HW1 Problem 2

22 November 2020

Problem 2

Figure 2a shows a highly simplified version of the automobile suspension system. As the car moves along the road, the vertical displacements at the tires excite the automobile suspension system. Assuming that the motion u at point P is the input to the system and the vertical motion y of the body is the output, obtain the transfer function $\frac{Y(s)}{U(s)}$. The displacement y is measured from the equilibrium position in the absence of the input u. *Obtain and plot the response* $y(t)$ assuming that the road profile $u(t)$ is depicted in the figure 2b. Find the response of the system for time $t = 4.0\ s$ given $m = 1000\ kg$, $b = 3250\ \frac{Ns}{m}$, and $k = 5281\ \frac{N}{m}$.

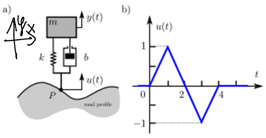

Figure 2: *a) Simplified model of a suspension; b) Input signal* $u(t)$

$$u(t) = t\mathbb{1} - 2\cdot(t-1)\mathbb{1}(t-1) + 2(t-3)\mathbb{1}(t-3) - (t-4)\mathbb{1}(t-4) \; / \; \mathcal{L}[\;]$$

$$U(s) = \frac{1}{s^2} - 2\frac{1}{s^2}e^{-s} + 2\frac{1}{s^2}e^{-3s} - \frac{1}{s^2}e^{-4s}$$

eq of motion

$$m\ddot{y} + b\dot{y} + ky = b\dot{u} + ku$$

equilibrium state

$y(0) = 0 \qquad u(0) = 0$

$\dot{y}(0) = 0$

$$ms^2Y(s) + bsY(s) + kY(s) = bsU(s) + kU(s)$$

$$\frac{Y(s)}{U(s)} = \frac{bs+k}{ms^2+bs+k} = G(s)$$

$3250 = \omega\zeta\frac{1}{2}$ =?

$3250 = \omega$

$$G(s) = \frac{3250s + 5281}{1000s^2 + 3250s + 5281}$$

$$Y(s) = G(s)U(s) = \left(\frac{1}{s^2} - 2\frac{1}{s^2}e^{-s} + 2\frac{1}{s^2}e^{-3s} - \frac{1}{s^2}e^{-4s}\right)\left(\frac{3250s + 5281}{1000s^2 + 3250s + 5281}\right)$$

$$y(t) = \mathcal{L}^{-1}[G(s)U(s)]$$

calculation in MATLAB:

Editor - C:\Users\kubaj\Downloads\bac1_hw1_ex2.m

bac1_ha2_ex1.m | bac1_hw1_ex2.m

```matlab
syms t;
syms s;

u(t) = t*heaviside(t) - 2*(t-1)*heaviside(t-1) + 2*(t-3)*heaviside(t-3)-(t-4)*heaviside(t-4);
U(s) = laplace(u(t),t,s);

G(s) = (3250*s+5281)/(1000*s^2+3250*s+5281);

Y(s) = G(s)*U(s);

y(t) = ilaplace(Y(s));

disp(y(t));

y_t = eval(y(4));

disp(y_t);

fplot(y(t), [0, 10])

title('Oscillation chart');
xlabel('Time [s]');
ylabel('Displacement [m]');
```

Command Window

```
>> bacl_hwl_ex2
t - 10562*heaviside(t - 1)*(t/5281 + (3250*exp(13/8 - (13*t)/8)*(cos((3*5^(1/2)*2347^(1/2)*(t - 1))/200) + (5^(1/2)*2347^(1/2)*sin((3*5^(1/2)*2347^(1/2)*(t - 1))/200))/2288

   -0.2515
```

$$y(4) = -0.2515$$

y(t) =

```
t - (5281heaviside(t - 1)((1000t)/5281 + (3250000exp(13/8 - (13t)/8)(cos((35^(1/2)2347^(1/2)(t - 1))/200) + (5^(1/2)2347^(1/2)sin((35^(1/2)2347^(1/2)(t -
1))/200))/2288325))/27888961 - 8531000/27888961))/500 + (5281heaviside(t - 3)((1000t)/5281 + (3250000exp(39/8 - (13t)/8)(cos((35^(1/2)2347^(1/2)(t - 3))/200) +
(5^(1/2)2347^(1/2)sin((35^(1/2)2347^(1/2)(t - 3))/200))/2288325))/27888961 - 19093000/27888961))/500 - (5281heaviside(t - 4)((1000t)/5281 + (3250000exp(13/2 -
(13t)/8)(cos((35^(1/2)2347^(1/2)(t - 4))/200) + (5^(1/2)2347^(1/2)sin((35^(1/2)2347^(1/2)(t - 4))/200))/2288325))/27888961 - 24374000/27888961))/1000 - (3250exp(-
(13t)/8)(cos((35^(1/2)2347^(1/2)t)/200) + (655^(1/2)2347^(1/2)sin((35^(1/2)2347^(1/2)t)/200))/7041))/5281 + (3250exp(-(13t)/8)(cos((35^(1/2)2347^(1/2)t)/200) +
(5^(1/2)2347^(1/2)sin((35^(1/2)2347^(1/2)t)/200))/2288325))/5281 + (13heaviside(t - 1)((1000exp(13/8 - (13t)/8)(cos((35^(1/2)2347^(1/2)(t - 1))/200) +
(655^(1/2)2347^(1/2)sin((35^(1/2)2347^(1/2)(t - 1))/200))/7041))/5281 - 1000/5281))/2 + (13heaviside(t - 4)((1000exp(13/2 - (13t)/8)(cos((35^(1/2)2347^(1/2)(t - 4))/200) +
(655^(1/2)2347^(1/2)sin((35^(1/2)2347^(1/2)(t - 4))/200))/7041))/5281 - 1000/5281))/4 - (13heaviside(t - 3)((1000exp(39/8 - (13t)/8)(cos((35^(1/2)2347^(1/2)(t - 3))/200) +
(655^(1/2)2347^(1/2)sin((35^(1/2)2347^(1/2)*(t - 3))/200))/7041))/5281 - 1000/5281))/2
```

Chart: