แบบฝึกหัด 5 INTERPOLATION AND EXTRAPOLATION II

1. กำหนดตารางความสัมพันธ์ระหว่างค่า x และ y ได้ดังตาราง

f(x)
↓

จุดที่	x	y
1	0	9.81
2	20,000	9.7487
3	40,000	9.6879
4	60,000	9.6879
5	80,000	9.5682

จงหาค่า y เมื่อ x = 42,000 ด้วยวิธี Lagrange Interpolation พร้อมเขียน code ดังต่อไปนี้

1.1 LINEAR INTERPOLATION (2 จุด จุดที่ 1, 5)

1.2 QUADRATIC INTERPOLATION (3 จุด จุดที่ 1, 3, 5)

1.3 POLYNOMIAL INTERPOLATION (5 จุด จุดที่ 1, 2, 3, 4, 5)

2. จงจัดรูปให้ Lagrange LINEAR INTERPOLATION L_0, L_1 มีรูปแบบ สมการตามนี้

$$f(x) = L_0(x)f(x_0) + L_1(x)f(x_1)$$

where $L_0(x) = \frac{x_1 - x}{x_1 - x_0}$ and $L_1(x) = \frac{x_0 - x}{x_0 - x_1}$

จุดที่	x	y
1	0	9.81
2	20,000	9.7487
3	40,000	9.6879
4	60,000	9.6879
5	80,000	9.5682

จงหาค่า y เมื่อ x = 42,000 ด้วยวิธี Lagrange Interpolation

1.1 LINEAR INTERPOLATION (2 จุด จุดที่ 1, 5)

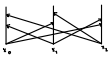
$f(x) = L_0(x)\, f(x_0) + L_1(x)\, f(x_1)$

$L_0 = \left(\frac{80000 - 42000}{80000 - 0}\right)(9.81) + \left(\frac{0 - 42000}{0 - 80000}\right)(9.6830559)$

$= 9.6830559$

1.2 QUADRATIC INTERPOLATION (3 จุด จุดที่ 1, 3, 5) point = [{x: 0, 40000, 80000}, {y: 9.81, 9.6879, 9.5682}]

$f(x) = L_0(x)\, f(x_0) + L_1(x)\, f(x_1) + L_2(x)\, f(x_2)$

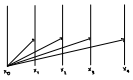


$\frac{(x_1 - x)(x_2 - x)}{(x_1 - x_0)(x_2 - x_0)}(f(x_0)) + \frac{(x_0 - x)(x_2 - x)}{(x_0 - x_1)(x_2 - x_1)}(f(x_1)) + \frac{(x_0 - x)(x_1 - x)}{(x_0 - x_2)(x_1 - x_2)}(f(x_2))$

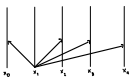
$= \underbrace{\frac{(40000 - 42000)(80000 - 42000)}{(40000 - 0)(80000 - 0)}(9.81)}_{-0.0237180} + \underbrace{\frac{(0 - 42000)(80000 - 42000)}{(0 - 40000)(80000 - 40000)}(9.6879)}_{0.4977000} + \underbrace{\frac{(0 - 42000)(40000 - 42000)}{(0 - 80000)(40000 - 80000)}(9.5682)}_{0.0263150}$

$= 9.6796626$

1.3 POLYNOMIAL INTERPOLATION (5 จุด จุดที่ 1, 2, 3, 4, 5) point = [{x: 0, 20000, 40000, 60000, 80000}, {y: 9.81, 9.7487, 9.6879, 9.6879, 9.5682}]

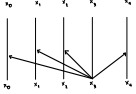


$L_0 = \frac{(x_1 - x)(x_2 - x)(x_3 - x)(x_4 - x)}{(x_1 - x_0)(x_2 - x_0)(x_3 - x_0)(x_4 - x_0)}$ $\frac{(20000 - 42000)(40000 - 42000)(60000 - 42000)(80000 - 42000)}{(20000 - 0)(40000 - 0)(60000 - 0)(80000 - 0)} = 0.0097838$

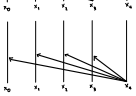


$L_1 = \frac{(x_0 - x)(x_2 - x)(x_3 - x)(x_4 - x)}{(x_0 - x_1)(x_2 - x_1)(x_3 - x_1)(x_4 - x_1)}$ $\frac{(0 - 42000)(40000 - 42000)(60000 - 42000)(80000 - 42000)}{(0 - 20000)(40000 - 20000)(60000 - 20000)(80000 - 20000)} = -0.0594850$

$L_2 = \frac{(x_0 - x)(x_1 - x)(x_3 - x)(x_4 - x)}{(x_0 - x_2)(x_1 - x_2)(x_3 - x_2)(x_4 - x_2)}$ $\frac{(0 - 42000)(20000 - 42000)(60000 - 42000)(80000 - 42000)}{(0 - 40000)(20000 - 40000)(60000 - 40000)(80000 - 40000)} = 0.4977525$



$L_3 = \frac{(x_0 - x)(x_1 - x)(x_2 - x)(x_4 - x)}{(x_0 - x_3)(x_1 - x_3)(x_2 - x_3)(x_4 - x_3)}$ $\frac{(0 - 42000)(20000 - 42000)(40000 - 42000)(80000 - 42000)}{(0 - 60000)(20000 - 60000)(40000 - 60000)(80000 - 60000)} = 0.0793150$



$L_4 = \frac{(x_0 - x)(x_1 - x)(x_2 - x)(x_3 - x)}{(x_0 - x_4)(x_1 - x_4)(x_2 - x_4)(x_3 - x_4)}$ $\frac{(0 - 42000)(20000 - 42000)(40000 - 42000)(60000 - 42000)}{(0 - 80000)(20000 - 80000)(40000 - 80000)(60000 - 80000)} = -0.0098663$

$= 0.0097838\,(9.81) + -0.0594850\,(9.7487) + 0.4977525\,(9.6879) + 0.0793150\,(9.6879) + (-0.0098663)(9.5682)$

$= 9.6962255$

```
const math = require("mathjs");

function lagrange_interpolation(xy,x){
    let l = math.clone(xy);
    //where Li(x)
    for(let i=0;i<math.size(xy);i++){
        let temp = 1;
        for(let j=0;j<math.size(xy);j++){
            if(j!=i){
                temp *= (x - xy[j].x) / (xy[i].x - xy[j].x);
            }
        }
        l[i].x = temp;
    }

    let y = math.clone(xy);
    let temp = 0;
    //General Form f(x)
    for(let i=0;i<math.size(xy);i++){
        temp += l[i].x * xy[i].y;
    }

    return temp;
}

let x = 42000;

let point = [
    {x: 0, y: 9.81},
    {x: 20000, y: 9.7487},
    {x: 40000, y: 9.6879},
    {x: 60000, y: 9.6879},
    {x: 80000, y: 9.5682}
]

let linear_interpolation = [
    {x: point[0].x, y: point[0].y},
    {x: point[4].x, y: point[4].y}
]

let quadratic_interpolation = [
    {x: point[0].x, y: point[0].y},
    {x: point[2].x, y: point[2].y},
    {x: point[4].x, y: point[4].y}
]

let polynomial_interpolation = [
    {x: point[0].x, y: point[0].y},
    {x: point[1].x, y: point[1].y},
    {x: point[2].x, y: point[2].y},
    {x: point[3].x, y: point[3].y},
    {x: point[4].x, y: point[4].y}
]

console.log("linear interpolation = "+lagrange_interpolation(linear_interpolation,x).toFixed(6))
console.log("quadratic interpolation = "+lagrange_interpolation(quadratic_interpolation,x).toFixed(6))
console.log("polynomial interpolation = "+lagrange_interpolation(polynomial_interpolation,x).toFixed(6))
```