Concise definitions

Is it problematic to say $\{v_1, \dots, v_n\}$ is in V? so it isn't a set

Spans

Let $\{v_1, \dots, v_n\}$ be a set of vectors in vector space V. We say that $\{v_1, \dots, v_n\}$ spans V, if there exist scalars $c_1, \dots, c_n$ in F, such that for any v in V

$$v = c_1 v_1 + \dots + c_n v_n$$

* $\text{Span}(\{v_1, \dots, v_n\}) = \{c_1 v_1 + \dots + c_n v_n : v_1, \dots, v_n \in V \text{ and } c_1, \dots, c_n \in F\}$, that is, it is the set of all possible linear combinations of vectors $\{v_1, \dots, v_n\}$.

Linear combination

Given a set of vector $\{v_1, \dots, v_n\}$ in the vector space V, a linear combination is a vector, which can be obtained by multiplying each vector in the set by a scalar and then summing the results. It means that, there exist v in V, such that

$$v = c_1 \cdot v_1 + \dots + c_n v_n$$

Subspace

A subspace U of V is a set, such that U is a subset of V that uses the same addition and scalar multiplication of V and

1. $0 \in U$
2. closed under addition
3. closed under scalar multiplication

Linear independence

Let $\{v_1, \dots, v_n\}$ be a set of vectors in V. We call it linear independent if a linear combination $v = \alpha_1 v_1 + \dots + \alpha_n v_n$ is equal 0 only when $\alpha_1 = \dots = \alpha_n = 0$.

$\{v_1, \dots, v_n\}$ spans V

$\underline{\alpha_1} v_1 + \dots + \underline{\alpha_n} v_n = 0$ $\quad \alpha_j v_j$

$v_j = \beta_1 v_1 + \dots + \beta_n v_n$

$v_1 = \alpha_1 v_1$

$v_2 =$

Theorem the set of non-zero vectors $x_1, \dots, x_n$ is linear dependent if and only if some x_n, $2 \leqslant k \leqslant n$ is a linear combination of the preceding ones.