Machine Learning for Official Statistics and SDGs

Third Live Lecture (Webinar):

Starts in **15** minutes

Christophe Bontemps, UN SIAP

UNITED NATIONS SIAP
Statistical Institute for Asia and the Pacific

Machine Learning for Official Statistics and SDGs

Third Live Lecture (Webinar):

Starts in **10** minutes

Christophe Bontemps, UN SIAP

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Machine Learning for Official Statistics and SDGs

Third Live Lecture (Webinar): Starts in **5** minutes

Christophe Bontemps, UN SIAP

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Machine Learning for Official Statistics and SDGs

On Regression

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[- Reminder -]

- Mute yourself always!

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- Ask questions in the chat

[- Agenda -]

- Introduction

[- AGENDA -]

- Introduction
- Regression (*in a Machine learning Framework*)

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- Next week

[Linear Regression]

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  Expressed as:

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  with possibly many regressors x_j
- βs are solutions of:

$$\min_{\beta} \frac{1}{n} \sum_{i=1}^{n} \left(y_i - \beta_0 - x_i'\beta\right)^2$$

[Linear Regression: Centering variables]

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Except: α_0 is the mean of y **if all x_j are equal their mean**

[Linear Regression: Scaling]

We can also scale each variable by its own standard deviation to obtain

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- The goal is to have variables and coefficients that are comparable

[Linear Regression: Example]

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- Example on a regression model with few variables

	Est.	S.E.	t val.	p
(Intercept)	4.390	0.136	32.204	0.000
Trans	-0.002	0.001	-1.603	0.110
HighTrans	0.013	0.003	3.945	0.000
Checks	0.008	0.005	1.534	0.126
Years	0.096	0.008	11.579	0.000

Initial regression with original values

[Linear Regression: Example]

- Example on a regression model with few variables

	Est.	S.E.	t val.	p
(Intercept)	5.927	0.039	150.243	0.000
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Regression with centred variables

[Linear Regression: Example]

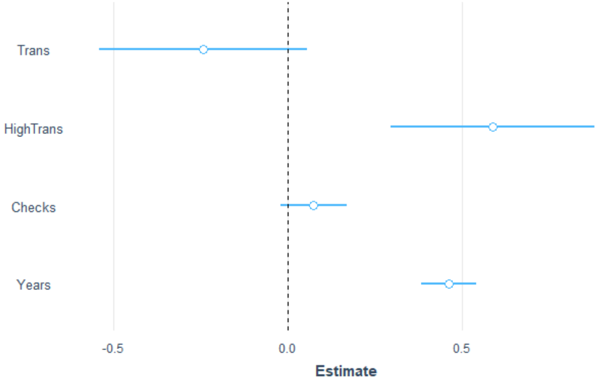
- Example on a regression model with few variables

	Est.	S.E.	t val.	p
(Intercept)	5.927	0.039	150.243	0.000
Trans	-0.243	0.152	-1.603	0.110
HighTrans	0.586	0.149	3.945	0.000
Checks	0.074	0.048	1.534	0.126
Years	0.462	0.040	11.579	0.000

Regression with scaled variables

[Linear Regression: Example]

- Example on a regression model with few variables
  The goal is to have *comparable* effects (same range)



Visual regression with scaled variables

[Problems in Linear Regression]

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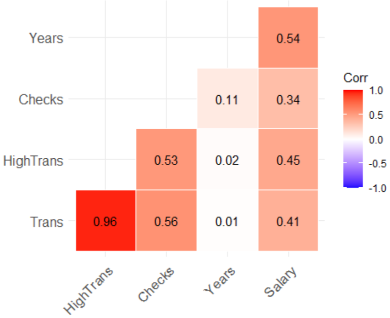

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$$VIF_j = \frac{1}{1 - R_j^2}$$

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$$VIF_j = \frac{1}{1 - R_j^2}$$

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↪ A $VIF \geq 10$ is considered as large and problematic

[Solutions to multi-collinearity]

2 solutions to multi-colinearity:

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- Remove some variables

[Using *VIF* to remove collinearity]

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Computing VIFs for all x_js

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Computing VIFs for all x_js

	Est.	S.E.	t val.	p	VIF
(Intercept)	5.93	0.04	150.24	0.00	NA
Trans	-0.24	0.15	-1.60	0.11	14.71
HighTrans	0.59	0.15	3.94	0.00	14.15
Checks	0.07	0.05	1.53	0.13	1.47
Years	0.46	0.04	11.58	0.00	1.02

[USING *VIF* TO REMOVE COLLINEARITY]

Computing VIFs for all x_js

	Est.	S.E.	t val.	p	VIF
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- *Trans* has the highest VIF

[USING *VIF* TO REMOVE COLLINEARITY]

Conclusion:

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Omitting one variable (*Trans:*)

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	Est.	S.E.	t val.	p	VIF
(Intercept)	5.93	0.04	149.79	0.00	NA
HighTrans	0.36	0.05	7.69	0.00	1.40
Checks	0.06	0.05	1.24	0.22	1.41
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- does not change the coefficients of the uncorrelated regressors
- reduces all the *VIFs*

[Real life Example]

In real life, one may have many variables

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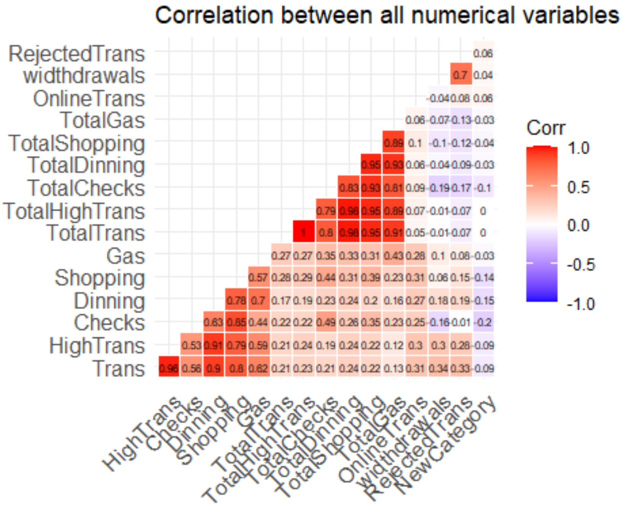

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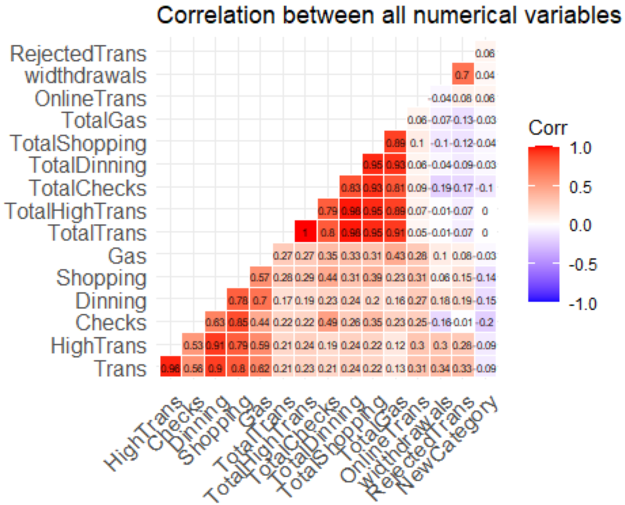

[Automatic selection of regressors]

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[AUTOMATIC SELECTION OF REGRESSORS]

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- The optimal number of regressors is unknown!

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- Compute the optimal nb of regressors before testing which regressors to include with Cross Validation

[Application on an Example]

To reduce the computational burden we restrict our choice to 8 variables in the final regression.

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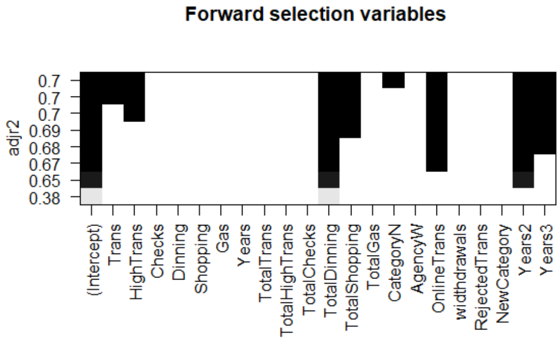

Visual representation of variables used (Forward)

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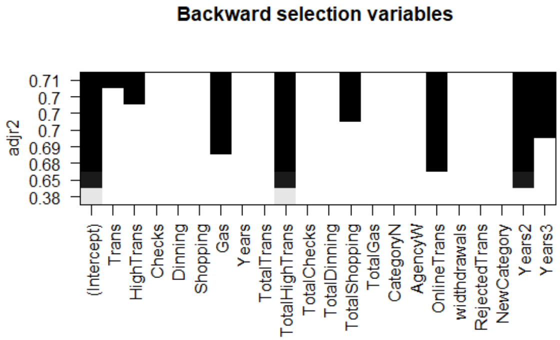

Visual representation of variables used (Backward)

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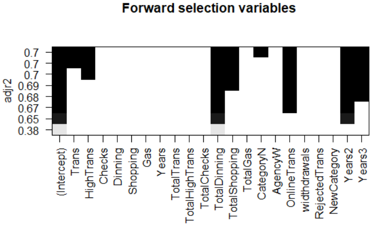

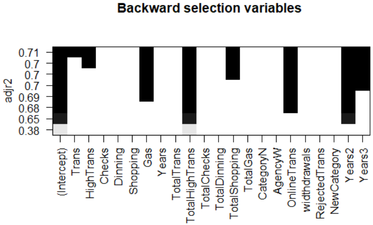

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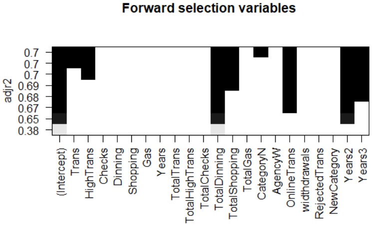

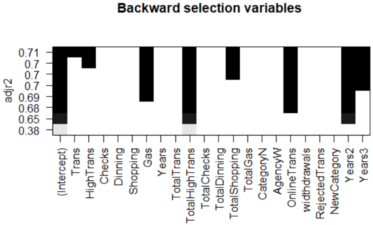

- *Great Need for Criteria*

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- Mallow's **Cp** $= \frac{RSS_p}{n} + \frac{2(p+1)}{n} \frac{RSS_k}{n-k-1}$

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- Akaike Information Criterion (**AIC**) $\propto C_p$ for linear regression
- Bayesian Information Criterion (**BIC**):

$$BIC \propto \frac{RSS_p}{n} + \frac{(p+1)\log n}{n} \frac{RSS_k}{n-k-1}$$

[Application on the Example]

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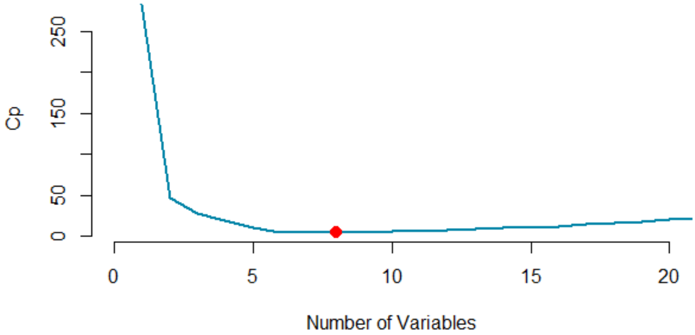
Selection using Mallow's C_p ($\rightarrow$ 8 variables)



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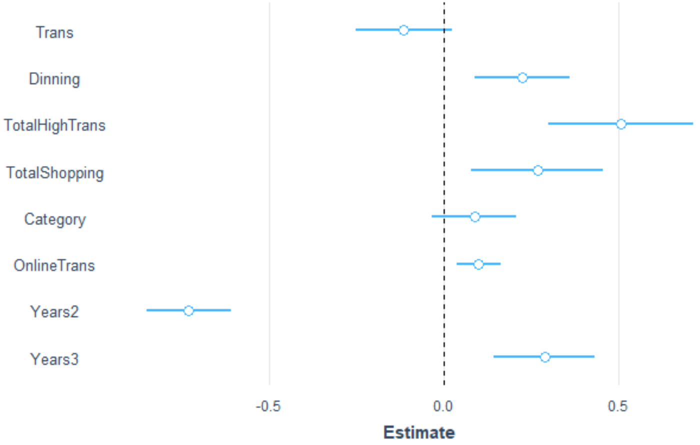
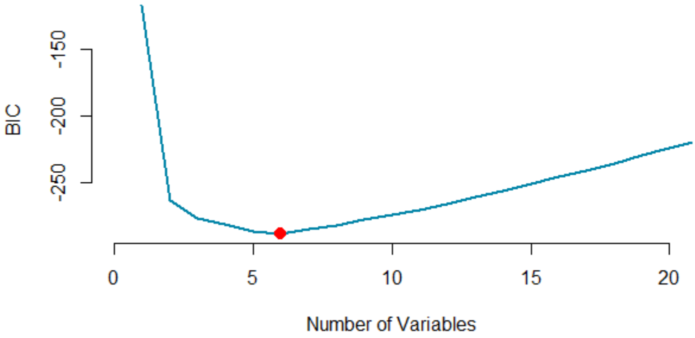


[APPLICATION ON THE EXAMPLE]

Selection using BIC ($\rightarrow$ 6 variables)



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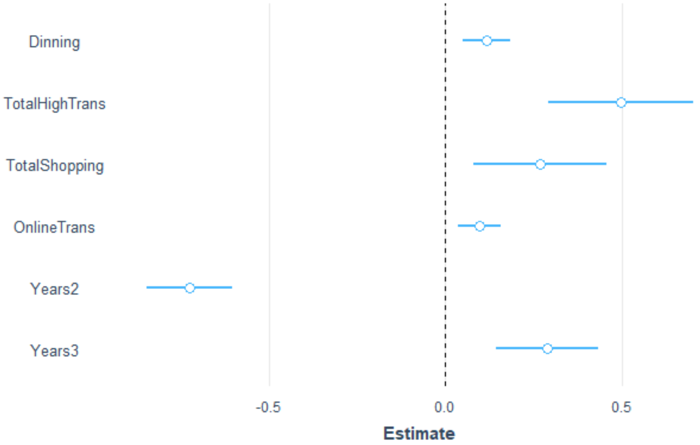

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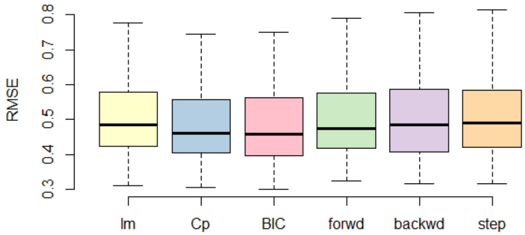

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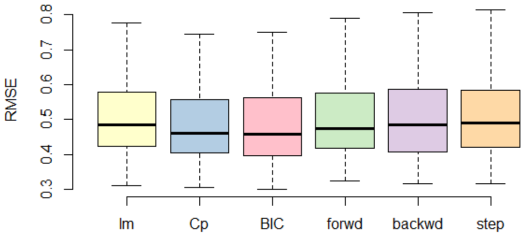

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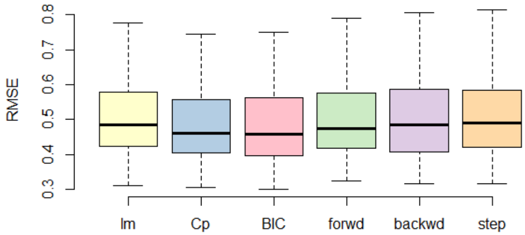

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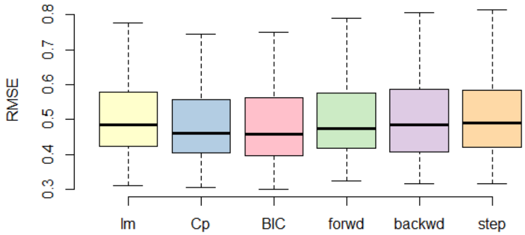

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↪ **Technical issue**: CV with stepwise selects different p-models for each K-fold

- Modern methods are available...

[Penalization methods]

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- *Many* problems when *many* variables are available:

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NB: *If* $\lambda = 0$ *the regression is just the classic OLS estimator*

[Penalization methods: Ridge regression]

Ridge regression is the solution of:

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↪ When there is collinearity, $X'X$ cannot be inverted, while if $\lambda > 0$ the matrix $(X'X + \lambda I)$ is invertible

[Ridge regression in Practice]

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- We compute the (CV-averaged) *RMSE* for many values of λ

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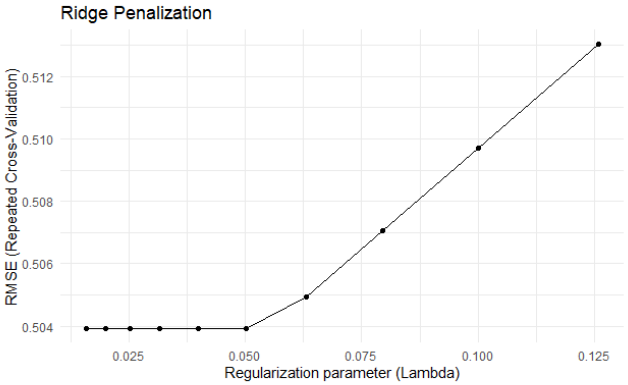

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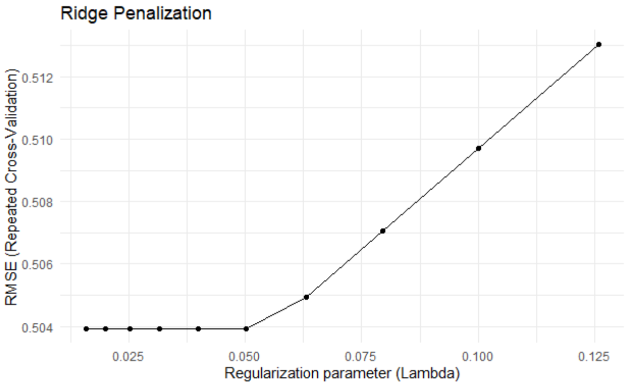

Remark: λ is typically small, and we usually select it on the log scale.

[Penalization methods: LASSO]

LASSO or *Least Absolute Shrinkage and Selection Operator*, is another common penalization method, is the solution of:

$$\min_{\beta} \frac{1}{2n} \sum_{i=1}^{n} \left(y_i - \beta_0 - x_i'\beta\right)^2 + \lambda|\beta| \qquad |\beta| = \sum_{j=1}^{k} |\beta_j|$$

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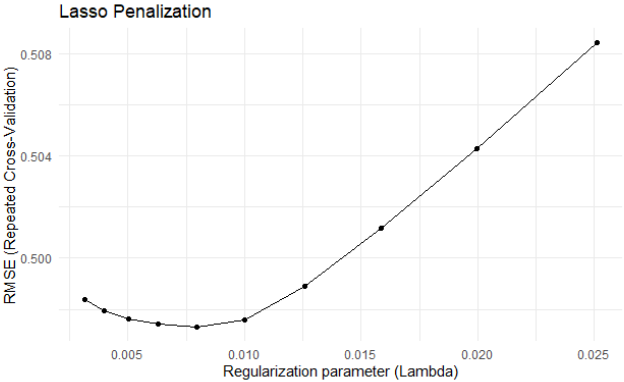

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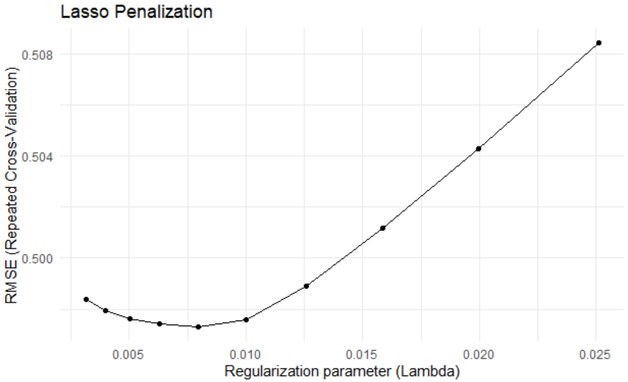

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Elastic Net combines both Lasso and Ridge regression:

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- If $\alpha = 1$ we have the Lasso estimator, if $\alpha = 0$, the Ridge regression.
- Through α we balance variable selection (Lasso) and coefficient reduction (Ridge)

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Compute the (CV-averaged) *RMSE* on a grid of $(\lambda\ ,\ \alpha)$

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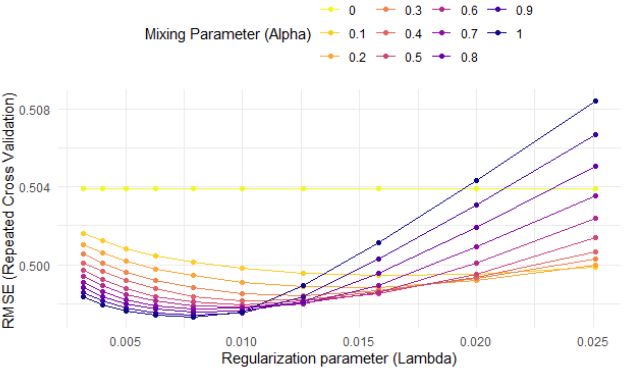

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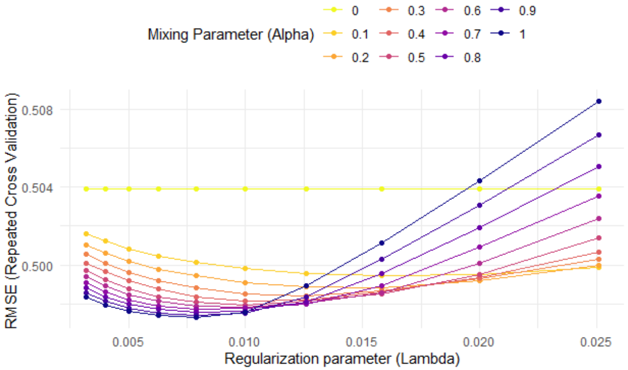

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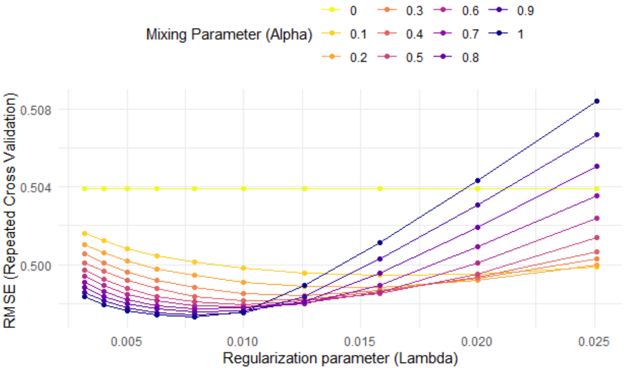

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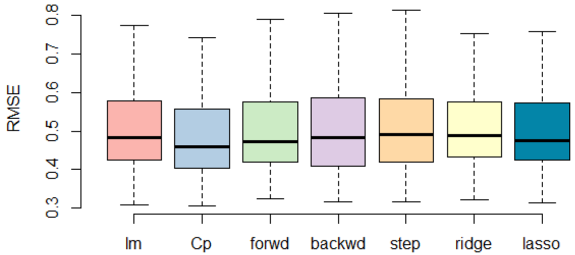

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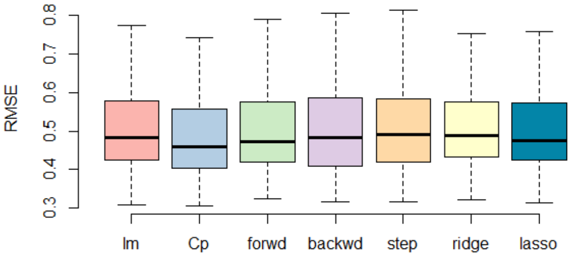

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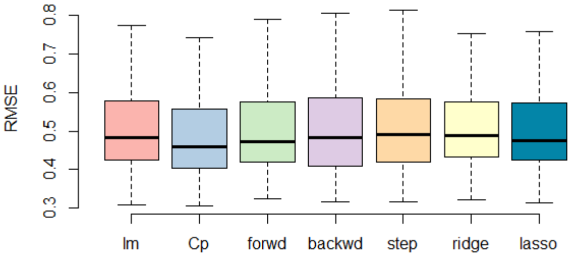

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- Model selected by Mallow's C_p (8 regressors) is almost as good!

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- Penalization methods are very popular in practice

[Q&A]

Write your questions in the chat

[Next week]

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Have a nice week!