $l = 11$ and $j = 15$. We know that,

$$j = \frac{3}{2} l - 2, \quad \text{or} \quad 15 = \frac{3}{2} \times 11 - 2 = 14.5, \text{ i.e., L.H.S.} > \text{R.H.S.}$$

not a kinematic chain





(*b*) Kinematic chain having binary and ternary joints.

Therefore total number of binary joints are $1 + 2 \times 6 = 13$.

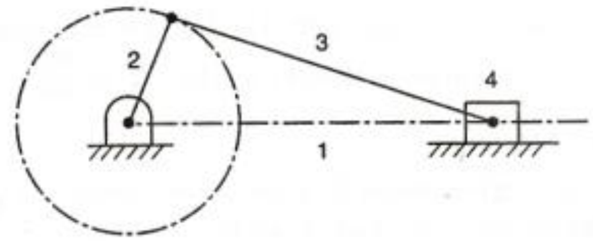
$$j = \frac{3}{2}l - 2, \quad \text{or} \quad 13 = \frac{3}{2} \times 10 - 2 = 13, \text{ i.e. L.H.S. = R.H.S.}$$

kinematic chain or constrained chain.

1.4. MECHANISM:

When one of the links of a kinematic chain is fixed, then the chain is known as mechanism. It may be used for transmitting motion..

Number of inputs which need to be provided in order to create a predictable output;* also: *the number of independent coordinates required to define its position.



Number of Degrees of Freedom for Plane Mechanisms

DEGREE FO FREEDOM (n) :

It is defined as the number of input parameters which must be Independently controlled in order to bring the mechanism in to a particular position.

Determining the number of degrees of freedom or movability (n)

$$\boxed{n = 3\,(l - 1) - 2\,j - h}$$

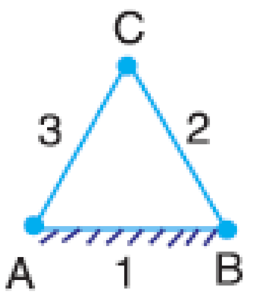
l = No of links, j = No of binary joint or No of lower pair, h = No of higher pair

This equation is called ***Kutzbach criterion for the movability*** of a mechanism having plane mechanism.

> *Note :* *If there are no higher pairs (i.e., two degree of freedom pairs), then h = 0, then,*
>
> *Kutzbach criterion,* $n = 3(l-1) - 2j$

Kutzbach criterion for plane Mechanism

Problems:



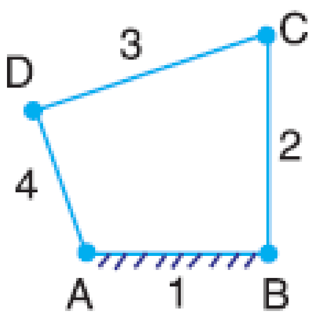
(*a*) Three-bar mechanism.

three links and three binary joints,

$l = 3$ and $j = 3$.

$$n = 3\,(3-1) - 2 \times 3 = 0$$

the mechanism forms a structure and no relative motion between the links is possible



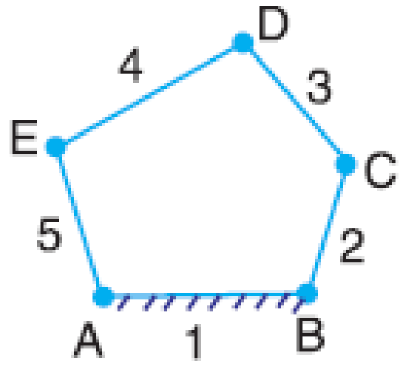
(*b*) Four bar mechanism.

four links and four binary joints,

$l = 4$ and $j = 4$.

$$n = 3\,(4-1) - 2 \times 4 = 1$$

the mechanism can be driven by a single input motion.



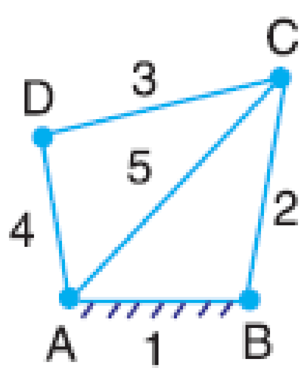
(*c*) Five bar mechanism.

five links and five binary joints,

$l = 5$, and $j = 5$.

$$n = 3\,(5 - 1) - 2 \times 5 = 2$$

then two separate input motions are necessary to produce constrained motion for the mechanism,



(*d*) Five bar mechanism.

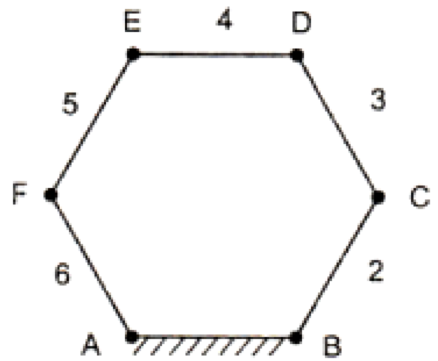
five links and six equivalent binary joints

$l = 5$ and $j = 6$.

$$n = 3\,(5 - 1) - 2 \times 6 = 0$$

the mechanism forms a structure and no relative motion between the links is possible

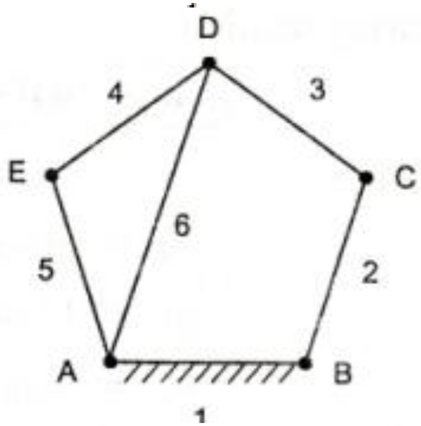
Six bar mechanism



$l = 6$; and $j = 6$

$n = 3(6-1) - 2 \times 6 = 3$

$n = 3$, then ***three separate input motions***



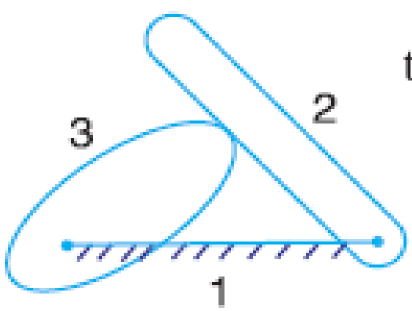
$l = 6$;

$j = 3 + (2 \times 2) = 7$;

$n = 3(6-1) - 2 \times 7 = 1$

$n = 1$, then the mechanism can be driven by a ***single input motion.***

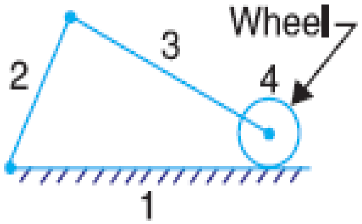
Mechanisms with Higher Pair :



there are three links, two binary joints and one higher pair, *i.e.* $l = 3, j = 2$ and $h = 1$.

$n = 3(3-1) - 2 \times 2 - 1 = 1$

When $n = 1$, then the mechanism can be driven by a ***single input motion.***



there are four links, three binary joints and one higher pair, *i.e.* $l = 4$, $j = 3$ and $h = 1$

$$n = 3\,(4 - 1) - 2 \times 3 - 1 = 2$$

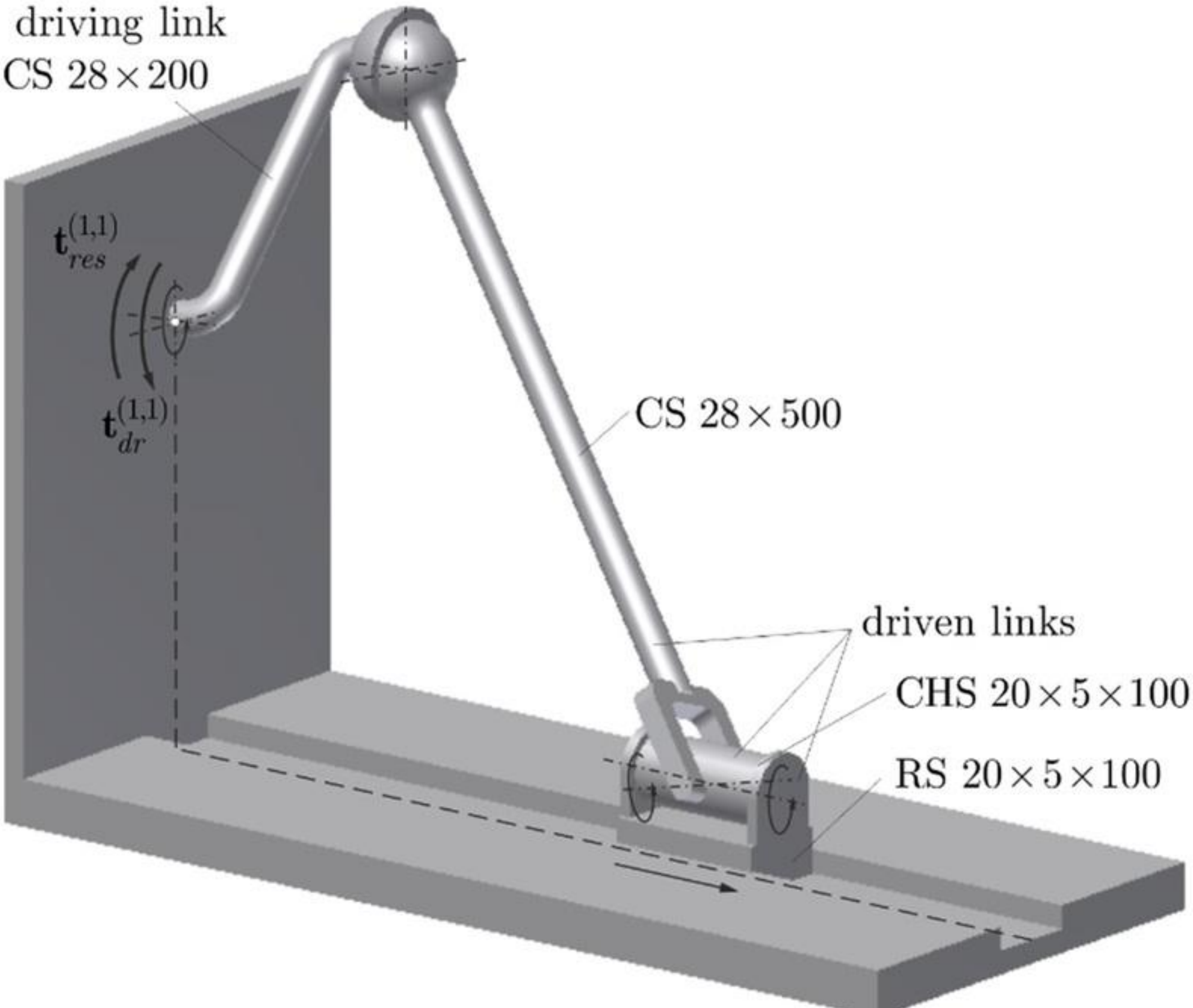
When **$n = 2$,** then to drive the ***mechanism two separate input motions*** are necessary.

All the mechanism discussed above is planar mechanism

In a ***planar mechanisms***, all of the relative motions of the rigid bodies are in one plane or in parallel planes.

In other words, *planar mechanisms* are essentially two dimensional.

If there is any relative motion that is not in the same plane or in parallel planes, the mechanism is called the ***spatial mechanism***.
Spatial mechanisms are three dimensional.



Kutzbach Criterion for Spatial Mechanisms

Degrees of freedom of a mechanism in space can be determined with the help of the following relation.

$$\boxed{n = 6(l-1) - 5p_1 - 4p_2 - 3p_3 - 2p_4 - 1p_5}$$

where,

n = Number of degrees of freedom,

l = Number of links in the mechanism,

p_1 = Number of pairs having one degree of freedom, and

p_2 = Number of pairs having two degrees of freedom and so on.

Grubler's Criterion for Plane Mechanism

Grubler's criterion for plane mechanisms is obtained by substituting $n = 1$ and $h = 0$ in Kutzbach criterion as below.

We know that,

$$l = 3(l-1) - 2j \quad \text{or}$$

$$\boxed{3l - 2j - 4 = 0}$$

This equation is known as ***Grubler's criterion for plane mechanism.*** Thus the Grubler's criterion applies to mechanisms with only single degree of freedom joints and the overall mobility of the mechanism is unity.

Grubler's Criterion for Spatial Mechanisms

We know that, Kutzbach's criterion for spatial mechanisms is

$$n = 6(l-1) - 5p_1 - 4p_2 - 3p_3 - 2p_4 - 1p_5$$

If we have all single-freedom pairs, and mobility of 1, then the Kutzbach equation is called as Grubler's criterion for spatial mechanisms.

i.e., Substitute $n = 1$, $p_2 = p_3 = \ldots\ldots\ldots = p_5 = 0$

then, $$1 = 6(l-1) - 5p_1$$

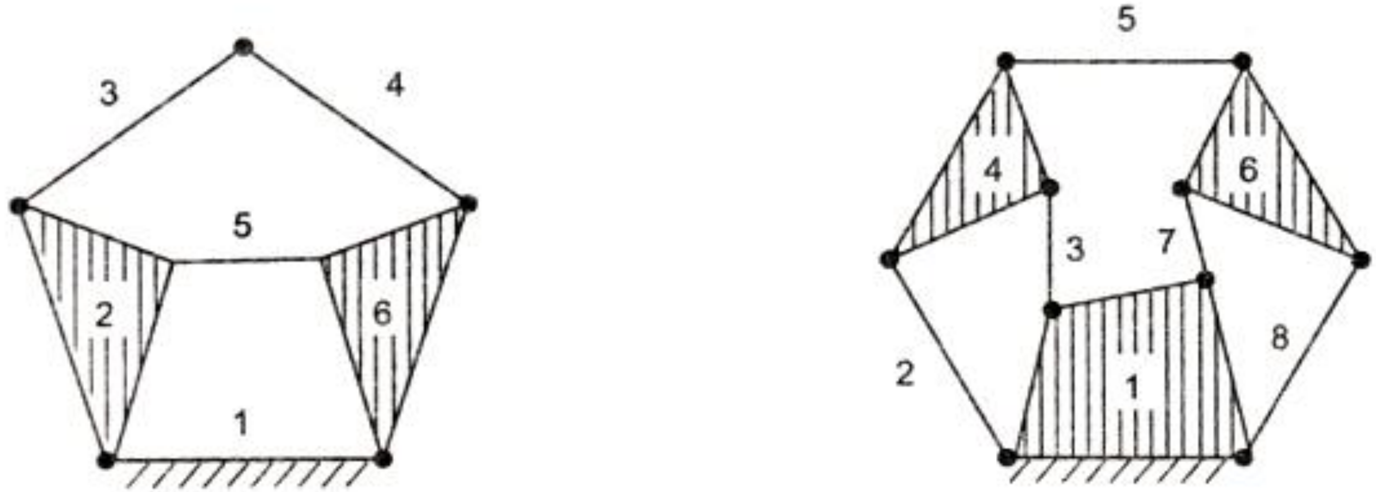
$$\boxed{6l - 5p_1 - 7 = 0}$$

This equation is known as the ***Grubler's equation for spatial mechanisms.***

Where, l = Total number of links

p_1 = Number of pairs having single degree of freedom

Example 1 ***Find the mobility (number of degrees of freedom) of each mechanisms shown in the figure below.***



(a) (b)

☺ ***Solution :*** Kutzbach's criterion for plane mechanism is given by,

$$n = 3(l-1) - 2j - h$$

From the Fig. (a),

Number of links, $l = 6$;

$h = 0$ (higher pair)

Number of binary joints, $j = 7$

$$n = 3(l-1) - 2j - h$$

$$= 3(6-1) - 2 \times 7 - 0$$

$\boxed{n = 1}$ **Ans.**

(b) From the Fig. (b),

Number of links, $l = 8$;

$h = 0$ (higher pair)

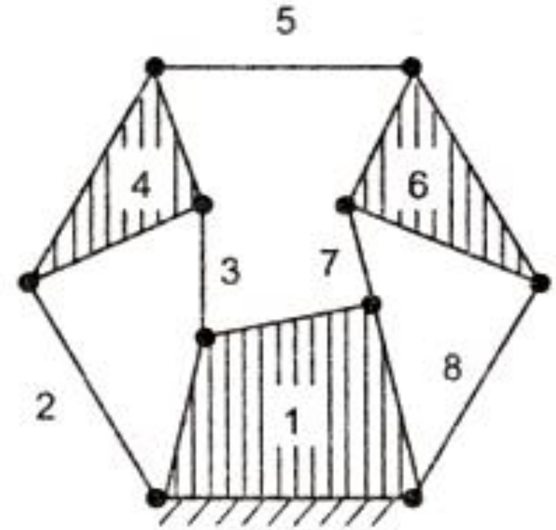
Number of binary joints, $j = 10$

Number of degrees of freedom,

$$n = 3(l-1) - 2j - h$$

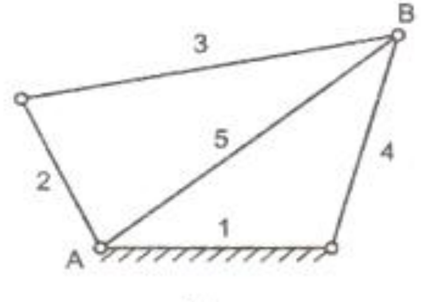
$$= 3(8-1) - 2 \times 10 - 0$$

$$\boxed{n = 1}$$ **Ans.**



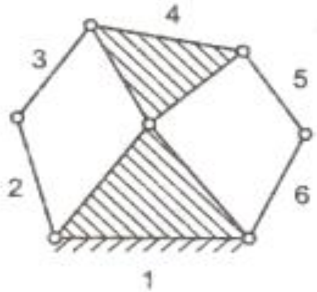
(b)

Example 2 ***Determine the mobility (number of degrees of freedom) of all the linkages shown in Fig.***



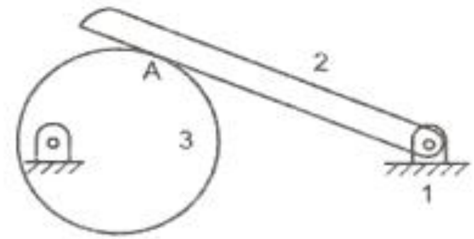
(a)

$$n = 3(l-1)-2j-h$$
$$\therefore n = 3(5-1)-2\times 6-0$$
$$n = 0$$



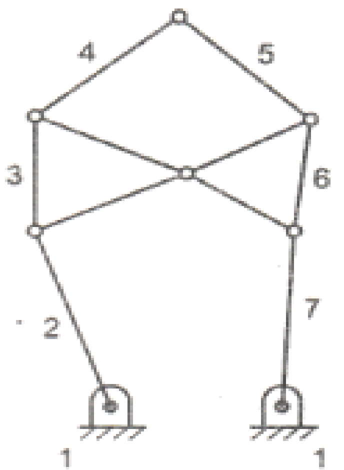
(b)

$$n = 3(l-1)-2j-h$$
$$n = 3(6-1)-2\times 7-0 = 1$$



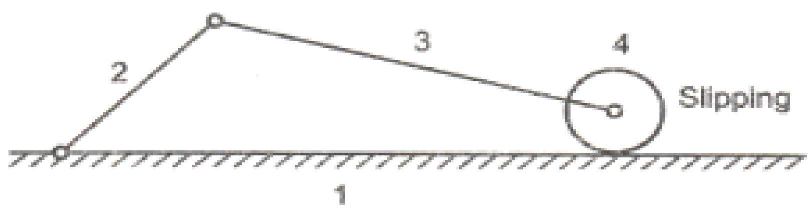
(e)

$$n = 3(l-1)-2j-h$$
$$= 3(3-1)-2\times 2-1 = 1$$



(c)

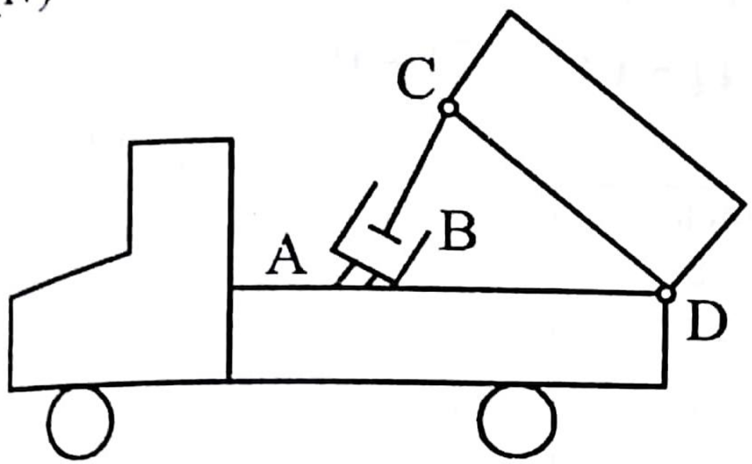
$$n = 3(l-1)-2j-h$$
$$n = 3(7-1)-2\times 8-0 = 2$$



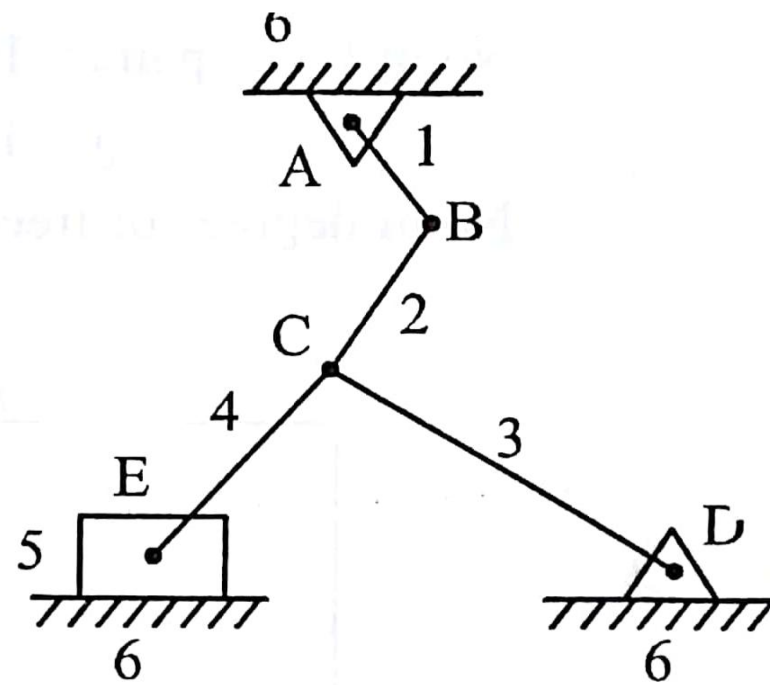
(d)

$$n = 3(l-1)-2j-h$$
$$= 3(4-1)-2\times 3-1 = 2$$

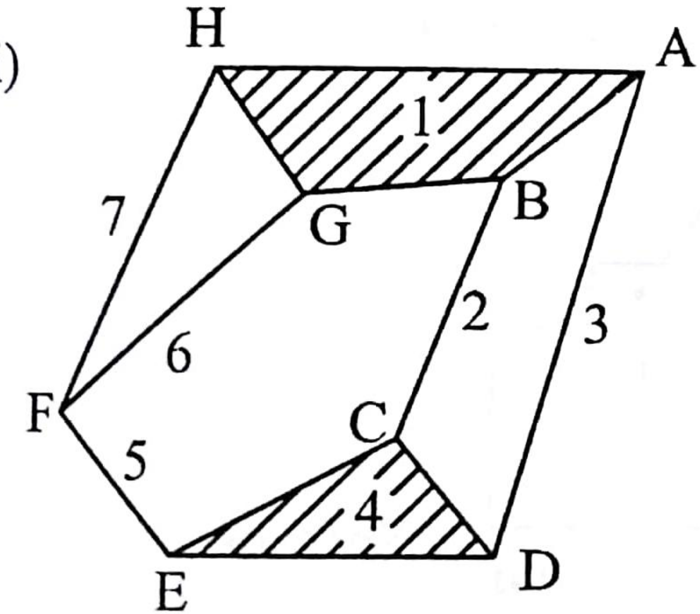
(iv)



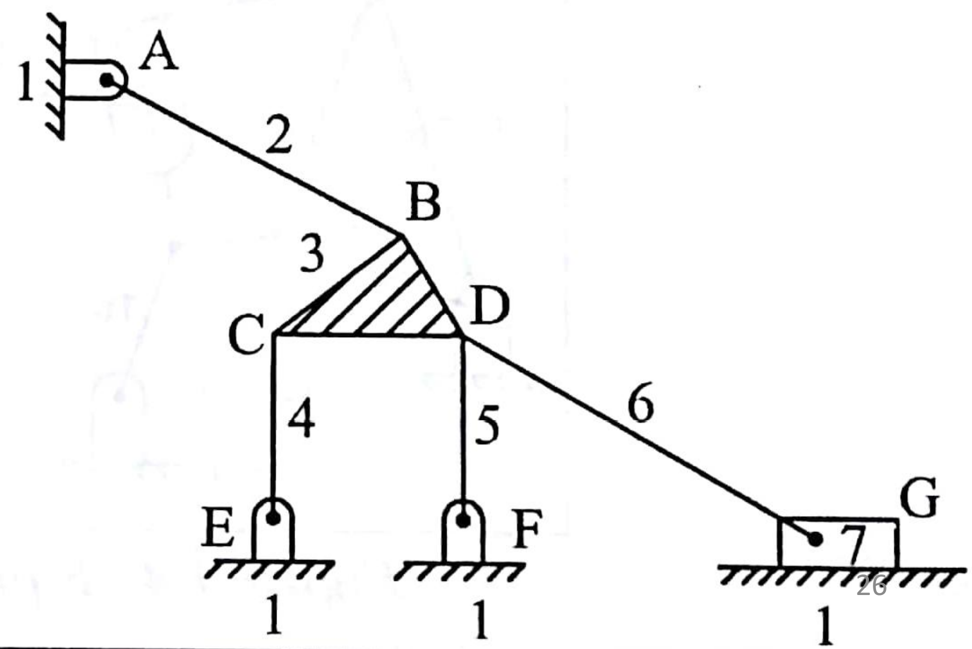
(v)

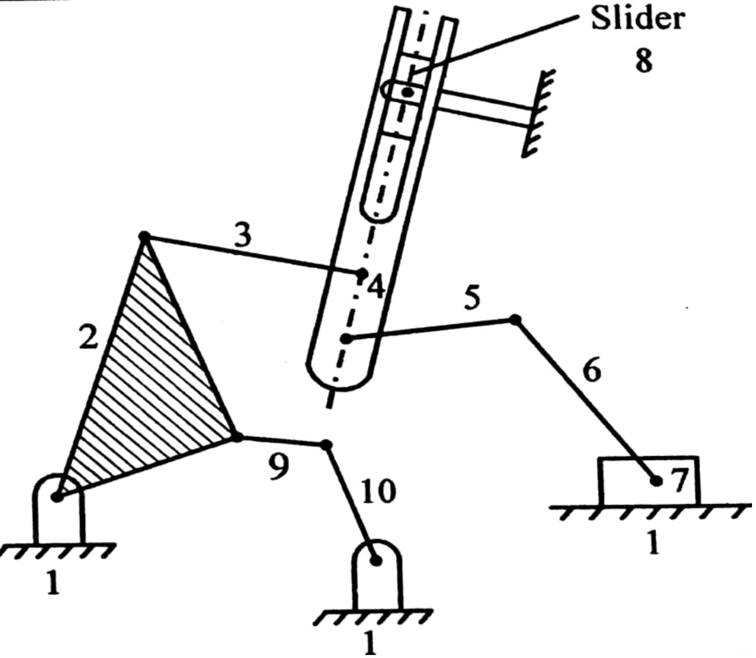


(vi)

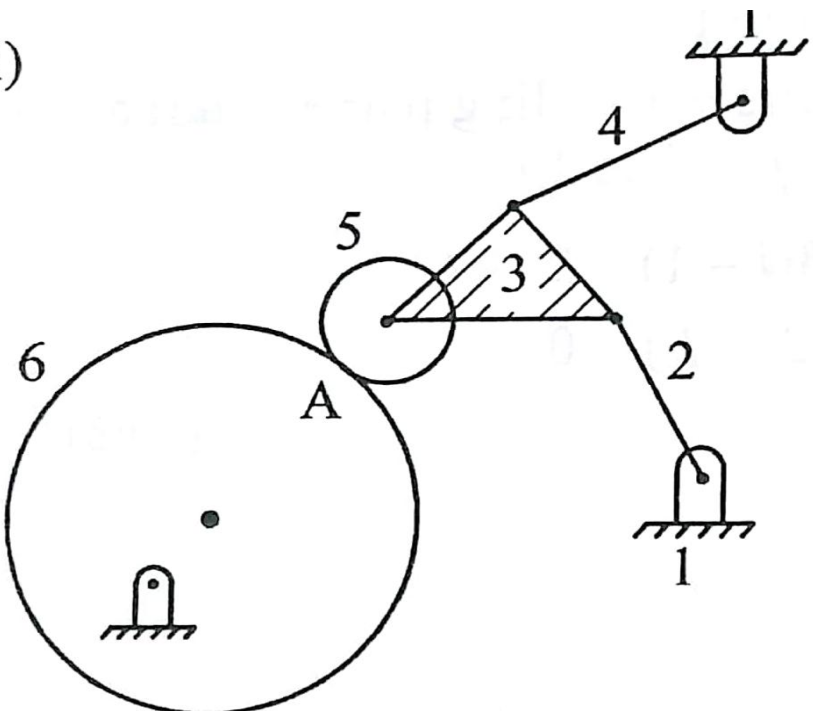


(vii)

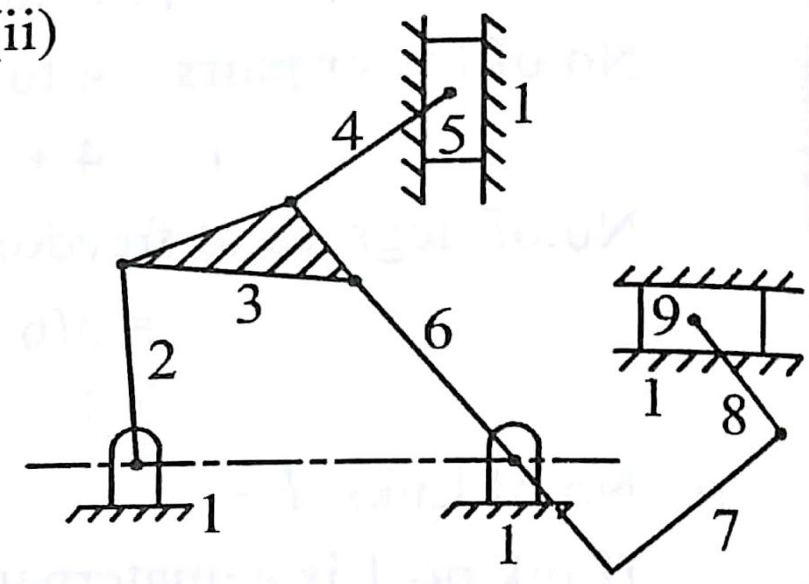


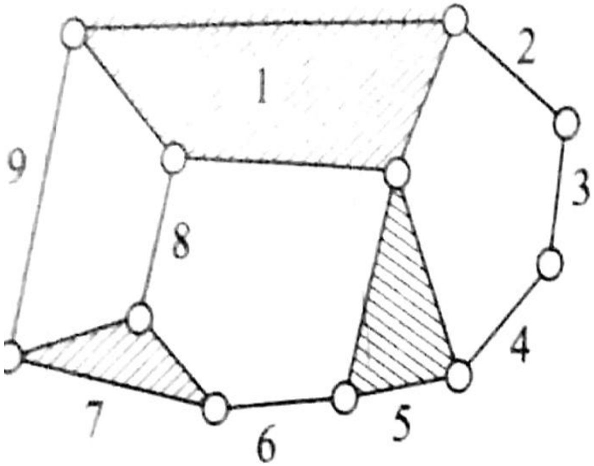


(i)

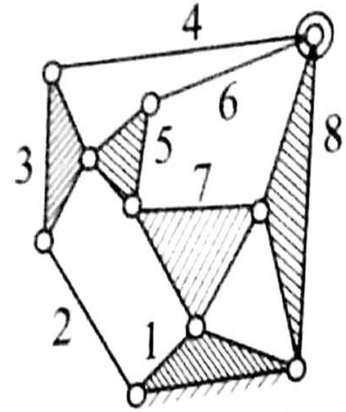


(ii)

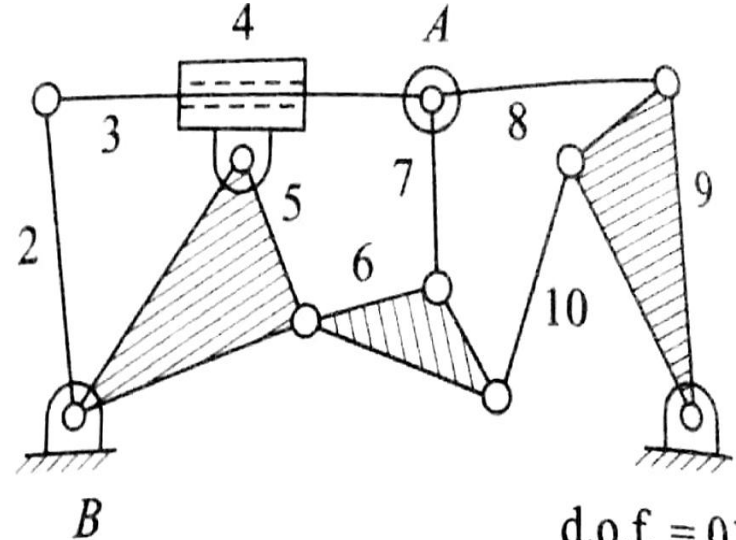




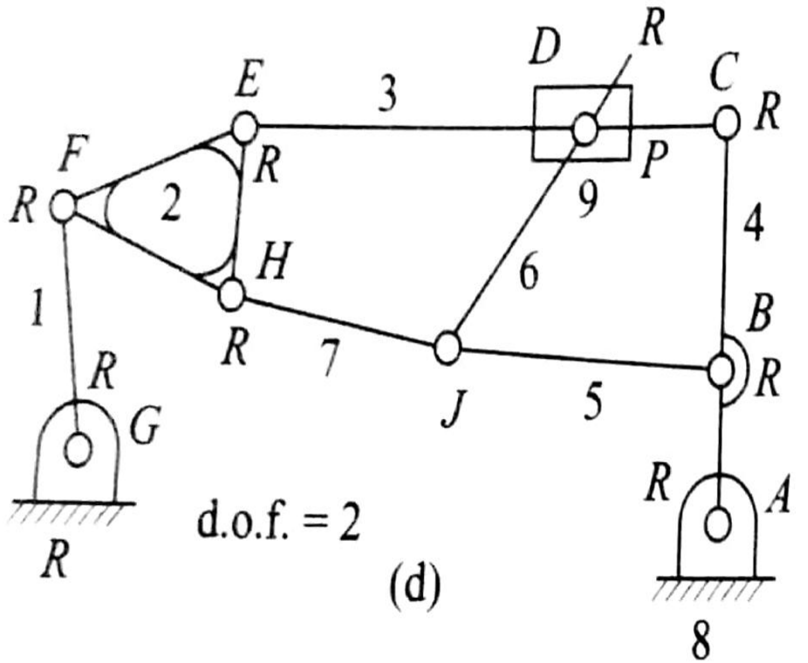
d.o.f. = 2
(a)



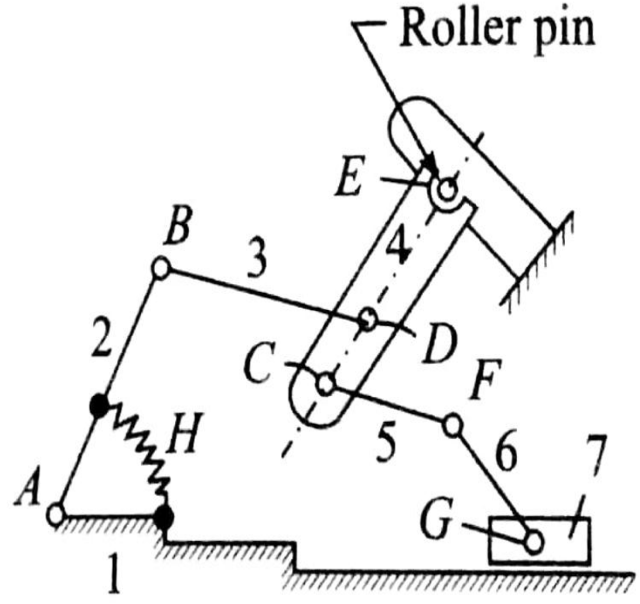
d.o.f. = −1
(b)



d.o.f. = 0)
(c)



d.o.f. = 2
(d)



d.o.f. = 3
(e)

Solution: (a) Here $n = 9$; $l = 11$

Therefore $\quad$ d.o.f. $= 3(9 - 1) - 2(11) = 2$

(b) Here $\quad n = 8$

$l = 9 + 2$ (on account of a double joint)

$= 11$

Therefore $\quad$ = d.o.f. $= 3(8 - 1) - 2(11)$

$= 21 - 22 = -1$ **Ans.**

i.e. the mechanism at Fig. 2.18(b) is a statically indeterminate structure.

(c) As in case (b), here too there are double joints at A & B. Hence

$$n = 10;\ l = 9 + 2(2) = 13$$

Therefore $\quad$ d.o.f. $= 3(10 - 1) - 2(13) = 1$ **Ans.**

(d) The mechanism at Fig. 2.18(d) has three ternary links (links 2, 3 and 4) and 5 binary links (links 1, 5, 6, 7 and 8) and one slider. It has 9 simple turning pairs marked R, one sliding pair marked P and one double joint at J. Since the double joint J joins 3 links, it may be taken equivalent to two simple turning pairs. Thus,

$$n = 9;\ l = 11$$

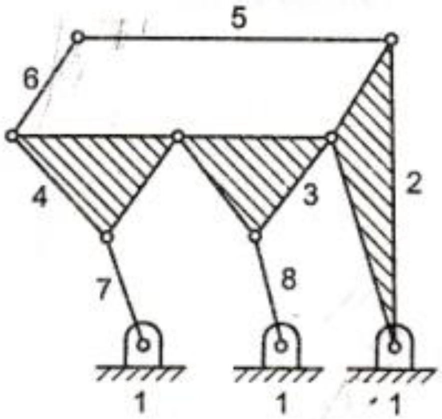
Therefore $\quad$ d.o.f. $= 3(9 - 1) - 2(11) = 2$ **Ans.**

(e) The mechanism at Fig. 2 .18(e) has a roller pin at E and a spring at H. The spring is only a device to apply force, and is not a link. Thus, there are 7 links numbered 1 through 7, one sliding pair, one rolling (higher) pairs at E besides 6 turning pairs

Thus $\quad n = 7$; $l = 7$ and $h = 1$

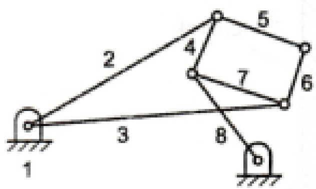
Therefore d.o.f., $\quad F = 3(7 - 1) - 2(7) - (1)$

$= 18 - 14 - 1 = 3$

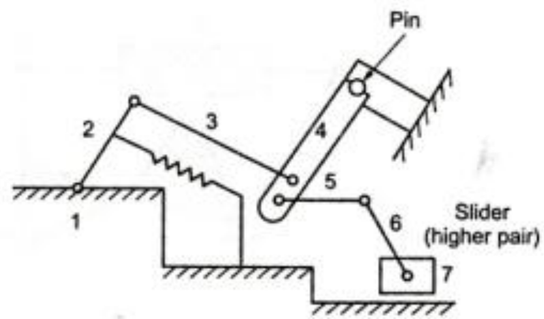


Degrees of freedom, $n = 3(l-1) - 2j$

$$n = 3(8-1) - 2 \times 10 = 1$$



(iii)



(iv)

(3) ***For Fig. (iii) :*** $l = 8$;

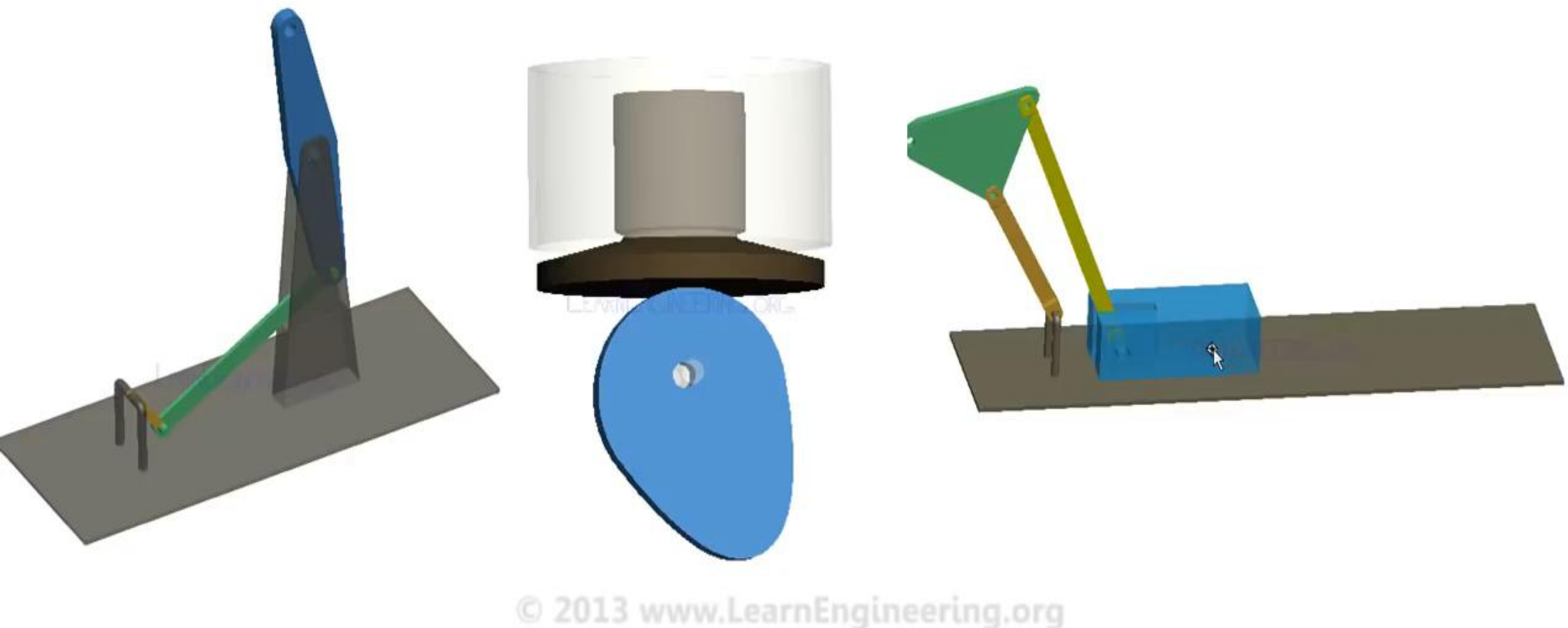
$j = 10$ [$\because$ two binary joints and four ternary joints, so $j = 2 + 4 \times 2 = 10$] ; $h = 0$

$$\therefore n = 3(l-1) - 2j - h$$
$$= 3(8-1) - 2 \times 10 - 0 = \mathbf{1} \quad \textbf{Ans.}$$

(4) ***For Fig (iv) :*** $l = 7$; $j = 7$; $h = 1$

$$\therefore n = 3(l-1) - 2j - h$$
$$= 3(7-1) - 2 \times 7 - 1 = \mathbf{3} \quad \textbf{Ans.}$$

DEGREES OF FREEDOM



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