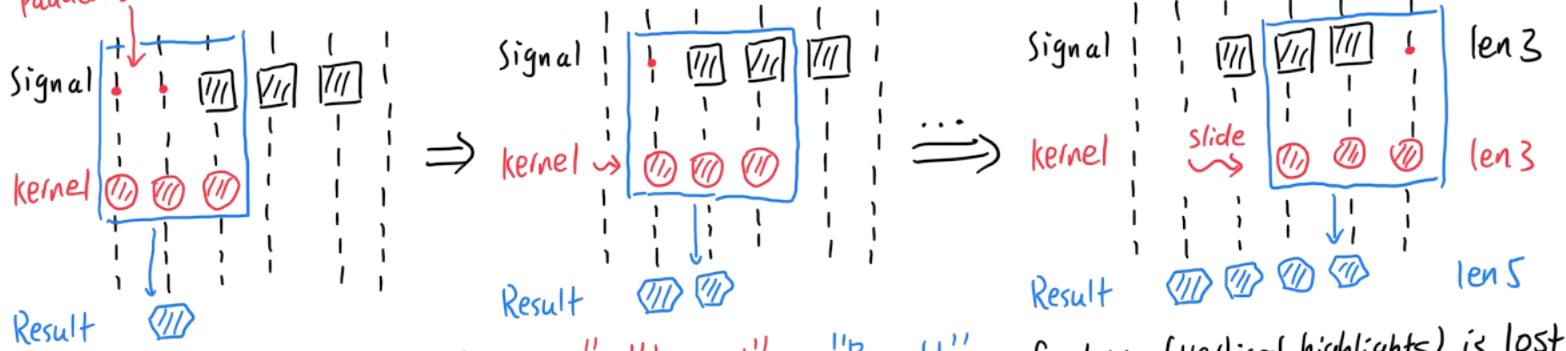


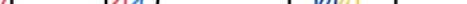
18) Convolution & Transformation

Convolution

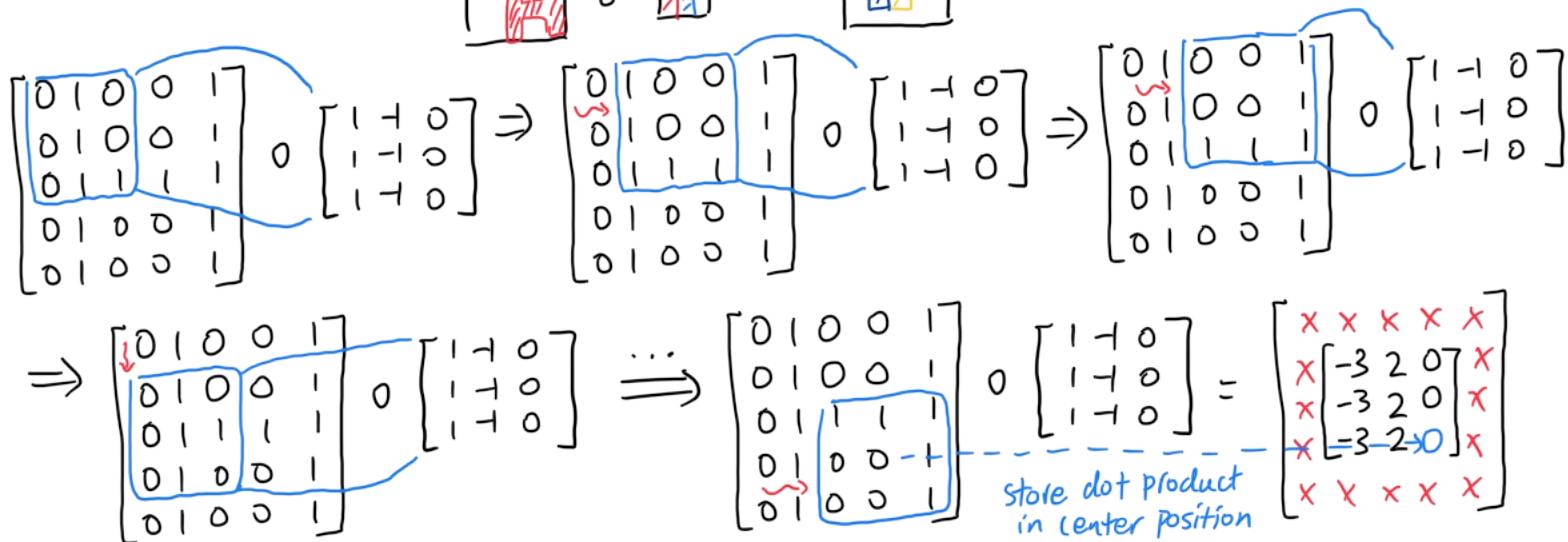
- dot product each image patch by a kernel

- Convolved (signals): "Signal" \circ "Kernel" = "Result"
padded zeros (optional) This averaging kernel
smoothens out edges or
reduces sizes



- `Convolve2d(images):` "Image" "kernel" = Result


feature (Vertical righting) is lost at the edges due to no padding



- , Convolve3d(Images): process is similar to 1D & 2D

- In DL, a bias term will be added to the "result"

More on Kernels & Feature Maps

→ which will then be used for classification

- kernel: small filter that extracts one feature across different images

- : In DL, kernels are learned through GD rather than scientific handcrafting

- Applying N -kernels outputs N feature maps, or layers/channels

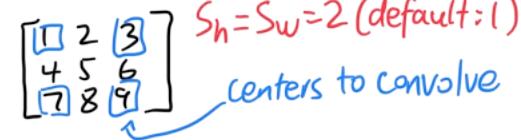
↑ shape: (# channels, height, width) in PyTorch

Convolution Parameters

- ① **Zero Padding**: insert P rows & cols of zeros to the image to increase the size of convolution result
- ② **Stride**: Skip convolving the image by S units to decrease the size of convolution result
- Formula on result size : $N_h = \left\lfloor \frac{M_h + 2P - K_h}{S_h} \right\rfloor + 1, N_w = \left\lfloor \frac{M_w + 2P - K_w}{S_w} \right\rfloor + 1$

. Denote N as output, M as input, h = (height), w = (width), K = kernel size $\propto 2n+1$

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 4 & 5 & 6 & 0 \\ 0 & 7 & 8 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix} \quad P=1 \quad (\text{default: } 0)$$



$$S_h = S_w = 2 \text{ (default: 1)}$$

centers to convolve

Transpose Convolution

- Scalar-multiply a kernel by each image pixel ; deconvolution

"Image" "Kernel" Result

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 1 & x \end{bmatrix} = \begin{bmatrix} 1 & 2 & x \\ x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 1 & x \end{bmatrix} = \begin{bmatrix} 1 & 2+1 & 2 \\ x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} \Rightarrow \text{keep multiplying until last pixel}$$

Sum results if overlap

- . Used for CNN autoencoders & Super-resolution CNN

- Formula on result size : $N_h = S_h(M_h - 1) + K_h - 2P, N_w = S_w(M_w - 1) + K_w - 2P$

Pooling

- Take each non-overlapping patch of image & compute their ___ :

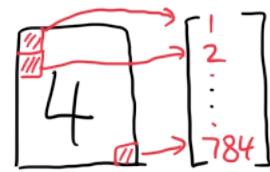
① Avg : result image is smoothed (prevent outliers/noisy data)

② Max : sharpened with important features (assist sparse data)

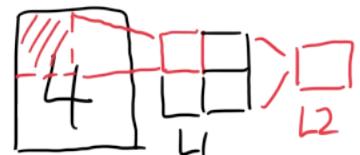
- Purpose: ① Downsampling & Feature Selection

- Purpose: ② Increase Receptive Field : # pixels that are mapped to a node

- FFN unit only has $RF = 1$ (treat each pixel independently)



Each pooling layer in CNN increases RF (attention) of image



- nn.MaxPool2d vs. nn.MaxPool3d on 3D-Image (latter removes depth dimension)

Pooling Parameters

- ① kernel size: # pixels in the pooling window (usually 2)
 - ② Stride: # pixels to skip for window (usually same as ① to prevent overlapping)
 - Formula on result size: $N = \left\lfloor \frac{M-K}{S} \right\rfloor + 1$ ↪ 'floor' mode: drops leftover window
 $N = \left\lceil \frac{M-K}{S} \right\rceil + 1$ ↪ 'ceil' mode: operate as normal
- } only matters if $K \neq S$

Image Transform

- Grayscale, Color change, flip, Zoom/crop, ...
- Why?: ① Adapt image size to pre-trained CNN ② Data augmentation

- Workflow:

