Velocity in Mechanisms

Relative velocity Method

Relative Velocity of Two Bodies Moving in Straight Lines

- the relative velocity of two bodies moving along parallel lines are shown in Fig.
- Consider two bodies *A and B moving along parallel* lines in the same direction with absolute velocities V_A *and* V_B *such that* $V_A > V_B$ *, as shown in Fig. (a). The relative* velocity of *A with respect to B,*

$$v_{AB} = \text{Vector difference of } v_A \text{ and } v_B = \overline{v_A} - \overline{v_B}$$

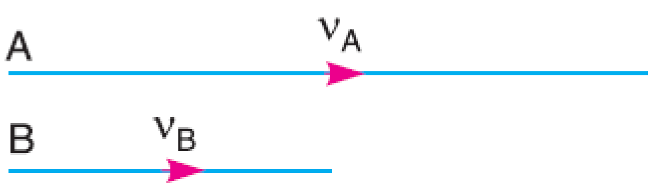
- the relative velocity of *A with respect to B (i.e.* V_{AB}*) may be written in the* vector form as follows :

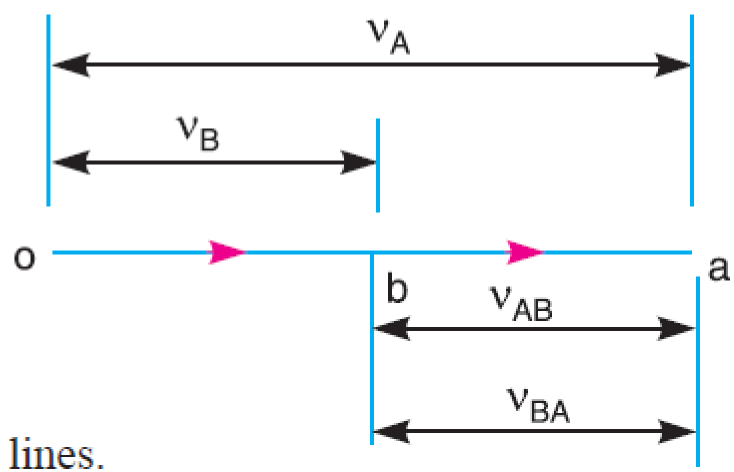
$$\overline{ba} = \overline{oa} - \overline{ob}$$

Similarly, the relative velocity of *B* with respect to *A* ,

$$v_{BA} = \text{Vector difference of } v_B \text{ and } v_A = \overline{v_B} - \overline{v_A}$$

$$\overline{ab} = \overline{ob} - \overline{oa}$$



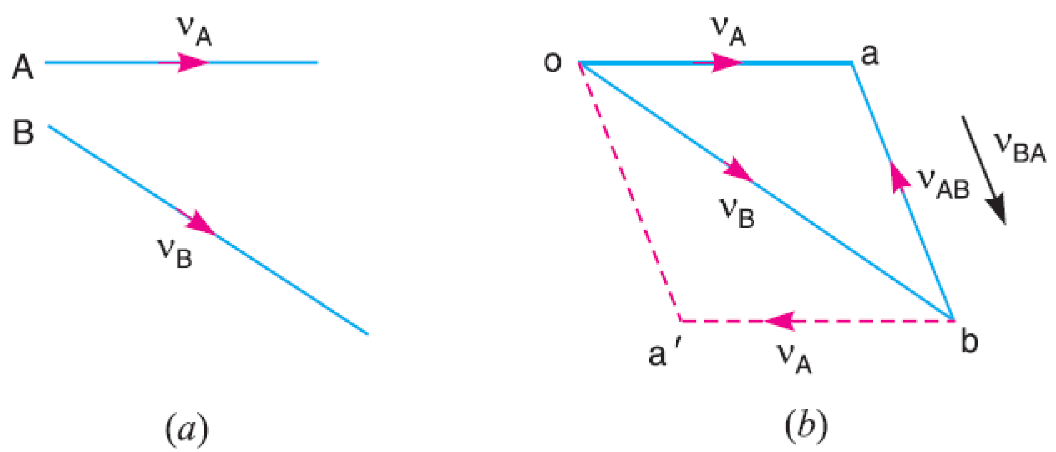


Relative velocity of two bodies moving along parallel lines.

(*a*) (*b*)

- consider the body *B moving in an* inclined direction as shown in Fig.
- *The* relative velocity of *A with respect to B may be* obtained by the law of parallelogram of velocities or triangle law of velocities.
- Take any fixed point *o and draw vector oa to represent* V_A *in* magnitude and direction to some suitable scale.
- Similarly, draw vector *ob to represent* V_B *in magnitude* and direction to the same scale.
- Then vector *ba represents the relative velocity of A with* respect to *B as shown in Fig.*
- *In the similar* way as discussed above, the relative velocity of *A with respect to B*

Note: It may be noted that to find v_{AB}, start from point b towards a and for v_{BA}, start from point a towards b.



Relative velocity of two bodies moving along inclined lines.

$$v_{AB} = \text{Vector difference of } v_A \text{ and } v_B = \overline{v_A} - \overline{v_B}$$

$$\overline{ba} = \overline{oa} - \overline{ob}$$

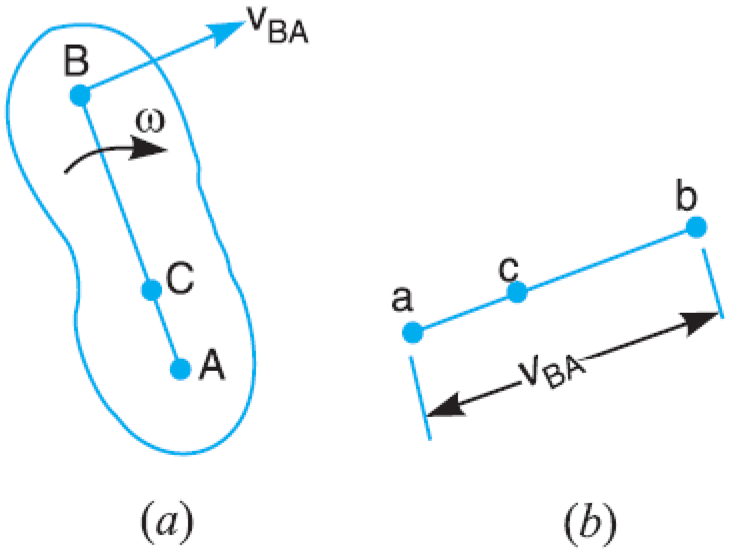
- we conclude that the relative velocity of point *A with respect to B (V_{AB}) and the* relative velocity of point *B with respect A (V_{BA}) are equal in magnitude but opposite in direction*

$$v_{\mathrm{AB}} = -v_{\mathrm{BA}} \quad \text{or} \quad \overline{ba} = -\overline{ab}$$

Motion of Link

- Consider two points *A and B on a rigid link AB, as* shown in Fig. (*a*).
- *Let one of the extremities (B) of the link* move relative to *A, in a clockwise direction.*
- *Since the distance* from *A to B remains the same, therefore there can be no* relative motion between *A and B, along the line AB.*
- *It is thus* obvious, that the relative motion of *B with respect to A must* be perpendicular to *AB.*

Hence ***velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.***



- The relative velocity of *B with respect to A (i.e. V_{BA}) is represented by the vector ab and is* perpendicular to the line *AB as shown in Fig.*
- we see from equation ***(iii), that the point c on the vector ab divides it in the same ratio*** as *C divides the link AB*

Let ω = Angular velocity of the link AB about A.

We know that the velocity of the point B with respect to A,

$$v_{BA} = \overline{ab} = \omega . AB \quad \ldots(i)$$

Similarly, the velocity of any point C on AB with respect to A,

$$v_{CA} = \overline{ac} = \omega . AC \quad \ldots(ii)$$

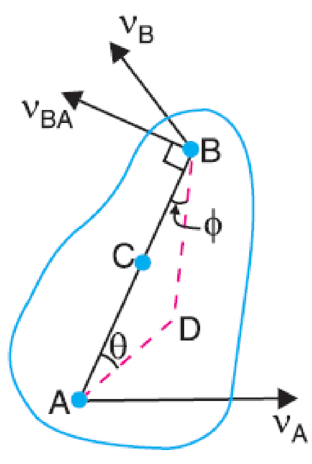
From equations (*i*) and (*ii*),

$$\frac{v_{CA}}{v_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{\omega . AC}{\omega . AB} = \frac{AC}{AB} \quad \ldots(iii)$$

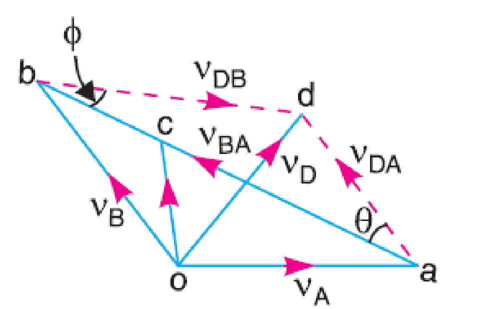
Note: The relative velocity of A with respect to B is represented by *ba*, although A may be a fixed point. The motion between A and B is only relative. Moreover, it is immaterial whether the link moves about A in a clockwise direction or about B in a clockwise direction.

Velocity of a Point on a Link by Relative Velocity Method

- Consider two points *A and B on a link as shown in Fig. (a).*
- *Let the absolute velocity of the* point *A i.e.*V_A *is known in magnitude and direction*
- *The absolute velocity of the point B i.e.*V_B *is* known in direction only.
- Then the velocity of *B may be determined by drawing the velocity diagram* as shown in Fig. (*b).*
- *The velocity diagram is drawn as follows :*
- Take some convenient point *o, known as the pole.*
- Through *o, draw oa parallel and equal to* V_A*, to some suitable scale.*
- Through *a, draw a line perpendicular to AB of Fig. a. This line will represent the*
- velocity of *B with respect to A, i.e.* V_{BA}*.*
- Through *o, draw a line parallel to* V_B *intersecting the line of* V_{BA} *at b*
- Measure *ob, which gives the required velocity of point B (*V_B*), to the scale*



Motion of points on a link.



(*b*) Velocity diagram.

Notes : 1. The vector *ab* which represents the velocity of *B* with respect to *A* (v_{BA}) is known as velocity of image of the link *A B*.

2. The absolute velocity of any point *C* on *A B* may be determined by dividing vector *ab* at *c* in the same ratio as *C* divides *A B* in Fig. 7.4 (*a*).

$$\frac{ac}{ab} = \frac{AC}{AB}$$

- Join *oc. The *vector oc represents the absolute velocity* of point *C (V_C) and the vector ac represents the velocity of C* with respect to *A i.e. V_{CA}.*

The absolute velocity of any other point *D outside*

- *AB, as shown in Fig. a), may also be obtained by completing* the velocity triangle *abd and similar to triangle ABD,* as shown in Fig. (*b).*

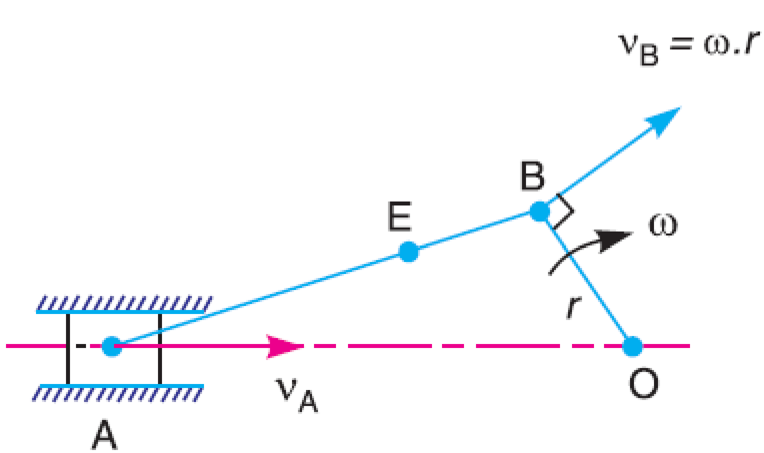
The angular velocity of the link *AB may be found*

- by dividing the relative velocity of *B with respect to A (i.e.*
- *V_{BA}) to the length of the link AB. Mathematically, angular*
- velocity of the link *AB*

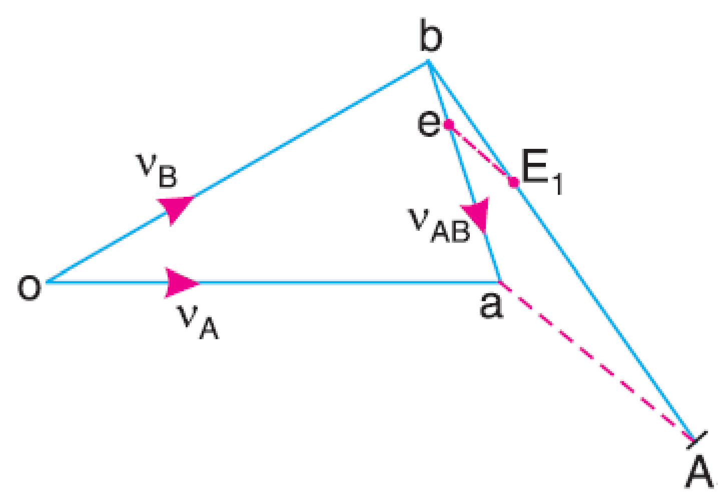
$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB}$$

Velocities in Slider Crank Mechanism

- The slider *A is attached to the connecting* rod *AB.*
- *Let the radius of crank OB be r and let it rotates in a clockwise direction, about the point O* with uniform angular velocity ω rad/s.
- Therefore, the velocity of *B i.e.* V_B *is known in magnitude and* direction. The slider reciprocates along the line of stroke *AO*



(*a*) Slider crank mechanism.



(*b*) Velocity diagram.

- From any point *o, draw vector ob parallel to the direction of V_B (or perpendicular to OB)*
- such that *ob = V_B = ω.r, to some suitable scale, as shown in Fig. (b)*
- Since *AB is a rigid link, therefore the velocity of A relative to B is perpendicular to AB.*
- Now draw vector *ba perpendicular to AB to represent the velocity of A with respect to B i.e. V_{AB}.*

From point *o, draw vector oa parallel to the path of motion of the slider A (which is along*

- *AO only). The vectors ba and oa intersect at a. Now oa represents the velocity of the slider A i.e. vA,*
- to the scale.

The angular velocity of the connecting rod *A B* (ω_{AB}) may be determined as follows:

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB} \qquad \text{(Anticlockwise about A)}$$

The direction of vector *ab* (or *ba*) determines the sense of ω_{AB} which shows that it is anticlockwise.

Note : The absolute velocity of any other point *E* on the connecting rod *AB* may also be found out by dividing vector *ba* such that $be/ba = BE/BA$. This is done by drawing any line bA_1 equal in length of BA. Mark $bE_1 = BE$. Join aA_1. From E_1 draw a line E_1e parallel to aA_1. The vector *oe* now represents the velocity of *E* and vector *ae* represents the velocity of *E* with respect to *A* .

Rubbing Velocity at a Pin Joint

The links in a mechanism are mostly connected by means of pin joints. The rubbing velocity is defined as **the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.**

Consider two links *OA* and *OB* connected by a pin joint at *O* as shown in Fig. 7.6.

Let ω_1 = Angular velocity of the link *OA* or the angular velocity of the point *A* with respect to *O*.

ω_2 = Angular velocity of the link *OB* or the angular velocity of the point *B* with respect to *O*, and

r = Radius of the pin.

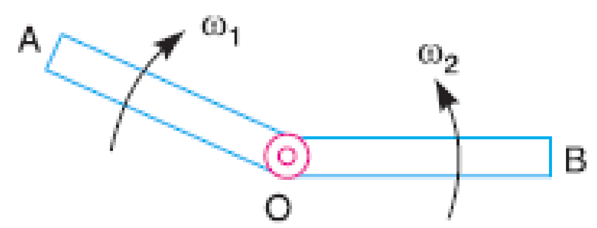


Fig. 7.6. Links connected by pin joints.

According to the definition,

Rubbing velocity at the pin joint *O*

$= (\omega_1 - \omega_2)\, r$, if the links move in the same direction

$= (\omega_1 + \omega_2)\, r$, if the links move in the opposite direction

Note : When the pin connects one sliding member and the other turning member, the angular velocity of the sliding member is zero. In such cases,

Rubbing velocity at the pin joint = $\omega . r$

where ω = Angular velocity of the turning member, and

r = Radius of the pin.