HW 2 Ex. 3

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Problem 3

A feedback control system has the structure shown in Fig. 3, where $G(s) = \frac{1}{s+42}$ Assume that $k_1 > 0$, $k_2 > 0$ and select the gains k_1 and k_2 such that the closed-loop response to a step input is critically damped, and the natural frequency is equal to $\omega_n = 63 \frac{rad}{sec}$. *Plot the step response.*

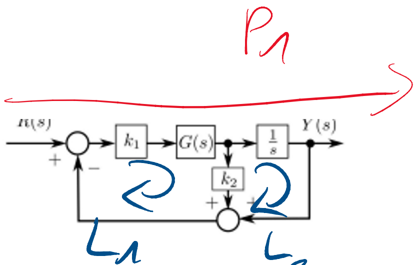

Figure 3: *Feedback control system in Problem 3*

Answer: $k_1 =$ ____________ $k_2 =$ ____________

$$L_1 = -k_1 k_2 G(s) \qquad P_1 = \frac{k_1 G(s)}{s}$$

$$L_2 = -\frac{k_1 G(s)}{s}$$

$$\Delta = 1 + \frac{k_1 G(s)}{s} + k_1 k_2 G(s) = $$

$$\frac{s + k_1 G(s) + k_1 k_2 s G(s)}{s}$$

$$\Delta_1 = 1$$

$$G(s) = \frac{P_1 \Delta_1}{\Delta} = \frac{k_1 G(s)}{\cancel{s}} \cdot \frac{\cancel{s}}{s + k_1 G(s) + k_1 k_2 s G(s)}$$

$$G(s) = \frac{k_1 G(s)}{s + k_1 G(s) + k_1 k_2 s G(s)}$$

$$G(s) = \frac{k_1 \frac{1}{s+42}}{}$$

$$s + k_1 \frac{1}{s+42} + k_1 k_2 s \frac{1}{s+42}$$

$$G(s) = \frac{k_1}{\cancel{s+42}} \cdot \frac{\cancel{s+42}}{s^2 + 42s + k_1 + k_1 k_2 s}$$

$$G(s) = \frac{k_1}{s^2 + s(k_1 k_2 + 42) + k_1}$$

$$G(s) = \frac{k_1}{\frac{s^2}{\sqrt{k_1}^2} + \frac{s}{\sqrt{k_1}} \cdot \frac{(k_1 k_2 + 42)}{\sqrt{k_1}} + 1} \cdot \frac{1}{k_1}$$

$$L(s) = \frac{1}{1 + 2\zeta\left(\frac{s}{\omega_n}\right) + \left(\frac{s}{\omega_n}\right)^2}$$

Critically damped case $\zeta = 1$

$\omega_n = \sqrt{k_1}$ $\qquad$ $\omega_n = 63 \Rightarrow k_1 = 3969$

$$2\zeta = \frac{k_1 k_2 + 42}{\sqrt{k_1}} = 2$$

$$\sqrt{k_1}$$

$$k_2 = \frac{2 \cdot 63 - 42}{3969} = 0.021164$$