Problem 1: SPAM, SPAM, HAM

In [1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import cvxopt
```

In [2]:
```
spam_train = np.loadtxt('spam_train.data', delimiter=',')
spam_validation = np.loadtxt('spam_validation.data', delimiter=',')
spam_test = np.loadtxt('spam_test.data', delimiter=',')
```

In [3]:
```
def preprocess(data):
    m, n = data.shape

    # X,y split
    X = data[:, :n-1]
    y = data[:, n-1:]

    # Set y = -1
    y = np.apply_along_axis(lambda x: -1 if x == 0 else 1, 1, y).reshape(-1, 1
)

    return X,y
```

In [4]:
```
X_train, y_train = preprocess(spam_train)
X_validate, y_validate = preprocess(spam_validation)
X_test, y_test = preprocess(spam_test)
```

In [5]:
```
def get_accuracy(X, y, w, b):
    m, n = X.shape
    z = np.dot(X,w) + b
    f = (y * z) > 0
    return np.sum(f.astype('float32')) * 100/m
```

In [6]:

```
# Primal problem
def SVM_primal(X, Y, c):
    print(f"Computing c = {c}")
    m, n = X.shape

    P = np.zeros((m+n+1, m+n+1))
    for i in range(n):
        P[i][i] = 1
    P = cvxopt.matrix(P)
    q = np.zeros((m+n+1, 1))
    for i in range(n, m+n):
        q[i][0] = c
    q = cvxopt.matrix(q)
    G = np.zeros((2*m, m+n+1))
    for i in range(m):
        for j in range(n):
            G[i][j] = -1 * Y[i] * X[i][j]
        G[i][n+i] = -1
        G[i][m+n] = -1 * Y[i]
        G[m+i][n+i] = -1
    G = cvxopt.matrix(G)
    h = np.zeros((2*m, 1))
    for i in range(m):
        h[i][0] = -1
    h = cvxopt.matrix(h)

    # CVXOPT Solver
    solution = cvxopt.solvers.qp(P, q, G, h)
    sol = np.array(solution['x'])
    w = sol[:n]
    b = sol[m+n]
    return w, b


# Dual problem
def SVM_dual(X, Y, c, σ2):

    print(f"Computing c = {c} and variance = {σ2}")
    m, n = X.shape

    # Create Gram matrix
    K = np.zeros((m, m))
    # Use gaussian kernel
    X_sq = -2 * np.dot(X, X.T)
    X_sq += (X ** 2).sum(axis=1).reshape(-1, 1)
    X_sq += (X ** 2).sum(axis=1)

    K = X_sq / (-2 * σ2)
    np.exp(K, K)
    # P = Combination of Yi Yj Xi Xj
    P = cvxopt.matrix(np.outer(Y,Y) * K)
    q = cvxopt.matrix(np.ones(m) * -1)

    # Constraints λY = 0
    A = cvxopt.matrix(Y, (1,m), 'd')
    b = cvxopt.matrix(0.0)
```

```
    # λ >= 0
    lhs = np.diag(np.ones(m) * -1)
    lhs2 = np.identity(m)
    G = cvxopt.matrix(np.vstack((lhs, lhs2)))
    rhs = np.zeros(m)
    rhs2 = np.ones(m) * c
    h = cvxopt.matrix(np.hstack((rhs, rhs2)))

    # CVXOPT Solver
    solution = cvxopt.solvers.qp(P, q, G, h, A, b)

    # Solver produces λ
    λ = np.array(solution['x'])
    idx = (λ > 1e-6).nonzero()[0]

    w = np.zeros(X.shape[1])
    for i in range(len(λ)):
        w += Y[i] * λ[i] * X[i]

    b = 0
    for i in idx:
        b = ((1 / Y[i]) - (np.dot(w, X[i])))

    support_vectors = X[idx]

    return w.reshape(-1, 1), b, support_vectors
```

In [7]:
```
def run_svm(c_list, σ2_list = [None], primal = True):
    data = {
        'c': [],
        'variance': [],
        'Training Data Accuracy': [],
        'Validation Data Accuracy': []
    }
    best_c, best_σ2, best_validation_acc, best_train_acc = c_list[0], σ2_list[
0], 0, 0
    best_w, best_b = [], 0
    for c in c_list:
        for σ2 in σ2_list:
            data['c'].append(c)
            data['variance'].append(σ2)
            if primal:
                w, b = SVM_primal(X_train, y_train, c)
            else:
                w, b, sv = SVM_dual(X_train, y_train ,c,σ2)
            train_acc = get_accuracy(X_train, y_train, w, b)
            data['Training Data Accuracy'].append(train_acc)
            validation_acc = get_accuracy(X_validate, y_validate, w, b)
            data['Validation Data Accuracy'].append(validation_acc)
            if validation_acc > best_validation_acc or (validation_acc == best
_validation_acc and train_acc > best_train_acc):
                best_validation_acc = validation_acc
                best_train_acc = train_acc
                best_c = c
                best_σ2 = σ2
                best_w = w
                best_b = b
    df = pd.DataFrame.from_dict(data)
    test_acc = get_accuracy(X_test, y_test, best_w, best_b)
    df['Testing Data Accuracy'] = df.apply(lambda row: test_acc if row['c'] ==
best_c and row['variance'] == best_σ2 else None, axis=1)
    if best_σ2 is None:
        df.drop(columns=['variance'], inplace=True)
    return df
```

1. Primal SVMs

- Using gradient descent or quadratic programming, apply the SVM with slack formulation to train a classifier for each choice of $c \in 1, 10, 10^2, 10^3, 10^4, 10^5, 10^6, 10^7, 10^8$ without using any feature maps.
- What is the accuracy of the learned classifier on the training set for each value of c?
- Use the validation set to select the best value of c. What is the accuracy on the validation set for each value of c?
- Report the accuracy on the test set for the selected classifier.

In [8]:
```
c_list = [1,10,10**2,10**3,10**4,10**5,10**6,10**7,10**8]
df_primal = run_svm(c_list, primal=True)
```

```
Computing c = 1
     pcost       dcost       gap    pres   dres
 0: -1.8124e+03  1.0398e+04  8e+04  6e+00  4e+04
 1:  5.6137e+03 -7.3332e+03  2e+04  1e+00  7e+03
 2:  3.7452e+03 -2.4540e+03  7e+03  4e-01  2e+03
 3:  2.3041e+03 -9.9333e+02  4e+03  2e-01  1e+03
 4:  1.6237e+03 -3.8918e+02  2e+03  1e-01  6e+02
 5:  1.3035e+03 -9.5546e+01  2e+03  6e-02  4e+02
 6:  1.1640e+03  1.5815e+01  1e+03  4e-02  3e+02
 7:  1.1113e+03  8.9810e+01  1e+03  3e-02  2e+02
 8:  1.0293e+03  1.6990e+02  9e+02  2e-02  1e+02
 9:  8.2001e+02  2.9315e+02  5e+02  1e-02  6e+01
10:  7.1530e+02  3.5051e+02  4e+02  6e-03  4e+01
11:  6.2597e+02  3.9397e+02  2e+02  3e-03  2e+01
12:  5.6753e+02  4.2284e+02  1e+02  2e-03  1e+01
13:  5.3370e+02  4.3939e+02  1e+02  1e-03  6e+00
14:  5.1168e+02  4.5038e+02  6e+01  5e-04  3e+00
15:  4.8730e+02  4.6134e+02  3e+01  3e-05  2e-01
16:  4.7777e+02  4.6801e+02  1e+01  7e-06  4e-02
     pcost       dcost       gap    pres   dres
 0: -2.4008e+07  3.1738e+07  1e+08  3e+02  1e+04
 1:  8.2844e+06 -1.2271e+07  4e+07  7e+01  3e+03
 2:  6.5713e+06 -6.2591e+06  2e+07  3e+01  1e+03
 3:  3.6902e+06 -2.7062e+06  9e+06  1e+01  5e+02
 4:  3.0269e+06 -1.8561e+06  7e+06  8e+00  3e+02
 5:  3.1457e+06 -1.4996e+06  6e+06  6e+00  2e+02
 6:  2.6834e+06 -7.3285e+05  4e+06  3e+00  1e+02
 7:  2.0652e+06 -3.5457e+05  3e+06  1e+00  5e+01
 8:  1.3716e+06 -1.4503e+05  2e+06  6e-01  3e+01
 9:  7.0906e+05 -1.7798e+04  8e+05  1e-01  6e+00
10:  3.8562e+05  1.0669e+04  4e+05  6e-02  3e+00
11:  1.3612e+05  2.4549e+04  1e+05  1e-02  5e-01
12:  1.1239e+05  2.8267e+04  9e+04  8e-03  3e-01
13:  9.7672e+04  3.0446e+04  7e+04  5e-03  2e-01
14:  8.1277e+04  3.3017e+04  5e+04  4e-03  2e-01
15:  6.8425e+04  3.5187e+04  3e+04  2e-03  8e-02
16:  6.1652e+04  3.6791e+04  3e+04  1e-03  5e-02
17:  5.8454e+04  3.7844e+04  2e+04  8e-04  4e-02
18:  5.5855e+04  3.8616e+04  2e+04  6e-04  2e-02
19:  5.2109e+04  3.9410e+04  1e+04  2e-04  7e-03
20:  5.0886e+04  3.9856e+04  1e+04  9e-05  4e-03
21:  4.7811e+04  4.1258e+04  7e+03  5e-05  2e-03
22:  4.6273e+04  4.1938e+04  4e+03  2e-05  9e-04
23:  4.5297e+04  4.2381e+04  3e+03  1e-05  4e-04
24:  4.4445e+04  4.2820e+04  2e+03  3e-06  1e-04
25:  4.4233e+04  4.2941e+04  1e+03  1e-06  6e-05
26:  4.3781e+04  4.3276e+04  5e+02  5e-07  2e-05
27:  4.3634e+04  4.3389e+04  2e+02  2e-07  9e-06
28:  4.3554e+04  4.3446e+04  1e+02  2e-08  8e-07
29:  4.3508e+04  4.3488e+04  2e+01  3e-09  1e-07
30:  4.3499e+04  4.3496e+04  3e+00  5e-10  2e-08
31:  4.3498e+04  4.3497e+04  6e-01  7e-11  3e-09
32:  4.3498e+04  4.3497e+04  2e-01  1e-11  5e-10
33:  4.3498e+04  4.3498e+04  5e-03  5e-15  4e-12
Optimal solution found.
Computing c = 1000
     pcost       dcost       gap    pres   dres
```

```
 0: -2.4062e+09  3.1134e+09  1e+10  3e+03  1e+04
 1:  7.8982e+08 -1.1970e+09  4e+09  7e+02  3e+03
 2:  6.3272e+08 -6.1346e+08  2e+09  3e+02  1e+03
 3:  3.6002e+08 -2.7249e+08  9e+08  1e+02  5e+02
 4:  2.9565e+08 -1.8769e+08  7e+08  8e+01  3e+02
 5:  3.0820e+08 -1.5094e+08  6e+08  6e+01  2e+02
 6:  2.6267e+08 -7.6671e+07  4e+08  3e+01  1e+02
 7:  1.9923e+08 -3.7043e+07  3e+08  1e+01  5e+01
 8:  1.3967e+08 -1.8220e+07  2e+08  7e+00  3e+01
 9:  6.7798e+07 -3.7433e+06  7e+07  1e+00  6e+00
10:  2.8408e+07 -6.9519e+05  3e+07  4e-01  2e+00
11:  8.8956e+06  8.2338e+04  9e+06  9e-02  4e-01
12:  1.8036e+06  2.2222e+05  2e+06  3e-03  1e-02
13:  1.5139e+06  2.5404e+05  1e+06  2e-03  9e-03
14:  1.0751e+06  2.9149e+05  8e+05  1e-03  5e-03
15:  9.8727e+05  3.0617e+05  7e+05  9e-04  4e-03
16:  8.6334e+05  3.2572e+05  5e+05  6e-04  3e-03
17:  7.3925e+05  3.4399e+05  4e+05  4e-04  2e-03
18:  6.4501e+05  3.6001e+05  3e+05  2e-04  1e-03
19:  6.0401e+05  3.7013e+05  2e+05  2e-04  7e-04
20:  5.6838e+05  3.8097e+05  2e+05  1e-04  5e-04
21:  5.4796e+05  3.8735e+05  2e+05  8e-05  4e-04
22:  5.1782e+05  3.9397e+05  1e+05  3e-05  1e-04
23:  5.0307e+05  3.9880e+05  1e+05  1e-05  6e-05
24:  4.8849e+05  4.0502e+05  8e+04  1e-05  4e-05
25:  4.6113e+05  4.1604e+05  5e+04  2e-06  9e-06
26:  4.5330e+05  4.1968e+05  3e+04  1e-06  5e-06
27:  4.4631e+05  4.2376e+05  2e+04  6e-07  3e-06
28:  4.3826e+05  4.2824e+05  1e+04  8e-08  4e-07
29:  4.3690e+05  4.2916e+05  8e+03  4e-08  2e-07
30:  4.3631e+05  4.2946e+05  7e+03  3e-08  1e-07
31:  4.3426e+05  4.3120e+05  3e+03  1e-08  4e-08
32:  4.3320e+05  4.3210e+05  1e+03  2e-09  9e-09
33:  4.3281e+05  4.3244e+05  4e+02  7e-10  3e-09
34:  4.3265e+05  4.3258e+05  7e+01  6e-11  2e-10
35:  4.3262e+05  4.3261e+05  7e+00  4e-12  7e-12
36:  4.3262e+05  4.3262e+05  9e-01  4e-13  5e-11
37:  4.3262e+05  4.3262e+05  1e-02  7e-15  1e-11
Optimal solution found.
Computing c = 10000
     pcost       dcost       gap    pres   dres
 0: -2.4067e+11  3.1074e+11  1e+12  3e+04  1e+04
 1:  7.8599e+10 -1.1940e+11  4e+11  7e+03  3e+03
 2:  6.3029e+10 -6.1221e+10  2e+11  3e+03  1e+03
 3:  3.5911e+10 -2.7265e+10  9e+10  1e+03  5e+02
 4:  2.9495e+10 -1.8789e+10  7e+10  8e+02  3e+02
 5:  3.0755e+10 -1.5102e+10  6e+10  6e+02  2e+02
 6:  2.6210e+10 -7.6997e+09  4e+10  3e+02  1e+02
 7:  1.9847e+10 -3.7178e+09  3e+10  1e+02  5e+01
 8:  1.3985e+10 -1.8571e+09  2e+10  7e+01  3e+01
 9:  6.7279e+09 -3.8999e+08  7e+09  1e+01  6e+00
10:  2.6147e+09 -7.9714e+07  3e+09  4e+00  2e+00
11:  7.9276e+08 -1.0508e+07  8e+08  8e-01  3e-01
12:  1.0368e+08  8.9575e+05  1e+08  9e-13  3e-12
13:  2.6536e+07  2.0710e+06  2e+07  3e-13  3e-12
14:  2.5444e+07  2.3041e+06  2e+07  2e-13  2e-12
15:  1.1428e+07  2.7606e+06  9e+06  9e-14  1e-12
```

```
16:  1.0035e+07  2.9990e+06  7e+06  6e-14  1e-12
17:  8.8805e+06  3.1786e+06  6e+06  5e-14  1e-12
18:  8.2463e+06  3.2979e+06  5e+06  5e-14  6e-13
19:  7.3386e+06  3.4435e+06  4e+06  4e-14  1e-12
20:  6.3689e+06  3.6086e+06  3e+06  2e-14  6e-13
21:  6.0183e+06  3.6991e+06  2e+06  2e-14  9e-13
22:  5.6566e+06  3.8072e+06  2e+06  1e-14  1e-12
23:  5.4820e+06  3.8590e+06  2e+06  1e-14  8e-13
24:  5.2193e+06  3.9383e+06  1e+06  1e-14  4e-13
25:  5.0408e+06  4.0037e+06  1e+06  8e-15  3e-12
26:  4.8314e+06  4.0545e+06  8e+05  6e-15  3e-12
27:  4.6116e+06  4.1517e+06  5e+05  6e-15  3e-12
28:  4.5093e+06  4.2011e+06  3e+05  4e-15  2e-13
29:  4.4244e+06  4.2498e+06  2e+05  6e-15  4e-12
30:  4.3873e+06  4.2755e+06  1e+05  6e-15  3e-12
31:  4.3612e+06  4.2914e+06  7e+04  4e-15  2e-13
32:  4.3391e+06  4.3102e+06  3e+04  5e-15  2e-12
33:  4.3265e+06  4.3211e+06  5e+03  4e-15  2e-12
34:  4.3245e+06  4.3229e+06  2e+03  4e-15  7e-12
35:  4.3238e+06  4.3235e+06  3e+02  4e-15  2e-11
36:  4.3237e+06  4.3237e+06  2e+01  7e-15  1e-12
37:  4.3237e+06  4.3237e+06  3e-01  5e-15  9e-12
Optimal solution found.
Computing c = 100000
     pcost       dcost       gap    pres   dres
 0: -2.4068e+13  3.1068e+13  1e+14  3e+05  1e+04
 1:  7.8561e+12 -1.1937e+13  4e+13  7e+04  3e+03
 2:  6.3005e+12 -6.1208e+12  2e+13  3e+04  1e+03
 3:  3.5902e+12 -2.7267e+12  9e+12  1e+04  5e+02
 4:  2.9488e+12 -1.8791e+12  7e+12  8e+03  3e+02
 5:  3.0749e+12 -1.5103e+12  6e+12  6e+03  2e+02
 6:  2.6204e+12 -7.7030e+11  4e+12  3e+03  1e+02
 7:  1.9839e+12 -3.7192e+11  3e+12  1e+03  5e+01
 8:  1.3986e+12 -1.8606e+11  2e+12  7e+02  3e+01
 9:  6.7225e+11 -3.9151e+10  7e+11  1e+02  6e+00
10:  2.5899e+11 -8.0644e+09  3e+11  4e+01  2e+00
11:  7.8146e+10 -1.2301e+09  8e+10  7e+00  3e-01
12:  9.9839e+09 -1.0962e+08  1e+10  7e-12  6e-13
13:  2.4943e+09 -4.8458e+06  2e+09  3e-12  6e-12
14:  2.3952e+09  7.8046e+06  2e+09  2e-12  5e-12
15:  2.2296e+08  2.2505e+07  2e+08  7e-13  4e-12
16:  1.5659e+08  2.6120e+07  1e+08  4e-13  6e-13
17:  1.1422e+08  2.9144e+07  9e+07  2e-13  2e-12
18:  9.9419e+07  3.0968e+07  7e+07  1e-13  7e-13
19:  8.5552e+07  3.3020e+07  5e+07  7e-14  1e-12
20:  7.8034e+07  3.4126e+07  4e+07  6e-14  2e-12
21:  6.6307e+07  3.5786e+07  3e+07  3e-14  9e-13
22:  6.0545e+07  3.6982e+07  2e+07  2e-14  1e-12
23:  5.7240e+07  3.7945e+07  2e+07  2e-14  6e-13
24:  5.5061e+07  3.8606e+07  2e+07  1e-14  7e-13
25:  5.2803e+07  3.9267e+07  1e+07  1e-14  7e-13
26:  5.0957e+07  3.9688e+07  1e+07  8e-15  1e-12
27:  4.8539e+07  4.0317e+07  8e+06  7e-15  3e-13
28:  4.6004e+07  4.1660e+07  4e+06  7e-15  3e-12
29:  4.5237e+07  4.2015e+07  3e+06  6e-15  3e-12
30:  4.4594e+07  4.2361e+07  2e+06  6e-15  2e-12
31:  4.3934e+07  4.2761e+07  1e+06  6e-15  9e-14
```

```
32:  4.3602e+07  4.2954e+07  6e+05  6e-15  1e-12
33:  4.3408e+07  4.3087e+07  3e+05  5e-15  5e-12
34:  4.3282e+07  4.3189e+07  9e+04  5e-15  5e-12
35:  4.3259e+07  4.3211e+07  5e+04  5e-15  5e-14
36:  4.3249e+07  4.3220e+07  3e+04  5e-15  8e-12
37:  4.3236e+07  4.3233e+07  3e+03  7e-15  7e-13
38:  4.3234e+07  4.3234e+07  3e+02  5e-15  1e-11
39:  4.3234e+07  4.3234e+07  5e+00  5e-15  3e-12
Optimal solution found.
Computing c = 1000000
     pcost       dcost       gap    pres   dres
 0: -2.4068e+15  3.1067e+15  1e+16  3e+06  1e+04
 1:  7.8557e+14 -1.1937e+15  4e+15  7e+05  3e+03
 2:  6.3003e+14 -6.1207e+14  2e+15  3e+05  1e+03
 3:  3.5901e+14 -2.7267e+14  9e+14  1e+05  5e+02
 4:  2.9487e+14 -1.8791e+14  7e+14  8e+04  3e+02
 5:  3.0748e+14 -1.5103e+14  6e+14  6e+04  2e+02
 6:  2.6203e+14 -7.7033e+13  4e+14  3e+04  1e+02
 7:  1.9839e+14 -3.7193e+13  3e+14  1e+04  5e+01
 8:  1.3987e+14 -1.8609e+13  2e+14  7e+03  3e+01
 9:  6.7220e+13 -3.9166e+12  7e+13  1e+03  6e+00
10:  2.5874e+13 -8.0736e+11  3e+13  4e+02  2e+00
11:  7.8030e+12 -1.2479e+11  8e+12  7e+01  3e-01
12:  9.9453e+11 -1.2934e+10  1e+12  8e-11  7e-12
13:  2.4786e+11 -2.4899e+09  3e+11  3e-11  3e-12
14:  2.3809e+11 -1.2337e+09  2e+11  2e-11  3e-12
15:  1.9536e+10  1.9347e+08  2e+10  8e-12  2e-12
16:  2.2073e+09  2.2729e+08  2e+09  1e-12  1e-12
17:  1.5904e+09  2.6162e+08  1e+09  6e-13  6e-13
18:  1.1312e+09  2.9247e+08  8e+08  3e-13  2e-12
19:  9.9715e+08  3.0965e+08  7e+08  2e-13  8e-13
20:  8.5660e+08  3.3027e+08  5e+08  1e-13  8e-13
21:  7.9603e+08  3.3926e+08  5e+08  1e-13  8e-13
22:  7.1533e+08  3.5075e+08  4e+08  7e-14  7e-13
23:  6.3410e+08  3.6430e+08  3e+08  5e-14  1e-12
24:  5.8423e+08  3.7584e+08  2e+08  3e-14  1e-12
25:  5.5083e+08  3.8607e+08  2e+08  2e-14  9e-13
26:  5.3499e+08  3.9025e+08  1e+08  1e-14  2e-12
27:  5.0854e+08  3.9676e+08  1e+08  9e-15  8e-13
28:  4.9598e+08  4.0028e+08  1e+08  8e-15  6e-13
29:  4.7632e+08  4.0771e+08  7e+07  5e-15  9e-13
30:  4.6514e+08  4.1228e+08  5e+07  8e-15  1e-12
31:  4.5149e+08  4.2015e+08  3e+07  7e-15  2e-12
32:  4.4640e+08  4.2311e+08  2e+07  8e-15  2e-12
33:  4.3994e+08  4.2698e+08  1e+07  6e-15  3e-12
34:  4.3666e+08  4.2896e+08  8e+06  6e-15  2e-12
35:  4.3395e+08  4.3100e+08  3e+06  5e-15  5e-12
36:  4.3261e+08  4.3211e+08  5e+05  6e-15  2e-12
37:  4.3248e+08  4.3222e+08  3e+05  6e-15  1e-12
38:  4.3242e+08  4.3226e+08  2e+05  5e-15  1e-11
39:  4.3237e+08  4.3231e+08  7e+04  6e-15  9e-13
40:  4.3236e+08  4.3232e+08  4e+04  4e-15  6e-13
41:  4.3234e+08  4.3233e+08  9e+03  6e-15  8e-12
42:  4.3234e+08  4.3234e+08  1e+02  6e-15  1e-11
Optimal solution found.
Computing c = 10000000
     pcost       dcost       gap    pres   dres
```

```
 0: -2.4068e+17  3.1067e+17  1e+18  3e+07  1e+04
 1:  7.8557e+16 -1.1937e+17  4e+17  7e+06  3e+03
 2:  6.3002e+16 -6.1207e+16  2e+17  3e+06  1e+03
 3:  3.5901e+16 -2.7267e+16  9e+16  1e+06  5e+02
 4:  2.9487e+16 -1.8791e+16  7e+16  8e+05  3e+02
 5:  3.0748e+16 -1.5103e+16  6e+16  6e+05  2e+02
 6:  2.6203e+16 -7.7033e+15  4e+16  3e+05  1e+02
 7:  1.9839e+16 -3.7193e+15  3e+16  1e+05  5e+01
 8:  1.3987e+16 -1.8609e+15  2e+16  7e+04  3e+01
 9:  6.7219e+15 -3.9168e+14  7e+15  1e+04  6e+00
10:  2.5871e+15 -8.0745e+13  3e+15  4e+03  2e+00
11:  7.8019e+14 -1.2497e+13  8e+14  7e+02  3e-01
12:  9.9414e+13 -1.3131e+12  1e+14  1e-09  1e-12
13:  2.4770e+13 -2.6893e+11  3e+13  3e-10  9e-12
14:  2.3795e+13 -1.4332e+11  2e+13  2e-10  8e-12
15:  1.9527e+12 -5.9445e+08  2e+12  7e-11  9e-12
16:  4.1605e+10  2.2102e+09  4e+10  9e-12  6e-12
17:  1.5499e+10  2.4714e+09  1e+10  2e-12  1e-12
18:  1.4119e+10  2.6788e+09  1e+10  2e-12  2e-12
19:  1.0792e+10  2.9444e+09  8e+09  1e-12  3e-13
20:  9.8967e+09  3.0813e+09  7e+09  9e-13  2e-12
21:  8.4825e+09  3.2995e+09  5e+09  5e-13  1e-12
22:  7.9001e+09  3.3884e+09  5e+09  4e-13  6e-13
23:  7.1549e+09  3.5005e+09  4e+09  3e-13  3e-13
24:  6.4152e+09  3.6274e+09  3e+09  1e-13  4e-13
25:  5.8999e+09  3.7400e+09  2e+09  9e-14  1e-12
26:  5.5360e+09  3.8489e+09  2e+09  6e-14  7e-13
27:  5.3425e+09  3.8992e+09  1e+09  4e-14  5e-13
28:  5.1796e+09  3.9325e+09  1e+09  3e-14  1e-12
29:  5.1223e+09  3.9412e+09  1e+09  2e-14  2e-13
30:  4.8578e+09  4.0532e+09  8e+08  1e-14  3e-13
31:  4.7130e+09  4.1039e+09  6e+08  9e-15  1e-12
32:  4.5717e+09  4.1785e+09  4e+08  7e-15  6e-13
33:  4.4992e+09  4.2163e+09  3e+08  6e-15  7e-13
34:  4.4100e+09  4.2543e+09  2e+08  5e-15  4e-12
35:  4.3875e+09  4.2713e+09  1e+08  5e-15  2e-12
36:  4.3558e+09  4.2947e+09  6e+07  5e-15  3e-12
37:  4.3337e+09  4.3142e+09  2e+07  5e-15  1e-12
38:  4.3252e+09  4.3218e+09  3e+06  5e-15  4e-13
39:  4.3239e+09  4.3229e+09  1e+06  5e-15  1e-11
40:  4.3235e+09  4.3233e+09  3e+05  5e-15  1e-11
41:  4.3235e+09  4.3233e+09  2e+05  4e-15  2e-12
42:  4.3234e+09  4.3233e+09  1e+05  5e-15  2e-11
43:  4.3234e+09  4.3234e+09  7e+03  9e-15  3e-12
44:  4.3234e+09  4.3234e+09  2e+02  6e-15  3e-12
Optimal solution found.
Computing c = 100000000
     pcost       dcost       gap    pres   dres
 0: -2.4068e+19  3.1067e+19  1e+20  3e+08  1e+04
 1:  7.8556e+18 -1.1937e+19  4e+19  7e+07  3e+03
 2:  6.3002e+18 -6.1207e+18  2e+19  3e+07  1e+03
 3:  3.5901e+18 -2.7267e+18  9e+18  1e+07  5e+02
 4:  2.9487e+18 -1.8791e+18  7e+18  8e+06  3e+02
 5:  3.0748e+18 -1.5103e+18  6e+18  6e+06  2e+02
 6:  2.6203e+18 -7.7033e+17  4e+18  3e+06  1e+02
 7:  1.9839e+18 -3.7193e+17  3e+18  1e+06  5e+01
 8:  1.3987e+18 -1.8609e+17  2e+18  7e+05  3e+01
```

```
 9:  6.7219e+17 -3.9168e+16  7e+17  1e+05  6e+00
10:  2.5871e+17 -8.0746e+15  3e+17  4e+04  2e+00
11:  7.8018e+16 -1.2499e+15  8e+16  7e+03  3e-01
12:  9.9410e+15 -1.3151e+14  1e+16  7e-09  3e-12
13:  2.4769e+15 -2.7092e+13  3e+15  3e-09  4e-12
14:  2.3794e+15 -1.4531e+13  2e+15  2e-09  3e-12
15:  1.9526e+14 -2.5818e+11  2e+14  7e-10  2e-11
16:  2.3047e+12  2.2004e+10  2e+12  7e-11  1e-12
17:  2.3063e+11  2.2599e+10  2e+11  6e-12  5e-12
18:  1.6338e+11  2.5951e+10  1e+11  3e-12  3e-13
19:  1.1957e+11  2.8802e+10  9e+10  2e-12  9e-13
20:  9.5079e+10  3.1221e+10  6e+10  1e-12  2e-12
21:  8.4848e+10  3.2912e+10  5e+10  7e-13  5e-13
22:  7.9305e+10  3.3815e+10  5e+10  5e-13  2e-12
23:  7.1973e+10  3.4941e+10  4e+10  4e-13  2e-12
24:  6.4330e+10  3.6231e+10  3e+10  2e-13  4e-13
25:  5.9402e+10  3.7298e+10  2e+10  1e-13  3e-12
26:  5.6046e+10  3.8281e+10  2e+10  1e-13  1e-12
27:  5.3169e+10  3.9129e+10  1e+10  6e-14  5e-13
28:  5.0854e+10  3.9779e+10  1e+10  3e-14  6e-13
29:  4.9577e+10  4.0233e+10  9e+09  2e-14  6e-13
30:  4.6828e+10  4.1290e+10  6e+09  1e-14  3e-12
31:  4.5644e+10  4.1895e+10  4e+09  9e-15  2e-12
32:  4.4963e+10  4.2178e+10  3e+09  7e-15  2e-12
33:  4.4324e+10  4.2445e+10  2e+09  5e-15  3e-12
34:  4.3806e+10  4.2805e+10  1e+09  5e-15  3e-12
35:  4.3568e+10  4.2945e+10  6e+08  5e-15  2e-12
36:  4.3333e+10  4.3147e+10  2e+08  6e-15  1e-12
37:  4.3252e+10  4.3218e+10  3e+07  6e-15  3e-12
38:  4.3240e+10  4.3229e+10  1e+07  6e-15  2e-11
39:  4.3235e+10  4.3233e+10  2e+06  6e-15  2e-11
40:  4.3234e+10  4.3234e+10  3e+05  5e-15  1e-11
41:  4.3234e+10  4.3234e+10  3e+05  5e-15  1e-11
42:  4.3234e+10  4.3234e+10  1e+05  4e-15  1e-11
43:  4.3234e+10  4.3234e+10  7e+03  7e-15  2e-12
Optimal solution found.
```

In [9]:
```
df_primal.style.apply(lambda x: ['background: lightgreen' if not np.isnan(x['Testing Data Accuracy']) else '' for i in x], axis=1)
```

Out[9]:

	c	Training Data Accuracy	Validation Data Accuracy	Testing Data Accuracy
0	1	94.466667	93.500000	nan
1	10	94.733333	93.875000	nan
2	100	94.866667	93.875000	62.172285
3	1000	94.833333	93.750000	nan
4	10000	94.833333	93.750000	nan
5	100000	94.833333	93.750000	nan
6	1000000	94.833333	93.750000	nan
7	10000000	94.833333	93.750000	nan
8	100000000	94.833333	93.750000	nan

2. Dual SVMs with Gaussian Kernels

- Using quadratic programming, apply the dual of the SVM with slack formulation to train a classifier for each choice of c $c \in 1, 10, 10^2, 10^3, 10^4, 10^5, 10^6, 10^7, 10^8$ using a Gaussian kernel with $\sigma^2 \in .1, 1, 10, 100, 1000$.
- What is the accuracy of the learned classifier on the training set for each pair of c and σ?
- Use the validation set to select the best value of c and σ. What is the accuracy on the validation set for each pair of c and σ?
- Report the accuracy on the test set for the selected classifier.

In [10]:
```
c_list = [1,10,10**2,10**3,10**4,10**5,10**6,10**7,10**8]
σ2_list = [.1,1,10,100,1000]
df_dual = run_svm(c_list, σ2_list, primal=False)
```

```
Computing c = 1 and variance = 0.1
     pcost       dcost       gap    pres   dres
 0: -1.1526e+03 -5.7019e+03  1e+04  2e+00  4e-16
 1: -1.1153e+03 -3.2841e+03  2e+03  9e-13  2e-16
 2: -1.2019e+03 -1.4313e+03  2e+02  2e-12  1e-16
 3: -1.2565e+03 -1.2861e+03  3e+01  2e-12  1e-16
 4: -1.2645e+03 -1.2702e+03  6e+00  6e-12  6e-17
 5: -1.2662e+03 -1.2669e+03  8e-01  2e-12  6e-17
 6: -1.2663e+03 -1.2665e+03  2e-01  5e-12  5e-17
 7: -1.2663e+03 -1.2664e+03  4e-02  5e-13  5e-17
 8: -1.2663e+03 -1.2663e+03  1e-02  8e-12  6e-17
 9: -1.2663e+03 -1.2663e+03  5e-03  5e-12  7e-17
10: -1.2663e+03 -1.2663e+03  8e-04  1e-12  5e-17
Optimal solution found.
Computing c = 1 and variance = 1
     pcost       dcost       gap    pres   dres
 0: -1.1106e+03 -5.6547e+03  1e+04  2e+00  6e-16
 1: -1.0753e+03 -3.2369e+03  2e+03  2e-13  3e-16
 2: -1.1581e+03 -1.3869e+03  2e+02  8e-13  2e-16
 3: -1.2083e+03 -1.2462e+03  4e+01  6e-14  1e-16
 4: -1.2182e+03 -1.2223e+03  4e+00  3e-13  1e-16
 5: -1.2193e+03 -1.2199e+03  6e-01  1e-12  1e-16
 6: -1.2194e+03 -1.2195e+03  8e-02  9e-13  1e-16
 7: -1.2194e+03 -1.2194e+03  2e-02  6e-13  9e-17
 8: -1.2194e+03 -1.2194e+03  4e-03  9e-13  1e-16
 9: -1.2194e+03 -1.2194e+03  1e-03  9e-13  1e-16
10: -1.2194e+03 -1.2194e+03  5e-04  9e-13  1e-16
Optimal solution found.
Computing c = 1 and variance = 10
     pcost       dcost       gap    pres   dres
 0: -9.0145e+02 -7.0933e+03  3e+04  2e+00  2e-15
 1: -8.0830e+02 -4.5438e+03  4e+03  4e-02  8e-16
 2: -8.8154e+02 -1.4577e+03  6e+02  6e-03  8e-16
 3: -9.4853e+02 -1.0618e+03  1e+02  4e-04  7e-16
 4: -9.6318e+02 -9.9244e+02  3e+01  5e-05  7e-16
 5: -9.6711e+02 -9.7323e+02  6e+00  5e-06  6e-16
 6: -9.6798e+02 -9.6913e+02  1e+00  7e-07  8e-16
 7: -9.6805e+02 -9.6884e+02  8e-01  3e-07  6e-16
 8: -9.6816e+02 -9.6830e+02  1e-01  5e-08  7e-16
 9: -9.6818e+02 -9.6821e+02  3e-02  2e-09  7e-16
10: -9.6819e+02 -9.6819e+02  4e-03  1e-10  7e-16
11: -9.6819e+02 -9.6819e+02  1e-03  2e-11  6e-16
12: -9.6819e+02 -9.6819e+02  2e-04  1e-12  7e-16
Optimal solution found.
Computing c = 1 and variance = 100
     pcost       dcost       gap    pres   dres
 0: -1.0701e+03 -7.2607e+03  3e+04  3e+00  1e-14
 1: -8.3029e+02 -4.6891e+03  4e+03  9e-02  1e-14
 2: -9.1798e+02 -1.6711e+03  8e+02  2e-02  1e-14
 3: -1.0154e+03 -1.2653e+03  3e+02  3e-03  1e-14
 4: -1.0520e+03 -1.1372e+03  9e+01  6e-04  1e-14
 5: -1.0642e+03 -1.0998e+03  4e+01  2e-04  1e-14
 6: -1.0710e+03 -1.0805e+03  1e+01  3e-05  1e-14
 7: -1.0730e+03 -1.0755e+03  3e+00  6e-06  1e-14
 8: -1.0733e+03 -1.0746e+03  1e+00  3e-13  1e-14
 9: -1.0736e+03 -1.0740e+03  4e-01  8e-13  1e-14
10: -1.0737e+03 -1.0738e+03  5e-02  6e-13  1e-14
```

```
11: -1.0737e+03 -1.0738e+03  2e-02  3e-13  1e-14
12: -1.0737e+03 -1.0737e+03  5e-03  4e-13  1e-14
13: -1.0737e+03 -1.0737e+03  1e-03  1e-12  1e-14
Optimal solution found.
Computing c = 1 and variance = 1000
     pcost       dcost       gap    pres   dres
 0: -1.4477e+03 -8.0782e+03  4e+04  3e+00  8e-14
 1: -1.0618e+03 -5.4574e+03  5e+03  1e-01  7e-14
 2: -1.1502e+03 -2.0003e+03  9e+02  2e-02  6e-14
 3: -1.2886e+03 -1.5205e+03  2e+02  4e-03  7e-14
 4: -1.3290e+03 -1.4280e+03  1e+02  1e-03  7e-14
 5: -1.3490e+03 -1.3837e+03  3e+01  3e-04  7e-14
 6: -1.3550e+03 -1.3721e+03  2e+01  8e-05  7e-14
 7: -1.3591e+03 -1.3644e+03  5e+00  2e-05  8e-14
 8: -1.3606e+03 -1.3618e+03  1e+00  2e-06  8e-14
 9: -1.3610e+03 -1.3613e+03  3e-01  2e-07  8e-14
10: -1.3611e+03 -1.3611e+03  5e-02  9e-13  8e-14
11: -1.3611e+03 -1.3611e+03  1e-02  8e-13  8e-14
12: -1.3611e+03 -1.3611e+03  2e-03  9e-14  8e-14
13: -1.3611e+03 -1.3611e+03  3e-04  2e-13  8e-14
Optimal solution found.
Computing c = 10 and variance = 0.1
     pcost       dcost       gap    pres   dres
 0:  9.6393e+03 -1.2892e+05  1e+05  2e-12  1e-15
 1:  2.0688e+03 -1.2927e+04  1e+04  3e-11  1e-15
 2: -9.7347e+02 -3.7561e+03  3e+03  2e-11  3e-16
 3: -1.2972e+03 -1.4801e+03  2e+02  4e-12  1e-16
 4: -1.3033e+03 -1.3167e+03  1e+01  3e-12  7e-17
 5: -1.3033e+03 -1.3066e+03  3e+00  3e-12  6e-17
 6: -1.3033e+03 -1.3038e+03  5e-01  4e-12  6e-17
 7: -1.3033e+03 -1.3035e+03  1e-01  2e-12  8e-17
 8: -1.3033e+03 -1.3034e+03  3e-02  4e-12  7e-17
 9: -1.3033e+03 -1.3034e+03  1e-02  2e-12  7e-17
10: -1.3033e+03 -1.3033e+03  4e-03  6e-12  6e-17
11: -1.3033e+03 -1.3033e+03  4e-04  6e-12  7e-17
Optimal solution found.
Computing c = 10 and variance = 1
     pcost       dcost       gap    pres   dres
 0:  9.9100e+03 -1.2972e+05  1e+05  6e-13  3e-15
 1:  2.1326e+03 -1.2845e+04  1e+04  5e-12  2e-15
 2: -9.2705e+02 -3.7056e+03  3e+03  2e-12  6e-16
 3: -1.2525e+03 -1.5911e+03  3e+02  4e-13  3e-16
 4: -1.2663e+03 -1.2973e+03  3e+01  2e-12  2e-16
 5: -1.2666e+03 -1.2697e+03  3e+00  1e-12  2e-16
 6: -1.2666e+03 -1.2672e+03  6e-01  2e-12  2e-16
 7: -1.2666e+03 -1.2667e+03  1e-01  1e-12  2e-16
 8: -1.2666e+03 -1.2667e+03  3e-02  5e-13  2e-16
 9: -1.2666e+03 -1.2666e+03  7e-03  8e-13  2e-16
10: -1.2666e+03 -1.2666e+03  2e-03  2e-12  1e-16
11: -1.2666e+03 -1.2666e+03  9e-04  2e-13  1e-16
Optimal solution found.
Computing c = 10 and variance = 10
     pcost       dcost       gap    pres   dres
 0:  9.3518e+03 -1.5637e+05  2e+05  3e-01  8e-15
 1:  4.3982e+03 -1.8333e+04  2e+04  1e-02  4e-15
 2:  1.8252e+02 -7.8933e+03  8e+03  3e-03  3e-15
 3: -8.8531e+02 -4.1207e+03  3e+03  8e-04  3e-15
```

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 4: -1.2262e+03 -2.3363e+03  1e+03  1e-04  2e-15
 5: -1.3014e+03 -1.5703e+03  3e+02  1e-05  2e-15
 6: -1.3153e+03 -1.3683e+03  5e+01  1e-06  2e-15
 7: -1.3183e+03 -1.3295e+03  1e+01  2e-07  2e-15
 8: -1.3190e+03 -1.3212e+03  2e+00  1e-08  2e-15
 9: -1.3191e+03 -1.3196e+03  5e-01  3e-09  2e-15
10: -1.3191e+03 -1.3193e+03  2e-01  6e-10  2e-15
11: -1.3191e+03 -1.3192e+03  4e-02  5e-11  2e-15
12: -1.3191e+03 -1.3191e+03  1e-02  8e-12  2e-15
13: -1.3191e+03 -1.3191e+03  2e-03  1e-12  2e-15
14: -1.3191e+03 -1.3191e+03  6e-04  4e-13  2e-15
Optimal solution found.
Computing c = 10 and variance = 100
     pcost       dcost       gap    pres   dres
 0:  3.2978e+03 -1.8823e+05  4e+05  5e-01  6e-14
 1:  2.2308e+03 -3.4364e+04  4e+04  1e-11  8e-14
 2: -1.8338e+03 -1.3452e+04  1e+04  3e-12  6e-14
 3: -3.1452e+03 -8.4596e+03  5e+03  4e-12  6e-14
 4: -3.4408e+03 -7.4868e+03  4e+03  2e-12  6e-14
 5: -3.9221e+03 -5.5403e+03  2e+03  1e-12  6e-14
 6: -4.1303e+03 -4.6756e+03  5e+02  5e-13  7e-14
 7: -4.2164e+03 -4.3609e+03  1e+02  3e-12  7e-14
 8: -4.2427e+03 -4.2748e+03  3e+01  1e-11  7e-14
 9: -4.2488e+03 -4.2570e+03  8e+00  6e-12  7e-14
10: -4.2506e+03 -4.2521e+03  2e+00  2e-13  7e-14
11: -4.2509e+03 -4.2514e+03  5e-01  4e-12  7e-14
12: -4.2510e+03 -4.2511e+03  8e-02  5e-12  7e-14
13: -4.2510e+03 -4.2510e+03  2e-02  2e-12  7e-14
14: -4.2510e+03 -4.2510e+03  2e-03  5e-12  7e-14
Optimal solution found.
Computing c = 10 and variance = 1000
     pcost       dcost       gap    pres   dres
 0: -1.4931e+03 -1.9224e+05  3e+05  4e-01  3e-13
 1: -2.2012e+03 -3.5480e+04  3e+04  1e-03  5e-13
 2: -5.3842e+03 -1.5855e+04  1e+04  3e-04  4e-13
 3: -6.1806e+03 -1.3912e+04  8e+03  2e-04  4e-13
 4: -6.9845e+03 -1.1587e+04  5e+03  9e-05  4e-13
 5: -7.4784e+03 -1.0318e+04  3e+03  4e-05  4e-13
 6: -7.7486e+03 -9.6050e+03  2e+03  3e-05  4e-13
 7: -7.9773e+03 -9.0184e+03  1e+03  1e-05  4e-13
 8: -8.1078e+03 -8.7038e+03  6e+02  7e-06  4e-13
 9: -8.2246e+03 -8.4388e+03  2e+02  2e-06  4e-13
10: -8.2685e+03 -8.3438e+03  8e+01  2e-07  5e-13
11: -8.2852e+03 -8.3145e+03  3e+01  4e-08  5e-13
12: -8.2943e+03 -8.2995e+03  5e+00  5e-09  5e-13
13: -8.2959e+03 -8.2970e+03  1e+00  3e-10  5e-13
14: -8.2963e+03 -8.2965e+03  2e-01  7e-12  5e-13
15: -8.2963e+03 -8.2964e+03  3e-02  4e-12  5e-13
16: -8.2963e+03 -8.2963e+03  8e-03  2e-12  5e-13
Optimal solution found.
Computing c = 100 and variance = 0.1
     pcost       dcost       gap    pres   dres
 0:  1.5556e+06 -9.2362e+06  1e+07  4e-11  2e-14
 1:  4.7323e+05 -8.0372e+05  1e+06  1e-10  2e-14
 2:  8.4529e+04 -1.4419e+05  2e+05  1e-10  4e-15
 3:  1.0715e+04 -3.2650e+04  4e+04  8e-12  2e-15
 4: -3.9469e+02 -5.1603e+03  5e+03  3e-11  8e-16
```

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 5: -1.4447e+03 -1.9699e+03  5e+02  4e-12  3e-16
 6: -1.4832e+03 -1.5018e+03  2e+01  3e-12  1e-16
 7: -1.4833e+03 -1.4863e+03  3e+00  3e-12  2e-16
 8: -1.4833e+03 -1.4839e+03  6e-01  2e-12  3e-16
 9: -1.4833e+03 -1.4835e+03  2e-01  2e-12  2e-16
10: -1.4833e+03 -1.4834e+03  4e-02  3e-12  1e-16
11: -1.4833e+03 -1.4834e+03  2e-02  4e-12  3e-16
12: -1.4833e+03 -1.4833e+03  6e-03  2e-12  2e-16
13: -1.4833e+03 -1.4833e+03  9e-04  1e-12  2e-16
Optimal solution found.
Computing c = 100 and variance = 1
     pcost       dcost       gap    pres   dres
 0:  1.5618e+06 -9.3091e+06  1e+07  3e-11  3e-14
 1:  4.7784e+05 -8.6965e+05  1e+06  5e-11  2e-14
 2:  8.4913e+04 -1.4818e+05  2e+05  2e-11  7e-15
 3:  1.1168e+04 -3.3650e+04  4e+04  1e-11  3e-15
 4: -3.0416e+02 -5.1636e+03  5e+03  2e-12  1e-15
 5: -1.4073e+03 -1.9433e+03  5e+02  2e-12  6e-16
 6: -1.4490e+03 -1.4877e+03  4e+01  3e-12  6e-16
 7: -1.4496e+03 -1.4528e+03  3e+00  3e-12  2e-16
 8: -1.4497e+03 -1.4502e+03  6e-01  1e-12  6e-16
 9: -1.4497e+03 -1.4498e+03  1e-01  2e-13  2e-16
10: -1.4497e+03 -1.4497e+03  3e-02  2e-12  5e-16
11: -1.4497e+03 -1.4497e+03  9e-03  2e-12  7e-16
12: -1.4497e+03 -1.4497e+03  2e-03  4e-13  2e-16
13: -1.4497e+03 -1.4497e+03  1e-03  2e-12  3e-16
Optimal solution found.
Computing c = 100 and variance = 10
     pcost       dcost       gap    pres   dres
 0:  1.3998e+06 -1.1772e+07  2e+07  1e-01  8e-14
 1:  6.3696e+05 -1.4101e+06  2e+06  1e-02  4e-14
 2:  1.5805e+05 -3.5054e+05  5e+05  2e-03  3e-14
 3:  4.0439e+04 -1.0954e+05  2e+05  4e-04  2e-14
 4:  9.5151e+03 -3.6152e+04  5e+04  1e-04  1e-14
 5:  1.8408e+02 -1.0024e+04  1e+04  1e-05  8e-15
 6: -1.3142e+03 -5.1831e+03  4e+03  3e-06  6e-15
 7: -1.8082e+03 -2.4774e+03  7e+02  5e-08  6e-15
 8: -1.8602e+03 -1.9762e+03  1e+02  5e-09  5e-15
 9: -1.8677e+03 -1.8959e+03  3e+01  6e-10  5e-15
10: -1.8695e+03 -1.8761e+03  7e+00  8e-11  5e-15
11: -1.8700e+03 -1.8710e+03  1e+00  6e-12  6e-15
12: -1.8700e+03 -1.8702e+03  2e-01  1e-12  5e-15
13: -1.8701e+03 -1.8701e+03  4e-02  3e-13  5e-15
14: -1.8701e+03 -1.8701e+03  5e-03  5e-13  5e-15
15: -1.8701e+03 -1.8701e+03  1e-03  3e-12  5e-15
Optimal solution found.
Computing c = 100 and variance = 100
     pcost       dcost       gap    pres   dres
 0:  8.7346e+05 -1.4744e+07  2e+07  3e-01  6e-13
 1:  5.9773e+05 -2.0172e+06  3e+06  1e-02  7e-13
 2:  1.7773e+05 -6.1817e+05  8e+05  3e-03  5e-13
 3:  4.3893e+04 -2.4483e+05  3e+05  3e-04  4e-13
 4:  7.7607e+03 -8.9548e+04  1e+05  7e-05  3e-13
 5: -4.7196e+03 -4.0225e+04  4e+04  4e-06  3e-13
 6: -8.5528e+03 -2.2915e+04  1e+04  8e-07  3e-13
 7: -1.0305e+04 -1.4849e+04  5e+03  2e-07  3e-13
 8: -1.0955e+04 -1.2121e+04  1e+03  2e-08  3e-13
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 9: -1.1131e+04 -1.1448e+04  3e+02  2e-12  3e-13
10: -1.1186e+04 -1.1263e+04  8e+01  5e-12  3e-13
11: -1.1199e+04 -1.1219e+04  2e+01  9e-12  3e-13
12: -1.1203e+04 -1.1208e+04  6e+00  4e-12  3e-13
13: -1.1204e+04 -1.1205e+04  2e+00  3e-14  3e-13
14: -1.1204e+04 -1.1204e+04  5e-01  3e-12  3e-13
15: -1.1204e+04 -1.1204e+04  8e-02  1e-12  3e-13
16: -1.1204e+04 -1.1204e+04  3e-02  7e-12  3e-13
17: -1.1204e+04 -1.1204e+04  4e-03  9e-12  3e-13
Optimal solution found.
Computing c = 100 and variance = 1000
     pcost       dcost       gap    pres   dres
 0:  5.8027e+05 -1.5137e+07  2e+07  3e-01  3e-12
 1:  3.9660e+05 -2.5720e+06  3e+06  2e-02  4e-12
 2:  1.2264e+05 -8.4084e+05  1e+06  6e-03  3e-12
 3:  3.4918e+04 -4.3076e+05  5e+05  2e-03  2e-12
 4: -7.7597e+03 -1.8129e+05  2e+05  5e-04  2e-12
 5: -2.5719e+04 -9.1261e+04  7e+04  1e-04  2e-12
 6: -3.0045e+04 -7.8143e+04  5e+04  8e-05  2e-12
 7: -3.4882e+04 -6.1460e+04  3e+04  4e-05  2e-12
 8: -3.6571e+04 -5.6154e+04  2e+04  3e-05  2e-12
 9: -3.9222e+04 -4.7614e+04  8e+03  7e-06  2e-12
10: -4.0576e+04 -4.3958e+04  3e+03  2e-06  2e-12
11: -4.1217e+04 -4.2359e+04  1e+03  4e-07  3e-12
12: -4.1482e+04 -4.1776e+04  3e+02  7e-08  3e-12
13: -4.1556e+04 -4.1622e+04  7e+01  3e-11  3e-12
14: -4.1578e+04 -4.1587e+04  9e+00  3e-11  3e-12
15: -4.1581e+04 -4.1583e+04  2e+00  4e-11  3e-12
16: -4.1581e+04 -4.1582e+04  6e-01  3e-12  3e-12
17: -4.1581e+04 -4.1581e+04  1e-01  1e-11  3e-12
18: -4.1581e+04 -4.1581e+04  1e-02  4e-11  3e-12
Optimal solution found.
Computing c = 1000 and variance = 0.1
     pcost       dcost       gap    pres   dres
 0:  1.6082e+08 -8.8720e+08  1e+09  1e-09  2e-13
 1:  4.9046e+07 -7.6638e+07  1e+08  9e-10  1e-13
 2:  7.4045e+06 -9.0297e+06  2e+07  1e-09  4e-14
 3:  1.1345e+06 -1.8891e+06  3e+06  2e-10  2e-14
 4:  1.5366e+05 -2.0696e+05  4e+05  2e-10  7e-15
 5:  1.6416e+04 -3.6766e+04  5e+04  8e-11  2e-15
 6: -1.3409e+03 -8.9702e+03  8e+03  3e-11  1e-15
 7: -3.1867e+03 -4.1041e+03  9e+02  9e-12  3e-16
 8: -3.2827e+03 -3.3295e+03  5e+01  6e-12  4e-16
 9: -3.2833e+03 -3.2881e+03  5e+00  9e-12  6e-16
10: -3.2833e+03 -3.2842e+03  9e-01  4e-12  2e-15
11: -3.2833e+03 -3.2836e+03  2e-01  3e-12  8e-16
12: -3.2833e+03 -3.2834e+03  5e-02  2e-12  2e-15
13: -3.2833e+03 -3.2834e+03  2e-02  3e-12  2e-15
14: -3.2833e+03 -3.2833e+03  8e-03  4e-12  4e-16
15: -3.2833e+03 -3.2833e+03  1e-03  4e-12  3e-16
Optimal solution found.
Computing c = 1000 and variance = 1
     pcost       dcost       gap    pres   dres
 0:  1.6124e+08 -8.9443e+08  1e+09  2e-10  2e-13
 1:  4.9476e+07 -8.2918e+07  1e+08  3e-10  2e-13
 2:  9.3261e+06 -1.3749e+07  2e+07  3e-10  7e-14
 3:  1.5772e+06 -2.2074e+06  4e+06  9e-11  3e-14
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 4:  2.4104e+05 -4.5676e+05  7e+05  3e-11  1e-14
 5:  2.8091e+04 -4.9767e+04  8e+04  2e-11  7e-15
 6:  1.4239e+02 -1.1821e+04  1e+04  1e-12  4e-15
 7: -3.0265e+03 -4.5660e+03  2e+03  2e-12  2e-15
 8: -3.2462e+03 -3.3662e+03  1e+02  7e-12  4e-15
 9: -3.2495e+03 -3.2640e+03  1e+01  2e-13  4e-15
10: -3.2497e+03 -3.2514e+03  2e+00  1e-12  3e-15
11: -3.2497e+03 -3.2501e+03  4e-01  1e-12  1e-15
12: -3.2497e+03 -3.2498e+03  7e-02  5e-12  3e-16
13: -3.2497e+03 -3.2497e+03  2e-02  1e-12  6e-16
14: -3.2497e+03 -3.2497e+03  4e-03  9e-13  1e-15
15: -3.2497e+03 -3.2497e+03  2e-03  2e-12  2e-15
Optimal solution found.
Computing c = 1000 and variance = 10
     pcost       dcost       gap    pres   dres
 0:  1.4409e+08 -1.1387e+09  2e+09  1e-01  8e-13
 1:  6.4949e+07 -1.4160e+08  2e+08  1e-02  1e-12
 2:  1.7015e+07 -3.5413e+07  5e+07  2e-03  3e-13
 3:  4.6111e+06 -1.0355e+07  2e+07  3e-04  2e-13
 4:  9.8933e+05 -2.8757e+06  4e+06  4e-11  9e-14
 5:  1.8598e+05 -4.2956e+05  6e+05  2e-11  5e-14
 6:  2.1441e+04 -5.2687e+04  7e+04  1e-11  5e-14
 7: -1.2562e+03 -1.3172e+04  1e+04  7e-12  5e-14
 8: -3.6883e+03 -8.2615e+03  5e+03  3e-12  5e-14
 9: -4.5543e+03 -6.1864e+03  2e+03  7e-12  5e-14
10: -4.6799e+03 -5.0472e+03  4e+02  1e-12  3e-14
11: -4.7021e+03 -4.7820e+03  8e+01  2e-12  3e-14
12: -4.7064e+03 -4.7270e+03  2e+01  8e-15  4e-14
13: -4.7077e+03 -4.7117e+03  4e+00  4e-12  3e-14
14: -4.7079e+03 -4.7087e+03  8e-01  1e-12  3e-14
15: -4.7080e+03 -4.7081e+03  2e-01  2e-12  4e-14
16: -4.7080e+03 -4.7080e+03  4e-02  3e-12  5e-14
17: -4.7080e+03 -4.7080e+03  1e-02  4e-12  4e-14
18: -4.7080e+03 -4.7080e+03  2e-03  8e-14  3e-14
Optimal solution found.
Computing c = 1000 and variance = 100
     pcost       dcost       gap    pres   dres
 0:  9.2224e+07 -1.4337e+09  2e+09  3e-01  6e-12
 1:  6.2627e+07 -1.9412e+08  3e+08  2e-02  1e-11
 2:  2.0854e+07 -6.0734e+07  8e+07  3e-03  5e-12
 3:  6.0716e+06 -2.1897e+07  3e+07  3e-04  3e-12
 4:  2.0139e+06 -7.4953e+06  1e+07  5e-05  2e-12
 5:  6.4908e+05 -2.8409e+06  3e+06  3e-10  2e-12
 6:  1.5753e+05 -7.1496e+05  9e+05  9e-11  1e-12
 7:  7.4489e+04 -4.3631e+05  5e+05  4e-11  9e-13
 8:  2.7160e+03 -1.4414e+05  1e+05  3e-11  9e-13
 9: -1.8194e+04 -6.0543e+04  4e+04  2e-11  9e-13
10: -2.4366e+04 -3.7220e+04  1e+04  3e-11  1e-12
11: -2.5991e+04 -3.1016e+04  5e+03  9e-12  1e-12
12: -2.6731e+04 -2.8019e+04  1e+03  1e-11  1e-12
13: -2.6928e+04 -2.7278e+04  3e+02  6e-12  1e-12
14: -2.6989e+04 -2.7070e+04  8e+01  3e-12  9e-13
15: -2.7003e+04 -2.7026e+04  2e+01  9e-12  1e-12
16: -2.7007e+04 -2.7011e+04  4e+00  4e-11  1e-12
17: -2.7008e+04 -2.7009e+04  1e+00  9e-12  1e-12
18: -2.7008e+04 -2.7009e+04  2e-01  4e-12  1e-12
19: -2.7008e+04 -2.7008e+04  4e-02  5e-11  1e-12
```

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20: -2.7008e+04 -2.7008e+04  5e-03  5e-11  1e-12
Optimal solution found.
Computing c = 1000 and variance = 1000
     pcost       dcost       gap    pres   dres
 0:  6.4620e+07 -1.4729e+09  2e+09  3e-01  3e-11
 1:  4.4773e+07 -2.4967e+08  3e+08  3e-02  4e-11
 2:  1.5961e+07 -7.2299e+07  9e+07  5e-03  3e-11
 3:  5.4477e+06 -3.0971e+07  4e+07  7e-04  2e-11
 4:  1.9883e+06 -1.1399e+07  1e+07  2e-04  2e-11
 5:  5.3748e+05 -3.5611e+06  4e+06  3e-05  1e-11
 6:  4.1338e+04 -1.0766e+06  1e+06  2e-10  1e-11
 7: -5.3709e+04 -6.8978e+05  6e+05  1e-10  1e-11
 8: -1.1912e+05 -3.8809e+05  3e+05  1e-10  1e-11
 9: -1.3301e+05 -3.4687e+05  2e+05  6e-11  1e-11
10: -1.5706e+05 -2.5790e+05  1e+05  2e-11  1e-11
11: -1.5775e+05 -2.5480e+05  1e+05  4e-11  1e-11
12: -1.6980e+05 -2.1554e+05  5e+04  7e-11  1e-11
13: -1.7125e+05 -2.0830e+05  4e+04  2e-10  1e-11
14: -1.7848e+05 -1.8901e+05  1e+04  2e-10  1e-11
15: -1.8059e+05 -1.8401e+05  3e+03  3e-10  1e-11
16: -1.8132e+05 -1.8235e+05  1e+03  2e-10  2e-11
17: -1.8163e+05 -1.8179e+05  2e+02  7e-11  2e-11
18: -1.8168e+05 -1.8171e+05  4e+01  2e-10  1e-11
19: -1.8169e+05 -1.8169e+05  6e+00  2e-10  1e-11
20: -1.8169e+05 -1.8169e+05  1e+00  1e-10  1e-11
21: -1.8169e+05 -1.8169e+05  3e-01  2e-12  1e-11
22: -1.8169e+05 -1.8169e+05  8e-02  3e-11  1e-11
Optimal solution found.
Computing c = 10000 and variance = 0.1
     pcost       dcost       gap    pres   dres
 0:  1.6134e+10 -8.8356e+10  1e+11  1e-08  2e-12
 1:  4.9208e+09 -7.6265e+09  1e+10  1e-08  1e-12
 2:  7.4697e+08 -8.8970e+08  2e+09  5e-09  6e-13
 3:  1.0794e+08 -1.2287e+08  2e+08  5e-09  2e-13
 4:  1.6652e+07 -2.5489e+07  4e+07  7e-10  6e-14
 5:  2.3552e+06 -2.7777e+06  5e+06  3e-10  2e-14
 6:  3.0729e+05 -4.2838e+05  7e+05  8e-11  2e-14
 7:  2.1804e+04 -8.6931e+04  1e+05  4e-11  2e-14
 8: -1.6513e+04 -3.2064e+04  2e+04  5e-11  4e-15
 9: -2.0963e+04 -2.2955e+04  2e+03  8e-12  1e-14
10: -2.1278e+04 -2.1452e+04  2e+02  2e-11  2e-14
11: -2.1283e+04 -2.1297e+04  1e+01  2e-11  1e-14
12: -2.1283e+04 -2.1286e+04  3e+00  6e-12  3e-14
13: -2.1283e+04 -2.1284e+04  5e-01  9e-12  3e-14
14: -2.1283e+04 -2.1283e+04  2e-01  2e-11  2e-15
15: -2.1283e+04 -2.1283e+04  3e-02  7e-12  2e-14
16: -2.1283e+04 -2.1283e+04  1e-02  4e-11  1e-14
Optimal solution found.
Computing c = 10000 and variance = 1
     pcost       dcost       gap    pres   dres
 0:  1.6175e+10 -8.9078e+10  1e+11  2e-10  3e-12
 1:  4.9636e+09 -8.2513e+09  1e+10  1e-09  2e-12
 2:  9.3682e+08 -1.3592e+09  2e+09  3e-09  8e-13
 3:  1.3998e+08 -1.7666e+08  3e+08  2e-09  3e-13
 4:  2.0614e+07 -2.6290e+07  5e+07  5e-10  1e-13
 5:  3.1095e+06 -4.6812e+06  8e+06  3e-10  6e-14
 6:  4.1234e+05 -5.3906e+05  1e+06  4e-11  3e-14
```

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 7:  3.6543e+04 -1.0653e+05  1e+05  7e-12  5e-14
 8: -1.4602e+04 -3.4896e+04  2e+04  1e-11  4e-14
 9: -2.0744e+04 -2.3435e+04  3e+03  1e-11  5e-14
10: -2.1237e+04 -2.1509e+04  3e+02  5e-12  5e-14
11: -2.1249e+04 -2.1276e+04  3e+01  3e-11  5e-14
12: -2.1250e+04 -2.1252e+04  3e+00  3e-12  5e-14
13: -2.1250e+04 -2.1250e+04  5e-01  2e-11  5e-14
14: -2.1250e+04 -2.1250e+04  1e-01  7e-12  7e-14
15: -2.1250e+04 -2.1250e+04  3e-02  7e-12  6e-14
16: -2.1250e+04 -2.1250e+04  7e-03  8e-12  9e-14
Optimal solution found.
Computing c = 10000 and variance = 10
     pcost       dcost       gap    pres   dres
 0:  1.4450e+10 -1.1348e+11  2e+11  1e-01  8e-12
 1:  6.5062e+09 -1.4161e+10  2e+10  1e-02  9e-11
 2:  1.7108e+09 -3.5393e+09  5e+09  2e-03  1e-11
 3:  4.8039e+08 -1.0720e+09  2e+09  4e-04  2e-12
 4:  1.0570e+08 -2.9620e+08  4e+08  6e-10  9e-13
 5:  1.9029e+07 -3.4349e+07  5e+07  8e-11  5e-13
 6:  2.9162e+06 -4.5700e+06  7e+06  6e-11  3e-13
 7:  3.8793e+05 -5.2642e+05  9e+05  1e-11  2e-13
 8:  3.3228e+04 -1.0780e+05  1e+05  2e-11  2e-13
 9: -1.3671e+04 -3.9901e+04  3e+04  1e-12  1e-13
10: -2.1250e+04 -2.7038e+04  6e+03  2e-12  2e-13
11: -2.2552e+04 -2.4457e+04  2e+03  1e-11  2e-13
12: -2.2721e+04 -2.3101e+04  4e+02  3e-13  1e-13
13: -2.2747e+04 -2.2833e+04  9e+01  3e-12  1e-13
14: -2.2752e+04 -2.2774e+04  2e+01  5e-12  2e-13
15: -2.2753e+04 -2.2758e+04  5e+00  7e-12  1e-13
16: -2.2753e+04 -2.2754e+04  9e-01  4e-12  1e-13
17: -2.2753e+04 -2.2753e+04  2e-01  4e-12  2e-13
18: -2.2753e+04 -2.2753e+04  5e-02  1e-12  9e-14
19: -2.2753e+04 -2.2753e+04  1e-02  2e-11  1e-13
Optimal solution found.
Computing c = 10000 and variance = 100
     pcost       dcost       gap    pres   dres
 0:  9.2706e+09 -1.4297e+11  2e+11  3e-01  5e-11
 1:  6.2906e+09 -1.9353e+10  3e+10  2e-02  3e-10
 2:  2.1149e+09 -6.0643e+09  8e+09  3e-03  8e-11
 3:  6.1959e+08 -2.1576e+09  3e+09  3e-04  4e-11
 4:  2.1344e+08 -7.6187e+08  1e+09  5e-05  2e-11
 5:  7.1323e+07 -2.7858e+08  3e+08  4e-10  2e-11
 6:  2.3996e+07 -9.1830e+07  1e+08  9e-10  1e-11
 7:  5.7294e+06 -1.7968e+07  2e+07  6e-11  6e-12
 8:  2.0638e+06 -8.1832e+06  1e+07  1e-10  5e-12
 9:  8.5265e+05 -3.8625e+06  5e+06  2e-11  4e-12
10:  1.3014e+05 -9.3841e+05  1e+06  4e-11  4e-12
11: -5.2044e+04 -2.9765e+05  2e+05  6e-12  4e-12
12: -9.1305e+04 -1.7175e+05  8e+04  1e-11  4e-12
13: -1.0402e+05 -1.3168e+05  3e+04  1e-11  5e-12
14: -1.0745e+05 -1.1827e+05  1e+04  9e-12  4e-12
15: -1.0895e+05 -1.1220e+05  3e+03  3e-11  4e-12
16: -1.0943e+05 -1.1033e+05  9e+02  4e-11  4e-12
17: -1.0959e+05 -1.0975e+05  2e+02  1e-10  4e-12
18: -1.0962e+05 -1.0965e+05  3e+01  9e-11  4e-12
19: -1.0963e+05 -1.0963e+05  9e+00  3e-11  5e-12
20: -1.0963e+05 -1.0963e+05  2e+00  9e-11  4e-12
```

```
21: -1.0963e+05 -1.0963e+05  6e-01  3e-11  4e-12
22: -1.0963e+05 -1.0963e+05  1e-01  5e-11  4e-12
23: -1.0963e+05 -1.0963e+05  2e-02  9e-12  4e-12
Optimal solution found.
Computing c = 10000 and variance = 1000
     pcost       dcost       gap    pres   dres
 0:  6.5272e+09 -1.4688e+11  2e+11  3e-01  3e-10
 1:  4.5272e+09 -2.4887e+10  3e+10  3e-02  4e-10
 2:  1.6434e+09 -7.2217e+09  9e+09  5e-03  3e-10
 3:  5.6936e+08 -3.0188e+09  4e+09  6e-04  2e-10
 4:  2.2329e+08 -1.1161e+09  1e+09  2e-04  2e-10
 5:  7.4225e+07 -3.4957e+08  4e+08  3e-05  1e-10
 6:  2.4226e+07 -1.1703e+08  1e+08  3e-06  1e-10
 7:  7.2833e+06 -3.8569e+07  5e+07  2e-10  9e-11
 8:  2.0402e+06 -1.2614e+07  1e+07  5e-10  7e-11
 9:  6.1121e+05 -6.6359e+06  7e+06  4e-10  6e-11
10: -8.7675e+04 -3.4411e+06  3e+06  4e-11  6e-11
11: -4.3327e+05 -1.9430e+06  2e+06  2e-10  7e-11
12: -6.0988e+05 -1.2123e+06  6e+05  3e-10  8e-11
13: -6.7554e+05 -9.8980e+05  3e+05  2e-10  8e-11
14: -7.2606e+05 -8.2800e+05  1e+05  2e-10  8e-11
15: -7.4499e+05 -7.7464e+05  3e+04  2e-10  8e-11
16: -7.5122e+05 -7.5857e+05  7e+03  3e-10  9e-11
17: -7.5270e+05 -7.5526e+05  3e+03  3e-12  8e-11
18: -7.5328e+05 -7.5407e+05  8e+02  6e-10  9e-11
19: -7.5350e+05 -7.5366e+05  2e+02  6e-10  9e-11
20: -7.5355e+05 -7.5358e+05  3e+01  7e-10  9e-11
21: -7.5356e+05 -7.5356e+05  5e+00  7e-10  9e-11
22: -7.5356e+05 -7.5356e+05  1e+00  9e-10  9e-11
23: -7.5356e+05 -7.5356e+05  4e-01  8e-11  9e-11
Optimal solution found.
Computing c = 100000 and variance = 0.1
     pcost       dcost       gap    pres   dres
 0:  1.6139e+12 -8.8320e+12  1e+13  8e-08  2e-11
 1:  4.9224e+11 -7.6227e+11  1e+12  8e-08  1e-11
 2:  7.4775e+10 -8.8867e+10  2e+11  1e-07  4e-12
 3:  1.0825e+10 -1.2232e+10  2e+10  7e-08  2e-12
 4:  1.5624e+09 -1.7779e+09  3e+09  3e-09  7e-13
 5:  2.3489e+08 -3.1917e+08  6e+08  2e-09  3e-13
 6:  3.3971e+07 -4.1258e+07  8e+07  1e-10  2e-13
 7:  4.6544e+06 -5.7399e+06  1e+07  3e-10  3e-13
 8:  4.7822e+05 -1.0192e+06  1e+06  8e-11  1e-13
 9: -1.0986e+05 -3.2784e+05  2e+05  3e-11  9e-14
10: -1.9029e+05 -2.2200e+05  3e+04  4e-11  2e-13
11: -2.0034e+05 -2.0465e+05  4e+03  1e-10  1e-13
12: -2.0125e+05 -2.0174e+05  5e+02  1e-11  4e-14
13: -2.0128e+05 -2.0130e+05  2e+01  2e-10  2e-13
14: -2.0128e+05 -2.0129e+05  3e+00  9e-11  6e-14
15: -2.0128e+05 -2.0128e+05  6e-01  2e-11  6e-14
16: -2.0128e+05 -2.0128e+05  2e-01  3e-11  1e-13
Optimal solution found.
Computing c = 100000 and variance = 1
     pcost       dcost       gap    pres   dres
 0:  1.6180e+12 -8.9041e+12  1e+13  2e-08  2e-11
 1:  4.9652e+11 -8.2473e+11  1e+12  6e-08  2e-11
 2:  9.3720e+10 -1.3576e+11  2e+11  2e-08  7e-12
 3:  1.4027e+10 -1.7614e+10  3e+10  2e-08  3e-12
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 4:  2.0278e+09 -2.2969e+09  4e+09  4e-09  1e-12
 5:  3.0302e+08 -4.0194e+08  7e+08  3e-09  6e-13
 6:  4.4357e+07 -5.5995e+07  1e+08  4e-10  6e-13
 7:  6.1437e+06 -7.4297e+06  1e+07  2e-10  2e-13
 8:  6.8948e+05 -1.2588e+06  2e+06  1e-12  4e-13
 9: -8.0285e+04 -3.6371e+05  3e+05  5e-11  6e-13
10: -1.8631e+05 -2.2760e+05  4e+04  1e-10  4e-13
11: -1.9985e+05 -2.0557e+05  6e+03  4e-11  6e-13
12: -2.0119e+05 -2.0187e+05  7e+02  1e-10  2e-13
13: -2.0125e+05 -2.0129e+05  4e+01  2e-10  2e-13
14: -2.0125e+05 -2.0125e+05  5e+00  2e-10  5e-13
15: -2.0125e+05 -2.0125e+05  8e-01  5e-11  5e-13
16: -2.0125e+05 -2.0125e+05  2e-01  3e-10  2e-13
Optimal solution found.
Computing c = 100000 and variance = 10
     pcost       dcost       gap    pres   dres
 0:  1.4454e+12 -1.1344e+13  2e+13  1e-01  8e-11
 1:  6.5073e+11 -1.4161e+12  2e+12  1e-02  8e-09
 2:  1.7117e+11 -3.5391e+11  5e+11  2e-03  1e-09
 3:  4.8225e+10 -1.0754e+11  2e+11  4e-04  2e-10
 4:  1.0634e+10 -2.9690e+10  4e+10  7e-09  9e-12
 5:  1.9625e+09 -3.6068e+09  6e+09  4e-09  4e-12
 6:  2.9659e+08 -3.8117e+08  7e+08  3e-10  2e-12
 7:  5.3787e+07 -9.4486e+07  1e+08  3e-10  1e-12
 8:  7.5086e+06 -8.7796e+06  2e+07  2e-10  1e-12
 9:  8.9032e+05 -1.5207e+06  2e+06  2e-10  1e-12
10: -5.2252e+04 -4.0058e+05  3e+05  2e-11  1e-12
11: -1.8338e+05 -2.3457e+05  5e+04  2e-10  2e-12
12: -1.9975e+05 -2.1014e+05  1e+04  3e-10  1e-12
13: -2.0181e+05 -2.0601e+05  4e+03  9e-11  9e-13
14: -2.0262e+05 -2.0403e+05  1e+03  4e-11  1e-12
15: -2.0273e+05 -2.0307e+05  3e+02  3e-11  1e-12
16: -2.0275e+05 -2.0282e+05  7e+01  1e-10  1e-12
17: -2.0275e+05 -2.0277e+05  2e+01  3e-11  9e-13
18: -2.0275e+05 -2.0276e+05  4e+00  1e-10  2e-12
19: -2.0275e+05 -2.0275e+05  7e-01  1e-10  1e-12
20: -2.0275e+05 -2.0275e+05  2e-01  5e-11  8e-13
Optimal solution found.
Computing c = 100000 and variance = 100
     pcost       dcost       gap    pres   dres
 0:  9.2754e+11 -1.4293e+13  2e+13  3e-01  6e-10
 1:  6.2933e+11 -1.9347e+12  3e+12  2e-02  3e-09
 2:  2.1179e+11 -6.0633e+11  8e+11  3e-03  7e-10
 3:  6.2101e+10 -2.1551e+11  3e+11  3e-04  4e-10
 4:  2.1465e+10 -7.6313e+10  1e+11  5e-05  2e-10
 5:  7.1919e+09 -2.7811e+10  4e+10  3e-09  2e-10
 6:  2.4980e+09 -9.4861e+09  1e+10  9e-10  1e-10
 7:  5.9435e+08 -1.7456e+09  2e+09  1e-09  6e-11
 8:  1.8179e+08 -6.2938e+08  8e+08  3e-10  5e-11
 9:  4.6441e+07 -1.4377e+08  2e+08  9e-12  3e-11
10:  1.0334e+07 -2.9267e+07  4e+07  6e-10  3e-11
11:  1.6748e+06 -5.5299e+06  7e+06  5e-11  3e-11
12:  2.7769e+03 -1.7064e+06  2e+06  3e-10  3e-11
13: -3.7437e+05 -9.6104e+05  6e+05  1e-10  2e-11
14: -4.9564e+05 -6.7461e+05  2e+05  1e-10  3e-11
15: -5.2574e+05 -5.6990e+05  4e+04  3e-10  3e-11
16: -5.3294e+05 -5.4646e+05  1e+04  2e-10  3e-11
```

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17: -5.3433e+05 -5.4194e+05  8e+03  3e-11  3e-11
18: -5.3541e+05 -5.3796e+05  3e+03  2e-10  3e-11
19: -5.3581e+05 -5.3650e+05  7e+02  4e-10  3e-11
20: -5.3593e+05 -5.3609e+05  2e+02  2e-10  2e-11
21: -5.3596e+05 -5.3600e+05  5e+01  2e-10  3e-11
22: -5.3596e+05 -5.3598e+05  2e+01  5e-10  3e-11
23: -5.3597e+05 -5.3597e+05  4e+00  5e-10  2e-11
24: -5.3597e+05 -5.3597e+05  7e-01  7e-10  3e-11
25: -5.3597e+05 -5.3597e+05  1e-01  3e-10  3e-11
Optimal solution found.
Computing c = 100000 and variance = 1000
     pcost       dcost       gap    pres   dres
 0:  6.5337e+11 -1.4684e+13  2e+13  3e-01  3e-09
 1:  4.5321e+11 -2.4879e+12  3e+12  3e-02  6e-08
 2:  1.6481e+11 -7.2209e+11  9e+11  5e-03  1e-08
 3:  5.7172e+10 -3.0108e+11  4e+11  6e-04  3e-09
 4:  2.2560e+10 -1.1144e+11  1e+11  2e-04  2e-09
 5:  7.6043e+09 -3.4874e+10  4e+10  2e-05  1e-09
 6:  2.5809e+09 -1.1757e+10  1e+10  4e-06  1e-09
 7:  8.8837e+08 -4.2043e+09  5e+09  4e-09  9e-10
 8:  3.2475e+08 -1.4157e+09  2e+09  3e-09  6e-10
 9:  1.2809e+08 -5.5179e+08  7e+08  2e-09  5e-10
10:  5.0181e+07 -2.3017e+08  3e+08  2e-10  5e-10
11:  1.4207e+07 -6.7005e+07  8e+07  7e-10  4e-10
12:  7.0905e+06 -4.2656e+07  5e+07  2e-10  3e-10
13:  8.2688e+05 -1.6368e+07  2e+07  4e-09  3e-10
14: -1.0575e+06 -9.4123e+06  8e+06  4e-10  3e-10
15: -2.1578e+06 -5.1262e+06  3e+06  2e-10  3e-10
16: -2.5091e+06 -3.8813e+06  1e+06  1e-11  3e-10
17: -2.7036e+06 -3.2466e+06  5e+05  4e-10  3e-10
18: -2.8031e+06 -2.9463e+06  1e+05  1e-09  3e-10
19: -2.8316e+06 -2.8752e+06  4e+04  2e-09  3e-10
20: -2.8411e+06 -2.8528e+06  1e+04  1e-09  3e-10
21: -2.8439e+06 -2.8466e+06  3e+03  7e-10  3e-10
22: -2.8442e+06 -2.8459e+06  2e+03  5e-09  3e-10
23: -2.8447e+06 -2.8450e+06  3e+02  4e-09  3e-10
24: -2.8448e+06 -2.8448e+06  7e+01  4e-10  3e-10
25: -2.8448e+06 -2.8448e+06  1e+01  2e-09  3e-10
26: -2.8448e+06 -2.8448e+06  4e+00  4e-10  3e-10
27: -2.8448e+06 -2.8448e+06  5e-01  2e-09  3e-10
Optimal solution found.
Computing c = 1000000 and variance = 0.1
     pcost       dcost       gap    pres   dres
 0:  1.6140e+14 -8.8316e+14  1e+15  1e-06  9e-11
 1:  4.9226e+13 -7.6224e+13  1e+14  1e-06  9e-11
 2:  7.4783e+12 -8.8857e+12  2e+13  1e-06  5e-11
 3:  1.0828e+12 -1.2228e+12  2e+12  3e-08  2e-11
 4:  1.5563e+11 -1.7273e+11  3e+11  2e-07  8e-12
 5:  2.2431e+10 -2.5338e+10  5e+10  7e-08  2e-12
 6:  3.3298e+09 -4.2765e+09  8e+09  1e-08  1e-12
 7:  4.8978e+08 -6.2019e+08  1e+09  8e-09  2e-12
 8:  6.8427e+07 -8.0767e+07  1e+08  2e-09  3e-12
 9:  8.0235e+06 -1.3267e+07  2e+07  5e-10  1e-12
10: -5.8795e+05 -3.6588e+06  3e+06  3e-09  2e-12
11: -1.8074e+06 -2.2525e+06  4e+05  2e-09  8e-13
12: -1.9766e+06 -2.0416e+06  6e+04  4e-09  1e-12
13: -1.9988e+06 -2.0080e+06  9e+03  7e-09  1e-12
```

```
14: -2.0012e+06 -2.0023e+06  1e+03  3e-09  5e-13
15: -2.0013e+06 -2.0013e+06  7e+01  4e-11  3e-12
16: -2.0013e+06 -2.0013e+06  5e+00  2e-09  9e-13
17: -2.0013e+06 -2.0013e+06  8e-01  1e-09  3e-12
Optimal solution found.
Computing c = 1000000 and variance = 1
     pcost       dcost       gap    pres   dres
 0:  1.6180e+14 -8.9038e+14  1e+15  1e-06  2e-10
 1:  4.9653e+13 -8.2469e+13  1e+14  5e-07  2e-10
 2:  9.3724e+12 -1.3575e+13  2e+13  3e-07  7e-11
 3:  1.4030e+12 -1.7609e+12  3e+12  1e-07  3e-11
 4:  2.0292e+11 -2.2952e+11  4e+11  5e-08  1e-11
 5:  2.9178e+10 -3.2567e+10  6e+10  3e-09  6e-12
 6:  4.3164e+09 -5.4648e+09  1e+10  1e-08  4e-12
 7:  6.3833e+08 -8.2049e+08  1e+09  4e-10  2e-12
 8:  8.9907e+07 -1.0557e+08  2e+08  8e-11  3e-12
 9:  1.1091e+07 -1.6657e+07  3e+07  5e-10  3e-12
10: -1.5108e+05 -4.1516e+06  4e+06  4e-11  2e-12
11: -1.7458e+06 -2.3249e+06  6e+05  3e-10  3e-12
12: -1.9682e+06 -2.0527e+06  8e+04  2e-10  5e-12
13: -1.9977e+06 -2.0098e+06  1e+04  5e-10  2e-12
14: -2.0010e+06 -2.0026e+06  2e+03  2e-09  3e-12
15: -2.0012e+06 -2.0014e+06  1e+02  9e-10  4e-12
16: -2.0012e+06 -2.0013e+06  1e+01  1e-09  2e-12
17: -2.0012e+06 -2.0013e+06  2e+00  4e-10  4e-12
Optimal solution found.
Computing c = 1000000 and variance = 10
     pcost       dcost       gap    pres   dres
 0:  1.4454e+14 -1.1344e+15  2e+15  1e-01  8e-10
 1:  6.5074e+13 -1.4161e+14  2e+14  1e-02  1e-06
 2:  1.7117e+13 -3.5390e+13  5e+13  2e-03  2e-07
 3:  4.8244e+12 -1.0758e+13  2e+13  4e-04  3e-08
 4:  1.0640e+12 -2.9697e+12  4e+12  2e-08  9e-11
 5:  1.9677e+11 -3.6207e+11  6e+11  5e-08  5e-11
 6:  2.9429e+10 -3.7206e+10  7e+10  4e-09  2e-11
 7:  4.2917e+09 -4.9783e+09  9e+09  1e-09  1e-11
 8:  6.9458e+08 -1.1820e+09  2e+09  4e-09  9e-12
 9:  9.8648e+07 -1.1308e+08  2e+08  1e-09  1e-11
10:  1.2360e+07 -1.8138e+07  3e+07  1e-09  1e-11
11:  3.5089e+04 -4.3828e+06  4e+06  6e-10  9e-12
12: -1.7185e+06 -2.3643e+06  6e+05  2e-10  6e-12
13: -1.9642e+06 -2.0611e+06  1e+05  6e-10  1e-11
14: -1.9967e+06 -2.0150e+06  2e+04  6e-10  1e-11
15: -2.0016e+06 -2.0063e+06  5e+03  9e-10  6e-12
16: -2.0026e+06 -2.0046e+06  2e+03  4e-10  1e-11
17: -2.0027e+06 -2.0031e+06  4e+02  1e-09  9e-12
18: -2.0027e+06 -2.0028e+06  9e+01  2e-09  6e-12
19: -2.0028e+06 -2.0028e+06  2e+01  5e-10  7e-12
20: -2.0028e+06 -2.0028e+06  4e+00  7e-10  8e-12
21: -2.0028e+06 -2.0028e+06  9e-01  9e-12  8e-12
Optimal solution found.
Computing c = 1000000 and variance = 100
     pcost       dcost       gap    pres   dres
 0:  9.2759e+13 -1.4292e+15  2e+15  3e-01  6e-09
 1:  6.2936e+13 -1.9346e+14  3e+14  2e-02  4e-06
 2:  2.1182e+13 -6.0632e+13  8e+13  3e-03  9e-07
 3:  6.2125e+12 -2.1552e+13  3e+13  3e-04  8e-08
```

```
 4:  2.1480e+12 -7.6329e+12  1e+13  5e-05  1e-08
 5:  7.1991e+11 -2.7812e+12  4e+12  4e-08  2e-09
 6:  2.5075e+11 -9.5142e+11  1e+12  2e-08  1e-09
 7:  5.9631e+10 -1.7403e+11  2e+11  2e-08  6e-10
 8:  1.7811e+10 -6.0396e+10  8e+10  4e-09  5e-10
 9:  5.1623e+09 -1.6756e+10  2e+10  8e-09  3e-10
10:  1.0357e+09 -2.4316e+09  3e+09  3e-09  2e-10
11:  1.6464e+08 -2.6686e+08  4e+08  1e-10  2e-10
12:  2.1494e+07 -3.0088e+07  5e+07  4e-10  1e-10
13:  2.5679e+06 -8.6843e+06  1e+07  3e-09  1e-10
14: -1.2950e+06 -3.9326e+06  3e+06  3e-09  1e-10
15: -1.9581e+06 -3.1296e+06  1e+06  8e-10  1e-10
16: -2.2685e+06 -2.7475e+06  5e+05  4e-10  1e-10
17: -2.3476e+06 -2.4725e+06  1e+05  1e-09  9e-11
18: -2.3675e+06 -2.4072e+06  4e+04  2e-09  1e-10
19: -2.3738e+06 -2.3865e+06  1e+04  2e-09  1e-10
20: -2.3755e+06 -2.3810e+06  6e+03  9e-10  8e-11
21: -2.3764e+06 -2.3777e+06  1e+03  2e-10  1e-10
22: -2.3766e+06 -2.3770e+06  4e+02  1e-09  1e-10
23: -2.3766e+06 -2.3767e+06  1e+02  1e-09  9e-11
24: -2.3767e+06 -2.3767e+06  6e+01  2e-10  1e-10
25: -2.3767e+06 -2.3767e+06  1e+01  1e-09  1e-10
26: -2.3767e+06 -2.3767e+06  2e+00  3e-09  9e-11
Optimal solution found.
Computing c = 1000000 and variance = 1000
     pcost       dcost       gap    pres   dres
 0:  6.5344e+13 -1.4684e+15  2e+15  3e-01  3e-08
 1:  4.5326e+13 -2.4879e+14  3e+14  3e-02  8e-06
 2:  1.6486e+13 -7.2208e+13  9e+13  5e-03  2e-06
 3:  5.7196e+12 -3.0100e+13  4e+13  6e-04  2e-07
 4:  2.2583e+12 -1.1142e+13  1e+13  2e-04  6e-08
 5:  7.6222e+11 -3.4865e+12  4e+12  2e-05  2e-08
 6:  2.5961e+11 -1.1761e+12  1e+12  4e-06  1e-08
 7:  9.0405e+10 -4.2398e+11  5e+11  2e-07  9e-09
 8:  3.3572e+10 -1.4309e+11  2e+11  3e-08  6e-09
 9:  1.3881e+10 -5.7627e+10  7e+10  4e-08  5e-09
10:  5.6122e+09 -2.3316e+10  3e+10  5e-08  4e-09
11:  1.9895e+09 -7.6870e+09  1e+10  2e-08  3e-09
12:  7.7957e+08 -3.0822e+09  4e+09  3e-09  3e-09
13:  2.1986e+08 -8.3350e+08  1e+09  1e-08  2e-09
14:  1.0543e+08 -4.6226e+08  6e+08  9e-09  2e-09
15:  4.5737e+07 -2.5061e+08  3e+08  1e-08  1e-09
16:  2.4879e+06 -7.4147e+07  8e+07  4e-10  2e-09
17: -8.4912e+06 -3.7300e+07  3e+07  1e-08  2e-09
18: -1.2685e+07 -2.3135e+07  1e+07  6e-09  2e-09
19: -1.4119e+07 -1.8448e+07  4e+06  7e-09  2e-09
20: -1.4717e+07 -1.6463e+07  2e+06  2e-09  2e-09
21: -1.4982e+07 -1.5619e+07  6e+05  1e-08  2e-09
22: -1.5099e+07 -1.5283e+07  2e+05  5e-09  2e-09
23: -1.5137e+07 -1.5187e+07  5e+04  8e-09  2e-09
24: -1.5148e+07 -1.5161e+07  1e+04  8e-09  2e-09
25: -1.5151e+07 -1.5154e+07  3e+03  1e-08  2e-09
26: -1.5151e+07 -1.5153e+07  2e+03  5e-09  2e-09
27: -1.5152e+07 -1.5152e+07  4e+02  5e-09  2e-09
28: -1.5152e+07 -1.5152e+07  1e+02  7e-09  2e-09
29: -1.5152e+07 -1.5152e+07  3e+01  8e-09  2e-09
30: -1.5152e+07 -1.5152e+07  3e+00  3e-09  2e-09
```

```
Optimal solution found.
Computing c = 10000000 and variance = 0.1
     pcost       dcost       gap    pres   dres
 0:  1.6140e+16 -8.8316e+16  1e+17  2e-06  1e-09
 1:  4.9226e+15 -7.6223e+15  1e+16  2e-05  1e-09
 2:  7.4783e+14 -8.8856e+14  2e+15  3e-06  6e-10
 3:  1.0828e+14 -1.2228e+14  2e+14  2e-06  2e-10
 4:  1.5564e+13 -1.7271e+13  3e+13  7e-07  6e-11
 5:  2.2326e+12 -2.4627e+12  5e+12  3e-08  2e-11
 6:  3.2128e+11 -3.6017e+11  7e+11  2e-07  2e-11
 7:  4.7270e+10 -5.8441e+10  1e+11  5e-08  8e-12
 8:  7.0392e+09 -9.2430e+09  2e+10  6e-08  3e-12
 9:  9.9435e+08 -1.1555e+09  2e+09  9e-09  2e-11
10:  1.2510e+08 -1.8046e+08  3e+08  5e-09  2e-11
11:  7.0162e+05 -4.3146e+07  4e+07  3e-08  1e-11
12: -1.7068e+07 -2.3375e+07  6e+06  6e-08  5e-13
13: -1.9593e+07 -2.0505e+07  9e+05  4e-08  9e-12
14: -1.9947e+07 -2.0080e+07  1e+05  1e-08  3e-11
15: -1.9995e+07 -2.0014e+07  2e+04  2e-08  3e-11
16: -2.0001e+07 -2.0003e+07  3e+03  9e-09  2e-12
17: -2.0001e+07 -2.0002e+07  2e+02  2e-08  2e-11
18: -2.0001e+07 -2.0001e+07  2e+01  8e-09  5e-12
Optimal solution found.
Computing c = 10000000 and variance = 1
     pcost       dcost       gap    pres   dres
 0:  1.6180e+16 -8.9037e+16  1e+17  4e-06  3e-09
 1:  4.9653e+15 -8.2468e+15  1e+16  1e-06  2e-09
 2:  9.3724e+14 -1.3575e+15  2e+15  2e-06  8e-10
 3:  1.4030e+14 -1.7608e+14  3e+14  1e-06  3e-10
 4:  2.0293e+13 -2.2950e+13  4e+13  4e-08  1e-10
 5:  2.9165e+12 -3.2437e+12  6e+12  1e-07  5e-11
 6:  4.1911e+11 -4.6671e+11  9e+11  6e-08  2e-11
 7:  6.1380e+10 -7.4430e+10  1e+11  5e-08  4e-11
 8:  9.1865e+09 -1.2244e+10  2e+10  1e-08  2e-11
 9:  1.3060e+09 -1.5192e+09  3e+09  6e-09  1e-11
10:  1.6974e+08 -2.2964e+08  4e+08  2e-08  2e-11
11:  7.0856e+06 -5.0205e+07  6e+07  3e-08  8e-12
12: -1.6157e+07 -2.4394e+07  8e+06  2e-08  5e-11
13: -1.9464e+07 -2.0654e+07  1e+06  2e-08  5e-11
14: -1.9929e+07 -2.0103e+07  2e+05  3e-09  4e-12
15: -1.9993e+07 -2.0018e+07  3e+04  9e-09  1e-11
16: -2.0001e+07 -2.0004e+07  3e+03  2e-08  7e-11
17: -2.0001e+07 -2.0002e+07  4e+02  3e-08  7e-11
18: -2.0001e+07 -2.0001e+07  3e+01  2e-08  2e-11
19: -2.0001e+07 -2.0001e+07  3e+00  2e-08  8e-11
Optimal solution found.
Computing c = 10000000 and variance = 10
     pcost       dcost       gap    pres   dres
 0:  1.4454e+16 -1.1344e+17  2e+17  1e-01  8e-09
 1:  6.5074e+15 -1.4161e+16  2e+16  1e-02  1e-04
 2:  1.7118e+15 -3.5390e+15  5e+15  2e-03  2e-05
 3:  4.8245e+14 -1.0758e+15  2e+15  4e-04  3e-06
 4:  1.0641e+14 -2.9698e+14  4e+14  8e-07  9e-10
 5:  1.9682e+13 -3.6221e+13  6e+13  3e-07  5e-10
 6:  2.9443e+12 -3.7215e+12  7e+12  7e-08  2e-10
 7:  4.2525e+11 -4.8386e+11  9e+11  1e-08  1e-10
 8:  6.2018e+10 -7.3875e+10  1e+11  4e-08  5e-11
```

```
 9:  9.4359e+09 -1.3320e+10  2e+10  3e-08  1e-10
10:  1.3378e+09 -1.5330e+09  3e+09  2e-08  2e-10
11:  1.7435e+08 -2.3498e+08  4e+08  1e-08  2e-10
12:  7.7698e+06 -5.1040e+07  6e+07  3e-10  1e-10
13: -1.6052e+07 -2.4541e+07  8e+06  9e-09  1e-10
14: -1.9447e+07 -2.0684e+07  1e+06  6e-09  1e-10
15: -1.9927e+07 -2.0109e+07  2e+05  3e-09  1e-10
16: -1.9991e+07 -2.0023e+07  3e+04  6e-09  1e-10
17: -2.0001e+07 -2.0008e+07  7e+03  5e-09  1e-10
18: -2.0002e+07 -2.0005e+07  3e+03  2e-08  9e-11
19: -2.0003e+07 -2.0003e+07  7e+02  4e-08  7e-11
20: -2.0003e+07 -2.0003e+07  1e+02  2e-08  9e-11
21: -2.0003e+07 -2.0003e+07  3e+01  6e-09  1e-10
22: -2.0003e+07 -2.0003e+07  7e+00  4e-10  2e-10
Optimal solution found.
Computing c = 10000000 and variance = 100
     pcost       dcost       gap    pres   dres
 0:  9.2759e+15 -1.4292e+17  2e+17  3e-01  5e-08
 1:  6.2936e+15 -1.9346e+16  3e+16  2e-02  3e-04
 2:  2.1182e+15 -6.0632e+15  8e+15  3e-03  6e-05
 3:  6.2127e+14 -2.1552e+15  3e+15  3e-04  5e-06
 4:  2.1482e+14 -7.6331e+14  1e+15  5e-05  9e-07
 5:  7.1998e+13 -2.7813e+14  4e+14  3e-07  2e-08
 6:  2.5084e+13 -9.5170e+13  1e+14  2e-07  1e-08
 7:  5.9651e+12 -1.7397e+13  2e+13  4e-09  7e-09
 8:  1.7945e+12 -6.0991e+12  8e+12  4e-08  4e-09
 9:  5.2936e+11 -1.7275e+12  2e+12  1e-07  3e-09
10:  1.0651e+11 -2.4974e+11  4e+11  7e-08  3e-09
11:  1.6812e+10 -2.5027e+10  4e+10  4e-08  1e-09
12:  3.1258e+09 -5.6304e+09  9e+09  4e-08  9e-10
13:  4.3531e+08 -5.2907e+08  1e+09  2e-08  1e-09
14:  4.4736e+07 -9.4242e+07  1e+08  7e-09  1e-09
15: -9.5282e+06 -3.3138e+07  2e+07  1e-08  9e-10
16: -1.8501e+07 -2.3018e+07  5e+06  2e-08  8e-10
17: -1.9825e+07 -2.1307e+07  1e+06  2e-09  9e-10
18: -2.0186e+07 -2.0892e+07  7e+05  1e-08  1e-09
19: -2.0327e+07 -2.0509e+07  2e+05  2e-08  1e-09
20: -2.0366e+07 -2.0405e+07  4e+04  1e-08  9e-10
21: -2.0373e+07 -2.0388e+07  1e+04  5e-09  1e-09
22: -2.0375e+07 -2.0383e+07  8e+03  6e-09  9e-10
23: -2.0376e+07 -2.0379e+07  3e+03  2e-08  1e-09
24: -2.0376e+07 -2.0377e+07  8e+02  4e-08  9e-10
25: -2.0377e+07 -2.0377e+07  2e+02  3e-08  1e-09
26: -2.0377e+07 -2.0377e+07  3e+01  6e-09  8e-10
27: -2.0377e+07 -2.0377e+07  8e+00  3e-08  9e-10
Optimal solution found.
Computing c = 10000000 and variance = 1000
     pcost       dcost       gap    pres   dres
 0:  6.5345e+15 -1.4684e+17  2e+17  3e-01  4e-07
 1:  4.5327e+15 -2.4879e+16  3e+16  3e-02  2e-04
 2:  1.6487e+15 -7.2208e+15  9e+15  5e-03  3e-05
 3:  5.7198e+14 -3.0099e+15  4e+15  6e-04  4e-06
 4:  2.2585e+14 -1.1142e+15  1e+15  2e-04  1e-06
 5:  7.6240e+13 -3.4864e+14  4e+14  2e-05  2e-07
 6:  2.5976e+13 -1.1761e+14  1e+14  4e-06  1e-07
 7:  9.0562e+12 -4.2433e+13  5e+13  1e-06  9e-08
 8:  3.3680e+12 -1.4324e+13  2e+13  7e-07  6e-08
```

```
 9:  1.3985e+12 -5.7861e+12  7e+12  6e-07  5e-08
10:  5.6658e+11 -2.3337e+12  3e+12  5e-07  5e-08
11:  2.0478e+11 -7.8269e+11  1e+12  1e-07  3e-08
12:  8.0728e+10 -3.0877e+11  4e+11  2e-08  3e-08
13:  2.3494e+10 -8.0858e+10  1e+11  2e-08  2e-08
14:  8.2111e+09 -2.9768e+10  4e+10  2e-08  1e-08
15:  2.2809e+09 -7.7268e+09  1e+10  5e-08  1e-08
16:  4.5000e+08 -1.5109e+09  2e+09  5e-08  1e-08
17:  1.0312e+08 -6.3429e+08  7e+08  3e-08  1e-08
18: -4.0781e+07 -1.8828e+08  1e+08  7e-08  1e-08
19: -6.9033e+07 -1.1711e+08  5e+07  3e-08  1e-08
20: -7.6283e+07 -9.8571e+07  2e+07  4e-08  1e-08
21: -7.9745e+07 -8.9936e+07  1e+07  2e-08  1e-08
22: -8.1462e+07 -8.5066e+07  4e+06  5e-09  1e-08
23: -8.2065e+07 -8.3339e+07  1e+06  6e-08  1e-08
24: -8.2282e+07 -8.2737e+07  5e+05  4e-09  1e-08
25: -8.2352e+07 -8.2544e+07  2e+05  1e-08  1e-08
26: -8.2392e+07 -8.2446e+07  5e+04  4e-11  1e-08
27: -8.2404e+07 -8.2418e+07  1e+04  7e-08  1e-08
28: -8.2408e+07 -8.2411e+07  3e+03  9e-08  1e-08
29: -8.2408e+07 -8.2410e+07  2e+03  3e-08  1e-08
30: -8.2409e+07 -8.2409e+07  3e+02  1e-07  1e-08
31: -8.2409e+07 -8.2409e+07  1e+02  3e-08  1e-08
32: -8.2409e+07 -8.2409e+07  7e+01  1e-08  1e-08
Optimal solution found.
Computing c = 100000000 and variance = 0.1
     pcost       dcost       gap    pres   dres
 0:  1.6140e+18 -8.8316e+18  1e+19  2e-04  2e-08
 1:  4.9226e+17 -7.6223e+17  1e+18  1e-04  1e-08
 2:  7.4783e+16 -8.8856e+16  2e+17  1e-04  5e-09
 3:  1.0828e+16 -1.2228e+16  2e+16  1e-04  2e-09
 4:  1.5564e+15 -1.7271e+15  3e+15  2e-05  7e-10
 5:  2.2327e+14 -2.4627e+14  5e+14  1e-06  4e-10
 6:  3.2009e+13 -3.5247e+13  7e+13  2e-06  1e-10
 7:  4.6003e+12 -5.1264e+12  1e+13  4e-07  7e-11
 8:  6.7208e+11 -8.0618e+11  1e+12  1e-07  4e-11
 9:  1.0086e+11 -1.3550e+11  2e+11  4e-07  1e-10
10:  1.4368e+10 -1.6684e+10  3e+10  6e-07  2e-10
11:  1.8869e+09 -2.4986e+09  4e+09  2e-07  1e-10
12:  9.8675e+07 -5.2962e+08  6e+08  2e-07  4e-12
13: -1.5733e+08 -2.4747e+08  9e+07  4e-07  4e-10
14: -1.9393e+08 -2.0688e+08  1e+07  1e-07  3e-10
15: -1.9915e+08 -2.0102e+08  2e+06  2e-07  2e-10
16: -1.9989e+08 -2.0016e+08  3e+05  2e-07  3e-11
17: -1.9999e+08 -2.0003e+08  4e+04  7e-09  6e-11
18: -2.0000e+08 -2.0001e+08  5e+03  5e-07  2e-10
19: -2.0000e+08 -2.0000e+08  6e+02  3e-08  4e-11
20: -2.0000e+08 -2.0000e+08  2e+01  5e-07  5e-11
21: -2.0000e+08 -2.0000e+08  3e+00  6e-07  6e-11
22: -2.0000e+08 -2.0000e+08  6e-01  2e-07  2e-11
23: -2.0000e+08 -2.0000e+08  1e-01  3e-07  2e-10
24: -2.0000e+08 -2.0000e+08  4e-02  4e-07  1e-10
25: -2.0000e+08 -2.0000e+08  2e-02  2e-07  1e-10
26: -2.0000e+08 -2.0000e+08  7e-03  7e-07  6e-11
27: -2.0000e+08 -2.0000e+08  1e-03  1e-07  1e-10
28: -2.0000e+08 -2.0000e+08  4e-05  6e-09  8e-11
Optimal solution found.
```

```
Computing c = 100000000 and variance = 1
     pcost       dcost       gap    pres   dres
 0:  1.6180e+18 -8.9037e+18  1e+19  8e-06  2e-08
 1:  4.9653e+17 -8.2468e+17  1e+18  4e-05  2e-08
 2:  9.3724e+16 -1.3574e+17  2e+17  2e-06  7e-09
 3:  1.4030e+16 -1.7608e+16  3e+16  4e-06  3e-09
 4:  2.0293e+15 -2.2950e+15  4e+15  8e-06  1e-09
 5:  2.9166e+14 -3.2436e+14  6e+14  4e-08  5e-10
 6:  4.1836e+13 -4.6173e+13  9e+13  1e-06  2e-10
 7:  6.0078e+12 -6.6698e+12  1e+13  4e-08  3e-10
 8:  8.7412e+11 -1.0301e+12  2e+12  2e-07  3e-11
 9:  1.3143e+11 -1.7733e+11  3e+11  2e-07  4e-10
10:  1.8837e+10 -2.2056e+10  4e+10  2e-07  4e-10
11:  2.5274e+09 -3.2047e+09  6e+09  6e-08  2e-10
12:  1.9043e+08 -6.3067e+08  8e+08  5e-08  4e-10
13: -1.4420e+08 -2.6196e+08  1e+08  7e-08  5e-10
14: -1.9206e+08 -2.0897e+08  2e+07  2e-08  4e-10
15: -1.9888e+08 -2.0132e+08  2e+06  1e-08  4e-10
16: -1.9985e+08 -2.0020e+08  4e+05  3e-07  3e-10
17: -1.9998e+08 -2.0003e+08  5e+04  4e-07  4e-10
18: -2.0000e+08 -2.0001e+08  7e+03  1e-07  9e-11
19: -2.0000e+08 -2.0000e+08  9e+02  4e-08  3e-10
20: -2.0000e+08 -2.0000e+08  4e+01  1e-08  7e-11
Optimal solution found.
Computing c = 100000000 and variance = 10
     pcost       dcost       gap    pres   dres
 0:  1.4454e+18 -1.1344e+19  2e+19  1e-01  7e-08
 1:  6.5074e+17 -1.4161e+18  2e+18  1e-02  1e-02
 2:  1.7118e+17 -3.5390e+17  5e+17  2e-03  1e-03
 3:  4.8246e+16 -1.0758e+17  2e+17  4e-04  3e-04
 4:  1.0641e+16 -2.9698e+16  4e+16  1e-05  9e-09
 5:  1.9682e+15 -3.6222e+15  6e+15  5e-06  5e-09
 6:  2.9445e+14 -3.7216e+14  7e+14  1e-06  2e-09
 7:  4.2529e+13 -4.8384e+13  9e+13  6e-07  1e-09
 8:  6.1109e+12 -6.8017e+12  1e+13  3e-07  8e-10
 9:  8.8867e+11 -1.0449e+12  2e+12  3e-08  6e-10
10:  1.3390e+11 -1.8206e+11  3e+11  7e-08  1e-09
11:  1.9191e+10 -2.2443e+10  4e+10  2e-07  1e-09
12:  2.5784e+09 -3.2616e+09  6e+09  7e-08  1e-09
13:  1.9779e+08 -6.3888e+08  8e+08  5e-08  1e-09
14: -1.4311e+08 -2.6328e+08  1e+08  1e-07  1e-09
15: -1.9189e+08 -2.0920e+08  2e+07  1e-07  1e-09
16: -1.9886e+08 -2.0137e+08  3e+06  5e-08  1e-09
17: -1.9984e+08 -2.0021e+08  4e+05  1e-07  1e-09
18: -1.9998e+08 -2.0004e+08  5e+04  1e-07  9e-10
19: -2.0000e+08 -2.0001e+08  1e+04  2e-07  1e-09
20: -2.0000e+08 -2.0001e+08  4e+03  1e-08  7e-10
21: -2.0000e+08 -2.0000e+08  1e+03  1e-07  8e-10
22: -2.0000e+08 -2.0000e+08  3e+02  1e-07  1e-09
23: -2.0000e+08 -2.0000e+08  8e+01  3e-08  5e-10
Optimal solution found.
Computing c = 100000000 and variance = 100
     pcost       dcost       gap    pres   dres
 0:  9.2759e+17 -1.4292e+19  2e+19  3e-01  6e-07
 1:  6.2936e+17 -1.9346e+18  3e+18  2e-02  5e-02
 2:  2.1182e+17 -6.0632e+17  8e+17  3e-03  1e-02
 3:  6.2127e+16 -2.1552e+17  3e+17  3e-04  9e-04
```

```
 4:  2.1482e+16 -7.6331e+16  1e+17  5e-05  1e-04
 5:  7.1999e+15 -2.7813e+16  4e+16  4e-06  2e-07
 6:  2.5085e+15 -9.5173e+15  1e+16  9e-06  1e-07
 7:  5.9653e+14 -1.7397e+15  2e+15  5e-07  6e-08
 8:  1.7959e+14 -6.1051e+14  8e+14  3e-07  4e-08
 9:  5.3063e+13 -1.7325e+14  2e+14  2e-06  3e-08
10:  1.0678e+13 -2.5030e+13  4e+13  2e-07  2e-08
11:  1.6877e+12 -2.5083e+12  4e+12  6e-08  1e-08
12:  2.7829e+11 -3.4719e+11  6e+11  2e-07  8e-09
13:  4.5319e+10 -7.8549e+10  1e+11  3e-08  1e-08
14:  6.3544e+09 -7.3267e+09  1e+10  5e-08  1e-08
15:  7.3798e+08 -1.2421e+09  2e+09  1e-07  1e-08
16: -6.6285e+07 -3.5090e+08  3e+08  2e-07  9e-09
17: -1.8126e+08 -2.2241e+08  4e+07  4e-07  1e-08
18: -1.9640e+08 -2.0556e+08  9e+06  1e-07  1e-08
19: -1.9937e+08 -2.0180e+08  2e+06  3e-08  1e-08
20: -1.9997e+08 -2.0108e+08  1e+06  2e-09  1e-08
21: -2.0028e+08 -2.0071e+08  4e+05  2e-08  1e-08
22: -2.0035e+08 -2.0046e+08  1e+05  3e-08  9e-09
23: -2.0037e+08 -2.0041e+08  4e+04  1e-07  8e-09
24: -2.0037e+08 -2.0039e+08  1e+04  6e-08  1e-08
25: -2.0038e+08 -2.0038e+08  5e+03  1e-07  9e-09
26: -2.0038e+08 -2.0038e+08  1e+03  1e-07  8e-09
27: -2.0038e+08 -2.0038e+08  4e+02  2e-07  1e-08
28: -2.0038e+08 -2.0038e+08  9e+01  1e-07  8e-09
29: -2.0038e+08 -2.0038e+08  6e+01  3e-08  7e-09
Optimal solution found.
Computing c = 100000000 and variance = 1000
     pcost       dcost       gap    pres   dres
 0:  6.5345e+17 -1.4684e+19  2e+19  3e-01  3e-06
 1:  4.5327e+17 -2.4879e+18  3e+18  3e-02  3e-02
 2:  1.6487e+17 -7.2208e+17  9e+17  5e-03  6e-03
 3:  5.7198e+16 -3.0099e+17  4e+17  6e-04  8e-04
 4:  2.2585e+16 -1.1142e+17  1e+17  2e-04  2e-04
 5:  7.6241e+15 -3.4864e+16  4e+16  2e-05  3e-05
 6:  2.5977e+15 -1.1761e+16  1e+16  2e-05  4e-06
 7:  9.0577e+14 -4.2437e+15  5e+15  9e-06  9e-07
 8:  3.3691e+14 -1.4325e+15  2e+15  1e-05  6e-07
 9:  1.3995e+14 -5.7885e+14  7e+14  3e-06  5e-07
10:  5.6711e+13 -2.3339e+14  3e+14  1e-06  5e-07
11:  2.0535e+13 -7.8403e+13  1e+14  1e-06  3e-07
12:  8.0977e+12 -3.0879e+13  4e+13  9e-07  2e-07
13:  2.3724e+12 -8.1180e+12  1e+13  1e-06  2e-07
14:  8.3650e+11 -2.9894e+12  4e+12  1e-06  1e-07
15:  2.3243e+11 -7.4508e+11  1e+12  5e-07  1e-07
16:  4.3392e+10 -9.7642e+10  1e+11  4e-07  8e-08
17:  1.0269e+10 -2.9445e+10  4e+10  1e-07  7e-08
18:  1.6816e+09 -4.9337e+09  7e+09  1e-07  7e-08
19:  1.4237e+08 -1.8099e+09  2e+09  1e-07  6e-08
20: -2.8654e+08 -7.8064e+08  5e+08  1e-07  7e-08
21: -3.8026e+08 -5.3320e+08  2e+08  1e-07  6e-08
22: -4.0479e+08 -4.7728e+08  7e+07  1e-07  6e-08
23: -4.1593e+08 -4.4206e+08  3e+07  2e-07  6e-08
24: -4.1977e+08 -4.3076e+08  1e+07  8e-08  6e-08
25: -4.2190e+08 -4.2439e+08  2e+06  3e-08  6e-08
26: -4.2232e+08 -4.2330e+08  1e+06  2e-08  6e-08
27: -4.2252e+08 -4.2277e+08  2e+05  7e-08  7e-08
```

```
28: -4.2257e+08 -4.2266e+08  9e+04  3e-07  7e-08
29: -4.2259e+08 -4.2262e+08  2e+04  4e-07  6e-08
30: -4.2260e+08 -4.2260e+08  7e+03  5e-08  6e-08
31: -4.2260e+08 -4.2260e+08  3e+03  2e-07  6e-08
32: -4.2260e+08 -4.2260e+08  6e+02  8e-08  6e-08
33: -4.2260e+08 -4.2260e+08  3e+02  4e-08  6e-08
Optimal solution found.
```

In [11]:
```
df_dual.style.apply(lambda x: ['background: lightgreen' if not np.isnan(x['Tes
ting Data Accuracy']) else '' for i in x], axis=1)
```

Out[11]:

	c	variance	Training Data Accuracy	Validation Data Accuracy	Testing Data Accuracy
0	1	0.100000	56.433333	87.125000	nan
1	1	1.000000	56.433333	87.125000	nan
2	1	10.000000	56.533333	87.125000	nan
3	1	100.000000	57.100000	87.125000	nan
4	1	1000.000000	60.233333	86.375000	nan
5	10	0.100000	56.433333	87.125000	nan
6	10	1.000000	56.433333	87.125000	nan
7	10	10.000000	56.733333	87.125000	nan
8	10	100.000000	57.166667	87.125000	90.387016
9	10	1000.000000	59.800000	86.875000	nan
10	100	0.100000	56.433333	87.125000	nan
11	100	1.000000	56.433333	87.125000	nan
12	100	10.000000	56.733333	87.125000	nan
13	100	100.000000	57.066667	87.125000	nan
14	100	1000.000000	61.933333	84.875000	nan
15	1000	0.100000	56.433333	87.125000	nan
16	1000	1.000000	56.433333	87.125000	nan
17	1000	10.000000	56.900000	87.125000	nan
18	1000	100.000000	56.966667	87.125000	nan
19	1000	1000.000000	44.900000	12.375000	nan
20	10000	0.100000	56.433333	87.125000	nan
21	10000	1.000000	56.433333	87.125000	nan
22	10000	10.000000	56.900000	87.125000	nan
23	10000	100.000000	56.533333	87.125000	nan
24	10000	1000.000000	57.000000	87.125000	nan
25	100000	0.100000	56.433333	87.125000	nan
26	100000	1.000000	56.433333	87.125000	nan
27	100000	10.000000	56.900000	87.125000	nan
28	100000	100.000000	63.866667	84.250000	nan
29	100000	1000.000000	56.433333	87.125000	nan
30	1000000	0.100000	56.433333	87.125000	nan
31	1000000	1.000000	56.433333	87.125000	nan
32	1000000	10.000000	56.900000	87.125000	nan
33	1000000	100.000000	57.533333	87.000000	nan
34	1000000	1000.000000	56.366667	87.125000	nan

	c	variance	Training Data Accuracy	Validation Data Accuracy	Testing Data Accuracy
35	10000000	0.100000	56.433333	87.125000	nan
36	10000000	1.000000	56.433333	87.125000	nan
37	10000000	10.000000	56.900000	87.125000	nan
38	10000000	100.000000	57.533333	87.000000	nan
39	10000000	1000.000000	56.200000	87.125000	nan
40	100000000	0.100000	56.433333	87.125000	nan
41	100000000	1.000000	56.433333	87.125000	nan
42	100000000	10.000000	56.900000	87.125000	nan
43	100000000	100.000000	57.533333	87.000000	nan
44	100000000	1000.000000	56.266667	87.125000	nan

3. k-Nearest Neighbors

- What is the accuracy of the k-nearest neighbor classifier for k = 1,5,11,15,21?

In [33]:

```
from collections import defaultdict

class kNN:

    def __init__(self, neighbors=1):
        self.train_data = []
        self.labels = []
        self.m = 0
        self.n = 0
        self.k = neighbors

    def fit(self, X_train, y_train):
        self.train_mean = X_train.mean(axis=0)
        self.train_std = X_train.std(axis=0)
        self.train_data = X_train
        self.train_data_normed = (self.train_data - self.train_mean) / self.tr
ain_std
        self.m, self.n = self.train_data.shape
        self.labels = y_train

    def get_distances(self, X_test):
        distances = -2 * self.train_data_normed.dot(X_test.T) + np.sum(X_test*
*2,axis=1) + np.sum(self.train_data_normed**2,axis=1)[:, np.newaxis]
        distances[distances < 0] = 0
        return distances

    def predict(self, X_test):
        X_test_normed = (X_test - self.train_mean) / self.train_std
        distances = self.get_distances(X_test_normed)
        idx = np.argsort(distances, axis=0)
        idx = idx[0:self.k, :]
        m, n = idx.shape
        labels = self.labels.ravel()
        y_pred = np.zeros((X_test.shape[0], 1))
        for col in range(n):
            classes = defaultdict(int)
            for row in range(m):
                label = labels[idx[row, col]]
                classes[label] += 1
            # Get the majority class
            y_pred[col] = max(classes, key=classes.get)
        return y_pred

    @staticmethod
    def get_accuracy(y_pred, y_test):
        return np.mean(y_pred.flatten() == y_test.flatten()) * 100
```

In [34]:
```
def run_kNN(k_list):
    data = {
        'k': [],
        'Validation Accuracy': [],
        'Test data Accuracy': []
    }
    for k in k_list:
        data['k'].append(k)
        classifier = kNN(k)
        classifier.fit(X_train, y_train)
        y_validate_pred = classifier.predict(X_validate)
        data['Validation Accuracy'].append(kNN.get_accuracy(y_validate_pred, y
_validate))
        y_test_pred = classifier.predict(X_test)
        data['Test data Accuracy'].append(kNN.get_accuracy(y_test_pred, y_test
))

    return pd.DataFrame.from_dict(data)
```

In [35]:
```
k_list = [1,5,11,15,21]
df = run_kNN(k_list)
display(df)
```

	k	Validation Accuracy	Test data Accuracy
0	1	88.500	72.659176
1	5	89.625	70.786517
2	11	88.500	72.159800
3	15	86.375	71.535581
4	21	86.500	70.911361

4. Which of these approaches (if any) should be preferred for this classification task? Explain

SVM with gaussian kernel should be prefered for this classification task as it has higher test data accuracy of 90.38% compared to kNN model which has average test data accuracy ~70%. SVM performs better in Higher dimensions compared to kNN.