Machine Learning for Official Statistics and SDGs

Classification

UNITED NATIONS
SIAP
Statistical Institute for Asia and the Pacific

[Classification]

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A **classifier** is a tool that provides a classification for y using (*or not*) additional information from other variables

[Supervised *vs* unsupervised Classification]

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- We'll focus on **supervised** classification

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- You observe households that are either in *Urban* or *Rural* areas (colors) and one variable (feature): *Education*.

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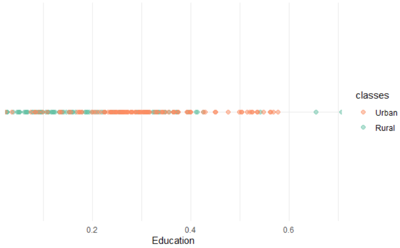

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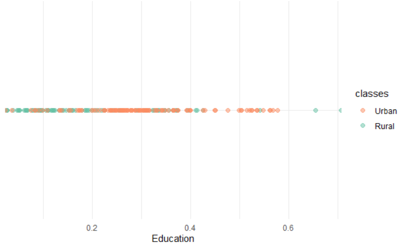

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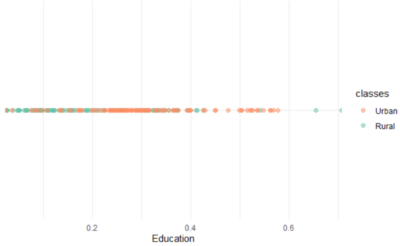

- A classifier determines the value of *Education* that separate "*Rural*" from "*Urban*"

  Typically with a threshold rule:

  "if $x \geq t$ then category is *Urban*"

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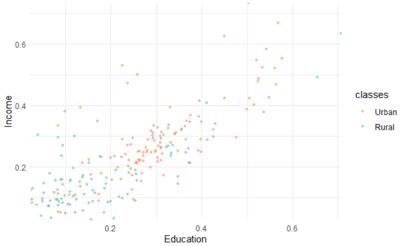

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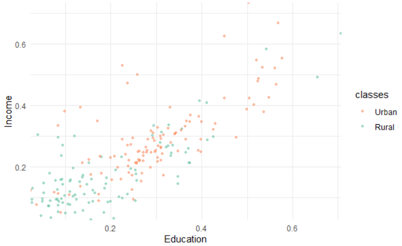

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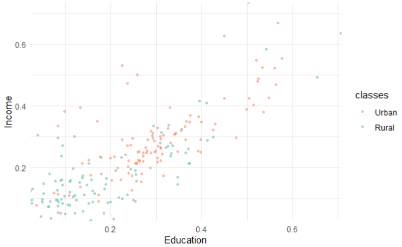

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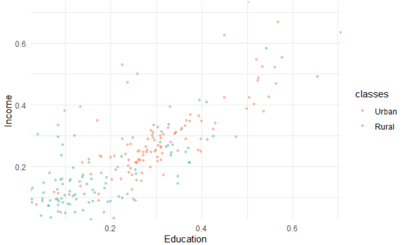

- The rule can be based on a linear relationship between *Education* and *Income* or can be non linear.

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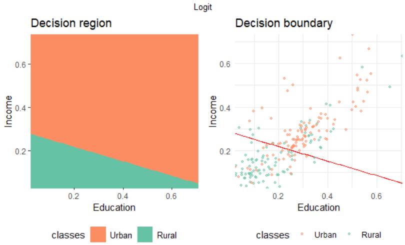

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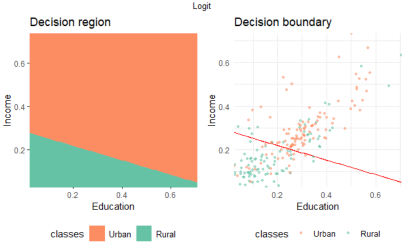

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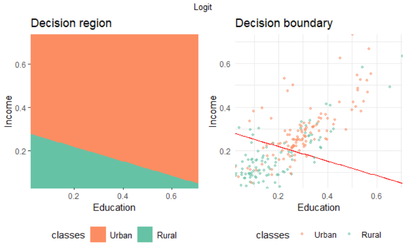

- The separation rule is $x'\beta \geq T_0$
  with T_0 a known threshold
  $e.g.\ \beta_0 + \beta_1 Education + \beta_2 Income \geq T_0 \Leftrightarrow Urban$

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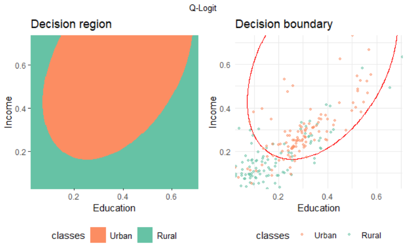

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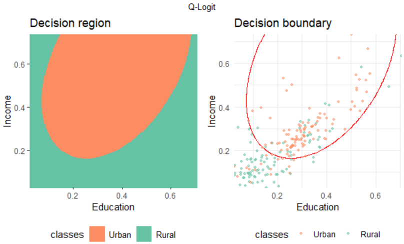

- The rule that separated the two classes is non linear in the variables *Education* and *Income*

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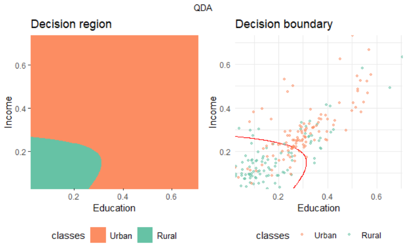

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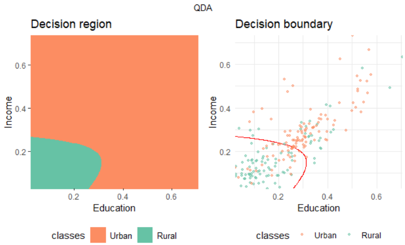

- The boundary is complex and uses *Education* and *Income* features.

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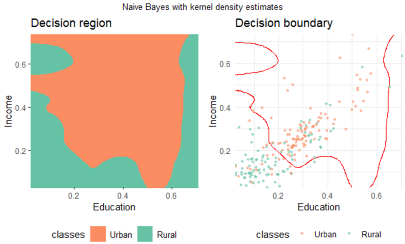

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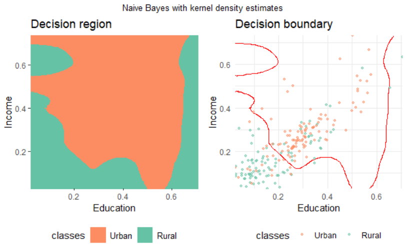

- It is hard to understand how the two classes are built using *Education* and *Income*

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- Measures of fit in classification are different and specific

[Measures of fit in classification]

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Each criterion answers to a different question

[Accuracy and Confusion Matrix]

Accuracy corresponds to the probability of being "accurate"

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$\hookrightarrow$ We want the maximum possible accuracy.

- Equivalently, we may want to minimize the *error rate* or *misclassification rate*

$$\Pr\left[y_0 \neq \widehat{f}(x_0)\right]$$

[Confusion Matrix & Accuracy]

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	Observed (True)	
Predicted	TP (True Positive)	FP (False Positive)
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$$Acuracy = \frac{TP + TN}{TP + TN + FP + FN}$$

$$= \frac{TruePositives + TrueNegatives}{N}$$

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- It is the proportion of accurate predictions

[Confusion Matrix & Accuracy]

In practice, with a classifier we have:

		Observed (True)	
		Urban	Rural
Predicted	Urban	87 (TP)	28 (FP)
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- We have an accurate prediction in 75% of the cases.

[Problem 1: Accuracy is one number]

Accuracy is not the panacea and may be misleading

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- They may go in different directions

[Sensitivity or *True Positive Rate*]

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[SENSITIVITY OR *True Positive Rate*]

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- *Sensitivity* focuses on "positives" (here *Urban*), *i.e.* on predicted positives *vs* the observed positives

$$Sensitivity = \frac{TP}{TP + FN}$$

$$= \frac{87}{87 + 24} = 0.78$$

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- On *Urban*, we correctly predict in 78% of the cases

[SPECIFICITY OR *True Negative Rate*]

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[Specificity or *True Negative Rate*]

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- *Sensitivity* focuses on negatives (*Rural*), *i.e.* on predicted negatives *vs* the observed negatives

$$Specificity = \frac{TN}{TN + FP}$$

$$= \frac{69}{69 + 28} = 0.71$$

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- On *Rural*, we predict correctly in **only** 71% of the cases

[Problem 2: Imbalanced outcomes]

Imagine you observe much more Urban than Rural

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Urban	Rural
95	5

[PROBLEM 2: IMBALANCED OUTCOMES]

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Accuracy = (TP + TN)/ 100 = 95 %

Sensitivity = TP/(TP + FN) = 95/95 = 100 %

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Its formal definition is given by

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- The larger κ is, the better the model for a given distribution of classes in a data set

[Quiz time]

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- When outcome is *imbalanced*, one may use **kappa** has a better measure for accuracy.

  Which measure you should consider depends on the context and your goal.

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- This can be transformed into:

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[Logit as You **Don't** know it]

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$\hookrightarrow$ *"The logit models log of odd ratios as linear in x"*

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$\hookrightarrow$ The logit classifier depends on the linear combination of the x's

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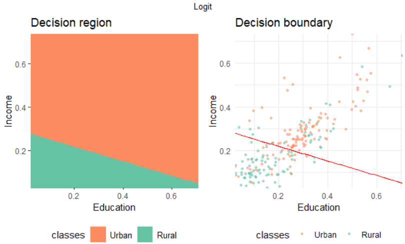

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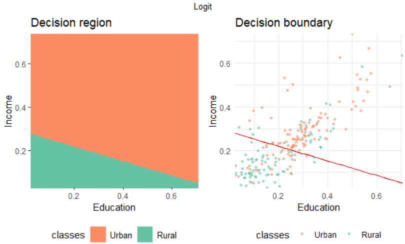

- This partition is sensitive to the choice of the threshold T_0 (and the t_0)

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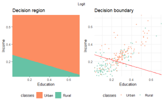

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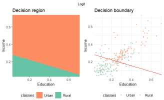

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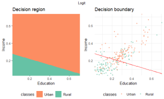

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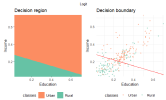

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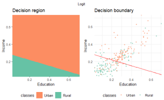

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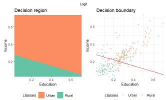

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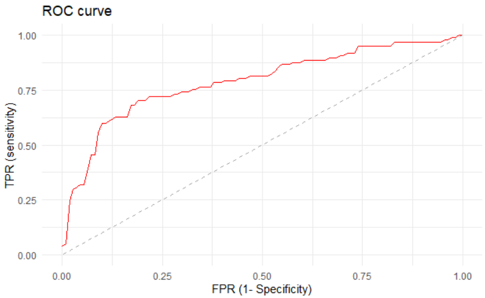

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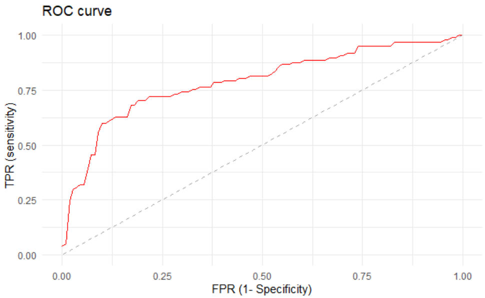

- *(sometimes x is sensitivity with inverted x-axis)*

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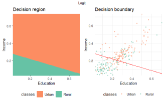

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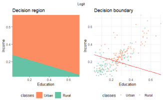

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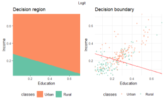

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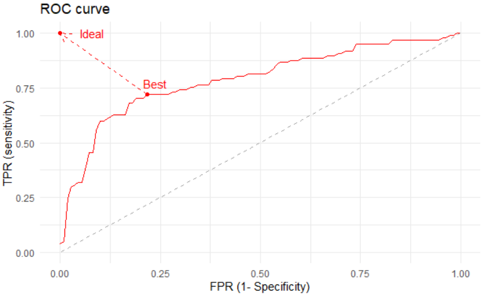

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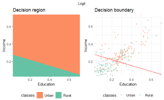

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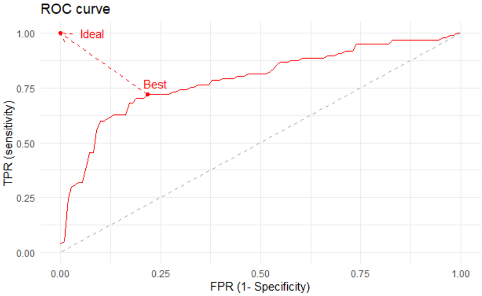

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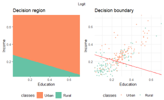

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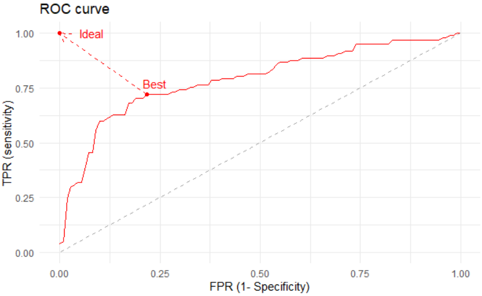

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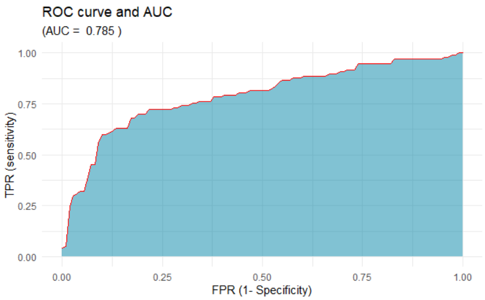

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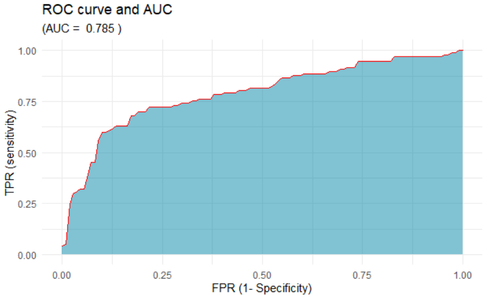

- The greater the area, the better the model

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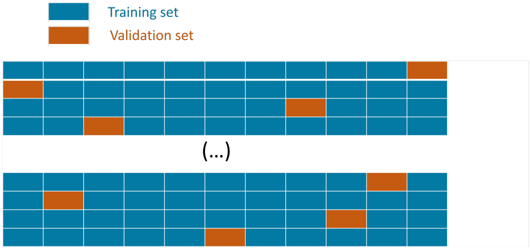

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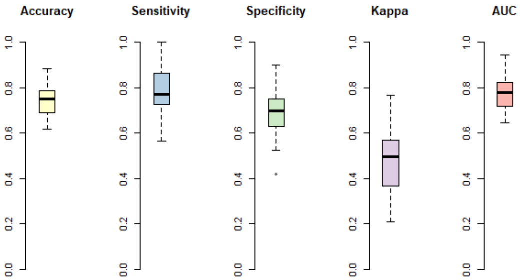

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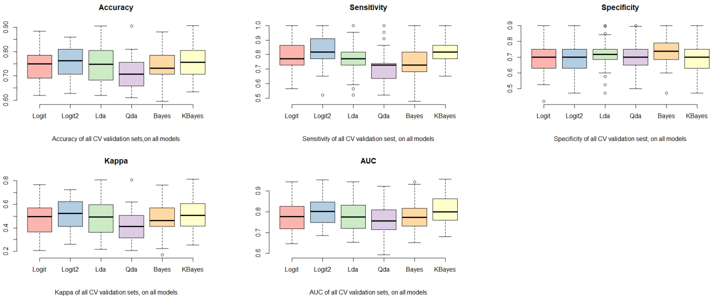

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  One may use the **ROC** to change the threshold parameter

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