The Penrose package

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1.0 from 2018/07/10

1 Introduction

This is a TikZ library for drawing Penrose tiles (kite/dart, rhombus, and pentagon versions). It provides two methods of drawing: one in which an automatic pattern is built, and one where the tiles can be placed "by hand". The tiles can be shaped and (hopefully!) still fit together. For user documentation, see the penrose.pdf file.

2 Implementation

2.1 Initialisation

```
1 (@@=penrose)
```

We use the spath3 library for manipulating the paths that will make up the tiles.

2 \RequirePackage{spath3}

Now we move in to the realm of LATEX3.

3 \ExplSyntaxOn

Start with some basic paths (lines) for the sides of the tiles so that we know that we have well-defined tiles at the outset.

```
4 \MakeSPath{Penrose path a}

5 {

6 \pgfsyssoftpath@movetotoken{0pt}{0pt}}

7 \pgfsyssoftpath@linetotoken{1pt}{0pt}

8 }

9 \SPathPrepare{Penrose path a}

10 \CloneSPath {Penrose path a}{Penrose path b}

11 \CloneSPath {Penrose path a}{Penrose path c}

12 \CloneSPath {Penrose path a}{Penrose path d}

13 \CloneSPath {Penrose path a}{Penrose path A}

14 \CloneSPath {Penrose path a}{Penrose path B}

15 \CloneSPath {Penrose path a}{Penrose path C}

16 \CloneSPath {Penrose path a}{Penrose path D}
```

```
\lambda_penrose_tmpa_fp \l_penrose_tmpb_fp \l_penrose_tmpc_fp \l_penrose_tmpa_tl \l_penrose_tmpb_tl \l_penrose_tmpc_tl
```

We need a few temporary variables to hold intermediate calculations.

```
17 \fp_new:N \l__penrose_tmpa_fp
18 \fp_new:N \l__penrose_tmpb_fp
19 \fp_new:N \l__penrose_tmpc_fp
20 \tl_new:N \l__penrose_tmpa_tl
21 \tl_new:N \l__penrose_tmpb_tl
22 \tl_new:N \l__penrose_tmpc_tl
```

2.2 Creating the Tiles

\penrose_normalise_path:n

When defining the path for a side, we normalise so that it starts at the origin and ends at (1pt,0pt).

```
23 \cs_new_nopar:Npn \penrose_normalise_path:n #1
24 f
```

Get the initial point of the path and convert to floating point.

```
\spath_get:nnN {#1} {initial point} \l__penrose_tmpa_tl

fp_set:Nn \l__penrose_tmpa_fp {\tl_head:N \l__penrose_tmpa_tl}

tl_set:Nx \l__penrose_tmpa_tl {\tl_tail:N \l__penrose_tmpa_tl}

fp_set:Nn \l__penrose_tmpb_fp {\tl_head:N \l__penrose_tmpa_tl}
```

Get the final point of the path, and compute the difference of the final and initial points. The resulting numbers, say a and b, will be put into a matrix to rotate and scale the path. The formula for the matrix is:

$$\frac{1}{a^2 + b^2} \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

```
\spath_get:nnN {#1} {final point} \l__penrose_tmpa_tl

\fp_set:Nn \l__penrose_tmpa_fp

\tl_head:N \l__penrose_tmpa_tl - \l__penrose_tmpa_fp}

\tl_set:Nx \l__penrose_tmpa_tl {\tl_tail:N \l__penrose_tmpa_tl}

\fp_set:Nn \l__penrose_tmpb_fp

\t\t\l_head:N \l__penrose_tmpa_tl - \l__penrose_tmpb_fp}

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```

Now compute the square of the length of the path for scaling.

```
35 \fp_set:Nn \l__penrose_tmpc_fp
36 {\l__penrose_tmpa_fp^2 + \l__penrose_tmpb_fp^2}
37 \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_tmpa_fp/\l__penrose_tmpc_fp}
38 \fp_set:Nn \l__penrose_tmpb_fp {\l__penrose_tmpb_fp/\l__penrose_tmpc_fp}
39 \fp_set:Nn \l__penrose_tmpc_fp {-\l__penrose_tmpb_fp}
```

Now construct the matrix.

Get the initial point back again for the translation part.

```
47 \spath_get:nnN {#1} {initial point} \l__penrose_tmpa_tl
```

```
But we need to premultiply by the matrix because of how the transformations are applied.

48 \fp_set:Nn \l__penrose_tmpa_fp

49 {
50 (-1) * \l__penrose_tmpa_fp * \tl_head:N \l__penrose_tmpa_tl
```

(-1) * \l_penrose_tmpa_fp * \tl_head:N \l_penrose_tmpa_tl
+ (-1) * \l_penrose_tmpb_fp * \tl_tail:N \l_penrose_tmpa_tl
}

fp_set:Nn \l_penrose_tmpb_fp

{
(-1) * \l_penrose_tmpa_fp * \tl_tail:N \l_penrose_tmpa_tl

\l_penrose_tmpb_fp * \tl_head:N \l_penrose_tmpa_tl

Finally, we apply the transformation to the path.

```
58  \tl_put_right:Nx \l_penrose_tmpb_tl {
59      {\fp_to_dim:N \l_penrose_tmpa_fp}}
60      {\fp_to_dim:N \l_penrose_tmpb_fp}
61  }
62  \spath_transform:nV {#1} \l_penrose_tmpb_tl
63 }
```

(End definition for \penrose_normalise_path:n.)

 $\verb|\SetPenrosePath|$

56 57

This sets the path corresponding to a particular side to the current path, and normalises it.

```
64 \NewDocumentCommand \SetPenrosePath { m }
65 {
66   \pgfsyssoftpath@getcurrentpath\l__penrose_tmpa_tl
67   \spath_clear_new:n {Penrose path #1}
68   \spath_put:nnV {Penrose path #1} {path} \l__penrose_tmpa_tl
69   \penrose_normalise_path:n {Penrose path #1}
70 }
```

(End definition for \SetPenrosePath.)

(End definition for \tikz_scan_point:n.)

\tikz_scan_point:n

This is a wrapper around \tikz@scan@one@point to make it easier to use with IATEX3 variables.

```
71 \cs_new_nopar:Npn \tikz_scan_point:n #1
72 {
73  \tikz@scan@one@point\pgfutil@firstofone#1\relax
74 }
75 \cs_generate_variant:Nn \tikz_scan_point:n {V}
```

\penrose_make_tile:nnn

This builds the tile path from its pieces. The arguments are the name of the tile, the descriptions of the sides, and a token list of the coordinates.

```
76 \cs_new_nopar:Npn \penrose_make_tile:nnn #1#2#3
77 {
```

Get the first coordinate and initialise the path with a move to this point.

```
78  \tl_set:Nn \l__penrose_tmpa_tl {#3}
79  \tl_set:Nx \l__penrose_tmpb_tl {\tl_head:N \l__penrose_tmpa_tl}
80  \tl_set:Nn \l__penrose_tmpa_tl {\pgfsyssoftpath@movetotoken}
81  \tikz_scan_point:V \l__penrose_tmpb_tl
82  \tl_put_right:Nx \l__penrose_tmpa_tl
```

```
83
        {\dim_use:N \pgf@x}{\dim_use:N \pgf@y}
 84
  85
      \spath_clear_new:n {Penrose path tile #1}
  86
      \spath_put:nnV {Penrose path tile #1} {path} \l__penrose_tmpa_tl
Now we have our path initialised, we can start appending the side paths according to the
specification in the second argument.
    We append the initial coordinate to the end of the list to make a closed cycle.
      \tl_set:Nn \l__penrose_tmpa_tl {#3}
      \tl_put_right:Nx \l__penrose_tmpa_tl {{\tl_head:N \l__penrose_tmpa_tl}}
Now we walk through the description of the sides, adding the specified paths to our tile
path.
      \tl_map_inline:nn {#2} {
  90
Clone the path for this side.
        \spath_clone:nn {Penrose path ##1} {Penrose path tmpa}
Strip off the next coordinate, and convert it to a point.
        \tl_set:Nx \l__penrose_tmpb_tl {\tl_head:N \l__penrose_tmpa_tl}
        \tl_set:Nx \l__penrose_tmpa_tl {\tl_tail:N \l__penrose_tmpa_tl}
        \tikz_scan_point:V \l__penrose_tmpb_tl
Store the resulting coordinate.
        \fp_set:Nn \l__penrose_tmpa_fp { \pgf@x }
        \fp_set:Nn \l__penrose_tmpb_fp { \pgf@y }
Now get the next coordinate.
 97
        \tl_set:Nx \l__penrose_tmpb_tl {\tl_head:N \l__penrose_tmpa_tl}
        \tikz_scan_point:V \l__penrose_tmpb_tl
We want the difference between the two coordinates.
        \fp_set:Nn \l__penrose_tmpa_fp {\pgf@x - \l__penrose_tmpa_fp}
        \fp_set:Nn \l__penrose_tmpb_fp {\pgf@y - \l__penrose_tmpb_fp}
 100
This is converted into a transformation matrix.
        \fp_set:Nn \l__penrose_tmpc_fp {-\l__penrose_tmpb_fp}
        \tl_set:Nx \l__penrose_tmpb_tl
 102
          {\fp_use:N \l__penrose_tmpa_fp}
 104
          {\fp_use:N \l__penrose_tmpc_fp}
 105
          {\fp_use:N \l__penrose_tmpb_fp}
 106
          {\fp_use:N \l__penrose_tmpa_fp}
 107
          {0}
 108
          {0}
 109
The transformation is applied to the cloned path.
        \spath_transform:nV {Penrose path tmpa} \l__penrose_tmpb_tl
And this is welded to the tile path.
        \spath_weld:nn {Penrose path tile #1} {Penrose path tmpa}
      }
At the end we close the path.
      \spath_close_path:n {Penrose path tile #1}
 115 }
```

(End definition for \penrose_make_tile:nnn.)

\penrose_make_tile:nn

A wrapper around the above which allows us to specify the second two arguments as two items in a token list.

```
116 \cs_new_nopar:Npn \penrose_make_tile:nn #1#2
 118
      \penrose_make_tile:nnn {#1} #2
 119 }
 120 \cs_generate_variant:Nn \penrose_make_tile:nn {nV}
(End definition for \penrose_make_tile:nn.)
```

Specifying the Tiles 2.3

The tile specifications are contained in a prop.

```
121 \prop_new:N \g_penrose_tiles_prop
```

\tl_add_coordinate: Nnn Process a coordinate through fp and adds it to a token list.

```
122 \cs_new_nopar:Npn \tl_add_coordinate:Nnn #1#2#3 {
     \fp_set:Nn \l__penrose_tmpa_fp{#2}
     \fp_set:Nn \l__penrose_tmpb_fp{#3}
124
     \tl_put_right:Nx #1
125
126
       {(\fp_use:N \l__penrose_tmpa_fp, \fp_use:N \l__penrose_tmpb_fp)}
    }
128
129 }
```

(End definition for \tl_add_coordinate:Nnn.)

Now we specify the tiles. The specification is a clockwise list of the vertices together with the labels of the corresponding sides. There are three basic paths, a, b, c, and their complements (which are capitalised).

• Thin Rhombus.

```
\tl_clear:N \l__penrose_tmpa_tl
        \tl_add_coordinate:Nnn \l__penrose_tmpa_t1 {0}{0}
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {cosd(18)}{sind(18)}
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {2*cosd(18)}{0}
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {cosd(18)}{-sind(18)}
134
135
        \prop_gput:\nx \g__penrose_tiles_prop {thin~ rhombus}
136
        {{a A B b} {\tl_use:N \l__penrose_tmpa_tl}}
```

• Thick Rhombus.

```
\tl_clear:N \l__penrose_tmpa_tl
138
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {0}{0}
139
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {cosd(36)}{sind(36)}
141
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {2*cosd(36)}{0}
142
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {cosd(36)}{-sind(36)}
143
        \prop_gput:Nnx \g__penrose_tiles_prop {thick~ rhombus}
144
        {{B a A b} {\tl_use:N \l__penrose_tmpa_tl}}
145
```

• Dart.

```
\tl_clear:N \l__penrose_tmpa_tl
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {0}{0}
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl
148
        {2*sind(18)*cosd(108)}{2*sind(18)*sind(108)}
149
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {2*sind(18)}{0}
150
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl
        {2*sind(18)*cosd(108)}{-2*sind(18)*sind(108)}
154
        \prop_gput:\nx \g__penrose_tiles_prop {dart}
        {{c a A C} {\tl_use:N \l__penrose_tmpa_tl}}
155
 • Kite.
        \tl_clear:N \l__penrose_tmpa_tl
156
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {0}{0}
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {cosd(36)}{sind(36)}
158
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {1}{0}
159
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {cosd(36)}{-sind(36)}
160
161
        \prop_gput:Nnx \g_penrose_tiles_prop {kite}
162
        {{a c C A} {\tl_use:N \l__penrose_tmpa_tl}}
   Golden Triangle.
        \tl_clear:N \l__penrose_tmpa_tl
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {0}{0}
165
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {cosd(18)}{sind(18)}
166
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {cosd(18)}{-sind(18)}
167
168
        \prop_gput:\nx \g__penrose_tiles_prop {golden~ triangle}
169
        {{a c b} {\tl_use:N \l__penrose_tmpa_tl}}
   Reverse Golden Triangle.
        \tl_clear:N \l__penrose_tmpa_tl
171
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {0}{0}
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {cosd(18)}{sind(18)}
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {cosd(18)}{-sind(18)}
174
175
        \prop_gput:\nx \g__penrose_tiles_prop {reverse~ golden~ triangle}
176
        {{B C A} {\tl_use:N \l_penrose_tmpa_tl}}
177
   Golden Gnomon
        \tl_clear:N \l__penrose_tmpa_tl
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {0}{0}
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {cosd(36)}{sind(36)}
180
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {2*cosd(36)}{0}
181
182
        \prop_gput:Nnx \g_penrose_tiles_prop {golden~ gnomon}
183
        {{C b A} {\tl_use:N \l_penrose_tmpa_tl}}
184
```

• Reverse Golden Gnomon

```
\tl_clear:N \l__penrose_tmpa_tl
                        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {0}{0}
                        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {2*cosd(36)}{0}
187
                        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {cosd(36)}{-sind(36)}
188
                        \prop_gput:Nnx \g__penrose_tiles_prop {reverse~ golden~ gnomon}
189
                        {{a B c} {\tl_use:N \l_penrose_tmpa_tl}}
190
          Primary Pentagon (pentagon 5)
                        \tl_clear:N \l__penrose_tmpa_tl
                        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {0}{0}
                        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {cosd(108)}{sind(108)}
                        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl
                        {1+\cos d(72)+\cos d(144)}{\sin d(72)+\sin d(144)}
195
                        \label{lem:loss} $$ \tilde{1}_{add\_coordinate:Nnn \l_penrose_tmpa_tl \{1+\cos d(72)\}\{\sin d(72)\} } $$
196
                        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {1}{0}
197
                        \prop_gput:Nnx \g_penrose_tiles_prop {pentagon~ 5}
198
                        {{a a a a a} {\tl_use:N \l__penrose_tmpa_tl}}
199
           Secondary Pentagon (pentagon 3)
                        \prop_gput:\nx \g__penrose_tiles_prop {pentagon~ 3}
                        {{A b a a b} {\tl_use:N \l__penrose_tmpa_tl}}
           Tertiary Pentagon (pentagon 2)
                        \prop_gput:Nnx \g__penrose_tiles_prop {pentagon~ 2}
202
                        {{d A c c A} {\tl_use:N \l__penrose_tmpa_tl}}
203
           Pentagram
                        \tl_clear:N \l__penrose_tmpa_tl
204
                        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {1}{0}
205
                        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {1-cosd(36)}{-sind(36)}
                        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl
207
                        {1-\cos d(36)-\cos d(108)}{-\sin d(36)-\sin d(108)}
208
                        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {cosd(108)}{-sind(108)}
209
                        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl
                        {-1+3*cosd(108)+cosd(36)}{-sind(36)-sind(108)}
                        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl
                        {-1+2*cosd(108)+cosd(36)}{-sind(36)}
                        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {-1+2*cosd(108)}{0}
214
                        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {2*cosd(108)}{0}
                        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {cosd(108)}{sind(108)}
                        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {0}{0}
                        \prop_gput:\nx \g_penrose_tiles_prop {pentagram}
218
                         \{ \{ \texttt{C} \ \texttt{
219
          Boat
                        \tl_clear:N \l__penrose_tmpa_tl
```

```
\tl_add_coordinate:Nnn \l__penrose_tmpa_tl {-1+2*cosd(108)}{0}
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {2*cosd(108)}{0}
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {cosd(108)}{sind(108)}
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {0}{0}
224
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {1}{0}
225
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl {1-cosd(36)}{-sind(36)}
226
        \tl_add_coordinate:Nnn \l__penrose_tmpa_tl
227
        \{-1+2*\cos d(108)+\cos d(36)\}\{-\sin d(36)\}
228
        \prop_gput:\nx \g__penrose_tiles_prop {boat}
229
        {{C C C C B D B} {\tl_use:N \l__penrose_tmpa_tl}}
230
```

• Diamond.

```
\tl_clear:N \l_penrose_tmpa_tl

\tl_add_coordinate:Nnn \l_penrose_tmpa_tl {0}{0}

\tl_add_coordinate:Nnn \l_penrose_tmpa_tl {cosd(18)}{sind(18)}

\tl_add_coordinate:Nnn \l_penrose_tmpa_tl {2*cosd(18)}{0}

\tl_add_coordinate:Nnn \l_penrose_tmpa_tl {cosd(18)}{-sind(18)}

\triadd_coordinate:Nnn \l_penrose_tmpa_tl {cosd(18)}{-sind(18)}

\triadd_coordin
```

\MakePenroseTile This is the user wrapper around the tile creation macros.

```
238 \NewDocumentCommand \MakePenroseTile {m}
239 {
240  \prop_get:\Nn\ \g_penrose_tiles_prop {#1} \l_penrose_tmpa_tl
241  \penrose_make_tile:\nV {#1} \l_penrose_tmpa_tl
242 }
```

(End definition for \MakePenroseTile.)

\UsePenroseTile This is the command that actually places a tile on the page. The first argument is optional and is for styling.

```
_{243} \NewDocumentCommand \UsePenroseTile {0{} m} _{244} {
```

We need to transform the tile to correspond to the current transformation matrix. To ensure that we only transform the current tile, we clone it first.

```
\spath_clone:nn {Penrose path tile #2} {Penrose path tmpa}
```

The transformation matrix returned by PGF appears to be transposed from what it should be. (This needs a little more investigation, it might be that I've implemented the multiplication incorrectly here.)

```
\pgfgettransform \l__penrose_tmpa_tl
246
     \tl_clear:N \l__penrose_tmpb_tl
247
     \tl_set:Nx \l__penrose_tmpb_tl {{\tl_head:N \l__penrose_tmpa_tl}}
248
     \tl_set:Nx \l__penrose_tmpa_tl {\tl_tail:N \l__penrose_tmpa_tl}
249
     \tl_put_right:Nx \l__penrose_tmpb_tl
250
251
       {\tl_item:Nn \l__penrose_tmpa_tl {2}}
     \tl_put_right:Nx \l__penrose_tmpb_tl
255
       {\tl_item:Nn \l__penrose_tmpa_tl {1}}
256
257
```

```
\tl_set:Nx \l__penrose_tmpa_tl {\tl_tail:N \l__penrose_tmpa_tl}
      \tl_set:Nx \l__penrose_tmpa_tl {\tl_tail:N \l__penrose_tmpa_tl}
 259
      \tl_put_right:NV \l__penrose_tmpb_tl \l__penrose_tmpa_tl
 260
Apply the transformation, protocol the path, and render it.
      \spath_transform:nV {Penrose path tmpa} \l__penrose_tmpb_tl
      \spath_protocol_path:n {Penrose path tmpa}
      \spath_tikz_path:nn {#1}{Penrose path tmpa}
 263
 264 }
This is a style for a user to take a path and make it into the path for one of the
sides. It needs to store both that side and the reverse.
 265 \tikzset{
      save~ Penrose~ path/.code={
        \tikz@addmode{
 267
Get the current path.
          \pgfsyssoftpath@getcurrentpath\l__penrose_tmpa_tl
Clear the receiving path, and store the current path in it.
          \spath_clear_new:n {Penrose path #1}
          \spath_put:nnV {Penrose path #1} {path} \l__penrose_tmpa_tl
 270
Normalise the path.
          \penrose_normalise_path:n {Penrose path #1}
Now create the reverse path. The name is the upper case version.
          \tex_uppercase:D {\tl_set:Nx \l__penrose_tmpa_tl {#1}}
Clone the path.
          \spath_clone:nn {Penrose path #1}
          {Penrose path \tl_use:N \l__penrose_tmpa_tl }
 274
Reverse it.
          \spath_reverse:n {Penrose path \tl_use:N \l__penrose_tmpa_tl}
Swap the start and end.
          \spath_transform:nnnnnnn {Penrose path \tl_use:N \l__penrose_tmpa_tl}
 276
          {-1} {0} {0} {-1} {1} {0}
 277
        }
 278
      },
 279
      expand~ key/.code={
 280
        \exp_args:NV \pgfkeysalso #1
 281
      }
 282
 283 }
    Create the basic tile shapes.
 284 \MakePenroseTile {thin~ rhombus}
 285 \MakePenroseTile {thick~ rhombus}
 286 \MakePenroseTile {dart}
 287 \MakePenroseTile {kite}
 288 \MakePenroseTile {golden~ triangle}
 289 \MakePenroseTile {reverse~ golden~ triangle}
 290 \MakePenroseTile {golden~ gnomon}
 291 \MakePenroseTile {reverse~ golden~ gnomon}
 292 \MakePenroseTile {pentagon~ 5}
```

```
293 \MakePenroseTile {pentagon~ 3}
294 \MakePenroseTile {pentagon~ 2}
295 \MakePenroseTile {pentagram}
296 \MakePenroseTile {boat}
297 \MakePenroseTile {diamond}
```

2.4 Lindenmayer System

This is an implementation of the Lindenmayer System description of Penrose tilings as a way of generating tilings from a specific starting seed.

The implementation uses props to store *rules* and *actions*. The rules are used to expand the starting seed to a certain level, after which the actions are carried out. The syntax is based on the PGF library, but as we're already using LaTeX3 it is reimplemented in that.

These are the rules for generating rhombus tilings.

```
298 \prop_new:N \g_penrose_rhombus_lms_rule_prop
299 \prop_put:Nnn \g_penrose_rhombus_lms_rule_prop {T} {[f*sT][f>g]}
300 \prop_put:Nnn \g_penrose_rhombus_lms_rule_prop {t} {[f_st][f>G]}
yprop_put:Nnn \g_penrose_rhombus_lms_rule_prop {G} {[f+sG][sf>g][sf*sT]}
302 \prop_put:Nnn \g_penrose_rhombus_lms_rule_prop {g} {[f-sg][sf>G][sf_st]}
   These are the rules for generating kite and dart tilings.
303 \prop_new:N \g__penrose_kite_lms_rule_prop
304 \prop_put:Nnn \g__penrose_kite_lms_rule_prop {T} {[f*sT][f>st][+sg]}
305 \prop_put:Nnn \g__penrose_kite_lms_rule_prop {t} {[f_st][f>sT][-sG]}
306 \prop_put:Nnn \g__penrose_kite_lms_rule_prop {G} {[f*+sG][sT]}
307 \prop_put:Nnn \g__penrose_kite_lms_rule_prop {g} {[f-_sg][st]}
   These are the rules for generating pentagon tilings.
308 \prop_new:N \g__penrose_pentagon_lms_rule_prop
309 \prop_put:Nnn \g__penrose_pentagon_lms_rule_prop {P}
310 {[s>P][1sF+Q][1+sF+Q][1*sF+Q][1-sF+Q][1_sF+Q]} % pentagon 5
311 \prop_put:Nnn \g_penrose_pentagon_lms_rule_prop {Q}
312 {[s>P][1+sFR][1*sF*R][1-sF+Q][1_sF+Q][1sF+Q][->fsD]} % pentagon 3
313 \prop_put:Nnn \g__penrose_pentagon_lms_rule_prop {R}
314 {[s>P][1-sF+Q][1+sF*R][1*sFR][1_sF*R][1sFR][_>fsD][>fsD]} % pentagon 2
315 \prop_put:Nnn \g_penrose_pentagon_lms_rule_prop {G}
316 {
317
     [se[>d+R][e1B]]
318
     [+se[>d+R][e1B]]
319
    [-se[>d+R][e1B]]
    [*se[>d+R][e1B]]
    [se[>d+R][e1B]]
323 } % pentagram
324 \prop_put:Nnn \g_penrose_pentagon_lms_rule_prop {B}
325 {
     [s>G]
326
    [se[>d+R][e1B]]
327
    [+se[>d+R][e1B]]
328
    [-se[>d+R][e1B]]
329
330 } % boat
331 \prop_put:Nnn \g_penrose_pentagon_lms_rule_prop {D}
332 {[s>d+R][s>eG][se1B]} % diamond
```

```
Each of the standard tilings can also be drawn using triangles using the same rules.
 333 \prop_set_eq:NN \g__penrose_rtriangle_lms_rule_prop
 334 \g_penrose_rhombus_lms_rule_prop
 335 \prop_set_eq:NN \g__penrose_ktriangle_lms_rule_prop
 336 \g_penrose_kite_lms_rule_prop
    These hold the various actions.
 \verb| 'prop_new:N 'g_penrose_default_lms_action_prop| \\
 338 \prop_new:N \g__penrose_rhombus_lms_action_prop
 339 \prop_new:N \g__penrose_kite_lms_action_prop
 341 \prop_new:N \g__penrose_ktriangle_lms_action_prop
 342 \prop_new:N \g__penrose_pentagon_lms_action_prop
    We need some parameters.
 343 \dim_new:N \l__penrose_step_dim
 344 \dim_set:Nn \l__penrose_step_dim {1cm}
    These are the defaults, which will be used in all the rule sets.
 345 \prop_put:Nnn \g__penrose_default_lms_action_prop {[} {\group_begin:}
 346 \prop_put:Nnn \g__penrose_default_lms_action_prop {]} {\group_end:}
 347 \prop_put:Nnn \g__penrose_default_lms_action_prop {f}
 348 {\pgftransformxshift{\l__penrose_step_dim}}
 349 \prop_put:Nnn \g__penrose_default_lms_action_prop {s} {
      \fp_set:Nn \l__penrose_tmpa_fp { 2 * sind(18) * \l__penrose_step_dim }
      \dim_set:Nn \l__penrose_step_dim {\fp_to_dim:N \l__penrose_tmpa_fp}
 351
 352 }
    The rhombus rules need a variety of turns.
 353 \prop_put:Nnn \g__penrose_rhombus_lms_action_prop {+}
 354 {\pgftransformrotate{144}}
 355 \prop_put:Nnn \g__penrose_rhombus_lms_action_prop {*}
 356 {\pgftransformrotate{108}}
 357 \prop_put:Nnn \g__penrose_rhombus_lms_action_prop {-}
 358 {\pgftransformrotate{216}}
 359 \prop_put:Nnn \g__penrose_rhombus_lms_action_prop {_}
 360 {\pgftransformrotate{252}}
 361 \prop_put:Nnn \g__penrose_rhombus_lms_action_prop {>}
 362 {\pgftransformrotate{180}}
    Up to now, the actions for the rhombus and its triangle replacement are the same.
 363 \prop_set_eq:NN \g__penrose_rtriangle_lms_action_prop
 364 \g__penrose_rhombus_lms_action_prop
    Now we do the actions that actually draw something.
   \group_begin:
As we go through, we keep track of how many tiles we've drawn.
     \int_gincr:N \l__penrose_tile_int
Set up the position, size, and angle correctly.
      \pgftransformrotate{198}
 368
      \fp_set:\n \l__penrose_tmpa_fp {\l__penrose_step_dim*2*cosd(18)}
 369
      \pgftransformxshift{-\fp_to_dim:N \l__penrose_tmpa_fp}
 370
      \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim/(1cm)}
 371
```

\pgftransformscale{\fp_use:N \l__penrose_tmpa_fp}

Now we draw the thin rhombus, applying every style we can possibly imagine. The Penrose tile style gets the current tile and total tile numbers passed to it.

```
\tl_set:Nx \l__penrose_tmpc_tl
373
374
     {
       {\int_use:N \l__penrose_tile_int}
375
       {\int_use:N \l__penrose_tiles_int}
376
377
     \UsePenroseTile[
378
       every~ Penrose~ tile/.try,
379
       every~ thin~ rhombus/.try,
380
       Penrose~ tile~ \int_use:N \l__penrose_tile_int/.try,
381
       Penrose~ tile/.try/.expand~ once=\l__penrose_tmpc_tl
     ]{thin~rhombus}
     \group_end:
385 }
   Same for the thick rhombus.
  \prop_put:Nnn \g_penrose_rhombus_lms_action_prop {G} {
     \group_begin:
     \int_gincr:N \l__penrose_tile_int
     \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim/(1cm)/(2*cosd(36))}
389
     \pgftransformscale{\fp_use:N \l__penrose_tmpa_fp}
     \tl_set:Nx \l__penrose_tmpc_tl
391
     {
392
       {\int_use:N \l__penrose_tile_int}
393
       {\int_use:N \l__penrose_tiles_int}
394
395
     \UsePenroseTile[
396
       every~ Penrose~ tile/.try,
398
       every~ thick~ rhombus/.try,
       Penrose~ tile~ \int_use:N \l__penrose_tile_int/.try,
300
       Penrose~ tile/.try/.expand~ once=\l__penrose_tmpc_tl
400
     ]{thick~rhombus}
401
     \group_end:
402
403 }
   Now we do the same for the kite and dart tiling.
\label{local_prop_put:Nnn} $$ \operatorname{\non-prop} {+} {\operatorname{\non-prop} {+} } $$
\label{local_prop_put:Nnn g_penrose_kite_lms_action_prop $$ {\footnotesize *} {\footnotesize 108}}
\label{local_prop_put:Nnn} $$ \operatorname{\prop\_put:Nnn \g\_penrose\_kite\_lms\_action\_prop {-} {\pgftransformrotate{-36}} } $$
407 \prop_put:Nnn \g__penrose_kite_lms_action_prop {_}
408 {\pgftransformrotate{-108}}
  \prop_put:Nnn \g__penrose_kite_lms_action_prop {>} {\pgftransformrotate{180}}
  \prop_set_eq:NN \g__penrose_ktriangle_lms_action_prop
  \g__penrose_kite_lms_action_prop
   \prop_put:Nnn \g_penrose_kite_lms_action_prop {T} {
412
     \group_begin:
413
     \int_gincr:N \l__penrose_tile_int
414
     \pgftransformrotate{36}
     \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim/(1cm)}
     \pgftransformscale{\fp_use:N \l__penrose_tmpa_fp}
417
     \tl_set:Nx \l__penrose_tmpc_tl
418
419
       {\int_use:N \l__penrose_tile_int}
420
```

```
{\int_use:N \l__penrose_tiles_int}
421
    }
422
     \UsePenroseTile[
423
      every~ Penrose~ tile/.try,
424
       every~ kite/.try,
425
      Penrose~ tile~ \int_use:N \l__penrose_tile_int/.try,
426
      Penrose~ tile/.try/.expand~ once=\l_penrose_tmpc_tl
427
     \group_end:
430 }
   \prop_put:Nnn \g_penrose_kite_lms_action_prop {g} {
431
     \group_begin:
432
     \int_gincr:N \l__penrose_tile_int
     \pgftransformrotate{144}
     \pgftransformxshift{-\l_penrose_step_dim * 2 * sin(18)}
     \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim/(1cm)}
     \pgftransformscale{\fp_use:N \l__penrose_tmpa_fp}
437
     \tl_set:Nx \l__penrose_tmpc_tl
438
439
       {\int_use:N \l__penrose_tile_int}
440
       {\int_use:N \l__penrose_tiles_int}
441
442
     \UsePenroseTile[
      every~ Penrose~ tile/.try,
       every~ dart/.try,
      Penrose~ tile~ \int_use:N \l__penrose_tile_int/.try,
446
      Penrose~ tile/.try/.expand~ once=\l__penrose_tmpc_tl
447
    ]{dart}
448
     \group_end:
449
450 }
   Now we set up the actions for the triangle variations.
  \prop_put:Nnn \g__penrose_rtriangle_lms_action_prop {T} {
     \group_begin:
452
     \int_gincr:N \l__penrose_tile_int
453
     \pgftransformrotate{18}
454
     \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim/(1cm)}
455
     \pgftransformscale{\fp_use:N \l__penrose_tmpa_fp}
456
     \tl_set:Nx \l__penrose_tmpc_tl
457
458
459
       {\int_use:N \l__penrose_tile_int}
       {\int_use:N \l__penrose_tiles_int}
460
     \UsePenroseTile[
       every~ Penrose~ tile/.try,
       every~ reverse~ golden~ triangle/.try,
      Penrose~ tile~ \int_use:N \l__penrose_tile_int/.try,
465
      Penrose~ tile/.try/.expand~ once=\l__penrose_tmpc_tl
    ]{reverse~ golden~ triangle}
467
     \group_end:
468
469 }
  \prop_put:Nnn \g_penrose_rtriangle_lms_action_prop {t} {
470
     \group_begin:
471
    \int_gincr:N \l__penrose_tile_int
472
```

```
\pgftransformrotate{-18}
473
     \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim/(1cm)}
474
     \pgftransformscale{\fp_use:N \l__penrose_tmpa_fp}
475
     \tl_set:Nx \l__penrose_tmpc_tl
476
477
       {\int_use:N \l__penrose_tile_int}
478
       {\int_use:N \l__penrose_tiles_int}
479
480
     \tl_set:Nx \l__penrose_tmpc_tl
481
482
     {
       {\int_use:N \l__penrose_tile_int}
483
       {\int_use:N \l__penrose_tiles_int}
484
485
     \UsePenroseTile[
486
       every~ Penrose~ tile/.try,
487
       every~ golden~ triangle/.try,
488
       Penrose~ tile~ \int_use:N \l__penrose_tile_int/.try,
       Penrose~ tile/.try/.expand~ once=\l__penrose_tmpc_tl
     ]{golden~ triangle}
     \group_end:
493 }
   \prop_put:Nnn \g__penrose_rtriangle_lms_action_prop {G} {
494
     \group_begin:
495
     \int_gincr:N \l__penrose_tile_int
496
     \pgftransformrotate{180}
497
     \pgftransformxshift{-\l_penrose_step_dim}
     \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim/(1cm)/(2*cosd(36))}
     \pgftransformscale{\fp_use:N \l__penrose_tmpa_fp}
501
     \tl_set:Nx \l__penrose_tmpc_tl
502
     {
       {\int_use:N \l__penrose_tile_int}
503
       {\int_use:N \l__penrose_tiles_int}
504
505
     \UsePenroseTile[
506
       every~ Penrose~ tile/.try,
507
       every~ reverse~ golden~ gnomon/.try,
508
       Penrose~ tile~ \int_use:N \l__penrose_tile_int/.try,
       Penrose~ tile/.try/.expand~ once=\l__penrose_tmpc_tl
     ]{reverse~ golden~ gnomon}
511
     \group_end:
512
513 }
   \prop_put:Nnn \g__penrose_rtriangle_lms_action_prop {g} {
514
     \group_begin:
515
     \int_gincr:N \l__penrose_tile_int
516
     \pgftransformrotate{180}
517
     \pgftransformxshift{-\l__penrose_step_dim}
     \label{localization} $$ \int_{-\infty}^{\infty} \left( \sum_{n=0}^{\infty} \frac{1_{penrose_step_dim}}{(1cm)/(2*cosd(36))} \right) $$
     \pgftransformscale{\fp_use:N \l__penrose_tmpa_fp}
520
521
     \tl_set:Nx \l__penrose_tmpc_tl
522
       {\int_use:N \l__penrose_tile_int}
523
       {\int_use:N \l__penrose_tiles_int}
524
525
```

```
\UsePenroseTile[
526
       every~ Penrose~ tile/.try,
527
       every~ golden~ gnomon/.try,
528
      Penrose~ tile~ \int_use:N \l__penrose_tile_int/.try,
529
       Penrose~ tile/.try/.expand~ once=\l__penrose_tmpc_tl
530
    ]{golden~ gnomon}
531
     \group_end:
532
533 }
  \prop_put:Nnn \g__penrose_ktriangle_lms_action_prop {T} {
534
     \group_begin:
535
     \int_gincr:N \l__penrose_tile_int
536
     \pgftransformrotate{18}
537
     \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim/(1cm)}
538
     \pgftransformscale{\fp_use:N \l__penrose_tmpa_fp}
539
     \tl_set:Nx \l__penrose_tmpc_tl
540
       {\int_use:N \l__penrose_tile_int}
542
       {\int_use:N \l__penrose_tiles_int}
543
    }
544
     \UsePenroseTile[
545
       every~ Penrose~ tile/.try,
546
       every~ reverse~ golden~ triangle/.try,
547
       Penrose~ tile~ \int_use:N \l__penrose_tile_int/.try,
548
       Penrose~ tile/.try/.expand~ once=\l__penrose_tmpc_tl
549
    ]{reverse~ golden~ triangle}
550
     \group_end:
  \prop_put:Nnn \g_penrose_ktriangle_lms_action_prop {t} {
553
     \group_begin:
554
     \int_gincr:N \l__penrose_tile_int
555
     \pgftransformrotate{-18}
556
     \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim/(1cm)}
557
     \pgftransformscale{\fp_use:N \l__penrose_tmpa_fp}
558
     \tl_set:Nx \l__penrose_tmpc_tl
559
560
       {\int_use:N \l__penrose_tile_int}
561
       {\int_use:N \l__penrose_tiles_int}
562
563
     \UsePenroseTile[
564
       every~ Penrose~ tile/.try,
565
       every~ golden~ triangle/.try,
566
       Penrose~ tile~ \int_use:N \l__penrose_tile_int/.try,
567
       Penrose~ tile/.try/.expand~ once=\l__penrose_tmpc_tl
568
    ]{golden~ triangle}
     \group_end:
570
571 }
572 \prop_put:\nn \g__penrose_ktriangle_lms_action_prop \{G\} \{
     \group_begin:
573
     \int_gincr:N \l__penrose_tile_int
574
     \pgftransformrotate{180}
575
     \pgftransformxshift{-\l_penrose_step_dim}
576
     \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim/(1cm)/(2*cosd(36))}
577
     \pgftransformscale{\fp_use:N \l__penrose_tmpa_fp}
```

```
579
     \tl_set:Nx \l__penrose_tmpc_tl
580
       {\int_use:N \l__penrose_tile_int}
581
       {\int_use:N \l__penrose_tiles_int}
582
583
     \UsePenroseTile[
584
       every~ Penrose~ tile/.try,
585
       every~ reverse~ golden~ gnomon/.try,
      Penrose~ tile~ \int_use:N \l__penrose_tile_int/.try,
587
       Penrose~ tile/.try/.expand~ once=\l__penrose_tmpc_tl
588
589
    []{reverse~ golden~ gnomon}
     \group_end:
590
591
   \prop_put:Nnn \g_penrose_ktriangle_lms_action_prop {g} {
592
     \group_begin:
593
     \int_gincr:N \l__penrose_tile_int
     \pgftransformrotate{180}
     \pgftransformxshift{-\l_penrose_step_dim}
     \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim/(1cm)/(2*cosd(36))}
     \pgftransformscale{\fp_use:N \l__penrose_tmpa_fp}
     \tl_set:Nx \l__penrose_tmpc_tl
599
600
       {\int_use:N \l__penrose_tile_int}
601
       {\int_use:N \l__penrose_tiles_int}
602
603
     \UsePenroseTile[
       every~ Penrose~ tile/.try,
       every~ golden~ gnomon/.try,
       Penrose~ tile~ \int_use:N \l__penrose_tile_int/.try,
607
608
       Penrose~ tile/.try/.expand~ once=\l__penrose_tmpc_tl
609
    ]{golden~ gnomon}
     \group_end:
610
611 }
   Now we do the same for the pentagonal tilings.
   The rules need a variety of turns.
^{612} \int_new:N \l__penrose_pentagon_parity_int
613 \seq_new:N \l__penrose_pentagon_parity_seq
614 \seq_set_from_clist: Nn \l__penrose_pentagon_parity_seq {odd,even}
615 \prop_put:Nnn \g__penrose_pentagon_lms_action_prop {1} {
    \int_set:Nn \l__penrose_pentagon_parity_int
616
    {3 - \l_penrose_pentagon_parity_int}
617
619 \prop_put:Nnn \g__penrose_pentagon_lms_action_prop {+}
620 {\pgftransformrotate{72}}
621 \prop_put:Nnn \g__penrose_pentagon_lms_action_prop {*}
622 {\pgftransformrotate{144}}
623 \prop_put:Nnn \g__penrose_pentagon_lms_action_prop {-}
624 {\pgftransformrotate{288}}
626 {\pgftransformrotate{216}}
627 \prop_put:Nnn \g_penrose_pentagon_lms_action_prop {>}
628 {\pgftransformrotate{180}}
629 \prop_put:Nnn \g__penrose_pentagon_lms_action_prop {|}
```

```
630 {\pgftransformxscale{-1}}
The scale factor is different.
    \prop_put:Nnn \g__penrose_pentagon_lms_action_prop {s} {
      \fp_set:Nn \l__penrose_tmpa_fp
 632
 633
        1/(2 + 2 * cosd(72)) * l_penrose_step_dim
 634
 635
      \dim_set:Nn \l__penrose_step_dim {\fp_to_dim:N \l__penrose_tmpa_fp}
 636
 637 }
And we tend to work better vertically.
 ^{638} \prop_put:\nn \g_penrose_pentagon_lms_action_prop {f} {
      \verb|\pgftransformyshift{\fp_to_dim:N \l__penrose_tmpa_fp}|
 640
 641 }
    \prop_put:Nnn \g__penrose_pentagon_lms_action_prop {F} {
 642
      \fp_set:Nn \l__penrose_tmpa_fp { tand(54) * \l__penrose_step_dim }
 643
      \pgftransformyshift{\fp_to_dim:N \l__penrose_tmpa_fp}
 644
 645 }
    \prop_put:Nnn \g__penrose_pentagon_lms_action_prop {d} {
      \fp_set:Nn \l__penrose_tmpa_fp
 647
 648
      {
        (tand(54)/2 - tand(72)/2 + sind(36)) * l_penrose_step_dim
 649
 650
      \pgftransformyshift{\fp_to_dim:N \l__penrose_tmpa_fp}
 651
 652
    \prop_put:Nnn \g__penrose_pentagon_lms_action_prop {e} {
 653
      \fp_set:Nn \l__penrose_tmpa_fp
 654
 655
        tand(54) * cosd(36) * \l__penrose_step_dim
 657
      \pgftransformyshift{\fp_to_dim:N \l__penrose_tmpa_fp}
 658
 659 }
    \prop_put:Nnn \g__penrose_pentagon_lms_action_prop {P} {
 660
      \group_begin:
 661
      \int_gincr:N \l__penrose_tile_int
 662
      \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim/2}
      \pgftransformxshift{-\fp_to_dim:N \l__penrose_tmpa_fp}
      \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim*tand(54)/2}
      \pgftransformyshift{-\fp_to_dim:N \l__penrose_tmpa_fp}
      \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim/(1cm)}
 667
      \pgftransformscale{\fp_use:N \l__penrose_tmpa_fp}
 668
      \tl_set:Nx \l__penrose_tmpc_tl
 669
 670
      ₹
        {\int_use:N \l__penrose_tile_int}
 671
        {\int_use:N \l__penrose_tiles_int}
 672
 673
      \UsePenroseTile[
 674
        every~ Penrose~ tile/.try,
 675
        every~ pentagon/.try,
 677
        \seq_item:Nn \l__penrose_pentagon_parity_seq
 678
        {\l__penrose_pentagon_parity_int}
 679
```

```
680
       \space pentagon/.try,
       every~ pentagon~ 5/.try,
681
      Penrose~ tile~ \int_use:N \l__penrose_tile_int/.try,
682
      Penrose~ tile/.try/.expand~ once=\l_penrose_tmpc_tl
683
    ]{pentagon~5}
684
     \group_end:
685
686
  \prop_put:Nnn \g__penrose_pentagon_lms_action_prop {Q} {
687
     \group_begin:
688
    \int_gincr:N \l__penrose_tile_int
689
    690
     \pgftransformxshift{-\fp_to_dim:N \l__penrose_tmpa_fp}
691
     \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim*tand(54)/2}
692
     \pgftransformyshift{-\fp_to_dim:N \l__penrose_tmpa_fp}
693
     \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim/(1cm)}
694
     \pgftransformscale{\fp_use:N \l__penrose_tmpa_fp}
     \tl_set:Nx \l__penrose_tmpc_tl
697
       {\int_use:N \l__penrose_tile_int}
698
       {\int_use:N \l__penrose_tiles_int}
699
700
    \UsePenroseTile[
701
      every~ Penrose~ tile/.try,
702
       every~ pentagon/.try,
703
704
       \seq_item: Nn \l__penrose_pentagon_parity_seq
       {\l__penrose_pentagon_parity_int}
       \space pentagon/.try,
       every~ pentagon~ 3/.try,
708
      Penrose~ tile~ \int_use:N \l__penrose_tile_int/.try,
709
      Penrose~ tile/.try/.expand~ once=\l__penrose_tmpc_tl
    ]{pentagon~3}
    \group_end:
713 }
714
  \prop_put:Nnn \g__penrose_pentagon_lms_action_prop {R} {
     \group_begin:
     \int_gincr:N \l__penrose_tile_int
    \fp_set:\n \l__penrose_tmpa_fp {\l__penrose_step_dim/2}
     \pgftransformxshift{-\fp_to_dim:N \l__penrose_tmpa_fp}
718
     fp_set:Nn l_penrose_tmpa_fp {l_penrose_step_dim*tand(54)/2}
719
     \pgftransformyshift{-\fp_to_dim:N \l__penrose_tmpa_fp}
720
     \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim/(1cm)}
     \pgftransformscale{\fp_use:N \l__penrose_tmpa_fp}
    \tl_set:Nx \l__penrose_tmpc_tl
724
       {\int_use:N \l__penrose_tile_int}
725
       {\int_use:N \l__penrose_tiles_int}
726
727
     \UsePenroseTile[
728
      every~ Penrose~ tile/.try,
729
730
      every~ pentagon/.try,
      everv~
731
       \seq_item: Nn \l__penrose_pentagon_parity_seq
732
```

```
{\l__penrose_pentagon_parity_int}
                   \space pentagon/.try,
734
                   every~ pentagon~ 2/.try,
735
                  Penrose~ tile~ \int_use:N \l__penrose_tile_int/.try,
736
                  Penrose~ tile/.try/.expand~ once=\l_penrose_tmpc_tl
             ]{pentagon~2}
738
             \group_end:
739
       \label{lem:condition} $$ \operatorname{penrose\_pentagon\_lms\_action\_prop } G $$ \{ G \in \mathcal{C} : \ C : \ C \in \mathcal{C} : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ C : \ 
741
             \group_begin:
742
             \int_gincr:N \l__penrose_tile_int
743
              \pgftransformrotate{198}
744 %
             \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim*cosd(72)}
745
             \pgftransformxshift{\fp_to_dim:N \l__penrose_tmpa_fp}
746
             \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim*tand(54)*cosd(72)}
747
             \pgftransformyshift{\fp_to_dim:N \l__penrose_tmpa_fp}
             \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim/(1cm)}
             \pgftransformscale{\fp_use:N \l__penrose_tmpa_fp}
             \tl_set:Nx \l__penrose_tmpc_tl
751
752
                   {\int_use:N \l__penrose_tile_int}
753
                   {\int_use:N \l__penrose_tiles_int}
754
755
             \UsePenroseTile[
756
                   every~ Penrose~ tile/.try,
757
                   every~ pentagram/.try,
                  Penrose~ tile~ \int_use:N \l__penrose_tile_int/.try,
                  Penrose~ tile/.try/.expand~ once=\l__penrose_tmpc_tl
761
             ]{pentagram}
             \group_end:
762
763
        \prop_put:Nnn \g__penrose_pentagon_lms_action_prop {B} {
764
             \group_begin:
765
             \int_gincr:N \l__penrose_tile_int
766
767 %
               \pgftransformrotate{198}
             \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim*cosd(72)}
             \pgftransformxshift{\fp_to_dim:N \l__penrose_tmpa_fp}
             \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim*tand(54)*cosd(72)}
             \pgftransformyshift{\fp_to_dim:N \l__penrose_tmpa_fp}
             \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim/(1cm)}
772
             \pgftransformscale{\fp_use:N \l__penrose_tmpa_fp}
773
             \tl_set:Nx \l__penrose_tmpc_tl
774
775
776
                   {\int_use:N \l__penrose_tile_int}
                   {\int_use:N \l__penrose_tiles_int}
777
778
             \UsePenroseTile[
779
                  every~ Penrose~ tile/.try,
780
781
                   every~ boat/.try,
                  Penrose~ tile~ \int_{\infty} \ln_{\infty} \ln_{\infty} \ln_{\infty} dx
782
                  Penrose~ tile/.try/.expand~ once=\l__penrose_tmpc_tl
783
             ]{boat}
784
             \group_end:
```

```
786 }
   \prop_put:Nnn \g_penrose_pentagon_lms_action_prop {D} {
787
788
     \group_begin:
     \int_gincr:N \l__penrose_tile_int
789
     \pgftransformrotate{90}
     \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim*cosd(18)}
     \pgftransformxshift{-\fp_to_dim:N \l__penrose_tmpa_fp}
     \fp_set:Nn \l__penrose_tmpa_fp {\l__penrose_step_dim/(1cm)}
793
     \pgftransformscale{\fp_use:N \l__penrose_tmpa_fp}
794
     \tl_set:Nx \l__penrose_tmpc_tl
795
796
       {\int_use:N \l__penrose_tile_int}
797
       {\int_use:N \l__penrose_tiles_int}
798
799
     \UsePenroseTile[
800
       every~ Penrose~ tile/.try,
       every~ diamond/.try,
       Penrose~ tile~ \int_use:N \l__penrose_tile_int/.try,
       Penrose~ tile/.try/.expand~ once=\l_penrose_tmpc_tl
     ]{diamond}
     \group_end:
806
807 }
```

\penrose_make_lms:Nnnn

822

This creates the token list of actions, starting with the seed. The arguments are: a token list to store the result in, the name of the system, the number of iterations, and the initial state.

```
808 \cs_new_nopar:Npn \penrose_make_lms:Nnnn #1#2#3#4
 809 {
      \group_begin:
 810
On the first time round, we start with the given seed.
      \tl_set:Nn \l__penrose_tmpb_tl {#4}
We repeat the specified number of times.
      \prg_replicate:nn {#3} {
Duplicate the current state.
        \tl_set_eq:NN \l__penrose_tmpa_tl \l__penrose_tmpb_tl
Clear the receiving token list.
        \tl_clear:N \l__penrose_tmpb_tl
Walk through the current list, appending to the receiving list according to the rules.
        \tl_map_inline:Nn \l__penrose_tmpa_tl
 816
If a rule exists, copy that.
          \prop_if_in:cnTF {g__penrose_#2_lms_rule_prop} {##1}
 817
             \tl_put_right:Nx \l__penrose_tmpb_tl
 819
             {\prop_item:cn {g_penrose_#2_lms_rule_prop} {##1} }
 820
 821
          {
```

Otherwise, just copy the token.

```
823 \tl_put_right:Nn \l_penrose_tmpb_tl {##1}

824 }

825 }

826 }
```

We've done all this inside a group, now pass the result outside.

 $(End\ definition\ for\ \verb|\penrose_make_lms:Nnnn.|)$

\penrose_invoke_lms:Nn

This carries out the actions specified by the resulting rules.

```
835 \cs_new_nopar:Npn \penrose_invoke_lms:Nn #1#2
836 {
837 \group_begin:
```

Walk through the given list, carrying out the corresponding action if it exists. If not, look at the default. Otherwise, just do nothing.

```
\tl_map_inline:Nn #1 {
838
       \prop_if_in:cnTF {g__penrose_#2_lms_action_prop} {##1}
839
840
          \prop_item:cn {g__penrose_#2_lms_action_prop} {##1}
842
       }
843
          \prop_if_in:cnT {g__penrose_default_lms_action_prop} {##1}
844
845
            \prop_item:cn {g__penrose_default_lms_action_prop} {##1}
846
847
848
849
     \group_end:
850
851 }
```

 $(End\ definition\ for\ \verb|\penrose_invoke_lms:Nn.|)$

We keep track of the number of tiles.

```
852 \int_new:N \l__penrose_tile_int
853 \int_new:N \l__penrose_tiles_int
```

\PenroseDecomposition

This is the user macro to invoke the decomposition. The arguments are: optional styles, the name, number of iterations, and starting seed.

```
\int_gset:Nn \l__penrose_pentagon_parity_int {2}
                               \penrose_invoke_lms:Nn \l__penrose_tmpa_tl {#2}
                          862
                               \group_end:
                          863
                         864 }
                        (End\ definition\ for\ \PenroseDecomposition.)
\penrose_count_lms:N
                        This counts the number of tiles in the string.
                            \cs_new_nopar:Npn \penrose_count_lms:N #1
                          866 {
                               \int_gzero:N \l__penrose_tiles_int
                          867
                               \tl_map_inline:Nn #1
                          868
                          869
                                 \tl_if_eq:nnT {##1} {T}
                          870
                          871
                                   \int_incr:N \l__penrose_tiles_int
                          872
                          873
                                 \tl_if_eq:nnT {##1} {t}
                          874
                          875
                                   \int_incr:N \l__penrose_tiles_int
                          876
                          877
                                 \tl_if_eq:nnT {##1} {G}
                          878
                          879
                                   \int_incr:N \l__penrose_tiles_int
                          880
                                 }
                          881
                                 \tl_if_eq:nnT {##1} {g}
                          882
                          883
                                   \int_incr:N \l__penrose_tiles_int
                          884
                          885
                          886
                               }
                          887 }
                        (End definition for \penrose_count_lms:N.)
                             This is a \tikzset mechanism for setting the dimensions of the tiling.
                            \tikzset{
                               Penrose~ step/.code={
                                 \dim_set:Nn \l__penrose_step_dim {#1}
                          890
                          891
                         892 }
                             We're done with LATEX3, so turn off the syntax.
                          893 \ExplSyntaxOff
```

2.5 TikZ Pictures

New in TikZ3.0 is the ability to make pictures that can be reused. Penrose tiles seems an obvious use for this. These pictures can be placed alongside other tiles, matching by edge type.

There are a variety of constants that are frequently used and reused, so we define them all here. These are the PGF versions.

```
894 \pgfmathsetmacro\pr@chphi{cos(18)}
895 \pgfmathsetmacro\pr@chphi{sin(18)}
896 \pgfmathsetmacro\pr@cphi{cos(36)}
897 \pgfmathsetmacro\pr@sphi{sin(36)}
```

```
\pgfmathsetmacro\pr@invphi{2/(sqrt(5)+1)}
  \pgfmathsetmacro\pr@phi{(sqrt(5)+1)/2}
  \pgfmathsetmacro\pr@invphisq{\pr@invphi*\pr@invphi}
  \pgfmathsetmacro\pr@ominvphisq{\pr@invphi - \pr@invphisq}
   \pgfmathsetmacro\pr@ominvphi{1 - \pr@invphi}
  \newif\if@edgealign
  \def\pr@getfirst#1#2\pr@stop{#1}%
  \def\test@edgealign#1{%
905
     \pgfkeysgetvalue{/tikz/Penrose/alignment edge}{\@penrose@tmpa}%
906
     \edef\@penrose@tmpb{#1}%
907
     \edef\@penrose@tmpa{\expandafter\pr@getfirst\@penrose@tmpa.\pr@stop}%
908
     \ifx\@penrose@tmpa\@penrose@tmpb
909
     \@edgealigntrue
910
     \else
912
     \@edgealignfalse
913
914 }%
   \newif\if@newedge
   \def\test@newedge#1{%
     \pgfkeysgetvalue{/tikz/Penrose/alignment new edge}{\@penrose@tmpa}%
917
     \edef\@penrose@tmpb{#1}%
918
     919
     \ifx\@penrose@tmpa\@penrose@tmpb
920
     \@newedgetrue
921
     \else
922
     \@newedgefalse
923
924
925 }%
```

The implementation is essentially the same for each, so only the first will be commented.

926 \tikzset{

The key align with=<tile> along <edge> is used to set the parameters for placing a tile next to an existing one. For most tiles, that's enough to specify how the new tile should be placed. Some, though, need more information. For those, use align with=<tile> along <edge> using <number>.

```
align with/.code args={#1 along #2}{%
       \pgfutil@in@{ using }{#2}
928
       \ifpgfutil@in@%
929
       \tikzset{
930
         Penrose/alignment location=#1,
931
         Penrose/alignment set edges=#2,
932
       }%
933
       \else
934
       \tikzset{
935
         Penrose/alignment location=#1,
         Penrose/alignment edge=#2,
937
       }%
938
       \fi
939
    },
940
     Penrose/alignment set edges/.code args={#1 using #2}{%
941
942
         Penrose/alignment edge=#1,
943
```

```
Penrose/alignment new edge=#2
 944
        },
 945
      },
 946
      Penrose/alignment location/.initial={},
 947
      Penrose/alignment edge/.initial=a,
 948
      Penrose/alignment new edge/.initial={},
 949
Default clipping style.
      every Penrose tile clip/.style={clip},
This is the code for setting up a pic.
      thin rhombus/.pic={
        \begin{scope}
 952
Were we given a tile to align ourselves against?
        \pgfkeysgetvalue{/tikz/Penrose/alignment location}{\prloc}
        \ifx\prloc\pgfutil@empty
 954
```

Yes, we were. So we adjust our position accordingly. The first job is to transform so that we're along the edge of the receiving tile.

956 \begingroup

We get the locations of the start and end of the receiving tile. As pic sets the node prefix, we have to temporarily suspend that (hence working in a group).

```
\tikzset{name prefix ..}%
957
       \tikz@scan@one@point\pgfutil@firstofone%
958
       (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} start)%
959
       \global\pgf@xa=\pgf@x
960
       \global\pgf@ya=\pgf@y
961
       \tikz@scan@one@point\pgfutil@firstofone%
962
       (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} end)%
963
       \global\pgf@xb=\pgf@x
964
       \global\pgf@yb=\pgf@y
       \endgroup
```

We store the initial points in \pgf@xa and \pgf@ya but we want \pgf@xb and \pgf@yb to be a vector along the edge.

```
967 \advance\pgf@xb by -\pgf@xa
968 \advance\pgf@yb by -\pgf@ya
```

We shift to the start of the edge.

\pgftransformshift{\pgfpoint{\pgf@xa}{\pgf@ya}}%

And normalise the vector along it.

```
970 \pgfpointnormalised{\pgfpoint{\pgf@xb}{\pgf@yb}}
971 \pgf@xb=\pgf@x
```

972 \pgf@yb=\pgf@y

Now rotate so that the x-axis lies along the edge.

```
973 \pgftransformtriangle%
974 {\pgfpoint{0pt}{0pt}}%
975 {\pgfpoint{\pgf@xb}{\pgf@yb}}%
976 {\pgfpoint{-\pgf@yb}{\pgf@xb}}
```

The next job is to shift and rotate the current tile so that the correct edge ends up against the receiving tile.

```
\if\pgfkeysvalueof{/tikz/Penrose/alignment edge}b\relax
977
       \pgftransformrotate{-18}%
978
       \pgftransformshift{\pgfpoint{-\pr@chphi cm}{\pr@shphi cm}}%
979
980
       \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}B\relax
981
       \pgftransformrotate{18}%
982
       \else
983
       \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}a\relax
       \pgftransformrotate{198}%
       \pgftransformshift{\pgfpoint{-2*\pr@chphi cm}{0 cm}}%
       \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}A\relax
       \pgftransformrotate{162}%
       \pgftransformshift{\pgfpoint{-\pr@chphi cm}{-\pr@shphi cm}}%
       \fi\fi\fi\fi
991
       \fi
992
```

Now that the transformation is finalised, we can render the tile. We clip against the tile path so that the tiles don't "bleed". If we didn't do this, drawing the tile would result in overlaps which can look a bit ugly. On the other hand, tight clipping can lead to "gaps" between the tiles so we make this optional by enclosing it in a style.

```
993    \UsePenroseTile[
994         every Penrose tile clip/.try,
995         every thin rhombus clip/.try
996    ]{thin rhombus}

997    \UsePenroseTile[
998         every Penrose tile/.try,
999         every thin rhombus/.try,
1000         pic actions
1001    ]{thin rhombus}
```

These draw the arcs that designate the joining rules. We draw full circles so that it doesn't matter what shape the tiles are.

```
1002 \UsePenroseTile[
1003 every Penrose arc clip/.try,
1004 every thin rhombus arc clip/.try
1005 ]{thin rhombus}
1006 \path[every circle arc/.try] (18:1) circle[radius=1/4];
1007 \path[every long arc/.try] (-18:1) circle[radius=1/4];
```

Lastly, we put coordinates at each vertex, labelled by which edge they are.

```
1008 \coordinate (-edge a start) at (0,0);
1009 \coordinate (-edge a end) at (18:1);
1010 \coordinate (-edge A start) at (18:1);
1011 \coordinate (-edge A end) at (2*\pr@chphi,0);
1012 \coordinate (-edge B start) at (2*\pr@chphi,0);
1013 \coordinate (-edge B end) at (-18:1);
1014 \coordinate (-edge b start) at (-18:1);
1015 \coordinate (-edge b end) at (0,0);
1016 \end{scope}
1017 },
```

```
This is a shortcut for installing the pic type.
      thin rhombus/.style={
1018
        every Penrose pic/.try,
1019
        pic type=thin rhombus,
1020
1021
Same again, but for the thick rhombus.
      thick rhombus/.pic={
1022
        \begin{scope}
1023
        \pgfkeysgetvalue{/tikz/Penrose/alignment location}{\prloc}
1024
        \ifx\prloc\pgfutil@empty
1025
        \else
        \begingroup
        \tikzset{name prefix ..}%
1028
        \tikz@scan@one@point\pgfutil@firstofone%
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} start)%
        \global\pgf@xa=\pgf@x
        \global\pgf@ya=\pgf@y
        \tikz@scan@one@point\pgfutil@firstofone%
        1034
        \global\pgf@xb=\pgf@x
1035
        \global\pgf@yb=\pgf@y
1036
        \endgroup
1037
        \advance\pgf@xb by -\pgf@xa
1038
        \advance\pgf@yb by -\pgf@ya
1039
        \pgftransformshift{\pgfpoint{\pgf@xa}{\pgf@ya}}%
        \pgfpointnormalised{\pgfpoint{\pgf@xb}{\pgf@yb}}
        \pgf@xb=\pgf@x
1042
        \pgf@yb=\pgf@y
1043
        \pgftransformtriangle%
1044
        {\pgfpoint{0pt}{0pt}}%
1045
        {\pgfpoint{\pgf@xb}{\pgf@yb}}%
1046
        {\pgfpoint{-\pgf@yb}{\pgf@xb}}
1047
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}b\relax
1048
        \pgftransformrotate{144}%
1049
        \pgftransformshift{\pgfpoint{-\pr@cphi cm}{-\pr@sphi cm}}%
        \else
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}B\relax
        \pgftransformrotate{36}%
1054
        \else
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}a\relax
1055
        \pgftransformrotate{-36}%
1056
        \pgftransformshift{\pgfpoint{-\pr@cphi cm}{\pr@sphi cm}}%
1057
        \else
1058
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}A\relax
1059
        \pgftransformrotate{216}%
1060
        \pgftransformshift{\pgfpoint{-2*\pr@cphi cm}{0 cm}}%
        \fi\fi\fi\fi
        \fi
        \UsePenroseTile[
1064
1065
          every Penrose tile clip/.try,
          every thick rhombus clip/.try
1066
        ]{thick rhombus}
1067
        \UsePenroseTile[
1068
```

```
1069
          every Penrose tile/.try,
          every thick rhombus/.try,
1070
          pic actions
1071
        ]{thick rhombus}
1072
        \UsePenroseTile[
1073
          every Penrose arc clip/.try,
1074
          every thick rhombus arc clip/.try
1075
        ]{thick rhombus}
1076
    \path[every circle arc/.try] (2*\pr@cphi,0) circle[radius=1/4];
    \path[every long arc/.try] (0,0) circle[radius=3/4];
    \coordinate (-edge B start) at (0,0);
    \coordinate (-edge B end) at (36:1);
    \coordinate (-edge a start) at (36:1);
    \coordinate (-edge a end) at (2*\pr@cphi,0);
1082
    \coordinate (-edge A start) at (2*\pr@cphi,0);
1083
    \coordinate (-edge A end) at (-36:1);
1084
    \coordinate (-edge b start) at (-36:1);
1085
    \coordinate (-edge b end) at (0,0);
1086
        \end{scope}
1087
      },
      thick rhombus/.style={
1089
        every Penrose pic/.try,
1090
        pic type=thick rhombus,
1091
      },
1092
Now the kite.
      kite/.pic={
        \begin{scope}
1094
        \pgfkeysgetvalue{/tikz/Penrose/alignment location}{\prloc}
1096
        \ifx\prloc\pgfutil@empty
        \else
1097
1098
        \begingroup
        \tikzset{name prefix ..}%
1099
        \tikz@scan@one@point\pgfutil@firstofone%
1100
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} start)%
        \global\pgf@xa=\pgf@x
        \global\pgf@ya=\pgf@y
        \tikz@scan@one@point\pgfutil@firstofone%
 1104
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} end)%
        \global\pgf@xb=\pgf@x
1106
        \global\pgf@yb=\pgf@y
        \endgroup
1108
        \advance\pgf@xb by -\pgf@xa
1109
        \advance\pgf@yb by -\pgf@ya
        \pgftransformshift{\pgfpoint{\pgf@xa}{\pgf@ya}}%
        \pgfpointnormalised{\pgfpoint{\pgf@xb}{\pgf@yb}}
1112
        \pgf@xb=\pgf@x
        \pgf@yb=\pgf@y
1114
        \pgftransformtriangle%
1115
        {\pgfpoint{0pt}{0pt}}%
        {\pgfpoint{\pgf@xb}{\pgf@yb}}%
1117
        {\pgfpoint{-\pgf@yb}{\pgf@xb}}%
1118
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}c\relax
1119
        \pgftransformrotate{-72}%
1120
        \pgftransformshift{\pgfpoint{-\pr@cphi cm}{\pr@sphi cm}}%
```

```
\else
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}C\relax
        \pgftransformrotate{-108}%
1124
        \pgftransformshift{\pgfpoint{-1 cm}{0 cm}}%
1125
1126
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}a\relax
        \pgftransformrotate{36}%
1128
        \else
1129
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}A\relax
1130
        \pgftransformrotate{144}%
1131
        \pgftransformshift{\pgfpoint{-\pr@cphi cm}{-\pr@sphi cm}}%
        \fi\fi\fi\fi
1133
1134
        \UsePenroseTile[
1135
          every Penrose tile clip/.try,
1136
          every kite clip/.try
1138
        \UsePenroseTile[
1139
          every Penrose tile/.try,
          every kite/.try,
          pic actions
        ]{kite}
1143
        \UsePenroseTile[
1144
          every Penrose arc clip/.try,
1145
          every kite arc clip/.try
1146
        ]{kite}
1147
    \path[every circle arc/.try] (0,0) circle[radius=\pr@invphi];
1148
    \path[every long arc/.try] (1,0) circle[radius=\pr@invphisq];
    \coordinate (-edge a start) at (0,0);
   \coordinate (-edge a end) at (36:1);
1152 \coordinate (-edge c start) at (36:1);
   \coordinate (-edge c end) at (1,0);
    \coordinate (-edge C start) at (1,0);
    \coordinate (-edge C end) at (-36:1);
    \coordinate (-edge A start) at (-36:1);
    \coordinate (-edge A end) at (0,0);
1157
        \end{scope}
1158
1159
The dart is next.
      dart/.pic={
1160
        \begin{scope}
1161
        \pgfkeysgetvalue{/tikz/Penrose/alignment location}{\prloc}
1162
        \ifx\prloc\pgfutil@empty
1163
        \else
1164
        \begingroup
1165
        \tikzset{name prefix ..}%
1166
        \tikz@scan@one@point\pgfutil@firstofone%
1167
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} start)%
1168
        \global\pgf@xa=\pgf@x
1170
        \global\pgf@ya=\pgf@y
        \tikz@scan@one@point\pgfutil@firstofone%
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} end)%
        \global\pgf@xb=\pgf@x
        \global\pgf@yb=\pgf@y
1174
```

```
\endgroup
       \advance\pgf@xb by -\pgf@xa
1176
       \advance\pgf@yb by -\pgf@ya
       \pgftransformshift{\pgfpoint{\pgf@xa}{\pgf@ya}}%
1178
       \pgfpointnormalised{\pgfpoint{\pgf@xb}{\pgf@yb}}
1179
       \pgf@xb=\pgf@x
1180
       \pgf@yb=\pgf@y
1181
       \pgftransformtriangle%
1182
       {\pgfpoint{0pt}{0pt}}%
       {\pgfpoint{\pgf@xb}{\pgf@yb}}%
1184
       {\pgfpoint{-\pgf@yb}{\pgf@xb}}%
1185
       \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}c\relax
1186
       \pgftransformrotate{108}%
1187
1188
       \else
       \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}C\relax
1189
       \pgftransformrotate{72}%
1190
       \pgftransformshift%
1191
       {\pgfpoint{\pr@invphi*\pr@shphi cm}{-\pr@invphi*\pr@chphi cm}}%
1192
       \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}a\relax
       \pgftransformrotate{-36}%
       \pgftransformshift%
       {\pgfpoint{\pr@invphi*\pr@shphi cm}{\pr@invphi*\pr@chphi cm}}%
1197
       \else
1198
       1199
       \pgftransformrotate{216}%
1200
       \pgftransformshift{\pgfpoint{-\pr@invphi cm}{0 cm}}%
1201
       \fi\fi\fi\fi
       \UsePenroseTile[
         every Penrose tile clip/.try,
         every dart clip/.try
1207
       ]{dart}
       \UsePenroseTile[
1208
         every Penrose tile/.try,
1209
         every dart/.try,
         pic actions
       ]{dart}
1213
       \UsePenroseTile[
         every Penrose arc clip/.try,
         every dart arc clip/.try
       ]{dart}
   \path[every circle arc/.try] (\pr@invphi,0) circle[radius=\pr@ominvphi];
   \path[every long arc/.try] (0,0) circle[radius=\pr@ominvphisq];
   \coordinate (-edge c start) at (0,0);
   \coordinate (-edge c end) at (108:\pr@invphi);
   \coordinate (-edge a start) at (108:\pr@invphi);
   \coordinate (-edge a end) at (\pr@invphi,0);
   \coordinate (-edge A start) at (\pr@invphi,0);
1223
   \coordinate (-edge A end) at (-108:\pr@invphi);
1224
   \coordinate (-edge C start) at (-108:\pr@invphi);
   \coordinate (-edge C end) at (0,0);
1227
       \end{scope}
     },
1228
```

```
kite/.style={
1229
        every Penrose pic/.try,
1230
        pic type=kite,
      },
      dart/.style={
        every Penrose pic/.try,
1234
        pic type=dart,
1235
      },
1236
The golden triangle.
      golden triangle/.pic={
        \begin{scope}
1238
        \pgfkeysgetvalue{/tikz/Penrose/alignment location}{\prloc}
1239
        \ifx\prloc\pgfutil@empty
1240
        \else
1241
        \begingroup
1242
        \tikzset{name prefix ..}%
        \tikz@scan@one@point\pgfutil@firstofone%
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} start)%
        \global\pgf@xa=\pgf@x
        \global\pgf@ya=\pgf@y
1247
        \tikz@scan@one@point\pgfutil@firstofone%
1248
        1249
        \global\pgf@xb=\pgf@x
1250
        \global\pgf@yb=\pgf@y
1251
        \endgroup
1252
        \advance\pgf@xb by -\pgf@xa
        \advance\pgf@yb by -\pgf@ya
        \pgftransformshift{\pgfpoint{\pgf@xa}{\pgf@ya}}%
        \pgfpointnormalised{\pgfpoint{\pgf@xb}{\pgf@yb}}
1256
1257
        \pgf@xb=\pgf@x
        \pgf@yb=\pgf@y
1258
        \pgftransformtriangle%
1259
        {\pgfpoint{0pt}{0pt}}%
1260
        {\pgfpoint{\pgf@xb}{\pgf@yb}}%
1261
        {\pgfpoint{-\pgf@yb}{\pgf@xb}}%
1262
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}B\relax
1263
        \pgftransformrotate{18}%
1264
        \else
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}C\relax
        \pgftransformrotate{-90}%
        \pgftransformshift{\pgfpoint{-\pr@chphi cm}{\pr@shphi cm}}%
        \else
1269
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}A\relax
        \pgftransformrotate{162}%
        \pgftransformshift{\pgfpoint{-\pr@chphi cm}{-\pr@shphi cm}}%
1272
        \fi\fi\fi
        \fi
1274
        \UsePenroseTile[
          every Penrose tile clip/.try,
1276
          every golden triangle clip/.try
        ]{golden triangle}
1278
        \UsePenroseTile[
1279
          every Penrose tile/.try,
1280
          every golden triangle/.try,
1281
```

```
pic actions
        ]{golden triangle}
1283
    \coordinate (-edge a start) at (0,0);
    \coordinate (-edge a end) at (18:1);
    \coordinate (-edge c start) at (18:1);
    \coordinate (-edge c end) at (-18:1);
    \coordinate (-edge b start) at (-18:1);
1288
    \coordinate (-edge b end) at (0,0);
1289
        \end{scope}
      },
1291
1292
      golden triangle/.style={
        every Penrose pic/.try,
1293
        pic type=golden triangle,
1294
      },
1295
The reverse golden triangle (is there a better name?).
      reverse golden triangle/.pic={
        \begin{scope}
        \pgfkeysgetvalue{/tikz/Penrose/alignment location}{\prloc}
        \ifx\prloc\pgfutil@empty
        \else
1300
        \begingroup
1301
        \tikzset{name prefix ..}%
1302
        \tikz@scan@one@point\pgfutil@firstofone%
1303
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} start)%
1304
        \global\pgf@xa=\pgf@x
1305
        \global\pgf@ya=\pgf@y
        \tikz@scan@one@point\pgfutil@firstofone%
1307
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} end)%
        \global\pgf@xb=\pgf@x
1309
        \global\pgf@yb=\pgf@y
1311
        \endgroup
        \advance\pgf@xb by -\pgf@xa
1312
        \advance\pgf@yb by -\pgf@ya
        \pgftransformshift{\pgfpoint{\pgf@xa}{\pgf@ya}}%
1314
        \pgfpointnormalised{\pgfpoint{\pgf@xb}{\pgf@yb}}
        \pgf@xb=\pgf@x
1316
        \pgf@yb=\pgf@y
        \pgftransformtriangle%
        {\pgfpoint{0pt}{0pt}}%
1319
        {\pgfpoint{\pgf@xb}{\pgf@yb}}%
        {\pgfpoint{-\pgf@yb}{\pgf@xb}}%
1321
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}b\relax
1322
        \pgftransformrotate{162}%
1323
        \pgftransformshift{\pgfpoint{-\pr@chphi cm}{-\pr@shphi cm}}%
1324
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}c\relax
1326
        \pgftransformrotate{-90}%
1327
        \pgftransformshift{\pgfpoint{-\pr@chphi cm}{\pr@shphi cm}}%
1328
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}a\relax
1330
        \pgftransformrotate{18}%
        \fi\fi\fi
1332
        \fi
        \UsePenroseTile[
1334
```

```
1335
          every Penrose tile clip/.try,
          every reverse golden triangle clip/.try
1336
        ]{reverse golden triangle}
        \UsePenroseTile[
1338
          every Penrose tile/.try,
1339
          every reverse golden triangle/.try,
1340
          pic actions
1341
        ]{reverse golden triangle}
    \coordinate (-edge B start) at (0,0);
    \coordinate (-edge B end) at (18:1);
    \coordinate (-edge C start) at (18:1);
    \coordinate (-edge C end) at (-18:1);
    \coordinate (-edge A start) at (-18:1);
1347
    \coordinate (-edge A end) at (0,0);
1348
        \end{scope}
1349
      },
1350
      reverse golden triangle/.style={
1351
        every Penrose pic/.try,
1352
        pic type=reverse golden triangle,
1353
      },
1354
The golden gnomon.
      golden gnomon/.pic={
1355
        \begin{scope}
1356
        \pgfkeysgetvalue{/tikz/Penrose/alignment location}{\prloc}
1357
        \ifx\prloc\pgfutil@empty
1358
        \else
        \begingroup
1360
        \tikzset{name prefix ..}%
        \tikz@scan@one@point\pgfutil@firstofone%
1362
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} start)%
1363
1364
        \global\pgf@xa=\pgf@x
        \global\pgf@ya=\pgf@y
1365
        \tikz@scan@one@point\pgfutil@firstofone%
1366
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} end)%
1367
        \global\pgf@xb=\pgf@x
1368
        \global\pgf@yb=\pgf@y
1369
        \endgroup
        \advance\pgf@xb by -\pgf@xa
        \advance\pgf@yb by -\pgf@ya
        \pgftransformshift{\pgfpoint{\pgf@xa}{\pgf@ya}}%
        \pgfpointnormalised{\pgfpoint{\pgf@xb}{\pgf@yb}}
1374
        \pgf@xb=\pgf@x
        \pgf@yb=\pgf@y
1376
        \pgftransformtriangle%
1377
        {\pgfpoint{0pt}{0pt}}%
1378
        {\pgfpoint{\pgf@xb}{\pgf@yb}}%
1379
        {\pgfpoint{-\pgf@yb}{\pgf@xb}}%
1380
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}c\relax
1381
        \pgftransformrotate{144}%
        \pgftransformshift{\pgfpoint{-\pr@cphi cm}{-\pr@sphi cm}}%
1383
1384
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}B\relax
1385
        \pgftransformrotate{-144}%
1386
        \pgftransformshift{\pgfpoint{-2*\pr@cphi cm}{0 cm}}%
1387
```

```
1388
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}a\relax
1389
        \fi\fi\fi
1390
        \fi
1391
        \UsePenroseTile[
1392
          every Penrose tile clip/.try,
1393
          every golden gnomon clip/.try
1394
        ]{golden gnomon}
1395
        \UsePenroseTile[
          every Penrose tile/.try,
          every golden gnomon/.try,
1399
          pic actions
        [golden gnomon]
1400
    \coordinate (-edge C start) at (0,0);
1401
    \coordinate (-edge C end) at (36:1);
1402
    \coordinate (-edge b start) at (36:1);
    \coordinate (-edge b end) at (2*\pr@cphi,0);
    \coordinate (-edge A start) at (2*\pr@cphi,0);
    \coordinate (-edge A end) at (0,0);
1407
        \end{scope}
      },
      golden gnomon/.style={
1409
        every Penrose pic/.try,
1410
        pic type=golden gnomon,
1411
      },
1412
The reverse golden gnomon.
      reverse golden gnomon/.pic={
        \begin{scope}
        \pgfkeysgetvalue{/tikz/Penrose/alignment location}{\prloc}
1415
1416
        \ifx\prloc\pgfutil@empty
1417
        \else
        \begingroup
1418
        \tikzset{name prefix ..}%
1419
        \tikz@scan@one@point\pgfutil@firstofone%
1420
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} start)%
1421
        \global\pgf@xa=\pgf@x
1422
        \global\pgf@ya=\pgf@y
        \tikz@scan@one@point\pgfutil@firstofone%
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} end)%
        \global\pgf@xb=\pgf@x
        \global\pgf@yb=\pgf@y
1427
        \endgroup
1428
        \advance\pgf@xb by -\pgf@xa
1429
        \advance\pgf@yb by -\pgf@ya
1430
        \pgftransformshift{\pgfpoint{\pgf@xa}{\pgf@ya}}%
1431
        \pgfpointnormalised{\pgfpoint{\pgf@xb}{\pgf@yb}}
1432
        \pgf@xb=\pgf@x
1433
        \pgf@yb=\pgf@y
        \pgftransformtriangle%
        {\pgfpoint{0pt}{0pt}}%
1436
        {\pgfpoint{\pgf@xb}{\pgf@yb}}%
1437
        {\pgfpoint{-\pgf@yb}{\pgf@xb}}%
1438
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}C\relax
1439
        \pgftransformrotate{36}%
1440
```

```
1441
        \else
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}b\relax
1442
        \pgftransformrotate{-36}%
1443
        \pgftransformshift{\pgfpoint{-\pr@cphi cm}{\pr@sphi cm}}%
1444
        \else
1445
        \if\pgfkeysvalueof{/tikz/Penrose/alignment edge}A\relax
1446
        \pgftransformrotate{180}%
1447
        \pgftransformshift{\pgfpoint{-2*\pr@cphi cm}{0 cm}}%
        \fi\fi\fi
        \fi
        \UsePenroseTile[
1451
          every Penrose tile clip/.try,
1452
          every reverse golden gnomon clip/.try
1453
1454
        ]{reverse golden gnomon}
        \UsePenroseTile[
1455
          every Penrose tile/.try,
1456
          every reverse golden gnomon/.try,
1457
          pic actions
        ]{reverse golden gnomon}
    \coordinate (-edge a start) at (0,0);
    \coordinate (-edge a end) at (2*\pr@cphi,0);
    \coordinate (-edge B start) at (2*\pr@cphi,0);
    \coordinate (-edge B end) at (-36:1);
    \coordinate (-edge c start) at (-36:1);
    \coordinate (-edge c end) at (0,0);
1465
        \end{scope}
1466
1467
1468
      reverse golden gnomon/.style={
        every Penrose pic/.try,
1469
1470
        pic type=reverse golden gnomon,
      },
1471
The primary pentagon.
      pentagon 5/.pic={
1472
        \begin{scope}
1473
        \pgfkeysgetvalue{/tikz/Penrose/alignment location}{\prloc}
1474
        \ifx\prloc\pgfutil@empty
1475
        \else
1476
        \begingroup
        \tikzset{name prefix ..}%
        \tikz@scan@one@point\pgfutil@firstofone%
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} start)%
        \global\pgf@xa=\pgf@x
1481
        \global\pgf@ya=\pgf@y
1482
        \tikz@scan@one@point\pgfutil@firstofone%
1483
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} end)%
1484
        \global\pgf@xb=\pgf@x
1485
        \global\pgf@yb=\pgf@y
1486
        \endgroup
        \advance\pgf@xb by -\pgf@xa
        \advance\pgf@yb by -\pgf@ya
1489
        \pgftransformshift{\pgfpoint{\pgf@xa}{\pgf@ya}}%
1490
        \pgfpointnormalised{\pgfpoint{\pgf@xb}{\pgf@yb}}
1491
        \pgf@xb=\pgf@x
1492
        \pgf@yb=\pgf@y
1493
```

```
\pgftransformtriangle%
       {\left\{ pfpoint{0pt}{0pt}\right\} }
       {\pgfpoint{\pgf@xb}{\pgf@yb}}%
       {\pgfpoint{-\pgf@yb}{\pgf@xb}}%
1497
       \test@newedge{1}%
1498
       \if@newedge
1499
       \pgftransformrotate{180}%
1500
       \pgftransformshift{\pgfpoint{-1 cm}{0 cm}}%
1501
       \test@newedge{2}%
       \if@newedge
       \pgftransformshift{\pgfpoint{1 cm}{0 cm}}%
1505
       \pgftransformrotate{108}%
1506
       \pgftransformshift{\pgfpoint{-1 cm}{0 cm}}%
1507
       \else
1508
       \test@newedge{3}%
1509
       \if@newedge
       \pgftransformshift%
1511
       {\pgfpoint{\pr@phi * \pr@shphi cm}{- \pr@phi * \pr@chphi cm}}%
       \pgftransformrotate{36}%
       \else
       \test@newedge{4}%
1515
       \if@newedge
1516
       \pgftransformrotate{-36}%
1518
       \else
1519
1520
       \test@newedge{5}%
1521
       \if@newedge
       \pgftransformrotate{-108}%
1522
       \fi\fi\fi\fi\fi
1524
       \UsePenroseTile[
1526
         every Penrose tile clip/.try,
1527
         every pentagon clip/.try,
         every pentagon 5 clip/.try
1528
       ]{pentagon 5}
1529
       \UsePenroseTile[
1530
         every Penrose tile/.try,
1531
         every pentagon/.try,
1532
         every pentagon 5/.try,
         pic actions
       ]{pentagon 5}
   \coordinate (-edge a1 start) at (0,0);
   \coordinate (-edge a1 end) at (1,0);
   \coordinate (-edge a2 start) at (1,0);
   \coordinate (-edge a2 end) at (1+\pr@shphi,\pr@chphi);
   \coordinate (-edge a3 start) at (1+\pr@shphi,\pr@chphi);
   \coordinate (-edge a3 end) at (\pr@cphi-\pr@shphi,\pr@sphi+\pr@chphi);
1541
   \coordinate (-edge a4 start) at (\pr@cphi-\pr@shphi,\pr@sphi+\pr@chphi);
   \coordinate (-edge a4 end) at (-\pr@shphi,\pr@chphi);
   \coordinate (-edge a5 start) at (-\pr@shphi,\pr@chphi);
   \coordinate (-edge a5 end) at (0,0);
1546
       \end{scope}
     },
1547
```

```
1548
     pentagon 5/.style={
        every Penrose pic/.try,
1549
1550
       pic type=pentagon 5,
     },
1551
The secondary pentagon.
     pentagon 3/.pic={
        \begin{scope}
        \pgfkeysgetvalue{/tikz/Penrose/alignment location}{\prloc}
1554
        \ifx\prloc\pgfutil@empty
1555
        \else
1556
        \begingroup
1557
        \tikzset{name prefix ..}%
1558
        \tikz@scan@one@point\pgfutil@firstofone%
1559
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} start)%
1560
        \global\pgf@xa=\pgf@x
1561
        \global\pgf@ya=\pgf@y
        \tikz@scan@one@point\pgfutil@firstofone%
        \global\pgf@xb=\pgf@x
        \global\pgf@yb=\pgf@y
        \endgroup
1567
        \advance\pgf@xb by -\pgf@xa
1568
        \advance\pgf@yb by -\pgf@ya
1569
        \pgftransformshift{\pgfpoint{\pgf@xa}{\pgf@ya}}%
1570
        \pgfpointnormalised{\pgfpoint{\pgf@xb}{\pgf@yb}}
1571
        \pgf@xb=\pgf@x
        \pgf@yb=\pgf@y
        \pgftransformtriangle%
1575
        {\pgfpoint{0pt}{0pt}}%
        {\pgfpoint{\pgf@xb}{\pgf@yb}}%
1576
1577
        {\pgfpoint{-\pgf@yb}{\pgf@xb}}%
        \test@edgealign{a}%
1578
        \if@edgealign
1579
        \pgftransformrotate{180}%
1580
        \pgftransformshift{\pgfpoint{-1 cm}{0 cm}}%
1581
1582
        \test@edgealign{B}%
1583
        \if@edgealign%
        \test@newedge{1}%
        \if@newedge
        \pgftransformshift{\pgfpoint{1 cm}{0 cm}}%
        \pgftransformrotate{108}%
1588
        \pgftransformshift{\pgfpoint{-1 cm}{0 cm}}%
1589
        \else
1590
        \pgftransformrotate{-108}%
1591
        \fi
1592
        \else
1593
        \test@newedge{1}%
        \if@newedge
        \pgftransformshift%
1596
        {\pgfpoint{\pr@phi * \pr@shphi cm}{- \pr@phi * \pr@chphi cm}}%
1597
        \pgftransformrotate{36}%
1598
1599
        \pgftransformshift{\pgfpoint{- \pr@shphi cm}{-\pr@chphi cm}}%
1600
```

```
1601
        \pgftransformrotate{-36}%
        \fi\fi\fi
1602
        \fi
1603
        \UsePenroseTile[
1604
          every Penrose tile clip/.try,
1605
          every pentagon clip/.try,
1606
          every pentagon 3 clip/.try
1607
        ]{pentagon 3}
1608
        \UsePenroseTile[
          every Penrose tile/.try,
1611
          every pentagon/.try,
          every pentagon 3/.try,
1612
1613
          pic actions
        ]{pentagon 3}
1614
    \coordinate (-edge A start) at (0,0);
1615
    \coordinate (-edge A end) at (1,0);
1616
    \coordinate (-edge b1 start) at (1,0);
1617
    \coordinate (-edge b1 end) at (1+\pr@shphi,\pr@chphi);
1618
    \coordinate (-edge a1 start) at (1+\pr@shphi,\pr@chphi);
    \coordinate (-edge a1 end) at (\pr@cphi-\pr@shphi,\pr@sphi+\pr@chphi);
    \coordinate (-edge a2 start) at (\pr@cphi-\pr@shphi,\pr@sphi+\pr@chphi);
    \coordinate (-edge a2 end) at (-\pr@shphi,\pr@chphi);
    \coordinate (-edge b2 start) at (-\pr@shphi,\pr@chphi);
1623
    \coordinate (-edge b2 end) at (0,0);
1624
        \end{scope}
1625
      },
1626
1627
      pentagon 3/.style={
1628
        every Penrose pic/.try,
1629
        pic type=pentagon 3,
      },
1630
The tertiary pentagon.
      pentagon 2/.pic={
1631
        \begin{scope}
1632
        \pgfkeysgetvalue{/tikz/Penrose/alignment location}{\prloc}
1633
        \ifx\prloc\pgfutil@empty
1634
        \else
1635
        \begingroup
1636
        \tikzset{name prefix ..}%
1637
        \tikz@scan@one@point\pgfutil@firstofone%
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} start)%
        \global\pgf@xa=\pgf@x
        \global\pgf@ya=\pgf@y
1641
        \tikz@scan@one@point\pgfutil@firstofone%
1642
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} end)%
1643
        \global\pgf@xb=\pgf@x
1644
        \global\pgf@yb=\pgf@y
1645
        \endgroup
1646
        \advance\pgf@xb by -\pgf@xa
1647
        \advance\pgf@yb by -\pgf@ya
        \pgftransformshift{\pgfpoint{\pgf@xa}{\pgf@ya}}%
1649
        \pgfpointnormalised{\pgfpoint{\pgf@xb}{\pgf@yb}}
1650
        \pgf@xb=\pgf@x
1651
        \pgf@yb=\pgf@y
1652
        \pgftransformtriangle%
1653
```

```
{\left\{ pfpoint{0pt}{0pt}\right\} }
1654
       {\pgfpoint{\pgf@xb}{\pgf@yb}}%
1655
       {\pgfpoint{-\pgf@yb}{\pgf@xb}}%
1656
       \test@edgealign{D}%
1657
       \if@edgealign
1658
       \pgftransformrotate{180}%
1659
       \pgftransformshift{\pgfpoint{-1 cm}{0 cm}}%
1660
       \else
1661
       \test@edgealign{a}%
       \if@edgealign%
       \test@newedge{1}%
       \if@newedge
1665
       \pgftransformshift{\pgfpoint{1 cm}{0 cm}}%
1666
       \pgftransformrotate{108}%
1667
       \pgftransformshift{\pgfpoint{-1 cm}{0 cm}}%
1668
       \else
1669
       \pgftransformrotate{-108}%
1670
       \fi
1671
       \else
       \test@newedge{1}%
       \if@newedge
       \pgftransformshift%
       \pgftransformrotate{36}%
1677
1678
       \pgftransformshift{\pgfpoint{- \pr@shphi cm}{-\pr@chphi cm}}%
1679
       \pgftransformrotate{-36}%
1680
       \fi\fi\fi
1681
       \fi
1682
       \UsePenroseTile[
         every Penrose tile clip/.try,
         every pentagon clip/.try,
1686
         every pentagon 2 clip/.try
1687
       ]{pentagon 2}
       \UsePenroseTile[
1688
         every Penrose tile/.try,
1689
         every pentagon/.try,
1690
         every pentagon 2/.try,
1691
1692
         pic actions
       ]{pentagon 2}
   \coordinate (-edge d start) at (0,0);
    \coordinate (-edge d end) at (1,0);
   \coordinate (-edge A1 start) at (1,0);
   \coordinate (-edge A1 end) at (1+\pr@shphi,\pr@chphi);
   \coordinate (-edge c1 start) at (1+\pr@shphi,\pr@chphi);
   \coordinate (-edge c1 end) at (\pr@cphi-\pr@shphi,\pr@sphi+\pr@chphi);
   \coordinate (-edge c2 start) at (\pr@cphi-\pr@shphi,\pr@sphi+\pr@chphi);
   \coordinate (-edge c2 end) at (-\pr@shphi,\pr@chphi);
   \coordinate (-edge A2 start) at (-\pr@shphi,\pr@chphi);
    \coordinate (-edge A2 end) at (0,0);
1703
       \end{scope}
1705
1706
     pentagon 2/.style={
       every Penrose pic/.try,
```

```
1708
       pic type=pentagon 2,
     },
1709
The pentagram.
     pentagram/.pic={
1710
        \begin{scope}
        \pgfkeysgetvalue{/tikz/Penrose/alignment location}{\prloc}
        \ifx\prloc\pgfutil@empty
1713
1714
        \else
1715
        \begingroup
        \tikzset{name prefix ..}%
1716
        \tikz@scan@one@point\pgfutil@firstofone%
1717
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} start)%
1718
        \global\pgf@xa=\pgf@x
1719
        \global\pgf@ya=\pgf@y
1720
        \tikz@scan@one@point\pgfutil@firstofone%
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} end)%
        \global\pgf@xb=\pgf@x
1724
        \global\pgf@yb=\pgf@y
        \endgroup
        \advance\pgf@xb by -\pgf@xa
1726
        \advance\pgf@yb by -\pgf@ya
1727
        \pgftransformshift{\pgfpoint{\pgf@xa}{\pgf@ya}}%
1728
        \pgfpointnormalised{\pgfpoint{\pgf@xb}{\pgf@yb}}
1729
        \pgf@xb=\pgf@x
1730
        \pgf@yb=\pgf@y
        \pgftransformtriangle%
        {\pgfpoint{0pt}{0pt}}%
1733
        {\pgfpoint{\pgf@xb}{\pgf@yb}}%
1735
        {\pgfpoint{-\pgf@yb}{\pgf@xb}}%
1736
        \test@newedge{2}%
1737
        \if@newedge
        \pgftransformshift{\pgfpoint{1 cm}{0 cm}}%
1738
        \pgftransformrotate{72}%
1739
        \else
1740
        \test@newedge{3}%
1741
        \if@newedge
1742
        \pgftransformrotate{-72}%
1743
        \pgftransformshift{\pgfpoint{2 * \pr@shphi cm}{0 cm}}%
        \else
        \test@newedge{4}%
        \if@newedge
1747
        1748
        \else
1749
        \test@newedge{5}%
1750
        \if@newedge
        \pgftransformrotate{216}%
1752
        \pgftransformshift{\pgfpoint{\pr@cphi cm}{\pr@sphi cm}}%
1753
1754
        \test@newedge{6}%
1756
        \if@newedge
        \pgftransformshift{\pgfpoint{1cm}{0cm}}%
1757
        \pgftransformrotate{-72}%
1758
        \pgftransformshift{\pgfpoint{\pr@cphi cm}{\pr@sphi cm}}%
1759
        \else
1760
```

```
\test@newedge{7}%
1761
       \if@newedge
1762
       \pgftransformrotate{144}%
1763
       \pgftransformshift{\pgfpoint{\pr@shphi cm}{\pr@chphi cm}}%
1764
       \else
1765
       \test@newedge{8}%
1766
       \if@newedge
1767
       \pgftransformshift{\pgfpoint{1 cm + 2*\pr@shphi cm}{0cm}}%
1768
       \pgftransformrotate{216}%
       \pgftransformshift{\pgfpoint{\pr@cphi cm}{\pr@sphi cm}}%
       \else
       \test@newedge{9}%
       \if@newedge
       \pgftransformshift{\pgfpoint{-2*\pr@shphi cm}{0cm}}%
1774
       \pgftransformrotate{72}%
1775
       \else
1776
       \test@newedge{10}%
       \if@newedge
1778
       \pgftransformrotate{144}%
       \pgftransformshift{\pgfpoint{-1 cm}{0 cm}}%
       \else
       \fi\fi\fi\fi\fi\fi\fi\fi
1782
       \fi
1783
       \UsePenroseTile[
1784
         every Penrose tile clip/.try,
1785
         every pentagram clip/.try
1786
1787
       ]{pentagram}
       \UsePenroseTile[
1788
         every Penrose tile/.try,
1789
         every pentagram/.try,
         pic actions
       ]{pentagram}
   \coordinate (-edge C1 start) at (1,0);
   \coordinate (-edge C1 end) at (0,0);
   \coordinate (-edge C2 start) at (0,0);
   \coordinate (-edge C2 end) at (-\pr@shphi,\pr@chphi);
   \coordinate (-edge C3 start) at (-\pr@shphi,\pr@chphi);
   \coordinate (-edge C3 end) at (-2*\pr@shphi,0);
    \coordinate (-edge C4 start) at (-2*\pr@shphi,0);
    \coordinate (-edge C4 end) at (-1-2*\pr@shphi,0);
   \coordinate (-edge C5 start) at (-1-2*\pr@shphi,0);
   \coordinate (-edge C5 end) at (-\pr@cphi,-\pr@sphi);
   \coordinate (-edge C6 start) at (-\pr@cphi,-\pr@sphi);
   \coordinate (-edge C6 end) at (-\pr@cphi-\pr@shphi,-\pr@sphi-\pr@chphi);
   \coordinate (-edge C7 start) at (-\pr@cphi-\pr@shphi,-\pr@sphi-\pr@chphi);
   \coordinate (-edge C7 end) at (-\pr@shphi,-\pr@chphi);
   \coordinate (-edge C8 start) at (-\pr@shphi,-\pr@chphi);
   \coordinate (-edge C8 end) at (1-\pr@cphi+\pr@shphi,-\pr@sphi-\pr@chphi);
   \coordinate (-edge C9 start) at (1-\pr@cphi+\pr@shphi,-\pr@sphi-\pr@chphi);
   \coordinate (-edge C9 end) at (1-\pr@cphi,-\pr@sphi);
   \coordinate (-edge C10 start) at (1-\pr@cphi,-\pr@sphi);
   \coordinate (-edge C10 end) at (1,0);
1813
       \end{scope}
     },
1814
```

```
1815
      pentagram/.style={
        every Penrose pic/.try,
1816
1817
        pic type=pentagram,
      },
1818
The boat.
      boat/.pic={
1819
        \begin{scope}
1820
        \pgfkeysgetvalue{/tikz/Penrose/alignment location}{\prloc}
1821
        \ifx\prloc\pgfutil@empty
1822
        \else
1823
        \begingroup
1824
        \tikzset{name prefix ..}%
1825
        \tikz@scan@one@point\pgfutil@firstofone%
1826
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} start)%
1827
        \global\pgf@xa=\pgf@x
1828
        \global\pgf@ya=\pgf@y
        \tikz@scan@one@point\pgfutil@firstofone%
        1831
        \global\pgf@xb=\pgf@x
1832
        \global\pgf@yb=\pgf@y
1833
        \endgroup
1834
        \advance\pgf@xb by -\pgf@xa
1835
        \advance\pgf@yb by -\pgf@ya
1836
        \pgftransformshift{\pgfpoint{\pgf@xa}{\pgf@ya}}%
1837
        \pgfpointnormalised{\pgfpoint{\pgf@xb}{\pgf@yb}}
1838
        \pgf@xb=\pgf@x
        \pgf@yb=\pgf@y
        \pgftransformtriangle%
1842
        {\pgfpoint{0pt}{0pt}}%
        {\pgfpoint{\pgf@xb}{\pgf@yb}}%
1843
        {\pgfpoint{-\pgf@yb}{\pgf@xb}}%
1844
        \test@edgealign{d}%
1845
        \if@edgealign
1846
        \pgftransformrotate{180}%
1847
        \pgftransformshift{\pgfpoint{\pr@cphi cm - 1cm}{\pr@sphi cm}}%
1848
1849
        \test@edgealign{b}%
1850
        \if@edgealign%
        \test@newedge{2}%
1852
        \if@newedge
        \pgftransformrotate{144}%
1854
        \pgftransformshift{\pgfpoint{-1 cm}{0 cm}}%
1855
1856
        \pgftransformrotate{216}%
1857
        \pgftransformshift{\pgfpoint{\pr@cphi cm}{\pr@sphi cm}}%
1858
        \fi
1859
        \else
1860
        \test@edgealign{c}%
1861
        \if@edgealign%
        \test@newedge{2}%
1863
        \if@newedge
1864
        \pgftransformshift{\pgfpoint{1 cm}{0 cm}}%
1865
        \pgftransformrotate{72}%
1866
        \else
1867
```

```
\test@newedge{3}%
        \if@newedge
1869
        \pgftransformrotate{-72}%
1870
        \pgftransformshift{\pgfpoint{2 * \pr@shphi cm}{0 cm}}%
1871
        \else
1872
        \test@newedge{4}%
1873
        \if@newedge
1874
        \pgftransformshift{\pgfpoint{1 cm + 2 * \pr@shphi cm}{0 cm}}%
1875
        \else
        \fi\fi\fi\fi\fi\fi
1877
        \fi
1878
        \UsePenroseTile[
1879
          every Penrose tile clip/.try,
1880
          every boat clip/.try
1881
        ]{boat}
1882
        \UsePenroseTile[
1883
          every Penrose tile/.try,
 1884
          every boat/.try,
          pic actions
        ]{boat}
    \coordinate (-edge C1 start) at (1,0);
    \coordinate (-edge C1 end) at (0,0);
    \coordinate (-edge C2 start) at (0,0);
    \coordinate (-edge C2 end) at (-\pr@shphi,\pr@chphi);
    \coordinate (-edge C3 start) at (-\pr@shphi,\pr@chphi);
    \coordinate (-edge C3 end) at (-2*\pr@shphi,0);
    \coordinate (-edge C4 start) at (-2*\pr@shphi,0);
    \coordinate (-edge C4 end) at (-1-2*\pr@shphi,0);
    \coordinate (-edge B1 start) at (-1-2*\pr@shphi,0);
    \coordinate (-edge B1 end) at (-\pr@cphi,-\pr@sphi);
    \coordinate (-edge D start) at (-\pr@cphi,-\pr@sphi);
    \coordinate (-edge D end) at (1-\pr@cphi,-\pr@sphi);
    \coordinate (-edge B2 start) at (1-\pr@cphi,-\pr@sphi);
    \coordinate (-edge B2 end) at (1,0);
1901
        \end{scope}
1902
1903
      boat/.style={
1904
        every Penrose pic/.try,
1905
1906
        pic type=boat,
      }.
The diamond.
      diamond/.pic={
1908
        \begin{scope}
1909
        \pgfkeysgetvalue{/tikz/Penrose/alignment location}{\prloc}
1910
        \ifx\prloc\pgfutil@empty
1911
        \else
1912
        \begingroup
1913
        \tikzset{name prefix ..}%
1914
        \tikz@scan@one@point\pgfutil@firstofone%
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} start)%
1916
        \global\pgf@xa=\pgf@x
1917
        \global\pgf@ya=\pgf@y
1918
        \tikz@scan@one@point\pgfutil@firstofone%
1919
        (\prloc-edge \pgfkeysvalueof{/tikz/Penrose/alignment edge} end)%
1920
```

```
1921
        \global\pgf@xb=\pgf@x
        \global\pgf@yb=\pgf@y
1922
        \endgroup
1923
        \advance\pgf@xb by -\pgf@xa
1924
        \advance\pgf@yb by -\pgf@ya
1925
        \pgftransformshift{\pgfpoint{\pgf@xa}{\pgf@ya}}%
1926
        \pgfpointnormalised{\pgfpoint{\pgf@xb}{\pgf@yb}}
1927
        \pgf@xb=\pgf@x
1928
        \pgf@yb=\pgf@y
        \pgftransformtriangle%
        {\pgfpoint{0pt}{0pt}}%
        {\pgfpoint{\pgf@xb}{\pgf@yb}}%
1932
        {\pgfpoint{-\pgf@yb}{\pgf@xb}}%
1933
1934
        \test@edgealign{d}%
        \if@edgealign%
1935
        \test@newedge{2}%
1936
        \if@newedge
1937
        \pgftransformshift{\pgfpoint{1cm}{0cm}}%
1938
        \pgftransformrotate{-162}%
        \else
        \pgftransformrotate{-18}%
        \fi
1943
        \else
1944
        \test@newedge{2}%
        \if@newedge
1945
        \pgftransformrotate{162}%
1946
        \pgftransformshift{\pgfpoint{- 2 * \pr@chphi cm}{0cm}}%
1947
1948
        \pgftransformshift{\pgfpoint{- \pr@cphi cm}{-\pr@sphi cm}}%
1949
        \pgftransformrotate{18}%
1951
        \fi\fi
        \fi
1953
        \UsePenroseTile[
          every Penrose tile clip/.try,
1954
          every diamond clip/.try
1955
       ]{diamond}
1956
        \UsePenroseTile[
1957
          every Penrose tile/.try,
1958
1959
          every diamond/.try,
          pic actions
       ]{diamond}
   \coordinate (-edge D1 start) at (0,0);
   \coordinate (-edge D1 end) at (\pr@chphi,\pr@shphi);
   \coordinate (-edge B1 start) at (\pr@chphi,\pr@shphi);
   \coordinate (-edge B1 end) at (2*\pr@chphi,0);
   \coordinate (-edge B2 start) at (2*\pr@chphi,0);
   \coordinate (-edge B2 end) at (\pr@chphi,-\pr@shphi);
   \coordinate (-edge D2 start) at (\pr@chphi,-\pr@shphi);
    \coordinate (-edge D2 end) at (0,0);
1969
        \end{scope}
1970
     },
     diamond/.style={
1973
       every Penrose pic/.try,
1974
       pic type=diamond,
```

1975 }, 1976 }