HW 2 Ex.4 corr

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Problem 4

A mechanical system is shown in Fig. 4. Assume that the input and output are the displacements $u(t)$ and $y(t)$, respectively. The displacement $y(t)$ is measured from the equilibrium position. Suppose that $u(t) = \sin \omega t$. What is the output $y(t)$ at steady-state? *Plot the response* $y(t)$ and find the magnitude (in dB) and phase angle (in degrees) for $\omega = 5.38816 \frac{rad}{sec}$. *Graph the Bode plots for the system.* Assume that the system is linear throughout the operating period and $m = 1kg, b = 1.20615 \frac{Ns}{m}$, and $k = 7.18421 \frac{N}{m}$.

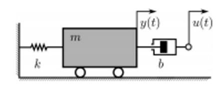

Figure 4: *Mechanical system in Problem 4*

Answer: $L(\omega) =$ ______________ dB $\qquad \varphi(\omega) =$ ______________ °

$$m\ddot{y} + b(\dot{y} - \dot{u}) + ky = 0$$

$$ms^2 Y(s) + bsY(s) + kY(s) = bsU(s)$$

$$Y(s)\left(ms^2 + bs + k\right) = bsU(s)$$

$$G(s) = \frac{bs}{ms^2 + bs + k}$$

$$G(j\omega) = \frac{bj\omega}{-m\omega^2 + j\omega b + k}$$

$$G(j\omega) = \frac{bj\omega}{k - m\omega^2 + j\omega b} \cdot \frac{k - m\omega^2 - j\omega b}{k - m\omega^2 - j\omega b}$$

$$G(j\omega) = \frac{bj\omega k - mj\omega^3 b + \omega^2 b^2}{(k - m\omega^2)^2 - \omega^2 b^2}$$

$$G(j\omega) = \underbrace{\frac{\omega^2 b^2}{(k - m\omega^2)^2 - \omega^2 b^2}}_{P(\omega)} + j \underbrace{\frac{b\omega k - m\omega^3 b}{(k - m\omega^2)^2 - \omega^2 b^2}}_{Q(\omega)}$$

predicting

$$y_{ss}(t) = A(\omega)\sin(\omega t + \varphi(\omega)) \cdot 1(t)$$

$$u(t) = \sin(\omega t) \cdot 1(t)$$

$$A(\omega) = \sqrt{P(\omega)^2 + Q^2(\omega)} = \frac{\sqrt{\omega^4 b^4 + (b\omega k - mb\omega^3)^2}}{(k - m\omega^2)^2 - \omega^2 b^2}$$

$$\varphi(\omega) = \arctan\left(\frac{Q(\omega)}{P(\omega)} = \frac{b\omega k - mb\omega^3}{\omega^2 b^2}\right)$$

calculations done in matlab