HW1 Problem 4

22 November 2020 21:58

Problem 4

A schematic diagram of a rotor mounted in bearings is shown in Fig. 4. The moment of inertia of the rotor about the axis of rotation is $J = 0.31\ kgm^2$. Let us assume that at $t = 0$ the rotor is rotating at the angular velocity $\omega(0) = \omega_0 = 121\ \frac{rad}{sec}$. We also assume that the friction in bearings is viscous friction, where the coefficient of friction is $b = 0.02\ \frac{Nm}{\frac{rad}{sec}}$.

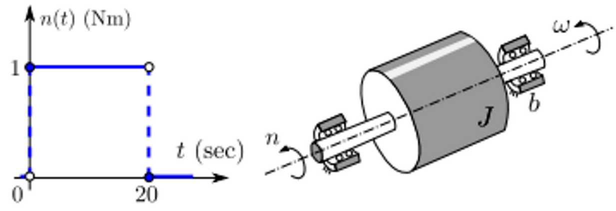

Figure 4: *Input signal nad rotor mounted in bearings*

Assume that the input to the system is the external applied torque $n(t)$ (Nm) in the form of a pulse signal shown in Fig. 4, while the output is the angular velocity $\omega(t)$ $(\frac{rad}{s})$. *Find and plot free* y_{free}, *forced* y_{forced}, *and total response* y_{total} of the system. Calculate the total response y_{total} for time $t = 20\ s$.

$$n(t) = 1 - 1(t-20)$$

$$Y(s) = G(s)N(s)$$

$$N(s) = \frac{1}{s} - \frac{1}{s} e^{-20s}$$

mathematical model

$$J\dot{\omega} = -b\omega + n$$

$$\omega(t) \Rightarrow Y(s)$$

$$JY(s)s + bY(s) - \omega_0 = N(s)$$

$$Y_{total}(s) = Y_{free} + Y_{forced}$$

$$Y_{free}: (Js+b)\underbrace{Y(s)} - \omega_0 = \overset{=0}{N(s)}$$

$$Y_{free}: \frac{\omega_0}{Js+b} = Y(s) = \frac{1}{J} \frac{\omega_0}{s + \frac{b}{J}}$$

$$Y_{forced}: JY(s)s + bY(s) - \overset{=0}{\omega_0} = \frac{1-e^{-20s}}{}$$

$$Y_{forced}: \quad J\,Y(s)\,s + b\,Y(s) - \overset{''}{\omega_0} = \frac{1-e^{-20s}}{s}$$

$$Y(s)\,(Js+b) = \frac{1-e^{-20s}}{s}$$

$$Y(s) = \frac{1-e^{-20s}}{s(Js+b)} = \frac{1}{J}\left(\frac{1-e^{-20s}}{s+\frac{b}{J}}\right)$$

$$Y_{total}(s) = \frac{\omega_0}{Js+b} + \frac{1-e^{-20s}}{s(Js+b)}$$

$$y_{total}(t) = y_{free}(t) + y_{forced}(t)$$

calculated using matlab

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syms t;
syms s;

n(t) = heaviside(t)-heaviside(t-20);

n(s) = laplace(n(t),t,s);

Yfree(s)=121*0.31/(0.31*s+0.02);
Yforced(s)= n(s)/(0.31*s+0.02);

yfree(t) = ilaplace(Yfree(s),s,t);
yforced(t) = ilaplace(Yforced(s),s,t);

disp(yfree(t));
disp(yforced(t));

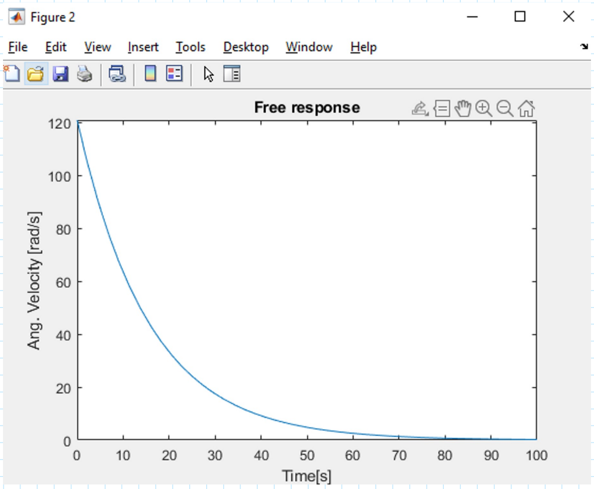
ytotal(t)= yfree(t)+yforced(t);

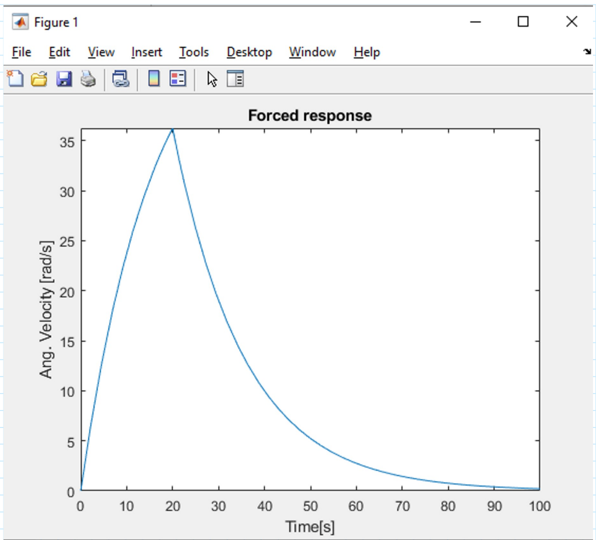
disp(ytotal(t));

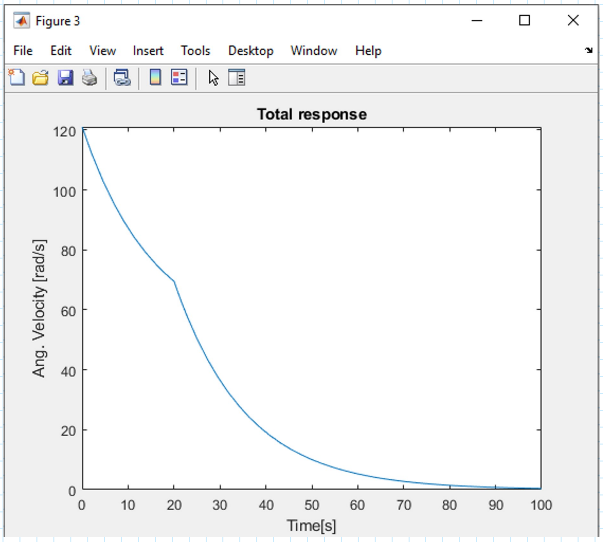
disp(eval(ytotal(20)));

fplot(yfree(t), [0,100]);
title('Free response');
xlabel('Time[s]');
ylabel('Ang. Velocity [rad/s]');

figure();
fplot(yforced(t), [0,100]);
```







$$y_{total}(20) = 69.5379 \left[\frac{rad}{s}\right]$$