HW1 Problem 1

03 November 2020 20:49

Problem 1

Consider the spring-loaded pendulum system shown in Fig. 1. Assuming that the spring force acting on the pendulum is zero when the pendulum is vertical ($\theta = 0$), a mathematical model for the system becomes:

$$ml^2\ddot{\theta} + mgl\sin\theta + 2ka^2\sin\theta\cos\theta = 0.$$

Find the state-space representation for the system and linearize the model around equilibrium point ($\theta = 0\ rad$, $\dot{\theta} = 0\ \frac{rad}{sec}$). *Find and plot the free response* for the system due to the initial conditions: $\theta_0 = \frac{\pi}{6}\ rad$, $\dot{\theta}_0 = 0\ \frac{rad}{sec}$. Calculate the response at time $t = 0.8761\ s$ given that $g = 9.81\ \frac{m}{s^2}$, $m = 2.5\ kg$, $l = 1.4\ m$, $a = 1.05\ m$, $k = 13\ \frac{N}{m}$.

Answer: $\theta_{free}(t = 0.8761) =$ ______________________

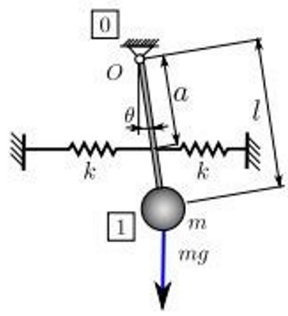

Figure 1: *Spring loaded pendulum*

$$y = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \qquad \dot{y} = \begin{bmatrix} \dot{\theta} \\ \dfrac{-mgl\sin\theta - 2ka^2\sin\theta\cos\theta}{ml^2} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ x & 0 \end{bmatrix}$$

$$x = \frac{-mgl\cos\theta - 2ka^2\left(\overbrace{\cos^2\theta - \sin^2\theta}^{\cos(2\theta)}\right)}{ml^2}$$

$$A_y = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l} - \frac{2ka^2}{ml^2} & 0 \end{bmatrix}$$

$$\dot{y} = \dot{y}_0 + A_y \Delta y$$

$$\dot{y} - \dot{y}_0 = A_y \Delta y$$

$$\Delta \dot{y} = A_y \Delta g$$

$$\ddot{y} = \left(-\frac{g}{l} - \frac{2ka^2}{ml^2} \right) y$$

$$s^2 Y(s) - \frac{\pi}{6} s = -\left(+\frac{g}{l} + \frac{2ka^2}{ml^2} \right) Y(s)$$

$$Y(s) = \frac{\frac{\pi}{6} s}{\quad}$$

$$Y(s) = \frac{\frac{\pi}{6}s}{\frac{g}{L} + \frac{2ka^2}{mL^2} + s^2}$$

using matlab

bac1_hw1_ex1.m

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syms a;
syms s;

u(s) = ((pi/6)*s)/((9.81/1.4)+(2*12*1.05^2)/(2.5*1.4^2)+s^2);

u(a) = ilaplace(u(s),s,a);

disp(u(a));

yt = eval(u(0.8761));

disp(yt);
```

$$y(0.8761) = -0.5228$$

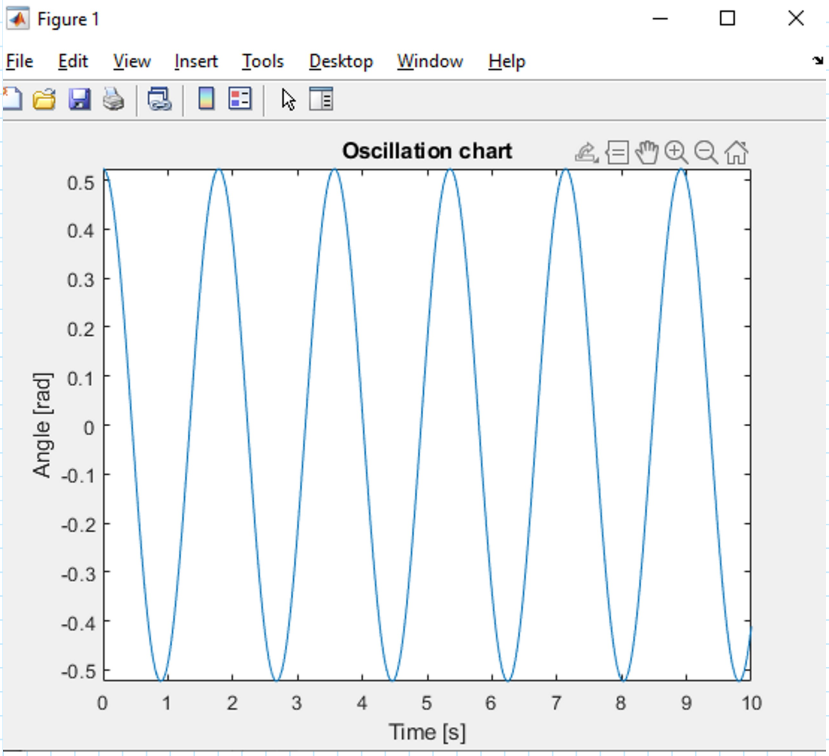
Figure 1
File
Edit
View
Insert
Tools
Desktop
Window
Help
Oscillation chart
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
Angle [rad]
0
1
2
3
4
5
6
7
8
9
10
Time [s]