Problem 2: Separability & Feature Vectors

Under which of the following feature vectors is the data linearly separable?

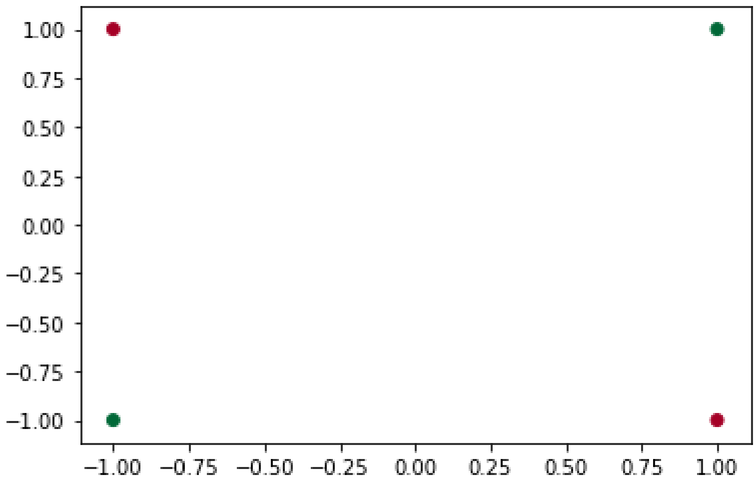
In [1]:

```
import pandas as pd
import math
import matplotlib.pyplot as plt
from matplotlib import cm
%matplotlib inline

# Initializing data
data = pd.DataFrame.from_dict({'x1': [-1,-1,1,1], 'x2': [-1,1,-1,1], 'y': [1,-1,-1,1]})
plt.scatter(data.x1, data.x2, c = data.y>0, cmap = cm.RdYlGn)
```

Out[1]:

```
<matplotlib.collections.PathCollection at 0x24ef61bd8c8>
```



Feature Vector (a)

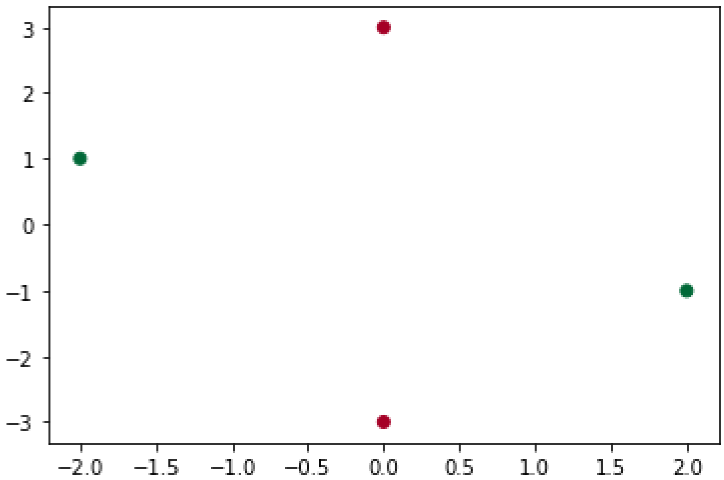
Plotting the first feature vector, it can be observed that under this feature vector the data is not linearly separable as no linear separator exists.

In [2]:

```
data['feature_a_x1'] = data.apply(lambda row: row['x1'] + (row['x2']), axis = 1)
data['feature_a_x2'] = data.apply(lambda row: row['x1'] - (2 * row['x2']), axis = 1)
plt.scatter(data.feature_a_x1, data.feature_a_x2, c = data.y>0, cmap = cm.RdYlGn)
```

Out[2]:

```
<matplotlib.collections.PathCollection at 0x24ef62b1908>
```



Feature Vector (b)

Plotting the second feature vector, it can be observed that under this feature vector the data is linearly separable and the plane z = 0 can be a linear separator.

In [3]:

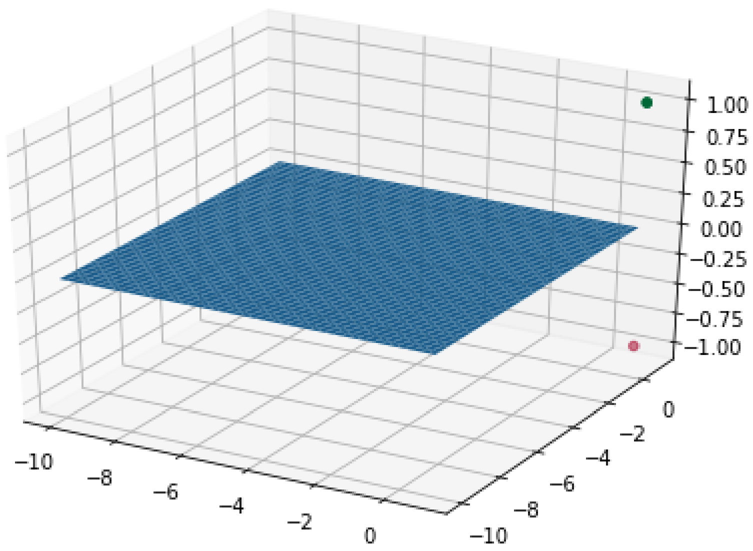
```
data['feature_b_x1'] = data['x1'].apply(lambda x: x ** 2)
data['feature_b_x2'] = data['x2'].apply(lambda x: x ** 2)
data['feature_b_x3'] = data.apply(lambda row: row['x1'] * (row['x2']), axis = 1)

from mpl_toolkits.mplot3d import Axes3D
import numpy as np
fig = plt.figure()
ax = Axes3D(fig)
ax.scatter(data.feature_b_x1, data.feature_b_x2, data.feature_b_x3, c = data.y>0, cmap
= cm.RdYlGn)

x = np.arange(-10, 1, 0.025)
y = np.arange(-10, 1, 0.025)
X,Y = np.meshgrid(x,y)
Z = 0 * X + 0 * Y + 0
ax.plot_surface(X, Y, Z)
```

Out[3]:

```
<mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x24ef63854c8>
```



Feature Vector (c)

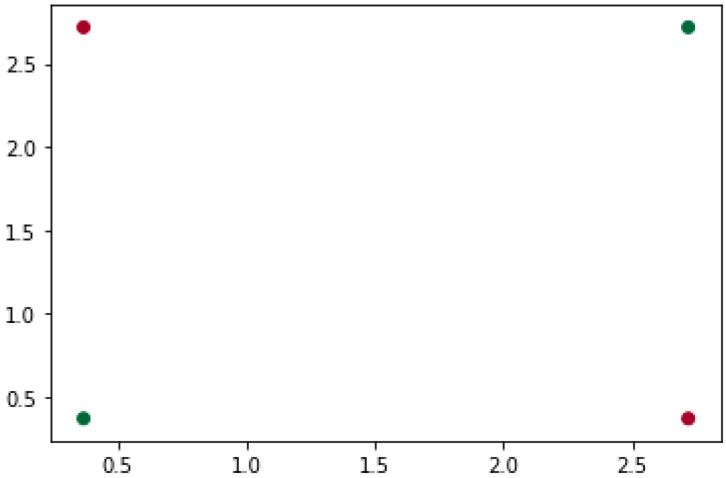
Plotting the third feature vector, it can be observed that under this feature vector the data is not linearly separable as no linear separator exists.

In [4]:

```
data['feature_c_x1'] = data['x1'].apply(math.exp)
data['feature_c_x2'] = data['x2'].apply(math.exp)
plt.scatter(data.feature_c_x1, data.feature_c_x2, c = data.y>0, cmap = cm.RdYlGn)
```

Out[4]:

```
<matplotlib.collections.PathCollection at 0x24ef6a682c8>
```



Feature Vector (d)

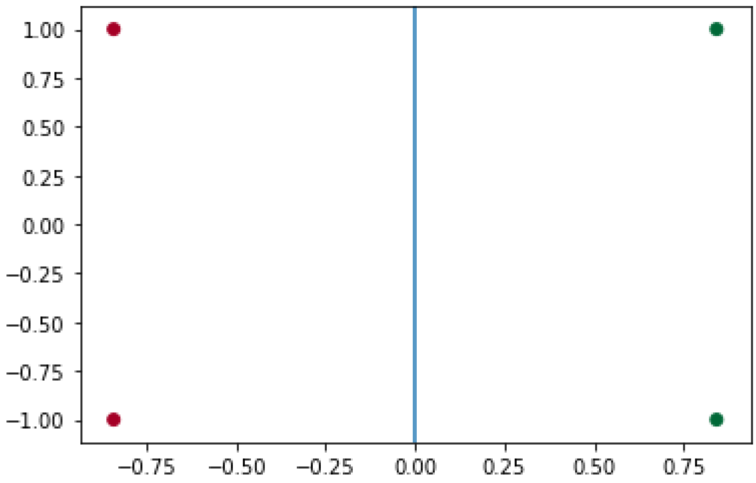
Plotting the second feature vector, it can be observed that under this feature vector the data is linearly separable and the plane x = 0 can be a linear separator.

In [5]:

```
data['feature_d_x1'] = data.apply(lambda row: row['x1'] * math.sin(row['x2']), axis = 1
)
data['feature_d_x2'] = data['x2']
plt.scatter(data.feature_d_x1, data.feature_d_x2, c = data.y>0, cmap = cm.RdYlGn)
plt.axvline(x=0)
```

Out[5]:

```
<matplotlib.lines.Line2D at 0x24ef6385dc8>
```



2. Suppose that you wanted to perform polynomial regression for 2-dimensional data points using gradient descent, i.e., you want to fit a polynomial of degree k to your data. Explain how to do this using feature vectors. What is the per iteration complexity of gradient descent as a function of the size of your feature representation and the number of training data points?

Feature vectors can be used to increase/decrease the dimensions of the data points which then can be used to fit the polynomial

$$a_k * x^k + a_{k-1} * x^{k-1} + \ldots + a_0$$

Let n be the number of input dimensions ($n = 1$ in this case), m be the number of data points and k be the degree of the polynomial

Per Iteration time complexity of gradient descent = Computing Loss + Computing (k+1) gradients

Computing Loss = $O(m(k+1))$

Computing kth gradient = $O(m(k+1))$

Computing k+1 gradients = $O(m(k+1)^2)$

Per Iteration Complexity = $O(m(k+1) + m(k+1)^2) = O(m(k+1)^2)$