UNIVERSITY OF CAMBRIDGE

ECONOMICS TRIPOS PART I

Monday 10 June 2013 9:00-12:00

Paper 1

MICROECONOMICS

Answer **ALL SIX** questions from Section A and **TWO** questions from Section B.

Section A and B will each carry 50% of the total marks for this paper.

Each question within each section will carry equal weight.

Write your **candidate number** not your name on the cover of each booklet.

Write legibly.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
20 Page booklet x 1	Approved calculators allowed
Rough work pads	
Tags	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 Suppose that a utility maximising individual has a utility function given by:

$$u(x, y) = Ax^{2/3}y^{2/3},$$

where x denotes the quantity of good X she consumes, y denotes the quantity of good Y she consumes, $p_X = 2$ is the price of one unit of good X, and $p_Y = 3$ is the price of one unit of good Y. Assume that her budget is £120. Solve for the optimal values of x and y using Lagrange multipliers.

2 Good A has a price elasticity of demand of $|\varepsilon| = 1$ when its price is $p_A = 10$, while good B has a price elasticity of demand of $|\varepsilon| = 2$ when its price is $p_B = 10$. Suppose that both prices go up so that $p_A = p_B = 11$. (i) Do you expect the demand for good B to go down by more units than the demand for good A? Briefly justify your answer. (ii) Would your answer change if you were told that at a price of $p_A = p_B = 10$ both sell 20 units? Briefly justify your answer.

3 Explain why a firm's long-run average cost curve is the envelope of its short-run average cost curves.

4 Find all the Nash equilibria of the following game.

	X_2	Y_2	Z_2
X_1	3,3	1,6	2,2
Y_1	2,1	0,0	1,2
Z_1	-1,-3	3,-2	-2,3

5 Person A values an indivisible object at v_A and person B values it at v_B. They each bid for it according to the following rules. First, A can bid either 0 or 1. Then B, after seeing A's bid, can bid either 0 or 1. The winner of the object is the higher bidder or, in the event of a tie, the winner is chosen by tossing a fair coin. Regardless of the bids, both the winner and the loser have to pay their bid. Both people want to maximize the expected value of their payoff - e.g., if both bid the same amount b, their utilities are $0.5v_A - b$ and $0.5v_B - b$.

For the case $v_A = 2, v_B = 3$, (i) represent this auction as a game in extensive form; (ii) describe it in strategic form; (iii) find all the pure strategy Nash equilibria; (iv) find all the pure strategy subgame-perfect equilibria.

6 Each one of a large number of people has to decide whether to dispose of rubbish legally or dump it illegally. For each person, the utility from legal disposal is 1 and the utility from dumping is $x^2 - x + 1.16$, where x is the proportion of people who choose to dump (you should treat x as a continuous variable ranging from 0 to 1).

(a) Sketch the payoff functions of a typical individual. What might account for the form of the functions?

(b) Determine which values of x are consistent with Nash equilibrium. Which equilibria are stable?

(TURN OVER)

SECTION B

7 Two firms (1 and 2) compete by setting quantities. They face the market inverse demand curve $p(Q) = a - Q$, where $a > 0$ is a constant and Q is total quantity. Firm i ($i = 1, 2$) has cost function $0.5cq_i^2$, where $c > 0$ is a constant and q_i is i's produced quantity.

(a) Suppose that the firms' interaction is one-shot. Find the Cournot quantities and price.

(b) Now suppose that the game is played infinitely often, with the firms observing all previous quantities before choosing quantity at any given stage. Both firms aim to maximize their discounted sum of profits, using a common discount factor δ.

(i) Explain what is meant by a subgame in this game. Explain what is meant by subgame-perfect equilibrium.

(ii) Show that, if δ is above a critical value, there is a subgame-perfect equilibrium in which each firm's payoff is half the monopoly profit. You should find an expression for the critical value in terms of the monopoly profit Π^m, the Cournot profit Π^c and the profit Π^* obtained by firm 1 from the pair of quantities $(b_1(0.5q^m), 0.5q^m)$, where $b_1(.)$ is firm 1's best-response function and q^m is the monopoly quantity. Explain carefully why this is a subgame-perfect equilibrium.

(c) In the infinitely-repeated game of part (b), is there a subgame-perfect equilibrium in which the payoffs are the same as in the one-shot Cournot game? If so, for what values of δ is it a subgame-perfect equilibrium?

8 Suppose that a monopolist produces and sells a good Z. There are two periods, and demand for the good in period 1 is given by z_1, while the demand for the good in period 2 is given by z_2. Furthermore, assume that:

$$\begin{aligned} z_1 &= 100 - p_1 \\ z_2 &= 100 - p_2 - z_1, \end{aligned}$$

where p_1 denotes the price in period 1 and p_2 denotes the price in period 2. The units sold in a period must be produced in that period, and the production function in period 1 is given by $f(K_1, L_1) = \sqrt{K_1 L_1}$, where K_1 denotes the amount of capital and L_1 denotes the amount of labour used in period 1. The production function in period 2 is given by $f(K_2, L_2) = \sqrt{K_2 L_2}$, where K_2 denotes the amount of capital and L_2 denotes the amount of labour used in period 2. Suppose that one unit of capital costs $r = 3$, while one unit of labour costs $w = 2$. The factor prices are the same in both periods, and both factors of production are flexible in both periods. The monopolist is assumed to maximise total profit, defined as $\pi = \pi_1 + \pi_2$, where π_1 is profit in period 1 and π_2 is profit in period 2.

(a) Explain why the demand functions might take the functional forms shown above.

(b) Find the cost functions for periods 1 and 2.

(c) Find the optimal values of z_1, z_2, p_1 and p_2. Briefly discuss your results.

9 Suppose that an individual lives for two periods and has the following utility function:

$$u(c_1, c_2) = \sqrt{c_1} + \beta\sqrt{c_2},$$

where c_1 denotes consumption in period 1, c_2 denotes consumption in period 2, $\beta = \frac{1}{1+r}$ and $r = 0.25$. The individual does not work, but the government provides her with an income of £10 in period 1 and £100 in period 2.

(a) Solve for the optimal values of c_1 and c_2. Please show your workings.

(b) Suppose that the individual can save, but she cannot borrow. Solve for the new optimal values of c_1 and c_2. Briefly discuss your result.

(c) Assume that although she cannot borrow, the individual can work to supplement her income (and keeps the money from the government). She has a time endowment of 10 units in period 1, but no time endowment in period 2 (e.g. she is too old to work or enjoy leisure). Her wage is $w = 1$ per unit of time worked. Her utility is now given by

$$u(c_1, c_2, l_1) = \sqrt{c_1} + \beta\sqrt{c_2} + (0.1)\sqrt{l_1}$$

where l_1 denotes the units of leisure she enjoys in period 1. (i) Write the individual's new optimisation problem and explain what each constraint means. (ii) How many units of time will she work in period 1, and what are the new optimal values of c_1 and c_2?

10 'If externalities exist in a market then the outcome will necessarily be inefficient if the government does not directly regulate quantities.' Discuss.

11 In a simple economy there are two goods, money and beer. There are two people, A and B. Person i $(i = A, B)$ has utility function $u_i(x_i, y_i) = y_i + \sqrt{x_i}$, where x_i is her consumption of beer and y_i is her consumption of money. A has an endowment of 1 unit of beer and 4 units of money, while B has 3 units of beer and 2 of money.

(a) Find the marginal rates of substitution and sketch the indifference curves. Explain how a person's marginal willingness to pay for beer varies with her wealth.

(b) Sketch the Edgeworth Box and find the equation of the contract curve.

(c) If A and B bargain over a trade what final allocation will they negotiate, assuming that A has all the bargaining power?

(d) Find the competitive equilibrium price of beer in terms of money and find the competitive equilibrium allocation.

(e) Sketch the aggregate excess demand function for beer, with the price of money fixed at 1. Is the equilibrium stable or unstable? Explain.

(f) If, instead of two people, there were 10,000 people with A's endowment and preferences, and 10,000 with B's endowment and preferences, explain why the prices you have found would still be competitive equilibrium prices.

12 Suppose that a firm produces and sells z units of a good Z using the production function $f(K, L) = 2\sqrt{KL}$, where K denotes the amount of capital and L denotes the amount of labour it uses. The cost of one unit of capital is $r = 1$, while the cost of one unit of labour is $w = 2$. The demand for the good is given by $z = p^{-2}$, where p is the price of one unit of Z.

(a) Suppose that capital is fixed. What is the output level that maximises the firm's profit?

(b) Suppose that both factors are flexible. What is the level of output that maximises the firm's profit?

(c) If the capital is fixed at $\overline{K} = 2$ will the firm stay in business? Would it stay in business if capital were flexible?

(d) Assume again that capital is fixed at $\overline{K} = 2$, but now the firm can pay £1.50 to make it flexible (e.g. it can pay a penalty that allows it to break a lease). Will it do so? What is the maximum amount it will be willing to pay to make capital flexible? Briefly discuss your results.

END OF PAPER