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University of Texas at Dallas
Department of Computer Science
CS 6364.002 – Artificial Intelligence

FINAL EXAM

Fall 2020
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Instructions: Do not communicate with anyone in any shape or form. This is an independent exam. Do not delete any problem formulation, just attach your answer in the space provided. If the problem is deleted and you send only the answer, you shall receive ZERO points.

Copy and paste the Final Exam into a Word document, enter your answers (either by typing in Word, or by inserting a VERY CLEAR picture of your hand-written solution) and transform the file of the exam into a PDF format. If we cannot clearly read the picture, you will get ZERO for that answer! If you create an enormous file for your final exam (i.e. larger than 5 Mbytes) you will receive ZERO for your entire exam. Please follow the instructions from the attached **instructions_submission_EXAM.pdf** file to make sure your final pdf file is of reasonable size.

If you will use a pencil instead of a black pen, you will receive a ZERO for the entire final exam.

Make sure that you insert EACH answer immediately after EACH question. Failure to do so will result in ZERO points for the entire exam! Submit the PDF file with the name **Final_Exam_netID.pdf**, where netID is your unique netid provided by UTD. If you submit your exam in any other format your will receive ZERO points.

The Final exam shall be submitted in eLearning before the deadline. No late submissions shall be graded! Any cheating attempt will determine the ENTIRE grade of the final exam to become ZERO.
Write your answers immediately after the problem statements.

Problem 1 Uncertainty (10 points)

Given the joint probability table:

	rain		*¬ rain*	
	sprinkle	*¬ sprinkle*	*sprinkle*	*¬ sprinkle*
Grasswet	0.1	0.02	0.023	0.007
¬ Grasswet	0.09	0.06	0.14	0.56

Compute:

1. $P(\neg Grasswet)$ (**2 points**)
2. $P(rain|Grasswet)$ (**3 points**)
3. $P(Grasswet|rain \vee sprinkle)$ (**5 points**)

Answers:

1. (**3 points**)

$P(\neg Grasswet)$ = 0.09 + 0.06 + 0.14 + 0.56 = 0.85

2. (**5 points**)

$P(rain|Grasswet)$ = P(rain ∧ Grasswet) / P(Grasswet) = (0.1 + 0.02) / (1 – 0.85) = 0.7999

3. **(12 points)**

$P(Grasswet|rain \vee sprinkle) = P(Grasswet \wedge rain \vee sprinkle)\ /\ P(rain \vee sprinkle)$

$\mathrm{P}(rain \vee sprinkle) = (0.1 + 0.02 + 0.09 + 0.06 + 0.023 + 0.14\ ) = 0.433$

$P(Grasswet \wedge (rain \vee sprinkle)) = 0.1 + 0.02 + 0.023 = 0.143$

$P(Grasswet|rain \vee sprinkle) = 0.143\ /\ 0.433 = 0.3302$

Problem 2 First Order Logic Representations (30 points)

(a) (**15 points**) Represent in First-Order Logic (FOL) the following sentences, using the semantics of the predicates provided for each sentence:

1. *Every artist is inspired by a painting.*
   Predicates: *artist(x)*- x is artist;
   inspire(x,y): x is inspired by y;
   painting(z): z is a painting.

2. *All musicians in an orchestra follow the instructions of their conductor.*
   Predicates: *musician(x)*- x is musician;
   inOchestra(x,y): x is in orchestra y;
   conductor(z,y): z is conductor of orchestra y;
   follows(x,y): x follows the instructions of y;

3. *In each concert, every musician plays some instrument and sings some song and no one gets bored.*
   Predicates: *concert(c)* – c is a concert;

musician(m) – m is a musician;
musicianInConcert(m,c) – m is a musician participating in concert c;
playsInstrument(m,i) – m plays instrument i;
singsSong(m,s) – m sings song s;
getBored(m) – m is getting bored;

Answers:

(a) 1. (**5 points**)

$\exists z\ [\ \forall x\ \text{artist}(x) \wedge \text{painting}(z) \Rightarrow \text{inspire}(x, z)]$

(a) 2. (**5 points**)

$\exists y\ \exists z\ [\ \forall x\ \textit{musician}(x) \wedge \textit{inOchestra}(x, z) \wedge \textit{conductor}(y, z) \Rightarrow \text{follows}(x, y)]$

(a) 3. (**5 points**)

$\forall c\ \forall m\ [\text{concert}(c) \wedge \text{musician}(m) \wedge \text{musicianInConcert}(m, c) \Rightarrow \exists i\ \text{playsInstrument}(m, i) \wedge \exists s\ \text{singsSong}(m,s) \wedge \forall x \neg \text{getBored}(x)\ ]$

(b) Generate the Conjunctive Normal Form (CNF) for the FOL expressions obtained in (a). Specify at each step of the conversion in CNF what you are doing!

Answers:

1. (**5 points**)

   STEP 1: Remove implication

   $\exists z\ [\neg (\forall x \text{ artist}(x) \wedge \text{painting}(z)) \vee \text{inspire}(x, z)]$

   STEP 2: Move $\neg$ inwards

   $\exists z\ [(\exists x \neg \text{artist}(x) \vee \neg \text{painting}(z)) \vee \text{inspire}(x, z)]$

   STEP 3: Standardize variables

   $\exists z\ [(\exists x \neg \text{artist}(x) \vee \neg \text{painting}(z)) \vee \text{inspire}(x, z)]$

   STEP 4: Skolemize

   $(\neg \text{artist}(A) \vee \neg \text{painting}(B)) \vee \text{inspire}(A, B)$

   STEP 5: Remove Universal quantifiers

   $(\neg \text{artist}(A) \vee \neg \text{painting}(B)) \vee \text{inspire}(A, B)$

   STEP 6: Distribute $\wedge$ over $\vee$

   $(\neg \text{artist}(A) \vee \neg \text{painting}(B)) \vee \text{inspire}(A, B)$

2. (**5 points**)

STEP 1: Remove implication

$\exists y \, \exists z \, [\neg (\forall x \, \mathit{musician}(x) \wedge \mathit{inOchestra}(x, z) \wedge \mathit{conductor}(y, z)) \vee \text{follows}(x, y)]$

STEP 2: Move $\neg$ inwards

$\exists y \, \exists z \, [\exists x \, \neg \mathit{musician}(x) \vee \neg \mathit{inOchestra}(x, z) \vee \neg \mathit{conductor}(y, z)) \vee \text{follows}(x, y)]$

STEP 3: Standardize variables

$\exists y \, \exists z \, [\exists x \, \neg \mathit{musician}(x) \vee \neg \mathit{inOchestra}(x, z) \vee \neg \mathit{conductor}(y, z)) \vee \text{follows}(x, y)]$

STEP 4: Skolemize

$[\neg \mathit{musician}(A) \vee \neg \mathit{inOchestra}(A, B) \vee \neg \mathit{conductor}(C, B)) \vee \text{follows}(A, B)]$

STEP 5: Remove Universal quantifiers

$[\neg \mathit{musician}(A) \vee \neg \mathit{inOchestra}(A, B) \vee \neg \mathit{conductor}(C, B)) \vee \text{follows}(A, B)]$

STEP 6: Distribute $\wedge$ over $\vee$

$[\neg \mathit{musician}(A) \vee \neg \mathit{inOchestra}(A, B) \vee \neg \mathit{conductor}(C, B)) \vee \text{follows}(A, B)]$

3. (**5 points**)

STEP 1: Remove implication

$\forall c\ \forall m\ [\neg\ (concert(c) \wedge musician(m) \wedge musicianInConcert(m, c)) \vee\ \exists i\ playsInstrument(m, i) \wedge \exists s\ singsSong(m,s) \wedge \forall x\ \neg\ getBored(x)\ ]$

STEP 2: Move $\neg$ inwards

$\forall c\ \forall m\ [\ (\neg\ concert(c) \vee \neg\ musician(m) \vee \neg\ musicianInConcert(m, c)) \vee\ \exists i\ playsInstrument(m, i) \wedge \exists s\ singsSong(m,s) \wedge \forall x\ \neg\ getBored(x)\ ]$

STEP 3: Standardize variables

$\forall c\ \forall m\ [\ (\neg\ concert(c) \vee \neg\ musician(m) \vee \neg\ musicianInConcert(m, c)) \vee\ \exists i\ playsInstrument(m, i) \wedge \exists s\ singsSong(m,s) \wedge \forall x\ \neg\ getBored(x)\ ]$

STEP 4: Skolemize

$\forall c\ \forall m\ [\ (\neg\ concert(c) \vee \neg\ musician(m) \vee \neg\ musicianInConcert(m, c)) \vee playsInstrument(m, F(m)) \wedge singsSong(m,G(m)) \wedge \forall x\ \neg\ getBored(x)\ ]$

STEP 5: Remove Universal quantifiers

$[\ (\neg\ concert(c) \vee \neg\ musician(m) \vee \neg\ musicianInConcert(m, c)) \vee\ (playsInstrument(m, F(m)) \wedge singsSong(m,G(m)) \wedge \neg\ getBored(x))\ ]$

STEP 6: Distribute $\wedge$ over $\vee$

$[\ (\neg\ concert(c) \vee \neg\ musician(m) \vee \neg\ musicianInConcert(m, c) \vee playsInstrument(m, F(m)) \wedge (\neg\ concert(c) \vee \neg\ musician(m) \vee \neg\ musicianInConcert(m, c) \vee singsSong(m,G(m))) \wedge (\neg\ concert(c) \vee \neg\ musician(m) \vee \neg\ musicianInConcert(m, c) \vee \neg\ getBored(x)))\ ]$

Problem 3 (20 points) Inference in First Order Logic

You are given the following description:

"*If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned*"

1. Transform the text in First-Order Logic (FOL) **(4 points).**
2. Convert each axiom in Conjunctive Normal Form (CNF) and produce a knowledge base (KB) containing all the clauses derived from the CNF. **(6 points).**
3. Given the KB, can you prove that a unicorn is mythical? How about magical? Or horned? Use refutation to prove each of your answers and show the entire proof. **(10 points)**

Answer:

1.

S1: *If the unicorn is mythical, then it is immortal*

S1: Mythical $\Rightarrow \neg$Mortal

S2: *if it is not mythical, then it is a mortal mammal*

S2: $\neg$ Mythical $\Rightarrow$ Mortal $\wedge$ Mammal

S3: *If the unicorn is either immortal or a mammal, then it is horned*

S3: $\neg$Mortal $\vee$ Mammal $\Rightarrow$ Horned

S4: *The unicorn is magical if it is horned*

S4: Horned $\Rightarrow$ Magical

2.

CNF:

S5: $\neg$Mythical $\vee$ $\neg$Mortal

S6: (Mythical $\vee$ Mortal) $\wedge$ (Mythical $\vee$ Mammal)

S7: (Mortal $\vee$ Horned) $\wedge$ ($\neg$Mammal $\vee$ Horned)

S8: $\neg$Horned $\vee$ Magical

KB clauses:

C1: $\neg$Mythical $\vee$ $\neg$Mortal

C2: Mythical $\vee$ Mortal

C3: Mythical $\vee$ Mammal

C4: Mortal $\vee$ Horned

C5: $\neg$Mammal $\vee$ Horned

C6: $\neg$Horned $\vee$ Magical

3.

Negating the goal:

G1: $\neg$Mythical

G2: $\neg$Magical

G3: $\neg$Horned

Applying resolution:

Pairing C1, C3:

X1: $\neg$Mortal $\vee$ Mammal

Pairing X1, C4:

X2: Mammal $\vee$ Horned

Pairing X2, C5:

X3: Horned

Pairing X3, C6:

X4: Magical

Pairing G2 and X3:

<u>X5: NIL</u> => G2 is proven

Pairing G3 and X4:

<u>X6: NIL</u> => G3 is proven

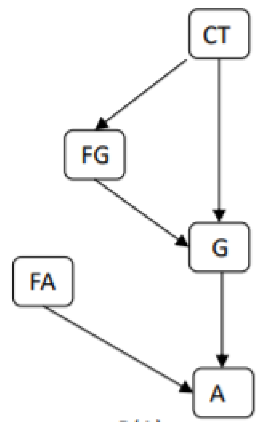
We cannot combine any clauses that resolve to Mythical to prove G1.

Hence Unicorn is Magical and horned but not Mythical.

Problem 4 (20 points) Probabilistic Reasoning

In a nuclear power plant, there is an alarm that senses when a temperature gauge exceeds a given threshold. The gauge measures the temperature of the core. Consider the Boolean random variables A (alarm sounds), FA (alarm is faulty), FG (gauge is faulty), G (gauge is reading) and CT (the core temperature is too high).

The following Bayesian network represents this domain:



P(CT)
m

CT	P(FG)
T	n
F	q

CT	FG	P(G)
T	T	y
F	T	$1-y$
T	F	x
F	F	$1-x$

P(FA)
fa

G	FA	P(A)
T	T	0
F	T	0
T	F	0
F	F	1

1. **(5 points)** Compute the probability that A=1, FA=0, FG=0 and CT=1.
2. **(10 points)** Compute the probability that the core temperature is too high when the alarm sounds, and the alarm and the gauge work well, i.e. P(CT|A, ¬FA, ¬FG).
3. **(5 points)** Decide whether A and FA have any effect on CT given G and explain why you reached that decision.

Answer:

1.

$P(A = 1, FA = 0, FG = 0, CT = 1) = P(A = 1) * P(FA = 0) * P(FG = 0) * P(CT = 1)$

$P(CT = 1) = m; P(\neg CT) = 1 - m$

$P(FA = 1) = fa; P(\neg FA) = 1 – fa$

$P(FG = 1) = mn + (1-m)q; P(\neg FG) = 1 – (mn + (1-m)q)$

To calculate $P(A = 1)$ we need to calculate $P(G = 1)$

$P(G = 1) = my(mn + (1-m)q) + (1-m)(mn + (1-m)q)(1-y) + mx(1 – (mn + (1-m)q)) + (1-m)(1-x)(1- (mn + (1-m)q))$

$P(\neg G) = 1 – P(G)$

$P(A = 1) = P(G)\ P(FA) * 0 + P(G)\ P(\neg FA) * 0 + P(\neg G)\ P(FA) * 0 + P(\neg G)\ P(\neg FA) * 1$

$= P(\neg G)\ P(\neg FA)$

$= (1-fa)\ [my(mn + (1-m)q) + (1-m)(mn + (1-m)q)(1-y) + mx(1 – (mn + (1-m)q)) + (1-m)(1-x)(1- (mn + (1-m)q))]$

$P(A = 1, FA = 0, FG = 0, CT = 1) = (1-fa)\ [my(mn + (1-m)q) + (1-m)(mn + (1-m)q)(1-y) + mx(1 – (mn + (1-m)q)) + (1-m)(1-x)(1- (mn + (1-m)q))] * (1-fa) * (1 – (mn + (1-m)q)) * (m)$

2. $P(CT|A, \neg FA, \neg FG)$

Case: Evidence is below the query

$P(CT|A, \neg FA, \neg FG) = \alpha\ P(A|\ CT)\ P(\neg FA\ |\ CT)\ P(\ \neg FG|CT)\ P(CT)$

$P(CT) = m; P(\neg CT) = 1-m$

From the tree, we know that FA is independent of CT,

$P(FA\ |\ CT) = P(FA) = fa; P(\neg FA\ |\ CT) = 1 - fa$

At Node FG:

CT	P(FG)	P(W)
T	n	1
F	q	0

P(FG | CT) = n; P(¬FG | CT) = 1-n

To compute P(A | CT), we need compute P(G | CT)

At Node G:

CT	FG	P(G)	P(W)
T	T	y	1 * n
F	T	1-y	0
T	F	x	1 * (1-n)
F	F	1-x	0

P(G | CT) = ny + x(1-n)

P(¬G | CT) = 1 – (ny + x(1-n))

At Node A:

G	FA	P(A)	P(W)
T	T	0	fa*[ny + x(1-n)]
F	T	0	[1 – (ny + x(1-n))]*fa
T	F	0	[ny + x(1-n)]*(1-fa)
F	F	1	[1 – (ny + x(1-n))] *(1-fa)

$P(A|CT) = [1 – (ny + x(1-n))] *(1-fa)$

$P(\neg A \mid CT) = 1 – [[1 – (ny + x(1-n))] *(1-fa)]$

$P(CT|A, \neg FA, \neg FG) = \alpha\, P(A| CT)\, P(\neg FA \mid CT)\, P(\neg FG|CT)\, P(CT)$

$= \alpha\, (1 – [[1 – (ny + x(1-n))] *(1-fa)])\, (1-fa)\, (1-n)\, m$

To Compute α, we need

$P(\neg CT|A, \neg FA, \neg FG) = \alpha\, P(A| \neg CT)\, P(\neg FA \mid \neg CT)\, P(\neg FG|\neg CT)\, P(\neg CT)$

At Node FG:

CT	P(FG)	P(W)
T	n	0
F	q	1

$P(FG \mid \neg CT) = q$; $P(\neg FG \mid \neg CT) = 1-q$

To compute $P(A \mid \neg CT)$, we need compute $P(G \mid \neg CT)$

At Node G:

CT	FG	P(G)	P(W)
T	T	y	0
F	T	1-y	1*q
T	F	x	0
F	F	1-x	1*(1-q)

$P(G \mid \neg CT) = (1-y)q + (1-x)(1-q)$

$P(\neg G \mid \neg CT) = 1 – ((1-y)q + (1-x)(1-q))$

At Node A:

G	FA	P(A)	P(W)
T	T	0	fa*[(1-y)q + (1-x)(1-q)
F	T	0	[1 – ((1-y)q + (1-x)(1-q))]*fa
T	F	0	[(1-y)q + (1-x)(1-q)]*(1-fa)
F	F	1	[1 – ((1-y)q + (1-x)(1-q))] *(1-fa)

$P(A \mid \neg CT) = [1 – ((1-y)q + (1-x)(1-q))] *(1-fa)$

$P(\neg A \mid \neg CT) = 1 – [[1 – ((1-y)q + (1-x)(1-q))] *(1-fa)]$

$P(\neg CT|A, \neg FA, \neg FG) = \alpha\, P(A| \neg CT)\, P(\neg FA \mid \neg CT)\, P(\neg FG|\neg CT)\, P(\neg CT)$

$= \alpha\, [1 – ((1-y)q + (1-x)(1-q))] *(1-fa) * (1-fa) * (1-q) * (1-m)$

$P(CT|A, \neg FA, \neg FG) + P(\neg CT|A, \neg FA, \neg FG) = 1$

$\alpha = 1 / \{[1 – ((1-y)q + (1-x)(1-q))] *(1-fa) * (1-fa) * (1-q) * (1-m)\} + \{(1 – [[1 – (ny + x(1-n))] *(1-fa)])\, (1-fa)\, (1-n)\, m\}$

$P(CT|A, \neg FA, \neg FG) = (1 – [[1 – (ny + x(1-n))] *(1-fa)])\, (1-fa)\, (1-n)\, m / \{[1 – ((1-y)q + (1-x)(1-q))] *(1-fa) * (1-fa) * (1-q) * (1-m)\} + \{(1 – [[1 – (ny + x(1-n))] *(1-fa)])\, (1-fa)\, (1-n)\, m\}$

$P(CT|A, \neg FA, \neg FG) =$

$$\frac{(1 – [[1 – (ny + x(1-n))] *(1-fa)])\, (1-fa)\, (1-n)\, m}{\{[1 – ((1-y)q + (1-x)(1-q))] *(1-fa) * (1-fa) * (1-q) * (1-m)\} + \{(1 – [[1 – (ny + x(1-n))] *(1-fa)])\, (1-fa)\, (1-n)\, m\}}$$

3.

$P(CT \mid G)$

Case: Evidence is Below the query

Computing:

$P(CT \mid G) = \alpha P(G \mid CT) * P(CT)$

To compute $P(G \mid CT)$ we only need CT and FG, as they are above the query.

To compute $P(CT)$ we only need CT, as it is the root.

Hence, A and FA do not have any effect on $P(CT \mid G)$ as they are much further below in the tree

Problem 5 (20 points) Inference in Propositional Logic

If your knowledge base (HB) contains the following Horn clauses:

1. $B \Rightarrow A$

2. $C \wedge D \wedge E \Rightarrow B$

3. $B \wedge F \Rightarrow C$

4. $F \wedge G \Rightarrow D$

5. $G \wedge H \Rightarrow E$

6. F

7. G

8. H

Use backward chaining to prove A.

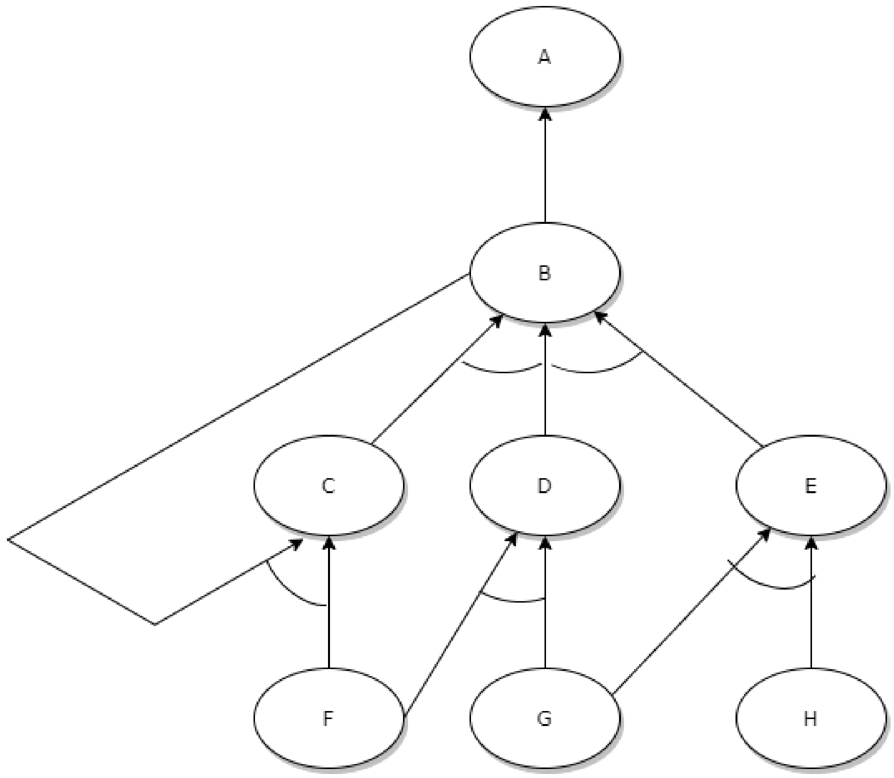
i. Draw the AND=OR graph for this KB (**5 points**).

ii. Show each step of your proof using backward chaining (**15 points**).

Answer:

i.

AND-OR DIAGRAM:



ii.

<u>Step 1:</u>

Suppose A is True,

According to S1: B is True

<u>Step 2:</u>

Suppose B is True,

According to S2: C, D, E are True

<u>Step 3:</u>

Suppose C is True,

According to S3: B, F are True

We know F is True from S6

<u>Step 4:</u>

Since F and C are True, from S3 in KB, we get B is True

Since B is True, From S1 we get A is True.

Therefore, our assumption that A is True was correct.