# Prep Work 2

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## Exercise 0.2 18

**Presentation:** Yes

**Solution Draft:** Suppose $P(x)$ is some predicate for which the statement $\forall x P(x)$ is true. Is it also the case that $\exists x P(x)$ is true? In other words, is the statement $\forall x P(x) \to \exists x P(x)$ always true? Is the converse always true? Assume the domain of discourse is non-empty.

$\forall x P(x) \to \exists x P(x)$ is always true. If for all $x$, $P(x)$ exists, then there must also be a $x$ that exists where $P(x)$ is true.

## Exercise 0.2 19

**Presentation:** Include here whether you'd be willing to present this one.

**Solution Draft:**
For each of the statements below, give a domain of discourse for which the statement is true, and a domain for which the statement is false.

a. $\forall x \exists y (y^2 = x)$

   For all positive real number $x$, $y$ exists.

   For negative real number $x$, there are no $y$s that exist that can satisfy $(y^2 = x)$

b. $\forall x \forall y (x < y \to \exists z (x < z < y))$

   For all numbers between $x$ and $y$, $z$ exists.

   When $x > y$, the statement is false.

c. $\exists x \forall y \forall z (y < z \to y \leq x \leq z)$

   For $x = y = z = 0$ the statement is true

   Including decimals, the statement is true.

   For only whole numbers, the statement is false because $y = 2$ and $z = 3$, there are no whole numbers for $x$ to exist.

## Exercise 0.3 Investigate!

**Solution Draft:**

1. Find the cardinality of each set below.

   (a) $A = \{3, 4, \ldots, 15\}$.

(b) $B = \{n \in \mathbb{N} : 2 < n \leq 200\}$.

(c) $C = \{n \leq 100 : n \in \mathbb{N} \wedge \exists m \in \mathbb{N}(n = 2m + 1)\}$.

2. Find two sets $A$ and $B$ for which $|A| = 5$, $|B| = 6$, and $|A \cup B| = 9$. What is $|A \cap B|$?

3. Find sets $A$ and $B$ with $|A| = |B|$ such that $|A \cup B| = 7$ and $|A \cap B| = 3$. What is $|A|$?

4. Let $A = \{1, 2, \ldots, 10\}$. Define $B_2 = \{B \subseteq A : |B| = 2\}$. Find $|B_2|$.

5. For any sets $A$ and $B$, define $AB = \{ab : a \in A \wedge b \in B\}$. If $A = \{1, 2\}$ and $B = \{2, 3, 4\}$, what is $|AB|$? What is $|A \times B|$?

**Answers**

1. The cardinality of each set:

   (a) $|A| = 13$ (counting from 3 to 15).

   (b) $|B| = 198$ (counting from 3 to 200).

   (c) $|C| = 50$ (all odd numbers less than or equal to 100).

2. For sets $A$ and $B$ where $|A| = 5$, $|B| = 6$, and $|A \cup B| = 9$, the intersection cardinality $|A \cap B|$ is 2.

3. For sets $A$ and $B$ with $|A| = |B|$ such that $|A \cup B| = 7$ and $|A \cap B| = 3$, the cardinality $|A|$ (or $|B|$) is 5.

4. For set $A = \{1, 2, \ldots, 10\}$ and $B_2$ being the 2-element subsets of $A$, the cardinality $|B_2|$ is 45.

5. For sets $A = \{1, 2\}$ and $B = \{2, 3, 4\}$, the cardinality of the product set $|AB|$ is 5 and the cardinality of the Cartesian product $|A \times B|$ is 6.

**Presentation:** Not this one