HW 2 Ex. 2

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**Problem 2**

Consider a unity feedback control system shown in Fig. 2 with the open-loop transfer function

$$G(s) = \frac{K}{(s+1/2)(s+7)(s+50)}.$$

Analitically find the gain $K$ such that the gain margin is less than $10dB$. *Plot the Nyquist plot for the calculated gain* $K$.

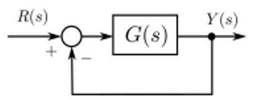


**Figure 2:** *Feedback control system in Problem 2*

**Answer:** $K =$ __________

$$G(s) = \frac{K}{\left(s^2 + 7s + \frac{1}{2}s + \frac{7}{2}\right)(s+50)}$$

$$G(s) = \frac{K}{s^3 + 50s^2 + 7s^2 + 350s + \frac{1}{2}s^2 + \frac{50}{2}s + \frac{7}{2}s + \frac{350}{2}}$$

$$G(s) = \frac{K}{s^3 + s^2\, 57\tfrac{1}{2} + 378\tfrac{1}{2}\, s + 175}$$

$$G(j\omega) = \frac{K}{-j\omega^3 - \omega^2\, 57\tfrac{1}{2} + j\omega\, 378\tfrac{1}{2} + 175}$$

$$G(j\omega) = \frac{K}{175 - 57\tfrac{1}{2}\omega^2 + j\left(\omega\, 378\tfrac{1}{2} - \omega^3\right)}$$

$$G(j\omega) = \underbrace{\frac{K\left(175 - 57\tfrac{1}{2}\omega^2\right)}{\left(175 - 57\tfrac{1}{2}\omega^2\right)^2 - \left(378\tfrac{1}{2}\omega - \omega^3\right)^2}}_{P(\omega)} - j\underbrace{\frac{K\left(378\tfrac{1}{2}\omega - \omega^3\right)}{\left(175 - 57\tfrac{1}{2}\omega^2\right)^2 - \left(378\tfrac{1}{2}\omega - \omega^3\right)^2}}_{Q(\omega)}$$

$$Q(j\omega) = 0 = 378\tfrac{1}{2}\omega - \omega^3 \qquad \omega > 0$$

$$0 = \omega\left(378\tfrac{1}{2} - \omega^2\right)$$

$$\omega = 0$$

$$\omega^2 = 378\tfrac{1}{2}$$

$$\omega = \pm\sqrt{378.5}$$

$$\omega = 19.4551\,\frac{rad}{s} \qquad \omega^2 = 378.5\,\frac{rad^2}{s^2}$$

$$P(\omega) = \frac{K\left(175 - 57\tfrac{1}{2}\omega^2\right)}{\left(175 - 57\tfrac{1}{2}\omega^2\right)^2 - \left(378\tfrac{1}{2}\omega - \omega^3\right)^2}$$

$$P(\omega) = K\,\frac{-21588.75}{4.6607\cdot10^8 - 0}$$

$$P(\omega) = K\,\underbrace{0.463208\cdot10^{-4}}$$

$$\Delta M = 20\log\alpha < 10\,dB$$

$$\log \alpha < \frac{\cancel{10}}{\cancel{20}} \frac{1}{2}$$

$$\alpha < 10^{\frac{1}{2}}$$

$$\sqrt{10} > \left|\frac{1}{G(j\omega)}\right| = \alpha \qquad G(j\omega) < 0$$

$$K \cdot A < \frac{1}{\sqrt{10}}$$

$$K < \frac{1}{\sqrt{10}\, A}$$

$$K < 0.682691 \cdot 10^{4}$$

$$K < 6826.91$$