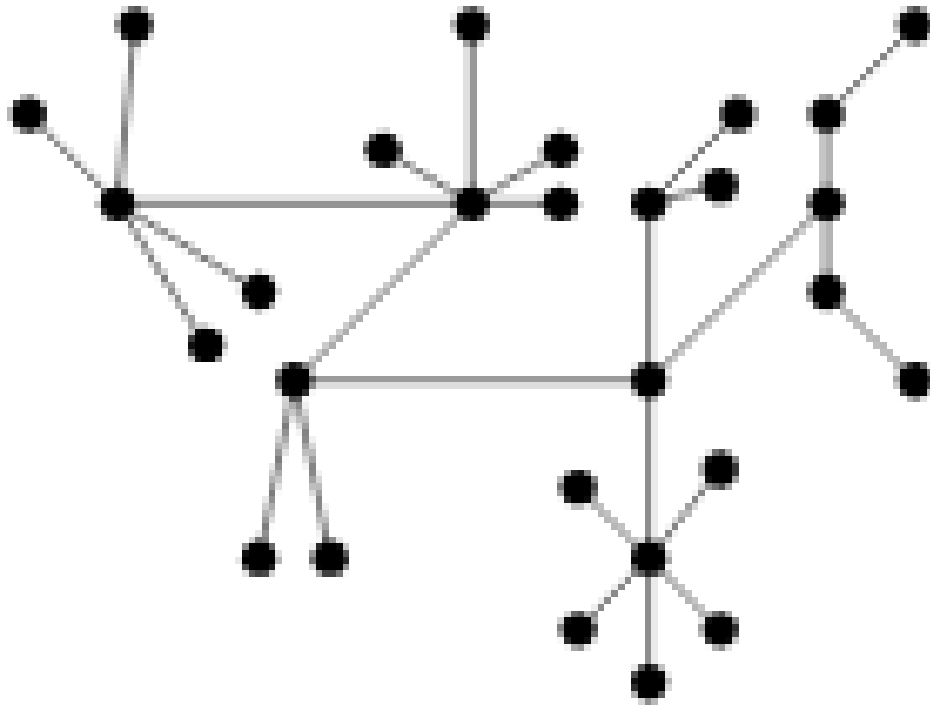# Prep Work 17

Xander

Apr 16

Presentation:

Exercise 6 4.4

Prove the chromatic number of any tree is two. Recall, a tree is a connected graph with no cycles.

(a) Describe a procedure to color the tree below.

(b) The chromatic number of C_n is two when n is even. What goes wrong when n is odd?

(c) Prove that your procedure from part (a) always works for any tree.

(d) Now, prove using induction that every tree has chromatic number 2.
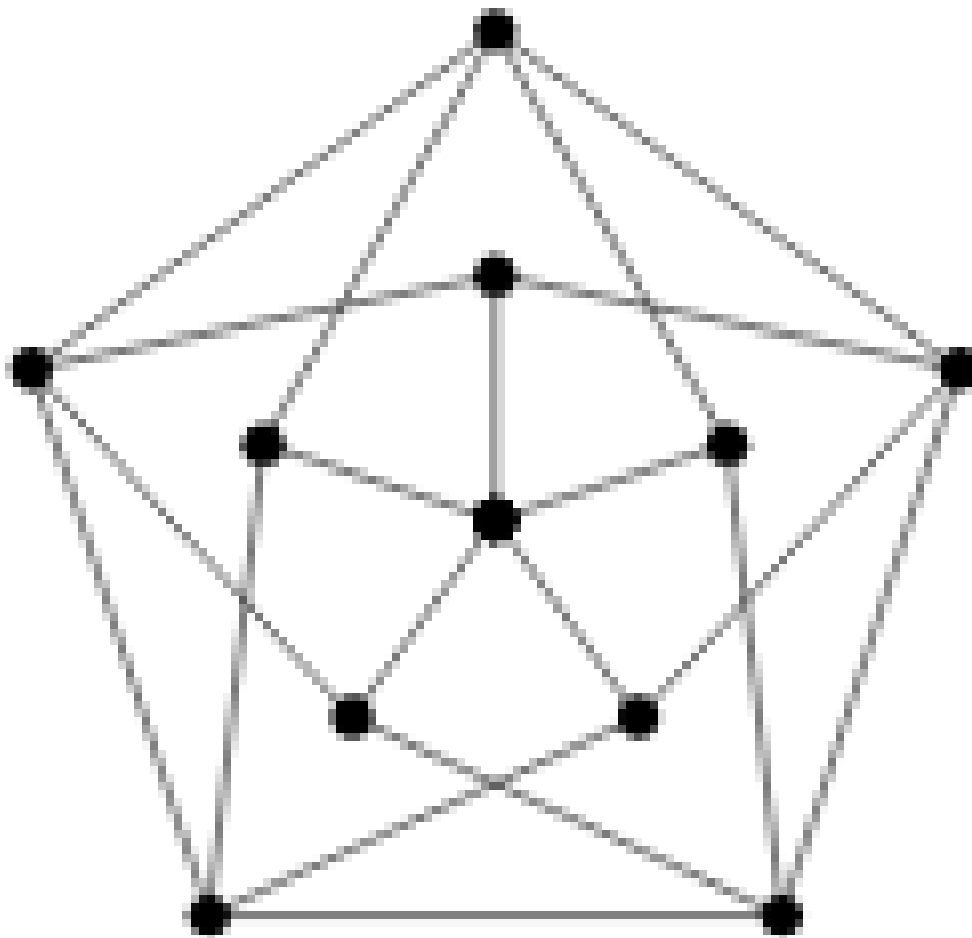
Solution Draft:

1. Start at one point, go to the next and switch colors, every other point connected to that once recieves the first color. Since this graph is a tree you will only need to use two colors

2. When n is odd, you will end up with two adjacent verticies of the same color since the cycles does not close evenly.

3. The algorithm stated above will work for all trees.

4. **Base Case** For a single vertex tree, the algorithm works since you will only need once color.

   Inductive Case Assume every tree with k verticies has a chromatic number of two. Consider a tree with $k + 1$ verticies. By removing a leaf, you have a tree with k verticies. No matter how many leafs you remove or add, the chromatic number will never change since it will never create a cycle.
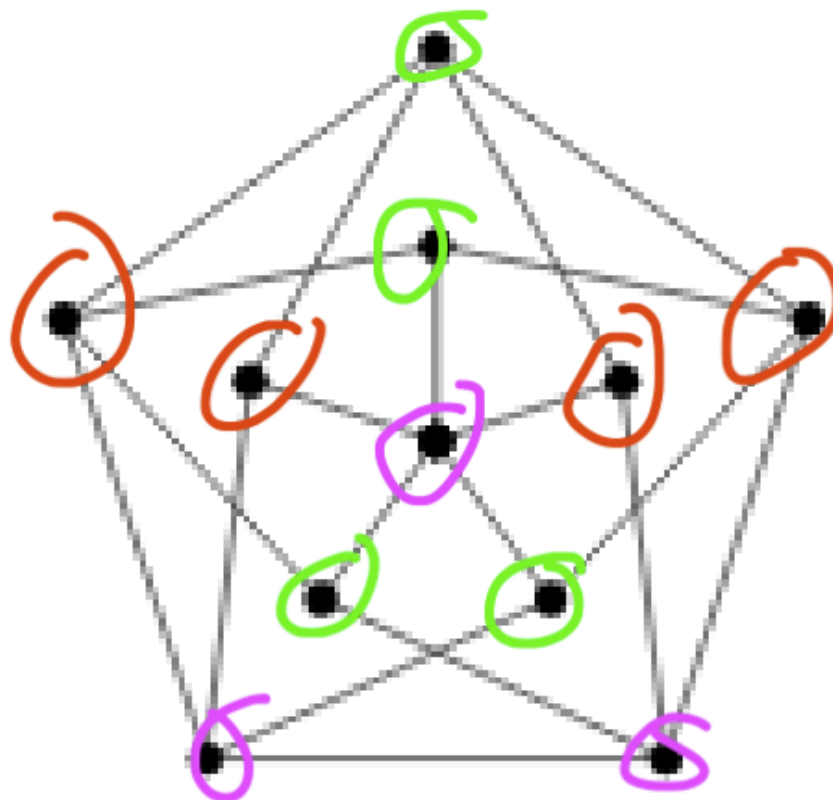
Exercise 10 4.4

Find the chromatic number of the graph below and prove you are correct.

Solution Draft:

   The chromatic number of the graph below is 3. You cannot use less colors than 3.

Size of Infinite Sets Problem

Determine which of the following sets are countable and which are uncountable.

a. $\{a, b, c\}$

b. All odd numbers in $\mathbb{N}$

c. $\mathbb{N}$

d. $\mathbb{Z}$

e. $\mathbb{R}$

Solution Draft:

a. Countable

b. Countable

c. Countable

d. Countable

e. Uncountable