Axiom of extension: Two sets are equal if and only if they have the same elements.

Axiom of specification: To every set A and to every condition $S(x)$ there corresponds a set B whose elements are exactly those elements x of A for which $S(x)$ holds.

↳ "condition", that is, a sentence specifying the elements in A that satisfies the conditions of S.

Example: $B = \{x \in A : S(x)\}$

Axiom of pairing: For any two sets there exists a set that they both belong to.

- A **collection** is a gathering of distints objects, or elements, with a well-defined property or condition

Axiom of unions: For every collection of sets there exists a set that contains all the elements that belong to at least one set of the given collection.

Example: $U = \{x : x \in X$ for some X in $\mathcal{C}\}$ ↳ union

Notation: $\bigcup\{X : X \in \mathcal{C}\}$ or $\bigcup_{X \in \mathcal{C}} X$. ↳ collection

Example: $\bigcup\{X : X \in \{A, B\}\} = A \cup B$
$= \{x : x \in A \text{ or } x \in B\}$

- **Intersection** is defined as

$$A \cap B = \{x \in A : x \in B\} \quad \leftarrow \text{special case}$$
$$= \{x : x \in A \text{ and } x \in B\}$$

Furthermore, for each collection $\mathcal{C}$, other than $\emptyset$, there exists a set V such that $x \in V$ if and only if $x \in X$ for every X in $\mathcal{C}$, that is

$V = \{x \in A : x \in X \text{ for every } X \text{ in } \mathcal{C}\}$

The set V has a special notation

$\bigcap\{X : X \in \mathcal{C}\}$ or $\bigcap_{X \in \mathcal{C}} X$.

- Let A and B be sets, we say the relative complement of B in A, is the set

$$A - B := \{x \in A : x \notin B\}$$

not $\in$

- The symmetric difference (or Boolean sum) of A and B is the set

$$A + B = (A - B) \cup (B - A)$$

$\triangle$

Axiom of powers: For each set there exists a collection of sets that contains among its elements all the subsets of the given set.

Let E be the complement of $\emptyset$, then there exists a set $\mathcal{P}$ such that if $X \subset E$, then $X \in \mathcal{P}$, that is

Notation: $\mathcal{P}(E) = \{X : X \subset E\}$

Example: $\mathcal{P}(\{a\}) = \{\emptyset, \{a\}\}$

- An ordered pair is a set defined as

$$(a,b) = \{\{a\},\{a,b\}\}$$

Consequence: with this definition $(a,b) = (x,y)$ if and only if $a = x$ and $b = y$. Notice that, if (a,b) is a set the equality holds for $\{\{a\},\{a,b\}\} = \{\{x\},\{x,y\}\}$

$\{a\} = \{x\} \Rightarrow a = x$

$\{a,b\} = \{x,y\} \Rightarrow b = y$, for $a = x$

- The cartesian product of A and B is a set defined as

$$A \times B = \{x : x = (a,b) \text{ for some } a \text{ in } A \text{ and for some } b \text{ in } B\}$$

- A relation is a set of ordered pairs

Notation: $x R y$, such that $(x,y) \in R$, with R being the set of ordered pairs.

- The domain is a set defined as

$\text{dom } R = \{x : \text{for some } y \ (xRy)\}$

- The range is a set defined as

$\text{ran } R = \{y : \text{for some } x \ (xRy)\}$

- A relation on X that is
  1. Reflexive $xRx, \forall x \in X$
  2. Symmetric $xRy \Rightarrow yRx, \forall x,y \in X$
  3. Transitive $(xRy) \wedge (yRz) \Rightarrow (xRz), \forall x,y,z \in X$

  is called an equivalence relation

- A partition of X is a disjoint collection $\mathcal{C}$ of non-empty subsets of X whose union is X.

- If X and Y are sets, a function from (on) X to (into) Y is a relation f such that $\text{dom } f = X$ and such that for each x in X there is a unique element y in Y with $(x,y) \in f$.

$f(x) = y \rightsquigarrow$ value that f assumes (takes on) argument x

Synonyms: map, mapping, transformation, correspondence, operator

* $f: X \longrightarrow X$ is an identity map.

- Let X be a subset of Y and f a function from Y to Z, we can construct a function $g: X \longrightarrow Z$, such that $g(x) = f(x)$ for every x in X. g is a restriction of f to X

Natural Numbers

Definition $0 = \emptyset$, $1 = \{\emptyset\}$ and $2 = \{\emptyset, \{\emptyset\}\}$, that is, 1 is $\{0\}$ and 2 is $\{0,1\}$

- If A is a subset of a set X, the caracteristic function of A is the function χ from X to 2 such that

$$\chi(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \in X - A \end{cases}$$

Notation: χ_A

- Let I and X be two sets and x a function from I to X.

This function is called a family.

$i \in I$ is an index

X is an indexed set

Notation: x_i

$\Leftrightarrow x(i)$ is a function

"a family $\{A_i\}$ of subsets of X" refers to a function A from I to $\mathcal{P}(X)$

For $x \in (\bigcup_i A_i) \cap (\bigcup_i B_j)$

$x \in (\bigcup_i A_i)$ and $x \in (\bigcup_j B_j)$

for some x, x is in A_i and B_j, for some $i \in I$ and $j \in J$

So, $x \in A_i \cap B_j$ as well as $x \in \bigcup_{i,j} (A_i \cap B_j)$