# LQR Control of a Planar VTOL

by

Jesse S. Wynn

Final Project for ECEN 773 (Linear Systems Theory)
Brigham Young University

April 20, 2017

# 1 Introduction

For this project I wanted to implement an LQR controller on a simple MIMO linear system. The dynamic system I chose to model and control was a simple two-dimensional UAV, or planar VTOL. The planar VTOL seemed to be an ideal system to study since it is relatively simple, yet it has many similarities to more complex rotor-craft systems such as quadrotor UAVs. In Dr. Beards undergraduate controls class, we used the planar VTOL as an ongoing case-study exercise, and this project is an extension of that work. The specifics of this project include re-modeling the linear state-space system, and then implementing LQR control, set-point control, and observer-based control in MATLAB and SIMULINK.

# 2 Linear Dynamic Model

The planar VTOL is characterized by a point mass $m_c$ at the vehicle center, and two additional point masses $m_l$ and $m_r$ at a distance $d$ on opposite sides of the vehicle. The term $m_c$ represents the vehicle's main body and electronics, and $m_l$ and $m_r$ represent electric motors and propellers. There are three forces acting on the VTOL which are $f_l$, $f_r$, and then the force of gravity $f_g$. The vehicle pose is defined by horizontal displacement $z$, angle $\theta$, and vertical displacement $h$. An illustration is given below.

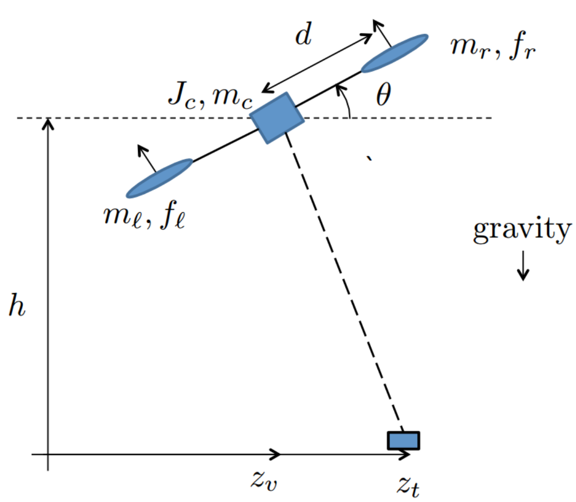


Figure 1: Planar VTOL physical System Definition

The dynamic model (or plant) that is used in simulation includes the full non-linear dynamics of the VTOL. For the controls design however, a much simpler linear model is used. Using principles of linearization from the textbook, the linear dynamics of the system were found to be

$$\ddot{z} = \frac{-F_e}{m_c + 2m_r} + \frac{-\mu\dot{z}}{m_c + 2m_r} \tag{1}$$

$$\ddot{\theta} = \frac{\tau}{J_c + 2m_r d^2} \tag{2}$$

$$\ddot{h} = \frac{\tilde{F}}{m_c + 2m_r} \tag{3}$$

where $F_e$ is an equilibrium force to counteract gravity, $\mu$ is a damping force to represent the propeller drag, $J_c$ is the VTOL's rotational moment of inertia, and $\tau$ and $\tilde{F}$ capture the combined force and torque from $f_l$ and $f_r$. These linear dynamics can be put into state-space form as follows:

$$\dot{x} = \begin{bmatrix} \dot{z} \\ \dot{\theta} \\ \dot{h} \\ \ddot{z} \\ \ddot{\theta} \\ \ddot{h} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \frac{-F_e}{m_c+2m_r} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{4}$$

# 3 Full-State Feedback LQR

# 4 Set-Point LQR Controller

# 5 Implementing an Observer