HW 2 Ex. 1

10 January 2021 16:05

Problem 1
Consider the feedback control system given in Fig. 1, where $T_1 = 1/60sec$, $T_2 = 1/70sec$, and $T_3 = 1/260sec$. Determine the value of the constant T_d of the PD controller when the system oscillates and calculate the angular frequency of these oscillations.

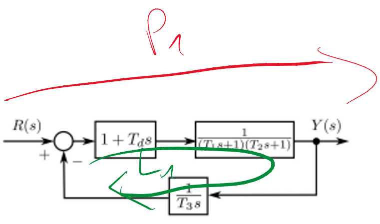


Figure 1: *Feedback control system in Problem 1*

Answer: $T_d =$ ______________________ sec $\omega_{osc} =$ ______________________ $\frac{rad}{sec}$

$$P_1 = G_1 G_2 = \frac{1+T_d s}{(T_1 s+1)(T_2 s+1)}$$

$$L_1 = -G_1 G_2 G_3 = -\frac{1+T_d s}{(T_1 s+1)(T_2 s+1)} \cdot \frac{1}{T_3 s}$$

$$\Delta = 1$$

$$\Delta_L = 1 + G_1 G_2 G_3 = 1 + \frac{1+T_d s}{(T_1 s+1)(T_2 s+1)T_3 s}$$

$$G(s) = \frac{P_1 \Delta^1}{\Delta_L} = \frac{P_1}{\Delta_L} = \frac{(1+T_d s)}{\cancel{(T_1 s+1)(T_2 s+1)}} \cdot \frac{\cancel{(T_1 s+1)(T_2 s+1)}(T_3 s)}{(T_1 s+1)(T_2 s+1)(T_3 s)+(T_d s)+1}$$

$$G(s) = \frac{(1+T_d s)(T_3 s)}{(T_1 s+1)(T_2 s+1)(T_3 s)+(T_d s)+1}$$

$$G(s) = \frac{T_d T_3 s^2 + T_3 s}{(T_1 T_2 s^2 + T_1 s + T_2 s + 1)(T_3 s) + T_d s + 1} = \frac{T_3 T_d s^2 + T_3 s}{T_1 T_2 T_3 s^3 + T_1 T_3 s^2 + T_2 T_3 s^2 + T_3 s + T_d s + 1}$$

$$G(s) = \frac{T_3 T_d s^2 + T_3 s}{s^3(T_1 T_2 T_3) + s^2(T_1 T_3 + T_2 T_3) + s(T_3 + T_d) + 1}$$

$$N(s) = s^3(T_1 T_2 T_3) + s^2(T_1 T_3 + T_2 T_3) + s(T_3 + T_d) + 1 s^0$$

$$a_3 = T_1 T_2 T_3 \qquad a_2 = T_1 T_3 + T_2 T_3 \qquad a_1 = T_3 + T_d \qquad a_0 = 1$$
$$> 0 \qquad\qquad > 0 \qquad\qquad\qquad\qquad > 0$$

≥ 0 ≥ 0 ≥ 0

$T_3 + T_d \geq 0$

$T_d \geq -T_3 \Rightarrow T_d > -\frac{1}{260}$

$$\Delta_3 = \begin{vmatrix} T_1T_3 + T_2T_3 & T_1T_2T_3 & 0 \\ 1 & T_3 + T_d & T_1T_2 + T_2T_3 \\ 0 & 0 & 1 \end{vmatrix}$$

all principal minors > 0

$\Delta_1 = T_3 + T_d$

$$\Delta_2 = \begin{vmatrix} T_1T_3 + T_2T_3 & T_1T_2T_3 \\ 1 & T_3 + T_d \end{vmatrix} =$$

$$(T_3 + T_d)(T_1T_3 + T_2T_3) - T_1T_2T_3 > 0$$

$$T_3 + T_d > \frac{T_1T_2T_3}{T_1T_3 + T_2T_3}$$

> 0 so no need to worry about potential inequality sign change when dividing

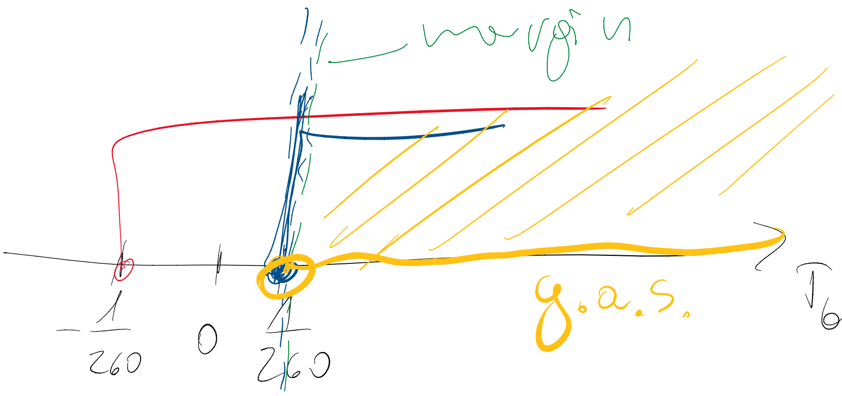
$T_1 > \frac{T_1T_2T_3}{}$ —

$$T_d > \frac{T_1 T_2 T_3}{T_1 T_3 + T_2 T_3} - T_3$$

$$T_d > \frac{\cancel{T_3}}{\cancel{T_3}} \cdot \frac{T_1 T_2}{T_1 + T_2} - T_3$$

$$T_d > \frac{\frac{1}{60} \cdot \frac{1}{70}}{\frac{1}{60} + \frac{1}{70}} - \frac{1}{260}$$

$$T_d > \frac{1}{260}$$



$$T_d = \frac{1}{260} \rightarrow \text{marginal stability}$$

$s \rightarrow j\omega$

$$N(s) = s^3 (T_1 T_2 T_3) + s^2 (T_1 T_3 + T_2 T_3) + s(T_3 + T_d) + 1 s^0$$

$$N(j\omega) = -j\omega^3 (T_1 T_2 T_3) - \omega^2 (T_1 T_3 + T_2 T_3) + j\omega (T_3 + T_d) + 1$$

$$N(j\omega) = 1 - \omega^2 (T_1 T_3 + T_2 T_3) + j(\omega (T_3 + T_d) - \omega^3 (T_1 T_2 T_3))$$

$$N(j\omega) = 1 - \omega^2(T_1T_3 + T_2T_3) + j\left(\omega(T_3 + T_d) - \omega^3(T_1T_2T_3)\right)$$

$$P(\omega) = 1 - \omega^2 T_3 (T_1 + T_2) = 0$$

$$Q(\omega) = \omega(T_3 + T_d) - \omega^3 T_{123} = 0$$

$$\omega^2 = \frac{1}{T_3(T_1 + T_2)}$$

$$\omega = \pm\sqrt{\frac{1}{T_3(T_1 + T_2)}}$$

considering only positive ω

$$\boxed{\omega_{osc} = \frac{1}{\sqrt{\frac{1}{260}\left(\frac{1}{60} + \frac{1}{70}\right)}} = 91.6515 \ \frac{rad}{s}}$$