$$P(x,\lambda) = P(x\mid\lambda)\cdot P(\lambda)$$

$$= \binom{m}{x}\left(1-e^{-\lambda}\right)^{x} e^{-\lambda(m-x)} \frac{\theta^2}{\theta+\alpha}(1+\alpha\lambda)e^{-\theta\lambda}$$

$$= \binom{m}{x}\sum_{j=0}^{x}\binom{x}{j}(-1)^{j} e^{-\lambda j} e^{-\lambda(m-x)} \frac{\theta^2}{\theta+\alpha}(1+\alpha\lambda)e^{-\theta\lambda}$$

$$= \binom{m}{x}\frac{\theta^2}{\theta+\alpha}\sum_{j=0}^{x}\binom{x}{j}(-1)^{j}(1+\alpha\lambda)e^{-\lambda(\theta+j+m-x)}$$

$$E(\lambda\mid x) = \frac{\int_0^\infty \lambda P(x,\lambda)\,d\lambda}{P(X=x)}$$

$$\int_0^\infty \lambda P(x,\lambda)\,d\lambda$$

$$= \int_0^\infty \lambda \binom{m}{x}\frac{\theta^2}{\theta+\alpha}\sum_{j=0}^{x}\binom{x}{j}(-1)^{j}(1+\alpha\lambda)e^{-\lambda(\theta+j+m-x)}\,d\lambda$$

$$= \binom{m}{x}\frac{\theta^2}{\theta+\alpha}\sum_{j=0}^{x}\binom{x}{j}(-1)^{j}\int_0^\infty (\lambda+\alpha\lambda^2)e^{-\lambda(\theta+j+m-x)}\,d\lambda$$

$$\lambda+\alpha\lambda^2 \quad e^{-\lambda(\theta+j+m-x)} \qquad +$$

$$\Big| \quad e^{-\lambda(\theta+j+m-x)} \qquad -$$

$$\begin{array}{ccc}
1+2\alpha\lambda & -\dfrac{1}{\theta+j+m-x}e^{-\lambda(\theta+j+m-x)} & + \\
2\alpha & \dfrac{1}{(\theta+j+m-x)^2}e^{-\lambda(\theta+j+m-x)} & - \\
0 & -\dfrac{1}{(\theta+j+m-x)^3}e & +
\end{array}$$

$$= \binom{m}{x}\frac{\theta^2}{\theta+\alpha}\sum_{j=0}^{x}\binom{x}{j}(-1)^j\left[-\frac{\lambda+\alpha\lambda^2}{\theta+j+m-x}-\frac{1+2\alpha\lambda}{(\theta+j+m-x)^2}\right.$$

$$\left.-\frac{2\alpha}{(\theta+j+m-x)^3}\right]e^{-\lambda(\theta+j+m-x)}$$

$$E(\lambda|x) = \frac{\cancel{\binom{m}{x}}\cancel{\frac{\theta^2}{\theta+\alpha}}\sum_{j=0}^{x}\binom{x}{j}(-1)^j\left[\qquad\qquad\right]e^{-\lambda(\theta+j+m-x)}}{\cancel{\binom{m}{x}}\cancel{\frac{\theta^2}{\theta+\alpha}}\sum_{j=0}^{x}\binom{x}{j}(-1)^j\frac{\theta+j+m-x+\alpha}{(\theta+j+m-x)^2}}$$

$$= \frac{\sum_{j=0}^{x}\binom{x}{j}(-1)^j\left[\qquad\qquad\right]e^{-\lambda(\theta+j+m-x)}}{\sum_{j=0}^{x}\binom{x}{j}(-1)^j\frac{\theta+j+m-x+\alpha}{(\theta+j+m-x)^2}}$$