(1)

a) Outliers are pixels which values doesn't follow the ones from the rest of the pixels. It can be due to noise, occlusion or alignment error for example.
The problem is that a single outlier can modify the estimation and make it worse for the rest of the points

b) $E(\theta) = \sum_{i=1}^{n} \rho_\sigma(d(x_{ij}, \theta))$
The standard least error equation is $\rho_\sigma(x) = x^2$ which is more sensitive to outliers

c) Geman-McClure function: $\rho_\sigma(x) = \frac{x^2}{x^2+\sigma^2}$

Its advantage is that it puts a limit of 1 to the error for outliers.
If we choose a large σ we may include outliers but if it is too small we may not include enough points so we can estimate σ as $\sigma = 1.5 \text{ med}(d(x_i; \theta_n))$

d) RANSAC is an iterative method to estimate the parameters of a mathematical model from a data set that contains outliers
It provides a reasonable result only with a certain probability that increases with the number of iterations.

e) The parameters of the RANSAC are:
n = number of points at each evaluation
d = minimum number of points needed
k = number of trials
t = distance to determine outliers

w = probability that a point is a inlier

f) Color, texture and location

Merge approach: start with each pixel in separate cluster
iteratively merge clusters.

Split approach: start with all pixels in one cluster
iteratively split clusters

g) k-means: select k (number of clusters) with an initial guess of k-means $\{m_i\}_1^k$

Assign $l_i = \underset{j \in \{1,k\}}{\operatorname{argmin}} \|f_i - m_j\|$ to each pixel and assign correspondent cluster

Recompute the mean: $m_j = \frac{\sum_{i \in S_j} f_i}{\# S_j}$

Stop when m_j doesn't change

Mixture of Gaussians: it is like k-means replacing

$d = \|f_i - m_j\|^2$ with $d = (f_i - m_j)^T \Sigma_j^{-1} (f_i - m_j)$

$m_j = \frac{\sum_{i \in S_j} f_i}{\# S_j}$ $\quad \Sigma_j = \frac{\sum (f_i - m_j)(f_i - m_j)^T}{\# S_j}$

h) It is similar to k-means too

Give a weight to each sample

$m_j = \frac{\sum_{i \in S_j} w(f_i - m_j) f_i}{\sum_{i \in S_j} w(f_i - m_j)}$ The closest a sample is to the mean the more it affects it

(2)

a) Forward projection: given a 3D world point project into the image using the projection matrix M.

Calibration: given P_i world points and corresponding p_i image points find M and the intrinsic and extrinsic parameters.

Reconstruction: given p_i points in image and M find corresponding P_i world points.

The easiest one is forward projection and the most difficult is reconstruction

b) A set of 3D world points and its image 2D corresponding points

c) Estimate the projection matrix M.

Find the intrinsic and extrinsic parameters

d) $p_i = M P_i = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 18 \\ 14 \\ 7 \end{pmatrix} \rightarrow \begin{pmatrix} 18/7 \\ 2 \end{pmatrix}$

e) $p_i = M P_i$; $\begin{pmatrix} 100 \\ 200 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 & 20 & 10 & 20 \\ 20 & 20 & 40 & 20 \\ & & & \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}$

f) Since we get 2 eqs. from each point and we have 11 unknowns we need 6 points $\Leftrightarrow$ 12 eqs.

g) The properties of the rotation matrix:

$r_1 \cdot r_1 = 1$ $\qquad r_1 \cdot r_2 = 0$

$r_2 \cdot r_2 = 1$ $\qquad r_1 \cdot r_3 = 0$

$r_3 \cdot r_3 = 1$ $\qquad r_2 \cdot r_3 = 0$

h) We need to compute the error

$$E(K^*, R^*, T^*) = \sum_{1}^{n} \left(x_i - \frac{m_1^T P_i}{m_3^T P_i} \right)^2 + \left(y_i - \frac{m_2^T P_i}{m_3^T P_i} \right)^2$$

i) In planar calibration we have a target and we want to know the coordinates in the calibration plane

The error is the euclidean distance

j) $p_i = M P_i$

$$M = K^* [r_1 \; r_2 \; r_3 \; T^*]$$

$$H = K^* [r_1 \; r_2 \; T^*] \quad \text{assumes } z = 0$$