

VC Dimension

Hypothesis space is set of parallel lines

- considers 1 point, the point can be shattered by the hypothesis space.

 $1/1$

$$V_C(H) \geq 1$$

- consider 2 points, the points can be shattered by the hypothesis space.

$$1 + 1$$
$$1 + 1 =$$
$$-1 + 1$$

11/11/11

$$V_C(H) \geq 2.$$

- Consider 3 points, the points can be shattered by the hypothesis space

1 + 1 = 2

$$V_C(H) \geq 3$$

and other variations

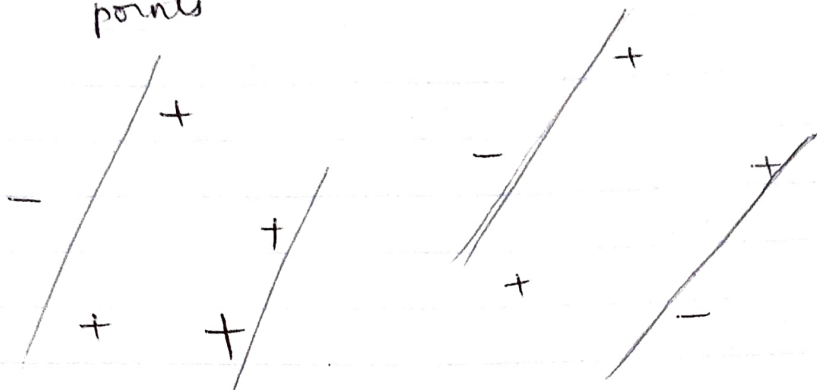
- Consider 4 points, the points can be shattered by the hypothesis space in all configurations

$$\begin{array}{c} \text{---} \\ | \\ + \\ | \\ - \\ | \\ + \\ | \end{array}$$

$$VC(H) \geq 4$$

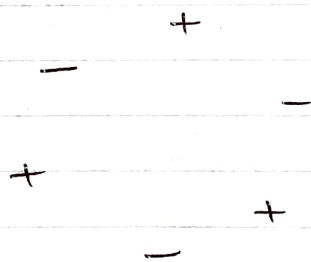
parallel lines can be drawn to enclose + labels

- Consider 5 points placed in a circle, the hypothesis space can shatter the points



$$VC(H) \geq 5$$

- Consider 6 points which labeled in alternate manner, it cannot be shattered by the hypothesis space



Hypothesis space parallel lines can only include $\frac{2}{3}$ of the '+' in this arrangement

$$\boxed{VC(H) = 5}$$

$$\delta = 1 - 0.95 = 0.05$$

$$e = 0.2$$

$$M \geq \frac{1}{0.2} \left(4 \ln \left(\frac{2}{0.05} \right) + 8(5) \ln \left(\frac{13}{0.2} \right) \right)$$

$$M \geq 908.65$$

$$M \geq 909$$

1.2.

VC Dimension

- Consider 1 point, the hypothesis space can shatter it. H_1 can shatter so H' can shatter

$$\begin{array}{|c|} \hline + \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline - \\ \hline \end{array}$$

Setting $\alpha_2 = 0$ $VC(H') = VC(H) \geq 1$

- Consider 2 points, the hypothesis space can shatter it.

$$\begin{array}{|c|c|} \hline + & + \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline - & - \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline + \\ \hline \end{array} -$$

$$- \begin{array}{|c|} \hline + \\ \hline \end{array}$$

H_1 can shatter so H' can shatter.

Setting $\alpha_2 = 0$ $VC(H') = VC(H) \geq 2$

- Consider 3 points, the hypothesis H_1 and H_2 cannot shatter individually if they are arranged as $+, -, +$

Let H_1 be

$$\begin{array}{|c|} \hline + \\ \hline \end{array} -$$

$\alpha_1 = 0.5$

Let H_2 be

$$+ - \begin{array}{|c|} \hline + \\ \hline \end{array}$$

$\alpha_2 = 0.5$

Combining 2 Hypotheses.

$$H' = \alpha_1 H_1 + \alpha_2 H_2$$

H' can shatter 3 points above

$$\begin{array}{|c|c|} \hline + & - \\ \hline \end{array} \begin{array}{|c|} \hline + \\ \hline \end{array}$$

$VC(H') \geq 3$

- By same argument for 3 points, 4 points can be shattered

$$\begin{array}{|c|c|} \hline + & - \\ \hline \end{array} \begin{array}{|c|c|} \hline + & - \\ \hline \end{array}$$

$VC(H') \geq 4$

- Consider 5 points arranged as $+-+--$
These points can not be shattered by
2 Hypotheses. Hence it cannot be used to
shatter with boosting

$$VC(H) = 4$$