Hoare Triples, wp/wlp, Syntactic Substitution

CS 536: Science of Programming, Fall 2019

Due Wed Oct 16

10/6

Problems [50 points]

Lectures 8 & 9: Hoare Triples [18 points]

1. [4 points] Let $\sigma \vDash \{p\}\ S\ \{q\}$. If $\sigma \nvDash p$, do we know whether $M(S, \sigma)$ contains $\perp$ or not? If $M(S, \sigma)$ - $\{\perp\} \vDash q$ or not? Repeat, if $\sigma \vDash p$.
2. [2 points] Briefly, how are $\vDash \{p\}\ S\ \{q\}$, $\vDash_{tot} \{p\}\ S\ \{q\}$, and $\vDash_{tot} \{p\}\ S\ \{\mathtt{T}\}$ related?
3. [2 points] If $\sigma \vDash_{tot} \{p\}\ S\ \{\mathtt{T}\}$ and $\perp \in M(S, \sigma)$, what is the relationship between σ and p?
4. [4 points] If $\sigma \nvDash \{p\}\ S\ \{q\}$ and S is deterministic, do we know whether $\sigma \vDash p$ or not? $M(S, \sigma) \vDash q$ or $\neg q$? $\perp \in$ or $\notin M(,\sigma)$? What if S is nondeterministic?
5. [3 points] What are the relationships between $\sigma \vDash$ (or $\nvDash$) $\{p\}\ S\ \{q\}$, $\sigma \vDash$ (or $\nvDash$) $\{p\}\ S\ \{\neg q\}$, $\sigma \vDash_{tot}$ (or $\nvDash_{tot}$) $\{p\}\ S\ \{q\}$, and $\sigma \vDash_{tot}$ (or $\vDash_{tot}$) $\{p\}\ S\ \{\neg q\}$?
6. [3 points] Suppose S is deterministic and $\sigma \nvDash_{tot} \{p\}\ S\ \{q\}$. Can we conclude anything about $\sigma \vDash$ (or $\nvDash$) $\{p\}\ S\ \{q$ or $\neg q\}$? (Break down your analysis into cases with $\perp \in$ or $\notin M(S, \sigma)$.)

Lectures 10 & 11: wp and wlp [20 points]

7. [3 points] For nondeterministic if, say $IF_N \equiv$ **if** $B_1 \rightarrow S_1 \square B_2 \rightarrow S_2$ **fi**, the basic calculation is $wp(IF_N, q) \equiv (B_1 \rightarrow wp(S_1, q)) \wedge (B_2 \rightarrow wp(S_2, q))$. Is it also the case $wp(IF_N, q) \Leftrightarrow (B_1 \wedge wp(S_1, q)) \vee (B_2 \wedge wp(S_2, q))$? Explain briefly.
8. [3 points] Can we always strengthen preconditions or weaken postconditions? Give an example of when it's useful and when it's not useful.
9. [4 points] Which of the following (four) statements behave differently depending on whether S is deterministic or nondeterministic. Explain briefly.
   - $wp(S, p \vee q) \rightarrow$ and $\leftarrow wp(S, p) \vee wp(S, q)$
   - $wp(S, p \wedge q) \rightarrow$ and $\leftarrow wp(S, p) \wedge wp(S, q)$
10. [4 points] Let $p_0 \rightarrow w \rightarrow p_1$ where $w \Leftrightarrow wp(S, q)$. Which of the following properties can fail? Explain briefly.
    - $\{p_0\}\ S\ \{q\}$, $\{p_1\}\ S\ \{q\}$, $\{\neg p_0\}\ S\ \{\neg q\}$, $\{\neg p_1\}\ S\ \{\neg q\}$
11. [6 points] Calculate the *wlp* or *wp* requested each of the following cases. Just do the syntactic calculation; don't also logically simplify the result.
    a. $wlp(\mathtt{x := x + y;\ y := x*z+y,\ x - y - z\ <\ f(x, y, z)})$
    b. $wlp(\mathbf{if}\ \mathtt{x \geq y}\ \mathbf{then}\ \mathtt{x := x-y}\ \mathbf{fi;}\ \mathtt{y := f(f(x/2, y), x-y), x < y})$
    c. $wp(\mathbf{if}\ \mathtt{x \geq y}\ \mathbf{then}\ \mathtt{x := x-y}\ \mathbf{fi;}\ \mathtt{y := f(f(x/2, y), x*y), x < y})$. Assume $D(\mathtt{f}(u, v)) \equiv u > v$

Lecture 12: Syntactic Substitution [12 points]

12. [12 points] Let $p \equiv (\texttt{z} < \texttt{2*x} \vee \texttt{x} \leq \texttt{y}) \wedge (\exists\, \texttt{x} \,.\, \texttt{x} \div \texttt{y} > \texttt{y} \div \texttt{z}) \wedge (\exists \texttt{y} \,.\, \texttt{g(z\^{}2 + z)} < \texttt{x} * \texttt{y})$. For the calculations below, show some detail if you want partial credit for a wrong answer. Do not logically simplify the results.

    a. [3 points] Calculate $p[\texttt{z}/\texttt{x}]$.

    b. [3 points] Calculate $p[\texttt{(z+a)}/\texttt{z}]$.

    c. [6 points] Calculate $p[\texttt{x+y}/\texttt{z}]$.

