```
>> demo_Polynomial_Dictionary_Learning
Starting to train the dictionary
solving the quadratic problem with YALMIP...

 num. of constraints = 65
 dim. of socp   var  = 66,   num. of socp blk  =  1
 dim. of linear var  = 800
*******************************************************************
   SDPT3: Infeasible path-following algorithms
*******************************************************************
 version  predcorr  gam  expon  scale_data
   HKM      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
 0|0.000|0.000|1.1e+00|3.5e+01|4.3e+06| 2.221660e+04  0.000000e+00| 0:0:00| chol  1↙
1
 1|1.000|0.979|6.1e-06|8.5e-01|1.2e+05| 2.217331e+04 -1.135326e+02| 0:0:00| chol  1↙
1
 2|0.706|0.836|4.8e-06|1.7e-01|4.6e+04| 2.404502e+04 -1.606244e+02| 0:0:00| chol  1↙
1
 3|0.845|1.000|4.8e-07|1.0e-02|1.8e+04| 1.706815e+04 -1.985758e+02| 0:0:00| chol  1↙
1
 4|0.971|0.978|7.6e-07|3.1e-03|5.4e+02| 3.426454e+02 -1.842973e+02| 0:0:00| chol  1↙
1
 5|0.266|0.421|9.3e-07|1.9e-03|5.0e+02| 3.548399e+02 -1.381015e+02| 0:0:00| chol  1↙
1
 6|0.376|1.000|6.0e-07|3.0e-05|4.2e+02| 3.183349e+02 -1.003525e+02| 0:0:00| chol  1↙
1
 7|1.000|1.000|1.5e-09|3.1e-06|2.7e+02| 1.826441e+02 -8.408682e+01| 0:0:00| chol  1↙
1
 8|1.000|1.000|1.3e-09|3.0e-07|1.5e+02| 7.447724e+01 -7.071554e+01| 0:0:00| chol  1↙
1
 9|0.980|1.000|3.4e-10|3.0e-08|6.0e+01|-5.085025e+00 -6.518300e+01| 0:0:00| chol  1↙
1
10|1.000|1.000|1.7e-13|3.1e-09|2.8e+01|-2.881547e+01 -5.640647e+01| 0:0:00| chol  1↙
1
11|1.000|1.000|2.0e-13|3.0e-10|1.1e+01|-4.318343e+01 -5.411663e+01| 0:0:00| chol  1↙
1
12|1.000|1.000|1.9e-14|3.1e-11|4.0e+00|-4.840806e+01 -5.239875e+01| 0:0:00| chol  1↙
1
13|1.000|1.000|2.5e-14|4.0e-12|1.4e+00|-5.048710e+01 -5.193013e+01| 0:0:00| chol  1↙
1
14|1.000|1.000|2.5e-14|1.3e-12|5.4e-01|-5.113374e+01 -5.167745e+01| 0:0:00| chol  1↙
1
15|1.000|1.000|4.1e-14|1.0e-12|1.7e-01|-5.143180e+01 -5.160514e+01| 0:0:00| chol  1↙
1
16|1.000|1.000|4.1e-14|1.0e-12|6.8e-02|-5.150606e+01 -5.157425e+01| 0:0:00| chol  1↙
1
17|1.000|1.000|3.1e-14|1.0e-12|1.9e-02|-5.154581e+01 -5.156465e+01| 0:0:00| chol  1↙
1
18|1.000|1.000|1.3e-12|1.0e-12|7.5e-03|-5.155401e+01 -5.156151e+01| 0:0:00| chol  1↙
1
19|0.976|0.924|4.1e-13|1.1e-12|1.6e-03|-5.155891e+01 -5.156048e+01| 0:0:01| chol  2↙
2
```

```
20|0.941|1.000|1.4e-12|1.0e-12|7.1e-04|-5.155961e+01 -5.156032e+01| 0:0:01| chol  2↙
2
21|1.000|0.954|8.4e-13|1.0e-12|1.6e-04|-5.156010e+01 -5.156026e+01| 0:0:01| chol  2↙
2
22|0.844|1.000|1.6e-12|1.0e-12|6.0e-05|-5.156018e+01 -5.156024e+01| 0:0:01| chol  2↙
2
23|0.848|0.756|6.4e-13|1.2e-12|1.7e-05|-5.156023e+01 -5.156024e+01| 0:0:01| chol  2↙
2
24|0.584|0.872|1.0e-12|1.2e-12|8.4e-06|-5.156023e+01 -5.156024e+01| 0:0:01|
  stop: max(relative gap, infeasibilities) < 1.00e-07
-------------------------------------------------------------------
 number of iterations   = 24
 primal objective value = -5.15602345e+01
 dual   objective value = -5.15602429e+01
 gap := trace(XZ)       = 8.35e-06
 relative gap           = 8.02e-08
 actual relative gap    = 8.02e-08
 rel. primal infeas     = 1.02e-12
 rel. dual   infeas     = 1.16e-12
 norm(X), norm(y), norm(Z) = 9.3e-01, 5.2e+01, 2.0e+01
 norm(A), norm(b), norm(C) = 6.9e+02, 1.4e+01, 7.7e+01
 Total CPU time (secs)  = 0.54
 CPU time per iteration = 0.02
 termination code       =  0
 DIMACS errors: 2.2e-12  0.0e+00  1.7e-12  0.0e+00  8.0e-08  8.0e-08
-------------------------------------------------------------------

ans =

   51.5602


 num. of constraints = 65
 dim. of socp   var  = 66,   num. of socp blk  =  1
 dim. of linear var  = 800
*******************************************************************
   SDPT3: Infeasible path-following algorithms
*******************************************************************
 version  predcorr  gam  expon  scale_data
   HKM      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
 0|0.000|0.000|1.0e+00|3.3e+04|1.3e+11| 6.760054e+08  0.000000e+00| 0:0:00| chol  2↙
2
 1|1.000|0.979|1.7e-07|6.9e+02|3.9e+09| 6.619566e+08 -5.877618e+05| 0:0:00| chol 22↙
16
 2|0.541|0.494|1.7e+00|3.5e+02|2.5e+09| 6.587451e+08 -1.274408e+06| 0:0:00| chol
  warning: symqmr failed: 0.3
  switch to LU factor. lu 30 ^20
 3|0.007|0.011|1.7e+00|3.5e+02|2.4e+09| 6.603596e+08 -2.742751e+06| 0:0:00| lu 23↙
30
 4|0.060|0.012|1.6e+00|3.4e+02|2.5e+09| 6.750523e+08 -1.650509e+06| 0:0:00| lu 17↙
3
 5|0.037|0.105|1.5e+00|3.1e+02|2.3e+09| 6.820281e+08 -4.399581e+06| 0:0:00| lu 30↙
```

```
5
 6|0.334|0.394|1.0e+00|1.9e+02|1.7e+09| 6.991993e+08 -2.072083e+06| 0:0:00| lu 17↙
3
 7|0.434|0.559|5.7e-01|8.2e+01|1.1e+09| 6.783419e+08 -3.157435e+06| 0:0:00| lu 18↙
4
 8|0.098|0.203|5.2e-01|6.6e+01|1.0e+09| 6.706612e+08 -3.541544e+06| 0:0:00| lu 30↙
4
 9|0.439|0.387|2.9e-01|4.0e+01|8.5e+08| 6.346809e+08 -5.663238e+06| 0:0:00| lu  8↙
2
10|0.120|0.663|2.6e-01|1.4e+01|6.9e+08| 6.123308e+08 -5.448146e+06| 0:0:00| lu 30↙
3
11|0.412|0.540|1.5e-01|6.2e+00|5.3e+08| 4.882831e+08 -7.275661e+06| 0:0:00| lu 30↙
3
12|0.281|0.257|1.1e-01|4.6e+00|4.8e+08| 4.482415e+08 -9.080632e+06| 0:0:00| lu 27↙
4
13|0.170|0.473|9.0e-02|2.4e+00|4.5e+08| 4.203751e+08 -8.992951e+06| 0:0:00| lu 28↙
6
14|0.183|0.287|7.3e-02|1.7e+00|4.1e+08| 3.895439e+08 -9.853246e+06| 0:0:00| lu 22↙
10
15|0.079|0.440|6.8e-02|9.8e-01|4.0e+08| 3.780815e+08 -1.056645e+07| 0:0:00| lu 19↙
4
16|0.151|0.210|5.7e-02|7.7e-01|3.9e+08| 3.608526e+08 -1.216636e+07| 0:0:00| lu 28↙
7
17|0.080|0.099|5.3e-02|6.9e-01|3.8e+08| 3.529262e+08 -9.011312e+06| 0:0:00| lu 21↙
4
18|0.029|0.121|5.1e-02|6.1e-01|3.9e+08| 3.578096e+08 -1.914576e+07| 0:0:00| lu 12↙
3
19|0.134|0.578|4.4e-02|2.6e-01|3.5e+08| 3.283242e+08 -1.171858e+07| 0:0:00| lu  8↙
2
20|0.321|0.474|3.0e-02|1.4e-01|3.0e+08| 2.735187e+08 -1.486433e+07| 0:0:00| lu 13↙
4
21|0.219|0.478|2.4e-02|7.1e-02|2.8e+08| 2.544167e+08 -1.660976e+07| 0:0:00| lu 30↙
5
22|0.293|0.269|1.7e-02|5.2e-02|2.5e+08| 2.230678e+08 -1.814397e+07| 0:0:00| lu 23↙
^13
23|0.265|0.447|1.2e-02|2.9e-02|2.4e+08| 2.093967e+08 -1.769108e+07| 0:0:00| lu 30↙
5
24|0.173|0.638|1.0e-02|1.0e-02|2.2e+08| 1.933299e+08 -1.782004e+07| 0:0:00| lu 30↙
4
25|0.574|0.805|4.3e-03|2.1e-03|1.8e+08| 1.619197e+08 -1.937200e+07| 0:0:00| lu 30 ^↙
4
26|1.000|1.000|1.1e-05|5.6e-04|1.4e+08| 1.206662e+08 -2.065620e+07| 0:0:00| lu 10↙
2
27|1.000|1.000|1.5e-06|2.1e-06|8.4e+07| 6.839149e+07 -1.536039e+07| 0:0:01| lu  5↙
2
28|1.000|1.000|3.7e-06|2.9e-07|3.0e+07| 2.417733e+07 -5.940438e+06| 0:0:01| lu  5↙
2
29|0.991|1.000|2.0e-06|4.4e-07|8.5e+06| 6.588798e+06 -1.884380e+06| 0:0:01| lu  5↙
2
30|1.000|1.000|2.7e-07|4.1e-07|4.4e+06| 3.501082e+06 -9.235481e+05| 0:0:01| lu  5↙
2
31|1.000|1.000|2.5e-08|5.4e-08|1.3e+06| 9.737676e+05 -3.280704e+05| 0:0:01| lu  4↙
2
32|1.000|1.000|4.2e-08|4.9e-09|5.6e+05| 4.336616e+05 -1.233840e+05| 0:0:01| lu  4↙
```

```
2
33|1.000|1.000|1.6e-08|7.4e-09|1.7e+05| 1.247408e+05 -4.576753e+04| 0:0:01| lu  4↙
2
34|1.000|1.000|5.5e-09|3.1e-09|7.0e+04| 5.432555e+04 -1.561049e+04| 0:0:01| lu  5↙
2
35|1.000|1.000|4.7e-09|1.1e-09|2.1e+04| 1.556171e+04 -5.934302e+03| 0:0:01| lu  6↙
2
36|1.000|1.000|3.8e-09|9.3e-10|8.8e+03| 6.858461e+03 -1.959056e+03| 0:0:01| lu  7↙
2
37|1.000|1.000|1.9e-09|7.6e-10|2.7e+03| 1.915254e+03 -7.641717e+02| 0:0:01| lu  5↙
1
38|1.000|1.000|1.2e-09|3.7e-10|1.1e+03| 8.435263e+02 -2.681359e+02| 0:0:01| lu  5↙
1
39|1.000|1.000|6.4e-10|2.4e-10|3.3e+02| 2.048397e+02 -1.220097e+02| 0:0:01| lu  6↙
1
40|1.000|1.000|1.3e-10|1.3e-10|1.4e+02| 7.361817e+01 -6.455270e+01| 0:0:01| lu  5↙
1
41|1.000|1.000|2.1e-10|2.7e-11|3.7e+01|-9.961826e+00 -4.706970e+01| 0:0:01| lu  5↙
1
42|1.000|1.000|1.7e-10|4.0e-11|1.7e+01|-2.484364e+01 -4.139329e+01| 0:0:01| lu  5↙
1
43|0.979|1.000|1.1e-10|3.3e-11|4.0e+00|-3.536892e+01 -3.939270e+01| 0:0:01| lu  7↙
1
44|1.000|1.000|5.4e-11|2.2e-11|1.8e+00|-3.711078e+01 -3.893061e+01| 0:0:01| lu  5↙
1
45|0.966|1.000|1.4e-11|1.1e-11|4.2e-01|-3.831617e+01 -3.873406e+01| 0:0:01| lu 26↙
1
46|1.000|1.000|6.4e-11|2.9e-12|1.9e-01|-3.850756e+01 -3.869604e+01| 0:0:01| lu 24↙
2
47|0.975|1.000|9.0e-11|4.3e-12|4.3e-02|-3.863358e+01 -3.867703e+01| 0:0:01| lu 30↙
^16
48|1.000|1.000|2.1e-10|6.5e-12|1.9e-02|-3.865419e+01 -3.867331e+01| 0:0:01| lu 30↙
30
49|0.982|1.000|2.6e-10|9.7e-12|4.3e-03|-3.866698e+01 -3.867127e+01| 0:0:01| lu 12↙
30
50|1.000|1.000|3.8e-10|1.5e-11|1.8e-03|-3.866912e+01 -3.867093e+01| 0:0:01|
  sqlp stop: maximum number of iterations reached
------------------------------------------------------------------
 number of iterations   = 50
 primal objective value = -3.86691192e+01
 dual   objective value = -3.86709291e+01
 gap := trace(XZ)       = 1.82e-03
 relative gap           = 2.32e-05
 actual relative gap    = 2.31e-05
 rel. primal infeas     = 3.77e-10
 rel. dual   infeas     = 1.45e-11
 norm(X), norm(y), norm(Z) = 5.1e+02, 6.4e+01, 2.8e+01
 norm(A), norm(b), norm(C) = 4.7e+06, 2.3e+06, 7.7e+01
 Total CPU time (secs)  = 0.74
 CPU time per iteration = 0.01
 termination code       = -6
 DIMACS errors: 9.7e-10  0.0e+00  2.1e-11  0.0e+00  2.3e-05  2.3e-05
------------------------------------------------------------------
```

```
ans =

   38.6709

Iteration   2   Total error is: 0.029093

 num. of constraints = 65
 dim. of socp   var  = 66,   num. of socp blk  =  1
 dim. of linear var  = 800
*******************************************************************
   SDPT3: Infeasible path-following algorithms
*******************************************************************
 version  predcorr  gam  expon  scale_data
   HKM      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
 0|0.000|0.000|1.0e+00|9.3e+04|3.3e+11| 1.731780e+09  0.000000e+00| 0:0:00| chol  2↙
2
 1|1.000|0.974|1.3e-07|2.4e+03|1.2e+10| 1.695396e+09 -1.633312e+06| 0:0:00| chol
  linsysolve: Schur complement matrix not positive definite
  switch to LU factor. lu 22 ^10
 2|0.838|0.719|3.2e-08|6.7e+02|5.5e+09| 1.816416e+09 -4.607623e+06| 0:0:00| lu 12↙
^18
 3|0.471|0.463|4.7e-08|3.6e+02|4.0e+09| 1.832423e+09 -8.404438e+06| 0:0:00| lu 30↙
^12
 4|0.019|0.036|1.2e-06|3.5e+02|4.0e+09| 1.830240e+09 -9.451966e+06| 0:0:00| lu 13↙
30
 5|0.027|0.055|1.2e-06|3.3e+02|3.9e+09| 1.833634e+09 -1.057925e+07| 0:0:00| lu 19↙
^16
 6|0.015|0.018|1.8e-06|3.2e+02|3.8e+09| 1.838933e+09 -1.224119e+07| 0:0:00| lu 30↙
30
 7|0.000|0.001|3.1e-06|3.2e+02|3.8e+09| 1.838169e+09 -1.305373e+07| 0:0:00| lu 30 ^↙
6
 8|0.010|0.017|2.5e-06|3.2e+02|3.8e+09| 1.839858e+09 -1.134956e+07| 0:0:00| lu 30↙
^24
 9|0.001|0.007|2.5e-06|3.1e+02|3.8e+09| 1.840784e+09 -1.309268e+07| 0:0:00| lu 30↙
^22
10|0.000|0.000|2.5e-06|3.1e+02|3.8e+09| 1.840595e+09 -1.223709e+07| 0:0:00| lu 12 ^↙
7
11|0.000|0.000|2.9e-06|3.1e+02|3.8e+09| 1.840302e+09 -1.485398e+07| 0:0:00| lu 17↙
^15
12|0.000|0.001|3.1e-06|3.1e+02|3.8e+09| 1.840280e+09 -1.452113e+07| 0:0:00| lu 30↙
^12
13|0.000|0.001|3.1e-06|3.1e+02|3.8e+09| 1.840628e+09 -1.322590e+07| 0:0:00| lu 30↙
^25
14|0.001|0.003|2.3e-06|3.1e+02|3.8e+09| 1.840344e+09 -1.486950e+07| 0:0:00|
 *** Too many tiny steps:  restarting with the following iterate.
 *** [X,y,Z] = infeaspt(blk,At,C,b,2,1e5); lu  2   1
15|0.981|0.981|1.9e-02|7.0e+02|1.5e+11| 4.013582e+07 -4.925948e+04| 0:0:00| lu  2↙
1
16|0.989|0.989|2.0e-04|7.7e+00|1.7e+09| 4.018032e+07 -5.014738e+04| 0:0:00| lu  4↙
1
17|1.000|0.995|4.1e-10|8.1e-02|5.3e+07| 3.616632e+07 -4.551267e+04| 0:0:00| lu  6↙
2
```

```
18|0.987|1.000|3.2e-09|2.3e-02|2.6e+06| 1.169096e+06 -4.974940e+03| 0:0:00| lu  4↙
2
19|1.000|1.000|6.7e-10|1.2e-02|6.7e+05| 3.261684e+05 -2.147660e+03| 0:0:00| lu  5↙
1
20|0.938|0.938|2.6e-10|1.8e-03|6.5e+04| 2.148236e+04 -8.958200e+02| 0:0:00| lu  9↙
2
21|1.000|0.999|3.5e-10|1.2e-04|2.2e+04| 1.727892e+04 -1.001158e+03| 0:0:00| lu  7↙
2
22|1.000|0.986|1.8e-09|1.3e-05|9.0e+03| 7.213592e+03 -1.006389e+03| 0:0:00| lu 22↙
3
23|1.000|1.000|1.4e-08|1.2e-06|5.1e+03| 3.822072e+03 -1.124813e+03| 0:0:00| lu 30↙
5
24|1.000|1.000|1.8e-08|1.2e-07|3.4e+03| 2.847272e+03 -5.505336e+02| 0:0:00| lu 12↙
2
25|0.954|1.000|2.8e-09|1.2e-08|9.9e+02| 6.389734e+02 -3.521156e+02| 0:0:00| lu 13↙
2
26|1.000|1.000|1.5e-09|1.5e-09|3.7e+02| 2.572891e+02 -1.156106e+02| 0:0:01| lu 30↙
2
27|0.963|1.000|1.7e-09|4.3e-10|1.1e+02| 3.534338e+01 -7.132930e+01| 0:0:01| lu 11↙
2
28|1.000|1.000|3.7e-10|3.6e-10|4.6e+01|-5.388182e+00 -5.179988e+01| 0:0:01| lu  9↙
2
29|0.994|1.000|6.1e-10|7.5e-11|1.2e+01|-3.335859e+01 -4.565548e+01| 0:0:01| lu 11↙
2
30|1.000|1.000|1.6e-10|1.1e-10|5.6e+00|-3.822120e+01 -4.377155e+01| 0:0:01| lu 26↙
2
31|0.988|1.000|1.7e-10|3.3e-11|1.4e+00|-4.167879e+01 -4.306735e+01| 0:0:01| lu 28↙
2
32|1.000|1.000|3.6e-10|3.3e-11|6.3e-01|-4.226095e+01 -4.288712e+01| 0:0:01| lu 30↙
2
33|0.975|1.000|2.7e-10|5.0e-11|1.5e-01|-4.266201e+01 -4.281215e+01| 0:0:01| lu 30 ^↙
6
34|1.000|1.000|8.8e-10|5.4e-11|6.3e-02|-4.273048e+01 -4.279380e+01| 0:0:01| lu 30↙
30
35|0.836|0.879|2.1e-09|8.7e-11|2.2e-02|-4.276711e+01 -4.278941e+01| 0:0:01| lu 30↙
^17
36|0.836|0.659|8.1e-09|1.5e-10|1.1e-02|-4.277640e+01 -4.278780e+01| 0:0:01| lu 24↙
^12
37|0.500|0.367|5.2e-09|2.8e-10|8.0e-03|-4.277890e+01 -4.278745e+01| 0:0:01| lu 30↙
^23
38|0.570|1.000|8.7e-08|2.7e-10|6.8e-03|-4.278011e+01 -4.278726e+01| 0:0:01| lu 30↙
30
39|1.000|0.813|1.3e-07|4.6e-10|4.9e-03|-4.278157e+01 -4.278719e+01| 0:0:01| lu 30↙
30
40|0.425|1.000|2.3e-08|6.1e-10|4.2e-03|-4.278248e+01 -4.278700e+01| 0:0:01| lu 30↙
30
41|1.000|1.000|1.6e-08|9.2e-10|2.4e-03|-4.278389e+01 -4.278688e+01| 0:0:01| lu 27 ^↙
9
42|0.879|0.796|1.0e-08|1.2e-09|9.6e-04|-4.278483e+01 -4.278674e+01| 0:0:01| lu 30↙
^13
43|0.929|0.967|9.4e-08|7.1e-10|6.1e-04|-4.278556e+01 -4.278673e+01| 0:0:01| lu 30↙
^20
44|0.762|0.879|2.3e-08|6.2e-10|4.9e-04|-4.278444e+01 -4.278672e+01| 0:0:01| lu 30↙
30
```

```
45|0.319|0.205|6.0e-08|1.0e-09|4.9e-04|-4.278489e+01 -4.278671e+01| 0:0:01| lu 17↙
30
46|0.268|0.286|5.7e-08|1.2e-09|4.5e-04|-4.278508e+01 -4.278671e+01| 0:0:01|
  stop: progress is too slow
  stop: progress is bad
  stop: progress is bad*
------------------------------------------------------------------
 number of iterations   = 46
 primal objective value = -4.27844350e+01
 dual   objective value = -4.27867184e+01
 gap := trace(XZ)       = 4.86e-04
 relative gap           = 5.62e-06
 actual relative gap    = 2.64e-05
 rel. primal infeas     = 2.34e-08
 rel. dual   infeas     = 6.21e-10
 norm(X), norm(y), norm(Z) = 1.0e+04, 6.0e+01, 2.4e+01
 norm(A), norm(b), norm(C) = 1.7e+07, 5.0e+06, 7.7e+01
 Total CPU time (secs)  = 0.93
 CPU time per iteration = 0.02
 termination code       = -5
 DIMACS errors: 5.0e-08  0.0e+00  8.9e-10  0.0e+00  2.6e-05  5.6e-06
------------------------------------------------------------------

ans =

   42.7867

Iteration   3   Total error is: 0.0291

ans =

   NaN

Iteration   4   Total error is: NaN
Error using svd
Input to SVD must not contain NaN or Inf.

Error in pinv (line 18)
[U,S,V] = svd(A,'econ');

Error in OMP_non_normalized_atoms (line 53)
        a = pinv(normalized_D(:,indx(1:j))) * x;

Error in Polynomial_Dictionary_Learning (line 170)
        CoefMatrix = OMP_non_normalized_atoms(Dictionary,Y, param.T0);

Error in demo_Polynomial_Dictionary_Learning (line 83)
[Dictionary_Pol, output_Pol, err]  = Polynomial_Dictionary_Learning(TrainSignal,↙
param, initial_sparsity_mx);

>>
```