Theoretical task 1

1. What is asymptotic complexity of training and application of LDA and QDA classifiers in terms of population size N and dimensionality size D?

2. Bayes minimum risk decision rule assigns object x to class $\widehat{c}$ according to the following rule:

$$\widehat{c} = \arg \max_{j \in 1,2,...C} \sum_{c \in C} \lambda_{cj} p(\omega_c | x)$$

   How the decision rule of Bayes minimum risk classifier will change if we let it reject objects with rejection cost λ_{c0}? How this classifier will behave if $\lambda_{ij} \geq 0 \, \forall i, j = 1, 2, ...C$ and $\lambda_{c0} = 0$?

3. Suppose individual features $x_1, x_2, ...x_D$ are independent within each class and belong to Poisson distribution:

$$p(x_i|\omega_1) \sim Poisson(a_i); \quad p(x_i|\omega_2) \sim Poisson(b_i), \quad i = 1, 2, ...D.$$

   Prove that discriminant function will be linear:

$$\widehat{c} = \begin{cases} 1, & \beta_0 + \beta_1 x_2 + ... + \beta_D x_D \geq 0 \\ 2, & \beta_0 + \beta_1 x_2 + ... + \beta_D x_D < 0 \end{cases}$$

   What are exact values of $\beta_0, \beta_1, ...\beta_D$?

4. In nearest mean classifier each class ω_c is associated a mean vector $\mu_c \in \mathbb{R}^D$ and x is assigned a class for which the distance to its mean is minimal. Prove that LDA reduces to nearest mean classifier when $\Sigma_1 = \Sigma_2 = ... = \Sigma_C = I$ (identity matrix) and prior class probabilities are equal: $p(\omega_1) = p(\omega_2) = ... = p(\omega_C)$.