Prep Work 2

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Exercise 0.2 18

Presentation: Yes

Solution Draft: Suppose $P(x)$ is some predicate for which the statement $\forall x P(x)$ is true. Is it also the case that $\exists x P(x)$ is true? In other words, is the statement $\forall x P(x) \to \exists x P(x)$ always true? Is the converse always true? Assume the domain of discourse is non-empty.

$\forall x P(x) \to \exists x P(x)$ is always true. If for all x, $P(x)$ exists, then there must also be a x that exists where $P(x)$ is true.

Exercise 0.2 19

Presentation: Include here whether you'd be willing to present this one.

Solution Draft:
For each of the statements below, give a domain of discourse for which the statement is true, and a domain for which the statement is false.

a. $\forall x \exists y (y^2 = x)$

   For all positive real number x, y exists.

   For negative real number x, there are no ys that exist that can satisfy $(y^2 = x)$

b. $\forall x \forall y (x < y \to \exists z (x < z < y))$

   For all numbers between x and y, z exists.

   When $x > y$, the statement is false.

c. $\exists x \forall y \forall z (y < z \to y \leq x \leq z)$

   For $x = y = z = 0$ the statement is true

   Including decimals, the statement is true.

   For only whole numbers, the statement is false because $y = 2$ and $z = 3$, there are no whole numbers for x to exist.

Exercise 0.3 Investigate!

Questions

1. Find the cardinality of each set below.

   (a) $A = \{3, 4, \ldots, 15\}$.

(b) $B = \{n \in \mathbb{N} : 2 < n \leq 200\}$.

(c) $C = \{n \leq 100 : n \in \mathbb{N} \wedge \exists m \in \mathbb{N}(n = 2m + 1)\}$.

2. Find two sets A and B for which $|A| = 5$, $|B| = 6$, and $|A \cup B| = 9$. What is $|A \cap B|$?
3. Find sets A and B with $|A| = |B|$ such that $|A \cup B| = 7$ and $|A \cap B| = 3$. What is $|A|$?
4. Let $A = \{1, 2, \ldots, 10\}$. Define $B_2 = \{B \subseteq A : |B| = 2\}$. Find $|B_2|$.
5. For any sets A and B, define $AB = \{ab : a \in A \wedge b \in B\}$. If $A = \{1, 2\}$ and $B = \{2, 3, 4\}$, what is $|AB|$? What is $|A \times B|$?

Solution Draft:

1. The cardinality of each set:
   (a) $|A| = 13$
   (b) $|B| = 198$
   (c) $|C| = 50$
2. For sets A and B where $|A| = 5$, $|B| = 6$, and $|A \cup B| = 9$, the intersection cardinality $|A \cap B|$ is 2.
3. For sets A and B with $|A| = |B|$ such that $|A \cup B| = 7$ and $|A \cap B| = 3$, the cardinality $|A|$ (or $|B|$) is 5.
4. For set $A = \{1, 2, \ldots, 10\}$ and B_2 being the 2-element subsets of A, the cardinality $|B_2|$ is 45.
5. For sets $A = \{1, 2\}$ and $B = \{2, 3, 4\}$, the cardinality of the product set $|AB|$ is 5 and the cardinality of the Cartesian product $|A \times B|$ is 6.

Presentation: Not this one