FACULTY OF ECONOMICS STUDY AIDS 2017

ECT1 Paper 1 Microeconomics

The Faculty Board has agreed to release outline solutions to the 2017 examinations as a study aid for exam revision. They are abridged solutions, and not 'definitive', and should therefore not be considered as an exemplar for 'complete' answers.

Also note that the Faculty will not respond to any queries regarding these solutions.

Part 1 Paper 1 suggested exam questions 2016-17

A4. An exchange economy has two agents, A and B, and two goods, x and y.

A's endowment is 1 unit of good x and 1 unit of y. B has 2 units of x and no y. A has utility function $U^A(x_A, y_A) = min\{x_A, y_A\}$; B has utility function $u^B(x_B, y_B) = x_B + 2y_B$. (i) Draw the Edgeworth Box; (ii) find the equation of the contract curve; (iii) find a pair (p_x, p_y) of competitive equilibrium prices. Is the equilibrium price ratio necessarily unique?

Ans: (i) see figure 1. (ii) Contract curve is $y_A = x_A$ for $x_A \leq 1$. (iii) For any pair of positive prices, A's optimal bundle is her endowment $(1, 1)$ (from the diagram). Hence, in equilibrium, B must want to consume the bundle $(2, 0)$. If $p_x = 0.5p_y$, $(2, 0)$ is an optimum bundle for A since her budget line coincides with an indifference curve. In equilibrium there is no trade - they both consume their endowment. There are other equilibrium prices: normalizing p_y to be 1, any $p_x \leq 0.5$ is an equilibrium price, since $(2, 0)$ will still be optimal for B at these prices.

A5. For an economy with one public good and one private good and two agents, suppose that units of the private good can be turned into units of the public good at the rate of one-to-one. Suppose also that the amount of the public good and the distribution of the private good between the two individuals is such that one person has a marginal rate of substitution (MRS) between private and public good of 0.4 while the other has a MRS of 0.7. Is this allocation efficient? If not, explain how a Pareto-improvement can be brought about. State a necessary condition for an allocation to be Pareto-efficient.

Ans. No, not Pareto-efficient. E.g., person A could give up 0.4 (infinitesimally small) units and person B could give up 0.7 units of the private good, creating 1.1 units of the public good. Each is strictly better off (since, e.g., A is indifferent to giving up 0.4 private for 1 public). Necessary condition for efficiency is the Samuelson Rule: Marginal rate of transformation between public good and private good equals sum over agents of marginal rate of substitution between that pair of goods. It's the sum (unlike in the case of two private goods) because they both benefit from the extra public good.

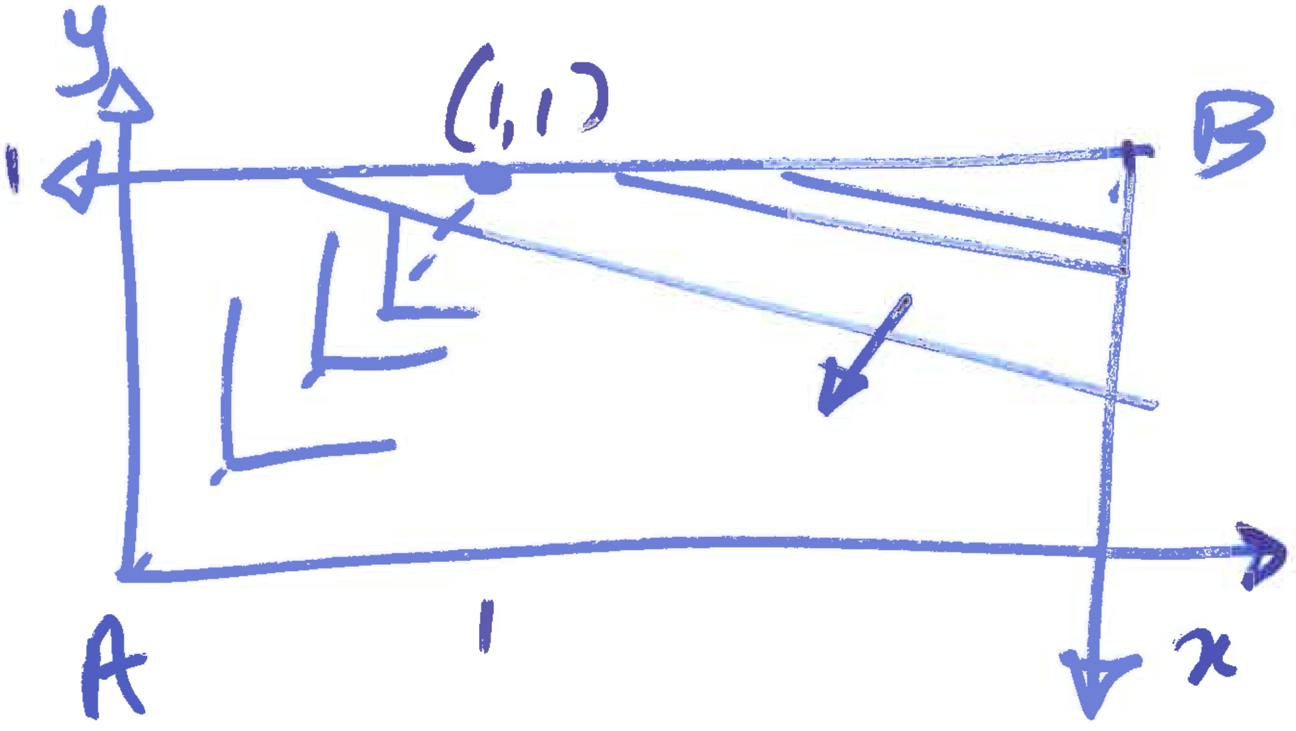

FIG. 1

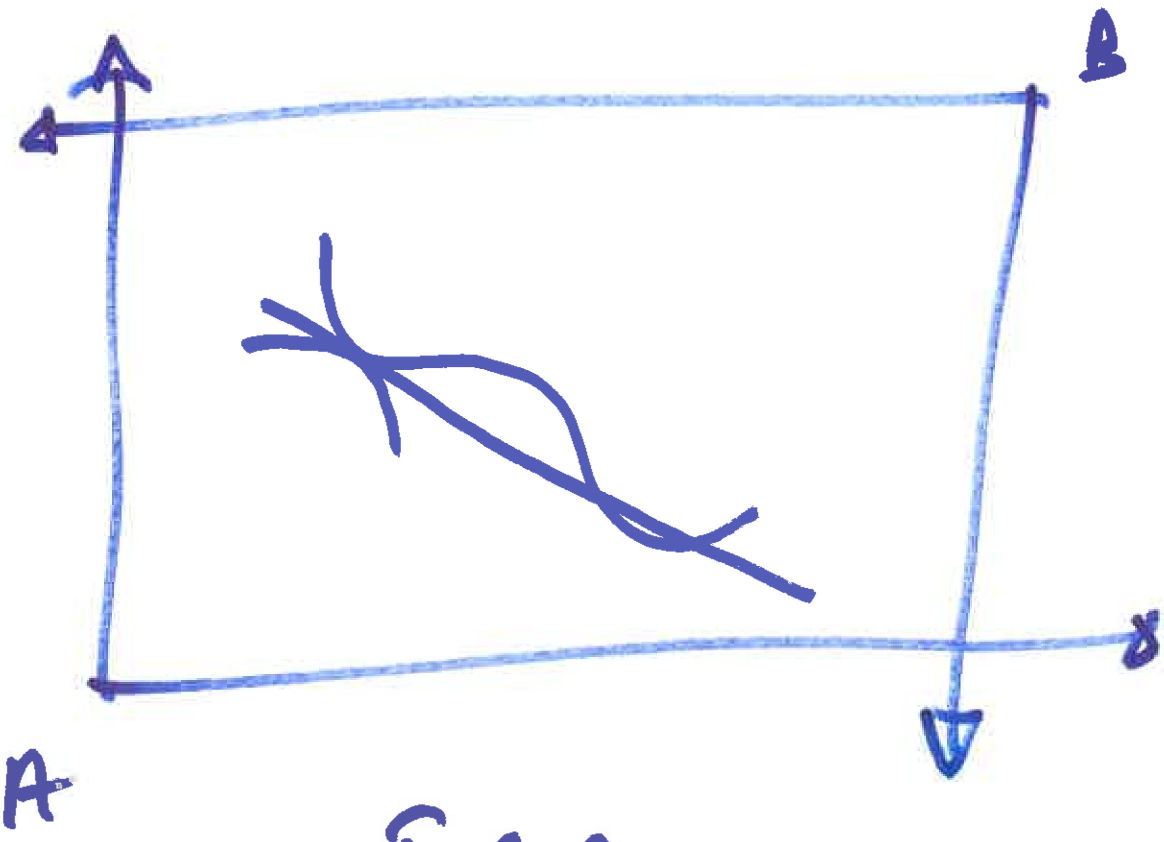

FIG. 2

Not satisfied here because $1 \neq 0.4 + 0.7$.

A6. 'As long as preferences satisfy the non-satiation property, any desirable allocation of goods can be achieved in competitive equilibrium by a suitable redistribution via income taxes'. Comment.

Ans. Should explain the logic of the second welfare theorem, e.g. by means of an Edgeworth Box. It requires convex preferences, not just non-satiation: should show a counter-example with non-satiated but non-convex preferences (see Fig.2). The tax should be lump-sum, not an income tax. An income tax is distortionary, hence inefficient (it implies buyers and sellers of labour have different marginal rates of substitution since they face different relative prices of labour). Could also discuss whether 'desirable' implies Pareto-efficient, as it would need to.

B10. 'An externality problem is really a problem of a missing market. Invari-ably the solution is to create the market'. Discuss.

Ans. Should explain that if all goods are priced and agents act as price-takers then the first welfare theorem applies. This can be illustrated in an Edgeworth Box with two goods, money and an externality, e.g. smoke pollution. If property rights in the externality are defined and there is a competitive market then the equilibrium is efficient. A Pigou tax can also be regarded in a sense as similar to creating a market since it prices the externality. Other, non-market, solutions should also be discussed, e.g. Coase bargaining, together with their limitations (e.g., problems when there are large numbers, or asymmetric information about costs and benefits). One natural application to discuss would be emission permit markets for greenhouse gas emissions. Should explain how the market works and perhaps compare with a Pigou tax. If the marginal cost of pollution abatement is unknown to the government then a Pigou tax is better than quantity regulation plus a permit market when the social marginal benefit of abatement is relatively flat in quantity of abatement.

B11. (i) A production economy with one input (labour) and one produced good has many consumers and many firms. All the firms have the same production function, which exhibits constant returns. Explain why any competitive equilibrium price pair of this economy is also a competitive equilibrium price pair of an economy with a single agent and a single firm.

(ii) Robinson Crusoe has utility function $u(f, l) = ln(f) + ln(l)$, where f is his consumption of food, l is his number of hours of leisure and $ln(.)$ is the natural logarithm. He has $T > 0$ hours available for leisure or labour. There is one firm, owned by Robinson, which produces food output y according to a production function $y(L)$, where L is number of hours of labour input. $y(.)$ is given by

$$y(L) = L^{\alpha}.$$

Suppose that $0 < \alpha < 1$.

(a) Find Robinson's optimal production plan and sketch a diagram to illustrate it.

(b) Suppose that the firm acts as a price-taker. Normalizing the food price to 1, find the firm's food supply function, labour demand function and profit function for the case $\alpha = 0.25$.

(c) Explain what is meant by a *competitive equilibrium* for this model.

(d) Use your result in (a) to deduce the equilibrium price pair, explaining which theorem you use to do this.

(d) Now suppose that $\alpha > 1$. Does a competitive equilibrium exist? Explain your answer diagrammatically.

Ans. (i) Constant returns implies that each firm makes zero profit in equilibrium and so is indifferent between all production plans (i.e., all amounts of labour input). In a one-person, one-firm economy, with these same prices the single firm again makes zero profit and is indifferent between all production plans. Hence the firm is willing to demand whatever labour the consumer supplies, so the labour market clears. This means that the goods market also clears, by Walras' Law.

(ii) (a) This question has nothing to do with competitive equilibrium, markets, prices, etc. It's asking what the Pareto-efficient (which is the same as optimal in this one-person model) outcome is. Just solve the problem

$$max_{L,f} ln(f) + ln(T - L)$$

subject to $f(L) = L^{\alpha}$.

$$L^* = \frac{\alpha T}{1+\alpha}, f^* = (\frac{\alpha T}{1+\alpha})^{\alpha}.$$

See fig. 3. Introducing prices, profits, etc. would cost marks here because it would suggest that you don't know the conceptual difference between a *Pareto-efficient* allocation and a *competitive equilibrium*. The First Welfare Theorem says that under certain conditions (satisfied here) a competitive equilibrium is Pareto-efficient (see (d) below) but they're different concepts.

(b) The firm's problem is

$$max_L 1(L^{0.25}) - wL.$$

$L^d(w) = (1/4w)^{4/3}$. $f^s(w) = (1/4w)^{1/3}$. $\pi(w) = 3w^{-1/3}(1/4)^{4/3}$.

(c) With p normalized to 1, a CE is a wage w such that the labour market and food market both clear, i.e. $L^d(w) = L^s(w)$ and $f^s(w) = f^d(w)$, where L^d and f^s are as above and f^d and L^s are the solutions to the consumer problem $max_{f,L} ln(f) + ln(T - L)$ subject to the budget constraint

$$f = \pi(w) + wL.$$

(d) By the First Welfare Theorem, the equilibrium allocation is optimal, so $L^d(w) = L^*$. Hence $w = (1/4)(5/T)^{3/4}$.

(e) Increasing returns. If there were an equilibrium, consumption would have to be strictly positive. For non-negative profit, the iso-profit line would intercept the vertical axis above zero. But then increasing production would increase profit, so it can't be an equilibrium (the firm isn't maximizing profit). See fig. 4.

B12. Consider a Ricardian trade model with two countries (A and B) and two produced goods (x and y). Consumers in country A are all identical and

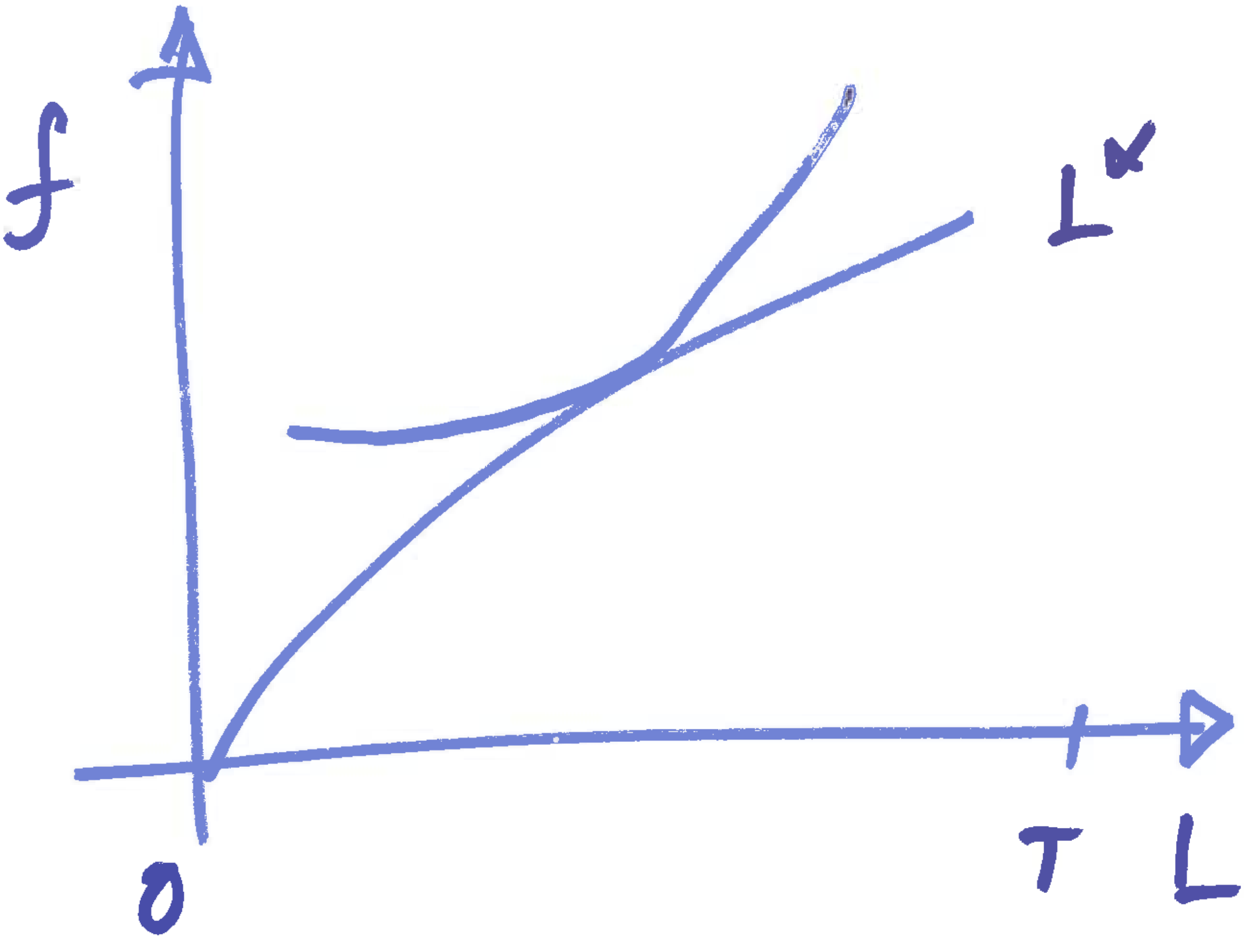

FIG. 3

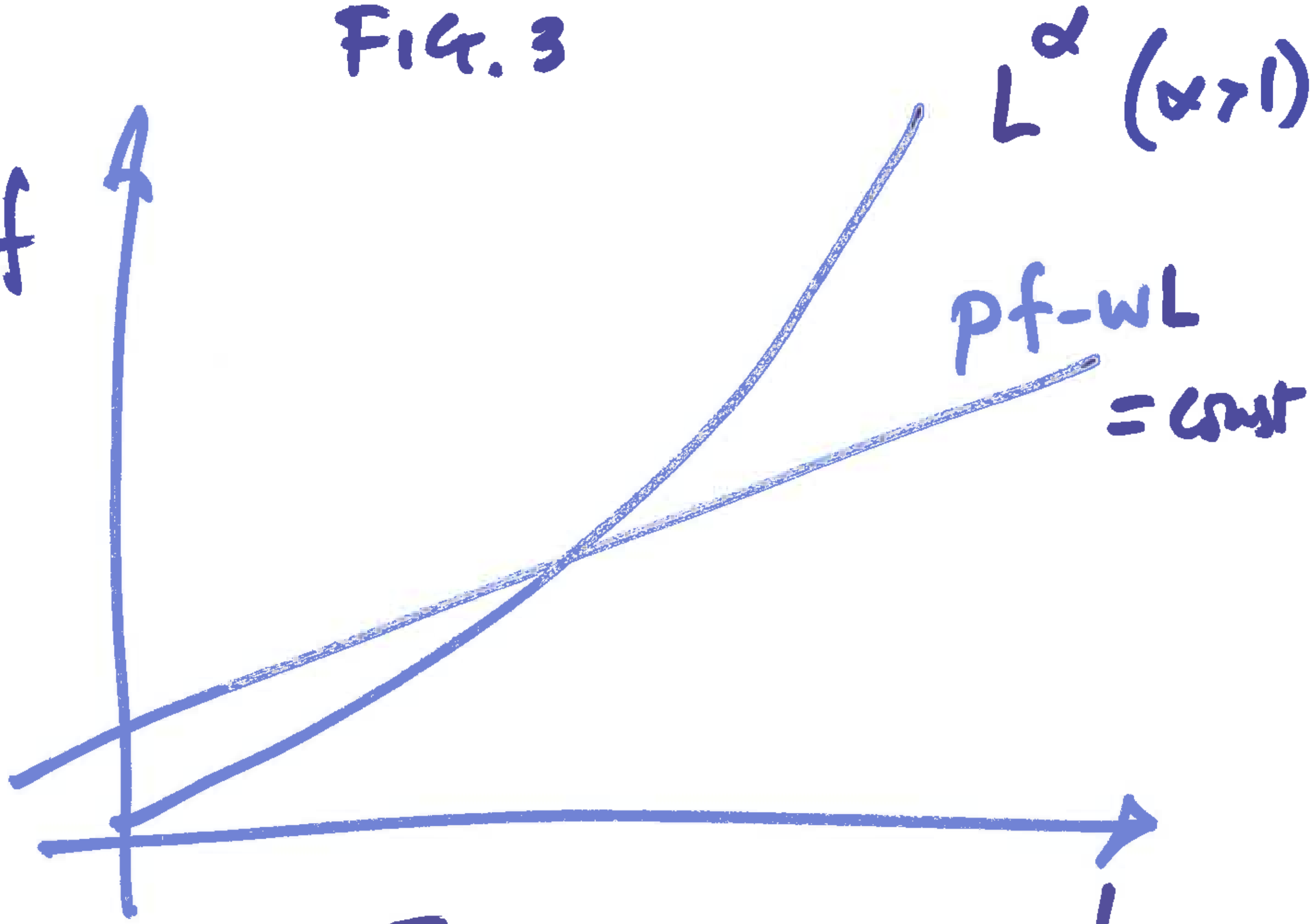

FIG. 4

have utility function $u_A(x_A, y_A) = x_A^{3/4} y_A^{1/4}$ where x_A and y_A are consumption of good x and good y respectively. Consumers in country B are all identical and have utility functions $u_B(x_B, y_B) = x_B^{1/4} y_B^{3/4}$. All consumers have 1 unit of labour and supply it inelastically. They have equal shares in all firms in their own country but have no other endowments. In country A all firms in the x industry have production function $f_A^x(L) = 6L$ where L is labour input. All firms in the y industry have production function $f_A^y(L) = 12L$. The corresponding production functions in country B are $f_B^x(L) = 10L$ and $f_B^y(L) = 4L$.

(a) Sketch the (per-person) production possibility set for each country when trade is not possible (autarky).

(b) Sketch the corresponding joint production possibility set when trade in goods is possible.

(c) Find competitive equilibrium prices for each country for the autarky case.

(d) Now suppose that trade in goods is possible. Assume that in competitive equilibrium each country specializes completely in one good. (i) For each country, write down an equation relating the wage to the price of the good which is produced in that country. (ii) For each country, write the budget constraint of a typical consumer at equilibrium prices. (iii) Hence find the competitive equilibrium prices and quantities, normalizing the price of good x as 1. What is the quantity of exports and imports of each good? Explain why this is an equilibrium.

(e) Briefly comment on the change in relative goods prices when trade is opened and on the effect on the welfare of the agents in the two countries.

Ans. Note that, since the two countries have the same population, and all consumers within a given country are identical, we can do everything in terms of per-person demands and supplies. Alternatively, we could treat the problem as if there's just one consumer in each country.

(c) Zero-profit conditions in each industry in country A are

$$6p_x - w = 0$$

and

$$12p_y - w = 0.$$

Normalized equilibrium prices are $(p_x = 1/6, p_y = 1/12, w = 1)$. Similarly, in country B, $(p_x = 1/10, p_y = 1/4)$.

(d) (i) A specializes in y since that's its comparative advantage, so zero-profit condition is

$$12p_y = w_A.$$

For B:

$$10p_x = w_B.$$

(ii) Country A BC:

$$p_x x + p_y y = w_A$$

since profit income is zero and one unit of labour is supplied. Country B:

$$p_x x + p_y y = w_B.$$

(iii) Solving the consumer problems, using equations in (i):

$$x_A = 9(p_y/p_x); y_A = 3; x_B = 5/2; y_B = (15p_x/2p_y).$$

Market-clearing in y-market:

$$3 + (15p_x/2p_y) = 12$$

so equilibrium prices (with normalization $p_x = 1$): $(p_x = 1, p_y = 5/6, w_A = 10, w_B = 10)$. Allocation: $(x_A = 15/2, y_A = 3, x_B = 5/2, y_B = 9)$. A exports 9 of y to B and imports 15/2 of x from B.

This is an equilibrium because both goods markets clear and the labour markets clear (y firms in A employ 1 unit per agent of labour, similarly x firms in B). y firms in A get zero profit whatever they produce, hence are profit-maximizing; x firms in A get negative profit per labour hour $(6p_x - w_A = 6 - 10 < 0)$, hence optimally produce zero. Other way round in B.

(e) Each agent in each country is strictly better off (e.g. show that the allocation in (d)(iii) is outside the PPF in (i)). Price of x relative to y in A is high in autarky (2) and falls when trade opens (6/5). Other way round in B.

SECTION A

1. (a) DM has complete and transitive preferences over these alternatives. That is, given any two alternatives, he can declare at least one of them as at least as good as the other. Secondly, given any three alternatives a, b, c, if he declares a is at least as good as b, and if he declares b is at least as good as c, then he must necessarily declare a is at least as good as c.

   (b) There exists a function u from the set of alternatives to $\mathbb{R}$ such that given any two alternatives a and b, DM weakly prefers a to b if and only if $u(a) \geq u(b)$.

   (c) Since the DM's utility function assigns a real number to each alternative, she can compare any two given alternatives by simply comparing the associated numbers. Hence, her preferences are complete. Secondly, if she finds a at least as good as b, and finds b at least as good as c, then the fact that she is a utility maximiser means $u(a) \geq u(b)$ and $u(b) \geq u(c)$. Of course, these two inequalities imply $u(a) \geq u(c)$, which means she finds a at least as good as c. Hence her preferences are transitive. Completeness and transitivity together mean she has rational preferences.

   (d) Thanks to the set of alternatives being finite, by pairwise comparisons, we can identify the least preferred alternatives, and set $u(a) = 0$ for all such alternatives. Next, among the remaining alternatives, we can identify the least preferred alternatives, and set $u(a) = 1$ for those alternatives. Continuing in this fashion we will exhaust all alternatives, and we will have a utility function representing DM's preferences.

   (e) A standard example is that of lexicographic preferences on the set of alternatives $\mathbb{R}_+ \times \mathbb{R}_+$. Verifying rationality is straightforward. Showing that these preferences do not admit a utility representation is beyond the scope of the course and the exam (and therefore is not needed).

2. (a) The critical aspect of the IC in this graph is that they are all concave (unlike the commonly depicted ones in textbooks). This is due to the fact that the preferences are concave in contrast with the commonly studied convex preferences.

(b) A casual examination of the indifference curves in part (a) shows that the optimal choices for this consumer will lie on the axes. In particular, $(2,5)$ cannot be an optimal choice. The fact that $MRS_{x,y} = p_x/p_y$ at this bundle does not imply that the bundle is the best. In fact, this is the least preferred bundle which exhausts her budget.

(c) For all price ratios, the consumer will spend all of her budget on a single good. Specifically, given budget m, let $p_x = 1$ and $\bar{y} = m/p_y$. If $\bar{y}^2 > m$, then she will spend all her budget on y. If $\bar{y}^2 < m$, then she will spend all her budget on x. If $\bar{y}^2 = m$, then she will either spend all her budget on x, or spend all her budget on y, but she will never consume a mixed bundle.

3. (a) Without interest, the problem can be treated as a consumer choice problem where $m = 250$, and $p_1 = p_2 = 1$. Cobb-Douglass preferences imply that he will spend 2/5 of his budget on period 1 consumption, and 3/5 of his budget on period 2 consumption. Hence optimal consumption is $(100, 150)$. That is, no borrowing, nor saving.

(b) With the interest rate of 50%, we can convert period 2 prices and incomes to their period 1 equivalent by dividing by 1.5. So, viewed from period 1, total income is $100 + 150/1.5 = 200$. Prices of consumption are $p_1 = 1$ and $p_2 = 1/1.5$. Preferences are the same as before, and therefore the budget will be split proportional to the powers 2 and 3. Hence $c_1^* = (200 \times \frac{2}{5})/p_1 = 80$ and $c_2^* = (200 \times \frac{3}{5})/p_2 = 180$. That is, he will save 20 units of his period 1 income and have 30 additional units for consumption in period 2.

SECTION B

7. (a) The Marshallian demand $x(p, m)$ is computed as a function of prices and income, where the Hicksian demand $h(p, u)$ is computed as a function of prices and utility level. For example, when evaluating the effects of a price change, x holds income constant, and re-calculates the most preferable bundle the consumer can afford with his original income at the new prices. h holds the utility level constant, and re-calculates the cheapest bundle which achieves his original utility level at the new prices.

A simple observation yields what's referred to as *duality*:

$$x(p, m) = h(p, u(x(p, m)))$$

(b) The Marshallian demand curve depicted on a p-x_1 plane will be downward sloping, meaning the consumer's demand for the good in question is decreasing in the price of the good. This is because the good is ordinary, that is, it is non-Giffen.

(c) The substitution effect is negative, meaning would depress the demand for this good. The income effect, however, is positive, because as a result of the price hike, the consumer will effectively be poorer, which would encourage him to increase his demand of inferior goods. Since this good is ordinary (non-Giffen), his demand goes down, and therefore the substitution effect must be dominating.

(d) See the drawing.

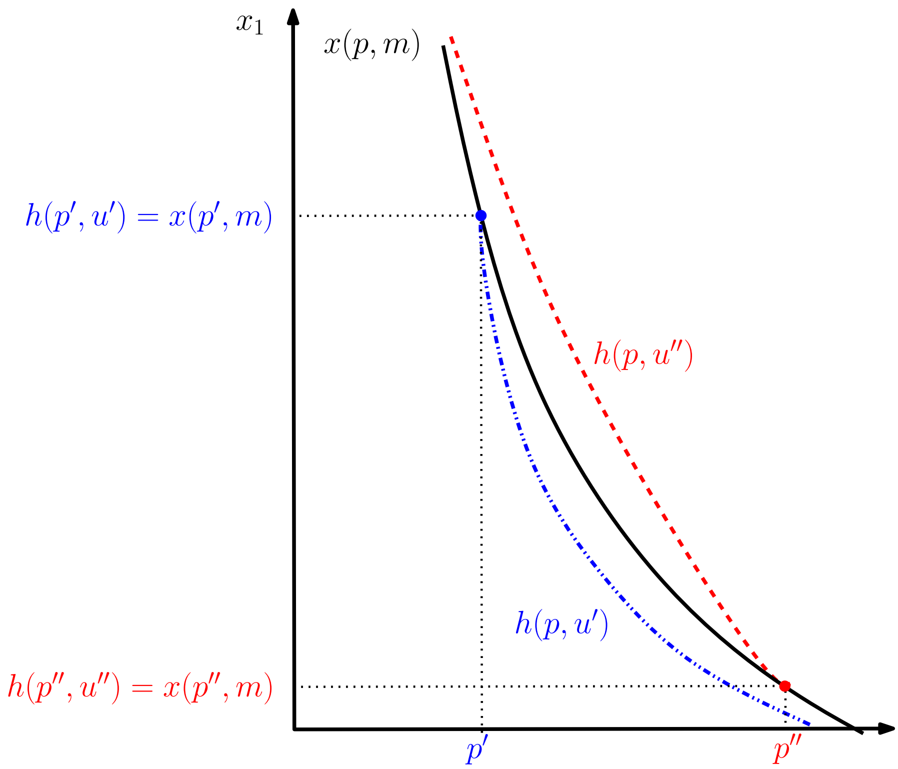

Figure 1: Answer to question 7 (d)

(e) The Hicksian demand function $h_1(p, u)$ is the consumer's demand as a function of prices, while holding the consumer at the utility level u. According to Hicks, this demand function is compensated to strip away the "income effects" of the price change, and thus keeps track of how the "substitution effect" is driving the change in the consumer's demand.

The actual demand under (p', m) is $x(p', m)$. This leads to utility u'. Since the Hicksian demand at (p', u') solves the dual problem, we know that $h(p', u') = x(p', m)$. As the price rises, the Substitution Effect brings the demand down despite the Income Effect counteracting. Without the income effect, the demand would go down even more quickly. Hence the dashed curve $(h(p, u'))$ decreases faster than the middle curve $(x(p, m))$ as a function of p.

Likewise, we have $x(p'', m) = h(p'', u'')$. Decreasing the price from p'', we have the substitution effect resulting in an increase in the demand for this good despite the income effect counteracting. Without the income effect, the demand would go up even more quickly. Hence the dashed curve $(h(p, u''))$ increases faster than the middle curve $(x(p, m))$ as a function of p as p decreases from p'' to p'.

8. (a) S_{n+1} will be flatter than S_n, because at the point Q, the curve S_n will correspond to each firm producing Q/n units, whereas S_{n+1} will imply each firm producing $Q/(n+1) < Q/n$ units. Decreasing returns to scale implies that the average cost of producing $Q/(n+1)$ units is lower than the AC of producing Q/n units.

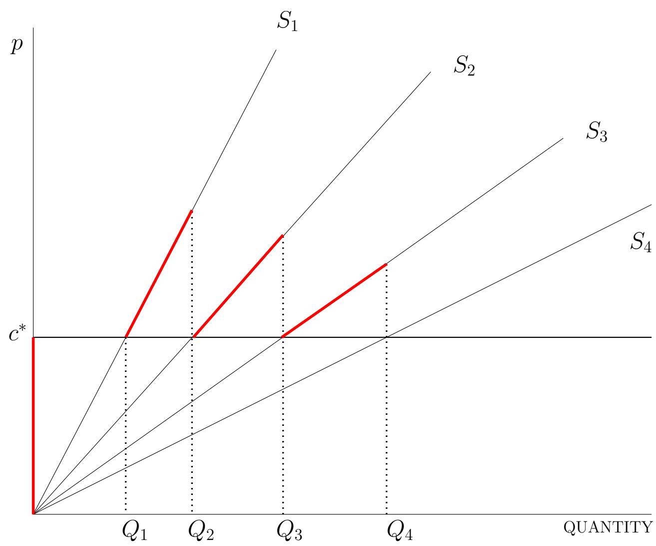

Figure 2: Answer to question 8 (a)+(b)

(b) The bold parts of the supply curves and the vertical segment on the p axis are those we can expect to observe. No point below the c^* line can be observed since they lead to negative profits for the firms. Two firms jointly producing a quantity between Q_1 and Q_2 units corresponds to a price below c^*, and therefore is not feasible. However a total supply above Q_2 can support two active firms. But not three active firms unless total supply is above Q_3. Thus we get this picture.

(c) The intersection of the demand curve with the supply curve obtained above occurs on the part belonging to S_2. Therefore we will have two firms operating in this market. This is the equilibrium outcome, because the level of demand allows two firms to co-exist and profitably operate. Hence, we won't expect a single producer. Three firms, however, cannot profitably co-exist with this given level of demand.

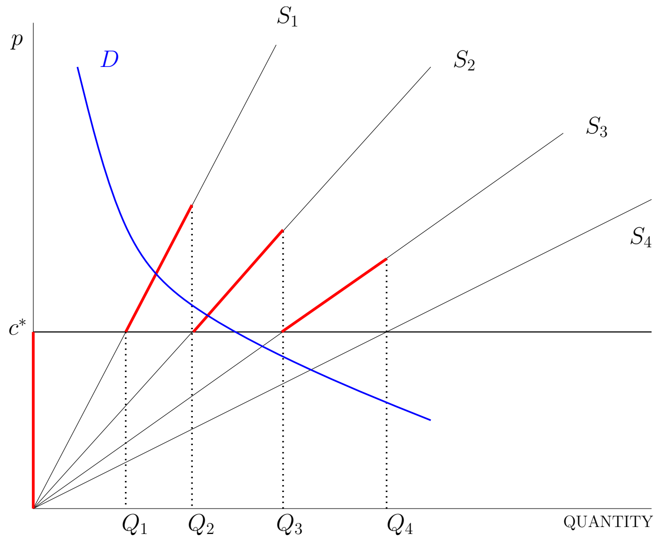

Figure 3: Answer to question 8 (c)

(d) When demand is large (or firms are small), the intersection of D with S will occur for larger values of Q, where S is given by S_n for higher values of n. As n grows, S_n gets flatter and flatter, the highest markup the firms can charge gets smaller and smaller, and the market becomes more and more competitive. In particular, the market price converges to c^* as the market grows (compared with the size of individual firms).

9. (a) The wage can be considered competitive if the revenue from employing a marginal worker is equal to the wage required to hire that worker. If the labour supply is given $P(\cdot)$, where $P(L)$ is the wage required for L units of labour to be supplied, then the competitive wage can be seen at the intersection of $MR(L)$ and $P(L)$ curves.

(b) Profit maximising monopsonist would hire L^* units of labour such that $MR(L^*) = MC(L^*)$.

(c) The profit from hiring L units of labour is $\Pi(L) = Revenue(L) - LP(L)$. Note that $MC(L) = P(L) + LP'(L)$. Since P is upward sloping, the MC curve is above the supply curve.

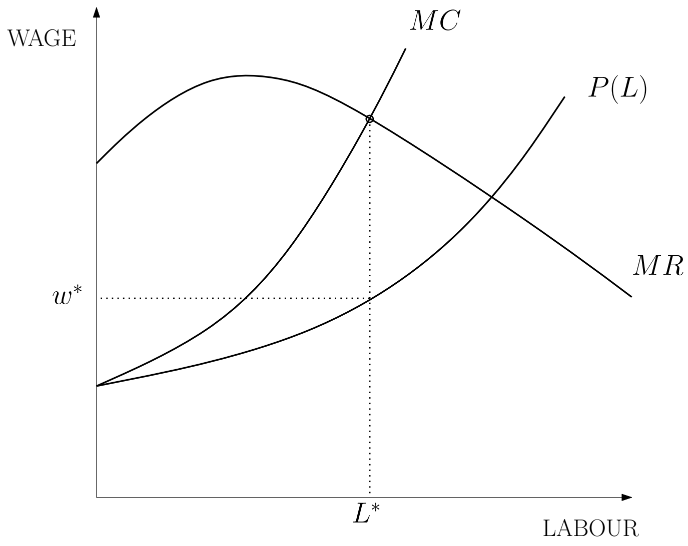

Figure 4: Answer to question 9 (c)

(d) If the minimum wage $\underline{w}$ is set between w^* and w^C, then the new marginal cost function for the firm is constant from 0 to $\underline{L}$. In particular, it is below the MR curve, and therefore the firm would employ at least $\underline{L}$. Additonal workers from this point on will cost the firm according to the original MC function which tracks well above the MR curve, and therefore the firm will not hire any further.

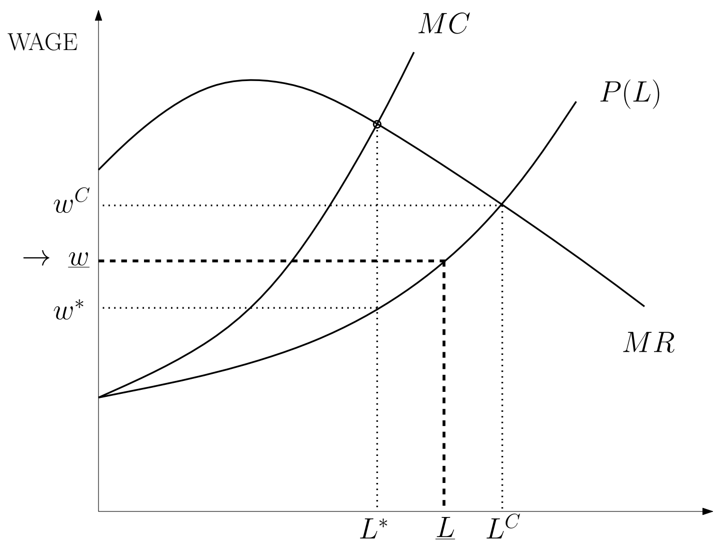

Figure 5: Answer to question 9 (d)