Métodos Multivariados: Tarea 5

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Ejercicio 1

Mostrar que la matriz de covarianzas ρ para las tres variables estandarizadas Z_1, Z_2, Z_3 puede ser generada por el modelo factorial con $m = 1$:

$$\rho = \begin{pmatrix} 1 & 0.63 & 0.45 \\ 0.63 & 1 & 0.35 \\ 0.45 & 0.35 & 1 \end{pmatrix}$$

Las ecuaciones del modelo factorial son:

$$Z_1 = 0.9F_1 + \varepsilon$$
$$Z_2 = 0.7F_1 + \varepsilon$$
$$Z_3 = 0.5F_1 + \varepsilon$$

donde $\mathrm{Var}(F_1) = 1$, $\mathrm{Cov}(F_1, \varepsilon) = 0$ y $\Psi =$

$$\begin{pmatrix} 0.19 & 0 & 0 \\ 0 & 0.51 & 0 \\ 0 & 0 & 0.75 \end{pmatrix}$$

tomando en cuenta la matriz de varianzas y covarianzas de las Z's

$Var(Z_1) = (0.9)^2 * 1 = 0.81 Var(Z_2) = (0.7)^2 * 1 = 0.49 Var(Z_3) = (0.5)^2 * 1 = 0.25 Cov(Z_1, Z_2) = 0.63 Cov(Z_1, Z_3)$

construimos la siguiente matriz y procedemos al método de factor principal con m=1

```r
Sigma <- matrix(c(0.81, 0.63, 0.45,
                  0.63, 0.49, 0.35,
                  0.45, 0.35, 0.25),
                nrow = 3, byrow = TRUE)
```

```r
psi <- matrix(c(0.19, 0, 0,
                0, 0.51, 0,
                0, 0, 0.75), nrow = 3, byrow = TRUE)

#Construimos L con un coeficiente de carga
L <- NULL
L <- cbind(L,eigen(Sigma)$vectors[,1]*sqrt(eigen(Sigma)$values[1]))

# LL' + psi
(L %*% t(L)) + psi
```

```
     [,1] [,2] [,3]
[1,] 1.00 0.63 0.45
[2,] 0.63 1.00 0.35
[3,] 0.45 0.35 1.00
```

Ejercicio 2

Se tiene la siguiente matriz de factores no rotada, obtenida utilizando el método de componentes principales y considerando 4 factores.

```r
ej2_factores <- matrix(c( 0.881, 0.828, 0.664, 0.792, 0.731, 0.476,
-0.347, 0.508, -0.711, 0.564, -0.647, 0.804,
-0.165, -0.070,0.154, -0.179, 0.117, 0.329,
0.268, -0.200, -0.031, -0.029, -0.125, 0.135), nrow = 6)
row.names(ej2_factores) <- paste0("X",1:6)
colnames(ej2_factores) <- paste0("F",1:4)
ej2_factores
```

```
      F1     F2     F3     F4
X1 0.881 -0.347 -0.165  0.268
X2 0.828  0.508 -0.070 -0.200
X3 0.664 -0.711  0.154 -0.031
X4 0.792  0.564 -0.179 -0.029
X5 0.731 -0.647  0.117 -0.125
X6 0.476  0.804  0.329  0.135
```

```r
R <- cor(ej2_factores)
p <- nrow(R)
```

```r
eigen_fact <- eigen(R)

# ¿Cuál es la proporición de varianza explicada por los factores?
cumsum(eigen_fact$values)/p
```

```
[1] 0.4921837 0.7433322 0.9933350 1.0000000
```

```r
eigen_fact$values
```

```
[1] 1.96873490 1.00459406 1.00001099 0.02666006
```

```r
L <- NULL

# Vamos a tomar en cuenta m=2
for(i in 1:2){
L <- cbind(L,round(eigen_fact$vectors[,i]*sqrt(eigen_fact$values[i]),2))
}

# ¿Cuáles son las comunalidades?
(h <- diag(L %*% t(L)))
```

```
[1] 0.9802 0.8597 0.9866 0.1490
```

```r
# ¿cuáles son las unicidades?
(psihat <- rep(1,4) - diag(L %*% t(L)))
```

```
[1] 0.0198 0.1403 0.0134 0.8510
```

a. A partir de estos resultados, ¿se puede saber cómo agrupar las variables? Explicar. Se pueden tomar en cuenta varios criterios como que el eigenvalor $\lambda_i > 1$ y que la varianza explicada sea un porcentaje aceptable. En este caso podríamos elegir $m = 2$ ya que explica un 74% de la varianza y sus eigenvalores son mayores a 1.

b. ¿Cuál es la variable que más se identifica con las características? Explicar porqué.

Sería el tercer factor original, ya que su unicidad es menor.

Ejercicio 3

Verificar las siguientes identidades:

a. $(I + L'\Psi^{-1}L)^{-1}(L'\Psi^{-1}L) = I - (I + L'\Psi^{-1}L)^{-1}$

$$(I + L'\Psi^{-1}L)^{-1}(L'\Psi^{-1}L) = I - (I + L'\Psi^{-1}L)^{-1}$$

Multiplicando ambos lados por ((I + L' ^{-1} L)) para deshacernos del inverso en el lado derecho, obtenemos:

$$(I + L'\Psi^{-1}L)(I + L'\Psi^{-1}L)^{-1}(L'\Psi^{-1}L) = (I + L'\Psi^{-1}L)(I - (I + L'\Psi^{-1}L)^{-1})$$

Esto se simplifica a:

$$L'\Psi^{-1}L = L'\Psi^{-1}L - (I + L'\Psi^{-1}L)^{-1}L'\Psi^{-1}L + (I + L'\Psi^{-1}L)^{-1}$$

Dado que ((I + L' ^{-1} L)(I + L' ^{-1} L)^{-1} = I), podemos simplificar aún más:

$$L'\Psi^{-1}L = L'\Psi^{-1}L - L'\Psi^{-1}L + I$$

QED

b. $L'(LL' + \Psi)^{-1} = (I + L'\Psi^{-1}L)^{-1}L'\Psi^{-1}$

Para verificar esta identidad, podemos usar la identidad de Woodbury y la inversión de matrices:

$$L'(LL' + \Psi)^{-1} = (I + L'\Psi^{-1}L)^{-1}L'\Psi^{-1}$$

Usamos la identidad de Woodbury para la inversión de matrices:

$$(LL' + \Psi)^{-1} = \Psi^{-1} - \Psi^{-1}L(I + L'\Psi^{-1}L)^{-1}L'\Psi^{-1}$$

Multiplicamos ambos lados por L' desde la izquierda:

$$L'(LL' + \Psi)^{-1} = L'\Psi^{-1} - L'\Psi^{-1}L(I + L'\Psi^{-1}L)^{-1}L'\Psi^{-1}$$

Dado que $L'\Psi^{-1}L(I + L'\Psi^{-1}L)^{-1} = I - (I + L'\Psi^{-1}L)^{-1}$, podemos simplificar:

$$L'(LL' + \Psi)^{-1} = (I + L'\Psi^{-1}L)^{-1}L'\Psi^{-1}$$

QED.

Ejercicio 4

El siguiente ejemplo muestra un caso que se conoce como el caso de Heywood. Consideren un modelo factorial con $m = 1$ para la población con matriz de covarianza

$$\Sigma = \begin{pmatrix} 1 & 0.4 & 0.9 \\ 0.4 & 1 & 0.7 \\ 0.9 & 0.7 & 1 \end{pmatrix}$$

Mostrar que hay una solución única para L y Ψ con $\Sigma = LL' + \Psi$, pero que $\psi_3 < 0$, así que la elección no es admisible.

como m=1, tenemos el siguiente sistema de ecuaciones:

$$ l_{11}^2 + 1 = 1, l{11}l_{12} = 0.4, l_{11}l_{12} = 0.9 \

l_{12}^2 + 2 = 1, l{12}*l_{13} = 0.7 \

l_{13}^2 + _3 = 1 $$

Continuamos con el sistema

$$\begin{gathered} \iff l_{11}^2 = 1 - \psi_1, \quad l_{12} = \frac{0.4}{l_{11}}, \quad l_{13} = \frac{0.9}{l_{11}} \\ l_{12}^2 = 1 - \psi_2, \quad \frac{0.4}{l_{11}} * \frac{0.9}{l_{11}} = 0.7 \\ l_{13}^2 = 1 - \psi_3 \end{gathered}$$

finalmente:

$$\iff l_{11} = 0.7171 \iff l_{12} = 0.5577 \iff l_{13} = 1.2549 \iff \psi_3 = -0.5747 < 0$$

como tenemos una varianza negativa, esta solución no es admisible.

Ejercicio 5

Este ejercicio se basa en los datos de monitoreo atmosférico (REDMA) que se encuentran en GitHub. Los datos son series diarias de los contaminantes que están en el aire medidos en diferentes estaciones de monitoreo. La descripción de los datos la pueden encontrar en: esta liga, y el catálogo de estaciones está aquí. El archivo .zip contiene hojas de Excel con las mediciones diarias de 2019 y por hora, para cada una de las estaciones.

a. Hacer un análisis de estos datos, creando una base de datos con las mediciones de contaminación para cada estación de todos los contaminantes disponibles.

```r
library(readxl)
```

```
Warning: package 'readxl' was built under R version 4.2.3
```

```r
library(dplyr)
```

```

Attaching package: 'dplyr'

The following objects are masked from 'package:stats':

    filter, lag

The following objects are masked from 'package:base':

    intersect, setdiff, setequal, union
```

```r
library(tidyr)

REDMA19PM10 <- read_xls("2019PM10.xls")
REDMA20PM10 <- read.csv("2020PM10.csv") #El archivo viene así de origen
REDMA19PM25 <- read_xls("2019PM25.xls")
REDMA20PM25 <- read_xls("2020PM25.xls")
REDMA19PST <- read_xls("2019PST.xls")
REDMA20PST <- read_xls("2019PST.xls")
```

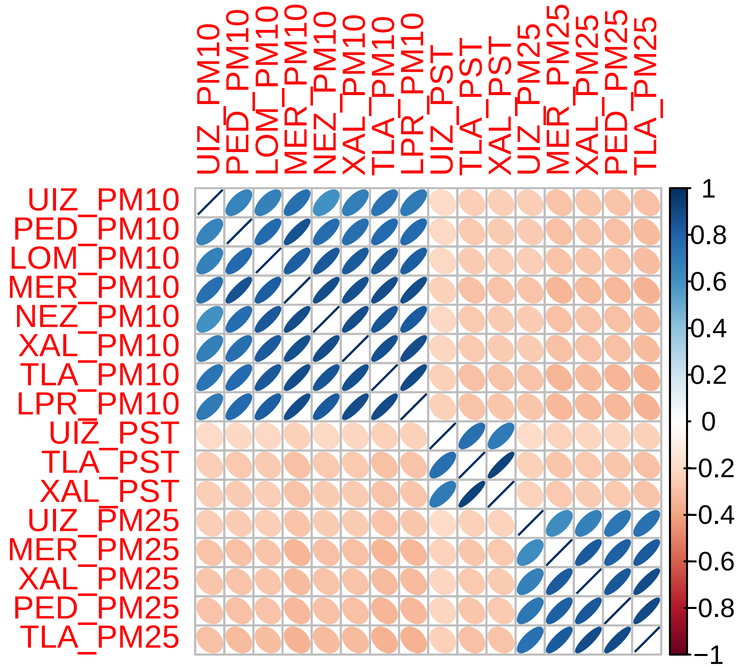
```r
REDMA20PM10 <- REDMA20PM10 %>%
                mutate(FECHA = as.Date(FECHA,format="%d/%m/%y"),
                TIPO = "PM10") %>%
                tibble()
REDMA19PM10 <- REDMA19PM10 %>%
                mutate(FECHA = as.Date(FECHA),
                       TIPO = "PM10")
REDMA19PM25 <- REDMA19PM25 %>%
                mutate(FECHA = as.Date(FECHA),
                       TIPO = "PM25")
REDMA20PM25 <- REDMA20PM25 %>%
                mutate(FECHA = as.Date(FECHA),
                       TIPO = "PM25")
REDMA19PST <- REDMA19PST %>%
                mutate(FECHA = as.Date(FECHA),
                       TIPO = "PST")
REDMA20PST <- REDMA20PST %>%
                mutate(FECHA = as.Date(FECHA),
                       TIPO = "PST")

REDMA <- full_join(REDMA19PM10,REDMA20PM10) %>%
             full_join(REDMA19PM25) %>%
             full_join(REDMA20PM25) %>%
             full_join(REDMA19PST) %>%
             full_join(REDMA20PST)
```

```
Joining with `by = join_by(FECHA, MER, PED, TLA, XAL, LOM, LPR, NEZ, SHA, UIZ,
TIPO)`

Joining with `by = join_by(FECHA, MER, PED, TLA, XAL, UIZ, TIPO)`
Joining with `by = join_by(FECHA, MER, PED, TLA, XAL, UIZ, TIPO, COY, SAG)`

Warning in full_join(., REDMA20PM25): Detected an unexpected many-to-many relationship betwe
i Row 169 of `x` matches multiple rows in `y`.
i Row 47 of `y` matches multiple rows in `x`.
i If a many-to-many relationship is expected, set `relationship =
  "many-to-many"` to silence this warning.

Joining with `by = join_by(FECHA, MER, PED, TLA, XAL, UIZ, TIPO)`
Joining with `by = join_by(FECHA, MER, PED, TLA, XAL, UIZ, TIPO)`
```

```r
REDMA <- REDMA[!is.na(REDMA$FECHA),]

REDMA[REDMA==-99] <- 0 # reemplazo -99 por NAs

REDMA2 <- REDMA %>%
           pivot_wider(names_from=TIPO,values_from=c(MER,PED,TLA,XAL,LOM,LPR,NEZ,UIZ))
REDMA2 <- REDMA2 %>%
select(-PED_PST,-LOM_PM25,-LOM_PST,-LPR_PM25,-LPR_PST,-NEZ_PM25,-NEZ_PST,-MER_PST) # quito

colnames(REDMA2)
```

```
 [1] "FECHA"    "SHA"      "COY"      "SAG"      "MER_PM10" "MER_PM25"
 [7] "PED_PM10" "PED_PM25" "TLA_PM10" "TLA_PM25" "TLA_PST"  "XAL_PM10"
[13] "XAL_PM25" "XAL_PST"  "LOM_PM10" "LPR_PM10" "NEZ_PM10" "UIZ_PM10"
[19] "UIZ_PM25" "UIZ_PST"
```

```r
REDMA2[is.na(REDMA2)] <- 0 # Reemplazamos -99 por 0 para el cálculo de scores

R <- cor(REDMA2 %>% select(-FECHA, -SHA, -COY, -SAG), use = "na.or.complete")


corrplot(R, method = "ellipse", order = "hclust")
```



Observamos que en general hay correlaciones postivas muy altas, cuando hay correlaciones negativas son muy bajas.

b. Hacer un análisis factorial exploratorio de estos datos. Interpretar y reportar los resultados: ¿Se pueden identificar factores?

```
p <- nrow(R)
eigen_REDMA <- eigen(R)

# ¿Cuál es la proporición de varianza explicada por los factores?
cumsum(eigen_REDMA$values)/p
```

```
 [1] 0.4833899 0.7510751 0.8379250 0.8657336 0.8917934 0.9122078 0.9308803
 [8] 0.9432400 0.9546628 0.9642013 0.9731490 0.9802081 0.9862054 0.9918264
[15] 0.9966092 1.0000000
```

```
eigen_REDMA$values
```

```
 [1] 7.73423765 4.28296334 1.38959882 0.44493854 0.41695626 0.32663088
 [7] 0.29875982 0.19775417 0.18276584 0.15261489 0.14316359 0.11294608
[13] 0.09595576 0.08993648 0.07652503 0.05425285
```

```r
L <- NULL

# Vamos a tomar en cuenta m=2
for(i in 1:2){
L <- cbind(L,round(eigen_REDMA$vectors[,i]*sqrt(eigen_REDMA$values[i]),2))
}

# ¿Cuáles son las comunalidades?
(h <- diag(L %*% t(L)))
```

```
 [1] 0.9090 0.7540 0.7417 0.7925 0.8864 0.8548 0.6568 0.8362 0.7816 0.6260
[11] 0.8010 0.8681 0.8185 0.6218 0.6025 0.4850
```

```r
# ¿cuáles son las unicidades?
(psihat <- rep(1,p) - diag(L %*% t(L)))
```

```
 [1] 0.0910 0.2460 0.2583 0.2075 0.1136 0.1452 0.3432 0.1638 0.2184 0.3740
[11] 0.1990 0.1319 0.1815 0.3782 0.3975 0.5150
```

Se puede observar que con 2 factores logramos explicar el 77% de la varianza y que sus eigenvalores son mayores a uno. Además, los valores de las comunalidades son bastante grandes.

c. Calcular los scores por el método de máxima verosimilitud y por el método de componentes principales.

```r
library(psych)
```

```
Warning: package 'psych' was built under R version 4.2.3
```

```r
# Máxima Verosimilitud
(m1 <-  fa(R, nfactors = 2, rotate = "none", fm = "ml", scores = "regression"))
```

```
Factor Analysis using method =  ml
Call: fa(r = R, nfactors = 2, rotate = "none", scores = "regression",
    fm = "ml")
Standardized loadings (pattern matrix) based upon correlation matrix
           ML1   ML2   h2    u2 com
MER_PM10  0.90  0.32 0.91 0.091 1.2
```

```
MER_PM25 -0.60  0.64 0.77 0.230 2.0
PED_PM10  0.80  0.28 0.72 0.276 1.2
PED_PM25 -0.61  0.69 0.86 0.141 2.0
TLA_PM10  0.89  0.31 0.88 0.116 1.2
TLA_PM25 -0.64  0.72 0.93 0.074 2.0
TLA_PST  -0.13 -0.55 0.32 0.676 1.1
XAL_PM10  0.87  0.34 0.86 0.137 1.3
XAL_PM25 -0.60  0.70 0.85 0.152 2.0
XAL_PST  -0.13 -0.54 0.30 0.697 1.1
LOM_PM10  0.83  0.32 0.79 0.209 1.3
LPR_PM10  0.88  0.31 0.87 0.127 1.2
NEZ_PM10  0.86  0.33 0.84 0.156 1.3
UIZ_PM10  0.72  0.23 0.57 0.430 1.2
UIZ_PM25 -0.51  0.56 0.57 0.426 2.0
UIZ_PST  -0.11 -0.45 0.21 0.786 1.1

                       ML1  ML2
SS loadings           7.52 3.76
Proportion Var        0.47 0.23
Cumulative Var        0.47 0.70
Proportion Explained  0.67 0.33
Cumulative Proportion 0.67 1.00

Mean item complexity =  1.5
Test of the hypothesis that 2 factors are sufficient.

df null model =  120  with the objective function =  19.74
df of  the model are 89  and the objective function was  2.93

The root mean square of the residuals (RMSR) is  0.08
The df corrected root mean square of the residuals is  0.1

Fit based upon off diagonal values = 0.97
Measures of factor score adequacy
                                                   ML1  ML2
Correlation of (regression) scores with factors   0.99 0.98
Multiple R square of scores with factors          0.98 0.96
Minimum correlation of possible factor scores     0.96 0.91
```

```r
(m2 <-  fa(R, nfactors = 2, rotate = "none", fm = "pa", scores = "Bartlett"))
```

```
Factor Analysis using method =  pa
```

```
Call: fa(r = R, nfactors = 2, rotate = "none", scores = "Bartlett",
    fm = "pa")
Standardized loadings (pattern matrix) based upon correlation matrix
           PA1   PA2   h2    u2 com
MER_PM10  0.93  0.21 0.91 0.093 1.1
MER_PM25 -0.53  0.66 0.73 0.273 1.9
PED_PM10  0.82  0.19 0.71 0.288 1.1
PED_PM25 -0.55  0.70 0.79 0.206 1.9
TLA_PM10  0.91  0.21 0.88 0.123 1.1
TLA_PM25 -0.57  0.73 0.86 0.138 1.9
TLA_PST  -0.21 -0.72 0.56 0.441 1.2
XAL_PM10  0.89  0.21 0.83 0.171 1.1
XAL_PM25 -0.54  0.69 0.77 0.231 1.9
XAL_PST  -0.20 -0.69 0.52 0.477 1.2
LOM_PM10  0.85  0.21 0.77 0.228 1.1
LPR_PM10  0.90  0.21 0.85 0.145 1.1
NEZ_PM10  0.87  0.21 0.80 0.199 1.1
UIZ_PM10  0.74  0.16 0.57 0.433 1.1
UIZ_PM25 -0.46  0.57 0.54 0.465 1.9
UIZ_PST  -0.17 -0.59 0.38 0.621 1.2

                       PA1  PA2
SS loadings           7.52 3.95
Proportion Var        0.47 0.25
Cumulative Var        0.47 0.72
Proportion Explained  0.66 0.34
Cumulative Proportion 0.66 1.00

Mean item complexity =  1.4
Test of the hypothesis that 2 factors are sufficient.

df null model =  120  with the objective function =  19.74
df of  the model are 89  and the objective function was  3.42

The root mean square of the residuals (RMSR) is  0.07
The df corrected root mean square of the residuals is  0.08

Fit based upon off diagonal values = 0.98
Measures of factor score adequacy
                                                   PA1  PA2
Correlation of (regression) scores with factors   0.99 0.97
Multiple R square of scores with factors          0.98 0.93
Minimum correlation of possible factor scores     0.96 0.87
```

d. ¿Se puede crear un índice de monitoreo ambiental que tome en cuenta todos los contaminantes? Si es así, ¿Cómo se puede interpretar su comportamiento a lo largo del tiempo?

Al haber realizado este análisis, se puede concluir que sí se puede crear un índice que tome en cuenta todos los contaminantes.

Ejercicio 6

En un estudio sobre pobreza, crimen y disuasión, Parker y Smith (1979) reportan ciertas estadísticas de crimen en varios estados para los años 1970 y 1973. Una porción de su matriz de correlación es de la forma:

$$R = \left[\begin{array}{c|c} R_{11} & R_{12} \\ \hline R_{21} & R_{22} \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 0.615 & -0.111 & -0.266 \\ 0.615 & 1 & -0.195 & -0.085 \\ \hline -0.111 & -0.195 & 1 & -0.269 \\ -0.266 & -0.085 & -0.269 & 1 \end{array}\right]$$

Las variables son:

- $X_1^{(1)}$ = homicidios no primarios en 1973.
- $X_2^{(1)}$ = homicidios primarios en 1973 (homicidios que involucran familia).
- $X_1^{(2)}$ = severidad de castigo en 1970 (meses promedio de prisión)
- $X_2^{(2)}$ = probabilidad de castigo en 1970 (número de encarcelados entre número de homicidios)

a. Encontrar las correlaciones canónicas muestrales.

```r
R <- matrix(c(
  1, 0.615, -0.111, -0.266,
  0.615, 1, -0.195, -0.085,
  -0.111, -0.195, 1, -0.269,
  -0.266, -0.085, -0.269, 1
), nrow = 4, byrow = TRUE)


RX1 <- R[1:2, 1:2]
RX2 <- R[3:4, 3:4]
R12 <- R[1:2, 3:4]
R21 <- R[3:4, 1:2]

A <- solve(RX1)%*% R12 %*% solve(RX2) %*% R21
```

```r
B <- solve(RX2)%*% R21 %*% solve(RX1) %*% R12
```

b. Determinar el primer par canónico $\widehat{U}_1, \widehat{V}_1$ e interpretar estas cantidades.

```r
eigen(A)$vectors[,1]
```

```
[1]  0.999996661 -0.002584248
```

```r
eigen(B)$vectors[,1]
```

```
[1] -0.5243952 -0.8514750
```

Sabemos que la combinación lineal $a'Z^{(1)} = (0.999)Z_1^{(1)} + (-0.003)Z_2^{(1)}$ y $b'Z^{(2)} = (-0.524)Z_1^{(2)} + (-0.851)Z_2^{(2)}$ maximiza la correlación entre $Z^{(1)}$ y $Z^{(2)}$

```r
sqrt(eigen(A)$values[1])
```

```
[1] 0.3266219
```

La correlación entre las dos primeras variables canónicas es 0.326.

```r
eigen(A)$values[1]/sum(eigen(A)$values)
```

```
[1] 0.7847333
```

La variabilidad explicada por las primeras variables canónicas es 0.785.

Ejercicio 7

Los datos que se usan en este ejercicio están relacionados con campañas de marketing directas de un banco portugués. Las campañas de marketing están basados en llamadas telefónicas. Con frecuencia, más de un contacto con el mismo cliente fue requerido, para accesar si el producto (depósito bancario a plazo) puede ser o no contratado. El archivo con la información relevante se puede obtener de la siguiente liga: bank.zip

Propongan un modelo para realizar CCA.

```r
ej7_datos <- read.csv("bank.csv", header = TRUE, sep = ";")

summary(ej7_datos)
```

```
      age            job              marital          education        
 Min.   :19.00   Length:4521        Length:4521        Length:4521       
 1st Qu.:33.00   Class :character   Class :character   Class :character  
 Median :39.00   Mode  :character   Mode  :character   Mode  :character  
 Mean   :41.17                                                           
 3rd Qu.:49.00                                                           
 Max.   :87.00                                                           
   default             balance        housing              loan          
 Length:4521        Min.   :-3313   Length:4521        Length:4521       
 Class :character   1st Qu.:   69   Class :character   Class :character  
 Mode  :character   Median :  444   Mode  :character   Mode  :character  
                    Mean   : 1423                                        
                    3rd Qu.: 1480                                        
                    Max.   :71188                                        
   contact               day           month              duration   
 Length:4521        Min.   : 1.00   Length:4521        Min.   :   4  
 Class :character   1st Qu.: 9.00   Class :character   1st Qu.: 104  
 Mode  :character   Median :16.00   Mode  :character   Median : 185  
                    Mean   :15.92                      Mean   : 264  
                    3rd Qu.:21.00                      3rd Qu.: 329  
                    Max.   :31.00                      Max.   :3025  
    campaign          pdays           previous          poutcome        
 Min.   : 1.000   Min.   : -1.00   Min.   : 0.0000   Length:4521       
 1st Qu.: 1.000   1st Qu.: -1.00   1st Qu.: 0.0000   Class :character  
 Median : 2.000   Median : -1.00   Median : 0.0000   Mode  :character  
 Mean   : 2.794   Mean   : 39.77   Mean   : 0.5426                     
 3rd Qu.: 3.000   3rd Qu.: -1.00   3rd Qu.: 0.0000                     
 Max.   :50.000   Max.   :871.00   Max.   :25.0000                     
      y            
 Length:4521       
 Class :character  
 Mode  :character  
```

```r
str(ej7_datos)
```

```
'data.frame':	4521 obs. of  17 variables:
 $ age      : int  30 33 35 30 59 35 36 39 41 43 ...
 $ job      : chr  "unemployed" "services" "management" "management" ...
 $ marital  : chr  "married" "married" "single" "married" ...
 $ education: chr  "primary" "secondary" "tertiary" "tertiary" ...
 $ default  : chr  "no" "no" "no" "no" ...
 $ balance  : int  1787 4789 1350 1476 0 747 307 147 221 -88 ...
 $ housing  : chr  "no" "yes" "yes" "yes" ...
 $ loan     : chr  "no" "yes" "no" "yes" ...
 $ contact  : chr  "cellular" "cellular" "cellular" "unknown" ...
 $ day      : int  19 11 16 3 5 23 14 6 14 17 ...
 $ month    : chr  "oct" "may" "apr" "jun" ...
 $ duration : int  79 220 185 199 226 141 341 151 57 313 ...
 $ campaign : int  1 1 1 4 1 2 1 2 2 1 ...
 $ pdays    : int  -1 339 330 -1 -1 176 330 -1 -1 147 ...
 $ previous : int  0 4 1 0 0 3 2 0 0 2 ...
 $ poutcome : chr  "unknown" "failure" "failure" "unknown" ...
 $ y        : chr  "no" "no" "no" "no" ...
```

```r
# Definir los conjuntos de variables
set1 <- ej7_datos[, c("age", "balance", "day", "duration", "campaign", "pdays", "previous"
set2 <- model.matrix(~ job + marital + education + default + housing + loan + contact + mo

# Realizar CCA
cca_result <- cancor(set1, set2)

# Ver los resultados
print(cca_result)
```

```
$cor
[1] 0.9067206 0.6521740 0.4964732 0.4253007 0.2686322 0.2123378 0.1648194

$xcoef
                  [,1]          [,2]          [,3]          [,4]          [,5]
age      -1.047629e-05  1.401623e-03 -6.580207e-06  1.797488e-05  1.298099e-04
balance  -2.875659e-08  1.524252e-07 -5.233865e-08  3.302582e-08 -3.116282e-06
day      -1.674222e-06  4.773370e-05  1.690783e-03 -6.792023e-04  1.342350e-04
duration  2.812921e-07 -5.621686e-07  2.124309e-05  5.283091e-05  4.955981e-06
campaign  1.554616e-04 -8.350079e-05 -1.235026e-03  2.281317e-04  3.384525e-03
pdays    -1.207785e-04 -2.169362e-06  3.635785e-06 -6.753052e-06  4.984125e-05
previous -2.404282e-03  1.663103e-04  3.652867e-04  4.457892e-05 -2.560205e-03
```

```
                 [,6]          [,7]
age       7.590947e-05  7.148585e-05
balance  -2.329665e-06 -3.075550e-06
day      -1.017649e-04 -1.078714e-04
duration -3.664857e-06 -3.640850e-06
campaign -3.052360e-03 -1.165897e-03
pdays     7.131971e-05 -1.057671e-04
previous -6.395150e-03  7.904625e-03

$ycoef
                            [,1]          [,2]          [,3]          [,4]
jobadmin.           0.0018534701 -7.815757e-03  4.418502e-03 -0.0018281833
jobblue-collar      0.0016635669 -1.083362e-02  5.980197e-03  0.0057823376
jobentrepreneur     0.0011620301 -6.140798e-03  4.035483e-03  0.0082810015
jobhousemaid        0.0029871217 -3.129399e-04  7.867748e-03  0.0090777160
jobmanagement       0.0017499339 -4.808141e-03  6.755225e-03  0.0031576906
jobretired          0.0029043678  2.915576e-02  3.686938e-03  0.0054375704
jobself-employed    0.0019956666 -4.716189e-03  7.377578e-03  0.0028701652
jobservices         0.0017186389 -1.017128e-02  4.664602e-03  0.0021342524
jobstudent          0.0025415920 -2.920148e-02  7.174977e-03 -0.0037218588
jobtechnician       0.0017343086 -7.269400e-03  7.244070e-03  0.0009844847
jobunemployed       0.0010783918 -8.397161e-03  7.703775e-03  0.0083127029
maritalmarried      0.0004648763 -3.915212e-03 -2.259474e-04 -0.0010424840
maritalsingle       0.0005226801 -1.984735e-02  7.816480e-04  0.0004272899
educationsecondary  0.0006944750 -8.365951e-03  1.184295e-03  0.0022244602
educationtertiary   0.0009878993 -1.090312e-02 -6.005131e-05 -0.0008198784
educationunknown    0.0006864731  9.914787e-05  1.432193e-03  0.0011337404
defaultyes         -0.0006881677 -2.094497e-03 -2.491623e-03 -0.0051532242
housingyes         -0.0004894376 -5.255890e-03 -3.119575e-03  0.0026835367
loanyes            -0.0001422637 -7.713328e-04 -2.070101e-03  0.0022196624
contacttelephone    0.0006364484  1.059215e-02  1.845087e-03 -0.0046691429
contactunknown      0.0026752584  1.636762e-03  1.157302e-02 -0.0019970181
monthaug            0.0005669021  2.431121e-03 -1.282315e-02  0.0013468762
monthdec            0.0017215075  1.548237e-03 -3.538785e-03  0.0115331040
monthfeb            0.0016585265  6.079801e-04 -4.083823e-02  0.0146207385
monthjan            0.0022021830  9.924127e-04  3.452833e-02 -0.0142008900
monthjul            0.0003748685 -1.519579e-03 -3.079377e-03  0.0016532781
monthjun           -0.0016587589 -2.676017e-04 -3.762607e-02  0.0116950163
monthmar            0.0015183218  5.014779e-03 -2.408695e-02 -0.0118392630
monthmay           -0.0025618471 -1.336029e-03 -1.383675e-02  0.0053430314
monthnov            0.0030241318  2.449321e-03  2.745421e-03  0.0006865639
monthoct           -0.0013699270  6.794177e-03 -3.059443e-03 -0.0124763087
monthsep            0.0010453853 -1.760703e-03 -2.354367e-02 -0.0043120957
```

```
poutcomeother       0.0017089395 -1.613112e-03 -5.413276e-04  0.0003871218
poutcomesuccess     0.0092627618  1.344017e-03 -6.227432e-03 -0.0067334455
poutcomeunknown     0.0397257356 -2.109376e-03  2.128035e-04  0.0035573083
yyes                0.0002064868  5.217253e-06  1.807511e-02  0.0442963545
                            [,5]          [,6]          [,7]          [,8]
jobadmin.          -5.613135e-04 -0.0082828564 -0.0017937820  1.658165e-02
jobblue-collar      1.641657e-03 -0.0106889855 -0.0015818241  1.458373e-02
jobentrepreneur    -4.316990e-03 -0.0090153385 -0.0083278476  1.398728e-02
jobhousemaid       -1.395639e-02 -0.0058853280 -0.0123053406  1.725353e-02
jobmanagement       8.269205e-05 -0.0107162165 -0.0099924117  1.666298e-02
jobretired         -8.351013e-03 -0.0119579513 -0.0063635147  1.843284e-02
jobself-employed    4.886382e-03 -0.0197698885  0.0032510511  2.263745e-02
jobservices        -1.508487e-04 -0.0101133104 -0.0016975818  5.865640e-02
jobstudent         -9.330068e-03 -0.0191391634  0.0074614938  2.036040e-03
jobtechnician      -3.386607e-03 -0.0080823440  0.0002165074  8.605744e-04
jobunemployed       5.573430e-03 -0.0074161192 -0.0021444213 -8.637199e-05
maritalmarried     -3.570050e-03 -0.0092024876 -0.0038930498  6.161492e-04
maritalsingle      -7.138867e-03 -0.0147319909 -0.0062030803  1.010582e-03
educationsecondary -2.268109e-03  0.0009610443 -0.0006487951  3.706420e-05
educationtertiary  -7.083400e-03 -0.0067922625 -0.0024111310  7.544684e-04
educationunknown   -4.373606e-03  0.0037207768 -0.0079137149 -4.522983e-05
defaultyes          1.082066e-02  0.0248737902  0.0256656979 -1.352915e-03
housingyes          6.357260e-03 -0.0042157752 -0.0063886836 -6.606708e-04
loanyes             3.962356e-03  0.0037525817  0.0097060856 -3.739879e-04
contacttelephone    3.398675e-03 -0.0114133834 -0.0039902674  4.517706e-04
contactunknown      1.551766e-03 -0.0087981987  0.0057543045  9.541066e-04
monthaug            2.829561e-02 -0.0349568452  0.0070279724 -7.643870e-04
monthdec           -1.625265e-02 -0.0299148335 -0.0253853798  1.601286e-03
monthfeb            1.261818e-04 -0.0083464932  0.0313332865 -7.882789e-04
monthjan            1.045745e-02 -0.0052486760  0.0230616854  1.791665e-03
monthjul            2.938555e-02 -0.0239736649  0.0141743023 -8.464362e-04
monthjun            6.064386e-03 -0.0181269847  0.0008266986 -1.311727e-03
monthmar            5.192380e-03 -0.0193210645  0.0137449520  2.875471e-04
monthmay            6.555242e-03 -0.0020179600  0.0141090124 -6.811428e-04
monthnov           -1.559144e-02 -0.0261340930  0.0086935654  2.783257e-03
monthoct           -1.720901e-02 -0.0265963779  0.0219662322  3.486978e-03
monthsep           -5.018669e-03 -0.0029256051  0.0285066877  4.332114e-04
poutcomeother      -3.014975e-03 -0.0280949635  0.0409968881  2.654163e-03
poutcomesuccess    -1.962112e-02 -0.0279117396  0.0545984439  4.128577e-03
poutcomeunknown    -1.097258e-02 -0.0017638433  0.0143793377  5.839334e-04
yyes                4.114628e-03 -0.0007912733 -0.0018215856 -7.539639e-04
                            [,9]         [,10]         [,11]         [,12]
jobadmin.           0.0060763744 -6.648840e-02  1.372103e-01 -0.0265735107
```

```
jobblue-collar      -0.0090432633 -7.120981e-02  1.396293e-01 -0.0054567207
jobentrepreneur     -0.0056334891 -6.960229e-02  1.410369e-01 -0.0059112803
jobhousemaid        -0.0048536897 -7.281668e-02  1.358031e-01 -0.0162032230
jobmanagement       -0.0069496514 -6.864375e-02  1.407599e-01 -0.0085477094
jobretired          -0.0130392289 -7.213255e-02  1.448602e-01  0.0062215925
jobself-employed     0.0022004426 -6.807561e-02  1.406282e-01 -0.0059311318
jobservices         -0.0057608803 -6.931968e-02  1.412872e-01 -0.0109772605
jobstudent          -0.1029123993 -6.734167e-02  1.401088e-01 -0.0313941482
jobtechnician        0.0031796622 -8.475175e-02  1.416248e-01 -0.0110672994
jobunemployed        0.0022844789  3.977629e-04  1.831812e-01 -0.0102874012
maritalmarried       0.0021748706  9.257268e-04 -2.469646e-04 -0.0329329335
maritalsingle        0.0072258843  2.316579e-03 -1.582164e-03 -0.0073091906
educationsecondary   0.0023580643  4.766062e-04 -7.428777e-04 -0.0032690795
educationtertiary    0.0042542802  1.324996e-03 -7.044343e-04 -0.0040301835
educationunknown    -0.0002123148  2.814687e-06  1.341023e-04 -0.0003824477
defaultyes          -0.0021391883 -1.651667e-03  4.453532e-05  0.0008351830
housingyes           0.0007092809  4.177135e-04 -6.790907e-04 -0.0017005241
loanyes             -0.0001936025 -4.619932e-04 -2.686446e-04 -0.0002111639
contacttelephone    -0.0022710674 -1.269334e-05  8.504953e-04  0.0042177318
contactunknown       0.0005770321  1.048259e-04 -2.295680e-04  0.0004680552
monthaug            -0.0004059382  7.768090e-04 -8.749803e-04  0.0024165377
monthdec             0.0034876418  1.961120e-03 -2.446774e-04 -0.0019598563
monthfeb             0.0019693787 -8.071484e-05 -2.132962e-04 -0.0002977530
monthjan             0.0001527023 -4.681773e-04 -4.203521e-04  0.0015783217
monthjul            -0.0001866221  1.887220e-04 -1.362116e-03  0.0008701660
monthjun             0.0010181079  8.521496e-04 -2.437952e-04 -0.0001244205
monthmar             0.0002774913  9.949735e-04  1.122885e-03  0.0041511426
monthmay             0.0003831216 -2.210256e-04 -3.978936e-04 -0.0003082537
monthnov             0.0043152210  1.423740e-03  8.442254e-05 -0.0003802435
monthoct             0.0038989674  1.542427e-03  1.269725e-03  0.0028453205
monthsep             0.0024653595  4.108182e-04  5.261460e-04  0.0004592747
poutcomeother        0.0055231537  9.496821e-04 -7.450562e-04 -0.0008807072
poutcomesuccess      0.0072260493  1.316527e-03  3.546504e-04  0.0003208858
poutcomeunknown      0.0024014059  4.510649e-05 -3.227666e-04 -0.0016368179
yyes                 0.0001654656 -1.796005e-03 -3.067104e-03 -0.0059346259
                            [,13]         [,14]         [,15]         [,16]
jobadmin.           -0.0201919117  2.665743e-02  2.274858e-02 -2.218941e-02
jobblue-collar       0.0019774843  1.649587e-02 -1.238245e-03 -3.131546e-02
jobentrepreneur     -0.0045706648  2.294940e-03  2.174255e-02 -2.148942e-02
jobhousemaid        -0.0072836239  7.621470e-03  7.191396e-03 -2.779416e-02
jobmanagement       -0.0088863421 -4.979067e-03  3.423674e-02 -2.130087e-02
jobretired           0.0159134614  4.026100e-03 -7.436550e-04 -2.492474e-02
jobself-employed    -0.0093083584  2.947674e-03  2.874296e-02 -2.897814e-02
```

```
jobservices        -0.0035845170  2.337878e-02  1.092033e-02 -2.176264e-02
jobstudent         -0.0280685098  1.336834e-02  1.577496e-02 -1.123383e-02
jobtechnician      -0.0044896880  1.650958e-02  1.764772e-02 -2.046056e-02
jobunemployed      -0.0003766101  1.276475e-02  1.262726e-02 -2.531508e-02
maritalmarried      0.0316064220 -2.952407e-04  1.839608e-03 -5.604139e-04
maritalsingle       0.0410751320 -1.784134e-04  1.065685e-02  8.002305e-04
educationsecondary -0.0032037203 -3.350681e-02 -2.439025e-02 -2.024264e-02
educationtertiary  -0.0065106008 -1.914770e-04 -5.024490e-02 -2.245815e-02
educationunknown   -0.0003720341  3.666081e-05 -7.367193e-05 -8.434659e-02
defaultyes          0.0088209072  3.554953e-03 -3.908639e-03 -1.839764e-03
housingyes         -0.0028184774  4.688879e-04  1.255501e-03 -1.217843e-05
loanyes             0.0018226448  9.737138e-04 -5.867285e-04 -9.127231e-04
contacttelephone    0.0014312342 -2.265180e-03 -1.182974e-03 -3.631917e-04
contactunknown     -0.0005960207 -2.279773e-04  3.819569e-04 -7.681561e-04
monthaug           -0.0033473835 -2.158941e-03  1.759565e-03 -3.825053e-03
monthdec           -0.0111199337 -4.359811e-03  5.412944e-03  1.855811e-03
monthfeb            0.0002632480 -1.820572e-03  1.810892e-03 -3.568430e-03
monthjan            0.0032593179  1.831451e-03 -1.495079e-03 -2.254111e-03
monthjul           -0.0014224371  2.064737e-04  9.379450e-04 -3.872387e-03
monthjun           -0.0042110927 -2.869340e-03  2.768285e-03 -1.679403e-03
monthmar           -0.0003909928 -4.073237e-03  7.839587e-04 -2.677650e-03
monthmay            0.0007769193  2.117630e-04  3.107844e-04 -1.806906e-03
monthnov           -0.0062643354 -3.675160e-03  4.039318e-03 -8.259892e-04
monthoct           -0.0032671199 -5.285406e-03  2.910217e-03 -1.973874e-03
monthsep            0.0007375712 -1.673215e-03  1.141513e-03 -2.590525e-03
poutcomeother      -0.0037987061 -2.067928e-03  3.960288e-03 -4.343720e-03
poutcomesuccess    -0.0031168746 -4.168143e-03  4.388348e-03 -4.708807e-03
poutcomeunknown    -0.0011337814 -1.569090e-04  1.254912e-03 -1.006871e-03
yyes               -0.0017870101  4.282685e-03  1.163895e-03  3.344059e-04
                           [,17]         [,18]         [,19]         [,20]
jobadmin.          -0.0039067877  1.964309e-02 -0.0086559222  0.0140018016
jobblue-collar     -0.0073123699  1.946189e-02 -0.0123133512  0.0055727347
jobentrepreneur    -0.0036123563  1.953044e-02 -0.0069556616  0.0112787757
jobhousemaid        0.0082406851  1.206227e-02  0.0039354253  0.0040666575
jobmanagement       0.0047665191  1.352466e-02 -0.0012269095 -0.0007119950
jobretired         -0.0011818860 -9.143204e-03 -0.0022071840 -0.0040598452
jobself-employed   -0.0055013012 -5.040009e-03 -0.0137272525 -0.0013218696
jobservices         0.0004473773  1.927472e-02 -0.0034273064  0.0029471227
jobstudent         -0.0002830992  4.925186e-03  0.0005237426  0.0041187207
jobtechnician      -0.0005177420  1.479160e-02 -0.0027967765  0.0036878176
jobunemployed      -0.0010125161  1.360335e-02 -0.0010914245  0.0027394661
maritalmarried      0.0047417709  1.464171e-05  0.0017601073 -0.0007697751
maritalsingle       0.0045382610  3.065639e-06  0.0037101527  0.0033284931
```

```
educationsecondary -0.0008552048  2.734984e-03 -0.0017420940  0.0046020101
educationtertiary   0.0023262775 -1.074864e-04  0.0009322959  0.0051532283
educationunknown    0.0012836465  3.032069e-04  0.0055132770  0.0013751440
defaultyes          0.1058942647 -9.576898e-04 -0.0149550594  0.0053071707
housingyes          0.0011920658 -2.931666e-02  0.0007485127  0.0024365399
loanyes            -0.0027936203 -8.749105e-04  0.0391634179  0.0009108636
contacttelephone    0.0029012587 -1.500625e-03  0.0012655828  0.0557133687
contactunknown      0.0004121151 -2.696154e-03 -0.0008177223 -0.0024451689
monthaug            0.0021455596 -2.226626e-03 -0.0031268086 -0.0053469389
monthdec            0.0113067255  1.105510e-03  0.0086680301 -0.0021161968
monthfeb           -0.0052834884 -4.855456e-03 -0.0062329533  0.0037388252
monthjan           -0.0040768574 -6.164087e-03 -0.0054462523 -0.0050261387
monthjul           -0.0010874629 -2.303247e-03 -0.0052343012 -0.0037037516
monthjun            0.0018720938  6.076088e-04 -0.0002456518  0.0013145957
monthmar           -0.0007143616 -4.633813e-03 -0.0028380912 -0.0030145669
monthmay           -0.0030386364 -1.183953e-03 -0.0035700772  0.0013192944
monthnov            0.0037491530 -5.920046e-03  0.0014255573 -0.0029053654
monthoct            0.0011238279 -9.104119e-03 -0.0010398133 -0.0047561005
monthsep           -0.0056484438 -5.517826e-03 -0.0054920790  0.0020735491
poutcomeother      -0.0034402308 -1.053903e-02 -0.0066210700 -0.0023133311
poutcomesuccess    -0.0054409879 -1.576760e-02 -0.0077507636 -0.0018294685
poutcomeunknown    -0.0030727957 -7.810717e-03 -0.0028579857  0.0031136349
yyes                0.0022920211  2.613159e-03  0.0009530107  0.0026137091
                           [,21]         [,22]         [,23]         [,24]
jobadmin.          -0.0157210075  0.0019103167 -0.0004222274 -5.485964e-03
jobblue-collar     -0.0085848098 -0.0113884304  0.0027399594 -5.507209e-03
jobentrepreneur    -0.0108631261 -0.0007145965 -0.0032173491 -4.518574e-02
jobhousemaid       -0.0014721780  0.0051762446 -0.0136521918  2.799146e-03
jobmanagement      -0.0045451816 -0.0218363581 -0.0045875416  4.379883e-03
jobretired         -0.0174908076  0.0036015442 -0.0015923420 -6.331997e-03
jobself-employed   -0.0083459883 -0.0094359750 -0.0009197570 -1.109750e-02
jobservices        -0.0035636614  0.0023203378 -0.0025662950  1.794088e-04
jobstudent         -0.0055176419  0.0066418452 -0.0061188298 -1.725751e-03
jobtechnician      -0.0005196271 -0.0028143716 -0.0032210718 -1.134969e-04
jobunemployed       0.0001018863  0.0034254417 -0.0052730553 -5.817178e-03
maritalmarried      0.0003703075 -0.0027270748 -0.0014734632  1.296434e-03
maritalsingle      -0.0021196730  0.0017733130 -0.0019820289  2.769049e-05
educationsecondary  0.0001761961  0.0014487803 -0.0003255652  7.255491e-05
educationtertiary   0.0014471970  0.0004871059 -0.0012826835 -5.021126e-04
educationunknown   -0.0017214522  0.0048309883 -0.0022960078  1.955155e-03
defaultyes         -0.0032923066  0.0039725227  0.0066712990 -9.522006e-03
housingyes         -0.0057345116  0.0053704845  0.0005166707  2.268462e-03
loanyes            -0.0003768991  0.0019224223  0.0020905893 -3.086147e-03
```

```
contacttelephone    0.0137981011  0.0004560715 -0.0006868656  5.114150e-03
contactunknown      0.0350587982  0.0043179654  0.0012098775  1.150875e-02
monthaug            0.0019420128  0.0242416208  0.0022049575  3.955670e-03
monthdec            0.0005934434 -0.0030410871  0.2181205759  1.050150e-02
monthfeb           -0.0079970040 -0.0057566446  0.0022287682  4.130414e-02
monthjan            0.0024514551 -0.0033585279  0.0027051868  1.579178e-02
monthjul            0.0013535150 -0.0186673767  0.0013560009 -1.048567e-03
monthjun           -0.0028988171 -0.0104909282 -0.0010204446 -1.930242e-02
monthmar           -0.0021487777 -0.0114291603  0.0016975736 -1.092728e-02
monthmay           -0.0017488751 -0.0035099980  0.0014500778 -1.061011e-02
monthnov           -0.0021040843 -0.0020987653 -0.0026648977  1.461981e-04
monthoct           -0.0015864685 -0.0024959212 -0.0006254517 -3.454676e-03
monthsep           -0.0064639888 -0.0010085380  0.0032256091 -1.794265e-02
poutcomeother      -0.0047729179 -0.0069112421  0.0012191569 -8.097519e-03
poutcomesuccess    -0.0100729160 -0.0026877077  0.0024393508 -1.378397e-02
poutcomeunknown    -0.0156213967 -0.0021101717  0.0023859185 -3.567532e-03
yyes                0.0016372825  0.0019439607 -0.0043897755  2.219677e-03
                           [,25]         [,26]         [,27]         [,28]
jobadmin.          -0.0107950660 -0.0130213796  0.0012563024  7.535687e-04
jobblue-collar     -0.0094544038 -0.0264862913  0.0006883563  7.945938e-05
jobentrepreneur    -0.0504564189  0.0094919803 -0.0363331028 -2.329590e-03
jobhousemaid       -0.0134534916 -0.0013503755 -0.0032220365  8.312994e-03
jobmanagement      -0.0137630240 -0.0212473006 -0.0105320543 -1.583411e-03
jobretired         -0.0094717638 -0.0009738294  0.0034230024 -3.888040e-03
jobself-employed   -0.0035472174 -0.0203452157 -0.0000886595 -2.828477e-03
jobservices        -0.0083630544 -0.0046439580  0.0023049284  3.126748e-03
jobstudent         -0.0049025728  0.0016032858  0.0063131796 -2.080492e-03
jobtechnician      -0.0042520217 -0.0016198658  0.0038713677  2.508078e-03
jobunemployed       0.0005050128 -0.0055092327  0.0027610529  3.992928e-03
maritalmarried     -0.0007871189  0.0010647862 -0.0020282229 -1.268038e-04
maritalsingle      -0.0010268041  0.0029790277 -0.0015913993  7.945882e-04
educationsecondary -0.0003838762  0.0016764981  0.0027808006  5.154997e-04
educationtertiary  -0.0017068152  0.0042090278  0.0003937119  2.975488e-04
educationunknown    0.0021363738  0.0045661483  0.0006149561  1.876238e-03
defaultyes          0.0013836700 -0.0091949875  0.0075162104 -2.274998e-04
housingyes         -0.0051035677  0.0014271934  0.0039447030  2.928447e-03
loanyes            -0.0010849481 -0.0094244471  0.0010474796  4.088346e-04
contacttelephone    0.0012529284 -0.0080113647 -0.0010697037 -3.143821e-03
contactunknown      0.0007661222  0.0067100614 -0.0112271781  2.791233e-03
monthaug           -0.0150017743 -0.0066510301 -0.0075933181  1.385460e-03
monthdec           -0.0155144205  0.0219335676 -0.0049606528  6.961719e-03
monthfeb           -0.0268256088  0.0131528645 -0.0140390068 -5.113946e-04
monthjan           -0.0650990269 -0.0018407064  0.0246205908  6.411504e-03
```

```
monthjul          -0.0029043440  0.0238724177 -0.0024545000  1.389073e-03
monthjun          -0.0216960199 -0.0010739794  0.0301455235 -1.811938e-03
monthmar          -0.0094471392 -0.0070082610 -0.0121564589  1.417672e-01
monthmay          -0.0067223738 -0.0035683213 -0.0065375512 -2.258952e-03
monthnov          -0.0002567841  0.0008648835 -0.0014669475 -1.220401e-03
monthoct           0.0009980170  0.0005524691 -0.0004835625 -2.878570e-03
monthsep          -0.0084157585 -0.0004476922 -0.0067164158 -3.988083e-03
poutcomeother      0.0004256563 -0.0110207345 -0.0003170540 -3.912644e-03
poutcomesuccess    0.0002055122 -0.0062692110  0.0014099376 -5.092995e-03
poutcomeunknown   -0.0009941640 -0.0080505674  0.0020397687 -1.579232e-03
yyes              -0.0005885058 -0.0014314547  0.0004314500  2.889086e-03
                          [,29]         [,30]         [,31]         [,32]
jobadmin.         -0.0021261682 -7.996279e-03  7.628465e-04 -1.680250e-03
jobblue-collar     0.0118393790  2.181906e-04  4.311914e-03 -3.140406e-03
jobentrepreneur    0.0178910447 -7.099737e-03  2.998800e-03  6.738105e-03
jobhousemaid      -0.0123085743 -7.630905e-03 -2.547220e-03 -1.114318e-02
jobmanagement     -0.0043329713 -1.181872e-02 -1.754695e-04 -6.945955e-03
jobretired        -0.0095187952 -8.513819e-03 -5.284066e-03 -1.892395e-03
jobself-employed  -0.0063681078 -1.984736e-02 -1.803218e-02  1.166821e-02
jobservices        0.0013587466 -5.353551e-03  8.873099e-04 -6.092923e-03
jobstudent        -0.0065063691 -7.163364e-03 -6.192423e-04  6.225076e-03
jobtechnician     -0.0016644507 -6.028783e-03 -6.935629e-04 -5.274415e-03
jobunemployed     -0.0013539273 -6.065928e-03  2.570389e-03 -3.964540e-03
maritalmarried    -0.0041490957 -1.110999e-03 -8.978364e-04  6.773292e-04
maritalsingle     -0.0055403513 -2.166491e-03 -6.673353e-04  1.072930e-03
educationsecondary -0.0011172401  1.183531e-04  8.916065e-04 -4.156015e-05
educationtertiary -0.0043163950 -2.184173e-03 -1.126649e-03  1.548233e-03
educationunknown  -0.0005614454  5.200885e-04  6.018736e-04 -2.100350e-05
defaultyes         0.0103899132  3.157020e-03  1.931541e-03  4.161057e-03
housingyes         0.0069642173  6.862241e-05  5.354477e-03 -3.744853e-03
loanyes            0.0019405101 -6.681443e-04 -5.666017e-05  1.758715e-03
contacttelephone  -0.0009135038 -2.876887e-03 -3.072581e-03 -1.335250e-03
contactunknown     0.0191680704 -1.767756e-03 -3.554733e-03  2.161297e-03
monthaug          -0.0153736334  3.186542e-02  2.295190e-02 -2.106130e-02
monthdec          -0.0313376002  2.594857e-02  2.200158e-02 -2.793105e-02
monthfeb          -0.0100830996  3.378943e-02  2.034834e-02 -1.093243e-02
monthjan          -0.0280749521  3.172220e-02  2.041787e-02 -2.423069e-02
monthjul          -0.0179939265  3.371710e-02  2.233628e-02 -2.007432e-02
monthjun          -0.0331958037  3.271456e-02  2.431963e-02 -1.936685e-02
monthmar          -0.0167009798  3.064842e-02  2.215315e-02 -1.799687e-02
monthmay          -0.0459366733  3.456705e-02  2.183943e-02 -1.831125e-02
monthnov          -0.0088961150  6.308334e-02  1.650041e-02 -1.974361e-02
monthoct          -0.0068809339 -7.678258e-03  1.160571e-01 -1.746284e-02
```

```
monthsep             0.0059992879 -1.089586e-03 -4.046369e-03 -1.421804e-01
poutcomeother       -0.0025413847 -6.984689e-03 -1.007268e-02  1.030777e-02
poutcomesuccess     -0.0054440145 -1.012652e-02 -1.371951e-02  1.471758e-02
poutcomeunknown     -0.0064058200 -6.437900e-03 -2.425342e-03  2.691932e-03
yyes                -0.0014301573  4.203021e-04  6.080254e-04 -3.630425e-03
                            [,33]         [,34]         [,35]         [,36]
jobadmin.            0.0090434448  0.0024596279  0.0433584083 -7.627751e-03
jobblue-collar      -0.0005706945  0.0006993307  0.0206568976 -1.495571e-02
jobentrepreneur     -0.0019085739 -0.0021767883  0.0090594982 -7.303735e-03
jobhousemaid         0.0017591679 -0.0096332463  0.0138103305 -9.598735e-02
jobmanagement        0.0026046506  0.0074979546  0.0161389082 -7.162577e-03
jobretired           0.0167895925  0.0053444435  0.0414893462 -3.467143e-03
jobself-employed    -0.0400201711 -0.0406388935  0.0255948804 -8.848967e-03
jobservices         -0.0001593014  0.0004616123  0.0070018853 -2.428608e-03
jobstudent          -0.0039228801 -0.0019599946  0.0219350508 -2.419421e-03
jobtechnician       -0.0008083707 -0.0009799858  0.0051701701 -1.399763e-03
jobunemployed        0.0007165508  0.0019316817  0.0054917869 -8.607467e-03
maritalmarried       0.0003423469 -0.0012373427  0.0017672789  3.447059e-03
maritalsingle        0.0028836168 -0.0001999274  0.0112255859 -6.015369e-04
educationsecondary   0.0021252890  0.0009102183  0.0049174514 -3.217656e-03
educationtertiary    0.0017451237 -0.0009428991  0.0070082659 -5.083032e-04
educationunknown     0.0037427017  0.0017043354 -0.0025748228  1.032310e-03
defaultyes          -0.0036184087 -0.0019596257  0.0098385209  2.169296e-03
housingyes           0.0002818436  0.0045103608 -0.0030662975 -1.393099e-03
loanyes             -0.0021966157 -0.0014101499  0.0021926820 -5.519819e-06
contacttelephone    -0.0050464270 -0.0007103864 -0.0064550972  3.597432e-03
contactunknown       0.0034727133  0.0002232560  0.0177325170  3.533655e-04
monthaug            -0.0073109444 -0.0030627116  0.0085567800  1.217887e-03
monthdec            -0.0033276682 -0.0262118146 -0.0061759027 -1.138285e-02
monthfeb            -0.0092789195 -0.0126101143  0.0117902781  2.232499e-03
monthjan            -0.0125240218 -0.0086497838  0.0060589252  1.653066e-03
monthjul            -0.0048328384 -0.0032547197  0.0136591457 -2.121785e-03
monthjun            -0.0061337357 -0.0029097969 -0.0006252471  2.616495e-03
monthmar            -0.0080011659 -0.0144726789  0.0073117877  8.781924e-03
monthmay            -0.0061088663 -0.0051294521  0.0015195525  1.699440e-03
monthnov            -0.0071165896 -0.0084666197  0.0099200879  2.958673e-03
monthoct            -0.0134513680 -0.0230306794  0.0036450385  1.364858e-03
monthsep            -0.0132884846 -0.0260119547  0.0120811943  9.323565e-03
poutcomeother        0.0551219315 -0.0175524376 -0.0227425187 -6.069425e-03
poutcomesuccess     -0.0214258398  0.0649912314 -0.0138266434 -1.628513e-02
poutcomeunknown      0.0104110351  0.0022306099 -0.0138921180 -1.957904e-03
yyes                 0.0003215176 -0.0042930676  0.0017695208  5.818915e-03
```

```
$xcenter
        age      balance          day     duration     campaign        pdays
 41.1700951 1422.6578191   15.9152842  263.9612917    2.7936297   39.7666445
   previous
  0.5425791

$ycenter
          jobadmin.     jobblue-collar    jobentrepreneur       jobhousemaid
         0.10572882         0.20924574         0.03715992         0.02477328
      jobmanagement         jobretired   jobself-employed        jobservices
         0.21433311         0.05087370         0.04047777         0.09223623
         jobstudent      jobtechnician      jobunemployed         jobunknown
         0.01857996         0.16987392         0.02831232         0.00840522
     maritalmarried      maritalsingle educationsecondary  educationtertiary
         0.61866844         0.26454324         0.51006415         0.29860650
   educationunknown         defaultyes         housingyes            loanyes
         0.04136253         0.01681044         0.56602522         0.15284229
   contacttelephone     contactunknown           monthaug           monthdec
         0.06657819         0.29285556         0.14001327         0.00442380
           monthfeb           monthjan           monthjul           monthjun
         0.04910418         0.03273612         0.15616014         0.11745189
           monthmar           monthmay           monthnov           monthoct
         0.01083831         0.30922362         0.08604291         0.01769520
           monthsep      poutcomeother    poutcomesuccess    poutcomeunknown
         0.01150188         0.04357443         0.02853351         0.81950896
               yyes
         0.11523999
```

Ejercicio 8

Se tienen tres medidas fisiológicas y tres variables de ejercicios medidas en 20 hombres de 30-40 años en un gimnasio. Los datos están en el archivo `FitnessClubdata.dat`.

Objetivo: determinar si las variables fisiológicas se relacionan de alguna forma con las variables de ejercicio.

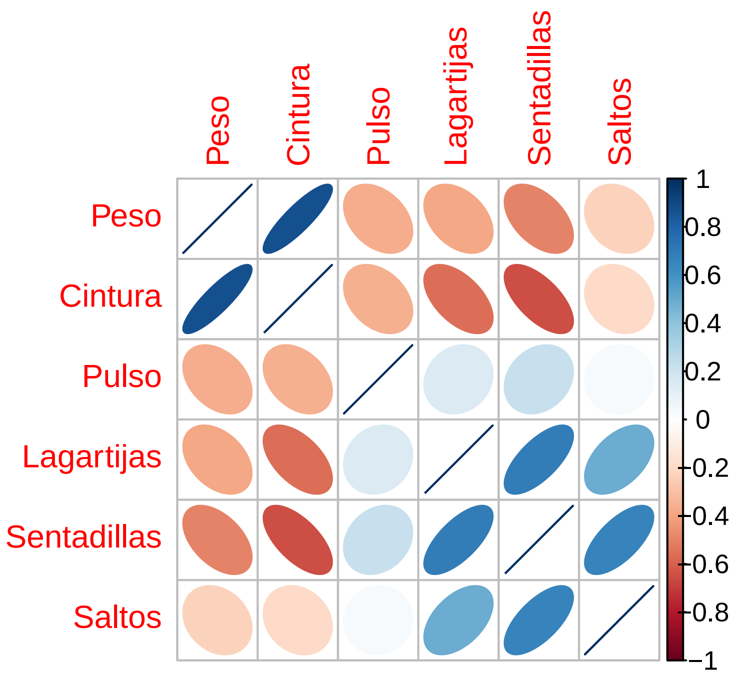
a. Analizar la matriz de correlaciones relevantes entre las variables de los dos grupos (dentro y entre grupos de variables).

```r
datos <- read.table("FitnessClubData.dat", header = TRUE)
X <- datos[,1:3]
Y <- datos[,4:6]
```

```
(R <- round(cor(datos),3))
```

```
             Peso Cintura  Pulso Lagartijas Sentadillas Saltos
Peso        1.000   0.870 -0.366     -0.390      -0.493 -0.226
Cintura     0.870   1.000 -0.353     -0.552      -0.646 -0.191
Pulso      -0.366  -0.353  1.000      0.151       0.225  0.035
Lagartijas -0.390  -0.552  0.151      1.000       0.696  0.496
Sentadillas -0.493 -0.646  0.225      0.696       1.000  0.669
Saltos     -0.226  -0.191  0.035      0.496       0.669  1.000
```

```
corrplot(R, method = "ellipse")
```



Las variables que parecen tener más correlación son la cintura con las lagartijas y las sentadillas.

b. Probar la hipótesis $H_0 : \Sigma_{xy} = 0$.

```
cca_xy <- cc(X,Y)
corr_1 <- cca_xy$cor[1]
corr_2 <- cca_xy$cor[2]
```

```r
n <- nrow(datos)
p <- 3; q <- 3

lambda <- -n*log((1-corr_1)*(1-corr_2))
1 - pchisq(lambda, df = p*q)
```

```
[1] 3.607206e-05
```

```r
# Con la corrección de Bartlett
m <- n-0.5*(p+q+3)
lambdaB <- -m*log((1-corr_1)*(1-corr_2))
1 - pchisq(lambdaB, df = p*q)
```

```
[1] 0.0009252632
```

Se rechaza H_0

Ejercicio 9

Una muestra aleatoria de $n = 70$ familias será encuestada para determinar la asociación entre ciertas variables 'demográficas' y ciertas variables de 'consumo'. Sea:

$$\text{Conjunto Criterio} \quad \begin{cases} X_1^{(1)} = \text{frecuencia anual de cena en restaurante} \\ X_2^{(1)} = \text{frecuencia anual ida al cine} \end{cases}$$

$$\text{Conjunto Predictor} \quad \begin{cases} X_1^{(2)} = \text{edad del jefe de familia} \\ X_2^{(2)} = \text{ingreso anual familiar} \\ X_3^{(2)} = \text{nivel de educación del jefe de familia} \end{cases}$$

Supongan que 70 observaciones de las variables precedentes dan una matriz de correlación muestral dada por:

$$R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} = \begin{bmatrix} 1 & & & & \\ 0.8 & 1 & & & \\ 0.26 & 0.33 & 1 & & \\ 0.67 & 0.59 & 0.37 & 1 & \\ 0.34 & 0.34 & 0.21 & 0.35 & 1 \end{bmatrix}$$

a. Determinar las correlaciones canónicas muestrales y probar la hipótesis $H_0 : \Sigma_{12} = 0$ (o equivalente $\rho_{12} = 0$ al nivel de 5%). Si se rechaza H_0, probar la significancia de la primera correlación canónica.

```
R <- matrix(c(1, 0.8, 0.26, 0.67, 0.34,
              0.8, 1, 0.33, 0.59, 0.34,
              0.26, 0.33, 1, 0.37, 0.21,
              0.67, 0.59, 0.37, 1, 0.35,
              0.34, 0.34, 0.21, 0.35, 1), nrow=5, byrow=TRUE)

R12 <- R[1:2, 3:5]
R21 <- R[3:5, 1:2]  # Transpuesta de R12

# Realizar correlación canónica
cca_result <- cancor(R12, t(R21))  # Usamos la transpuesta de R21 para ajustar las dimensi

# Mostrar las correlaciones canónicas
print(cca_result$cor)
```

```
[1] 1
```

```
# Calcular los estadísticos para probar la hipótesis de correlaciones nulas
n <- 70  # tamaño de la muestra
m <- min(nrow(R12), ncol(R12))
k <- 1  # para probar la primera correlación canónica
# Asegurarse de que la correlación canónica no sea exactamente 1
cor_squared <- cca_result$cor[1:k]^2
cor_squared[cor_squared >= 1] <- 1 - .Machine$double.eps  # Restar un pequeño valor para e

chi_sq_value <- -(n - 1 - (1/2) * (nrow(R12) + ncol(R12) + 1)) * sum(log(1 - cor_squared))
p_value <- pchisq(chi_sq_value, df = m * nrow(R12) * ncol(R12), lower.tail = FALSE)

# Imprimir resultados del test
cat("Chi-squared Value:", chi_sq_value, "\n")
```

```
Chi-squared Value: 2378.881
```

```
cat("p-Value:", p_value, "\n")
```

```
p-Value: 0
```

b. Usando las variables estandarizadas, construir las variables canónicas correspondientes a las correlaciones canónicas significativas.

c. Usando los resultados de las partes (a) y (b), preparar una tabla mostrando los coeficientes de las variables canónicas y las correlaciones muestrales de las variables canónicas con sus variables componentes.

d. Dada la información en (c), interpretar las variables canónicas.

e. ¿Tienen las variables demográficas algo que ver con las variables de consumo? ¿Las variables de consumo proveen mucha información sobre las variables demográficas?

Ejercicio 10

(Correlaciones para medidas angulares) Algunas observaciones tales como la dirección del viento, son en forma de ángulos. Un ángulo θ_2 puede ser representado como el par $x = (\cos(\theta_2), \sin(\theta_2))'$.

a. Mostrar que $x = \sqrt{b_1^2 + b_2^2}(\cos(\theta_2 - \beta)$ donde $b_1/\sqrt{b_1^2+b_2^2} = \cos(\beta)$ y $b_2/\sqrt{b_1^2+b_2^2} = \sin(\beta)$.

(Hint: $\cos(\theta_2 - \beta) = \cos(\theta_2)\cos(\beta) + \sin(\theta_2)\sin(\beta)$).

expresamos $x = (\cos(\theta_2), \sin(\theta_2))'$ y queremos mostrar que:

$$x = \sqrt{b_1^2 + b_2^2}(\cos(\theta_2 - \beta)),$$

donde β se define tal que:

$$\cos(\beta) = \frac{b_1}{\sqrt{b_1^2+b_2^2}} \quad \text{y} \quad \sin(\beta) = \frac{b_2}{\sqrt{b_1^2+b_2^2}}.$$

Esto implica que:

$$\cos(\theta_2 - \beta) = \cos(\theta_2)\cos(\beta) + \sin(\theta_2)\sin(\beta),$$

lo cual se puede sustituir en las definiciones de $\cos(\beta)$ y $\sin(\beta)$ para obtener:

$$\cos(\theta_2 - \beta) = \cos(\theta_2)\frac{b_1}{\sqrt{b_1^2+b_2^2}} + \sin(\theta_2)\frac{b_2}{\sqrt{b_1^2+b_2^2}}.$$

Multiplicando ambos lados por $\sqrt{b_1^2+b_2^2}$:

$$\sqrt{b_1^2+b_2^2}\cos(\theta_2-\beta)=b_1\cos(\theta_2)+b_2\sin(\theta_2).$$

Esto muestra que $x=\sqrt{b_1^2+b_2^2}(\cos(\theta_2-\beta))$ como se deseaba demostrar.

b. Sea $X^{(1)}$ con una única componente $X_1^{(1)}$. Mostrar que la correlación canónica simple es

$$\rho_1'=\max_{\beta}\operatorname{Corr}(X_1^{(1)},\cos(\theta_2-\beta))$$

Selecciona la variable canónica V_1 tomando en cuenta seleccionar un nuevo origen β para el ángulo θ_2.

La correlación entre $X_1^{(1)}$ y $\cos(\theta_2-\beta)$ se puede expresar como sigue:

$$\operatorname{Corr}(X_1^{(1)},\cos(\theta_2-\beta))=\frac{\operatorname{Cov}(X_1^{(1)},\cos(\theta_2-\beta))}{\sigma_{X_1^{(1)}}\sigma_{\cos(\theta_2-\beta)}}$$

La función $\cos(\theta_2-\beta)$ se puede reescribir utilizando la identidad trigonométrica para ángulos sumados, que es:

$$\cos(\theta_2-\beta)=\cos(\theta_2)\cos(\beta)+\sin(\theta_2)\sin(\beta)$$

Al cambiar el valor de β, modificamos la relación entre θ_2 y $X_1^{(1)}$, optimizando la correlación entre estas dos variables. El valor óptimo de β que maximiza esta correlación puede ser encontrado mediante:

$$\rho_1'=\max_{\beta}\operatorname{Corr}(X_1^{(1)},\cos(\theta_2-\beta))$$

Este valor máximo, ρ_1', representa la correlación canónica simple entre las dos variables y se obtiene ajustando β para alinear lo más posible las variaciones de θ_2 expresadas a través de $\cos(\theta_2-\beta)$ con las variaciones en $X_1^{(1)}$.

c. Sea $X_1^{(1)}$ el ozono (en partes por millón) y θ_2 = dirección de viento medida desde el norte. Se tomaron 19 observaciones en el centro de Milwaukee, Wisconsin, dando la matriz de correlaciones:

$$R = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0.166 & 0.694 \\ 0.166 & 1 & -0.051 \\ 0.694 & -0.051 & 1 \end{pmatrix}$$

Encontrar la correlación canónica muestral r_1' y la variable canónica $\hat{V}_1$, representando el nuevo origen β.