ECONOMICS TRIPOS PART I

MOCK EXAMINATION 2020

Paper 1 MICROECONOMICS - 3 HOUR EXAMINATION

Answer **ALL SIX** questions from Section A and **TWO** questions from Section B.

Section A and B will each carry 50% of the total marks for this paper.

Students are permitted to use an approved calculator.

SECTION A *Answer all six questions from this Section.*

1. (a) Give definitions of:

   (i) monotonic preferences;

   (ii) strictly convex preferences.

   (b) Suppose Ann has preferences between two divisible goods. It is known that one of Ann's indifference curves is described by the equation $x_2 = \sqrt{x_1}$, where x_1 and x_2 are the amounts of the first and second good, and $x_1, x_2 \geq 0$.

   (i) Is it possible for Ann's preferences to be monotonic? Explain your answer.

   (ii) Is it possible for Ann's preferences to be strictly convex? Explain your answer.

2. (a) Give a definition of first-degree price discrimination.

   (b) Suppose that a monopolist has cost function $c(q) = \frac{1}{100}q^2$. It engages in first-degree price discrimination, and its optimal choice of quantity is $q = 50$. One of its customers, Bob, has demand $D_B(p) = 5 - p$ for the monopolist's good. How much of the good does the monopolist sell to Bob, as it engages in first-degree price discrimination?

3. (a) Give definitions of:

   (i) a Giffen good;

   (ii) an inferior good.

   (b) Why is a Giffen good necessarily an inferior good?

   (c) Suppose Cindy has preferences between two goods. How many (i.e. 0, 1 or 2) of these goods can be inferior? Explain your answer.

4. "Walras' Law says that as long as all agents have convex preferences then, at equilibrium prices, the values of excess demands add to zero." Comment.

5. In a pure exchange economy with two agents, A and B, and two goods, x and y, agent A has 2 units of x and 1 of y while agent B has 2 units of y and 1 of x. The utility functions of A and B respectively are

$$u_A(x_A, y_A) = \min\{x_A, y_A\}$$

and

$$u_B(x_B, y_B) = x_B + y_B.$$

By a process of negotiation A and B agree on a Pareto-efficient final allocation. Find the set of allocations which they might agree on. Which of these allocations, if any, are competitive equilibrium allocations?

6. In a Robinson Crusoe economy, the production function is $f(L - \frac{1}{2})$ for $L \geq \frac{1}{2}$ and zero for $L < \frac{1}{2}$, where L is labour input and f is a strictly increasing, differentiable and strictly concave function such that $f(0) = 0$. Suppose that

$$f(L^* - \tfrac{1}{2}) < L^* f'(L^* - \tfrac{1}{2}),$$

where L^* is the labour input which maximizes Robinson's utility function subject to feasibility. Robinson has convex preferences. Sketch a diagram which illustrates the determination of L^*. Is there a competitive equilibrium of this economy? Explain why or why not.

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SECTION B *Answer two questions from this Section.*

7. Jane has the utility function

$$u_J(x_1, x_2) = x_1 + x_2,$$

where x_1 and x_2 are the amounts of the first and second good respectively. Jane's income is $m = 10$, the price of the first good is p and the price of the second good is 1.

   (a) Assume the price of the first good changes from $p' = 2$ to $p'' = \frac{1}{2}$. Find Jane's optimal consumption bundle both before and after the price change.

   (b) Find the equivalent and compensating variations, EV and CV, corresponding to the price change.

   (c) Draw a graph of Jane's optimal consumption of the first good as a function of p on the interval $[\frac{1}{2}, 2]$.

   (d) What is the change in Jane's consumer surplus, CS, corresponding to the price change?

   (e) Compare EV, CV and CS. How would the comparison change if, instead, the price changed from $p' = 2$ to $p''' = \frac{4}{3}$?

8. Paul lives for two periods, and his utility is given by

$$u_P(c_1, c_2) = \ln(c_1) + \ln(c_2),$$

where c_1 denotes consumption in period 1 and c_2 denotes consumption in period 2. Paul receives an income $x_1 = 12$ in period 1 and $x_2 = 5$ in period 2. The interest rate r is 25%.

   (a) Assuming Paul can save and borrow at the interest rate r, find his optimal choice of c_1 and c_2. Is he a lender or a borrower?

   (b) Now assume that Paul can spend some time in education to increase his human capital. His new income is $x_1 = 12 - H_1$ in period 1 and $x_2 = 5 + 5\sqrt{H_1}$ in period 2, where H_1 is time spent in education. Assuming he can still borrow and save at the interest rate r, find his optimal choice of H_1, c_1 and c_2. Is he a lender or a borrower?

   (c) Now assume that a different person, Mary, faces almost the same problem as in part (b). The only difference is that Mary has utility function $u_M(c_1, c_2) = c_1^{1/5} c_2^{4/5}$. How does Mary's optimal choice of education, H_1, compare with that of Paul?

9. Suppose that a firm has a production technology $f(K, L) = \min\{K, L\}$.
   (a) Does the firm have increasing, constant or decreasing returns to scale? Justify your answer.
   (b) Assume the total cost of hiring labor is $w(L) = 2L$. Assume the firm can rent capital only from factory A, at the total cost $r_1(K) = 2K^2$. Derive the cost function of the firm. If the firm can sell its good on a competitive market, what is its supply curve?
   (c) Now assume the firm can instead rent capital only from factory B, at the total cost
   $$r_2(K) = \left\{ \begin{array}{cc} 12 + K^2 & \text{if } K > 0 \\ 0 & \text{if } K = 0 \end{array} \right\}.$$
   Derive the firm's cost function, and its supply curve.
   (d) Now assume the firm can rent capital both from factory A and factory B. The total amount of capital, K, equals the sum of the amounts rented from the two factories. Derive the firm's cost function, and its supply curve.

10. Three agents (A, B, C) have to decide whether to acquire a public good which will give a benefit to each of them. The utility value of the good for agent i $(i = A, B, C)$ is $v_i \geq 0$.
   (a) Suppose that the good costs $3c$ $(c > 0)$. Each agent starts with an amount of money $\bar{m} > 3c$. If the good is acquired then agent i's utility is $v_i + m_i$ and, if the good is not acquired, then her utility is m_i. (In each case, m_i is the amount of money she ends up with.) Under what condition is it Pareto-inefficient not to buy the public good? Justify your answer.
   (b) Now suppose that the agents have no money and cannot borrow any, but they can choose to exert effort, which has a utility cost c. (Effort is binary, i.e. it is either exerted or not.) The public good is acquired if and only if all three exert effort. If it is acquired then agent i gets utility $v_i - c$ and, if it is not acquired, then she gets utility zero or $-c$ (depending on whether she exerted effort or not). Under what condition is not acquiring the good Pareto-inefficient?
   (c) In the model in part (a), suppose that the utility value of an agent is known only to that agent. Suppose they each announce their value for the good and the good is acquired if and only if it is efficient to acquire it, on the assumption that the agents told the truth. Consider the following two schemes. Scheme I: If it is acquired, they each pay c. If not, they pay nothing. Scheme II: If it is acquired, each agent i pays $(3c - \hat{v}_j - \hat{v}_k)$, where $\hat{v}_j$ and $\hat{v}_k$ are the announced values of the two agents other than i. If not, they pay nothing. For each scheme, is it in the interest of a given agent to reveal her true valuation, assuming the others do so? Explain your answer. Discuss the intuition for the result and the practicality of these schemes.

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11. Consider the following economy in which firms catch fish using labour as input. There is a large number of identical agents and one firm per agent. Each agent has an endowment of 1 unit of potential labour time and has utility function

$$U(f, l) = f^{\frac{1}{2}} l^{\frac{1}{2}},$$

where f is consumption of fish and l is leisure time.

Each firm is owned equally by all the agents and has production function

$$y_f = A\, L$$

where y_f is output of fish, L is labour input at the firm concerned and $A > 0$ is a constant.

(a) Define what is meant by a *competitive equilibrium* for this economy.

(b) Find the competitive equilibrium prices and allocation (assuming that all firms employ the same amount of labour).

(c) Assume now that, instead of a competitive allocation, there is a social planner who wants to choose an allocation so as to maximize a typical agent's utility, subject to each agent having the same bundle of fish and leisure. Write down the associated maximization problem and solve it. Comment on your result.

(d) Now suppose that the production function of a typical firm is $y_f = \alpha(\bar{L})\, L$, where $\bar{L}$ is the average labour input of all firms in the economy and α is a strictly positive function with a strictly negative derivative α'. Assume that a given firm is negligible in size in the sense that changing its input L does not affect $\bar{L}$.

(i) Find expressions for the competitive equilibrium prices and allocation.

(ii) Write down the social planner maximization problem corresponding to the one in part (c). Show that the competitive equilibrium allocation does not satisfy the first-order condition for this problem. Comment.

12. "Free trade benefits workers in poor countries but the added competition reduces the living standards of workers in rich countries." Discuss these claims in the context of a two-country, two-good Ricardian trade model in which each country has a single agent and one country has an absolute advantage over the other in both goods.

END OF PAPER