HW1 Problem 5

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Problem 5
For the system shown in Fig. 5, find the output, $y(t)$, if the input $r(t)$ is a unit step, where $G(s) = \frac{13}{s(s+2)}$. *Plot the response and provide* $y(t)$ for $t = 0.9069\ s$.

Answer: $y(t = 0.9069) =$ ____________

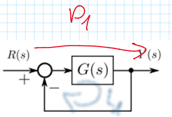


Figure 5: *Feedback control system*

$$r(t) = 1(t)$$

$$R(s) = \frac{1}{s}$$

$$P_1 = G_1$$

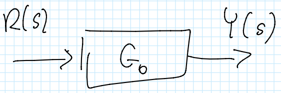
$$L_1 = -G_1$$

$$\Delta = 1 + G_1$$

$$\Delta_1 = 1$$

$$G_0 = \frac{G_1}{1+G_1} = \frac{13}{s(s+2)} \cdot \frac{1}{1+\frac{13}{s^2+2s}} =$$

$$\frac{13}{\cancel{s^2+2s}} \cdot \frac{\cancel{s^2+2s}}{s^2+2s+13} = \frac{13}{s^2+2s+13}$$



$$Y(s) = R(s)\,G_0(s)$$

$$y(t) = \mathcal{L}^{-1}[R_s G_0(s)] = \mathcal{L}^{-1}\left[\frac{1}{s}\frac{13}{s^2+2s+13}\right] = 1 - \frac{1}{\sqrt{1-(0.2774)^2}}\, e^{-0.2774\cdot\sqrt{13}\cdot t} \cdot \sin\left(\sqrt{13}\sqrt{1-(0.2774)^2}\cdot t + 1.2897\right)$$

$$\omega^2 = 13$$

$$\omega = \pm\sqrt{13}$$

$$\zeta \in (0;1)$$

$$2\zeta\omega = 2 \Rightarrow \omega > 0 \Rightarrow \omega = \sqrt{13}$$

$$+ \quad \zeta = \frac{1}{\omega} \approx 0.2774 \in (0;1)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

$$\phi \approx 1.2897 \text{ rad} \in \left(0;\frac{\pi}{2}\right) \quad +$$

Calculations done using matlab:

Laplace inv. transform check:

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bac1_hw1_ex5.m

```
syms t;
syms s;

r(t) = heaviside(t);

R(s) = laplace(r(t),t,s);

disp(R(s));

G(s) = 13/(s^2+2*s+13);

Y(s) = R(s)*G(s);

y(t) = ilaplace(Y(s),s,t);

disp(y(t));
```

Command Window

```
>> bac1_hw1_ex5
1/s

>> bac1_hw1_ex5
1/s

1 - exp(-t)*(cos(2*3^(1/2)*t) + (3^(1/2)*sin(2*3^(1/2)*t))/6)

>>
```

— So manual transform is wrong

Result of g(t) and y(0.9069)

```
>> bac1_hw1_ex5
1 - exp(-t)*(cos(2*3^(1/2)*t) + (3^(1/2)*sin(2*3^(1/2)*t))/6)

    1.4038

>>
```