Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2019

Marking Scheme

Mathematics

Higher Level

Paper 2

Marking Scheme – Paper 2, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 2, 5	0, 2, 3, 5		
10 mark scales			0, 4, 7, 10	0, 4, 5, 8, 10	
15 mark scales			0, 5, 10, 15	0, 5, 7, 11, 15	
20 mark scales					
25 mark scales					

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

Note: In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in *scale 10C*, 9 marks may be awarded.

Rounding and units penalty to be applied only once in each section (a), (b), (c) etc.

Throughout the scheme indicate by use of # where an arithmetic error occurs.

Summary of mark allocations and scales to be applied

Section A

Question 1

(a)(i)	10C
(a)(ii)	10D
(b)	5C

Question 2

(a)	10C
(b)(i)	5B
(b)(ii)	10D

Question 3

(a)	10C
(b)	15D

Question 4

(a)	10C
(b)(i)	15D

Question 5

(a)	15D
(b)(i)	10D

Question 6

(a)(i)	10C
(b)	15D

Section B

Question 7 (50)

(a)(i)	10C
(a)(ii)	5C
(a)(iii)	10D
(a)(iv)	5C
(b)(i)	15D
(b)(ii)	5C

Question 8 (45)

(a)(i)	10C
(a)(ii)	15D
(a)(iii)	10C
(b)(i)	5B
(b)(ii)	5C

Question 9 (55)

(a)	10C
(b)	10C
(c)	15C
(d)(i)	5B
(d)(ii)	5C
(d)(iii)	5C
(e)	5C

Model Solutions & Detailed Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 25 Marks	Marking Notes
(a) (i)	$\frac{12}{20}\times\frac{8}{19}+\frac{8}{20}\times\frac{12}{19}$ $=\frac{96}{380}+\frac{96}{380}$ or $2\left(\frac{96}{380}\right)$ $\frac{12}{20}\times\frac{8}{19}+\frac{8}{20}\times\frac{12}{19}=\frac{192}{380}$ or $\frac{48}{95}$ Or $\frac{\binom{12}{1}\binom{8}{1}}{\binom{20}{2}}=\frac{96}{190}$ or $\frac{48}{95}$ Or $1-\left[\frac{12}{20}\times\frac{11}{19}+\frac{8}{20}\times\frac{7}{19}\right]=1-\frac{188}{380}$ $=\frac{192}{380}$ or $\frac{48}{95}$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* 1 probability given e.g. $\frac{12}{20}$ or equivalent 1 combination indicated e.g. $\binom{12}{1}$ or $\binom{8}{1}$ or $\binom{20}{2}$ $\frac{12}{20}\times\frac{8}{19}$ or $\frac{8}{20}\times\frac{12}{19}$ or equivalent and stops $\frac{\binom{12}{1}}{\binom{20}{2}}$ or $\frac{\binom{8}{1}}{\binom{20}{2}}$ and stops $1-\frac{12}{20}\times\frac{11}{19}$ or $1-\frac{8}{20}\times\frac{7}{19}$ and stops *High Partial Credit:* $\frac{12}{20}\times\frac{8}{19}$ or $\frac{8}{20}\times\frac{12}{19}$ or equivalent and continues $\frac{\binom{12}{1}}{\binom{20}{2}}$ or $\frac{\binom{8}{1}}{\binom{20}{2}}$ and continues $1-\frac{12}{20}\times\frac{11}{19}$ or $1-\frac{8}{20}\times\frac{7}{19}$ and continues
(a) (ii)	$\frac{12}{20}\times\frac{11}{19}\times\frac{10}{18}\times\frac{8}{17}=\frac{10560}{116280}$ or $\frac{88}{969}$ **Or** $\frac{\binom{12}{1}}{\binom{20}{1}}\times\frac{\binom{11}{1}}{\binom{19}{1}}\times\frac{\binom{10}{1}}{\binom{18}{1}}\times\frac{\binom{8}{1}}{\binom{17}{1}}$ $=\frac{10560}{116280}$ or $\frac{88}{969}$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* 1 probability given 1 combination indicated *Mid Partial Credit* 3 or 4 correct probabilities indicated *High Partial Credit:* 3 correct probabilities with multiplication completed 4 probabilities with correct operator

(b)	$\binom{6}{3} \times \binom{8}{4} = 1400$ or $\binom{1}{1} \times \binom{6}{3} \times \binom{8}{4} = 1400$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* $\binom{6}{3}$ or $\binom{8}{4}$ or $\binom{1}{1}$ *High Partial Credit:* $\binom{6}{3} \times \binom{8}{4}$ and stops

<table>
<tr><th>Q2</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>
$m = \frac{b-0}{0-a} = \frac{-b}{a}$
$y - 0 = \frac{-b}{a}(x-a)$
$ay = -bx + ab$
$bx + ay = ab$
Now divide across by ab
$$\frac{x}{a} + \frac{y}{b} = 1$$
Or
$$m = \frac{b-0}{0-a} = \frac{-b}{a}$$
$$y = mx + c \ \ => \ y = \frac{-b}{a}x + c\,.$$
$$\text{But } (o, b) \text{ is on this line, thus}$$
$$b = \ \frac{-b}{a}(o) + c$$
$$\therefore \ b = c$$
Equation $y = \frac{-b}{a}x + b$
$$ay = -bx + ab$$
$bx + ay = ab$
Now divide across by ab
$$\frac{x}{a} + \frac{y}{b} = 1$$
Or
$$(a, 0) \in \ y = mx + c => 0 = ma + c$$
$$=> -ma = c$$
$$(0, b) \in \ y = mx + c => b = c$$
$$\therefore -ma = b => m = \frac{-b}{a}$$
Equation $y = \frac{-b}{a}x + b$
$$ay = -bx + ab$$
$bx + ay = ab$
Now divide across by ab
$$\frac{x}{a} + \frac{y}{b} = 1$$
Or
$$\frac{x}{a} + \frac{y}{b} = 1$$
LHS: $\frac{x}{a} + \frac{y}{b}$
$(a, 0)$: $\frac{a}{a} + \frac{0}{b}$ = 1=1 or RHS
$(0, b)$: $\frac{0}{a} + \frac{b}{b}$ = 1=1 or RHS
</td><td>
Scale 10C (0, 4, 7, 10)
Low Partial Credit:
Slope formula with some substitution

High Partial Credit:
Equation of line formula fully substituted

Low Partial Credit:
Slope formula with some substitution

High Partial Credit:
m expressed in terms of a and b, and c in terms of b

Low Partial Credit:
$(a, 0)$ or $((0, b)$ correctly substituted e.g. $\frac{a}{a} + \frac{0}{b}$

High Partial Credit:
$(a, 0)$ and $(0, b)$ correctly substituted
</td></tr>
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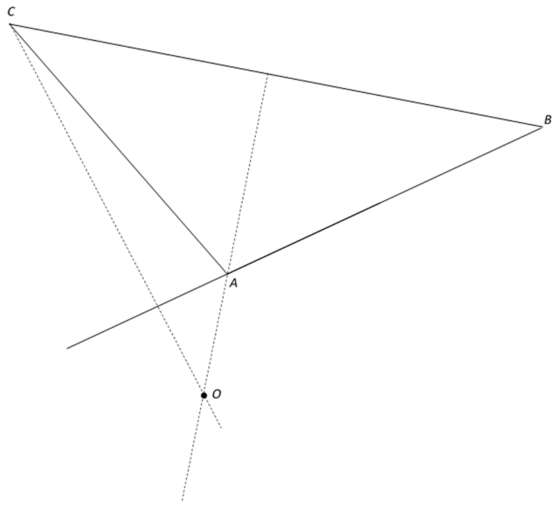
(b) (i)	$y - 0 = m(x - 6)$ <u>or</u> $y = m(x - 6)$ Or $$y = mx - 6m$$ Or $$y = mx + c$$ $\therefore\ 0 = 6m + c \ \Rightarrow\ c = -6m$	**Scale 5B (0, 2, 5)** *Mid Partial Credit:* Equation of line formula with some relevant substitution
(b) (ii)	$$y = m(x - 6)$$ $$4x + 3y = 25$$ => $4x + 3m(x - 6) = 25$ =>$x = \frac{25+18m}{3m+4}$ Substitute this into $y = m(x - 6)$ $$y = m\left(\frac{25 + 18m}{3m + 4}\right) - 6m$$ $$= \frac{25m + 18m^2 - 18m^2 - 24m}{3m + 4}$$ $$= \frac{m}{3m + 4}$$ Or $$4x + 3y = 25\ \cap mx - y = 6m$$ $$4x + 3y = 25$$ $$\underline{3mx - 3y = 18m}$$ $$4x + 3mx = 18m + 25$$ $$x = \frac{25+18m}{3m+4}$$ $$4mx + 3my = 25m$$ $$\underline{4mx - 4y = 24m}$$ $$(3m + 4)y = m$$ $$\therefore\ y = \frac{m}{3m + 4}$$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* Indication of use of simultaneous equations *Mid Partial Credit* One relevant substitution *High Partial Credit:* x or y value found *Low Partial Credit:* Indication of use of simultaneous equations *Mid Partial Credit* One successful elimination in equations *High Partial Credit:* x or y value found

Q3	Model Solution – 25 Marks	Marking Notes
(a)	$(-2-2)^2 + (k-3)^2 = 65$ $16 + (k-3)^2 = 65$ $(k-3)^2 = 49$ $k-3 = \pm\sqrt{49} = \pm 7$ $k = 10$ and $k = -4$ Or $k^2 - 6k + 9 = 49$ $k^2 - 6k - 40 = 0$ $(k-10)(k+4) = 0$ $k = 10$ and $k = -4$ Or $x^2 - 4x + 4 + y^2 - 6y + 9 = 65$ $x^2 + y^2 - 4x - 6y = 52$ $4 + k^2 + 8 - 6k = 52$ $k^2 - 6k - 40 = 0$ $(k-10)(k+4) = 0, \therefore k = 10,\ k = -4$ Or Centre (2, 3) , radius $\sqrt{65}$ $\sqrt{(2+2)^2 + (3-k)^2} = \sqrt{65}$ and proceed as above	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* Some relevant substitution Centre or radius *High Partial Credit:* Equation in k^2

(b)	Both axes are tangents to the circle. centre (-g,-g) and radius is g Perpendicular distance (-g,-g) to $3x - 4y + 6 = 0$ is equal to the radius $$\frac{-3g + 4g + 6}{5} = -g$$ $$g + 6 = \pm 5g$$ $g + 6 = -5g, \therefore\ -g = 1$ Centre (1, 1) and radius 1 Equation: $(x-1)^2 + (y-1)^2 = 1$ or $x^2 + y^2 - 2x - 2y + 1 = 0$ Or s is a tangent to both axes therefore $c = g^2 = f^2$ So equation is in the form $x^2 + y^2 + 2gx + 2gy + g^2 = 0$ $3x - 4y + 6 = 0 \Rightarrow y = \frac{3x+6}{4}$ Substitute into circle: $x^2 + \left(\frac{3x+6}{4}\right)^2 + 2gx + \frac{2g(3x+6)}{4} + g^2 = 0$ => $25x^2 + x(36 + 56g) + 36 + 48g + 16g^2 = 0$ Tangent therefore $b^2 = 4ac$ $(36 + 56g)^2 = 4(25)(36 + 48g + 16g^2)$ $2g^2 - g - 3 = 0$ $g = -1$ and $g = \frac{3}{2}$ But can't have positive g as the co-ordinate $-g$ is in first quadrant. => $g = -1$. Therefore equation is $x^2 + y^2 - 2x - 2y + 1 = 0$ or $(x-1)^2 + (y-1)^2 = 1$	**Scale 15D (0, 5, 7, 11, 15)** *Low Partial Credit:* centre *(-g,-g)* or equivalent *Mid Partial Credit:* Substitution into perpendicular distance formula completed Perpendicular distance of centre to tangent equals radius with some substitution *High Partial Credit:* equation in g or equivalent *Low Partial Credit:* $c = g^2$ or f^2 Effort to express $\boldsymbol{x}$ in terms of $\boldsymbol{y}$ or equivalent *Mid Partial Credit:* Substitution into circle equation completed *High Partial Credit:* Quadratic equation in g or f

Q4	Model Solution – 25 Marks	Marking Notes
(a)	$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\cos 2A = \cos^2 A - \sin^2 A$ $\cos 2A = (1-\sin^2 A) - \sin^2 A$ $\cos 2A = 1 - 2\sin^2 A$ Or Taking RHS $1-2\sin^2 A = 1-2(1-\cos^2 A)$ $= -1+2\cos^2 A$ $= -(\cos^2 A+\sin^2 A)+2\cos^2 A$ $= \cos^2 A - \sin^2 A$ $= \cos A\cos A - \sin A\sin A = \cos 2A$ Or $(\cos A + i\sin A)^2 = \cos 2A + i\sin 2A$ $(\cos A + i\sin A)^2$ $= \cos^2 A + 2i\sin A\cos A + (i\sin A)^2$ $\cos 2A = \cos^2 A - \sin^2 A$ $\cos 2A = (1-\sin^2 A) - \sin^2 A$ $\cos 2A = 1 - 2\sin^2 A$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* $\cos(A+B)$ formula with some substitution $\cos^2 A + \sin^2 A = 1$ indicated or clearly implied *High Partial Credit:* $\cos 2A = \cos^2 A - \sin^2 A$ *Low Partial Credit:* $\cos^2 A + \sin^2 A = 1$ indicated or clearly implied $(\cos A + i\sin A)^2$ expanded *High Partial Credit:* $\cos 2A = \cos^2 A - \sin^2 A$

(b)	Let length of side be x Diagonal of any face $=\sqrt{x^2+x^2}=\sqrt{2}x$ Internal diagonal $=x^2+\left(\sqrt{2}x\right)^2=\sqrt{3}x$ By cosine rule: $x^2=\left(\frac{\sqrt{3}x}{2}\right)^2+\left(\frac{\sqrt{3}x}{2}\right)^2-2\frac{\sqrt{3}x}{2}\frac{\sqrt{3}x}{2}\cos A$ $$\cos A=\frac{\left(\frac{\sqrt{3}x}{2}\right)^2+\left(\frac{\sqrt{3}x}{2}\right)^2-x^2}{2\left(\frac{\sqrt{3}x}{2}\right)\left(\frac{\sqrt{3}x}{2}\right)}$$ $$\cos A=\frac{1}{3}$$	**Scale 15D (0, 5, 7, 11, 15)** *Low Partial Credit:* Length of any diagonal formulated *Mid Partial Credit* Internal diagonal found *High Partial Credit:* Fully substituted cosine rule **Note:** Accept and mark work where a consistent numerical value is assigned to one side of the cube.
	Or Drop perpendicular from intersecting diagonals to side of cube, thereby creating angle $A/2$ at vertex in a right-angled triangle. $$\sin\frac{A}{2}=\frac{\frac{x}{2}}{\frac{\sqrt{3}x}{2}}=\frac{1}{\sqrt{3}}$$ $$\therefore \cos\frac{A}{2}=\frac{\sqrt{2}}{\sqrt{3}}$$ $$\cos A=2\cos^2\frac{A}{2}-\ 1=2\left(\frac{2}{3}\right)-1=\frac{1}{3}$$ Also: $\sin\frac{A}{2}=\ \frac{1}{\sqrt{3}}\ \rightarrow\frac{A}{2}=35{\cdot}2643896^{\circ}$ $$A=70{\cdot}5287792^{\circ}$$ $$\cos A=0{\cdot}33236$$	*Low Partial Credit:* Length of any diagonal formulated *Mid Partial Credit* Internal diagonal found *High Partial Credit:* $\sin\frac{A}{2}$ fully substituted

Q5	Model Solution – 25 Marks	Marking Notes
(a)	Standard Orthocentre Construction  	**Scale 15D (0, 5, 7, 11,15)** *Low Partial Credit:* Some correct element of construction Some evidence of understanding of term orthocentre *Mid Partial Credit* One correct altitude *High Partial Credit:* Two correct altitudes but not intersecting.
(b)	$\|DC\| = \|OB\|$ Given $\Rightarrow \|DC\| =$ Radius $\Rightarrow \Delta ODC$ is equilateral $\Rightarrow \angle ODC = 60$ $\Rightarrow \angle AOD = 60$ Alternate ΔAOD is isosceles as $\|OA\| = \|OD\|$ $\angle OAD = \angle ODA = \frac{120}{2} = 60$ $\|\angle ABE\| = 90°$ as BE tangent $\|\angle BEA\| = 180 - 90 - 60 = 30°$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* 1 relevant step listed or shown on diagram *Mid Partial Credit* 3 relevant steps listed or shown on diagram *High Partial Credit:* All valid steps included but with no justification

Q6	Model Solution – 25 Marks	Marking Notes
(a)	$P(F \cap S) = P(F) \times P(S)$ since the events are independent. $$\frac{1}{5} = \frac{9}{20} \times P(S)$$ => $P(S) = \frac{4}{9}$ So $P(S \backslash F) = \frac{4}{9} - \frac{1}{5} = \frac{11}{45} = x$ Or $$P(S) = \frac{1}{5} + x$$ $$\frac{1}{5} = \frac{9}{20}\left(\frac{1}{5} + x\right) => \frac{11}{45} = x$$ Or $$P(F\|S) = \frac{P(F \cap S)}{P(S)} = P(F)$$ $$\frac{\frac{1}{5}}{\frac{1}{5} + x} = \frac{9}{20} \quad => x = \frac{11}{45}$$ Or $$P(S\|F) = \frac{P(S \cap F)}{P(F)} = P(S)$$ $$\frac{\frac{1}{5}}{\frac{9}{20}} = \frac{1}{5} + x => x = \frac{11}{45}$$ $$y = 1 - \frac{11}{45} - \frac{1}{5} - \frac{1}{4} = \frac{11}{36}$$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* $P(F \cap S) = P(F) \times P(S)$ or equivalent $P(F) = \frac{1}{4} + \frac{1}{5}$ $P(S) = x + \frac{1}{5}$ $\frac{1}{4} + \frac{1}{5} + x + y = 1$ *High Partial Credit* x found

(b)		
	If n Germans then $2n$ Irish and $3n+10$ children in total $\frac{n}{3n+10} \times \frac{2n+10}{3n+9} = \frac{1}{6}$ $\frac{2n^2+10n}{9n^2+57n+90} = \frac{1}{6}$ $3n^2 + 3n - 90 = 0$ $n^2 + n - 30 = 0$ $(n+6)(n-5) = 0$ $n = 5$ German children. There are 10 Irish (and 10 Spanish) so 25 children in the club. Or 25 by trial and improvement method: 5 German, 10 Irish, 10 Spanish and verified to indicate $\frac{5}{25} \times \frac{20}{24} = \frac{1}{6}$	**Scale 15D (0, 5, 7, 11, 15)** *Low Partial Credit:* *2n* *3n+10* One correct probability e.g. $\frac{n}{3n+10}$ *Mid Partial Credit:* $\frac{n}{3n+10}$ and $\left(\frac{2n+10}{\blacksquare} \text{ or } \frac{\blacksquare}{3n+9}\right)$ *High Partial Credit:* $\frac{n}{3n+10} \times \frac{2n+10}{3n+9} = \frac{1}{6}$ *Low Partial Credit:* Some correct element in approach *Mid Partial Credit* Tests more than one value *High Partial Credit:* Correct number of each nationality but not verified that probability is $\frac{1}{6}$ Correct answer (25) with no supporting work

Section B

Q7	Model Solution – 50 Marks	Marking Notes
(a) (i)	$\lvert AD\rvert^2 = 90^2 - 60^2$ $90^2 = 60^2 + \lvert AD\rvert^2$ $\lvert AD\rvert = \sqrt{8100-3600} = \sqrt{4500} = 30\sqrt{5}$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* $\lvert OD\rvert = 60$ Pythagoras formulated Effort to find angle other than $\angle ODA$ *High Partial Credit:* $\sqrt{8100-3600}$ or equivalent
(a) (ii)	$\cos(\angle DOA) = \frac{60}{90}$ $\cos^{-1}\left(\frac{6}{9}\right) = 0{\cdot}84$ Or $\sin(\angle DOA) = \frac{30\sqrt{5}}{90} = \frac{\sqrt{5}}{3} = 0{\cdot}745356$ $\lvert\angle DOA\rvert = 48{\cdot}189^{\circ}$ $\lvert\angle DOA\rvert = 0{\cdot}84139 = 0{\cdot}84$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* Relevant trigonometric ratio formulated *High Partial Credit:* Relevant trigonometric ratio fully substituted
(a) (iii)	Area of sector: $\frac{1}{2}r^2\theta$ $\frac{1}{2}(0{\cdot}9)^2 \times 2(0{\cdot}84) = 0{\cdot}6804\ \text{m}^2$ Area $\triangle$ ACO : $\frac{1}{2}\lvert AC\rvert\lvert OD\rvert = \frac{1}{2}(60\sqrt{5})60\ \text{cm}^2$ $\frac{1}{2}(1{\cdot}34164)(0{\cdot}6) = 0{\cdot}40\ \text{m}^2$ Or Area $\triangle$ ACO : $\frac{1}{2}\lvert AO\rvert\lvert OC\rvert \sin(\angle AOC) =$ $\frac{1}{2}(90)(90)\sin 2(48{\cdot}189^{\circ})$ $= 4024{\cdot}9174\ \text{cm}^2 = 0{\cdot}40\ \text{m}^2$ Area of segment = 0·6804 – 0·40 = 0·28	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* Formula for area of sector with some substitution Formula for area of $\triangle$ ACO with some substitution *Mid Partial Credit:* One relevant area fully substituted *High Partial Credit:* Both relevant areas fully substituted Mishandling conversion of units
(a) (iv)	Volume = $0{\cdot}28 \times 2{\cdot}5 = 0{\cdot}7$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* Formula for **volume** of trough with some substitution Indicates some relevant use of 2·5 *High Partial Credit:* Formula fully substituted

(b) (i)	Volume = $\pi\left[\left(\left(\frac{2}{3}\right)1{\cdot}25^3\right)\right]$ $+\pi[(1{\cdot}25^2 \times 3{\cdot}5)]$ $+\pi\left[\left(\left(\frac{1}{3}\right)1{\cdot}25^2 \times 1{\cdot}5\right)\right]$ $= 4{\cdot}0906 + 17{\cdot}1805 + 2{\cdot}4544$ $= 23{\cdot}73$	**Scale 15D (0, 5, 7, 11, 15)** *Low Partial Credit:* 1 volume formula with some substitution *Mid Partial Credit* 2 volumes fully substituted *High Partial Credit:* 3 volumes fully substituted
(b) (ii)	$23{\cdot}73 \times 0\ 02 = 0{\cdot}4746$ cm^3 $\frac{r}{h} = \frac{1{\cdot}25}{1{\cdot}5} = \frac{5}{6}$ $r = \frac{5h}{6}$ Volume in cone $= \frac{1}{3}\pi\left(\frac{5h}{6}\right)^2 \times h = 0{\cdot}4746$ $h^3 = \frac{0{\cdot}4746.3.6}{25\pi} = 0{\cdot}65262$ $h = \sqrt[3]{0{\cdot}65262} = 0{\cdot}8674$ $h = 0{\cdot}87$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* volume $\times$ 0·98 or equivalent volume multiplied by 2% effort at *r* : *h* *High Partial Credit:* Volume formula expressed in one variable

Q8	Model Solution – 45 Marks	Marking Notes
(a) (i)	Confidence interval $$0{\cdot}2175 \pm 1{\cdot}96\sqrt{\frac{\left(\frac{174}{800}\right)\left(\frac{626}{800}\right)}{800}}$$ $$0{\cdot}2175 - 1{\cdot}96\sqrt{\frac{(0{\cdot}2175)(0{\cdot}7825)}{800}} < p$$ $$< 0{\cdot}2175 + 1{\cdot}96\sqrt{\frac{(0{\cdot}2175)(0{\cdot}7825)}{800}}$$ $$0{\cdot}2175 - 1{\cdot}96\sqrt{0{\cdot}00021274} < p$$ $$< 0{\cdot}2175 + 1{\cdot}96\sqrt{0{\cdot}00021274}$$ $$0{\cdot}188913 < p < 0{\cdot}246087$$ $$0{\cdot}1889 < p < 0{\cdot}2461$$ or $$18{\cdot}89\% < p < 24{\cdot}61\%$$	**Scale 10C(0, 4, 7,10)** *Low Partial Credit:* $0{\cdot}2175$ or $\frac{174}{800}$ CI formulated with some substitution *High Partial Credit:* CI fully substituted
(a) (ii)	$$\frac{x - \bar{x}}{\sigma}$$ $\frac{95-87{\cdot}3}{12} = 0{\cdot}64167$ (z score) => $p(Z \leq 0{\cdot}64167) = 0{\cdot}7389$ $$P(z \geq 0{\cdot}64) = 1 - 0{\cdot}7389$$ $$= 0{\cdot}2611 \text{ or } 26{\cdot}11\%$$	**Scale 15D(0, 5, 7, 11, 15)** *Low Partial Credit:* μ or σ identified *Mid Partial Credit:* z found *High Partial Credit:* $P(z < 0{\cdot}64)$ and stops or continues incorrectly
(a) (iii)	$$z = -0{\cdot}52 = \frac{x - 87{\cdot}3}{12}$$ $$=> x = 81{\cdot}06 \text{ km/h}$$ $$x = 81 \text{ km/h}$$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* $\frac{x - 87{\cdot}3}{12}$ $z \in [0{\cdot}52, 0{\cdot}53]$ or $z \in [-0{\cdot}52, -0{\cdot}53]$ and stops *High Partial Credit:* Formula for x fully substituted

(b) (i)	Average speed has changed p-value $< 0{\cdot}05$	**Scale 5B (0, 2, 5)** *Mid Partial Credit:* Answer or reason correct
(b) (ii)	$$0{\cdot}024 = 2\big(1 - P(z \leq \lvert T \rvert)\big)$$ => $P(z \leq \lvert T \rvert) = 0{\cdot}988$ Therefore $z = 2{\cdot}26$ or $-2{\cdot}26$. Because the mean has reduced $z = -2{\cdot}26$ $$-2{\cdot}26 = \frac{x - 87{\cdot}3}{\frac{12}{\sqrt{100}}}$$ => $x = 84{\cdot}588$ km/h => $x = 84{\cdot}6$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* $0{\cdot}024 = 2(0{\cdot}012)$ Value(s) of *z* found *High Partial Credit:* Formula for x fully substituted

Q9	Model Solution – 55 Marks	Marking Notes
(a)	$\lvert SG\rvert^2 = 30^2 + 58^2 - 2(30)(58)(\cos 68)$ $= 2960{\cdot}369$ $\lvert SG\rvert = 54{\cdot}409$ m $\lvert SG\rvert = 54{\cdot}4$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* Some relevant substitution into correct cosine formula *High Partial Credit:* Formula fully substituted
(b)	$\dfrac{54{\cdot}4}{\sin 68} = \dfrac{30}{\sin \angle HSG}$ $\sin \angle HSG = 0{\cdot}51131$ $\lvert\angle HSG\rvert = 30{\cdot}75$ Or $\cos \angle HSG = \dfrac{54{\cdot}4^2 + 58^2 - 30^2}{2(54{\cdot}4)(58)}$ $= 0 \cdot 859432$ $\lvert\angle HSG\rvert = 30{\cdot}747^\circ = 30{\cdot}75$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* Some relevant substitution into relevant formula *High Partial Credit:* Formula fully substituted Note: Finds $\lvert\angle HGS\rvert$ => ✔#
(c)	Area $\Delta GSH = \frac{1}{2}(30)(58)\sin 68 = 806{\cdot}65$ Also Area ΔGSH: $\dfrac{1}{2}(54{\cdot}4)(58)\sin 30{\cdot}75$ and $\frac{1}{2}(54{\cdot}4)(30)\sin 81{\cdot}25$	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* Some substitution into area formula *High Partial Credit:* Formula fully substituted
(d) (i)	$\dfrac{1}{2}(58)(r)$ or $29r$	**Scale 5B (0, 2, 5)** *Mid Partial Credit:* Right angle indicated Relevant triangle indicated on diagram Area of triangle formula with some substitution

(d) (ii)	Area ΔGHS $= \frac{1}{2}(30)(r) + \frac{1}{2}(54{\cdot}4)(r) + \frac{1}{2}(58)(r)$ $= 15r + 27{\cdot}2r + 29r = 71{\cdot}2r$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* Relevant use of previous answer in this part Indication of 3 relevant triangle areas to be added Area of 1 additional triangle (in terms of r) *High Partial Credit:* Addition of 2 areas (each written in terms of r)
(d) (iii)	$71{\cdot}2r = 806{\cdot}62$ $r = \frac{806{\cdot}62}{71{\cdot}2}$ $= 11{\cdot}3289 = 11{\cdot}3$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* Both relevant answers presented *High Partial Credit:* Areas equated
(e) (ii)	$\tan 14 = \frac{\lvert TS\rvert}{\lvert PS\rvert}$ $\sin 15{\cdot}375 = \frac{11{\cdot}3}{\lvert PS\rvert} = 42{\cdot}51$ $=> \lvert PS\rvert = 42{\cdot}619$ $\tan 14 = \frac{\lvert TS\rvert}{42{\cdot}619}$ $\lvert TS\rvert = 10{\cdot}626 = 10{\cdot}6$ Or $\lvert\angle HPS\rvert = 180 - 15{\cdot}375 - 34$ $= 130{\cdot}625^\circ$ $\frac{\sin 130{\cdot}625}{58} = \frac{\sin 34}{\lvert PS\rvert}$ $\lvert PS\rvert = 42{\cdot}73$ $\tan 14 = \frac{\lvert TS\rvert}{42{\cdot}73}$ $\lvert TS\rvert = 10{\cdot}653 = 10{\cdot}7$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* Some relevant substitution *High Partial Credit:* Formula fully substituted