Dimension

Let V be a vector space over a field F. Let $U \subseteq V$ be finite. Let U be a basis of V. Define a funcion $\dim \in U^{\mathbb{N}}$, such that $\dim U := |U|$.

$1.1 \in S$

$1.1 := \max\{[n]\} = n$

$(1, a), (2, b)$

?

Basis

Let V be a vector space over a field F. Let $U \subseteq V$ be a finite dimension. If U is lin. ind. and $\operatorname{span} U = V$, then U is a basis of V.

Isomorphism

...

Subspaces

Let V be a vector space over F. Let U be a non-empty set. If $0 \in U$ and $\forall \alpha \in F$ and $\forall v \in U$, $\alpha \cdot v \in U$ and $\forall u, v \in U$ $u + v \in U$, then U is a subspace of V.

$0 \in U$

$\forall \alpha, v \in U \quad \alpha \cdot v \in U$

$\forall u, v \in U \quad \alpha + v \in U$

If U is a plane or a line that passes through the origin $(0 \in U)$, then $\bigcap_{U \subseteq V} U = 0$?

$\hookrightarrow \neq 0$, if $U_1 \subseteq V$ and $U_2 \subseteq V$ are equal, but then they are the same set.

Theorem: The intersection of any collection of subspaces is a subspace.

Let V be a vector space over F. If $\{U_v\}$ is a set of subspaces of V, then $\bigcap_{U \subseteq V} U_v$ is a subspace.

- The intersection reduces the "dimensions"? *

1. $0 \in \bigcap U_v$ ✓ trivial
2. $\forall \alpha \in F$, $\forall v \in \bigcap U_v$, $\alpha . v \in \bigcap U_v$?
3. $\forall u, v \in \bigcap U_v$, $u + v \in \bigcap U_v$?

for any vector v if $v \in U_1$ and $v \in U_2$, by the definition, $\forall \alpha \in F$, $\alpha . v \in U_1$ as well as $\alpha . v \in U_2$

If $\bigcap U_v = \{0\}$, it is also a subspace

* **Theorem:** A subspace M in an n-dimensional vector space V is a vector space of dimension $\leq n$.

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Theorem: Let V be a vector space over a field F. Let B and B' be bases of V. Then $|B| = |B'|$

B and B' linearly independent and span V

$$\mathrm{span}(B) = \left\{ \sum_{v \in B} \alpha(v) v : \alpha \in F^{v} \right\} = V$$

$$\mathrm{span}(B') = \left\{ \sum_{v \in B'} \beta(v) v : \beta \in F^{v} \right\}$$

$|B| \neq |B'|$, w.l.o.g. $B, B' \subseteq V$

$|B| < |B'| \Rightarrow \exists v \in B', v \notin B' \Rightarrow$ since B' is lin. ind. $v \notin \mathrm{span}(B)$, a contradiction

Let B and B' be two bases of a vector space V over a field F.

$B, B' \subseteq V$

B and B' are lin. ind.

B span V and B' span V

Suppose, by contradiction, $|B| \neq |B'|$

Without loose of generality, if $|B| < |B'|$. Then, $\exists v \in B'$ and $v \notin B$ ~~at least~~ one element more than B

Since B' is lin. ind., $v \notin \mathrm{span}(B)$

However $v \in V$ and B span V, then $v \in \mathrm{span}\, B$, a contradiction.

Hence, $|B| = |B'|$

Let V be a vector space over a field F.
Let $\mathcal{B} := \{B \subseteq V : B \text{ is a basis of } V\}$, then $|\{|B| : B \in \mathcal{B}\}| = 1$

Let B and B' be two basis of V.