Name and surname (legibly): Kuba Jatoszyński
Student's registration book number: 304557
Field of study: Power Engineering

Please write your answers to three decimal places and enclose the solutions to each problem.

PROBLEM SET 57

Problem 1

Consider the spring-loaded pendulum system shown in Fig. 1. Assuming that the spring force acting on the pendulum is zero when the pendulum is vertical ($\theta = 0$), a mathematical model for the system becomes:

$$ml^2\ddot{\theta} + mgl\sin\theta + 2ka^2\sin\theta\cos\theta = 0.$$

Find the state-space representation for the system and linearize the model around equilibrium point ($\theta = 0\ rad$, $\dot{\theta} = 0\ \frac{rad}{sec}$). *Find and plot the free response* for the system due to the initial conditions: $\theta_0 = \frac{\pi}{6}\ rad$, $\dot{\theta}_0 = 0\ \frac{rad}{sec}$. Calculate the response at time $t = 0.8761\ s$ given that $g = 9.81\ \frac{m}{s^2}$, $m = 2.5\ kg$, $l = 1.4\ m$, $a = 1.05\ m$, $k = 13\ \frac{N}{m}$.

Answer: $\theta_{free}(t = 0.8761) =$ -0.5228 [rad]

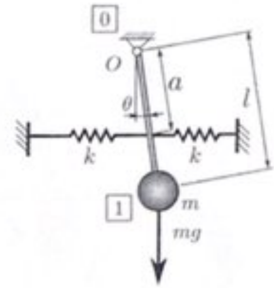


Figure 1: *Spring loaded pendulum*

Problem 2

Figure 2a shows a highly simplified version of the automobile suspension system. As the car moves along the road, the vertical displacements at the tires excite the automobile suspension system. Assuming that the motion u at point P is the input to the system and the vertical motion y of the body is the output, obtain the transfer function $\frac{Y(s)}{U(s)}$. The displacement y is measured from the equilibrium position in the absence of the input u. *Obtain and plot the response* $y(t)$ assuming that the road profile $u(t)$ is depicted in the figure 2b. Find the response of the system for time $t = 4.0\ s$ given $m = 1000\ kg$, $b = 3250\ \frac{Ns}{m}$, and $k = 5281\ \frac{N}{m}$.

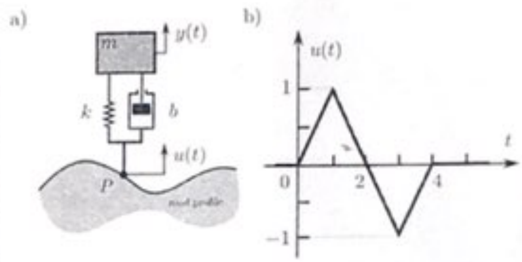
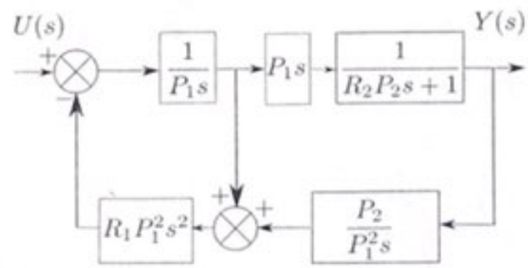


Figure 2: *a) Simplified model of a suspension; b) Input signal* $u(t)$

Answer: $y(t = 4.0) =$ -0.2515 [-]

Problem 3

Reduce the block diagram shown in figure 3 to a single transfer function, $G(s) = \frac{Y(s)}{U(s)}$. Assume that $P_1 = 6$, $P_2 = 21$, $R_1 = 7$ and $R_2 = 18$.



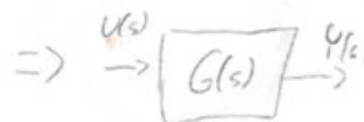


Figure 3: *Block diagram*

Answer: $G(s) = \frac{1}{15876 s^2 + 504 s + 148}$

Problem 4

A schematic diagram of a rotor mounted in bearings is shown in Fig. 4. The moment of inertia of the rotor about the axis of rotation is $J = 0.31\ kgm^2$. Let us assume that at $t = 0$ the rotor is rotating at the angular velocity $\omega(0) = \omega_0 = 121\ \frac{rad}{sec}$. We also assume that the friction in bearings is viscous friction, where the coefficient of friction is $b = 0.02\ \frac{Nm}{\frac{rad}{sec}}$.

Figure 4: *Input signal nad rotor mounted in bearings*

Assume that the input to the system is the external applied torque $n(t)$ (Nm) in the form of a pulse signal shown in Fig. 4, while the output is the angular velocity $\omega(t)$ $(\frac{rad}{s})$. *Find and plot free y_{free}, forced y_{forced}, and total response y_{total}* of the system. Calculate the total response y_{total} for time $t = 20$ s.

Answer: $y_{total}(t = 20) =$ 69.5379 $\left[\frac{rad}{s}\right]$

Problem 5

For the system shown in Fig. 5, find the output, $y(t)$, if the input $r(t)$ is a unit step, where $G(s) = \frac{13}{s(s+2)}$. *Plot the response and provide* $y(t)$ for $t = 0.9069$ s.

Answer: $y(t = 0.9069) =$ 1.4038

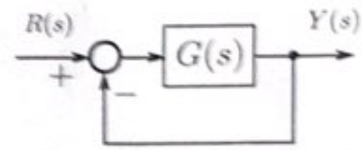


Figure 5: *Feedback control system*

Problem	No. 1	No. 2	No. 3	No. 4	No. 5	Total
Points						

[a]The problem set has been generated November 3, 2020 n., g. 13:52:28