Problem 2: Medical Diagnostics

2. Now, suppose that the hypothesis space consists of only height 1 decision trees for this data set (only one attribute split).

In [1]:
```
import numpy as np
import pandas as pd
from matplotlib import pyplot as plt
import math
from tqdm.notebook import tqdm, trange

%matplotlib inline
```

In [2]:
```
heart_train = pd.read_csv('heart_train.data', header=None)
heart_test = pd.read_csv('heart_test.data', header=None)

# Changing 0 class to -1 to predict using Sign function
heart_train.loc[heart_train[0] == 0, 0] = -1
heart_test.loc[heart_test[0] == 0, 0] = -1

# Split X, Y
y_train, X_train = heart_train.iloc[:, 0], heart_train.iloc[:, 1:]
y_test, X_test = heart_test.iloc[:, 0], heart_test.iloc[:, 1:]

attributes = X_train.columns
m, n = X_train.shape
classes = y_train.unique()
```

In [3]:
```
def generate_1_attr_hypotheses(attributes, classes):
    hypotheses = []
    for l0 in attributes:
        for leaf1 in classes:
            for leaf2 in classes:
                h = {}
                h[l0] = {}
                h[l0][0] = leaf1
                h[l0][1] = leaf2
                hypotheses.append(h)
    return hypotheses

H = generate_1_attr_hypotheses(attributes, classes)
assert len(attributes) * (2**2) == len(H)
```

(a) Use coordinate descent to minimize the exponential loss function for this hypothesis space over the training set. You can use any initialization and iteration order that you would like other than the one selected by adaBoost. What is the optimal value of α that you arrived at? What is the corresponding value of the exponential loss on the training set?

In [10]:
```
def predict(X, h):
    for l0 in h.keys():
        val = X[l0]
        pred = h[l0][val]
        if not isinstance(pred, dict):
            return pred
        else:
            for l1 in pred.keys():
                val = X[l1]
                return pred[l1][val]


def boosting_predict(a, H=None, x=None, h_x=None):
    if h_x is None:
        h_x = []
        for h in H:
            y_pred = x.apply(lambda row: predict(row, h), axis=1)
            h_x.append(y_pred)
        h_x = np.array(h_x)
    return np.sign(a.dot(h_x))

def accuracy(y_truth, y_pred):
    return np.mean(y_truth == y_pred) * 100

def coordinate_descent(y_train, ht_x, H):
    T = len(H)
    alphas = np.array([1/len(y_train)] * T)
    alpha_change = 1
    changes = []
    iter_counter = 0
    print("Change in alpha: ")
    while alpha_change > 0.01:
        alpha_change = 0
        for t_prime in range(T):
            tmp = np.array(alphas[:])
            tmp[t_prime] = 0
            y_t_prime = np.array(ht_x[t_prime])
            y_t = np.array(ht_x)
            mask = (y_train == y_t_prime)
            loss_n = np.sum(mask * np.exp(-y_train * tmp.dot(y_t)))
            loss_d = np.sum((~mask) * np.exp(-y_train * tmp.dot(y_t)))
            alpha_t_prime = 0.5 * np.log(loss_n / loss_d)
            old_alpha = alphas[t_prime]
            alphas[t_prime] = alpha_t_prime
            alpha_change += abs(alpha_t_prime - old_alpha)

        iter_counter += 1
        changes.append(alpha_change)
        if iter_counter % 10 == 0:
            print(f"Iteration {iter_counter} - Alpha difference {alpha_change}
")
    return alphas
```

In [5]:
```
ht_x = []
for h in H:
    y_pred = X_train.apply(lambda row: predict(row, h), axis=1)
    ht_x.append(y_pred)
ht_x = np.array(ht_x)
```

In [6]:

```
alphas = coordinate_descent(y_train, ht_x, H)
```

```
Change in alpha:
Iteration 10 - Alpha difference 0.7157898613298839
Iteration 20 - Alpha difference 0.42098805160270647
Iteration 30 - Alpha difference 0.2939106042233337
Iteration 40 - Alpha difference 0.2284805512187042
Iteration 50 - Alpha difference 0.18791210504579767
Iteration 60 - Alpha difference 0.16070233814860924
Iteration 70 - Alpha difference 0.14117305359578078
Iteration 80 - Alpha difference 0.1261143700744089
Iteration 90 - Alpha difference 0.11411099531368328
Iteration 100 - Alpha difference 0.10429088814982893
Iteration 110 - Alpha difference 0.0962353070727448
Iteration 120 - Alpha difference 0.0894655342628396
Iteration 130 - Alpha difference 0.08362998089641584
Iteration 140 - Alpha difference 0.078494531195247
Iteration 150 - Alpha difference 0.07393380636194959
Iteration 160 - Alpha difference 0.06985807115686238
Iteration 170 - Alpha difference 0.06619514489473755
Iteration 180 - Alpha difference 0.06288632447929013
Iteration 190 - Alpha difference 0.05988340000652787
Iteration 200 - Alpha difference 0.057146430430905296
Iteration 210 - Alpha difference 0.05464206043598378
Iteration 220 - Alpha difference 0.05234223013969443
Iteration 230 - Alpha difference 0.05022317458762325
Iteration 240 - Alpha difference 0.04826464000896416
Iteration 250 - Alpha difference 0.04644926419446315
Iteration 260 - Alpha difference 0.04476208248854688
Iteration 270 - Alpha difference 0.043190130861454804
Iteration 280 - Alpha difference 0.04172212467613327
Iteration 290 - Alpha difference 0.040348196957291534
Iteration 300 - Alpha difference 0.03905968378805347
Iteration 310 - Alpha difference 0.03784894729647631
Iteration 320 - Alpha difference 0.03670922882298508
Iteration 330 - Alpha difference 0.03563452647125516
Iteration 340 - Alpha difference 0.03461949247496876
Iteration 350 - Alpha difference 0.0336593467585343
Iteration 360 - Alpha difference 0.03274980380248303
Iteration 370 - Alpha difference 0.031887010494950255
Iteration 380 - Alpha difference 0.031067493098874348
Iteration 390 - Alpha difference 0.03028811181774985
Iteration 400 - Alpha difference 0.029546021723503995
Iteration 410 - Alpha difference 0.02883863903344302
Iteration 420 - Alpha difference 0.028163611903392855
Iteration 430 - Alpha difference 0.027518795047877527
Iteration 440 - Alpha difference 0.0269022276161656
Iteration 450 - Alpha difference 0.026312113847743623
Iteration 460 - Alpha difference 0.025746806108606128
Iteration 470 - Alpha difference 0.025204789974121775
Iteration 480 - Alpha difference 0.0246846710760464
Iteration 490 - Alpha difference 0.02418516347550771
Iteration 500 - Alpha difference 0.02370507935928504
Iteration 510 - Alpha difference 0.023243319887367102
Iteration 520 - Alpha difference 0.02279886704423604
Iteration 530 - Alpha difference 0.022370776368255876
Iteration 540 - Alpha difference 0.02195817045043697
Iteration 550 - Alpha difference 0.021560233109601278
Iteration 560 - Alpha difference 0.02117620416310958
```

```
Iteration 570 - Alpha difference 0.0208053747235735
Iteration 580 - Alpha difference 0.020447082960660442
Iteration 590 - Alpha difference 0.020100710275471025
Iteration 600 - Alpha difference 0.01976567784128206
Iteration 610 - Alpha difference 0.01944144347052017
Iteration 620 - Alpha difference 0.019127498772555897
Iteration 630 - Alpha difference 0.018823366571482116
Iteration 640 - Alpha difference 0.018528598556364025
Iteration 650 - Alpha difference 0.01824277314029608
Iteration 660 - Alpha difference 0.017965493506464175
Iteration 670 - Alpha difference 0.017696385823023783
Iteration 680 - Alpha difference 0.017435097609485313
Iteration 690 - Alpha difference 0.017181296240336916
Iteration 700 - Alpha difference 0.016934667572251857
Iteration 710 - Alpha difference 0.016694914683472895
Iteration 720 - Alpha difference 0.016461756714582325
Iteration 730 - Alpha difference 0.016234927801490433
Iteration 740 - Alpha difference 0.016014176092104662
Iteration 750 - Alpha difference 0.01579926283911641
Iteration 760 - Alpha difference 0.01558996156223242
Iteration 770 - Alpha difference 0.015386057273613541
Iteration 780 - Alpha difference 0.015187345761147344
Iteration 790 - Alpha difference 0.014993632924560945
Iteration 800 - Alpha difference 0.014804734159756906
Iteration 810 - Alpha difference 0.014620473787648849
Iteration 820 - Alpha difference 0.014440684523415383
Iteration 830 - Alpha difference 0.014265206983194535
Iteration 840 - Alpha difference 0.014093889225009345
Iteration 850 - Alpha difference 0.01392658632128007
Iteration 860 - Alpha difference 0.013763159960329582
Iteration 870 - Alpha difference 0.013603478074670014
Iteration 880 - Alpha difference 0.013447414494060159
Iteration 890 - Alpha difference 0.01329484862132563
Iteration 900 - Alpha difference 0.013145665129250717
Iteration 910 - Alpha difference 0.012999753677035706
Iteration 920 - Alpha difference 0.0128570086447306
Iteration 930 - Alpha difference 0.012717328884498402
Iteration 940 - Alpha difference 0.012580617487369236
Iteration 950 - Alpha difference 0.012446781564327684
Iteration 960 - Alpha difference 0.012315732040907197
Iteration 970 - Alpha difference 0.012187383464082477
Iteration 980 - Alpha difference 0.012061653820892965
Iteration 990 - Alpha difference 0.011938464367680637
Iteration 1000 - Alpha difference 0.011817739469449803
Iteration 1010 - Alpha difference 0.011699406448565448
Iteration 1020 - Alpha difference 0.011583395442160358
Iteration 1030 - Alpha difference 0.011469639267728043
Iteration 1040 - Alpha difference 0.011358073296274142
Iteration 1050 - Alpha difference 0.011248635332594181
Iteration 1060 - Alpha difference 0.011141265502260847
Iteration 1070 - Alpha difference 0.01103590614474099
Iteration 1080 - Alpha difference 0.010932501712402709
Iteration 1090 - Alpha difference 0.010830998675014407
Iteration 1100 - Alpha difference 0.010731345429284897
Iteration 1110 - Alpha difference 0.010633492213238704
Iteration 1120 - Alpha difference 0.010537391025190398
Iteration 1130 - Alpha difference 0.010442995546799907
```

```
Iteration 1140 - Alpha difference 0.010350261070250971
Iteration 1150 - Alpha difference 0.010259144429061019
Iteration 1160 - Alpha difference 0.010169603932511661
Iteration 1170 - Alpha difference 0.010081599303341014
Iteration 1180 - Alpha difference 0.009995091618474112
```

In [7]:
```
import pickle
with open('coor-alpha.pkl', 'wb') as pkl:
    pickle.dump(alphas, pkl)
```

In [11]:
```
print("Alpha values")
print(alphas)
loss = np.sum(np.exp(-y_train * alphas.dot(ht_x)))

print("************************************************************")
print("Training Loss: ", loss)
print("************************************************************")
```

```
Alpha values
[ 0.0125      2.56450499  0.0125      0.9397913   0.0125      0.49712744
  0.0125      0.42087416  0.0125      5.33319822  0.0125      3.43956813
  0.0125     -0.80077938  0.0125     -0.51476228  0.0125     -0.30639216
  0.0125     -0.12937515  0.0125     -0.11811744  0.0125     -0.09058471
  0.0125     -0.59497221  0.0125     -0.30993912  0.0125     -4.91762638
  0.0125     -3.30379305  0.0125      3.63608266  0.0125      2.81664809
  0.0125     -2.40589643  0.0125     -0.90478602  0.0125     -0.37833946
  0.0125     -0.24624989  0.0125      0.46849362  0.0125      0.29120756
  0.0125     -0.78119306  0.0125     -0.3731185   0.0125     -3.07731466
  0.0125     -2.35307728  0.0125     -1.21925251  0.0125     -1.19535567
  0.0125     -0.72422825  0.0125     -0.57953009  0.0125     -3.1952759
  0.0125     -3.13311246  0.0125     -2.37589849  0.0125     -2.35623757
  0.0125     -0.03859159  0.0125     -0.02098391  0.0125     -0.31266444
  0.0125     -0.2022478   0.0125     -0.15147537  0.0125     -0.10043005
  0.0125     -0.36479571  0.0125     -0.16285177]
************************************************************
Training Loss:  39.475146659581576
************************************************************
```

(b) What is the accuracy of the resulting classifier on the test data?

In [12]:
```
coor_pred = boosting_predict(alphas, H=H, x=X_test)
print("************************************************************")
print("Coordinate Test Accuracy", accuracy(y_test.ravel(), coor_pred.flatten
()))
print("************************************************************")
```

```
************************************************************
Coordinate Test Accuracy 69.5187165775401
************************************************************
```

(c) What is the accuracy of adaBoost after 20 rounds for this hypothesis space on the test data?

In [38]:
```
def adaboost(H, X_train, y_train):
    m, n = X_train.shape
    w = np.array([1/m] * m)
    alphas = [0] * T
    epsilons = [0] * T
    selected_H = [None] * T
    y_predictions = [None] * T
    best_idxs = []
    print("Running Adaboost")
    for t in range(T):
        e_t = 1
        h_t = None
        y_t = None
        best_i = 0
        h_i = 0
        tq = tqdm(H)
        tq.set_description(f"Round {t+1}")
        for h in tq:
            h_i += 1
            y_pred = X_train.apply(lambda row: predict(row, h), axis=1)
            mask = (y_pred != y_train).astype(np.float64)
            e_h = np.sum(mask * w)
            if e_h < e_t:
                e_t = e_h
                h_t = h
                y_t = y_pred
                best_i = h_i
        print(f"Round {t+1} - Best hypothesis index {best_i}")
        best_idxs.append(best_i)
        selected_H[t] = h_t
        y_predictions[t] = y_t
        epsilons[t] = e_t

        a_t = 0.5 * math.log((1-e_t)/e_t) # Log base e
        alphas[t] = a_t

        # Weight update
        normalize = 2 * np.sqrt(e_t * (1-e_t))
        w = w * np.exp(-1 * y_train * y_t * a_t)/normalize
    return np.array(alphas), np.array(epsilons), selected_H, np.array(y_predic
tions), best_idxs

T = 20
a, e, h_, y_, idxs = adaboost(H, X_train, y_train)
```

```
Running Adaboost

Round 1 - Best hypothesis index 51

Round 2 - Best hypothesis index 43

Round 3 - Best hypothesis index 1

Round 4 - Best hypothesis index 27

Round 5 - Best hypothesis index 1

Round 6 - Best hypothesis index 31

Round 7 - Best hypothesis index 10

Round 8 - Best hypothesis index 87

Round 9 - Best hypothesis index 1

Round 10 - Best hypothesis index 63

Round 11 - Best hypothesis index 1

Round 12 - Best hypothesis index 79

Round 13 - Best hypothesis index 10

Round 14 - Best hypothesis index 31

Round 15 - Best hypothesis index 1

Round 16 - Best hypothesis index 43

Round 17 - Best hypothesis index 10

Round 18 - Best hypothesis index 15

Round 19 - Best hypothesis index 1

Round 20 - Best hypothesis index 67
```

In [39]:
```
pred = boosting_predict(a, H=h_, x=X_test)
print("************************************************************")
print("AdaBoost Test Accuracy", accuracy(y_test.ravel(), pred.flatten()))
print("************************************************************")
```

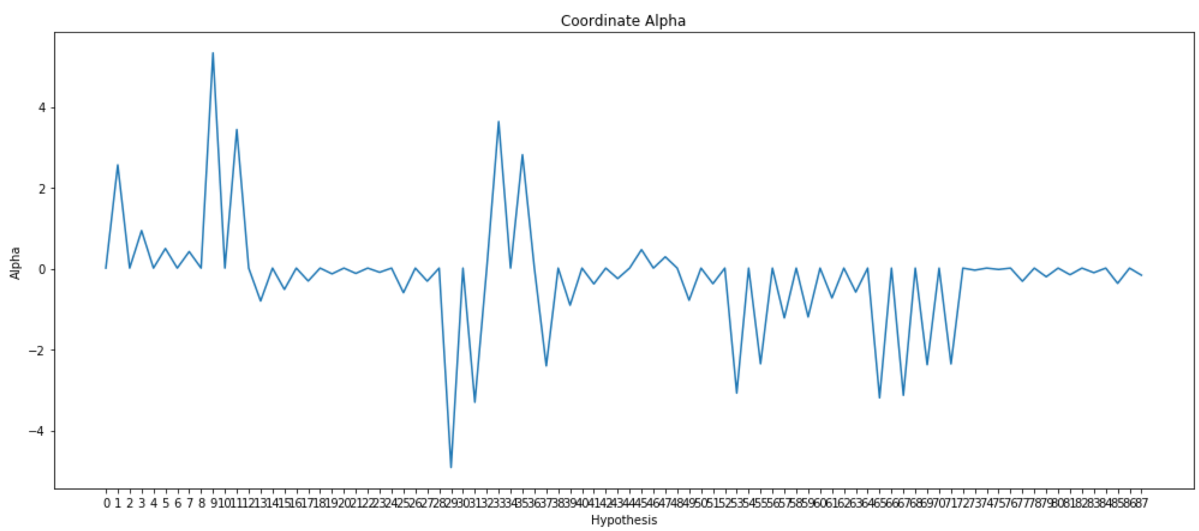
```
************************************************************
AdaBoost Test Accuracy 66.84491978609626
************************************************************
```

(c) How does the α learned by adaBoost compare to the one learned by coordinate descent/gradient descent?

α values from adaboost are positive and slowly converge, whereas in the case of coordinate descent α values are eratic with contribution of all the hypothesis. Even for best hypotheses picked by Adaboost, α values of both methods vary diversely.

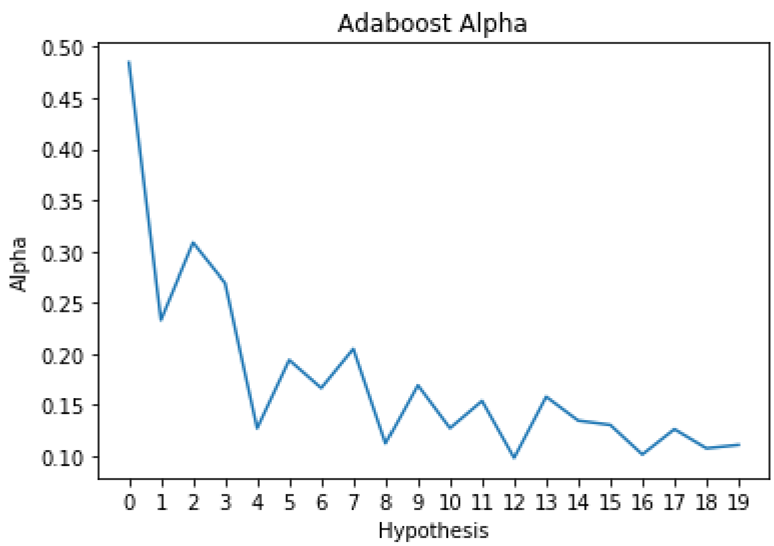
In [90]:
```
coordinate_alpha = alphas

plt.figure(figsize=(17,7))
plt.title("Coordinate Alpha")
plt.xlabel("Hypothesis")
plt.ylabel("Alpha")
plt.xticks(range(len(coordinate_alpha)))
plt.plot(range(len(coordinate_alpha)), coordinate_alpha)
```

Out[90]: [<matplotlib.lines.Line2D at 0x1e94985d388>]



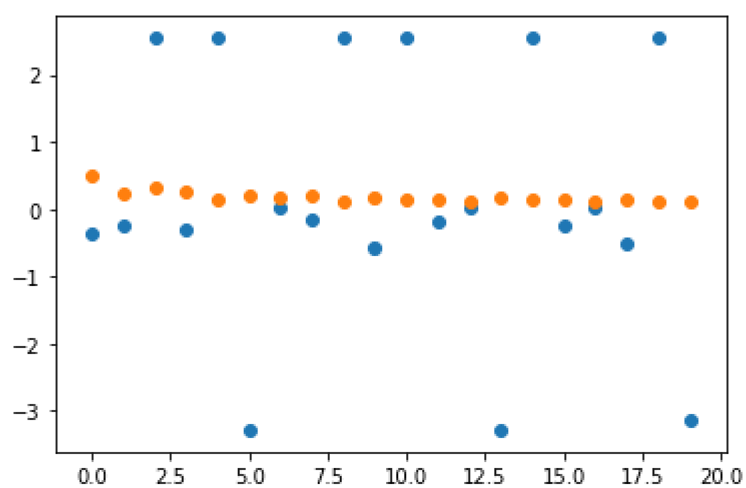
In [81]:
```python
adaboost_alpha = a
plt.title("Adaboost Alpha")
plt.xlabel("Hypothesis")
plt.ylabel("Alpha")
plt.xticks(range(len(adaboost_alpha)))
plt.plot(range(len(adaboost_alpha)), adaboost_alpha)
```

Out[81]:
```
[<matplotlib.lines.Line2D at 0x1e948d9d7c8>]
```



In [97]:
```python
plt.scatter(range(len(coordinate_alpha[idxs])), coordinate_alpha[idxs])
plt.scatter(range(len(adaboost_alpha)), adaboost_alpha)
```

Out[97]:
```
<matplotlib.collections.PathCollection at 0x1e949f5ae08>
```



(d) Use bagging, with 20 bootstrap samples, to produce an average classifier for this data set. How does it compare to the previous classifiers in terms of accuracy on the test set?

In [64]:

```python
def fit_decision_stump(data, attributes):
    m, n = data.shape
    best_accuracy = 0
    best_h = 0
    for h in H:
        y_pred = data.apply(lambda row: predict(row, h), axis=1)
        acc = accuracy(data[0].ravel(), y_pred.ravel())
        if acc > best_accuracy:
            best_accuracy = acc
            best_h = h
    return best_h
```

In [65]:

```python
B = 20
# Random forest
T = []
attrs = attributes.to_list()
for b in range(B):
    bootstrap_sample = heart_train.iloc[np.random.randint(m, size=m)]
#     T_b, best_split = fit_decision_stump_ig(bootstrap_sample, attrs)
    T_b = fit_decision_stump(bootstrap_sample, attrs)
    T.append(T_b)
```

In [66]:

```python
from collections import defaultdict

def stump_predict(T_b, x_row):
    for root in T_b.keys():
        val = x_row[root]
        return T_b[root][val]

def random_forest_predict(T, data):
    m, n = data.shape
    y_pred = np.array([0] * m)
    for i in range(m):
        row = data.loc[i, :]
        preds = defaultdict(int)
        for t_b in T:
            preds[stump_predict(t_b, row)] += 1
        if preds[-1] > preds[1]:
            y_pred[i] = -1
        else:
            y_pred[i] = 1
    return y_pred

test_accuracy = accuracy(random_forest_predict(T, X_test), y_test.ravel())

print("***********************************************************")
print("Bagging Test accuracy", test_accuracy)
print("***********************************************************")
```

```
***********************************************************
Bagging Test accuracy 61.49732620320856
***********************************************************
```

(e) Which of these 3 methods should be preferred for this data set and why

Adaboost and Coordinate descent methods can be used for this data set with this hypothesis space. Decision stumps are not expressive, so bagging would not be much useful for improving the accuracy.