Prep Work 3

Xander

Jan 30

Exercise 0.3 10

Presentation: Yes

Question Let $A = \{x \in 3 \leq x \leq 13\}$, $B = \{x \in x \text{ is even}\}$ is even, and $C = \{x \in x\}$ is odd.

1. Find $A \cap B$.
2. Find $A \cup B$.
3. Find $B \cap C$.
4. Find $B \cup C$.

Solution Draft:

1. $A \cap B = \{4, 6, 8, 10, 12\}$

   Since A is the set of integers from 3 to 13, and B is the set of all even integers, $A \cap B$ will be the even numbers from 4 to 12.

2. $A \cup B = A$

   Since A already includes both even and odd numbers from 3 to 13, and B includes all even numbers, $A \cup B$ will essentially be the same as the set of all integers because every integer is either within the range of A or is even.

3. $B \cap C = \emptyset$

   This set contains elements that are both even and odd, which is an impossibility because no integer can be both even and odd

4. $B \cup C = \{x \in x\}$

   Since every integer is either even or odd, the set contains all integers.

Exercise 0.3 24

Presentation: No

Question Let $X = \{n \in \mathbb{N} : 10 \leq n < 20\}$. Find examples of sets with the properties below and very briefly explain why your examples work.

a. A set $A \subseteq \mathbb{N}$ with $|A| = 10$ such that $X \setminus A = \{10, 12, 14\}$.

b. A set $B \in \mathcal{P}(X)$ with $|B| = 5$.

c. A set $C \subseteq \mathcal{P}(X)$ with $|C| = 5$.

d. A set $D \subseteq X \times X$ with $|D| = 5$

e. A set $E \subseteq X$ such that $|E| \in E$.

Solution Draft:

a. $A = \{11, 13, 15, 16, 17, 18, 19\}$.

This works because A contains 7 elements from X, and when you remove A from X, you're left with $\{10, 12, 14\}$, as required.

b. $B = \{10, 11, 12, 13, 14\}$.

B is a subset of X and has 5 elements, satisfying the condition $|B| = 5$.

c. $C = \{\{10\}, \{11\}, \{12\}, \{13\}, \{14\}\}$.

Each subset in C is an element of $\mathcal{P}(X)$, and there are 5 subsets, so $|C| = 5$.

d. $D = \{(10, 11), (11, 12), (12, 13), (13, 14), (14, 15)\}$.

This subset of $X \times X$ has 5 ordered pairs, so $|D| = 5$.

e. Set $E = \{11\}$. Here, $|E| = 1$ and $1 \notin X$, but since $11 \in X$ and $11 \in E$, the condition $|E| \in E$ is met.

Exercise 0.3 25

Presentation: No

Question 25. Let A, B and C be sets.

a. Suppose that $A \subseteq B$ and $B \subseteq C$. Does this mean that $A \subseteq C$? Prove your answer. Hint: to prove that $A \subseteq C$ you must prove the implication, "for all x, if $x \in A$ then $x \in C$".

b. Suppose that $A \in B$ and $B \in C$. Does this mean that $A \in C$? Give an example to prove that this does NOT always happen (and explain why your example works). You should be able to give an example where $|A| = |B| = |C| = 2$.

Solution Draft:

a. Yes, if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

We assume that $A \subseteq B$ and $B \subseteq C$, and we need to show that for any x, if $x \in A$ then $x \in C$. Since $A \subseteq B$, for any $x \in A$, it follows that $x \in B$. And since $B \subseteq C$, it must be that $x \in C$. Therefore, $A \subseteq C$.

b. No, $A \in B$ and $B \in C$ does not necessarily mean that $A \in C$.

Let $A = \{1, 2\}$, $B = \{\{1, 2\}, 3\}$, and $C = \{\{\{1, 2\}, 3\}, 4\}$. Here, $A \in B$ because A is one of the elements of B, and $B \in C$ because B is one of the elements of C. However, A is not an element of C; rather, A is a subset of one of the elements of C, which is B. Thus, $A \notin C$.

Exercise 0.3 29

Presentation: No

Question Explain why there is no set A which satisfies $A = \{2, |A|\}$.

Solution Draft:

Assuming such a set A exists, it must satisfy the condition that $A = \{2, |A|\}$. This means that the set A has two elements, namely 2 and the cardinality of A itself.

If A has two elements, then $|A| = 2$. But then A would have to be $\{2, 2\}$, which simplifies to $\{2\}$ since sets do not account for duplicate elements, and hence $|A|$ would be 1, not 2. Thus, A cannot simultaneously have two distinct elements and have its cardinality as one of its elements. Therefore, there is no such set A that satisfies the given condition.

Investiage!

Presentation: No

Question 4 Let $A = \{1, 2, \ldots, 10\}$. Define $B_2 = \{B \subseteq A : |B| = 2\}$. Find $|B_2|$.

Solution Draft:

The set $A = \{1, 2, \ldots, 10\}$ contains 10 elements. The number of 2-element subsets of A is given by the binomial coefficient $\binom{10}{2}$, which represents the number of ways to choose 2 elements from 10 without regard to order. This can be calculated as:

$$\binom{10}{2} = \frac{10!}{2!(10-2)!} = \frac{10 \times 9}{2 \times 1} = 45.$$

Therefore, $|B_2| = 45$.

Presentation: No

Question 5 For any sets A and B, define $AB = \{ab : a \in A \wedge b \in B\}$. If $A = \{1, 2\}$ and $B = \{2, 3, 4\}$, what is $|AB|$? What is $|A \times B|$?

Solution Draft:

Given $A = \{1, 2\}$ and $B = \{2, 3, 4\}$, the set AB is defined as $AB = \{ab : a \in A \wedge b \in B\}$. Thus, we have:

$$AB = \{1 \times 2, 1 \times 3, 1 \times 4, 2 \times 2, 2 \times 3, 2 \times 4\} = \{2, 3, 4, 4, 6, 8\} = \{2, 3, 4, 6, 8\}.$$

Therefore, $|AB| = 5$.

The Cartesian product $A \times B$ is the set of all ordered pairs where the first element comes from A and the second element comes from B. Thus:

$$A \times B = \{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4)\}.$$

Therefore, $|A \times B| = 6$.