Problem 2

$$\min \quad \frac{1}{2}\|w\|^2 + c\sum_i \xi_i^2$$

st.

$$y^{(i)}\left(w x^{(i)} + b\right) \geq 1 - \xi_i$$

$$1 - \xi_i - y_{(i)}\left(w x^{(i)} + b\right) \leq 0$$

$$L(w, b, \xi, \lambda) = \frac{1}{2}\|w\|^2 + c\sum_i \xi_i^2 + \sum_i \lambda_i \left(1 - \xi_i - y_i \left(w x^{(i)} + b\right)\right)$$

$$= \frac{1}{2}\left[w^T w\right] + c\sum_i \xi_i^2 + \sum_i \lambda_i \left(1 - \xi_i - y^{(i)}\left(w^T x^{(i)} + b\right)\right)$$

$$\frac{\partial L}{\partial w_k} = w_k - \sum_i \lambda_i y^{(i)} x^{(i)} = 0$$

$$w_k = \sum_i \lambda_i y^{(i)} x^{(i)}$$

$$\frac{\partial L}{\partial b} = -\sum_i \lambda_i y^{(i)} = 0$$

$$\sum_i \lambda_i y^{(i)} = 0$$

$$\frac{\partial L}{\partial \xi_k} = 2c\xi_k + (-\lambda_k) = 0$$

$$\xi_k = \frac{\lambda_k}{2c}$$

$$\xi_k^2 = \frac{\lambda_k^2}{4c^2}$$

Dual :

$$G(\lambda) = \frac{1}{2}\sum_i\sum_j \lambda_i \lambda_j y^{(i)} y^{(j)} \langle x^{(i)} x^{(j)} \rangle$$

$$+ \sum_i \frac{\lambda_i^2}{4C}$$

$$+ \sum_i \lambda_i \left(1 - \frac{\lambda_i}{2C} - y_i \left(\left(\sum_j \lambda_j y_j x_j \right) x_i + b \right) \right)$$

$$= \frac{1}{2}\sum_i\sum_j \lambda_i \lambda_j y^{(i)} y^{(j)} x^{(i)} x^{(j)} + \sum_i \cancel{\frac{\lambda_i^2}{4C}} + \left(\sum - \frac{\lambda_i^2}{2C} \right)$$

$$+ \sum_i \lambda_i \cancel{- \sum \frac{\lambda_i^2}{4C}} - \sum_i\sum_j \lambda_i \lambda_j y_i y_j x_i x_j + 0$$

$$G(\lambda) = -\frac{1}{2}\sum_i\sum_j \lambda_i \lambda_j y_i y_j x_i x_j + \sum_i \lambda_i - \sum_i \frac{\lambda_i^2}{2C}$$

Kernel Trick can be applied as $\langle x_i, x_j \rangle$ is present in the dual.