Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2018

Marking Scheme

Mathematics

Higher Level

Paper 2

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	
5 mark scales		0, 2, 5	0, 2, 4, 5		
10 mark scales			0, 3,7, 10	0, 3, 5, 8, 10	
15 mark scales			0, 4, 11, 15	0, 4, 7, 11, 15	
20 mark scales			0, 7, 13, 20	0, 5, 10, 15, 20	

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

Marking Scheme

Section A

Question 1

(a)	15C
(b)	10D

Question 2

(a)	5B
(b)(i)	10C
(b)(ii)	5C
(b)(iii)	5C

Question 3

(a)(i)	15C
(a)(ii)	5B
(b)	5C

Question 4

(a)	20C
(b)	5C

Question 5

(a)	10C
(b)	10D
(c)	5C

Question 6

(a)	15D
(b)	10C

Section B

Question 7

(a)	10D
(b)(i)	10C
(b)(ii)	10D
(c)	15C
(d)	5B

Question 8

(a)(i)	20D
(a)(ii)	15C
(b)(i)	5C
(b)(ii) (iii)	10D
(c)	10D

Question 9

(a)	10C
(b)(i)	5C
(b)(ii)	5C
(b)(iii)	10C
(b)(iv)	5B
(c)	5C

Note: In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded.
Throughout the scheme indicate by use of * where an arithmetic error occurs.

Model Solutions & Detailed Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

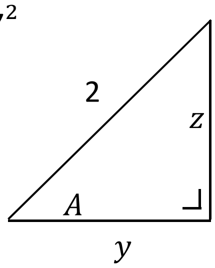
Q1	Model Solution – 25 Marks	Marking Notes
(a)	$\frac{1}{20}(9000)+\frac{1}{10}(7000)+\frac{1}{4}(3000)$ $= 1900$ $E(x)=2000-1900=100$ **Or** $E(x)=\frac{1}{20}(-7000)+\frac{1}{10}(-5000)+\frac{1}{4}(-1000)+\frac{3}{5}(2000)$ $=-350-500-250+1200=100$ So expected gain for organisers of competition and therefore a loss for Mary of 100	**Scale 15C (0, 4, 11, 15)** *Low Partial Credit:* $E(x)$ partially formulated (1 or 2 terms) *High Partial Credit:* $E(x)$ fully formulated (sum of all three/all four terms)

(b)	$$\frac{1}{20}(9000+x)+\frac{1}{10}(7000+x)$$ $$+\frac{1}{4}(3000+x)=2000$$ $$\left(1900+\frac{8}{20}x\right)=2000$$ $$\frac{8}{20}x=100$$ $$x=250$$ **Or** From **(a)** to break even it will take €100. $$\frac{x}{20}+\frac{x}{10}+\frac{x}{4}=100$$ $$\frac{x+2x+5x}{20}=100$$ $$\frac{8}{20}x=100$$ $$x=250$$ **Or** $$E(x)=\frac{1}{20}(-7000-x)$$ $$+\frac{1}{10}(-5000-x)$$ $$+\frac{1}{4}(-1000-x)+\frac{3}{5}(2000)=0$$ $$-7000-x-10\,000-2x-5000-5x$$ $$+24\,000=0$$ $$2000=8x\Rightarrow 250=x$$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Any relevant use of x, excluding $(9000+x)$ *Mid Partial Credit:* $E(x)$ fully formulated (LHS). $\left(1900+\frac{8}{20}x\right)$ or equivalent and stops. $\frac{x}{20}+\frac{x}{10}+\frac{x}{4}$ *High Partial Credit* Relevant equation in x *Low Partial Credit:* Any relevant use of x e.g. $(-7000+x)$ *Mid Partial Credit:* $E(x)$ fully formulated (LHS). $\left(100-\frac{8}{20}x\right)$ or equivalent and stops. *High Partial Credit* Relevant equation in x

Q2	Model Solution – 25 Marks	Marking Notes
(a)	$P(z < z_1) = 0{\cdot}67$ $z = 0{\cdot}44$	**Scale 5B (0, 2, 5)** *Partial Credit:* $P(z < z_1) = 0{\cdot}67$
(b) (i)	Mary Maths $\frac{65-70}{15} = -\frac{1}{3}$ Mary English $\frac{68-72}{10} = -\frac{2}{5}$ $-\frac{1}{3} > -\frac{2}{5}$ Mary did better in Maths Justification: $-\frac{1}{3} > -\frac{2}{5}$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* Relevant formula with some correct substitution $\frac{65-70}{15}$ or $\frac{68-72}{10}$. *High Partial Credit:* $\frac{65-70}{15}$ and $\frac{68-72}{10}$
(b) (ii)	$P(z > z_1) = 0{\cdot}15$ $z = \frac{x-72}{10} = 1{\cdot}04$ $x = 82{\cdot}4\%$ $x = 83$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* $0{\cdot}15$ $1{\cdot}04$ Relevant formula with some correct substitution *High Partial Credit:* Relevant equation in x

(b) (iii)	82 is 1 st. dev. above mean $\Rightarrow \approx \frac{68}{2}\%$ above 52 is 2 st. dev. below mean $\Rightarrow \approx \frac{95}{2}\%$ below $34+47{\cdot}5=81{\cdot}5\%$ **Or** From tables: 82 is 1 deviation off mean $\Rightarrow \frac{0{\cdot}6826}{2}=0{\cdot}3413$ 52 is 2 dev. off mean $\Rightarrow \frac{0{\cdot}9544}{2}=0{\cdot}4772$ $0{\cdot}3413+0{\cdot}4772=0{\cdot}8185=81{\cdot}85\%$ **Or** $z=\frac{52-72}{10}=-2$ $\quad$ $z=\frac{82-72}{10}=1$ $P(-2<z<1)$ $P(z<1)-[1-P(z<2)]$ $0{\cdot}8413-[1-0{\cdot}9772]$ $=0{\cdot}8185$ $=81{\cdot}85\%$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* Evidence of relevant linking of deviation and mean. $\frac{68}{2}$ or $\frac{95}{2}$ $\frac{52-72}{10}$ or $\frac{82-72}{10}$ $\frac{0{\cdot}6826}{2}$ or $\frac{0{\cdot}9544}{2}$ *High Partial Credit:* $\frac{68}{2}$ and $\frac{95}{2}$ $\frac{52-72}{10}$ and $\frac{82-72}{10}$ $\frac{0{\cdot}6826}{2}$ and $\frac{0{\cdot}9544}{2}$

Q3	Model Solution – 25 Marks	Marking Notes
(a) (i)	$10^5 \times 1$ or $100\ 000$	**Scale 15C (0, 4, 11, 15)** *Low Partial Credit:* Some use of 10. Identifies that 5 other digits are required to complete code. *High Partial Credit* 9^5 or equivalent 10^6
(a) (ii)	$1 \times 10 \times 10 + 10 \times 1 \times 10 + 10 \times 10 \times 1$ $3 \times 10 \times 10$ or 3×10^2 or 300	**Scale 5B (0, 2, 5)** *Partial Credit:* 10 × 10
(b)	$\frac{(n+3)!\ (n+2)!}{(n+1)!\,(n+1)!} =$ $(n+3)(n+2)(n+2) =$ $n^3 + 7n^2 + 16n + 12$ **Or** $\frac{(n+3)!\,(n+2)!}{(n+1)!\,(n+1)!} = an^3 + bn^2 + cn + d$ $n = 0 \rightarrow \frac{3!.2!}{1!\,1!} = 12 = d$ $n = 1 \rightarrow a + b + c + d = 36$ $n = 2 \rightarrow 8a + 4b + 2c + d = 80$ $n = 3 \rightarrow 27a + 9b + 3c + d = 150$ Solving the simultaneous equations $a = 1, b = 7, c = 16, d = 12$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* Factorial expansion (e.g. $(n+3)! =$ $(n+3)(n+2)(n+1) \ldots \ldots \ldots .1)$ Effort at a numerical value for *n* on both LHS and RHS (method 2) *High Partial Credit:* $(n+3)(n+2)(n+2)$ Four simultaneous equations

Q4	Model Solution – 25 Marks	Marking Notes
(a)	$2x = 150 + 360n$ or $2x = 210 + 360n$ $x = 75 + 180n$ $\quad x = 105 + 180n$ $n = 0 \Longrightarrow x = 75°$ $\quad n = 0 \Longrightarrow x = 105°$ $n = 1 \Longrightarrow x = 255°$ $\quad n = 1 \Longrightarrow x = 285°$	**Scale 20C (0, 7, 13, 20)** *Low Partial Credit:* 30° or 150° or 210° *High Partial Credit:* 2 relevant values of x
(b)	$2^2 = y^2 + z^2$ $z = \sqrt{4 - y^2}$ $\sin 2A = 2\sin A\cos A$  $2\left(\frac{\sqrt{4 - y^2}}{2}\right)\left(\frac{y}{2}\right)$ $= \frac{y\sqrt{4-y^2}}{2}$ **Or** $\sin 2A = \frac{2\tan A}{1 + \tan^2 A}$ $\frac{2\frac{\sqrt{4-y^2}}{y}}{1+\frac{4-y^2}{y^2}} = \frac{2y\sqrt{4-y^2}}{y^2+4-y^2} = \frac{y\sqrt{4-y^2}}{2}$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* $\sqrt{4 - y^2}$ $2\sin A\cos A$ without substitution sin2*A* expressed in tan *A* format Relevant labelled diagram (2, y, A) *High Partial Credit:* Substitution for sin *A* or cos *A* in formula sin $A = \left(\frac{\sqrt{4-y^2}}{2}\right)$ tan $A = \frac{\sqrt{4-y^2}}{y}$

<table>
<tr><th>Q5</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>2(-2) + 3(1) + 1 = 0
or $-4+3+1=0$</td><td>Scale 10C (0, 3, 7, 10)
Low Partial Credit:
Substitution for x or y in equation of line

High Partial Credit:
Substitution for x and y in eq. of line (LHS when no indication of 0)</td></tr>
<tr><td>(b)</td><td>Slope of m or n = $\frac{-2}{3}$
Slope of AB is $\frac{3}{2}$ and (−2, 1) is on AB
$$y-1=\frac{3}{2}\left(x-\ (-2)\right)$$
equation of AB is $3x-2y+8=0$
Solve for (x, y) between
$3x-2y+8=0$ and $2x+3y-51=0$
$n\cap AB=(6,13)=B$

Or

coordinates of $B(x, y)$
$$|AB|=\sqrt{(x+2)^2+(y-1)^2}$$
Perp. distance (−2, 1) to $2x+3y-51=0$
$$\left|\frac{-4+3-51}{\sqrt{13}}\right|=\frac{52}{\sqrt{13}}=4\sqrt{13}$$
$$\therefore (x+2)^2+(y-1)^2=\ (4\sqrt{13})^2$$
Substituting $x=\frac{1}{2}(-3y+51)$
$$(\frac{-3y+55}{2})^2+(y-1)^2=\ (4\sqrt{13})^2$$
$13y^2-338y+2197=0$
$$y^2-26y+169=0$$
$$(y-13)^2=0\rightarrow y=13$$
$n\cap AB=(6,13)=B$</td><td>Scale 10D (0, 3, 5, 8, 10)
Low Partial Credit:
Slope of AB
Equation of line formula with some substitution

Mid Partial Credit:
Equation of AB

High Partial Credit:
Effort at finding intersection of lines

Note: Point of intersection, found correctly, of n and a relevant AB (with errors) merits Mid Partial Credit at least.

<u>Method 2</u>
Low Partial Credit:
Perpendicular distance formula with some substitution
Distance formula with some substitution

Mid Partial Credit:
Quadratic equation in x and y

High Partial Credit:
Quadratic equation in either x or y</td></tr>
</table>

<table>
<tr><td>(c)</td><td>
$\overrightarrow{AB} = x$ up 8 and y up 12
Centre of s is $\frac{1}{8}(8) - 2 = -1 = h$
and $\frac{1}{8}(12) + 1 = 2{\cdot}5 = k$
Eqn s: $(x+1)^2 + (y-2{\cdot}5)^2 = \left(\frac{\sqrt{13}}{2}\right)^2$
</td><td>
Scale 5C (0, 2, 4, 5)
Low Partial Credit:
8 up or 12 up
Indication $4\sqrt{13}$ from(b) of relevance

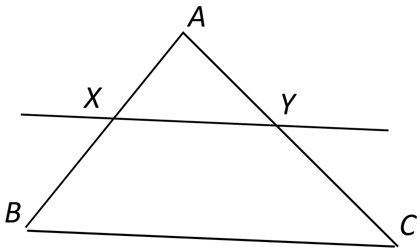
High Partial Credit:
Centre and radius of circle
</td></tr>
<tr><td></td><td>
Or

$s \cap t$
$\left(\frac{3(-2)+1(6)}{3+1}, \frac{3(1)+1(13)}{3+1}\right) = (0,4)$
Centre s: $\left(\frac{0-2}{2}, \frac{4+1}{2}\right) = (-1,\ 2{\cdot}5)$
Radius : distance $(-1, 2{\cdot}5)$ to either $(-2,1)$or $(0,4)$ or calculated otherwise $\sqrt{3{\cdot}25}$ or $\frac{\sqrt{13}}{2}$
$(x+1)^2 + (y-2{\cdot}5)^2 = \left(\frac{\sqrt{13}}{2}\right)^2$
</td><td>
Low Partial Credit:
Some relevant use of 1 : 3
Midpoint of AB found once but no further work of relevance
Formula with some relevant substitution

High Partial Credit:
Centre and radius of circle
</td></tr>
<tr><td></td><td>
Or

using ratio 1 : 7 centre s:
$\left(\frac{1(6)+7(-2)}{1+7}, \frac{1(13)+7(1)}{1+7}\right) = (-1, 2{\cdot}5)$
Radius as above or $\frac{\mathbf{1}}{\mathbf{8}}|AB|= \frac{\sqrt{\mathbf{13}}}{\mathbf{2}}$
$(x+1)^2 + (y-2{\cdot}5)^2 = \left(\frac{\sqrt{13}}{2}\right)^2$
</td><td>
Low Partial Credit:
Some relevant use of 1 : 7
Formula with some relevant substitution

High Partial Credit:
Centre and radius of circle
</td></tr>
</table>

<table>
<tr><th>Q6</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>Diagram:


Given:
A triangle ABC and a line XY parallel to BC which cuts AB in the ratio $s : t$ where $s, t \in \mathbb{N}$.
To Prove:
[AY] : [YC] $= s : t$

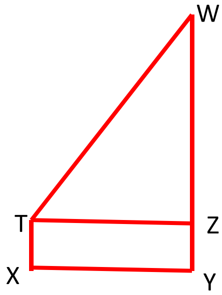
Construction:
Divide [AB] into $s + t$ equal parts, s of them lying along [AX] and t of them lying along [XB].
Through each point of division draw a line parallel to [BC]
Proof:
By a previous theorem the parallel lines cut off segments of equal length along [AC].
Therefore [AC] is divided into $s + t$ equal parts with s of them forming [AY] and t of them forming [YC].
Let k be the length of one segment on [AC].
[AY] : [YC] $= ks : kt \quad = s : t$</td><td>Scale 15D (0, 4, 7, 11, 15)
Low Partial Credit:
Relevant diagram drawn

Mid Partial Credit:
Construction clearly indicated

High Partial Credit:
Proof missing 1 relevant step</td></tr>
</table>

(b)	$$\lvert XY\rvert = \sqrt{4^2+3^2} = 5$$ $$\lvert ZC\rvert = 5$$ $$\lvert BZ\rvert = 10\text{cm}$$ **Or** $$\frac{8}{4} \text{ or } \frac{2}{1} = \frac{\lvert BZ\rvert}{5} \rightarrow \lvert BZ\rvert = 10\text{cm}$$ **Or** $$\frac{4}{12} = \frac{5}{5+\lvert BZ\rvert}$$ $$4\lvert BZ\rvert + 20 = 60 \rightarrow \lvert BZ\rvert = 10 \text{ cm}$$ Similarly: $\frac{3}{9} = \frac{5}{5+\lvert BZ\rvert}$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* $\lvert XY\rvert$ or $\lvert BX\rvert$ or $\lvert CY\rvert$ found Pythagoras with some substitution *High Partial Credit:* $\lvert ZC\rvert$ or $\lvert BC\rvert$ found Ratios formulated with $\lvert BZ\rvert$ the sole unknown

Q7	Model Solution – 50 Marks	Marking Notes
(a)	$V = \frac{4}{3}\pi 3^3 = 36\pi = 113{\cdot}1$ $\frac{113{\cdot}1(1 - 1{\cdot}75^5)}{1 - 1{\cdot}75} = 2324{\cdot}29$ $= 2324$ **or** Volume A = $113{\cdot}1$ Volume B = $197{\cdot}925$ Volume C = $346{\cdot}36875$ Volume D = $606{\cdot}1453125$ Volume E = $1060{\cdot}754296875$ Total: 2324·293359375= 2324	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Volume formula with some substitution *Mid Partial Credit:* Volume of 2 spheres GP formula with some substitution *High Partial Credit:* Volume of 5 spheres G P formula fully substituted
(b) (i)	$4\pi r^2 = 503 \Rightarrow r = \sqrt{\frac{503}{4\pi}} = 6{\cdot}33$ Height $=120 - 2(6{\cdot}33) = 107{\cdot}3$ **Or** $\frac{4}{3}\pi r^3 = 1060{\cdot}754 \text{ from(a)}$ $r = 6{\cdot}326$ Height : $120 - 2(6{\cdot}326) = 107{\cdot}348 = 107{\cdot}3$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* $4\pi r^2 = 503$ $\frac{4}{3}\pi r^3 =$ volume from (a) *High Partial Credit:* *r* found
(b) (ii)	A: $\pi 1^2 h = 71{\cdot}3\pi \Rightarrow h = 71{\cdot}3$ Height difference: $107{\cdot}3 - 71{\cdot}3 = 36$ $\frac{36}{4} = 9$ step up in each bar. **Or** $T_5 = 71{\cdot}3 + 4d = 107{\cdot}3 \rightarrow d = 9$ Height of each bar (in cm) $71{\cdot}3,\ 80{\cdot}3,\ 89{\cdot}3,\ 98{\cdot}3,\ 107{\cdot}3$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Vol formula with some substitution $\pi r^2 h = 71{\cdot}3\pi$ *Mid Partial Credit:* Height of bar A *High Partial Credit:* Difference in height between bar A and bar E

(c)	$150 - (20 + 20 + 9(2)) = 92$ $\frac{92}{8}$ cm or $11{\cdot}5$ cm	**Scale 15C (0, 4, 11, 15)** *Low Partial Credit:* Recognises 8 equal divisions Indicates subtraction of one relevant length 9×2 *High Partial Credit:* $150 - 40 - 18$ or equivalent
(d)	$V_B = 1{\cdot}75\left(\frac{4}{3}\pi 3^3\right) = 63\pi$ $V_B = \frac{4}{3}\pi r^3 = 63\pi \Rightarrow r_b = 3{\cdot}62$ cm  $\lvert XY\rvert = 1 + 11{\cdot}5 + 1 = 13{\cdot}5$ $\lvert ZW\rvert = (9 - 3) + 3{\cdot}62 = 9{\cdot}62$ $\lvert TW\rvert = \sqrt{13{\cdot}5^2 + 9{\cdot}62^2} = 16{\cdot}576$ **Or** $\tan\angle WTZ = \frac{9{\cdot}62}{13{\cdot}5} \rightarrow \lvert\angle WTZ\rvert = 35{\cdot}459°$ $\cos\angle WTZ = \frac{13{\cdot}5}{\lvert TW\rvert} \rightarrow \lvert TW\rvert = 16{\cdot}576$ The rod is: $\lvert TW\rvert - 3 - 3{\cdot}62$ $= 16{\cdot}576 - 3 - 3{\cdot}62 = 9{\cdot}95$ $\lvert TW\rvert = 10$	**Scale 5B (0, 2, 5)** *Partial Credit:* V_B formulated with some substitution $\lvert XY\rvert$ formulated $\lvert TW\rvert$ evaluated rod $= \lvert TW\rvert - r_b - r_a$ formulated with 2 relevant values

Q8	Model Solution – 60 Marks	Marking Notes
(a) (i)	$z_1 = \frac{4{\cdot}6 - 4{\cdot}64}{\frac{0{\cdot}12}{\sqrt{10}}} = -1{\cdot}05409$ $z_2 = \frac{4{\cdot}7 - 4{\cdot}64}{\frac{0{\cdot}12}{\sqrt{10}}} = 1{\cdot}581138$ $p(-1{\cdot}05 < z < 1{\cdot}58)$ $= 0{\cdot}9429 - (1 - 0{\cdot}8531)$ $= 0{\cdot}796$ or 79·6%	**Scale 20D (0, 5, 10, 15, 20)** *Low Partial Credit:* z_1 formulated with some correct substitution z_2 formulated with some correct substitution *Mid Partial Credit:* z_1 and z_2 fully substituted *High Partial Credit:* –1·05 and 1·58 or equivalents
(a) (ii)	Confidence Interval: $\hat{p} \pm 1{\cdot}96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $0{\cdot}81 - 1{\cdot}96\sqrt{\frac{0{\cdot}81 \times 0{\cdot}19}{400}} \leq p$ $\leq 0{\cdot}81 + 1{\cdot}96\sqrt{\frac{0{\cdot}81 \times 0{\cdot}19}{400}}$ $0{\cdot}77155 \leq p \leq 0{\cdot}848445$ $0{\cdot}77 \leq p \leq 0{\cdot}85$	**Scale 15 C (0, 4, 11, 15)** *Low Partial Credit:* CI formulated with some correct substitution $1{\cdot}96$ $\hat{p} \pm \frac{1}{\sqrt{n}}$ incomplete or completed *High Partial Credit:* CI fully substituted

(b) (i)

Statement	**Always True**	**Sometimes True**	**Never True**
1. When forming confidence intervals (for fixed *n* and $\hat{p}$), an increased confidence level implies a wider interval.	✓		
2. As the value of $\hat{p}$ increases (for fixed *n*), the estimated standard error of the population proportion increases.		✓	
3. As the value of $\hat{p}(1-\hat{p})$ increases (for fixed *n*), the estimated standard error of the population proportion increases.	✓		
4. As *n*, the number of people sampled, increases (for fixed $\hat{p}$), the estimated standard error of the population proportion increases.			✓

Scale 5C (0, 2, 4, 5)
Low Partial Credit:
Any 1 correct

High Partial Credit:
Any 2 correct

(b) (ii)+ (iii)	$M = \hat{p} - \hat{p}^2$ $\frac{dM}{d\hat{p}} = 1 - 2\hat{p}$ $\frac{dM}{d\hat{p}} = 1 - 2\hat{p} = 0$ $\hat{p} = \frac{1}{2}$ $M_{max} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ **Or** $f(p) = \hat{p} - \hat{p}^2 = -(\hat{p}^2\ \hat{p})$ $= -[(\hat{p}^2 - \hat{p} + (-\frac{1}{2})^2) - (-\frac{1}{2})^2]$ $= \frac{1}{4} - \left(\hat{p} - \frac{1}{2}\right)^2$ $\hat{p} = \frac{1}{2} \quad M_{max} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ $1{\cdot}96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $= 1{\cdot}96\sqrt{\frac{1}{4n}} = 0{\cdot}03464 = 3{\cdot}46\%$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Any relevant calculus $\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{800}}$ Effort at completing the square *Mid Partial Credit* $\hat{p} = \frac{1}{2}$ or equivalent *High Partial Credit:* Maximum value
(c)	$20\,000 + \frac{20\,000(1{\cdot}01)}{1{\cdot}024} + \frac{20\,000(1{\cdot}01^2)}{1{\cdot}024^2} + \frac{20\,000(1{\cdot}01^3)}{1{\cdot}024^3} + \cdots + \frac{20\,000(1{\cdot}01^{25})}{1{\cdot}024^{25}}$ $20\,000\left[1 + \frac{1{\cdot}01}{1{\cdot}024} + \frac{1{\cdot}01^2}{1{\cdot}024^2} + \frac{1{\cdot}01^3}{1{\cdot}024^3} + \cdots + \frac{1{\cdot}01^{25}}{1{\cdot}024^{25}}\right]$ $a = 1,\ r = \frac{1{\cdot}01}{1{\cdot}024} = \frac{505}{512}\ , n = 26$ $20000\left[\frac{\left(1 - \frac{505^{26}}{512^{26}}\right)}{1 - \frac{505}{512}}\right] = €440\,132{\cdot}40$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* 20000(1·01) or $\frac{20000}{1{\cdot}024}$ *Mid Partial Credit:* $\frac{20\,000(1{\cdot}01)}{1{\cdot}024}$ or similar term Correctly handles inflation element or completes correctly present values element and finishes *High Partial Credit:* GP with *a*, *r* and *n* identified **Note**: Treat $n = 25$ as a misreading

<table>
<tr><th>Q9</th><th>Model Solution – 40 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>

$$\frac{10}{\sin 15} = \frac{30}{\sin x}$$

$$\sin x = \frac{30 \ \sin 15}{10}$$

$$\sin x = 0{\cdot}77645$$

$$x = 51°$$

</td><td>

Scale 10C (0, 3, 7, 10)
Low Partial Credit:
Sine rule formulated with some substitution

High Partial Credit:
$\sin x$

</td></tr>
<tr><td>(b)
(i)</td><td>period = 2π   Range = [10, 30]</td><td>

Scale 5C (0, 2, 4, 5)
Low Partial Credit:
Period **or** range correct

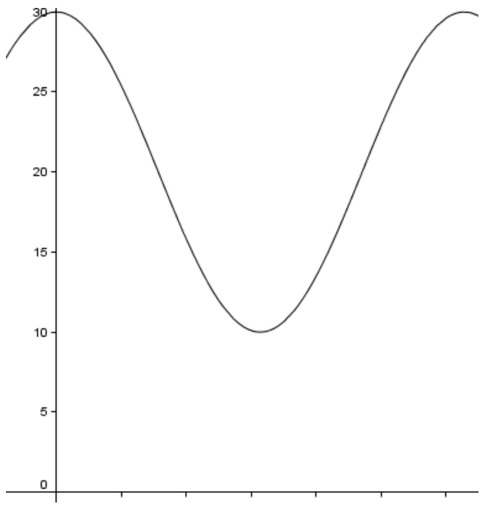
High Partial Credit:
Period correct and range partly correct
Period **and** range in incorrect order

</td></tr>
<tr><td>(b)
(ii)</td><td colspan="2">
<table>
<tr><td>α</td><td>$0°$</td><td>$90°$</td><td>$180°$</td><td>$270°$</td><td>$360°$</td></tr>
<tr><td>$f(\alpha)$
(cm)</td><td>30</td><td>18·28</td><td>10</td><td>18·28</td><td>30</td></tr>
</table>
</td></tr>
<tr><td></td><td></td><td>

Scale 5C (0, 2, 4, 5)
Low Partial Credit:
1 correct new value

High Partial Credit:
2 correct new values

</td></tr>
</table>

Q9	Marking Notes	
(b) (iii)		**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* 1 point from table plotted *High Partial Credit:* 3 points from table plotted
(b) (iv)	Answer: diagram 2 refer to the steepness of their graph at the three corresponding points **or** rely on the original geometry of the situation: the closer ∠CDO is to a right angle the more the connecting rod will get pulled or pushed by a small change in the crank angle	**Scale 5B (0, 2, 5)** *Partial Credit:* Diagram 2 identified but without reason or with invalid reason

(c)	$r^2 = 36^2 + (31+r)^2$ $- 2(36)(31+r)\cos 10°$ $r^2 = 1296 + 961 + 62r + r^2$ $-(2232\cos 10° - 72r\cos 10°)$ $8{\cdot}906r = 58{\cdot}91$ $r = 6{\cdot}62$ $r = 7$ **Or** $\lvert BX\rvert^2 = 36^2 + 31^2 - 2 \times 36 \times 31\cos 10°$ $\lvert BX\rvert^2 = 58{\cdot}91$ $\lvert BX\rvert = 7{\cdot}675$ $\dfrac{\sin 10°}{7{\cdot}675} = \dfrac{\sin \angle BXA}{36}$ $\angle BXA = 125{\cdot}462° \Rightarrow \angle BXO = 54{\cdot}53795°$ ΔBXO is isosceles $\Rightarrow \angle BOX = 70{\cdot}924°$ $\dfrac{\sin 70{\cdot}924\,°}{7{\cdot}675} = \dfrac{\sin 54{\cdot}53795\,°}{r}$ $r = 6{\cdot}6145$ $r = 7$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* Cosine rule formulated with some substitution $(31 + r)$ *High Partial Credit:* Relevant equation in r