

7/10

Zeeman Effect

State the result and compare to accepted value.

REVISED: 10/

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The report should be written in the format of a scientific article for a journal publication.

It is not intended as a homework assignment in which you simply answer questions. The questions in the writeups are only guidelines to help you discuss and put the theory and experimental results in a broader context.

Please consult the journals in the lab or see for example: Physical Review Letters, Science, Nature <http://journals.aps.org/prl/>. The experiments you perform in this class are intended to challenge your understanding of physics and to give you the means to explore some physical principles. They are not recipes to be followed blindly. More thought should go into understanding the theory, the limitation of the experiment, implicit approximations etc. Effort should be put into planning the experiment, its execution, data collection, analysis, experimental controls etc. .

ABSTRACT

We investigate the optical Zeeman effect through electrical discharge tubes containing samples of neon. We record spectra of the samples and report on the 404.7 nm and 585.3nm line in neon, in both σ and π polarization. We also report the effective Lande' g factor. We also report the electron charge to mass, or e/m ratio.

INTRODUCTION

We investigate the splitting of the energy levels of an atom in an applied magnetic field, known in atomic physics as the Zeeman effect. This effect is observed experimentally as a splitting in the lines of an atomic emission spectrum. The splitting occurs due to the interaction of the magnetic moment (μ) of the atom with the magnetic field \mathbf{B} , which shifts the energy of the atomic levels by Eq. 1:

$$\Delta E = -\mu B$$

From a quantum mechanical standpoint, the electron experiences the effects of orbital angular momentum as well as intrinsic angular momentum, or spin. These are denoted in atomic notation as L and S respectively, giving rise to a total angular momentum J . The electron traveling in a circular orbit in a plane perpendicular to the z axis has a magnetic moment (μ) _{Lz} that is related to the orbital angular momentum L by Eq. 2:

$$\mu_{Lz} = (e/2m) L_z$$

For electron spin S , the magnetic moment is given by Eq. 3:

$$\mu_{Sz} = g(e/2m) S_z$$

where g is taken to be exactly 2 for the purpose of this experiment. Therefore the total magnetic moment results in Eq. 4:

$$\mu_{Jz} = g_L(e/2m) J_z$$

where J_z is the z -component of the total angular momentum. g_L is the Landé g factor, calculated by Eq. 5:

$$g_L = 1 + (J(J+1) + S(S+1) - L(L+1))/(2J(J+1))$$

In this lab, g_L for mercury is calculated to be 1, and for neon it is calculated to be 3/2.

explain how this is derived. what approximations go into this expression? State the result and compare to accepted value.

To predict Zeeman splitting, we anticipate an energy shift of, finally, Eq. 6:

$$\Delta E = -g_L(eh/4\pi m)m_J B = -g_L \mu_B m_J B$$

where μ_B is the Bohr magneton, equal to 5.788e-5 ev/T.

In Zeeman's original theory he predicted that energy levels split into multiple levels. He also predicted that the splitting would depend on the strength of the magnetic field. This is the same splitting that we see throughout this lab.

In this lab we are interested specifically in the optical Zeeman effect, in which samples in an electrical discharge tube are excited by the external B field. In this setup, both the excited and ground state energy levels experience splitting, and the photons emitted during transition are produced at slightly different energies. The expected change in photon energy is therefore Eq. 7:

$$\Delta h\nu = g_{eff}\mu_B B$$

where g_{eff} refers to the effective Lande' g factor resulting from the shift in g during transition.

In this report we investigate four important transitions: $6s7s-3S \rightarrow 3P_2$, $6s7s-3S_1 \rightarrow 3P_1$, and $6s7s-3S_1 \rightarrow 3P_0$ in Mercury (corresponding to 546.1 nm, 435.8nm, and 404.7nm lines), and the 585.3nm line in Neon. A magnetic field is applied to the discharge tubes, and they are observed through sigma (circular) and pi (linear) polarization; we also report measurements of the spectra in zero external field. From this we are able to determine the effective Lande' g-factor as well as the charge-mass ratio of the electron.

APPARATUS

The main components of the apparatus include a Spex 1000M 1 meter grating Spectrometer system, Ne and Hg discharge tubes, high voltage power supply, electromagnet, a Polaroid filter, a Gaussmeter, and spectrum analysis computer software.

To produce light emitted from the element of each gas, a gas discharge tube is attached to a high voltage supply, which produces a large voltage across the discharge tube that accelerates free electrons up to some kinetic energy that is enough to excite the electrons in the gas to higher orbital energies, when these electrons return to the lower energy state they release photons whose energy (and corresponding wavelength) are dependent on the difference between orbitals. These differences in orbital energies will differ for different elements, allowing for identification of the element based off of what wavelength photons it expels when returning from being excited. For this lab setup, we processed these excited photons through a polarizer that can be set to either 24° , for σ spectra's, or 114° , for π spectra's allowing for a more focused look at the different types of photons being emitted. The polarized light from the gas travels through the diffraction grating in the spectrometer, and is separated into its component wavelengths. The diffraction grating is rotated slightly, so that only a specific wavelength window of light reaches the exit slit and then lastly this information was transferred between our spectrometer system to our computer so that our "Jabon Yvon-Spex "Spectramex" control and data acquisition software" could be used to analyze the resulting data. So that we could observe the Zeeman effect (which requires an external magnetic field) an iron core electromagnet was connected to a Varian 6121 30 ampere Variac magnet power supply that allowed us to vary the

magnetic field by setting the current. Lastly a RFL Industries model 904 Gaussmeter with Hall probe was used to measure the magnetic field of the electromagnet. The setup is displayed in Figure 0.

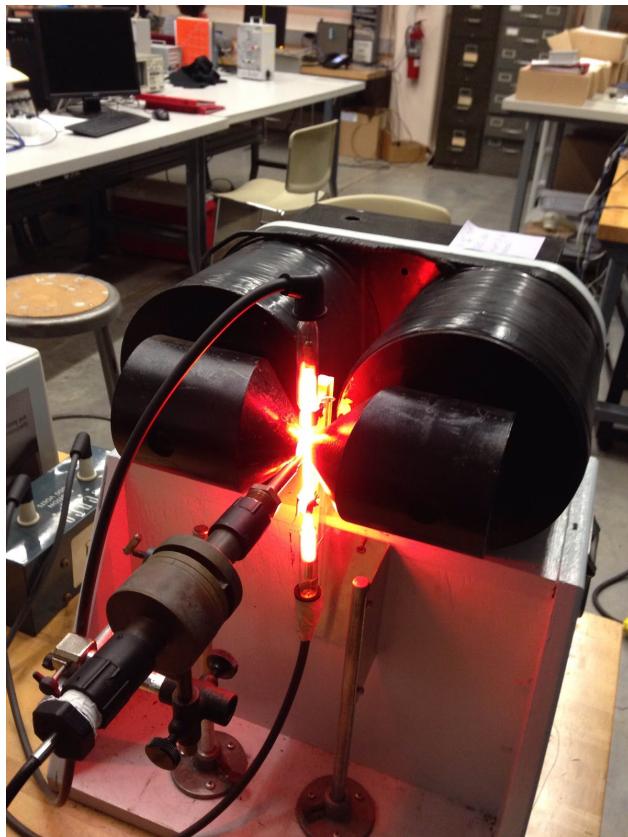


Figure 0, Neon gas discharge tube glowing from the connected high voltage supply connected to either endcap while mounted in between the electromagnet. In the bottom left corner you can observe the polarizer that then connects to the spectrometer system.

DATA AND ANALYSIS

Q1. 

In this lab, we concern ourselves largely with calculation and measurement of the anomalous Zeeman effect observed in the Hg spectra under the influence of a magnetic field. For the Hg spectra we consider three transitions in particular, namely from the 3S_1 state to 3P_0 , 3P_1 , and 3P_2 states. These three transitions account for the 404.7 nm (violet), 435.8 nm (blue) and 546.1 nm (green) lines in the Hg spectrum. We can calculate the energy splitting of these three cases individually using Equation XXX. In doing so, and after considering that the changes in values of m_j must be either 0, -1, or 1, we arrive at the results displayed in Figures 1, 2, and 3 which present the g_{eff} values for the three aforementioned transitions. Figures 1 through 3 are color coded such that the g_{eff} values for the described transitions are given in the red and blue cells, where blue cells correspond to light emitted with π (parallel) polarization and the red cells correspond to light emitted with σ (circular) polarization. The cells which are marked with an X

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are those transitions which are forbidden by quantum mechanical selection rules. In these cases $\Delta m_l \neq 0$, or $+/- 1$, thus the transition is forbidden.

Figure 1: g_{eff} Values for 3S_1 to 3P_0 (violet, 404.7 nm) Transition (Hg)

			<u>m_l Initial</u>	
		-1	0	1
<u>m_l Final</u>	0	2	0	-2

Figure 2: g_{eff} Values for 3S_1 to 3P_1 (blue, 435.8 nm) Transition (Hg)

		<u>m_l Initial</u>		
		-1	0	1
	-1	1/2	-3/2	X
<u>m_l Final</u>	0	2	0	-2
	1	X	3/2	-1/2

Figure 3: g_{eff} Values for 3S_1 to 3P_2 (green, 546.1 nm) Transition (Hg)

		<u>m_l Initial</u>		
		-1	0	1
	-2	-1	X	X
	-1	1/2	-3/2	X
<u>m_l Final</u>	0	2	0	-2
	1	X	3/2	-1/2
	2	X	X	1

Q2.

Measuring the g_{eff} of Hg and Ne

Since we know that the energy splitting of the Zeeman effect is given by Equation XXX, we set about to measure this value experimentally and compare to our calculated values. We did so for both of the Hg 407. nm (violet) line and the Ne 585.3 nm (yellow) line. In the presence of a ?? why only these lines? observed the splitting of these spectral lines into two peaks. As we increased

the magnitude of the field we noticed the spread in the positions of the centers of these peaks. In order to measure the dependence of splitting of the energy levels on the strength of the magnetic field, we first measured the wavelength at which the spectral emission was peaked. A quick manipulation of Equation XXX yields

$$y(B) = hc(1/\lambda_f - 1/\lambda_i)/\mu_B = g_{\text{eff}} B$$

Thus we see that by taking λ_i to be the position of the peak with no magnetic field applied and by measuring how far from this peak one of the two split peaks is we can measure the dependence of splitting on B. To measure the displacement of these peaks we varied the B field and resolved the spectra of each of the samples. We then utilized the peak-fitting function in Origin8 to fit Gaussian functions to the two peaks of our spectra. We took the location of these peaks to be the center of the Gaussian functions. We took the uncertainty in λ values to be the uncertainty of the position of the peaks given by the fit function. Using these values we arrived at the estimated uncertainty of the left hand side of the equation. Namely since:

$$\sigma_{y(b)} = \frac{hc}{\mu_B} \sqrt{\left(\frac{\delta f}{\lambda_f}\right)^2 (\sigma_{\lambda_f})^2 + \left(\frac{\delta f}{\lambda_i}\right)^2 (\sigma_{\lambda_i})^2} \Rightarrow \sigma_{y(b)} = \frac{hc}{\mu_B} \sqrt{\left(\frac{1}{\lambda_f}\right)^4 * (\sigma_{\lambda_f})^2 + \left(\frac{1}{\lambda_i}\right)^4 * (\sigma_{\lambda_i})^2}$$

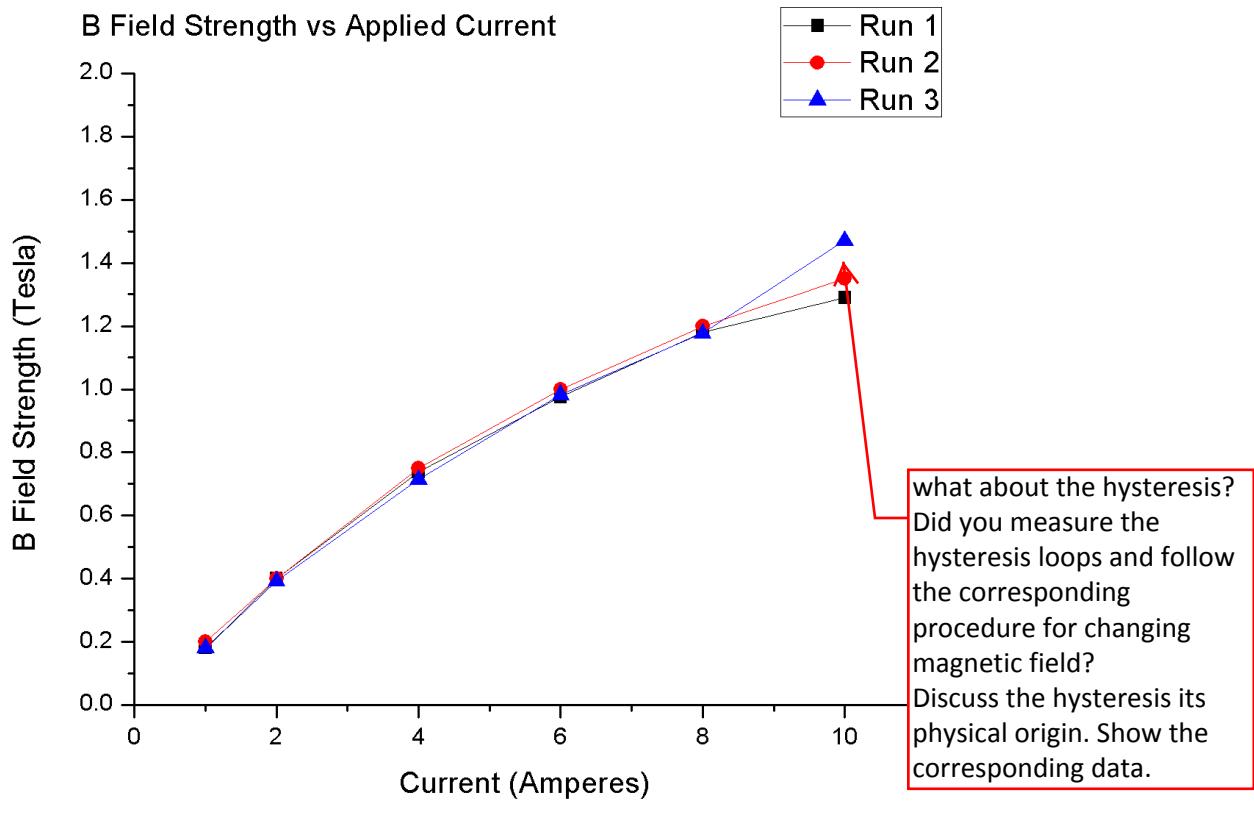
Fitting a line to our $y(B)$ data points serves the function of finding out g_{eff} as this value would be given by the slope of our $y(B)$ line. The plots corresponding to Hg 404.7 nm line and the Ne 585.3 nm line are shown in Figures 4 and 5 respectively.¹ Figure 4 shows the response of the magnetic field strength to the applied current. We see that for three separate runs where the current through the magnet was left for applied for various amounts of time the measurements of the B field agree very well for values of amperage ≤ 8 amps.²

Figure 4: The B Field Response vs Applied Current

¹ Note that there is an inherent ambiguity in the choice of λ_f : since the splitting occurs for both positive and negative values of m_j with the same magnitude, we could have chosen the position of the peak which shifted to a higher wavelength or the one with lower wavelength. In order to do a systematic study of the energy level splitting we chose to always use λ_f as the higher of the two wavelengths (namely the one which corresponds to a lower frequency and therefore energy shift). This choice accounts for the minus sign in the measurement of g_{eff} .

² In order to make sure that the magnetic fields strength is similar for various measurements we only made measurements of even current values from 0 to 8 amps

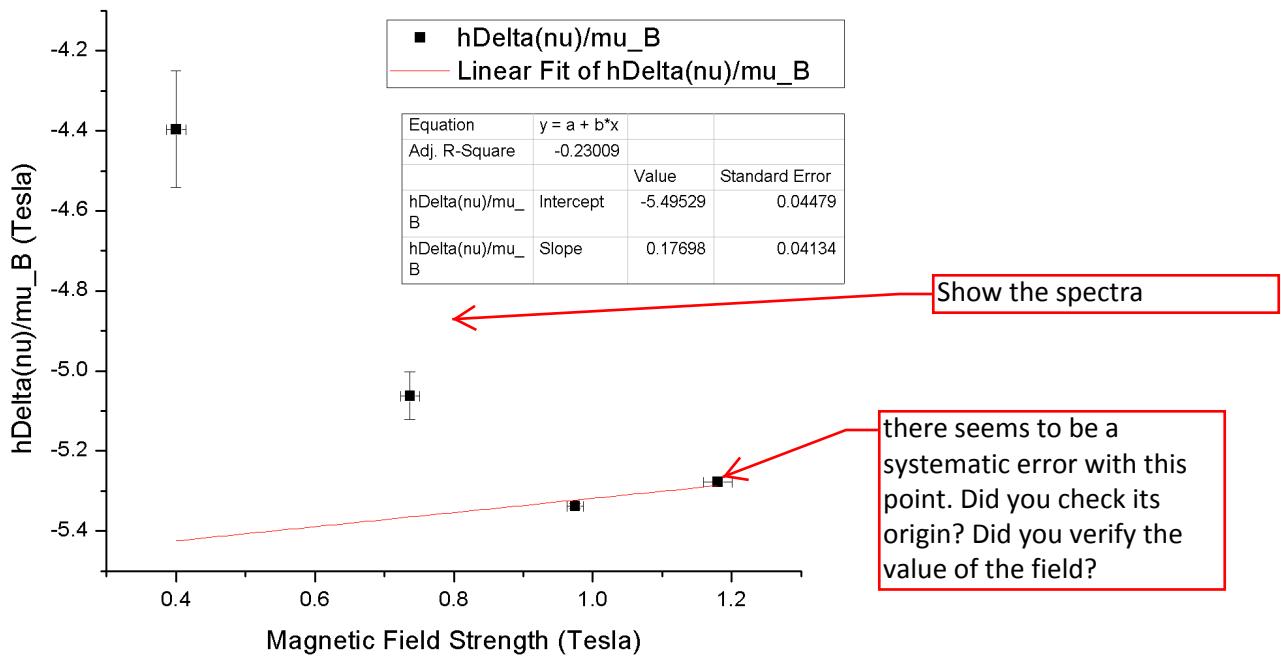
what about the hysteresis?
How was this taken into account?



Immediately we can compare our measured and calculated values of g_{eff} for the 404.7 nm Hg. Upon inspection we notice that the measured value of $g_{\text{eff}} = 0.17698 \pm 0.04134$ doesn't agree within experimental error with the calculated value of 2. In fact in Figure XXX it appears as though there was an error in measuring the last datum, the one where the B field was increased to ~ 1.2 Tesla.

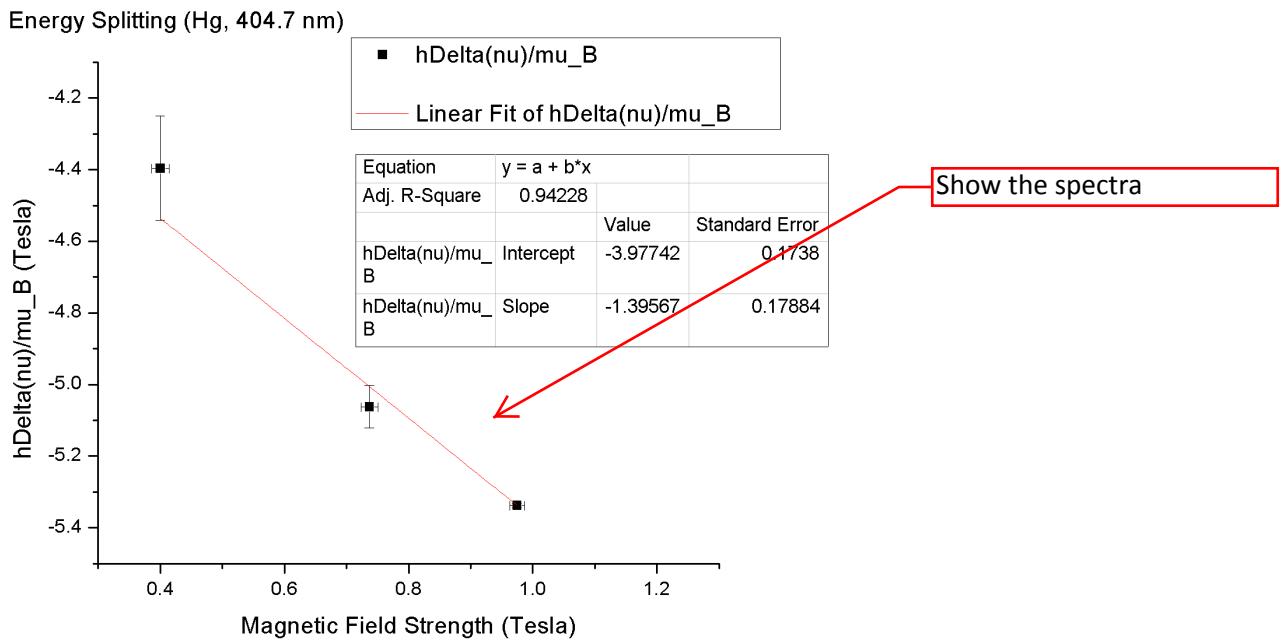
Figure 5: Splitting of Energy of 404.7 nm line in Hg (All Data)

Energy Splitting (Hg, 404.7 nm)



If we exclude this point which dominates the linear fit and fit a line to our data again we resolve a far better, but still disagreeable value of $g_{\text{eff}} = -1.398567 \pm 0.17884$. Although this value still disagrees with the expected value of 2, we see that the linear fit to our data is at least qualitatively better than before. This fit is plotted in Figure 6.

Figure 6: Splitting of Energy of 404.7 nm line in Hg (One Datum Removed)



One source of systematic error was from the improper calibration of the spectrum. Upon measuring the unshifted peak in the wavelength spectrum at $404.64 \pm 1.6165 \times 10^{-5}$ nm we expected that, when applied, the B field would split this peak *symmetrically* around this value. However as we measured, we found that this was not the case. Instead of having one of the peaks shifting to a lower wavelength and one to a higher wavelength symmetrically around 404.64 nm, we found that peaks were both shifted upwards of 404.64 nm. In measuring the energy shift (which depends on the shifted wavelength and the initial wavelength, we introduced a large systematic uncertainty to the measurement of g_{eff} , ultimately influencing its quality. Yet another source of uncertainty in the outcome is the improper measurement of the B field. Since the B field strength will directly influence the energy level splitting, any uncertainty in the measurement of the B field will lead directly to uncertainty in the measurement of the g_{eff} .

Q3. ????

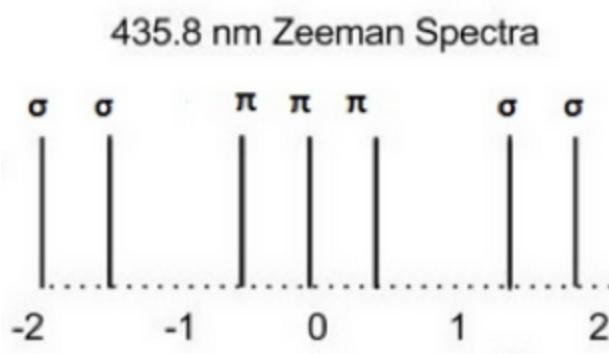
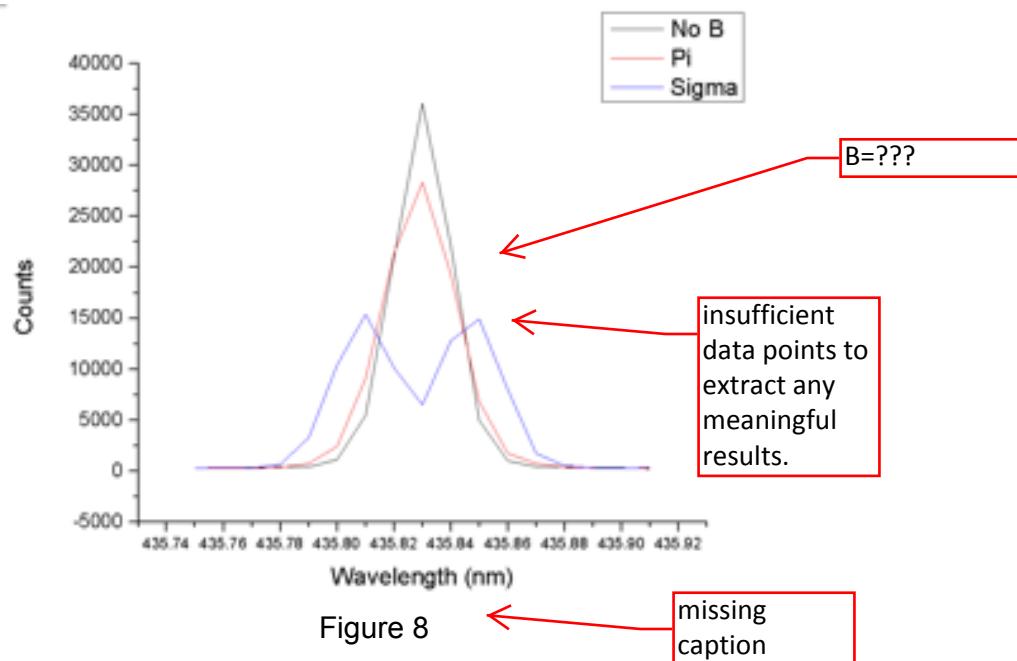


Figure 7



Experimentally for the sigma splitting we observed two wide splits instead of the expected four, however based off of how wide the two splits are, it is easy to reason that the resolution of our experiment prevented us from being able to tell the difference between the two peaks that are close to each other theoretically (with Geff values of +2 and +3/2) if the two peaks were to be combined into one peak, we should observe them to have a Geff of 1.75 and -1.75 (the average of the two).

For the Pi splitting, we observe that with our resolution we still see only one peak as we did without the magnetic field, but the width of the peak widens, which occurs because the single peak at 435.8 nm is actually splitting into three close peaks that we observe as only one peak. If theoretically the three peaks were to combine, we would expect this large peak to have a Geff of 0.

To calculate the Geff the following equation was used:

$$g_{eff} = |\Delta\lambda| \left(\frac{hc}{\lambda^3 \mu_B B} \right)$$

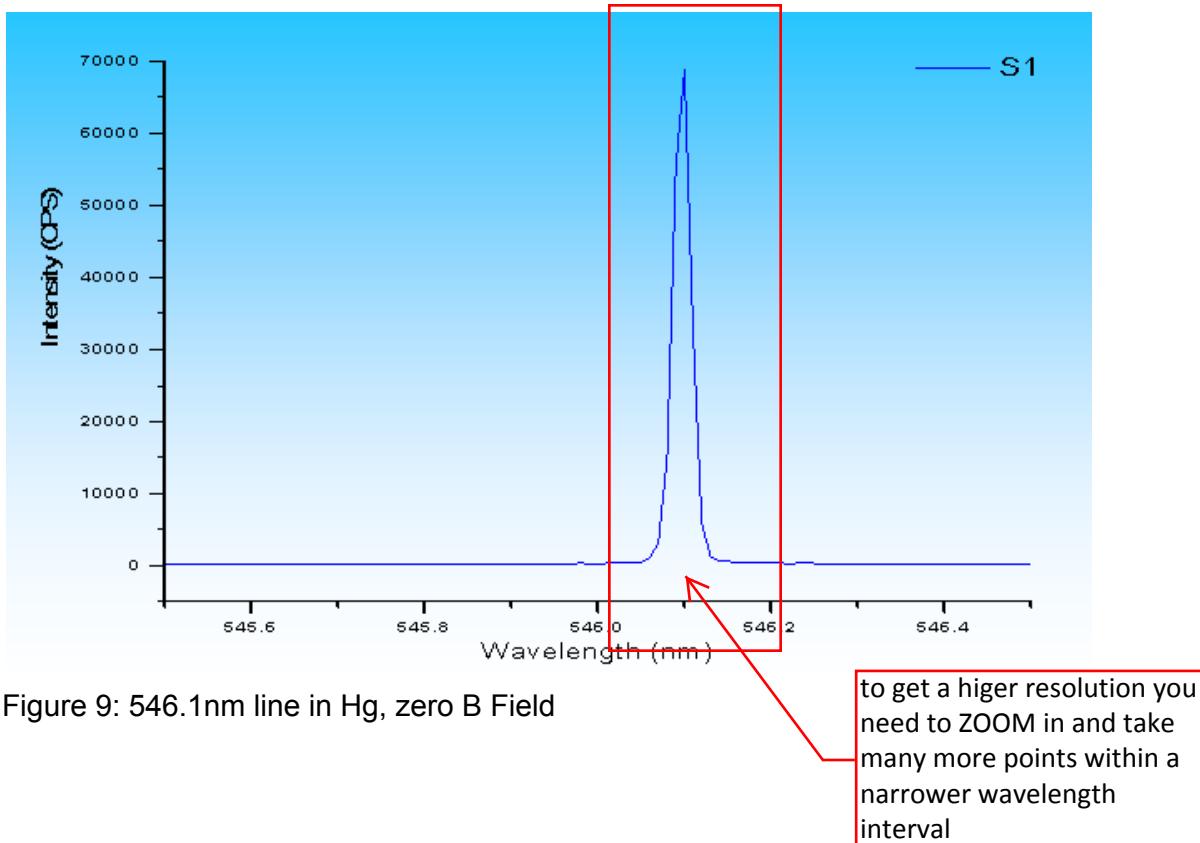
how was this calculated?
How many data points (field values) were used?

Polarization	Peak Location	g_{eff}
Pi	435.83nm +- 0.011 nm	0 +- 0.827
Sigma	435.81nm +- 0.008 nm	-1.503 +- 0.601
Sigma	435.85nm +- 0.008 nm	1.503 +- 0.601

Table 1

We find that for all three of the calculated “averaged” peaks, our Geff's are within a standard deviation of the expected values, however the error in these calculations are very high. To get a more precise calculation it is important to have a much higher resolution so that the uncertainty can be smaller.

Q4. ????



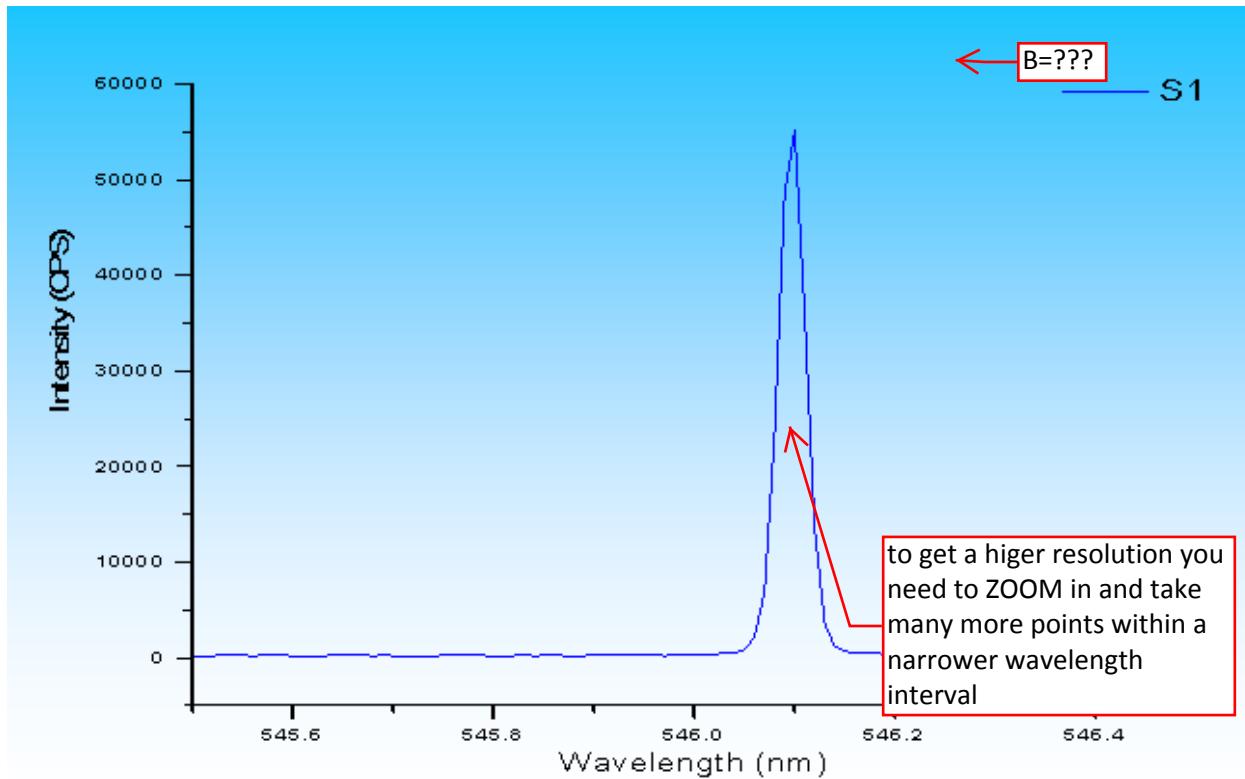


Figure 10: 546.1nm line in Hg, Pi polarization

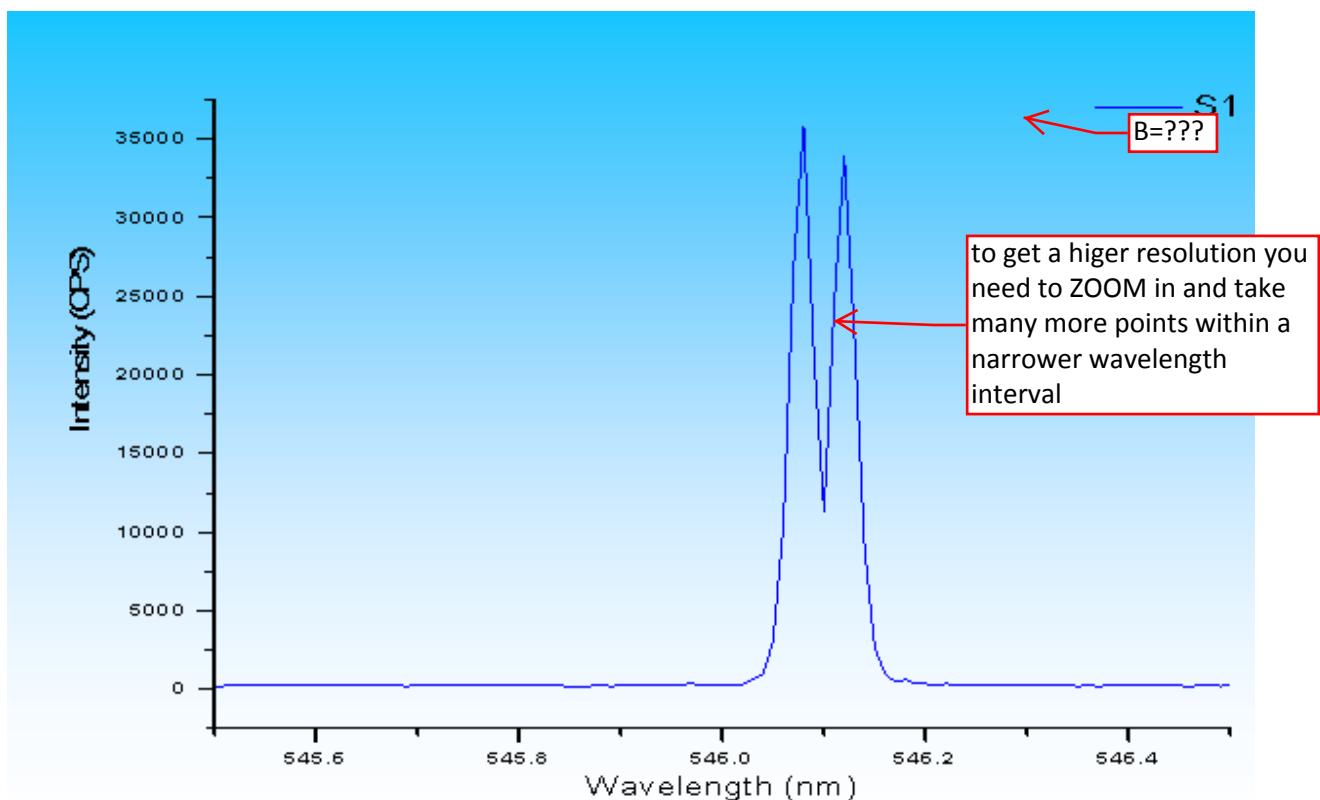


Figure 11: 546.1nm line in Hg, Sigma Polarization

Figure 9 shows the 546.1 nm line of Hg, or the $6s7s-3S1 \rightarrow 6p6s-3Ps$ transition in zero B field, figure 10 shows the same transition in the pi polarization with an applied B field, and figure 11 displays this transition in the sigma polarization. For this line we expect to see quite a bit of splitting in the presence of a B field - six sigma lines, and three pi lines. However, due to poor resolution of our equipment, we detect only two very broad (appx. Gaussian) peaks of several nearby split lines. The measurement in the zero B field displays the line in its proper location, at 546.1nm, and the pi polarization, shows a slight broadening in the peak width but ultimately no splitting. The sigma polarization very clearly shows splitting into at least two energy levels of a roughly 0.5 Angstrom distance from the original peak (corresponding to about 2.5 eV), which is approximately correct according to equation $\Delta E = \frac{eB}{2m_e}$. Though we do see splitting, a substantial amount of error due to experimental set up prevents us from qualitatively analyzing these spectra.

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Q5.

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Next, one can inspect the results of the measurement of the g_{eff} of the transition which produces the 585.3 nm yellow line in Ne using the same method as described for measuring the g_{eff} of Hg. After doing so we again saw the same difficulties as with Hg. Although this time the fit converged for the data, we see that the final datum doesn't follow the same linear correlation as the other data in the Ne measurements. With these data we measure the $g_{eff} = 1.25736 \pm 0.04719$. The plot of the measurements of the energy splitting of the Ne 585.3 nm line is show in Figure 12.

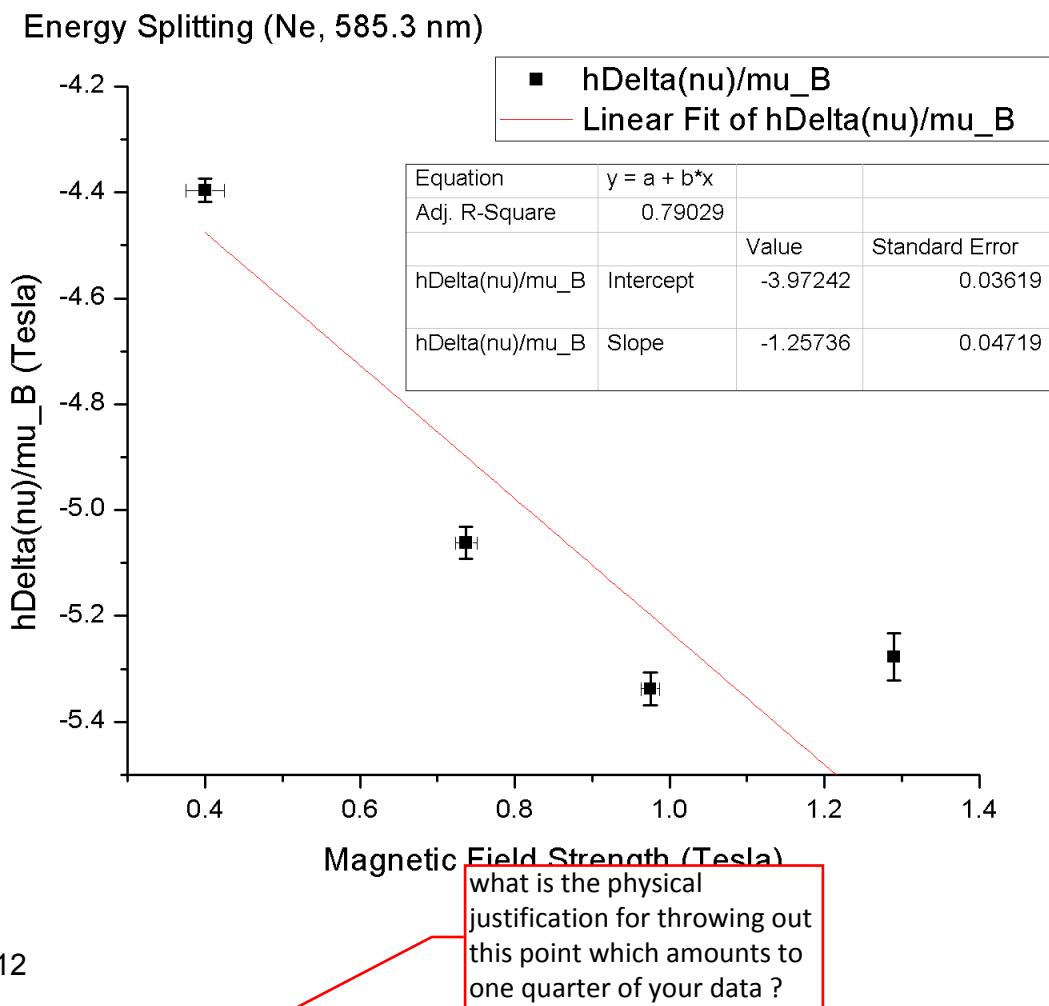


Figure 12

Once more, upon excluding this data point we see that we resolve a linear fit which agrees nicely with our data. This fit is shown in Figure 13 below. If we remove this datum and re-fit the line to our data, we notice that the measured value of g_{eff} changes slightly to $g_{\text{eff}} = 1.69205 \pm 0.0634$. The same uncertainties occur in our measurements here as in the measurements of the Hg spectrum.

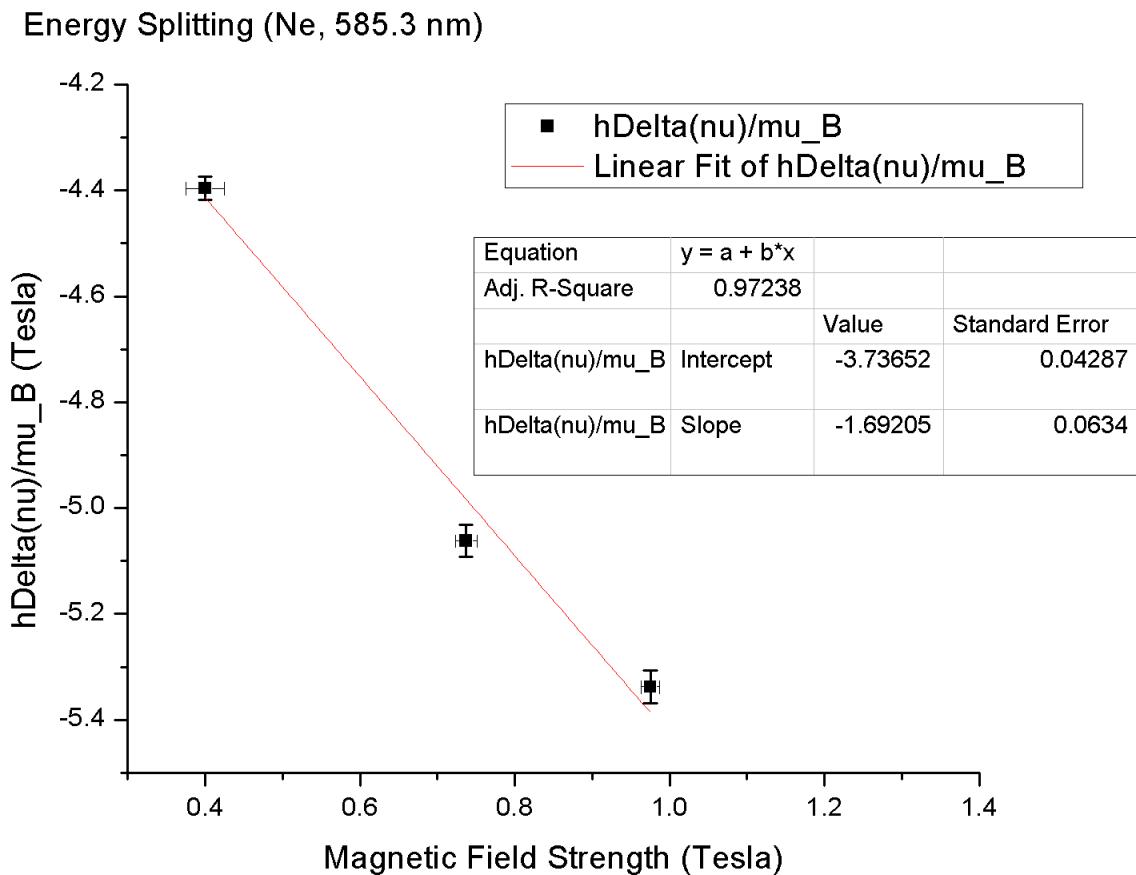


Figure 13

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Q6.

We can calculate the charge to mass ratio of the electron using the definition of the Bohr magneton and our measured shifts in wavelength (and their corresponding energies).

We can manipulate Equation 7:

$$\Delta h\nu = g_{eff}\mu_B B$$

where we use the definition of the Bohr magneton, $\mu_B = (e\hbar)/(4\pi m)$, so that

$$e/m = 4\pi\Delta\nu/(g_{eff}B)$$

where we take the effective Lande g factor to be 2.

In the table below, we display values for the magnetic field measurements as well as the measured shift in frequency and the corresponding calculation for e/m with an estimated error.

B (T)	0.393	0.714	0.982	1.177
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$ \Delta v $ (1e10 s ⁻¹)	6.130 ± 0.203	7.059 ± 0.222	7.443 ± 0.891	7.359 ± 0.865
e/m (1e11 C/kg)	-9.800 ± 0.325	-6.210 ± 0.195	-4.760 ± 0.570	-3.920 ± 0.462

Table 2.

This gives us an estimated charge to mass ratio of $-5.748 \pm 2.288 \text{ e}11 \text{ C/kg}$, which is a factor of 3.3 different from the expected value. Since this is within the same order of magnitude and we had significant issues in the calibration of our equipment, we reason that this is a reasonable measurement.

not quite. One
can do much
better in this lab.



CONCLUSION

In this lab we were able to effectively examine the Normal and Anomalous Zeeman splitting effect for mercury and neon. From this, we were able to determine the charge to mass ratio of the electron and demonstrate the importance of quantum effects on atomic spectra. Sources of error were likely due to human error as well as imprecision in the calibration of the apparatus in addition to several physical issues with the machines. Overall, we were able to investigate to a fair certainty the effect of Zeeman energy level splitting.

