FACULTY OF ECONOMICS STUDY AIDS 2021

ECT1 Paper 1 Microeconomics

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Note that the Faculty will not respond to any queries regarding these solutions.

SECTION A

1. For the following utility functions determine whether the corresponding preferences are i) monotonic, ii) strictly convex. Explain your answer.

   (a) $u_1(x,y) = \min\{x, 2y\}$, $x, y \geq 0$;
   (b) $u_2(x,y) = \min\{x, 2y\} + \max\{x, 2y\}$, $x, y \geq 0$.

   Answer. (a). $u_1(x,y) = \min\{x, 2y\}$ corresponds to perfect complements. It is neither strictly convex nor monotonic. Students should either draw a diagram or provide counterexamples - say, $u_1(1,10) = u_1(1,12) = 1$ means preferences are not monotonic. The average of these bundles yields the same utility, $u_1(1,11) = 1$, hence not strictly convex.

   (b) Note that $u_2(x,y) = \min\{x, 2y\} + \max\{x, 2y\} = x + 2y$ - that is, it corresponds to perfect substitutes. It is monotonic but not strictly convex.

2. Suppose Bob has £100 that he can spend to buy sweets from shops A and B. Shop A charges an entrance fee of £10; and sells chocolate candies and ice-cream at prices of £5 and £10 per kilogram, respectively. Shop B charges an entrance fee of £20; and sells chocolate candies and ice-cream (identical to the ones sold by shop A) at prices of £10 and £5 per kilogram. Plotting the weight of chocolate candies on the horizontal axis, and weight of ice-cream on the vertical axis, draw Bob's budget set.

   Answer. Let's refer to x, y as weights of chocolate candies and ice-cream, respectively. If Bob goes to shop A, he pays 10£, and gets budget line $x+2y = 18$. If Bob goes to shop B, he pays 20£, and gets budget line $2x + y = 16$. Also, Bob can go to both shops, pay the total fee of 30£, and buy x and y at price of 5 each. The corresponding budget line will be $x + y = 14$.

   The resulting budget line is the 'maximal' of the three budget lines derived before. It consists of three parts: for $x \in [0,2]$, it equals $2x + y = 16$; for $x \in [2,10]$, it equals $x + y = 14$, and for $x \in [10,18]$, it equals $x + 2y = 18$. The budget set is bounded by these three segments.

3. Consider a monopolist with a cost function $c(q) = 0.5q^2$, who faces an inverse demand $p(q) = 100 - q$. Suppose the government forces the monopolist to produce at least $q^* = 60$ units, otherwise it will not allow the monopolist to operate.

   (a) Find the price p that the monopolist will choose.
   (b) Find the size of the deadweight loss as compared to the efficient level of production.

   Answer.
   (a) The marginal cost equals $MC(q) = q$, and it crosses inverse demand at efficient quantity of $q = 50$. Hence if the monopolist wants to produce

a positive quantity, it will not go beyond the value $q^* = 60$ enforced by government. The monopolist will charge the price as per demand, $p(q^* = 60) = 100 - 60 = 40$. The monopolist's profit will be positive, hence the monopolist will indeed want to produce 60 units and sell at price $p = 40$, rather than quit the market.

(b) The produced quantity of $q^* = 60$ units is larger than the efficient quantity of $q = 50$ units. Hence, the deadweight loss happens due to marginal costs exceeding the demand. Since both MC and demand are linear functions, the area corresponding to deadweight loss equals the excess quantity of $60 - 50 = 10$, times the difference in MC and inverse demand at $q^* = 60$, $MC(60) - p(60) = 20$, times one half - which is equal to 100.

4. Are the following statements true, false or uncertain? Explain your answer.

(a) In a pure exchange economy, price levels are determined in a general equilibrium system.

(b) In a pure exchange economy, Walras' law states that the value of aggregate excess demand is zero for any prices.

Answer.

(a) False. Price levels are not determined in a general equilibrium system (only relative prices are).

(b) True. Implied by Walras law (need to show).

5. Agent A has 3 units of cheese and 2 units of wine. Agent B has 1 unit of cheese and 6 units of wine. A and B have identical utility functions. The utility of agent i is:

$$u_i(x_c^i, x_w^i) = x_c^i x_w^i,$$

where $i \in \{A, B\}$, and x_c^i and x_w^i are person i's consumption of cheese and wine respectively.

(a) Draw the Edgeworth box illustrating the initial endowments and indifference curves of both agents.

(b) Find an expression for the MRS of each agent and hence find the equation of the contract curve. Draw the contract curve on the Edgeworth box.

(c) Find the equilibrium price ratio and equilibrium allocation (final consumptions). Mark the competitive equilibrium allocation on the Edgeworth box.

Answer.

At any Pareto optimal allocation where both consume some of each good, A's marginal rate of substitution between cheese and wine must equal B's. An equation that states this condition in terms of the consumptions of each good by each person is $x_w^B/x_c^B = x_w^A/x_c^A$. At any Pareto efficient allocation, where both persons consume both goods, the slope of A's indifference curve will be -2. Therefore, since we know that competitive equilibrium must be Pareto efficient, we know that at a competitive equilibrium, $p_c/p_w = 2$. In competitive equilibrium, A's consumption bundle must be 2 cheese, 4 wine. B's consumption bundle is 2 cheese, 4 wine. Figure 1 shows the Edgeworth box and illustrates the C.E..

6. Consider an exchange economy consisting of two persons living in a household, A and B. There are two goods, consumption (x) and leisure (y). Let x_i and y_i be the amounts of good x and y consumed by individual $i = A, B$. The household pools consumption together, so each person values the total consumption of the household and his/her own leisure. A has utility function $u_A(x_A, y_A, x_B) = x_A + x_B + \ln(y_A)$ and B has utility function $u_B(x_B, y_B, x_A) =$

$x_A + x_B + \sqrt{y_B}$. The endowments are $(e_A^x, e_A^y) = (6, 1)$ and $(e_B^x, e_B^y) = (2, 0)$. Set the price of x equal to 1 and denote by p the price of good y.

(a) Solve for the competitive equilibrium price p and the equilibrium allocation.
(b) Show all the Pareto efficient allocations.
(c) Is the competitive equilibrium allocation derived in (a) Pareto efficient or not? Explain why.

Answer.
(a) A solves $max_{x_A,y_A} x_A + x_B + ln(y_A)$ subject to

$$py_A + x_A = p + 6,$$

taking x_B as given. B solves $max_{x_B,y_B} \sqrt{y_B} + x_A + x_B$ subject to

$$py_B + x_B = 2.$$

taking x_A as given. Demand functions:

$$y_A = \frac{1}{p} \qquad x_A = p + 5,$$

$$y_B = \frac{1}{4p^2} \qquad x_B = 2 - \frac{1}{4p}.$$

Market-clearing condition for y-market:

$$\frac{1}{p} + \frac{1}{4p^2} = 1$$

so equilibrium p is $\sqrt{2}/2 + 0.5$.

(b) Taking x away from A and giving it to B makes no difference to either A or B's utility. So A cares only about consumption of y_A, and B only cares about consumption of y_B. Therefore any allocation is Pareto-efficient.

(c) The CE is Pareto-efficient.

SECTION B

7. Jane has inherited a box with $w_1 = 6$ kg of tea and $w_2 = 1$ kg of coffee. She has no other source of income. Her utility of consuming x kg of tea and y kg of coffee is $u(x, y) = xy$.

   (a) Assume that the price of tea is £1 per kg, and the price of coffee is $p_0 = £4$ per kg. Jane can trade tea and coffee at these prices. Find Jane's optimal consumption, and utility u_0 after trading.

   Answer. Jane has Cobb-Douglas utility. The total monetary value of her inheritance is $w_1 + p_0 w_2 = 6 + p_0$, and her optimal demands are $\frac{6+p_0}{2}$ for tea, and $\frac{6+p_0}{2p_0}$ for coffee. Substituting $p_0 = 4$, obtain 5 kg of tea, $5/4$ kg of coffee, and $u_0 = 5 \times 5/4 = 25/4$.

   (b) Assume that the Supreme Leader of Jane's country changes the price of coffee, and that Jane has to trade her initial endowment given the new price. Among all possible prices of coffee, find the price p_1 that minimizes Jane's utility after trading.

   Answer. Jane would reach minimal utility if she cannot benefit from trading, starting from her initial endowment. That is, the price ratio for tea and coffee has to correspond to MRS at the initial endowment. That is, $p_1 = \frac{MU_2}{MU_1} = \frac{w_1}{w_2} = 6$.

   (c) Given the new price p_1, find how much additional income Jane needs to be given, so that after trading she reaches her old level of utility, u_0.

   Answer. At $p_1 = 6$, monetary valuation of Jane's endowment is $6 + 6 = 12$. If Jane gets an additional income of m, her demand on tea will be $\frac{m+12}{2}$, and on coffee is $\frac{m+12}{12}$. The final utility will be $\frac{m+12}{2} \times \frac{m+12}{12}$, and one needs to find m such that the final utility equals $u + 0 = 25/4$. Calculation reveal that $m = 5\sqrt{6} - 12 \approx 0.25$

   (d) Now assume that the Supreme Leader has instead increased the price of coffee to $p_2 > p_0$, such that when Jane optimally trades her initial endowment given price p_2, she reaches her old level of utility, u_0. Find p_2.

   Answer. Given price p_2, monetary value of Jane's endowment is $6 + p_2$; demand on tea is $\frac{6+p_2}{2}$, on coffee is $\frac{6+p_2}{2p_2}$, and final utility is $\frac{6+p_2}{2} \times \frac{6+p_2}{2p_2} = u_0 = 25/4$. This formula is equivalent to $p_2 - 5\sqrt{p_2} + 6 = 0$, which has two roots: 4 and 9. The answer is $p_2 = 9$.

8. Suppose that a firm has production technology

$$f(K, L) = \sqrt{KL}$$

(a) Assume that prices on K, L equal $r = 2, w = 1$, respectively. Derive the firm's cost function, $c(q)$.

Answer. To derive cost function $c(q)$, the firm solves problem of minimizing $rK + wL = 2K + L$, subject to $q = \sqrt{KL}$. The solution is $L = 2K = \sqrt{2}q$, and the related cost is $c(q) = 2K + L = 2\sqrt{2}q$.

(b) Now assume that the firm cannot buy capital from outside. Instead, the firm can produce capital using labour, with production technology $K(L) = \sqrt{L}$. That is, when the firm hires labour, it uses part of it to produce capital, and then uses this capital with the rest of labour to produce the final good. Derive firm's cost function $c(q)$ of producing the final good.

Answer. Assume the firm hires L_1 units of labor to produce capital, and L_2 units of labor to produce the final good. Then one has $q = \sqrt{KL_2} = \sqrt{\sqrt{L_1}L_2} = L_1^{1/4}L_2^{1/2}$ - that is, a Cobb-Douglas production; with cost of both $L1, L2$ equal $w = 1$. Solving the cost-minimisation problem, one gets $L_2 = 2L_1 = \frac{2q^{4/3}}{2^{2/3}}$, and the cost is $c(q) = L_1 + L_2 = \frac{3q^{4/3}}{2^{2/3}}$.

(c) Now assume that the firm can both buy capital as in part (a), and produce it as in part (b). Derive firm's cost function $c(q)$.

Answer. Let's derive the minimal cost of having K units of capital. If the firm produces K_1 units of capital using labor, then by spending small amount dx on labor, the firm hires $\frac{dx}{w}$ units of labor. Since marginal product of producing capital using labor is $\frac{1}{2\sqrt{K_1}}$, the firm produces $\frac{dx}{2w\sqrt{K_1}}$ units of capital. The firm would stop producing capital with its own labor and starts buying it from the market once the value $\frac{dx}{2w\sqrt{K_1}}$ equals $\frac{dx}{r} = \frac{dx}{2}$ - that corresponds to additional quantity of capital by spending dx on the market. Hence, there is a cutoff value of capital that satisfies $\frac{dx}{2w\sqrt{K_1}} = \frac{dx}{2}$, or $K_1 = 1$ - so that the firm produces the first K_1 units using labor, and the rest - buying from the market. The cost $c(K)$ of producing K units of capital equals K^2 for $K \leq K_1 = 1$, and equals $2K - 1$ for $K \geq 1$.
Respectively, for $K \leq 1$, the firm's solution is the same as in part (b) - when it does not buy any capital from the market. The amount of q that corresponds to $K = 1$ is $q = \sqrt{2}$. That is, for $q \leq \sqrt{2}$, the cost is $c(q) = \frac{3q^{4/3}}{2^{2/3}}$. For $K \geq 1$, (that is, for $q \geq \sqrt{2}$), the firm buys some of capital from the market, and its solution satisfies the same first-order conditions as in part (a) - that is, $L = 2K = \sqrt{2}q$, and the cost function is the same as in (a) except the new cost for capital: $wL + 2K - 1 = 2\sqrt{2}q - 1$.

9. Consider a monopolist with a cost function $c(q) = q^2$, who faces two markets A and B, with inverse demand functions $p_A(q_A) = 10 - q_A$ and $p_B(q_B) = 13 - 2q_B$, respectively.

(a) Assume that the monopolist has to charge the same price on both markets. Find the profit-maximizing choice of price.

Answer. If the monopolist charges price p on both markets then the respective quantities equal $q_A = 10 - p$, and $q_B = \frac{13-p}{2}$. The monopolist's profit is $\pi(p) = p(10 - p + \frac{13-p}{2}) - (10 - p + \frac{13-p}{2})^2$. Solving for the optimal p, one gets $p = 8.8$. Both markets will be served.

(b) Assume that the monopolist can engage in third-degree price discrimination. Find the profit-maximizing choices of prices in both markets.

Answer. If the monopolist sells q_A units at price $10 - q_A$ on market A, and sells q_B units at price $13 - 2q_B$, then its profit is $q_A(10 - q_A) + q_B(13 - 2q_B) - (q_A + q_B)^2$. Writing the FOC yields $10 - 2q_A = 2q_A + 2q_B$, and $13 - 4q_B = 2q_A + 2q_B$. Solving, one gets $q_A = 1.7$, $p_A = 8.3$; and $q_B = 1.6$, $p_B = 9.8$.

(c) Now assume that the government imposes a tax t per unit of sale on market A. Find the value of t, such that the monopolist who engages in third-degree price discrimination, will serve both markets and will charge the same price.

Answer. As compared to part (b), the monopolist's profit is now $q_A(10 - t - q_A) + q_B(13 - 2q_B) - (q_A + q_B)^2$. Writing the FOC yields $10 - t - 2q_A = 2q_A + 2q_B$, and $13 - 4q_B = 2q_A + 2q_B$. The monopolist will charge the same price if $10 - q_A = 13 - 2q_B$. Solving yields $t = 3$, $q_A = 0.8$, $q_B = 1.9$, $p_A = p_B = 9.2$.

10. Are the following statements true, false or uncertain? Explain your answer.

(a) If preferences are quasilinear, the efficient amount of a consumption externality will be independent of the assignment of property rights.
(b) In an economy with production and no externalities, Pareto efficiency requires that each individual's marginal rate of substitution be equal to the marginal rate of transformation.
(c) Every Pareto efficient allocation can be achieved as a competitive equilibrium.

Answer. (a) True. Students need to derive it. The result that under certain conditions the efficient amount of the good involved in the externality is independent of the distribution of property rights is sometimes known as the Coase Theorem. So if preference is quasilinear, Coase theorem is valid.

(b) True (need to show to get full credits). Students also get full credits if he/she points out that MRS is not equal to MRT if efficient production is zero.

(c) False. Need convex preferences and in an economy with production, need production set to be convex (need to give examples).

11. An airport is located next to a large tract of land owned by a housing developer. The developer would like to build houses on this land, but noise from the airport reduces the value of the land, and, respectively, it reduces the profit of the developer. Let X be the number of planes that fly per day and let Y be the number of houses that the developer builds. The airport's total profits are $48X - X^2$, and the developer's total profits are $60Y - Y^2 - XY$. Let us consider the outcome under various assumptions about institutional rules and about bargaining between the airport and the developer.

(a) Suppose that no bargains can be struck between the airport and the developer, and that each can decide on its own level of activity. Find the number of planes per day that maximizes profits for the airport and the number of houses that maximizes the developer's profits.
(b) Suppose that a law is passed that makes the airport liable for all damages to the developer's property values. Find the airplane traffic per day that the airport will choose and the number of houses that the developer will choose to build.
(c) Suppose that the housing developer purchases the airport. Find the airplane traffic and the number of houses as chosen optimally by the developer.
(d) Which institution rule(s) proposed above fail(s) to achieve an efficient outcome? Explain.

Answer. (a) No matter how many houses the developer builds, the number of planes per day that maximizes profits for the airport is 24. Given that the airport is landing this number of planes, the number of houses that maximizes the developer's profits is 18.

(b) the total amount of damages awarded to the developer will be XY. To maximize his net profits, the developer will choose to build 30 houses no matter how many planes are flown. To maximize its profits, net of damages, the airport will choose to land 9 planes.

(c) To maximize joint profits, it should build 24 houses and let 12 planes land.

(d) In (a), the airport does not bear the cost of damages from landing planes. In (b), the airport bears the full cost of damages from landing but the developer bears none of the cost. Therefore the developer overbuilds. Total combined profit is highest in (c).

12. Consider the Ricardian trade model where there are two countries (country A and country B), two output goods (wine and cheese) and one input (labour). Suppose that A has an absolute advantage in both goods and a comparative advantage in cheese. In each country, there are two sectors. One sector produces wine and the other sector produces cheese. Each sector uses constant returns to scale production functions with labour as the only input. In A the labour requirement is $\frac{1}{\alpha_A^w}$ for one unit of wine and $\frac{1}{\alpha_A^c}$ for one unit of cheese. The corresponding quantities in B are $\frac{1}{\alpha_B^w}$ and $\frac{1}{\alpha_B^c}$.

(a) Suppose that trade is not allowed. P_c and P_w denote the prices of cheese and wine in country A, respectively. In what range of prices will both goods will be produced in country A? Show the derivation.

(b) Suppose that trade is allowed. p_c and p_w denote the prices of cheese and wine in the world market, respectively.

i. In what range of prices will country A specialize in cheese production and country B specialize in wine production? Show the derivation.

ii. In what range of prices will country A be producing both cheese and wine and country B specialize in wine production? Show the derivation.

iii. In what range of prices will wine not be produced in either of the two countries? Show the derivation.

(c) Which country has a higher wage in equilibrium? Explain your answer.

(d) An international labour union claims that there should be one common wage in both countries and sectors. For instance, assume that the wage rate must be fixed at a level such that profit in the cheese sector in country A is zero. Is this compatible with a competitive equilibrium? Explain your answer.

Answer. (a) There is no profit in the one factor constant return model, so hourly wage in wine (cheese) sector will equal the value of marginal product of labor in the wine (cheese) sector. For both goods to be produced, it must be that wages are equalized between the two sectors. So we must have $\frac{P_c}{P_w} = \frac{\alpha_A^c}{\alpha_A^w}$.

(b) Specialization requires $\frac{\alpha_B^c}{\alpha_B^w} < \frac{P_w}{P_c} < \frac{\alpha_A^c}{\alpha_A^w}$; no wine produced if $\frac{P_w}{P_c} < \frac{\alpha_B^c}{\alpha_B^w} < \frac{\alpha_A^c}{\alpha_A^w}$; country A will be producing both some cheese and some wine and country B specializes in wine production if $\frac{\alpha_B^c}{\alpha_B^w} < \frac{P_w}{P_c} = \frac{\alpha_A^c}{\alpha_A^w}$

(c) the equilibrium wage in country A is higher than the wage in country B.

(d) Zero profit in coconuts in country A, so $p_c c_A - wL_A^c = 0$, so $p_c \alpha_A^c = w$ (notation follows the lecture notes)

Suppose that the given prices and wage form a competitive equilibrium. For plausible utility functions, a strictly positive amount of each good must be produced in equilibrium, so we assume that. It must be that

$$\frac{\alpha_A^c}{\alpha_A^w} > \frac{p_w}{p_c} > \frac{\alpha_B^c}{\alpha_B^w}(*)$$

However, this implies that $p_c\alpha_A^c - w > p_w\alpha_A^w - w$ and, so, from absolute advantage, $p_c\alpha_A^c - w > p_w\alpha_B^w - w$, so profits are negative in the wine industry in both countries (if a strictly positive amount is produced). So no wine will be produced in equilibrium, which is not compatible with equilibrium. Absolute advantage also implies that profits are negative in the coconut industry in B. The problem is that the wage is the same in both countries. In equilibrium, p_c and p_w have adjusted so that (*) is true - this means that cheese are more profitable than wine in A and vice versa in B. But the wage in A has to adjust so that profit in cheese production in A is zero and, similarly, the wage in B has to adjust so that wine give zero profit in B. This requires wages to be different in the two countries (and lower in the less productive country, B).

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