MATHEMATICS

What prior to use given a Poisson likelihood?

Asked 5 years ago Active 5 years ago Viewed 2k times

1

I am trying to incorporate a prior into a model I am working on. From available data, I have found that the likelihood follows a Poisson distribution with $\lambda = 1.5$.

I have then used R to generate random Poisson values with $\lambda = 1.5$.

However, I suspect that the λ value is more likely to be $\lambda = 2$. How would I turn this into a prior I could use and what would the posterior be?

Thanks in advance, Neil

statistics bayesian

edited Apr 6 '15 at 22:45
Math1000
28.5k 4 22 59

asked Apr 6 '15 at 22:21
Neil
61 4

1 Answer

Active Oldest Votes

3

If you have a large set of data which you believe comes from a Poisson distribution. The assumption is that Y_i iid $\sim$ Poisson(λ).

$$f(y|\lambda) = \frac{\lambda^y e^{-\lambda}}{y!}$$

The idea behind the Bayesian approach of estimation / regression is that there is uncertainty in the parameter λ for each Y_i. And that for each observation, there may be a natural variation of λs such that they have their own distribution $g(\lambda|\nu)$ with hyper-parameters ν. As mentioned, a convenient choice of prior for the Poisson distribution is the gamma distribution because with hyper-parameters $\lambda \sim \Gamma(\upsilon, r)$:

$$f(\lambda|y) \propto f(y|\lambda) \cdot g(\lambda|\nu)$$

$$\propto \left(\frac{\lambda^y e^{-\lambda}}{y!}\right) \cdot g(\lambda|\nu)$$

$$\propto \left(\frac{\lambda^y e^{-\lambda}}{y!}\right) \cdot \left(\frac{\upsilon^r}{\Gamma(r)} \lambda^{r-1} e^{-\upsilon\lambda}\right)$$

$$\propto \frac{\upsilon^r}{\Gamma(r)} \lambda^{r+y-1} e^{-(\upsilon+1)\lambda} \sim \Gamma(r' = r + y, \upsilon' = \upsilon + 1)$$

Now you use the fact that $E[\lambda] = \frac{r'}{\upsilon'}$, $\text{Var}[\lambda] = \frac{r'}{\upsilon'^2}$ you can solve for the regressed maximum likelihood estimate of λ, where r, and υ come from a historical set of data to inform the prior, and y is the next observation or $\sum y_i$ the 'next' set of information.

edited Apr 7 '15 at 2:00

answered Apr 6 '15 at 22:54

jameselmore

4,891 3 22 35

Yes. I've been trying a Gamma Distribution but couldn't figure out how to choose the shape and rate parameters that would resemble my prior – Neil Apr 6 '15 at 22:58

That's a common difficulty--with various solutions. Do you have a mean in mind for your prior, and maybe an interval that should contain most of the probability? Or do you want a noninformative prior? Tell me what info you have and I'll try to show you how to pick a prior. (Put @ followed by my name in your comment when you reply and I'll be notified to look at it.) – BruceET Apr 7 '15 at 1:38