FACULTY OF ECONOMICS STUDY AIDS 2017

ECT1 Paper 2 Macroeconomics

The Faculty Board has agreed to release outline solutions to the 2017 examinations as a study aid for exam revision. They are abridged solutions, and not 'definitive', and should therefore not be considered as an exemplar for 'complete' answers.

Also note that the Faculty will not respond to any queries regarding these solutions.

SECTION A

1. Suppose that the job separation and finding rates in an economy for any given month are 2% and 18% respectively. What is the natural rate of unemployment? Explain how information about, as well as the number of, job vacancies may affect the natural rate of unemployment.

   ANSWER: The natural rate is

   $$u = \frac{s}{s+f} = \frac{0.02}{0.02+0.18} = 0.1 = 10\%.$$

   Both information (or lack of) and the number of vacancies affects the job finding rate. With more information and more vacancies, the finding rate goes up and long run unemployment goes down.

2. Consider an economy described by the Solow growth model in which aggregate production is given by a Cobb-Douglas production function. What relationship must the saving rate satisfy in order for the economy to be at the golden rule steady state? Explain your answer.

   ANSWER: The saving rate s must be equal to the capital share of income α. Note this relationship is always true for CD production function, with or withour population or technology growth. This is because (e.g. in the baseline version)

   $$k^* = \left(\frac{\delta}{s}\right)^{\frac{1}{\alpha-1}}$$

   and the GR relation is

   $$MPK = \delta \Longleftrightarrow \alpha\left(k^*\right)^{\alpha-1} = \delta \Longleftrightarrow \alpha\frac{\delta}{s} = \delta \Longleftrightarrow \alpha = s$$

   It reflects the best allocation between consumption and investment and says that, if all income made by renting capital (on aggregate) is saved (=invested), then the long run consumption per capita is the highest possible.

3. Suppose that there is a permanent increase in long run real aggregate income of an economy. What is the effect of this on the aggregate price level? What medium-run policy(ies) could the central bank implement to prevent this from happening?

   ANSWER: If real income increases then the demand for money will increase. From money market equilibrium, this will put downward pressure on aggregate prices, which will fall if no action is taken. To prevent a drop of aggregate prices, the monetary authority will have to shift the money supply to a higher level enough to meet the demand for money generated by the presence of

additional income in the economy. Some students may also suggest that the central bank may implement a change in interest rate policy.

4. Explain what is meant by '*crowding out*' in the context of fiscal policy analysis. In a small open economy that satisfies the Classical dichotomy, what component(s) of expenditure are crowded out by a tax-financed increase in government spending?

ANSWER: Crowding out refers to the argument that if the government consumes a larger share of resources, other components of expenditure will be forced to contract so as to keep total aggregate demand equal to aggregate supply. This arises in economies where output is supply-determined, as in the Classical model. Writing the GDP identity for an open-economy version of this model, and making the usual assumptions for the determinants of the different expenditure categories:

$$\overline{Y} = \overline{C} + c\left(\overline{Y} - T\right) + I\left(\overline{r}\right) + G + NX\left(\varepsilon\right)$$

where $\overline{r}$, the world real interest rate, is fixed. A tax-financed increase in G can be interpreted as raising T one-for-one with G. This reduces consumption by c per unit, with net exports falling by the remaining $(1-c)$, via an exchange-rate appreciation.

5. How do the following factors matter for the position and slope of the aggregate demand curve in inflation-output space?
   (a) The marginal propensity to save of consumers.
   (b) The long-run real interest rate.

ANSWER:
   (a) A higher MPS is equivalent to a lower MPC. This implies a steeper IS curve in (r, Y) space, and hence less of a reduction in equilibrium income for any given change in the policy rate. This implies a steeper AD curve, with point of pivot arbitrary.
   (b) If the central bank correctly identifies any changes in long-term real rates this should not matter: the AD curve should have the same slope, and always intersect $\left(\overline{Y}, \pi^T\right)$ for any $\overline{r}$. But a low long-term real rate can make the zero bound more likely to bind. This can be represented as a kink in the AD curve, which becomes vertical at some maximal attainable output level. This maximal level falls the lower is $\overline{r}$, holding constant inflation expectations.

6. Suppose that consumer spending in a fixed-price economy depends on long-run disposable income, according to the function $C = \gamma\left(\overline{Y}^e - \overline{T}^e\right)$, where $\overline{Y}^e$ and $\overline{T}^e$ are expected values for long-run income and taxation respectively, and $0 < \gamma < 1$. Assuming that the real interest rate is constant, what is the effect

on output, in the short run of:

(a) a temporary increase in government spending by ΔG?

(b) a permanent increase in government spending by ΔG?

ANSWER:

(a) This should raise short-run output by the amount of the spending increase ΔG, with no Keynesian multiplier effect. There may be a small offsetting effect through consumption from higher long-run taxation.

(b) This should raise output by the amount of the spending increase ΔG, but if the spending increase is permanent then taxes must rise by an equivalent (or near-equivalent) amount in the long run. This reduces consumption by an amount $\gamma \Delta G$, so that the overall impact on short-run output is $\Delta G\,(1 - \gamma)$.

SECTION B

7. Following the referendum for EU membership in June 2016, there was a significant drop of the nominal exchange rate of Euros per Pound (€/£), and in the last year it has been about 10% lower relative to where it has been in the year before June 2016.
   (a) Which of the two main price indeces, namely the GDP deflator and the CPI, do you expect to see reacting more to this drop in the exchange rate of €/£, and why?
   (b) Inflation in the Eurozone in the last two years has been very low and close to zero. Assuming that purchasing power parity (PPP) holds, how much would inflation in the UK be, following the recent Pound devaluation? To what extent is this number realistic? Explain your reasoning.

ANSWER: The question combines knowledge from different parts of the course, and links it to events relating to the UK economy.

(a) Give brief definitions of CPI and GDP deflator, highlighting the important difference between them that is relevant to this question, i.e. that CPI contains imported goods, while the deflator only prices of domestically produced goods. Both indeces will react, but CPI will probably react faster and by more, since all prices of goods imported from Europe will be affected directly. The GDP deflator will also respond to the devaluation, but only to the extend that the devaluation affects prices of imported intermediate goods used in domestic production.

(b) For this, we need the definition of the real exchange rate:

$$\varepsilon = \frac{eP}{P^*}$$

where ε is the real exchange rate, e is the nominal £/€ rate and P and P^* are prices in the UK and Eurozone respectively. Taking logs and differences we have that

$$\Delta \ln \varepsilon = \Delta \ln e + \Delta \ln P - \Delta \ln P^*.$$

PPP suggests that $\Delta \ln \varepsilon = 0$ always, therefore

$$\Delta \ln e + \Delta \ln P - \Delta \ln P^* = 0$$

i.e.

$$\frac{\Delta e}{e} = \pi^* - \pi$$

Given that $\frac{\Delta e}{e}$ is approximately 10% and inflation in the Eurozone has been about $\pi^* = 0$, the law of one price suggests that prices in the UK will rise by about 10%. This is clearly not realistic. The law of one price (and PPP) do not hold true generally because of goods/services that

are not tradeable, etc. In the particular scenario, of course, the rest of the world is not just the Eurozone, but also other countries from where UK imports/exports goods. Finally, this relationship does not show causality. In fact, the Bank of England actively intervenes to prevent this large increase in inflation from happening.

8. Consider the following production function

$$F(K,L) = \left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha) L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{1-\sigma}},$$

where K is capital stock, L is labour supply and $0 < \alpha < 1$, $\sigma > 0$ are parameters.

(a) Give an interpretation for the parameters α and σ. Does F exhibit increasing, constant or decreasing returns to scale? Let $k = K/L$ and $y = Y/L$. Derive the production function in per worker terms.

(b) Let $\alpha = 1/2$ and $\sigma = 2$. Suppose that $F(K,L)$ describes the aggregate production in an economy in which labour force is equal to the total constant population L, capital depreciates at a 10% rate per year, and households save a constant fraction of 10% of their income per year.
   i. Write the equation of capital accumulation and use it to demonstrate that this economy will have a unique steady state.
   ii. What is the steady state capital per worker k^*, output per worker y^*, and consumption per worker c^* in this case?

(c) Keep assuming that $\alpha = 1/2$ and $\sigma = 2$. Will this economy always have a steady state for any savings or depreciation rate? Explain.

ANSWER:

(a) The parameter α shows the share of each input to production. The parameter σ captures the degree of substitutability between inputs of production. The production function has constant returns to scale. This is because

$$\begin{aligned}
F(\lambda K, \lambda L) &= \left[\alpha (\lambda K)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(\lambda L)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{1-\sigma}} \\
&= \left[\lambda^{\frac{\sigma-1}{\sigma}} \alpha K^{\frac{\sigma-1}{\sigma}} + \lambda^{\frac{\sigma-1}{\sigma}} (1-\alpha) L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{1-\sigma}} \\
&= \left(\lambda^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha) L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{1-\sigma}} \\
&= \lambda \left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha) L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{1-\sigma}} \\
&= \lambda F(K,L).
\end{aligned}$$

The production function in per capita terms is:

$$f(k) = \left[\alpha k^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\right]^{\frac{\sigma}{1-\sigma}}$$

(b) Since there is no population growth, we have that $(\Delta K)/L = \Delta(K/L) = \Delta k$ and therefore capital accumulates according to

$$\Delta k = sf(k) - \delta k.$$

For the given parameter values this becomes

$$\Delta k = sf(k) - \delta k = 0.1\left(\frac{1}{2}k^{\frac{1}{2}} + \frac{1}{2}\right)^2 - 0.1k.$$

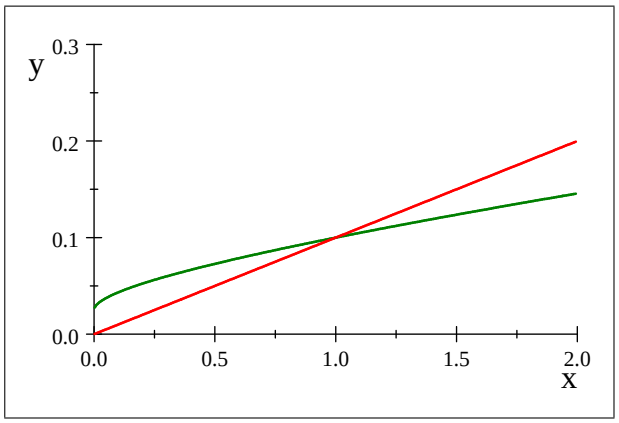
The economy has a steady state if the equation $\Delta k = 0$ has a solution which the economy converges to. First we solve

$$\begin{aligned} 0.1\left(\frac{1}{2}k^{\frac{1}{2}} + \frac{1}{2}\right)^2 - 0.1k &= 0 \\ \left(\frac{1}{2}k^{\frac{1}{2}} + \frac{1}{2}\right)^2 - k &= 0 \\ 2k^{\frac{1}{2}} - 3k + 1 &= 0 \end{aligned}$$

To solve this, set $x = k^{1/2}$, then we solve

$$2x - 3x^2 + 1 = 0$$

which has two solutions, namely $x = 1, -\frac{1}{3}$. Since $x = \sqrt{k}$, the negative solution is not relevant, i.e. there is only one (unique) solution to the equation $\boxed{k^* = 1}$. This is indeed a steady state to which the economy converges to, because whenever $k < k^*$ we have that $sf(k) > \delta k$, i.e. investment is more than depreciation and capital accumulates. Similarly, when $k > k^*$, depreciation is higher and capital decumulates towards k^*. See graph below. Also $\boxed{y^* = 1, \quad c^* = 0.9.}$



(c) To find a steady state, the equation to solve is

$$\begin{aligned} \Delta k &= 0 \Longleftrightarrow s\left(\frac{1}{2}k^{\frac{1}{2}} + \frac{1}{2}\right)^2 - \delta k = 0 \Longleftrightarrow \\ \left(1 - \frac{4\delta}{s}\right)k + 2\sqrt{k} + 1 &= 0 \Longleftrightarrow \left(1 - \frac{4\delta}{s}\right)x^2 + 2x + 1 = 0 \qquad (1) \end{aligned}$$

where we have set $x = \sqrt{k}$. For the transformed equation we have,

$$\sqrt{4 - 4\left(1 - \frac{4\delta}{s}\right)} = 4\sqrt{\frac{\delta}{s}} > 0$$

so that (1) always has real solutions. If

$$1 - \frac{4\delta}{s} > 0$$

then its solutions are always negative, and there will be *no steady state*, since we need to have $k^* = \sqrt{x^*}$. This condition is satisfied for large enough savings rate, take e.g. $s = 0.5$, then one can easily check that capital will be increasing perpetually over time. The intuition is that because labour and capital are substitutable in production, whenever capital is low, production (output) with the help of labour can replenish it, if the saving rate is high enough (relative to depreciation). For the same reason, capital can keep growing if savings rate is high enough.

Notes for marking: Students that answer parts (a)-(b) well, should be at the high 2.I or low 1. Part (c) is harder. Students may answer this part of the question *mathematically*, with the *use of a graph* or just with their *intuition*; as long as the answer is correct either way is acceptable, and this would guarantee the solid 1st.

9. What is the role of banks in the monetary system? Describe a simple model of money supply and use it to explain to what extent banks and the monetary authority determine money supply in an economy.

ANSWER: Straight, textbook material, chapter 4 of Mankiw. Banks are financial intermediaries that transfer funds from savers to borrowers. This creates money. The monetary authority issues currency, controls nominal interest rates via open market operations and determines capital and reserve requirements for commercial banks. The model to be used starts with the assumption that Money Supply = Currency + Deposits and Monetary Base = Currency + Reserves, i.e.

$$\begin{aligned} M &= C + D \\ B &= C + R \end{aligned}$$

and therefore

$$M = \frac{C/D + 1}{C/D + R/D} B = mB$$

where

$$m = \frac{cr + 1}{cr + rr}$$

is the money multiplier. The banks and monetary authority can control (subject of course to exogenous shocks) the monetary base B as well as the reserve deposit ratio rr, but not the currency deposit ratio, which depends on individuals' preferences and other economic conditions.

10. The economy of Zenland has a large number of firms, each responsible for setting the price of their products. The ideal price for an individual firm to set, p^*, depends on the economy-wide aggregate price level, P, and real marginal costs, c, as follows:

$$p^* = P + \lambda (c - \overline{c}),$$

where $\lambda > 0$ is a parameter and $\overline{c}$ is the expected value for c. Marginal costs, in turn, relate to the economy's aggregate output gap $\left(Y - \overline{Y}\right)$ according to the following linear relationship:

$$c = \overline{c} + \gamma \left(Y - \overline{Y}\right),$$

for some parameter $\gamma > 0$. Before observing the true marginal cost c, all firms make an initial choice for the price that they plan to set, which they equate to their expected value of p^*.

(a) Let the fraction of firms that revise their price be a parameter $0 < \theta < 1$. Show that the aggregate supply curve is of the linear form:

$$\pi = \pi^e + \alpha \left(Y - \overline{Y}\right),$$

where $\pi = P - P_{-1}$ is the inflation rate, π^e is expected inflation, and the slope α depends on parameters θ, λ and γ.

(b) Now suppose that the fraction of firms revising their price is given by:

$$\frac{(c - \overline{c})^2}{\phi + (c - \overline{c})^2},$$

where $\phi > 0$ is a parameter.

i. Provide a brief interpretation of this expression.

ii. Derive an expression for the aggregate supply curve in this case, and sketch it in inflation-output space.

(c) Discuss the implications of your result in (b) for the sensitivity of output and inflation to demand shocks.

ANSWER:

(a) This is textbook material, covered in detail in lectures. We have:

$$P = \theta p^* + (1 - \theta) P^e$$

where P^e is expected inflation. Hence:

$$\begin{aligned} P &= P^e + \frac{\theta\lambda}{1-\theta}(c - \overline{c}) \\ &= P^e + \frac{\theta\lambda\gamma}{1-\theta}\left(Y - \overline{Y}\right) \\ \pi &= \pi^e + \frac{\theta\lambda\gamma}{1-\theta}\left(Y - \overline{Y}\right) \end{aligned}$$

which is the form required, with $\alpha := \frac{\theta\lambda\gamma}{1-\theta}$

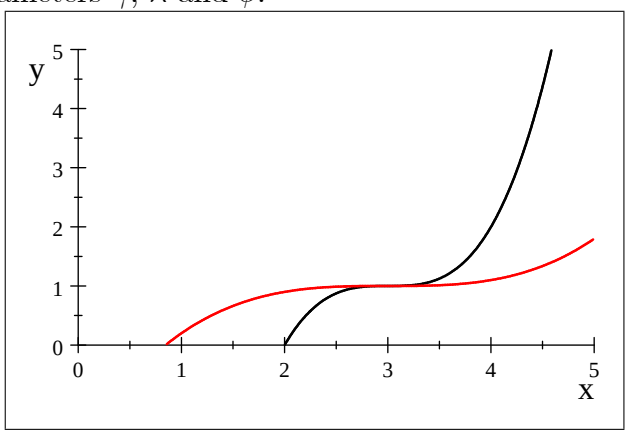
(b)

i. The fraction of firms resetting their price is increasing in the deviation of marginal costs from their expected value. This is a way to capture the heuristic idea that observed price stickiness should depend on the relative marginal benefits to a firm from resetting prices. When marginal costs deviate little from their expected value, the benefits to revising price are likely to be small. With a large departure, revision becomes more desirable.

ii. Using the given expression to sub out for θ in the Phillips curve derived in (a), we have:

$$\begin{aligned} \pi &= \pi^e + \frac{(c - \overline{c})^2}{\phi}\lambda\gamma\left(Y - \overline{Y}\right) \\ &= \pi^e + \frac{\lambda\gamma^3}{\phi}\left(Y - \overline{Y}\right)^3 \end{aligned}$$

Hence a cubic, with point of inflection at $\left(\overline{Y}, \pi^e\right)$ and slope controlled by the parameters γ, λ and ϕ:



(c) The SRAS curve is upward-sloped throughout, but extremely flat in the neighbourhood of full-capacity output and steeper elsewhere. This means that small demand shocks are likely to have a larger relative impact on output versus inflation than larger demand shocks in the short run. Intuitively, price-setters may be content to allow small output variations to absorb minor fluctuations in economic conditions, but for more severe

disturbances they decide to reset their prices, and so the aggregate level of output is far less elastic.

Notes for marking: Part (a) is easy here, whereas (b) is a little different from what the students have already seen. Most should be able to solve out for the correct expression, and first-class marks should only be awarded to those additionally able to interpret the model correctly in (b)i and (c).

11. A small open economy with fixed prices and a fixed exhange rate is characterised by the following aggregate relationships:

$$\begin{aligned} C &= 100 + 0.7\,(Y - T)\,, \\ I &= 120 - 10r, \\ G &= 150, \\ T &= 150, \\ NX &= 160 - 0.1Y - 50e, \\ e &= \overline{e}, \end{aligned}$$

where Y is aggegate income, C is consumption, I is investment, G is government spending, T is taxation, NX is net exports, e is the nominal exchange rate and r is the real interest rate. Net exports are assumed to equal the current account balance throughout.

(a) Suppose first that capital is perfectly mobile internationally, with a world real interest rate $r^w = 2$. Derive expressions for the equilibrium income and net exports in terms of $\overline{e}$. For what values of $\overline{e}$ does international capital flow out of this economy?

(b) The central bank decides that the income level in (a) is too low. It bans private-sector capital flows, and chooses r to maximise income, subject to a restriction that the official sector cannot borrow internationally.
   i. Solve for the value of Y that is chosen, as a function of $\overline{e}$. For what values of $\overline{e}$ will the strategy be successful?
   ii. Derive the relationship between the chosen value of r and $\overline{e}$, and explain its form intuitively.

(c) Assess the different policy options that are available in the event that the output level in (b) remains too low.

ANSWER:

(a) Straightforward algebra:

$$\begin{aligned} Y &= C + I + G + NX \\ &= 100 + 0.7\,(Y - 150) + 120 - 20 + 150 + 160 - 0.1Y - 50\overline{e} \\ 0.4Y &= 405 - 50\overline{e} \\ Y &= 1012.5 - 125\overline{e} \\ NX &= 160 - 0.1\,[1012.5 - 125\overline{e}] - 50\overline{e} \\ &= 58.75 - 37.5\overline{e} \end{aligned}$$

Capital outflows will occur if net exports are positive. This happens when:

$$\begin{aligned} 58.75 - 37.5\overline{e} &> 0 \\ \overline{e} &< \frac{58.75}{37.5} = \frac{47}{30} \end{aligned}$$

For all larger values of $\overline{e}$ there will be capital outflows.

(b)

i. The current account (net exports) is worsening in Y, so the central bank will increase output until net exports equal zero. This implies:

$$Y = 1600 - 500\overline{e}$$

This will be greater than the level of output in (a) iff:

$$\begin{aligned} 1012.5 - 125\overline{e} &< 1600 - 500\overline{e} \\ 375\overline{e} &< 587.5 \\ \overline{e} &< \frac{47}{30} \end{aligned}$$

which is exactly the case consistent with a trade surplus under free capital flows. Economies that start with a trade surplus are able to pursue expansionary monetary policy with capital controls, up to the point where trade is balanced.

ii. Inserting the previous expression into the GDP identity gives:

$$\begin{aligned} 1600 - 500\overline{e} &= \frac{10}{4}\,[265 - 10r] \\ &= 662.5 - 25r \\ 937.5 + 25r &= 500\overline{e} \\ r &= 20\overline{e} - 37.5 \end{aligned}$$

A more appreciated exchange rate implies a higher real interest rate in order to prevent international borrowing from being required. This is because an appreciated e.r. worsens the trade balance, and this can only be offset by a lower level of output – and hence import demand.

(c) The most obvious solution is to devalue the exchange rate. This would raise the level of output for any r, as desired. But good answers should point out that if this sort of devaluation is anticipated ex-ante, it is likely to imply a risk premium on international borrowing prior to the introduction of capital controls. Thus there may be an incentive for governments to bind their hands, ruling out devaluations ex-ante (for instance, through membership of a common currency area). Expansionary fiscal policy is an obvious alternative to consider when stimulating output. In this case it will stimulate output without capital controls (classic Mundell-Fleming argument), but will worsen the trade balance and may come at the risk of longer-term sustainability issues. Relative to the eqm with controls fiscal policy cannot help, since output is already at the maximum value consistent with no international borrowing.

Notes for marking: Part (c) should be used heavily to distinguish among students with otherwise strong answers. A good, clear discussion is necessary for first-class marks.

12. What policy conclusions should be drawn from the observation that monetary policy only affects the economy after 'long and variable lags'?

ANSWER:

A number of points could be made in response to this question. The well-known quote comes from Friedman, and so an obvious starting point is his view that monetary policy should be conducted 'passively' according to simple money growth rules because of such lags. The students have been introduced to the Friedman rule (setting the nominal interest rate to zero), and a good answer could then explain why this particular passive rule might be desirable conditional on a passive policy strategy. In addition, it would help to provide clarity on exactly what the 'lags' in the question are – the usual distinction being between 'inside' and 'outside' lags.

Against this is the consensus view that monetary activism is both possible and desirable. The 'possible' part requires an assessment of whether the lags in the question are really so significant. Inside lags may be far less significant when large, easily-identifiable shocks hit the economy, with the UK referendum result (and swift monetary response to it) providing a neat recent example. Outside lags are the source of much dispute in the literature, because the precise effects of monetary policy are so hard to identify. Most macroeconomists believe that activist monetary policy played an important role in producing the Great Moderation, for instance. If it is *possible* to pursue an activist policy, it is fairly easy to make a case that it is also desirable, even if the ideal form for the policy rule could be the source of debate.

Notes for marking: Clearly there is much scope for variability in structure etc here, but good answers should certainly frame the central debate as between 'passive' and 'activist' approaches.

END OF PAPER