

# Coimisiún na Scrúduithe Stáit State Examinations Commission

# Leaving Certificate Examination, 2012 Sample Paper

# Mathematics (Project Maths – Phase 1)

Paper 2

Higher Level

Time: 2 hours, 30 minutes

300 marks

Examination number	For exa	For examiner							
	Question	Mark							
	1								
	2								
	3								
Centre stamp	4								
	5								
	6								
	7								
	8								
Running total	Total								

Grade

# There are **two** sections in this examination paper. Section A 150 marks Concepts and Skills 6 questions 150 marks Section B Contexts and Applications 2 questions Answer all eight questions, as follows: In Section A, answer: Ouestions 1 to 5 and either Question 6A or Question 6B. In Section B, answer Question 7 and Question 8. Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part. The superintendent will give you a copy of the booklet of Formulae and Tables. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answers should include the appropriate units of measurement, where relevant.

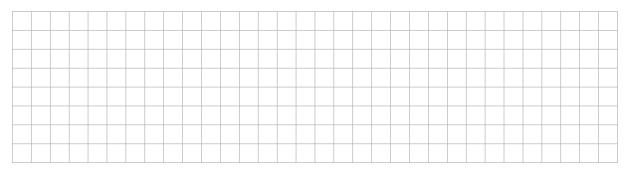
**Instructions** 

Answer all six questions from this section.

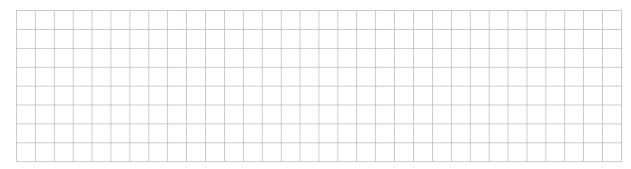
Question 1 (25 marks)

The events A and B are such that P(A) = 0.7, P(B) = 0.5 and  $P(A \cap B) = 0.3$ .

(a) Find  $P(A \cup B)$ 



**(b)** Find P(A|B)



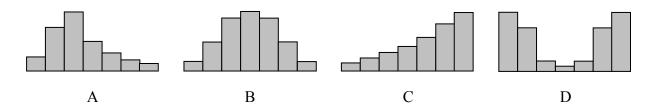
(c) State whether A and B are independent events, and justify your answer.



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Question 2 (25 marks)

The shapes of the histograms of four different sets of data are shown below.



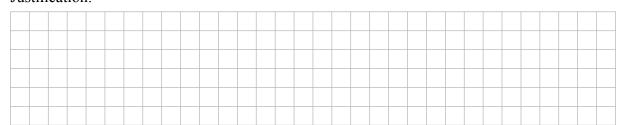
(a) Complete the table below, indicating whether the statement is correct (✓) or incorrect (✗) with respect to each data set.

	A	В	С	D
The data are skewed to the left				
The data are skewed to the right				
The mean is equal to the median				
The mean is greater than the median				
There is a single mode				

(b) Assume that the four histograms are drawn on the same scale. State which of them has the largest standard deviation, and justify your answer.

Answer:

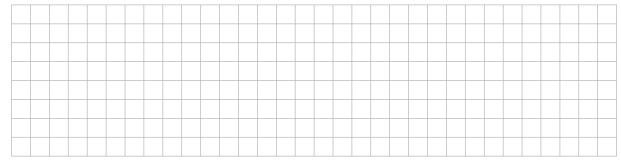
Justification:



Question 3 (25 marks)

The co-ordinates of three points A, B, and C are: A(2, 2), B(6, -6), C(-2, -3). (See diagram on facing page.)

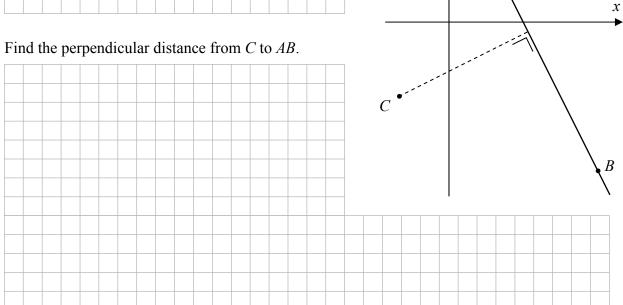
(a) Find the equation of AB.



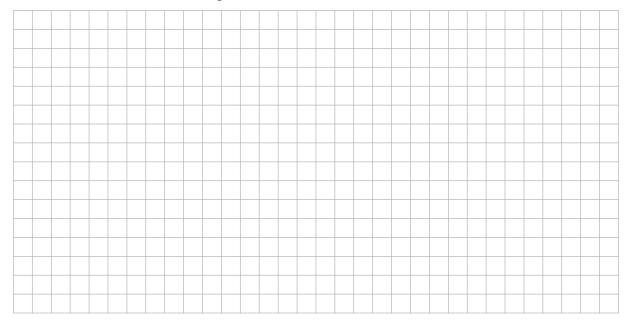
The line AB intersects the y-axis at D. **(b)** Find the coordinates of D.



**(c)** 



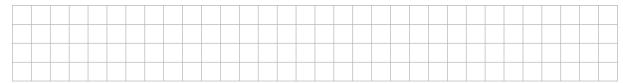
Hence, find the area of the triangle ADC. (d)



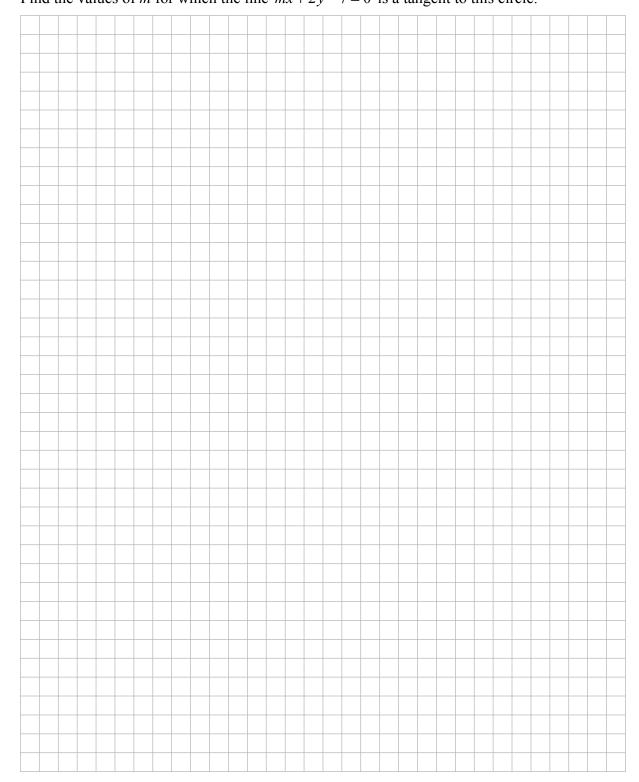
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Question 4 (25 marks)

(a) Write down the equation of the circle with centre (-3, 2) and radius 4.



**(b)** A circle has equation  $x^2 + y^2 - 2x + 4y - 15 = 0$ . Find the values of m for which the line mx + 2y - 7 = 0 is a tangent to this circle.



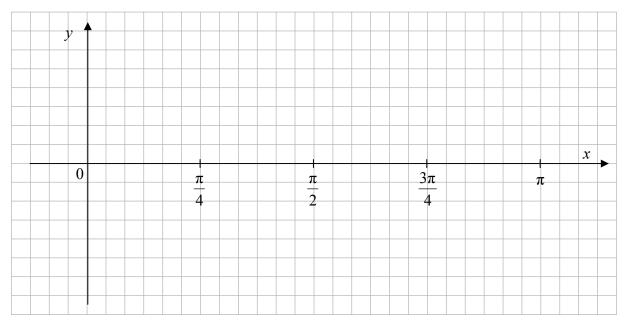
Question 5 (25 marks)

The function  $f: x \mapsto 3\sin(2x)$  is defined for  $x \in \mathbb{R}$ .

(a) Complete the table below

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
2x					
$\sin(2x)$					
$3\sin(2x)$					

**(b)** Draw the graph of y = f(x) in the domain  $0 \le x \le \pi$ ,  $x \in \mathbb{R}$ .



(c) Write down the range and the period of f.

Range =

Period =

Question 6 (25 marks)

Answer either 6A or 6B.

#### **Question 6A**

Explain, with the aid of an example, what is meant by proof by contradiction.

Note: you do not need to provide the full proof involved in your example. Give sufficient outline to illustrate how contradiction is used.

## Explanation:



## Example:



#### OR

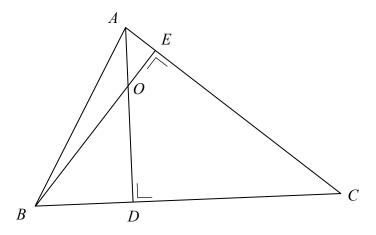
### **Question 6B**

ABC is a triangle.

D is the point on BC such that  $AD \perp BC$ . E is the point on AC such that  $BE \perp AC$ .

AD and BE intersect at O.

Prove that  $|\angle DOC| = |\angle DEC|$ .





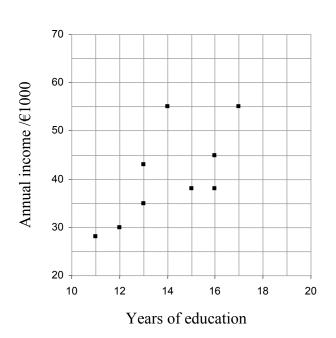
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Answer Question 7 and Question 8.

Question 7 (75 marks)

(a) An economics student wants to find out whether the length of time people spend in education affects the income they earn. The student carries out a small study. Twelve adults are asked to state their annual income and the number of years they spent in full-time education. The data are given in the table below, and a partially completed scatter plot is given.

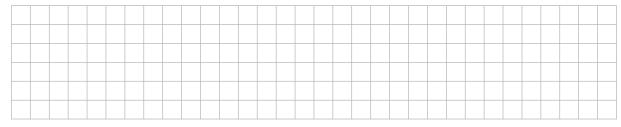
Years of	Income
education	/€1,000
11	28
12	30
13	35
13	43
14	55
15	38
16	45
16	38
17	55
17	60
17	30
19	58



- (i) The last three rows of data have not been included on the scatter plot. Insert them now.
- (ii) Calculate the correlation coefficient.

Answer:

(iii) What can you conclude from the scatter plot and the correlation coefficient?



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- (iv) Add the line of best fit to the completed scatter plot above.
- (v) Use the line of best fit to estimate the annual income of somebody who has spent 14 years in education.

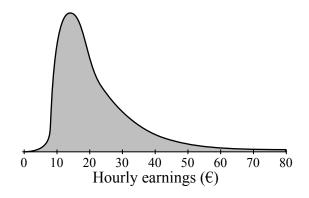
Leaving Certificate 2012 – Sample Paper

Answer:

viii) Explain how to interpret this slope in this context.    Viii) The student collected the data using a telephone survey. Numbers were randomly chosen from the Dublin area telephone directory. The calls were made in the evenings between 7 and 9 pm. If there was no answer, or if the person who answered did not agree to participate, then another number was chosen at random.  List three possible problems regarding the sample and how it was collected that might make the results of the investigation unreliable. In each case, state clearly why the issu you mention could cause a problem.  roblem 1:  roblem 2:	(vi)				g s est	uita fit.	able	e re	adi	ing	s fr	om	ı yo	our	dia	agra	am	, or	ot	her	wis	se,	cal	cul	ate	the	e sl	оре	e of	th	e
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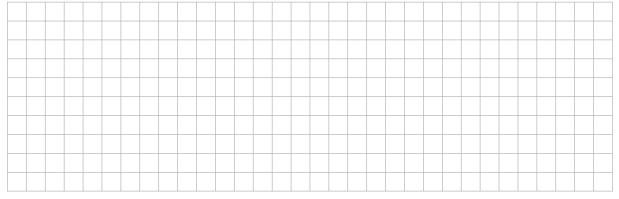
(b) The distribution of the hourly earnings of all employees in Ireland in October 2009 is shown in the diagram. It can be seen that the distribution is positively skewed.

The mean is €22·05. The median is €17·82. The standard deviation is €10·64. The lower quartile is €12·80. The upper quartile is €26·05.



(Source: adapted from: CSO. National Employment Survey 2008 and 2009)

(i) If six employees are selected at random from this population, what is the probability that exactly four of them had hourly earnings of more than €12·80?



In a computer simulation, random samples of size 200 are repeatedly selected from this population and the mean of each sample is recorded. A thousand such sample means are recorded.

(ii) Describe the expected distribution of these sample means. Your description should refer to the shape of the distribution and to its mean and standard deviation.



(iii) How many of the sample means would you expect to be greater than €23?

Question 8 (75 marks)

(a) Two surveyors want to find the height of an electricity pylon. There is a fence around the pylon that they cannot cross for safety reasons. The ground is inclined at an angle. They have a clinometer (for measuring angles of elevation) and a 100 metre tape measure. They have already used the clinometer to determine that the ground is inclined at 10° to the horizontal.

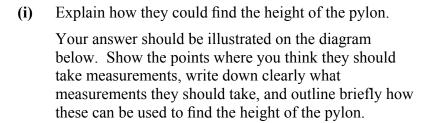
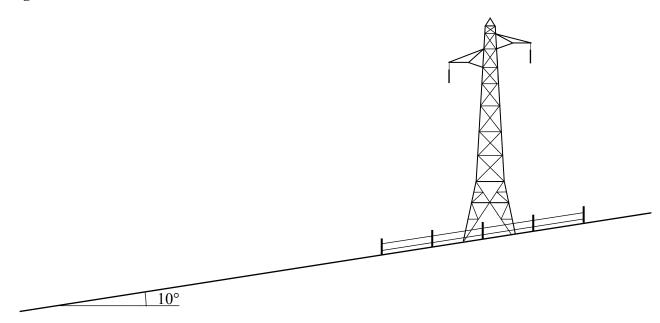
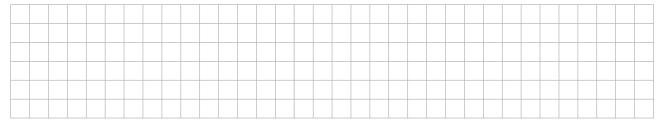




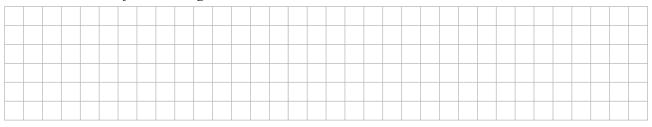
Diagram:



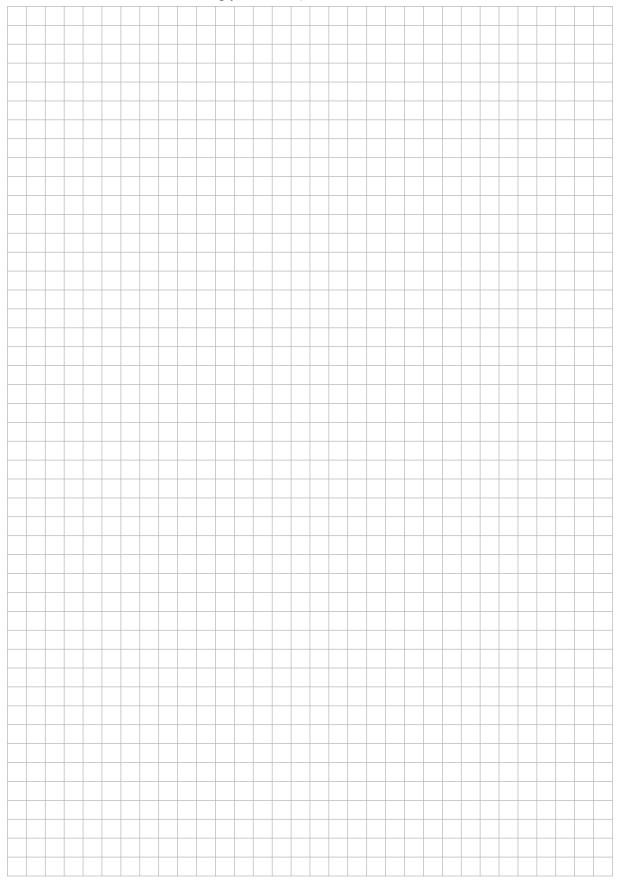
Measurements to be taken:



Procedure used to find the height:

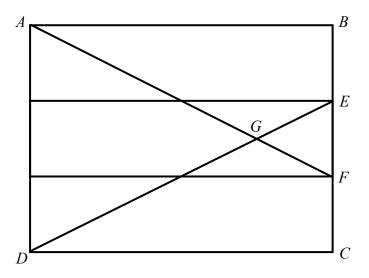


(ii) Write down possible values for the measurements taken, and use them to show how to find the height of the pylon. (That is, find the height of the pylon using your measurements, and showing your work.)



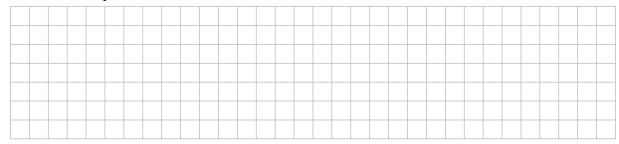
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**(b)** Anne is having a new front gate made and has decided on the design below.



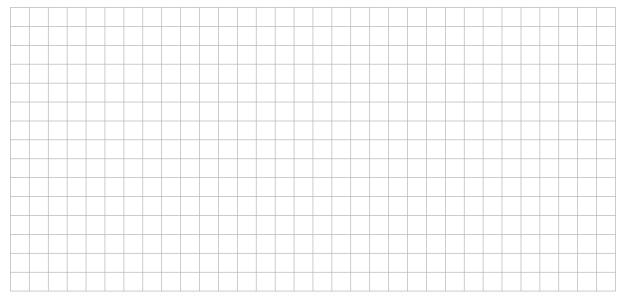
The gate is 2 metres wide and 1.5 metres high. The horizontal bars are 0.5 metres apart.

(i) Calculate the common length of the bars [AF] and [DE], in metres, correct to three decimal places.

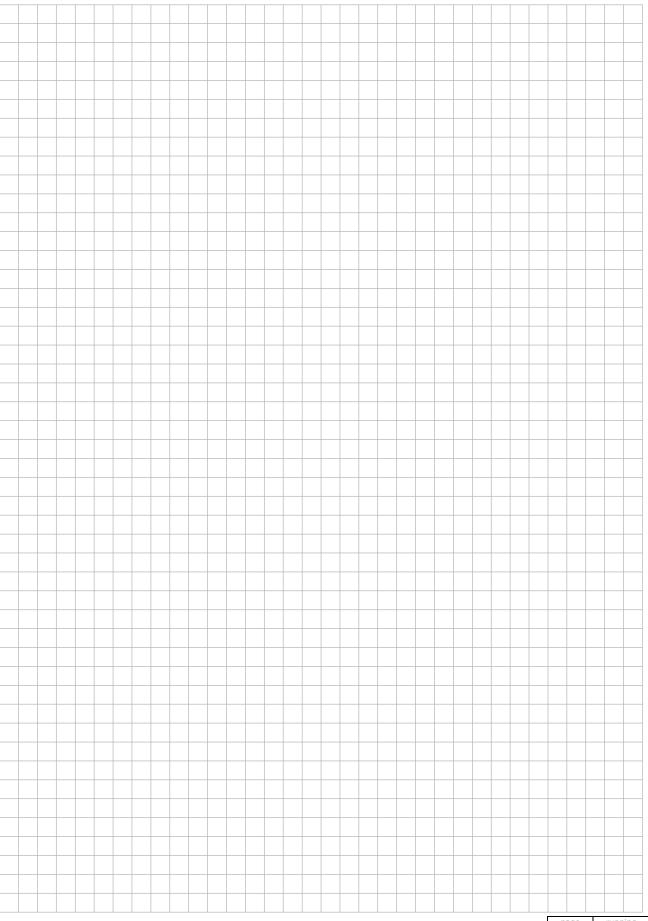


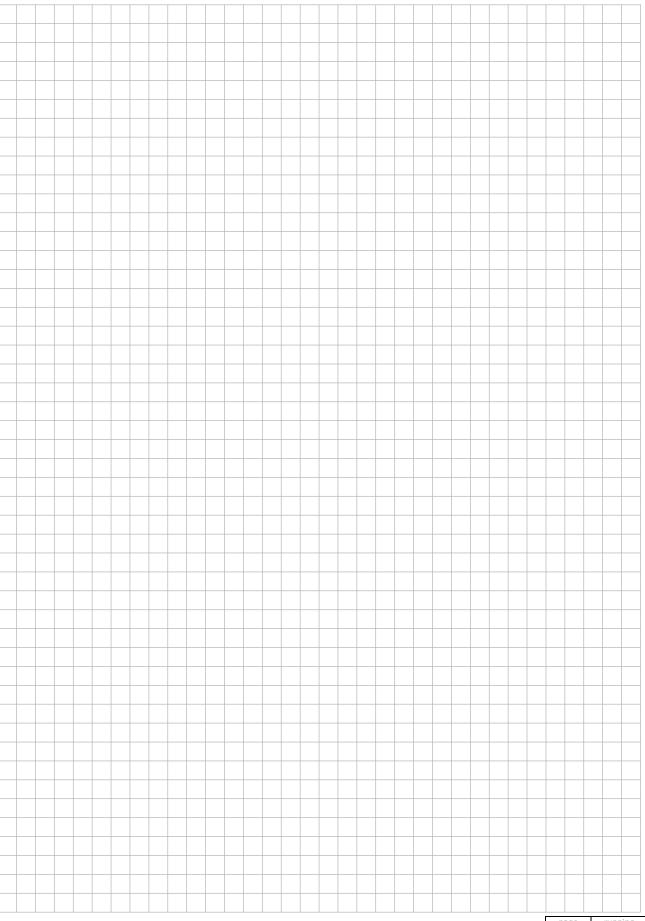
- (ii) In order to secure the bar [AF] to [DE], the manufacturer needs to know:
  - the measure of the angle EGF, and
  - the common distance |AG| = |DG|.

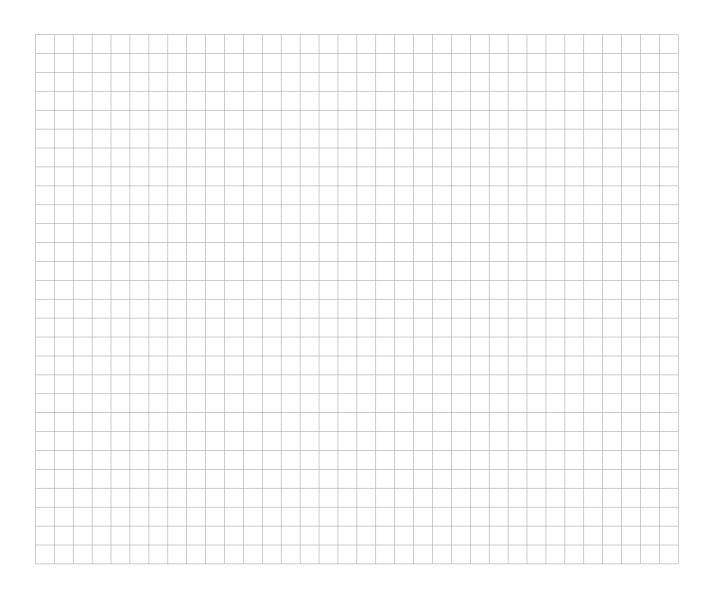
Find these measures. Give the angle correct to the nearest degree and the length correct to three decimal places.



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*Note to readers of this document:* 

This sample paper is intended to help teachers and candidates prepare for the June 2012 examination in *Mathematics* under Phase 1 of *Project Maths*. The content and structure do not necessarily reflect the 2013 or subsequent examinations.

For the examination of 2012, Paper 1 remains unchanged in both content and format.

Leaving Certificate – Higher Level

Mathematics (Project Maths – Phase 1) – Paper 2

Sample Paper, 2012 Time: 2 hours 30 minutes