FACULTY OF ECONOMICS STUDY AIDS 2018

ECT1 Paper 1 Microeconomics

The Faculty Board has agreed to release outline solutions to the 2018 examinations as a study aid for exam revision. They are abridged solutions, and not 'definitive', and should therefore not be considered as an exemplar for 'complete' answers.

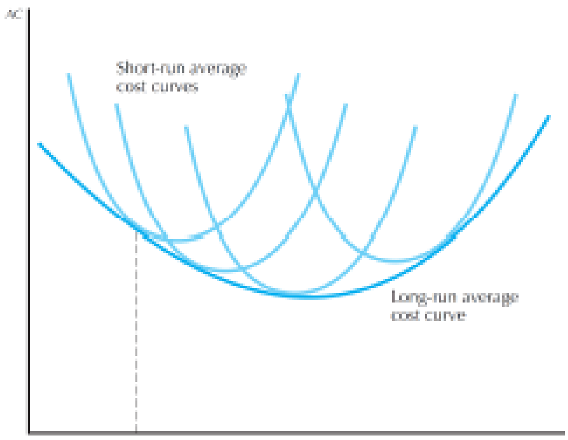
Also note that the Faculty will not respond to any queries regarding these solutions.

PART I – PAPER 1 – MICROECONOMICS – JUNE 2018

Draft Answers

SECTION A

1. The following graph is from lecture notes, and those students who draw it with some explanation deserve at least half the mark for this question.



A good discussion of this statement might acknowledge

- In the short run, some factors are fixed (e.g., K), whereas others are variable (e.g., L).
- In the long run, all factors (e.g., both K and L when the production function looks like $f(K, L)$) are variable. The firm has more flexibility when cost minimising, and as such the long-run AC curve will always lie below the short-run curve.
- Note that if (K^*, L^*) is an optimal bundle (minimiser of long-run cost function) to produce q units, then the short-run curve whose K is fixed at K^* must touch the long-run curve at $L = L^*$.
- Continuing with that idea, every point on the long-run cost curve is point of contact with some short-run cost curve.
- Finally, if we were to draw all short-run cost curves (one for each K), then the highest curve we can fit underneath (that is, the lower envelope) would be nothing but the long-run cost curve.

2. (a) He is worse off as his new choice is strictly within his old set of opportunities. *Revealed preference* says his previous choice beats every point on the new budget line northwest of the endowment point. (Note that the new choice does not have to be to the southwest of his original choice.)

(a) We can think of consumption in periods 1 and 2 priced as $1 + r$ and 1, respectively. A decrease in r leads to a decrease in income. Also, the consumer would like to substitute away from the good that becomes relatively more expensive (i.e., consumption in period 2).
He is a net supplier of good 1 (period 1 consumption) which becomes cheaper. Thus his "income" goes down.

   i. Choice moves south-west. x_1 is normal and subject to a negative income effect and positive substitution effect. Income effect dominates. x_2 is subject to a negative substitution effect. It might be inferior, but if so, the income effect is dominated by the substitution effect.

   ii. Seems unlikely, but possible. x_1 is normal, subject to a negative income effect and positive substitution effect; and the income effect dominates. x_2 must be inferior since it is subject to a negative substitution effect which should be dominated by a positive income effect.

   iii. Again seems unlikely but possible. If period 1 consumption becomes significantly cheaper, and thus attractive versus period 2, he might prefer to reduce x_2 below m_2, and might even be better off.

3. Bliss-point preferences: they are *rational*, they are *not increasing*, they are *convex*. Students are expected to display that they understand the meaning of these notions.
Can be represented by the utility function $u(x, y) = (x - 2)^2 + (y - 5)^2$.
Part (e) might be tricky to many of them. If the set A is closed, then, yes, she has a most preferred alternative. The anomaly of not being able to find an optimal choice can be illustrated by setting A to be all possible bundles excluding $(2, 5)$.

4. 'In a pure exchange economy with two agents, A and B, and two goods, x and y, if A is able to make a take-it-or-leave-it offer to B it makes no difference if the offer is an allocation or a pair of prices; the final allocation will be the same in both cases, and Pareto-efficient'. Comment

Ans: If A makes a take-it-or-leave-it offer of an allocation, it will be efficient and give B approx. the same utility as B's endowment. If the offer is a price (and B acts as a price-taker, with A taking the other side of the market, i.e. both markets clear) then the allocation won't be efficient. A will pick the best point (for her) on B's offer curve. This means that A's indifference curve will be tangent to B's offer curve, not to B's indifference curve.

5. 'In a pure exchange economy with two agents and two goods a competitive equilibrium will exist as long as both agents have non-satiated preferences, since they will negotiate a trade in which both markets clear'. Comment.

Ans. Non-satiated preferences are not enough to guarantee existence of an equilibrium. Non-convex preferences could give rise to discontinuous excess demand curves, hence non-existence. This can be explained intuitively, using non-convex indifference curves suggesting why excess demands could jump, and a diagram of excess demand for one good as a function of own price, with a jump across the zero axis.

Secondly, they don't negotiate trades in competitive equilibrium.

6. In a Robinson Crusoe economy with a single produced good and one input, labour, the production function is $y = L^2$, where L is labour input. The utility function is $U(c, l) = c^{2/3} l^{1/3}$, where c is consumption of the produced good and l is the amount of leisure. The amount of time available is T. Find the optimal amount of output and leisure. If this optimal allocation were the outcome of a competitive equilibrium, what would the equilibrium wage and profit have to be? Could the allocation be the outcome of a competitive equilibrium? Explain why or why not, using a diagram.

Ans. $l = T/5, c = (4T/5)^2$. Normalizing output price as 1, the wage in equilibrium would have to be marginal product of labour, i.e., $2L = 8T/5$. Profit would be $(4T/5)^2 - (8T/5)(4T/5)$. Since this is negative, it cannot be an equilibrium outcome. Should draw the production function, with indifference curve and iso-profit line.

1. **SECTION B**

7. a. $f(\lambda K, \lambda L) = \sqrt{\lambda^2 KL} = \lambda\sqrt{KL} = \lambda f(K, L)$. Thus, constant returns to scale production.

   (a) Cobb-Douglas production function with equal weights for each input implies the firm spends equal amounts of money on each input. Since L is four times as expensive, the firm uses K four times as much as it uses L.[1] Therefore, the cost of producing 1 unit can be computed by solving $\sqrt{k\ell} = 1$ and $k = 4\ell$, yielding $\ell = 1/2$ and $k = 2$, and $c(1) = 4\ell + k = 4$. Since production is constant returns to scale, $c(q) = qc(1)$, and hence $c(q) = 4q$.

   (b) Since the cost function is linear, the firm can treat the two markets separately. (If students don't make this point, they should not get full mark on this bit.) The profit from the students' market is

$$\Pi(p_S) = (p_S - 4)(100 - 2p_S)$$

[1] Alternatively, the optimal choice of inputs must satisfy $MP_K/p_K = MP_L/p_L$ at the optimal choice (K^*, L^*), which implies

$$\frac{\frac{\sqrt{L}}{2\sqrt{K}}}{1} = \frac{\frac{\sqrt{K}}{2\sqrt{L}}}{4} \quad \Longrightarrow \quad 4L = K.$$

This expression is maximised when $p_S = 27$.
Likewise, the profit from the employed's market is

$$\Pi(p_E) = (p_E - 4)(100 - p_E)$$

This expression is maximised when $p_E = 52$.

(c) $\varepsilon = \frac{p}{D}\frac{dD}{dp}$. Computing it in each market

$$\varepsilon_S = \frac{27}{46} \times (-2) = -\frac{27}{23} \qquad \varepsilon_E = \frac{52}{48} \times (-1) = -\frac{13}{12}$$

(d) The joint market demand is $D_M = 200 - 3p$. The profit function

$$\Pi(p_M) = (p_M - 4)(200 - 3)$$

is maximised when $p_M = 106/3$. The price elasticity of demand is

$$\varepsilon_M = \frac{106/3}{94} \times (-3) = -\frac{53}{47}$$

The monopolist always sets prices so the demand is elastic (i.e. $\varepsilon < -1$) since otherwise he would be wasting opportunities for profit-making by increasing prices.
Comparing the three markets, the more elastic the demand, the lower the price should be which can be verified in this case by noticing

$$p_S < p_M < p_E \qquad |\varepsilon_S| > |\varepsilon_M| > |\varepsilon_E|$$

since

$$27 < \frac{106}{3} < 52 \qquad \frac{54}{46} > \frac{53}{47} > \frac{52}{48}$$

8. (a) Observing

$$u(x, y) \quad = \quad x + 2y\sqrt{x} + y^2 \quad = \quad (\sqrt{x} + y)^2$$

would simplify the analysis, but is not necessary.
At an interior solution, the consumer's choice satisfies

$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y} \qquad \text{that is} \qquad \frac{1}{2\sqrt{x}p_x} = \frac{1}{p_y}$$

So x^* will never be 0, but if spending all m on x still yields higher MU_x then $\frac{1}{p_y}$, then y^* will be 0.
Assuming an interior solution (m large enough), these preferences are quasi-linear with the demand function

$$x^*(p_x, p_y, m) = \frac{p_y^2}{4p_x^2} \qquad y^*(p_x, p_y) = \frac{1}{p_y}\left(m - \frac{p_y^2}{4p_x}\right)$$

(a) Students are expected to describe Slutsky and Hicks approaches to measuring the substitution/income effects. Slutsky fixes the original choice, pivots the budget line around this point to get the new price ratio, and marks the utility maximising bundle on this new (hypothetical) budget line as the consequence of the substitution effect. Hicks, on the other hand, fixes the original indifference curve, and identifies at which point (bundle) on this curve in the slope of the curve equal to the new price ratio. That bundle, according to Hicks, is the consequence of the substitution effect.

(b) Quasi-linearity in x can be defined via quasi-linear utility functions of the form $U(x, y) = V(x) + y$. Or can be defined as indifference curves being parallel shifts along the y axis (the choice x^* being independent of income effect m).

(c) This part is harder: No income effects with quasi-linear preferences, and therefore $EV = CV = CS$.
CS: Her willingness to pay (measured by her demand function) minus what she pays.
EV: Instead of implementing the new state (i.e., lowering p_x), what adjustment to the consumer's income in status quo would lead her to achieve the utility she would otherwise reach in the new state?
CV: Once she is in the new state, facing new prices, what amount removed from her income would lead her to achieve the same utility she used to have in status quo?

9. A good answer can articulate broadly on the rational choice framework as a modelling framework within which the modeller tackles a particular economic context, and aims to construct a preference structure whose maximisation is consistent with the observed choices. Once a clear understanding of what we mean by rational choice (utility maximising) framework, one can remark strengths and/or weaknesses of the framework.

- As such, altruistic behaviour may well be incorporated. Likewise, framing effects, in some cases, can be systematically analysed to the extent that what appears "irrational" in a colloquial sense can be explained by properly adjusting the hypothesised preferences.
  In this perspective, utility maximisation does not require a leap of assumption. To the extent that choices can be rationalised, under fairly permissive assumptions, preferences can be represented by utility functions.
- While modeller's flexibility within the rational choice framework allows her to rationalise a wide range of phenomena, a universally applicable model of individual preferences becomes practically impossible. In contrast with physical sciences, economic models might have vary significantly across applications, and it is usually impossible to refer to a single model of economic agent.
- While one can rationalise framing effects to explain observed phenomena,

the nature of certain cognitive biases might make some choices almost arbitrary. As such, predictability of some biases and/or framing effects might be rather limited. Thus tractable models of decision making in those contexts are particularly difficult to build.

Answers should be judged on (1) whether they display an accurate understanding of what the "rational choice framework" means, and (2) how they articulate reasons in favour or against the applicability and use of models within this framework.

More points for those answers which acknowledge both the flexibility and the limitations of the framework.

10. B1. (a) Consider a Ricardian trade model with the following features. In country A, labour produces twice as much per hour in industry y as in industry x. Country B labour is twice as productive as country A labour in industry x and 50% more productive than country A labour in industry y. There are constant returns to scale in all firms and each consumer supplies one unit of labour inelastically.

(i) Describe what is meant by a *competitive equilibrium* of the model, when trade in goods is possible but labour migration is not.

(ii) Suppose that in the competitive equilibrium there is complete specialization of production. Find upper and lower bounds for the ratio of the price of good x to the price of good y, and explain why specialization occurs in equilibrium if the ratio is between these bounds. Find the ratio of the wage in B to the wage in A, in terms of the output prices. What is the lowest possible value of this ratio?

(b) 'Free Trade exploits a country and makes it worse off if its workers make much lower wages than workers in other countries'. Discuss in the context of the model in (a).

Ans. (a) (i) Price vector (p_x, p_y, w_A, w_B) such that, when each firm in each industry chooses labour demand and output to maximize profit, taking the prices as given, and each consumer chooses demand for x and y to max utility, given their budget constraint, the four markets all clear (world markets for x and y, local labour markets).

(ii) p_x/p_y lies between the slopes of the two PPFs, so $3/2 < (p_x/p_y) < 2$. A specializes in y, so zero-profit condition: $w_A = p_y(2z)$ where z is the output of x from 1 unit of labour, in A. Similarly, B specializes in x and $w_B = p_x(2z)$. So

$$\frac{w_B}{w_A} = \frac{p_x}{p_y}$$

and $w_B \geq 1.5w_A$. Specialization occurs because profit is negative in industry x in country A and industry y in country B.

(b) In this example, the wage in A is less than the wage in B. However, trade improves welfare in both countries. This can be shown by comparing the autarky case, in which A's (per-person) consumption is on A's PPF, with the case in which trade is allowed. A diagram could show a price line with slope steeper than $-3/2$ through the point $(2z, 0)$: under trade, consumption corresponds to the optimal bundle on this line, hence better than autarky. A's wage is low because of lower productivity, not trade.

11. B2. Consider an economy with two types of firms which emit greenhouse gases. Each firm emits 1 thousand tonnes. The benefit to society of reducing emissions by Q tonnes is γQ. The cost of reducing emissions by q is αq^2 for a type 1 and βq^2 for a type 2 firm (independently of any other production decisions). There are n firms of each type. Assume that $\gamma < 2\alpha$ and $\gamma < 2\beta$.

(i) Find the socially optimal amount of emission reduction for each firm.

(ii) Find a Pigou tax (or carbon tax) which would achieve the social optimum, and explain why it does so.

(iii) Suppose the government adopts a cap-and-trade approach, issuing emission permits which are subsequently tradeable in a competitive market. Suppose the government issues permits for free, k to each firm. (a) How many permits should it issue?

(b) For a given type 1 firm, write down the profit, from abatement and corresponding sale of permits, if the market price of permits is p and abatement is q_1. Hence find the total demand for permits at price p.

(c) Write down the market-clearing condition. Find the equilibrium price of permits when the government issues the optimal number of permits. Briefly discuss your result.

(d) Briefly discuss the best way for the government to distribute permits in a cap-and-trade scheme.

Ans. (i) Social planner's problem:

$Max_{(q_1,q_2)} n[(q_1)\gamma + (q_2)\gamma - \alpha q_1^2 - \beta q_2^2]$.

Solution: $[q_1^* = (\gamma/2\alpha), q_2^* = (\gamma/2\beta)]$. This is marginal benefit (γ) equals marginal cost ($2\alpha q_1$ or $2\beta q_2$) of abatement.

(ii) Given tax t, a type 1 firm's problem is $min_{q_1}(1 - q_1)t + \alpha q_1^2$, so $q_1 = (t/2\alpha)$. Hence set $t = \gamma$. Similarly for type 2. Should discuss how the Pigou tax charges the polluter for social cost of pollution, i.e., marginal environmental damage cost γ.

(iii)(a) Optimal number of permits is $n[2-(q_1^*+q_2^*)] = n[2-(\gamma/2)((1/\alpha)+(1/\beta))]$.

(b) If $1-q_1 > k$ the firm buys $1-q_1-k$ permits and if $1-q_1 < k$ it sells $k-1+q_1$ permits. Hence it chooses q_1 to maximize $p(k - 1 + q_1) - \alpha q_1^2$. Hence $q_1 = (p/2\alpha)$. Similarly, $q_2 = (p/2\beta)$. So demand for permits is $n[2 - (p/2)((1/\alpha) + (1/\beta)))]$.

(c) Market-clearing: $nk = n[2 - (p/2)((1/\alpha) + (1/\beta)))]$. Hence if k is given by the expression in (a), $p = \gamma$. So the equilibrium price equals the social marginal cost of emissions.

(iv) Handing out permits for free gives rents to firms. If proportional to emissions, it may give incentives for firms to increase emissions. Auctioning the permits means that the government captures the rents. Equivalent to a carbon tax in that case (though if govt has imperfect info about abatement costs the two schemes have different effects).

12. B3. (i) n people eat a meal together in a restaurant. Person i $(i = 1, 2, ..., n)$ has utility function $U_i(y_i, x_i) = y_i + a_i ln(x_i)$, where y_i is the amount of money i ends up with, x_i is the amount of food she eats, $ln(.)$ is the natural logarithm and $a_i > 0$. One unit of food costs $p > 0$. Assume that each person's initial wealth is large.

(a) For each person, derive an expression for the Pareto-efficient amount of food consumption.

(b) Suppose the diners know that, at the end of the meal, the bill is to be divided equally between them. How much will each person decide to consume?

(c) Discuss these results.

(ii) n people each attend a firework display. Person i $(i = 1, 2, ..., n)$ has utility function $U_i(y_i, X) = y_i + aln(X)$, where X is the total quantity of fireworks, $a > 0$ and otherwise notation is as in (i). A quantity X of fireworks costs pX.

(a) Derive an expression for the Pareto-efficient amount of fireworks and show that it corresponds to the Samuelson Rule.

(b) If each person individually buys fireworks for the display at unit cost p, what will the total amount of fireworks be?

(c) Discuss what your results in (ii)(a) and (ii)(b) suggest about private provision of public goods.

Ans. (i)(a) Quasi-linear, so choose $(x_1, x_2, .., x_n)$ to max sum of utilities $\sum_i Y_i - px_i + a_i ln(x_i)$, where Y_i is initial wealth. This gives $x_i = (a_i/p)$.

(b) i takes each x_j $(j \neq i)$ as given and solves $Max_{x_i} Y_i - (p/n)(x_i + \sum_{i \neq j} x_j) + a_i ln(x_i)$. This gives $x_i = (na_i/p)$.

(c) In each case, solution is increasing in a_i and decreasing in p. In (b), i over-consumes relative to the Pareto-efficient quantity. Negative externality: i imposes costs on the other diners by increasing consumption. Only takes into account a fraction n of the incremental cost. Should discuss difference between social and private costs. Charging each diner for what he or she consumes will result in the Pareto-efficient allocation.

(ii) (a) $max_X \sum_i Y_i - pX + naln(X)$ gives $X = (na/p)$. $|MRT|$ is p and (a/X) is $|MRS|$ for i.

(b) $max_{x_i} Y_i - p(x_i) + aln(x_i + \sum_{j \neq i} x_j)$ gives $X = (a/p)$.

(c) Under-provision of public good. Each ignores positive externality on others if X increases. Should discuss the possibility that public goods can in some circumstances be supplied efficiently by private providers. E.g. an entrepreneur might provide the firework display and find a way to exclude non-payers. In some cases, with non-excludable goods, it may be possible to bundle with other goods which can be charged for.

END OF PAPER