The University of Texas at Dallas
CS 6364
Artificial Intelligence
Fall 2020
Instructor: Dr. Sanda Harabagiu
Grader/ Teaching Assistant: Maxwell Weinzierl

Homework 4: 100 points
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Submit only in eLearning

Name: Shankaranarayanan Kallidaikuruchi Ramakrishnan
NetId: sxk190109

PROBLEM 1: Qualifying Uncertainty (**20 points**)
Problem 13.21 from the Textbook at page 509. (It should start with (Adapted from Pearl (1988)). Suppose you are a witness to a nightime….

(a) 10 points

Solution:

Let,
B = Boolean Random variable for taxi is Blue and LB = Boolean Random variable for taxi looks like Green under Dim Light.
G = Boolean Random variable for taxi is Green

Given under the dim lighting conditions, discrimination between blue and green is 75% reliable.
P (LB | B) = 0.75
P (¬LB | ¬B) = 0.75

Probability of the taxi to be blue given it looks blue

P (B | LB) = αP(LB | B) . P(B) = α (0.75) P(B)

Probability of the taxi not blue given it looks blue

$P(\neg B \mid LB) = \alpha P(LB \mid \neg B) P(\neg B) = \alpha (0.25)(1 - P(B))$

$P(B \mid LB)$ cannot be obtained with the given information.

(b) 10 points

Solution:

$P(G) = P(\neg B) = 0.9$
$P(B) = 0.1$

We know,

$\alpha P(LB \mid B). P(B) + \alpha (0.25)(1 - P(B)) = 1$

$\alpha (0.75 (0.1) + 0.25 - 0.25 (0.1)) = 1$

$\alpha = 1 / 0.3 = 3.33$

$P(B \mid LB) = (3.33)*(0.75)*(0.1) = .25$

PROBLEM 2: Naïve Bayesian Reasoning (**25 points**)

Consider a traveler that wants to climb the Everest. He gets to Nepal in summer and also finds an experienced guide. Use Naive Bayesian reasoning to decide if the traveler will climb to 1000 ft from the top of the Everest based (**15 points**) on the following information:

1. 10% of all climbers get to 1000 ft from the top of the Everest.

2. Among all travelers who get to 1000 ft from the top of the Everest, 90% went to Nepal in summer and 80% used an experienced guide.

3. 50% of climbers that cannot get to 1000 ft from the top of the Everest went to Nepal in summer and 30% were able to find an experienced guide.

Explain your conclusion. (**10 points)**

Solution:

Let the Boolean random variables be,

C – Climber climbing to 1000ft from top of Everest
N – Climber went to Nepal
G – Climber finds an experienced guide

From 1, $P(C) = 0.1; P(\neg C) = 0.9$
From 2, $P(N \mid C) = 0.9; P(G \mid C) = 0.8$
From 3, $P(N \mid \neg C) = 0.5; P(G \mid \neg C) = 0.3$

Probability that the climber will climb 1000ft by going to Nepal and with an experienced guide.

$$
\begin{aligned}
P(C \mid N, G) &= \alpha\, P(N \mid C)\, P(G \mid C)\, P(C) \\
&= \alpha * 0.9 * 0.8 * 0.1 \\
&= \alpha * 0.072
\end{aligned}
$$

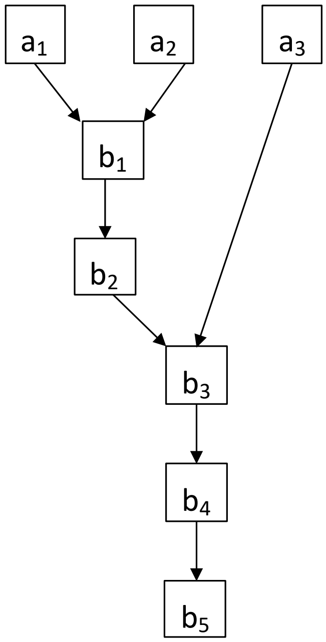
$$
\begin{aligned}
P(\neg C \mid N, G) &= \alpha\, P(N \mid \neg C)\, P(G \mid \neg C)\, P(\neg C) \\
&= \alpha * 0.5 * 0.3 * 0.9 \\
&= \alpha * 0.135
\end{aligned}
$$

We know,
$P(C \mid N, G) + P(\neg C \mid N, G) = 1$
$\alpha * 0.072 + \alpha * 0.135 = 1$
$\alpha = 1 / 0.207 = 4.83$

$P(C \mid N, G) = 4.83 * 0.072 = 0.347$

Since $P(C \mid N, G) > P(C)$, the given traveler will be able to climb 1000 ft from top of Mt. Everest.

PROBLEM 3: Inference with Bayesian Networks (**55 points**) Given the following Bayesian Network:



<u>where:</u>

at node a1 : <u>P(a1)</u> at node a2: <u>P(a2)</u> at node a3: <u>P(a3)</u>

0.7 0.8 0.9

at node b1 :

a1	a2	P(b1)
0	0	0.2
0	1	0.6
1	0	0.7
1	1	0.9

at node b2: at

b1	P(b2)
0	0.6
1	0.8

node b3:

a3	b2	P(b3)
0	0	0
0	1	0.7
1	0	0.8
1	1	1

at node b4 :

b3	P(b4)
0	0.1
1	0.7

at node

b5:

b4	P(b5)
0	0
1	1

You are asked to compute several probabilities by considering the above Bayesian network. In each case your answer can be a number or an expression that can be converted into a number by a pocket calculator.

A) (**5 points**) Compute the probability that: a1 = 1, a2 = 1, a3 = 1, b1 = 0, b2 = 0, b3 = 0, b4 = 0, b5 = 0

<u>Solution:</u>

P(a1 = 1, a2 = 1, a3 = 1, b1 = 0, b2 = 0, b3 = 0, b4 = 0, b5 = 0)
= 0.7 * 0.8 * 0.9 * (1-0.9) * (1-0.6) * (1 – 0.8) * (1 – 0.1) * (1 – 0)

= 0.003628

B) (**15 points**) Compute the probability that b5 = 1

Solution:

Using possible worlds method:

Computing P(b1):

a1	a2	P(b1)	P(W)
0	0	0.2	0.3 * 0.2
0	1	0.6	0.3 * 0.8
1	0	0.7	0.7 * 0.2
1	1	0.9	0.7 * 0.8

P(b1) = 0.2 * 0.06 + 0.6 * 0.24 + 0.7 * 0.14 + 0.9 * 0.56 = 0.758
P(¬b1) = 1 – 0.758 = 0.242

Computing P(b2):

b1	P(b2)	P(W)
0	0.6	0.242
1	0.8	0.758

P(b2) = 0.6 * 0.242 + 0.8 * 0.758 = 0.7516
P(¬b2) = 1 – 0.7516 = 0.2484

Computing P(b3):

a3	b2	P(b3)	P(W)
0	0	0	0.1 * 0.2484
0	1	0.7	0.1 * 0.7516
1	0	0.8	0.9 * 0.2484
1	1	1	0.9 * 0.7516

P(b3) = 0.7 * 0.07516 + 0.8 * 0.223 + 0.9 * 0.7516 = 0.90745
P(¬b3) = 1 - 0.90745 = 0.09255

Computing P(b4):

b3	P(b4)	P(w)
0	0.1	0.09255
1	0.7	0.90745

P(b4) = 0.1 * 0.09255 + 0.7 * 0.90745 = 0.64447
P(¬b4) = 1 - 0.64447 = 0.35553

<u>Computing P(b5):</u>

b4	P(b5)	P(w)
0	0	0.35553
1	1	0.64447

P(b5 = 1) = 0.64447

C) (**10 points**) Compute the probability that b5 = 1 given that:
a1 = 1, a2 = 1, a3 = 1,
b1= 0, b2 = 0, b3 = 0

<u>Solution:</u>

<u>Case 2: Evidence is above the query</u>

<u>Using Possible worlds method</u>

<u>Computing P(b1):</u>

a1	a2	P(b1)	P(W)
0	0	0.2	0
0	1	0.6	0
1	0	0.7	0
1	1	0.9	1

P(b1) = 0.9 * 1 = 0.9
P(¬b1) = 1 – 0.9 = 0.1

<u>Computing P(b2):</u>

b1	P(b2)	P(W)
0	0.6	1

1	0.8	0

P(b2) = 0.6
P(¬b2) = 1 – 0.6 = 0.4

Computing P(b3):

a3	b2	P(b3)	P(W)
0	0	0	0
0	1	0.7	0
1	0	0.8	1
1	1	1	0

P(b3) = 0.8
P(¬b3) = 1 – 0.8 = 0.2

Computing P(b4):

b3	P(b4)	P(w)
0	0.1	1
1	0.7	0

P(b4) = 0.1
P(¬b4) = 1 – 0.1 = 0.9

Computing P(b5):

b4	P(b5)	P(w)
0	0	0.9
1	1	0.1

P(b5 = 1) = 0.1

D) (**5 points**) Compute the probability that b3=0 given that: b5=1

Solution:

Case 3: Evidence is below the query

P(¬b3 | b5) = α P(b5 | ¬b3) P(¬b3)

Computing P(b5 | ¬b3)

P(b5 | ¬b3) = 0.1 [From Problem 3.(C)]

<u>Computing P(¬b3)</u>
P(¬b3) = 0.09255 [From Problem 3.(B)]

<u>Substituting the values:</u>

P(¬b3 | b5) = α (0.1) (0.09255) = (0.009255)α

<u>To Compute α we also need to compute:</u>

P(b3 | b5) = α P(b5 | b3) P(b3)

<u>Computing P(b5 | b3):</u>

<u>Computing P(b4):</u>

b3	P(b4)	P(w)
0	0.1	0
1	0.7	1

P(b4) = 0.7
P(¬b4) = 1 – 0.7 = 0.3

<u>Computing P(b5):</u>

b4	P(b5)	P(w)
0	0	0.3
1	1	0.7

P(b5) = 0.7

<u>Computing P(b3):</u>
P(b3) = 0.90745 [From Problem 3.(B)]

<u>Substituting the values:</u>
P(b3 | b5) = α P(b5 | b3) P(b3) = α (0.7) (0.90745) = 0.635215 α

We know,
P(¬b3 | b5) + P(b3 | b5) = 1
0.009255 α + 0.635215 α = 1
α = 1 / 0.64447 = 1.551

Substituting the value of α

P(¬b3 | b5) = 0.009255 * 1.551 = 0.01435

E) (**20 points**) The CPT in node a3 is changed to:

at node a3 : P(a3)

x

where the value of x is unknown. What values of x would make it make it more likely that b5 happened than that b5 did not happen?

Solution:

P(b1) and P(b2) can be obtained from Problem 3.(B)

Computing P(b3):

a3	b2	P(b3)	P(W)
0	0	0	(1-x) * 0.2484
0	1	0.7	(1-x) * 0.7516
1	0	0.8	x * 0.2484
1	1	1	x * 0.7516

P(b3) = [0.7 * 0.7516 (1-x)] + [0.8 * 0.2484 x] + [0.7516 x]

= 0.52612 + 0.4242 x

P(¬b3) = 0.47388 - 0.4242 x

Computing P(b4):

b3	P(b4)	P(w)
0	0.1	0.47388 - 0.4242 x
1	0.7	0.52612 + 0.4242 x

P(b4) = [0.1 * (0.47388 - 0.4242 x) + 0.7 * (0.52612 + 0.4242 x)]

= 0.415672 + 0.25452 x

P(¬b4) = 0.584328 - 0.25452 x

Computing P(b5):

b4	P(b5)	P(w)
0	0	0.584328 - 0.25452 x
1	1	0.415672 + 0.25452 x

P(b5) = 0.415672 + 0.25452 x

For b5 more likely to happen:

$P(b5) > 0.5$
=>$0.415672 + 0.25452\,x > 0.5$
=> $x > 0.331322$

$P(b5) < 1$
=>$0.415672 + 0.25452\,x < 1$
=> $x < 2.2958$
Since x is a value of probability it should be <= 1

Hence, for b5 to be more likely to happen x should be in the range $0.331322 < x <= 1$