用户A、B、C、D对商品1、2、3、4、5、6的评分记录如下表1，其中评分数值介于1~5，请使用基于用户的协同过滤算法，参考公式1和公式2：

评分	商品1	商品2	商品3	商品4	商品5	商品6
用户A	4	3	2	?	4	2
用户B	?	3	2	?	1	2
用户C	4	4	4	5	3	?
用户D	?	4	3	3	2	?

$$s(u,v)=\frac{\sum_{i\in I_u\cap I_v}(r_{u,i}-\overline{r}_u)(r_{v,i}-\overline{r}_v)}{\sqrt{\sum_{i\in I_u\cap I_v}(r_{u,i}-\overline{r}_u)^2}\sqrt{\sum_{i\in I_u\cap I_v}(r_{v,i}-\overline{r}_v)^2}} \quad (公式1)$$

$$p_{u,i}=\overline{r}_u+\frac{\sum_{u'\in N}s(u,u')(r_{u',i}-\overline{r}_{u'})}{\sum_{u'\in N}|s(u,u')|} \quad (公式2)$$

（1）为用户B推荐一个他没评价过的商品（写出过程，保留四位小数）

$\overline{r}_A=\frac{4+3+2+4+2}{5}=3,\overline{r}_B=\frac{3+2+1+2}{4}=2,\overline{r}_C=\frac{4+4+4+5+3}{5}=4,\overline{r}_D=\frac{4+3+3+2}{4}=3$

1. **对于商品1：**

用户B的邻居集合$N=\{A,C\}$；

① 对于用户B、A，共同评价过的商品集合$I_B\cap I_A=\{2,3,5,6\}$

$$\begin{aligned}s(B,A)&=\frac{(r_{B,2}-\overline{r}_B)(r_{A,2}-\overline{r}_A)+(r_{B,3}-\overline{r}_B)(r_{A,3}-\overline{r}_A)+(r_{B,5}-\overline{r}_B)(r_{A,5}-\overline{r}_A)+(r_{B,6}-\overline{r}_B)(r_{A,6}-\overline{r}_A)}{\sqrt{(r_{B,2}-\overline{r}_B)^2+(r_{B,3}-\overline{r}_B)^2+(r_{B,5}-\overline{r}_B)^2+(r_{B,6}-\overline{r}_B)^2}\sqrt{(r_{A,2}-\overline{r}_A)^2+(r_{A,3}-\overline{r}_A)^2+(r_{A,5}-\overline{r}_A)^2+(r_{A,6}-\overline{r}_A}}\\&=\frac{(3-2)(3-3)+(2-2)(2-3)+(1-2)(4-3)+(2-2)(2-3)}{\sqrt{(3-2)^2+(2-2)^2+(1-2)^2+(2-2)^2}\sqrt{(3-3)^2+(2-3)^2+(4-3)^2+(2-3)^2}}\\&=\frac{0+0-1+0}{\sqrt{1+0+1+0}\sqrt{0+1+1+1}}\\&=\frac{-1}{\sqrt{2}\sqrt{3}}\\&\approx-0.4082\end{aligned}$$

②对于用户B、C，共同评价过的商品集合$I_B\cap I_C=\{2,3,5\}$

$$\begin{aligned}s(B,C)&=\frac{(r_{B,2}-\overline{r}_B)(r_{C,2}-\overline{r}_C)+(r_{B,3}-\overline{r}_B)(r_{C,3}-\overline{r}_C)+(r_{B,5}-\overline{r}_B)(r_{C,5}-\overline{r}_C)}{\sqrt{(r_{B,2}-\overline{r}_B)^2+(r_{B,3}-\overline{r}_B)^2+(r_{B,5}-\overline{r}_B)^2}\sqrt{(r_{C,2}-\overline{r}_C)^2+(r_{C,3}-\overline{r}_C)^2+(r_{C,5}-\overline{r}_C)^2}}\\&=\frac{(3-2)(4-4)+(2-2)(4-4)+(1-2)(3-4)}{\sqrt{(3-2)^2+(2-2)^2+(1-2)^2}\sqrt{(4-4)^2+(4-4)^2+(3-4)^2}}\\&=\frac{0+0+1}{\sqrt{1+0+1}\sqrt{0+0+1}}\\&=\frac{1}{\sqrt{2}}\\&\approx0.7071\end{aligned}$$

③ 预测用户B对商品1的评分：

$$\begin{aligned}p_{B,1}&=\overline{r}_B+\frac{s(B,A)(r_{A,1}-\overline{r}_A)+s(B,C)(r_{C,1}-\overline{r}_C)}{|s(B,A)|+|s(B,C)|}\\&=2+\frac{(-0.4082)\times(4-3)+(0.7071)\times(4-4)}{|-0.4082|+|0.7071|}\\&=2-\frac{0.4082}{0.4082+0.7071}\\&\approx1.6339\end{aligned}$$

2. **对于商品4：**

用户B的邻居集合$N=\{C,D\}$；

① 对于用户B、C，由上述结果②得$s(B,C)\approx0.7071$

② 对于用户B、D，共同评价过的商品集合$I_B\cap I_D=\{2,3,5\}$

$$
\begin{aligned}
s(B,D) &= \frac{(r_{B,2}-\overline{r}_B)(r_{D,2}-\overline{r}_D)+(r_{B,3}-\overline{r}_B)(r_{D,3}-\overline{r}_D)+(r_{B,5}-\overline{r}_B)(r_{D,5}-\overline{r}_D)}{\sqrt{(r_{B,2}-\overline{r}_B)^2+(r_{B,3}-\overline{r}_B)^2+(r_{B,5}-\overline{r}_B)^2}\sqrt{(r_{C,2}-\overline{r}_C)^2+(r_{C,3}-\overline{r}_D)^2+(r_{D,5}-\overline{r}_D)^2}} \\
&= \frac{(3-2)(4-3)+(2-2)(3-3)+(1-2)(2-3)}{\sqrt{(3-2)^2+(2-2)^2+(1-2)^2}\sqrt{(4-3)^2+(3-3)^2+(2-3)^2}} \\
&= \frac{1+0+1}{\sqrt{1+0+1}\sqrt{1+0+1}} \\
&= \frac{2}{\sqrt{2}\sqrt{2}} \\
&= 1
\end{aligned}
$$

③ 预测用户B对商品4的评分：

$$
\begin{aligned}
p_{B,4} &= \overline{r}_B + \frac{s(B,C)(r_{C,4}-\overline{r}_C)+s(B,D)(r_{D,4}-\overline{r}_D)}{|s(B,C)|+|s(B,D)|} \\
&= 2 + \frac{(0.7071)\times(5-4)+(1)\times(3-3)}{|0.7071|+|1|} \\
&= 2 + \frac{0.7071}{0.7071+1} \\
&\approx 2.4142
\end{aligned}
$$

3. **由于$p_{B,4} > p_{B,1}$，所以向用户B推荐商品4****

（2）预测用户C对商品6的评分（保留四位小数）

$\overline{r}_A = \frac{4+3+2+4+2}{5} = 3, \overline{r}_B = \frac{3+2+1+2}{4} = 2, \overline{r}_C = \frac{4+4+4+5+3}{5} = 4, \overline{r}_D = \frac{4+3+3+2}{4} = 3$

对于商品6：

用户C的邻居集合$N = \{A, B\}$;

① 对于用户C、A，共同评价过的商品集合$I_B \cap I_A = \{1,2,3,5\}$

$$
\begin{aligned}
s(C,A) &= \frac{(r_{C,1}-\overline{r}_C)(r_{A,1}-\overline{r}_A)+(r_{C,2}-\overline{r}_C)(r_{A,2}-\overline{r}_A)+(r_{C,3}-\overline{r}_C)(r_{A,3}-\overline{r}_A)+(r_{C,5}-\overline{r}_C)(r_{A,5}-\overline{r}_A)}{\sqrt{(r_{C,1}-\overline{r}_C)^2+(r_{C,2}-\overline{r}_C)^2+(r_{C,3}-\overline{r}_C)^2+(r_{C,5}-\overline{r}_C)^2}\sqrt{(r_{A,1}-\overline{r}_A)^2+(r_{A,2}-\overline{r}_A)^2+(r_{A,3}-\overline{r}_A)^2+(r_{A,5}-\overline{r}_A)^2}} \\
&= \frac{(4-4)(4-3)+(4-4)(3-3)+(4-4)(2-3)+(3-4)(4-3)}{\sqrt{(4-4)^2+(4-4)^2+(4-4)^2+(3-4)^2}\sqrt{(4-3)^2+(3-3)^2+(2-3)^2+(4-3)^2}} \\
&= \frac{0+0+0-1}{\sqrt{0+0+0+1}\sqrt{1+0+1+1}} \\
&= \frac{-1}{\sqrt{1}\sqrt{3}} \\
&\approx -0.5774
\end{aligned}
$$

②对于用户C、B，由(1)中计算结果得$s(C,B) \approx 0.7071$

③ 预测用户C对商品6的评分：

$$
\begin{aligned}
p_{C,6} &= \overline{r}_C + \frac{s(C,A)(r_{A,6}-\overline{r}_A)+s(C,B)(r_{B,6}-\overline{r}_B)}{|s(C,A)|+|s(C,B)|} \\
&= 4 + \frac{(-0.5774)\times(2-3)+(0.7071)\times(2-2)}{|-0.5774|+|0.7071|} \\
&= 4 + \frac{0.5774}{0.5774+0.7071} \\
&\approx 4.4495
\end{aligned}
$$