HW1 Problem 3

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$P_1 = 6$

$P_2 = 21$

$R_1 = 7$

$R_2 = 18$

Problem 3

Reduce the block diagram shown in figure 3 to a single transfer function, $G(s) = \frac{Y(s)}{U(s)}$. Assume that $P_1 = 6$, $P_2 = 21$, $R_1 = 7$ and $R_2 = 18$.

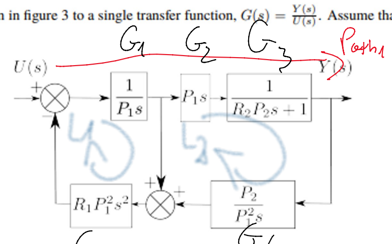

Figure 3: *Block diagram*

$$\text{Path}_1 = \frac{1}{\cancel{P_1 s}} \cdot \cancel{P_1 s} \cdot \frac{1}{R_2 P_2 s + 1} = \frac{1}{R_2 P_2 s + 1} = G_1 G_2 G_3$$

$$L_1 = -G_1 \cdot G_5 = -\frac{R_1 P_1^{\cancel{2}} \cancel{s^2}}{1 \; \cancel{P_1 s}}$$

$$L_2 = -G_1 G_2 G_3 G_4 G_5 = -\frac{1}{\cancel{P_1 s}} \cdot \cancel{P_1 s} \cdot \frac{1}{R_2 P_2 s + 1} \cdot \frac{P_2}{\cancel{P_1^2 s}} \cdot R_1 \cancel{P_1^2} s^{\cancel{2}}$$

$$= -\frac{P_2 R_1}{R_2 P_2 s + 1}$$

$$\Delta = 1 - (L_1 + L_2)$$

$$\Delta = 1 + (G_1 G_2 G_3 G_4 G_5 + G_1 G_5) = 1 + \frac{1 \; 1 \; s}{R \; P_1} + \frac{2 R_1}{R_2 P_2 s + 1}$$

$$= \frac{R_2 P_2 s + 1 + (R_2 P_2 s + 1) P_1 R_1 s + R_1 P_2}{R_2 P_2 s + 1}$$

$$\Delta_1 = 1$$

$$G(s) = \frac{\text{Path}_1 \, \Delta_1}{\Delta} = \frac{G_1 \; G_2 \; G_3}{1 + G_1 G_2 G_3 G_4 G_5 + G_1 G_5}$$

1

$$G(s) = \frac{\cancel{(R_2P_2s+1)}}{\dfrac{R_2P_2s+1+(R_2P_2s+1)P_1R_1s+R_1P_2}{\cancel{(R_2P_2s+1)}}} = \frac{1}{R_2P_2s+1+P_1P_2R_1R_2s^2+P_1R_1s+R_1P_2}$$

$$= \frac{1}{s^2P_1P_2R_1R_2+s(R_2P_2+P_1R_1)+R_1P_2+1} = \begin{bmatrix} P_1 = 6 \\ P_2 = 21 \\ R_1 = 7 \\ R_2 = 18 \end{bmatrix} -$$

$$= \frac{1}{s^2\cdot 15876+s(378+126)+1+147} = \boxed{\frac{1}{15876s^2+504s+148}}$$