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CS512
1st SEMESTER
Computer Vision

A. $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $C = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$

1. $2A - B$

$\Rightarrow 2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$

2. $\|A\|$ and the angle of A relative to the positive X axis

$\|A\| = \sqrt{1^2 + 2^2 + 3^2}$
$= \sqrt{14}$
$= 3.741$

$\cos\alpha = 1/\sqrt{14}$
$\alpha = \cos^{-1}(1/\sqrt{14})$
$\alpha = 1.3$

3. $\hat{A}$, a unit vector in the direction of A

$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\|A\| = \sqrt{14}$

$u_A = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right)$

4. the direction cosine of A

$$\cos\alpha = \frac{A_x}{\|A\|} = \frac{1}{\sqrt{14}}$$

$$\cos\beta = \frac{A_y}{\|A\|} = \frac{2}{\sqrt{14}}$$

$$\cos\gamma = \frac{A_z}{\|A\|} = \frac{3}{\sqrt{14}}$$

5. A . B and B . A

A . B

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$4(1) + 2(5) + 3(6)$

$4 + 10 + 18$

32

B . A

$$\begin{bmatrix} 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$4(1) + 5(2) + 6(3)$

$4 + 10 + 18$

32

6. the angle between A and B

$|A| = \sqrt{14}$ $\qquad |B| = \sqrt{73}$

$a . b = |a| . |b| \cos\theta$

$$\cos\theta = \frac{a . b}{|a| . |b|}$$

$$\cos\theta = \frac{32}{\sqrt{14}\sqrt{7\cancel{7}}}$$

continued

$\cos\theta = 1$

$\theta = \cos^{-1}(1)$

$\theta = 0$

7. a vector which is perpendicular to A

$\vec{w} \cdot \vec{v} = 0$

$$[a \quad b \quad c] \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$a + 2b + 3c = 0$

if $a = 1$, $b = c$ then

$1 + 2 + 3c = 0$

$c = -1$

$$[1 \quad 1 \quad -1]$$

8. $A \times B$ $\qquad$ $B \times A$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \qquad \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2(6) - 3(5) \\ 3(4) - 1(6) \\ 1(5) - 2(4) \end{bmatrix} \qquad \begin{bmatrix} 5(3) - 6(2) \\ 6(1) - 4(3) \\ 4(3) - 6(1) \end{bmatrix}$$

$$\begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix} \qquad \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}$$

9. a vector which is perpendicular to both A & B use cross product.

$-3i + 6j - 3k = 0$

$i = 1$, $j = 1$

$-3 + 6 - 3K = 0$

$K = 1$

10. the linear dependency between A, B, C

$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $C = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$

$\begin{vmatrix} 1 & 4 & -1 \\ 2 & 5 & 1 \\ 3 & 6 & 3 \end{vmatrix} \Rightarrow 1(15-6) + 4(3-6) - 1(12-15)$

$9 - 12 + 3$

0

They are dependent as determinant is 0

$0 = x \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + z \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$

$0 = \begin{bmatrix} 1 & 4 & -1 \\ 2 & 5 & 1 \\ 3 & 6 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$1x + 4y - z = 0 \rightarrow ①$

$2x + 5y + z = 0 \rightarrow ②$

$3x + 6y + 3z = 0 \rightarrow ③$

① + ②

$3x + 9y = 0$

$x + 3y = 0 \rightarrow ④$

③ - ④

$x + 2y + z = 0$

$- x - 3y$

$- y + z = 0 \rightarrow ⑤$

conti

$y = z \longrightarrow$ A

$x = -3y \longrightarrow$ B

$$-3y\,A + y\,B + y\,C = 0$$

$$-3A + B + C = 0$$

II. $A^T B$ and AB^T

$$A^T B = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 4 + 10 + 18 \end{bmatrix}$$

$$= 32$$

$$AB^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}$$

B. $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix}$

1. $2A - B$

$$\begin{bmatrix} 2 & 4 & 6 \\ 8 & -4 & 6 \\ 0 & 10 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 10 \\ -3 & 12 & -3 \end{bmatrix}$$

2. AB and BA

AB

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+4+9 & 2+2-6 & 1-8+3 \\ 4+4+9 & 8-2-6 & 4+8+3 \\ 0+10-3 & 0+5+2 & 0+20-1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$$

BA

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+8+0 & 2-4+5 & 3+6-1 \\ 2+4+0 & 4-2-20 & 6+3+4 \\ 3-8+0 & 6+4+5 & 9-6-1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$$

3. $(AB)^T$ and B^TA^T

$(AB)^T$

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$$

$$(AB)^T \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

B^TA^T

$$B^TA^T \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1+4+9 & 4-4+9 & 0+10-3 \\ 2+2-6 & 8-2-6 & 0+5+2 \\ 1-8+3 & 4+8+3 & 0-20-1 \end{bmatrix} \Rightarrow \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

4. $|A|$ and $|C|$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{vmatrix} \Rightarrow 1(2-15) + 2(0+4) + 3(20+0)$$

$$\Rightarrow -13 + 8 + 60$$

$$\Rightarrow 55$$

$$|C| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{vmatrix} \Rightarrow 1(15-6) + 2(-6-12) + 3(4+5)$$

$$\Rightarrow 9 - 32 + 27$$

$$\Rightarrow 0.$$

5. the matrix (A, B or C) in which the row vectors form an orthogonal set.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$$

1. $4 - 4 + 9 \neq 0$ ✗

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

1. $2 + 2 - 4 = 0.$
2. $6 - 2 - 4 = 0$
3. $3 - 4 + 1 = 0$

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$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix}$$

1. $4 + 10 + 18 = 32$ ✗

B forms an orthogonal set

6. A^{-1} and B^{-1}

$$A^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}^{-1} = \frac{1}{\|A\|} \times \begin{bmatrix} +(2-15) & -(-4-0) & +(20+0) \\ -(-2-15) & +(-1) & -(5) \\ +(6+6) & -(3-12) & +(-2-8) \end{bmatrix}^T$$

$$= \frac{1}{55} \begin{bmatrix} -13 & 4 & 20 \\ +17 & -1 & -5 \\ 12 & 9 & -10 \end{bmatrix}^T$$

$$= \frac{1}{55} \begin{bmatrix} -13 & +17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$$

conti

$$\Rightarrow \begin{bmatrix} -0.23 & 0.30 & 0.21 \\ 0.07 & -0.01 & 0.16 \\ 0.36 & -0.09 & -0.18 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}^{-1} = \frac{1}{\|B\|} \begin{bmatrix} +(1-8) & -(2+12) & +(-4-3) \\ -(2+2) & +(1-3) & -(-2-6) \\ +(-8-1) & -(-4-2) & +(1-4) \end{bmatrix}^{T}$$

$$= \frac{1}{\|B\|} \begin{bmatrix} -7 & -14 & -7 \\ -4 & -2 & +8 \\ -9 & 6 & -3 \end{bmatrix}^{T}$$

$$\|B\| = 1(1-8) + 2(-12-2) + 1(-4-3)$$
$$= -7 + 2(-14) + -7$$
$$= -42$$

$$= \frac{1}{-42} \begin{bmatrix} -7 & -4 & -9 \\ -14 & -2 & 6 \\ -7 & 8 & -3 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 0.16 & 0.09 & 0.21 \\ 0.33 & 0.04 & -0.14 \\ 0.16 & -0.19 & 0.07 \end{bmatrix}.$$

c. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$, find:

1. the eigenvalues and corresponding eigenvectors of A.

$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

(1) $\lambda I = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

(2) $A - \lambda I = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix}$

(3) $\det \begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix} = (1-\lambda)(2-\lambda) - 3(2)$

cont

$$(1-\lambda)(2-\lambda) - 3(2)$$
$$2 - \lambda - 2\lambda + \lambda^2 - 6$$
$$\lambda^2 - 3\lambda - 4$$

4. $\lambda^2 - 3\lambda - 4 = 0$

$pq \quad p+q$
$-4 \quad -3$
$-4 \quad 1$

$$\cancel{\lambda(\lambda - 3)}$$
$$\lambda^2 + \lambda - 4\lambda - 4 = 0$$
$$\lambda(\lambda+1) - 4(\lambda+1) = 0$$
$$(\lambda - 4)(\lambda + 1) = 0$$
$$\lambda - 4 = 0 \qquad \lambda + 1 = 0$$
$$\boxed{\lambda = 4} \qquad \boxed{\lambda = -1}$$ eigen values

5. $\begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix}$

$\lambda = 4$

$$\begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix}$$

$B\bar{x} = \bar{0}$

$$\begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-3x_1 + 2x_2 = 0 \rightarrow (1)$

$3x_1 - 2x_2 = 0 \rightarrow (2)$

$\lambda = -1$

$$\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}$$

$B\bar{x} = \bar{0}$

$$\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$2x_1 + 2x_2 = 0$

$3x_1 + 3x_2 = 0$

$2x_2 = 3x_1$

if $x_1 = 2$ or -0.55 (after normalizing)

$x_2 = 3$ or -0.83

$x_1 = -x_2$

if $x_1 = 1$ or 0.70

$x_2 = -1$ or -0.70

$$V = \begin{bmatrix} -0.55 & 0.70 \\ -0.83 & -0.70 \end{bmatrix}$$

2. the matrix $V^{-1}AV$ where V is composed of the eigen vectors of A

$$V^{-1} = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}^{-1} = \frac{1}{-5}\begin{bmatrix} 3 & -2 \\ -1 & -1 \end{bmatrix}$$
$$= \begin{bmatrix} -0.6 & 0.4 \\ 0.2 & 0.2 \end{bmatrix}$$

$$V^{-1} = \frac{1}{\underset{0.96}{(0.38 + 0.58)}} \begin{bmatrix} -0.70 & -0.70 \\ +0.83 & -0.55 \end{bmatrix}$$

$$V^{-1} = \begin{bmatrix} -0.72 & -0.72 \\ -0.84 & 0.56 \end{bmatrix}$$

$$V^{-1}A = \begin{bmatrix} -0.72 & -0.72 \\ -0.84 & 0.56 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2.88 & -2.88 \\ 0.84 & -0.56 \end{bmatrix}$$

$$V^{-1}AV = \begin{bmatrix} -2.88 & -2.88 \\ 0.84 & -0.56 \end{bmatrix} \begin{bmatrix} -0.55 & 0.70 \\ -0.83 & -0.70 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix}$$

3. The dot product of Eigen vectors of A

$$\begin{bmatrix} 0.70 & -.70 \end{bmatrix} \begin{bmatrix} -0.55 \\ -0.83 \end{bmatrix} = -0.385 + 0.58$$

$$= 0.2$$

4. The dot product of Eigen vectors of B

$$B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$$

1) $\lambda I = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

2) $A - \lambda I = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 2-\lambda & -2 \\ -2 & 5-\lambda \end{bmatrix}$

3) determinat $(2-\lambda)(5-\lambda) - 4$

$$10 - 2\lambda - 5\lambda + \lambda^2 - 4$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$\lambda^2 - \lambda - 6\lambda + 6 = 0$$

$$\lambda(\lambda - 1)\, 6(\lambda - 1) = 0$$

$$(\lambda - 6)(\lambda - 1) = 0$$

6 7

-6 -1

$\lambda = 6$ $\qquad\qquad\qquad$ $\lambda = 1$

Eigen values.

$$\begin{bmatrix} 2-\lambda & -2 \\ -2 & 5-\lambda \end{bmatrix}$$

$\lambda = 6$

$$\begin{bmatrix} -4 & -2 \\ -2 & -1 \end{bmatrix}$$

$\bar{B}\bar{X} = \bar{0}$

$$\begin{bmatrix} -4 & -2 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-4x_1 - 2x_2 = 0$

$-2x_1 - 1x_2 = 0$

$\boxed{2x_1 = -x_2}$

Normalized

$x_1 = 1$ or 0.44

$x_2 = -2$ or -0.89

$\lambda = 1$

$$\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_1 - 2x_2 = 0$

$-2x_1 + 4x_2 = 0$

$\boxed{2x_2 = x_1}$

$x_1 = 2$ or 0.89

$x_2 = 1$ or -0.44

$$Vect = \begin{bmatrix} 0.44 & -0.89 \\ -0.89 & -0.44 \end{bmatrix}$$

$$\begin{bmatrix} 0.44 & -0.89 \end{bmatrix}\begin{bmatrix} -0.89 \\ -0.44 \end{bmatrix}$$

$= (0.44)(-0.89) + (-0.89)(-0.44)$

$= 0$

5. The property of eigen vectors of B and its reason.

As the Dot product is 0 they must be perpendicular.

D) Let $f(x) = x^2 + 3$, $g(x,y) = x^2 + y^2$, find

1. the first and second derivative of $f(x)$ with respect to x: $f'(x)$ and $f''(x)$

$f(x) = x^2 + 3$ $\qquad$ $x^n \Rightarrow f'(x) = nx^{n-1}$

$f'(x) = 2x$

$f''(x) = 2$ $\qquad$ $nx \Rightarrow f'(x) = n$

2. The partial derivatives : $\frac{\partial g}{\partial x}$ and $\frac{\partial g}{\partial y}$.

$g(x,y) = x^2 + y^2$

$\frac{\partial g}{\partial x} = 2x$ $\qquad$ $\frac{\partial g}{\partial y} = 2y$

3. The gradient vector $\nabla g(x,y)$

$\nabla g(x,y) = \langle g_x(x,y), g_y(x,y) \rangle . \langle a, b \rangle$

$\Rightarrow \nabla g(x,y) = \langle g_x(x,y), g_y(x,y) \rangle$

$= \langle 2x, 2y \rangle$

4. The probability density func of a univariate Gaussian (normal) distribution.

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

μ = mean

σ^2 = variance