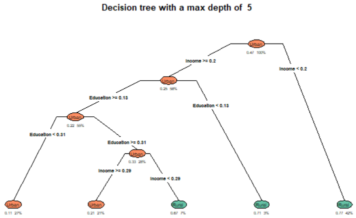
Machine Learning for Official Statistics & SDGs

Decision Trees

UNITED NATIONS SIAP
Statistical Institute for Asia and the Pacific

[Decision Trees]

Trees are method for classification or regression analysis.
↪ Focus on classification



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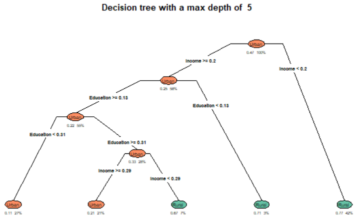
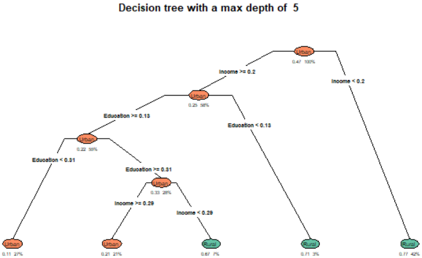


- Trees split the space into non-overlapping spaces
- Used to assign/predict a *class* following conditions
- Optimally, final classification is homogenous

[What's in a tree?]

Trees have a very simple structure and are easy to understand:



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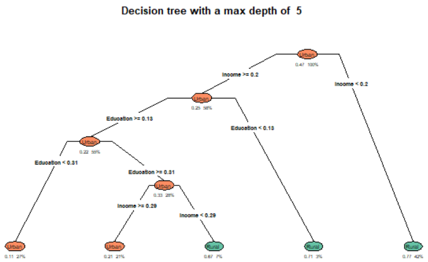

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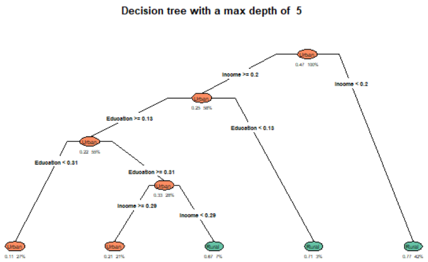

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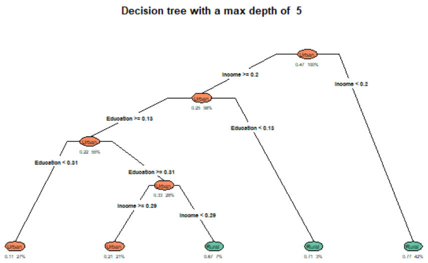
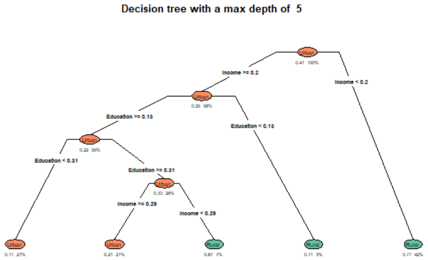


Trees have:

- **Nodes** where splitting decisions are done
- **Branches** following conditions
- **Leaves** are terminal nodes of the classification

[What's in a tree?]

How to build a tree?



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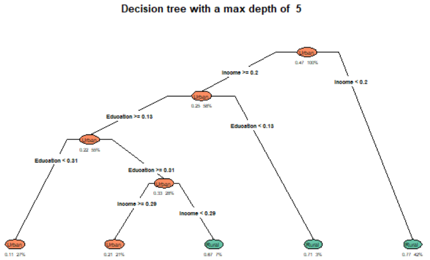
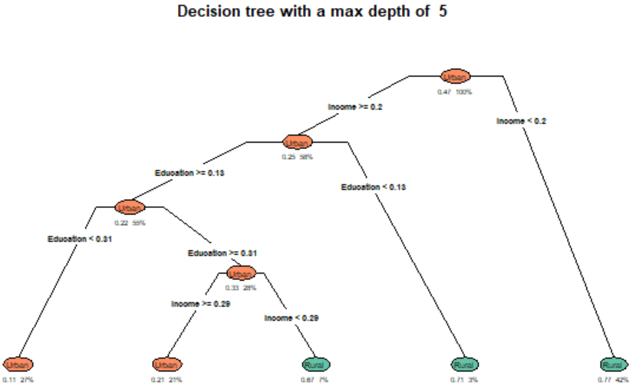


- Trees are based on recursive binary splits
- Each node uses a threshold on a variable
- Each node separates the observations in two sets

[Example on a simple tree]

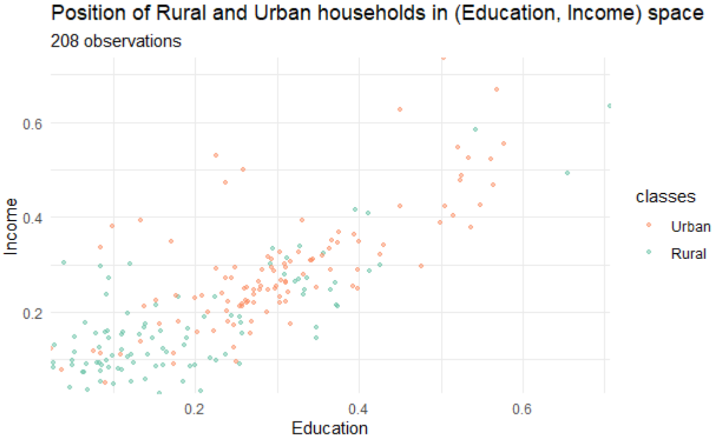
Let us see how this tree is constructed:



4 nodes leading to final leaves $\hookrightarrow$ Depth = 5

[Example on a simple tree]

The problem is a 2D space



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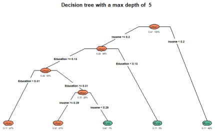

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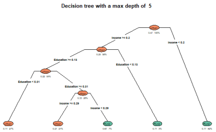
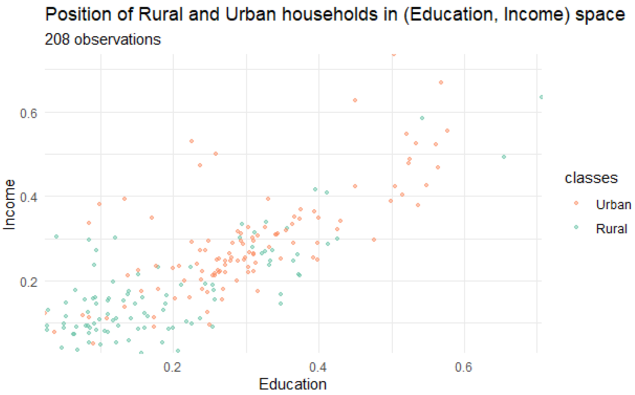
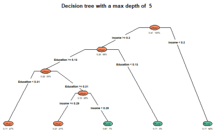
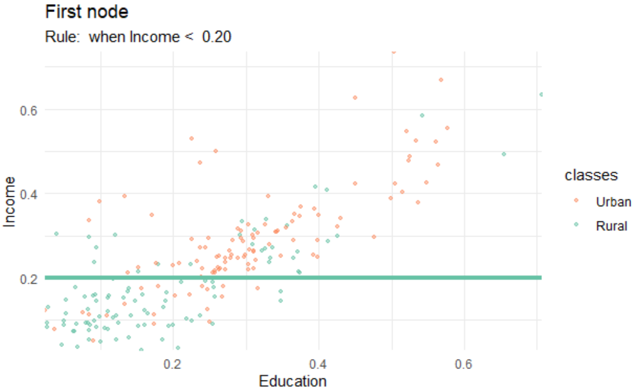




How to split the (Education,Income) space?

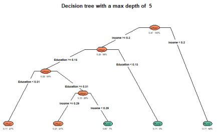
[Example on a simple tree]

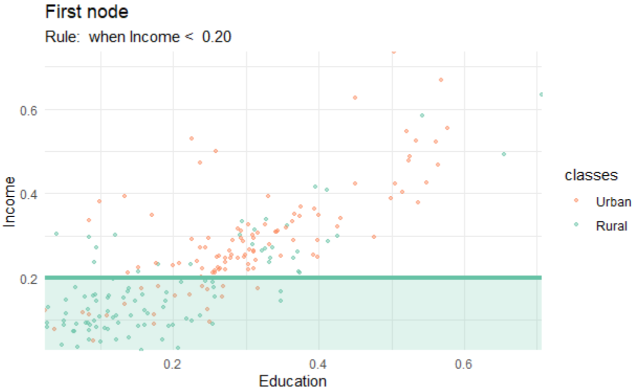




The first boundary decision line

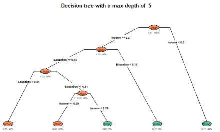
[Example on a simple tree]

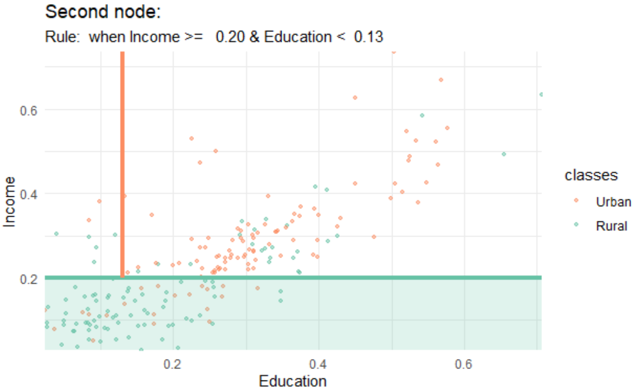




The space below the line is classified as rural

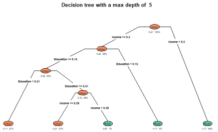
[EXAMPLE ON A SIMPLE TREE]

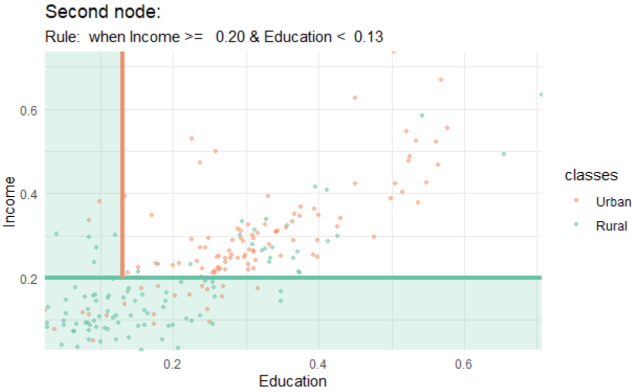




Second boundary decision line

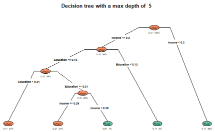
[EXAMPLE ON A SIMPLE TREE]

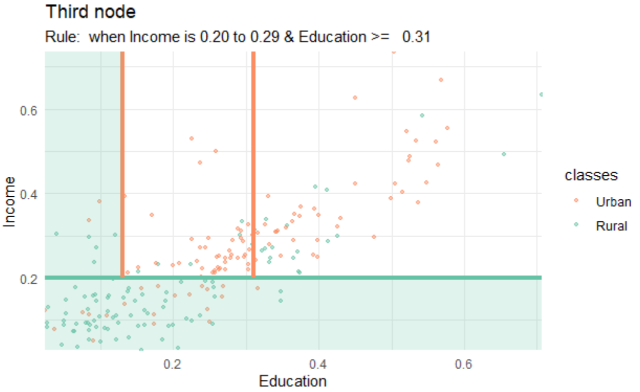




The space on the left of the line is classified as rural

[Example on a simple tree]





Third boundary decision line

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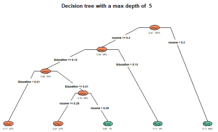
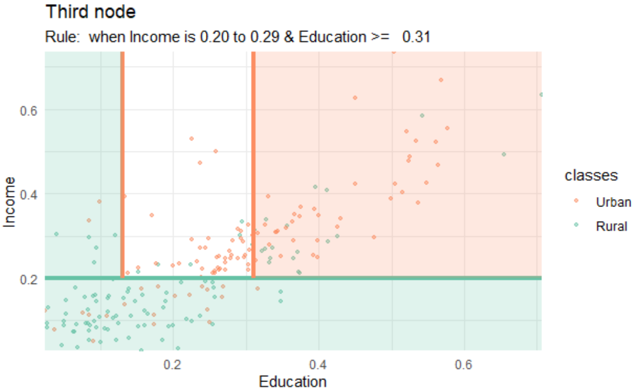
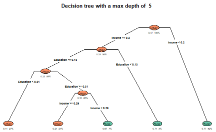
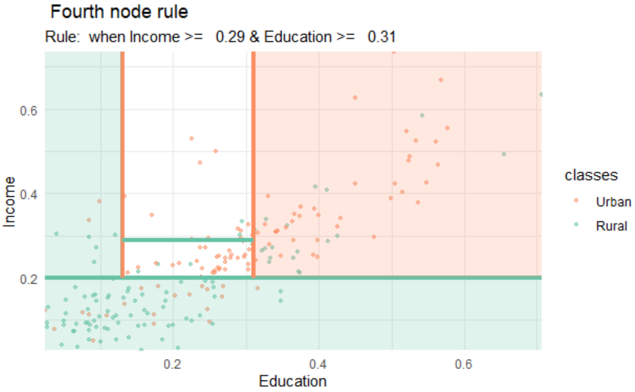




The space on the right of the line is classified as Urban

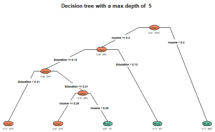
[Example on a simple tree]

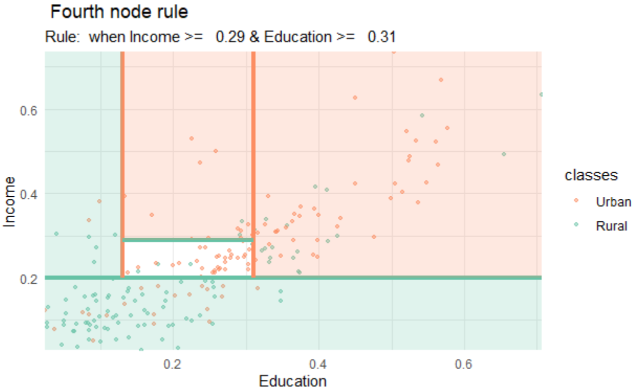




Fourth boundary decision line

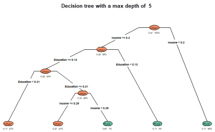
[Example on a simple tree]

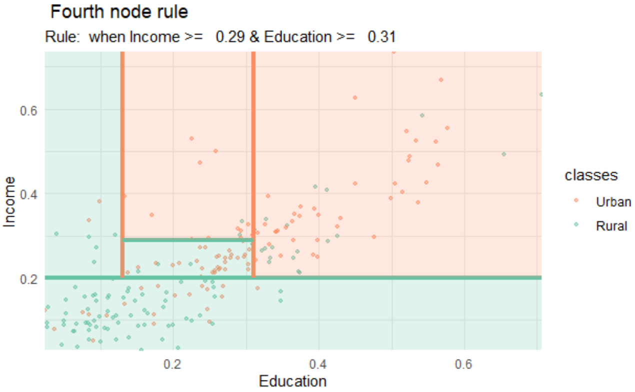




The space above the line is classified as Urban

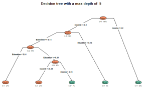
[Example on a simple tree]





Finally, the remaining space is classified as rural

[How to build a tree?]



We need several tools to build a tree:

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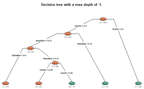

We need several tools to build a tree:

- A method to choose the decision variable (one per node)

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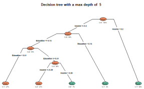

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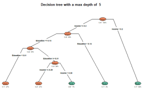

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- A criterion to measure the quality of each split

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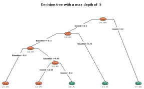

We need several tools to build a tree:

- A method to choose the decision variable (one per node)
- A criterion to define best threshold
- A criterion to measure the quality of each split
- A way to decide when to stop (the terminal node becomes a leaf)

[Tools to build a tree]

At each node, one can measure the *purity* of each split

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- The Gini coefficient measures the purity in each node κ

$$D_\kappa = \sum_{m=1}^{M} \widehat{p}_{m\kappa}(1 - \widehat{p}_{m\kappa})$$

where $\widehat{p}_{m\kappa}$ is the proportion of class m in node κ.

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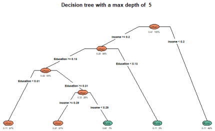
- We expect that there is an Information gain from the splitting

$$\mathit{Information\ Gain} = \mathit{Entropy}_{\,\mathit{Before}} - \mathit{Entropy}_{\,\mathit{After}}$$

[Example on a simple tree]



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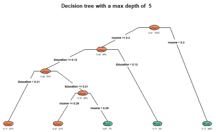
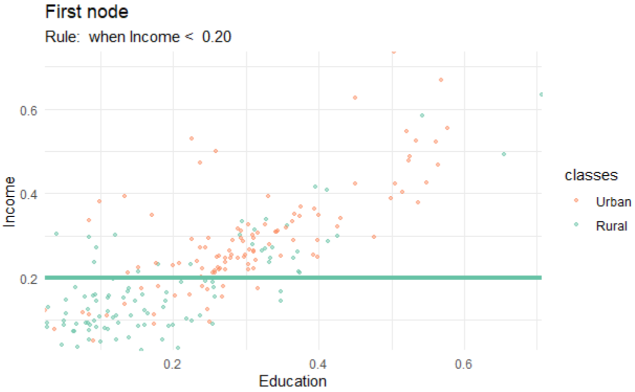
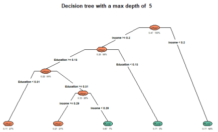
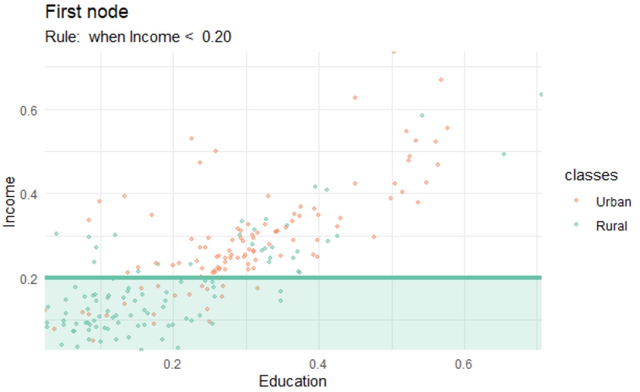




First the boundary decision line

[Example on a simple tree]





The space below the line is classified as rural

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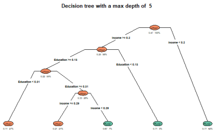

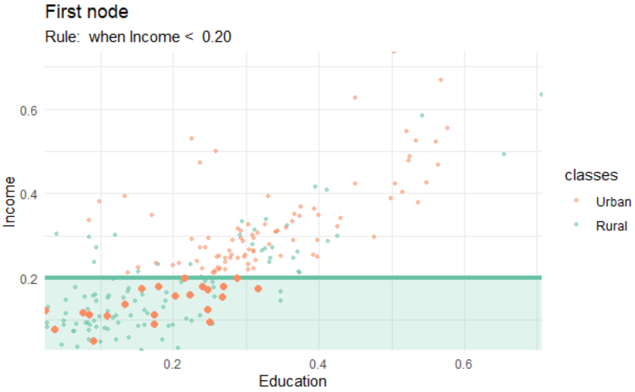

Some "Urban" are classified as rural ↪ Impurity

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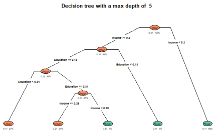

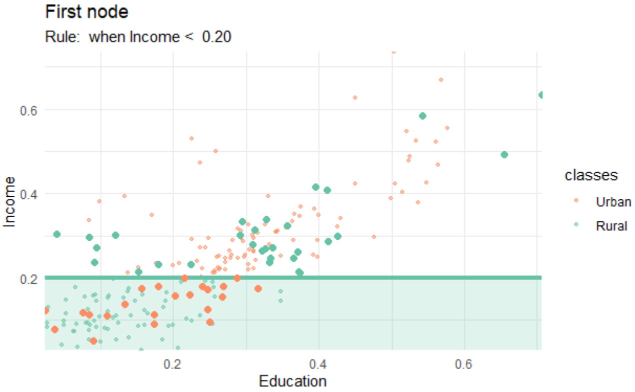

Some "Rural" are classified as Urban $\hookrightarrow$ Impurity

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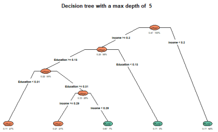
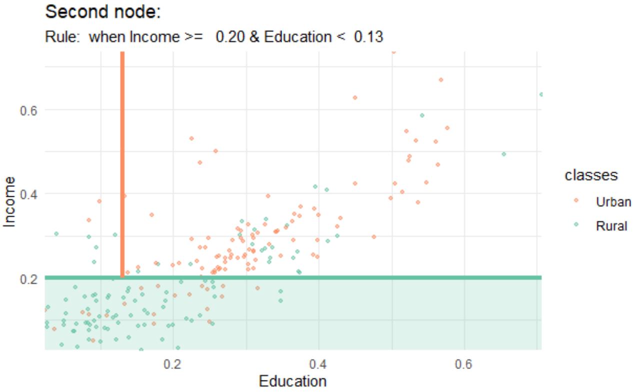
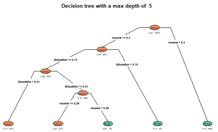
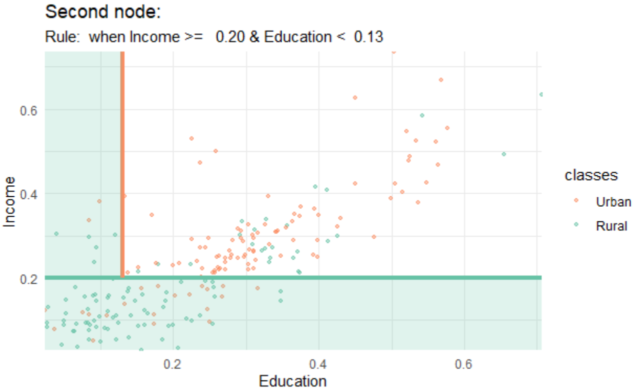




Second boundary decision line

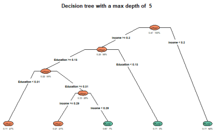
[Example on a simple tree]

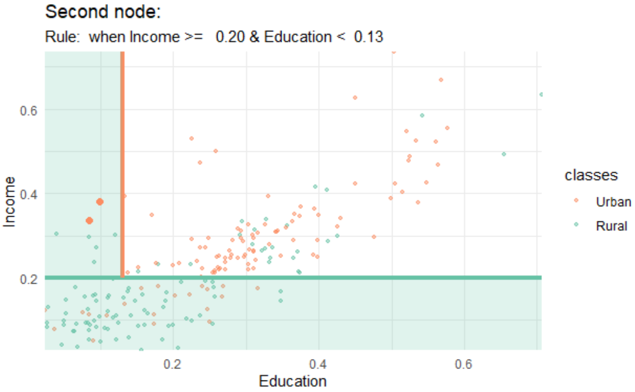




The space on the left of the line is classified as rural

[Example on a simple tree]





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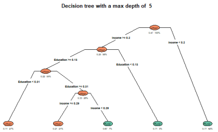

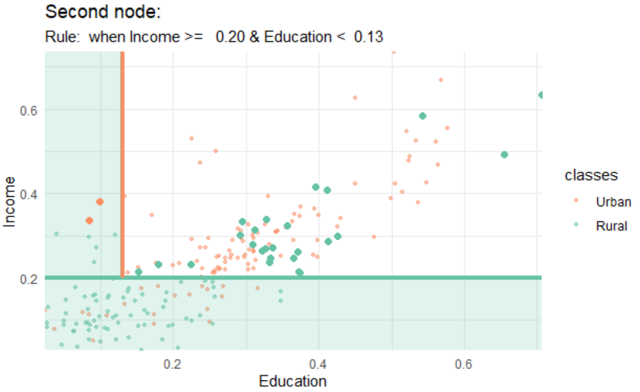

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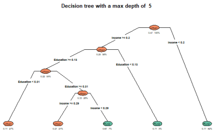
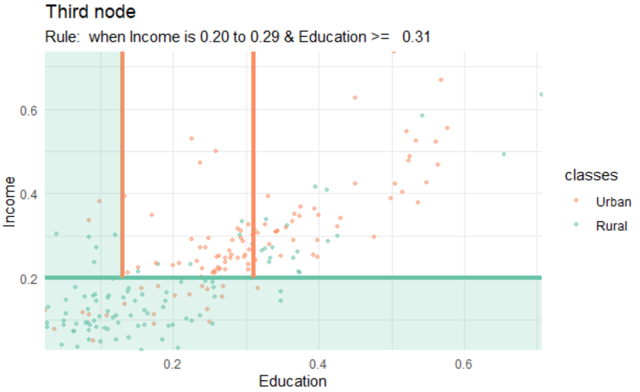
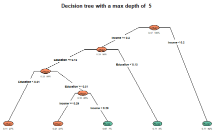
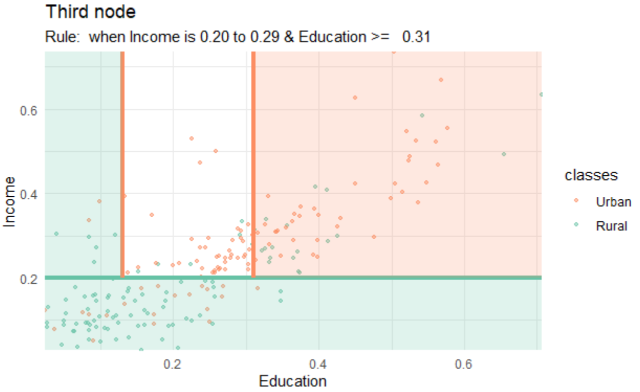




Third boundary decision line

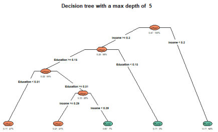
[Example on a simple tree]

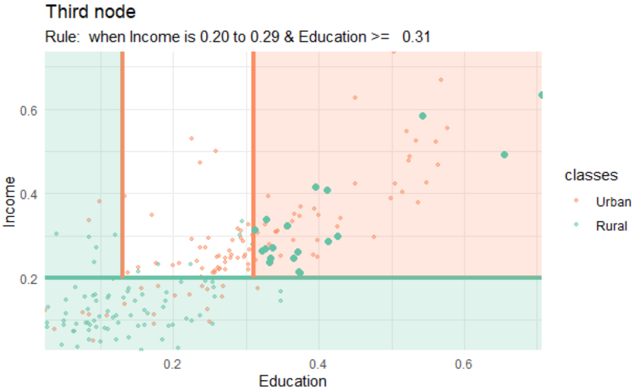




The space on the right of the line is classified as Urban

[Example on a simple tree]





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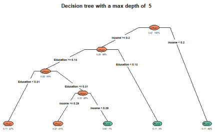

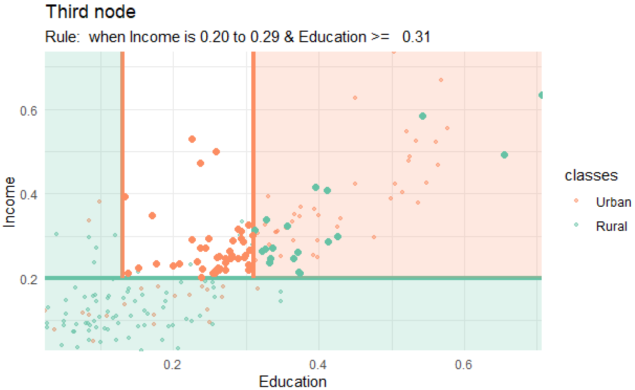

"Urban" seem well classified in the remaining space

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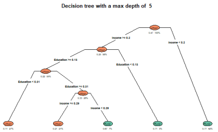
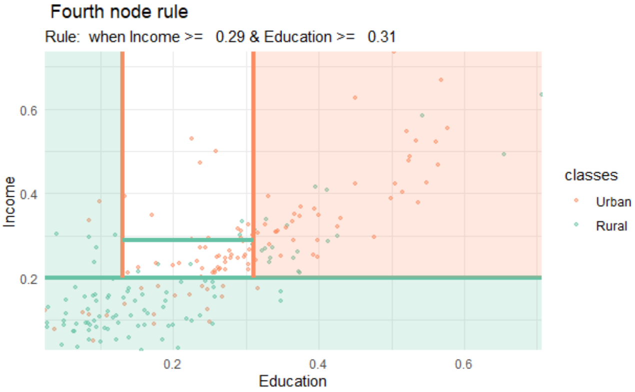
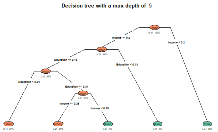
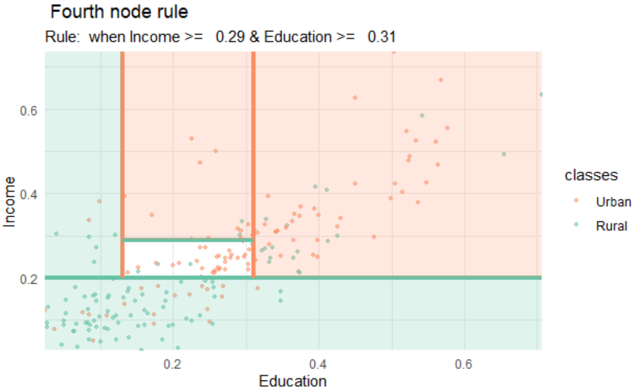




Fourth boundary decision line

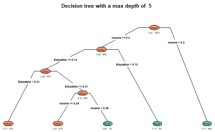
[Example on a simple tree]

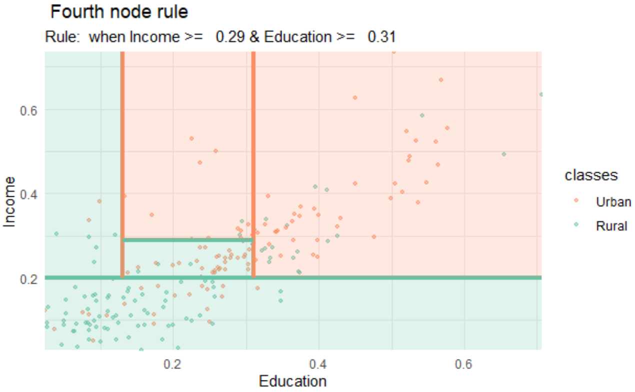




The space above the line is classified as Urban

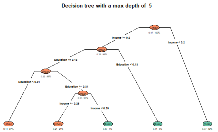
[Example on a simple tree]

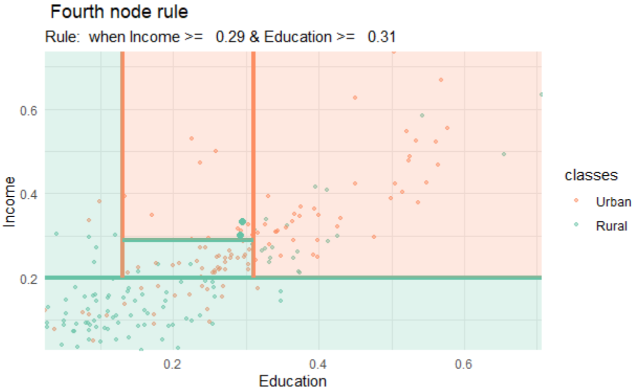




Finally, the remaining space is classified as rural

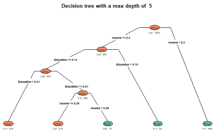
[Example on a simple tree]

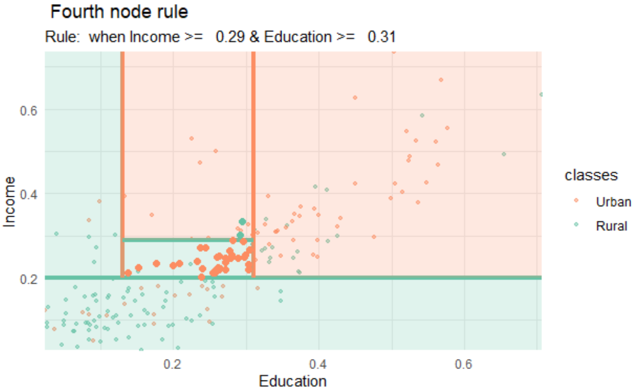




Few "Rural" are classified as Urban

[Example on a simple tree]





Many "Urban" are classified as Rural

[How to build a tree?]

The construction is based on recursive binary splits

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- The goal is to increase the quality of the classification at the each stage

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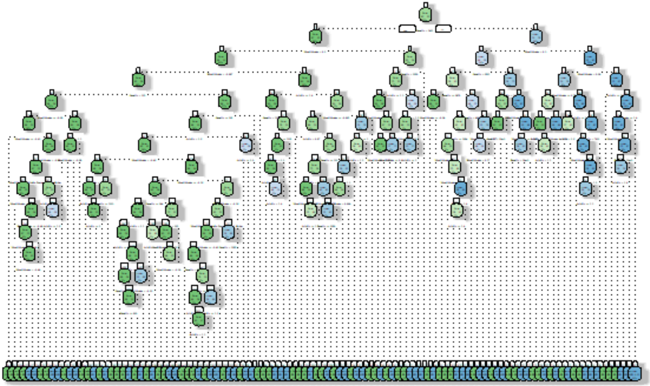
  - ...

[Trees can be complex]

Trees can decompose the space in very specific zones.
$\hookrightarrow$ Example with the full set of variables



Decision tree with no constrains

[Selecting the **Depth** of a tree]

Using CV, we can select the *maximum depth* parameter

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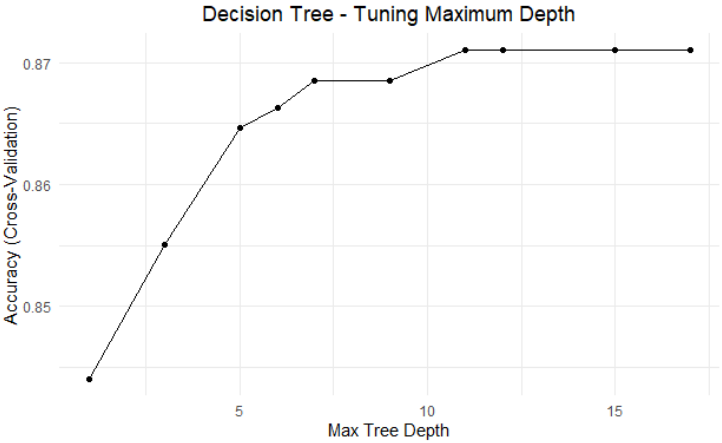
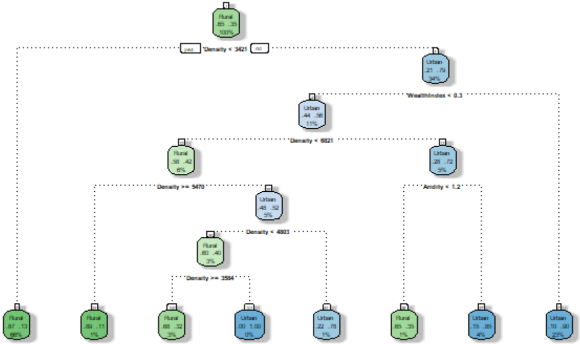


[Selecting the **Depth** of a tree]

[SELECTING THE DEPTH OF A TREE]

The resulting tree

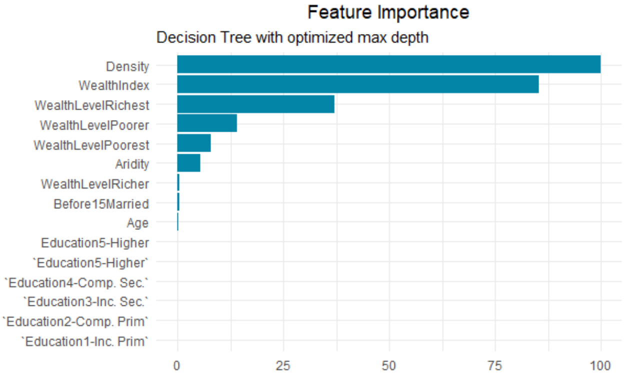


Tree Selected (max depth = 11)

Tree with optimal depth

[Selecting the Depth of a tree]

The resulting tree



Feature importance (also confusion matrix, kappa, etc..)

[Selecting the **Complexity** of a tree]

The complexity of a tree is a parameter C_p governing the trade-off between tree size $|T|$ and its overall accuracy $D(T)$:

$$D_{C_p}(T) = D(T) + C_p \cdot |T|$$

- $D(T) = \sum_{\kappa=1}^{K} D_\kappa$: the total *impurity* of the tree

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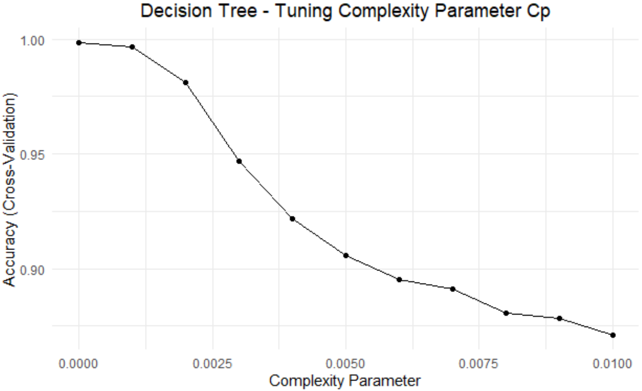
$\hookrightarrow$ A value of $C_p = 1$ only **one** terminal (and initial) node.

[Selecting the **Complexity** of a tree]

[SELECTING THE **COMPLEXITY** OF A TREE]

The result:



Grid search

[Selecting the Complexity of a tree]

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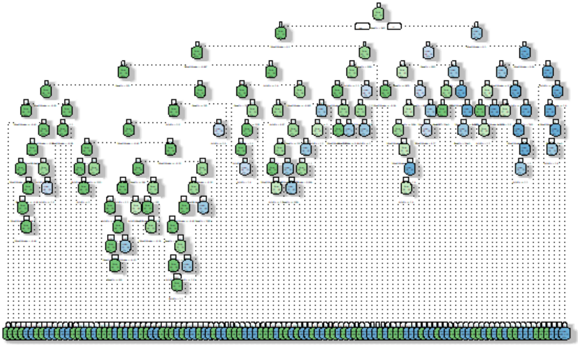

Tree with optimized Cp

Final tree with optimal complexity parameter

[Selecting the Complexity of a tree]

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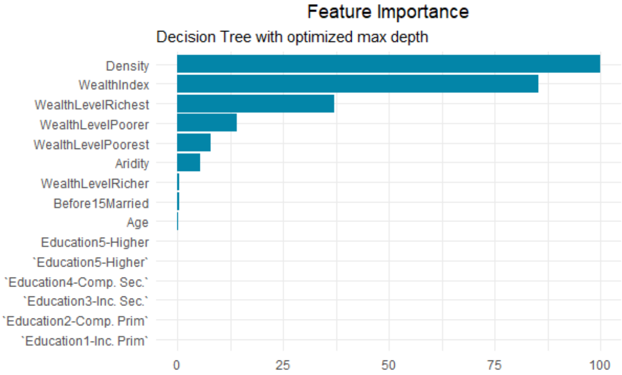

Feature importance (also confusion matrix, kappa, etc..)

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$\hookrightarrow$ Remove least important nodes

[Pruned tree]

After "*pruning*":

[PRUNED TREE]

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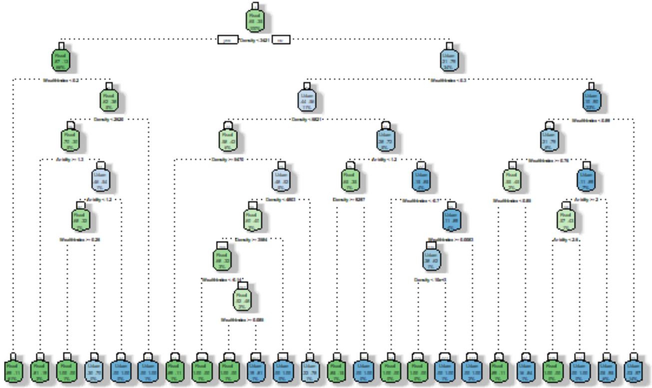

Pruned tree

[PRUNED TREE]

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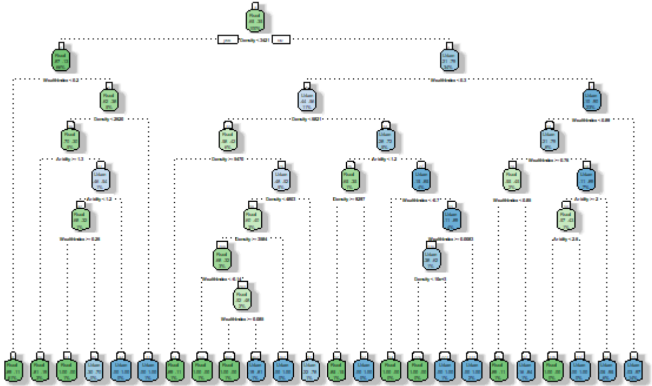

Pruned tree

↪ Easier to interpret & no loss of accuracy

[Quiz time]

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[Takeaways]

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↪ There are powerful methods using many trees...