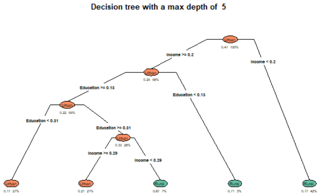
Machine Learning for Official Statistics & SDGs

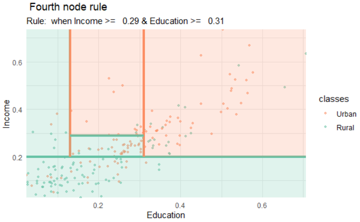
Random Forest

UNITED NATIONS
SIAP
Statistical Institute for Asia and the Pacific

[From Trees to Forest]

Trees are methods for classification or regression analysis.





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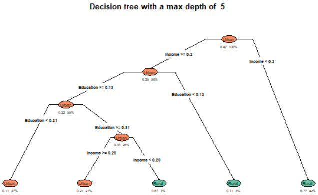

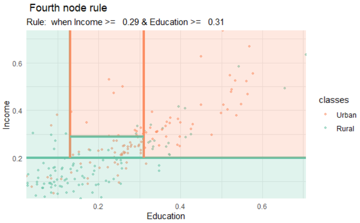

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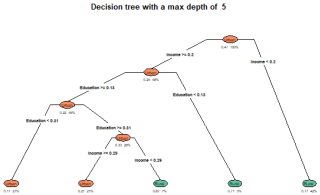

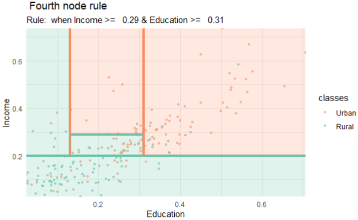

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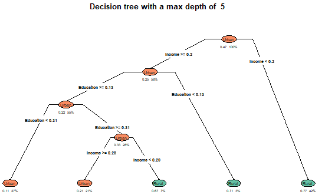

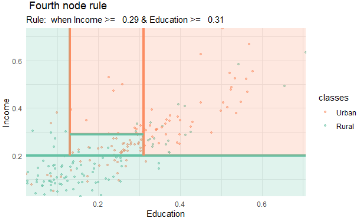

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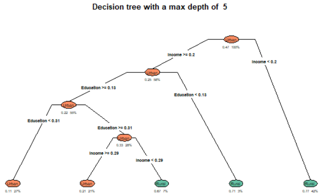

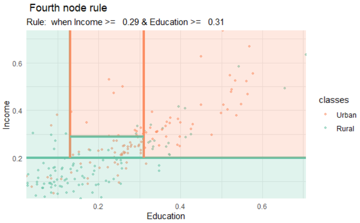

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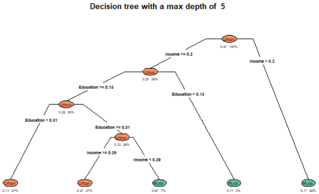
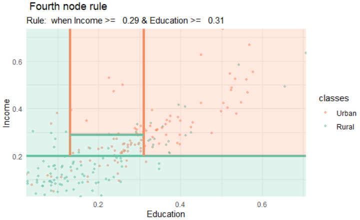
- Trees can be very detailed and prone to **over fitting**

[Problems with trees]





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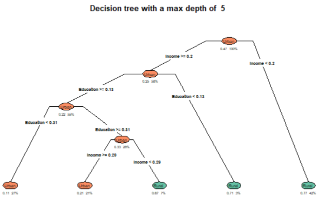

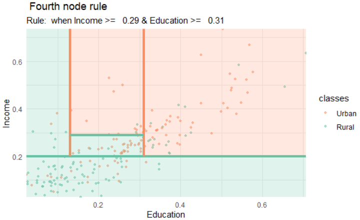

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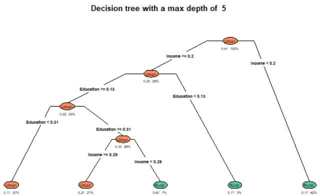

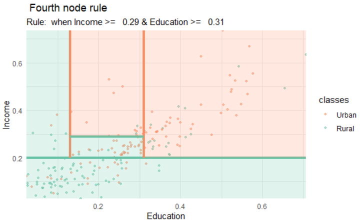

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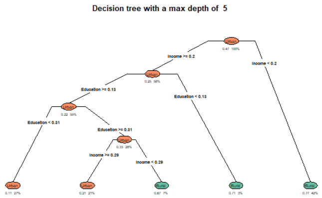

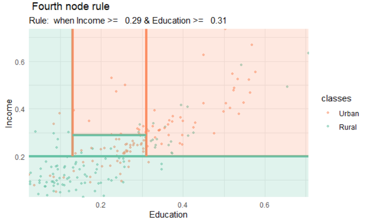

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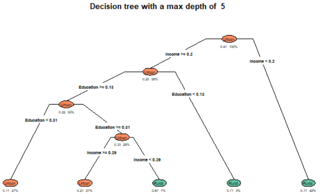

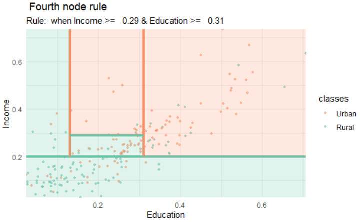

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$\hookrightarrow$ The two trees will likely be very different

[Example on a simple tree]

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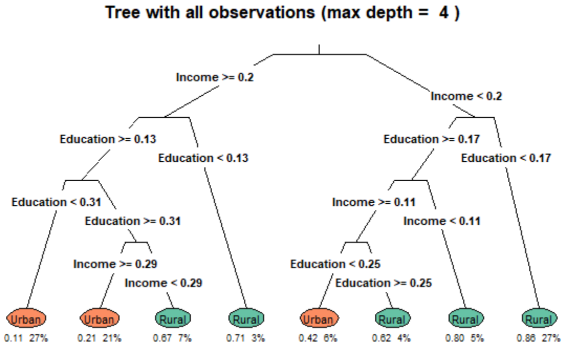
Which tree do you trust?



Tree using all observations

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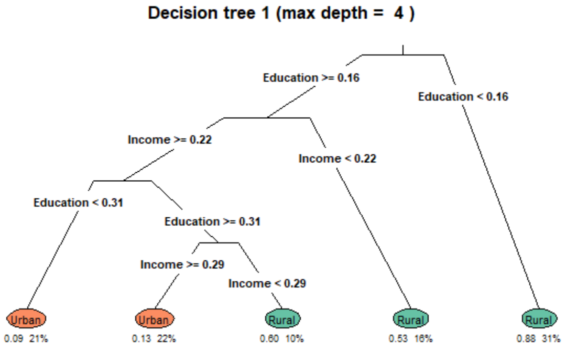

First tree with 50% of observations

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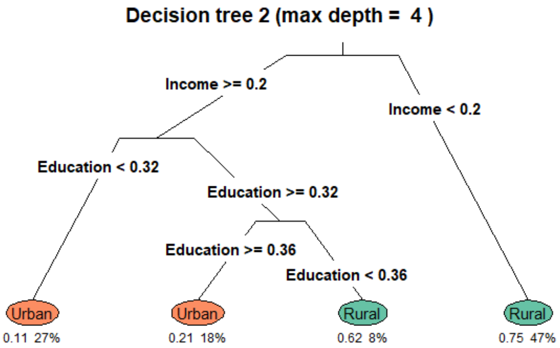

Second tree with 50% of observations

[Example on a simple tree]

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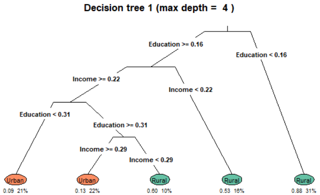
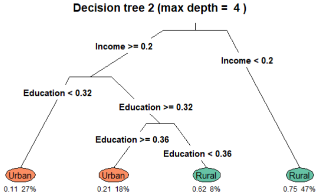
Outcomes are different from one tree to another





$\hookrightarrow$ Instability of the outcome (or high variance)

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- This is a simple principle that is used here

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In classification, we do not average but take the *majority vote* rule

[*Bagging*]

Bagging scheme:

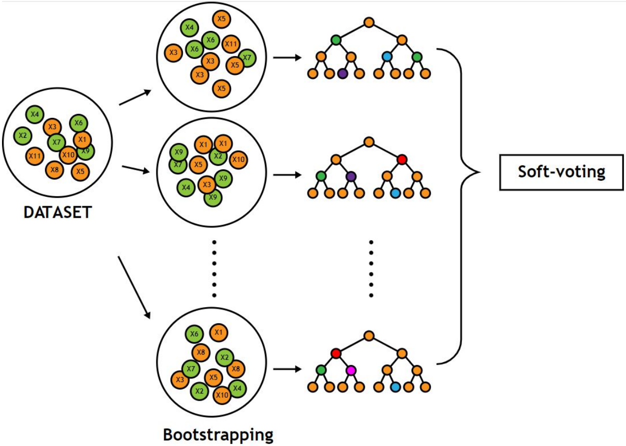

Image from igem.org

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$\hookrightarrow$ *Random forest* use a *decorrelation* algorithm by adding ***randomness*** in the set of variables used

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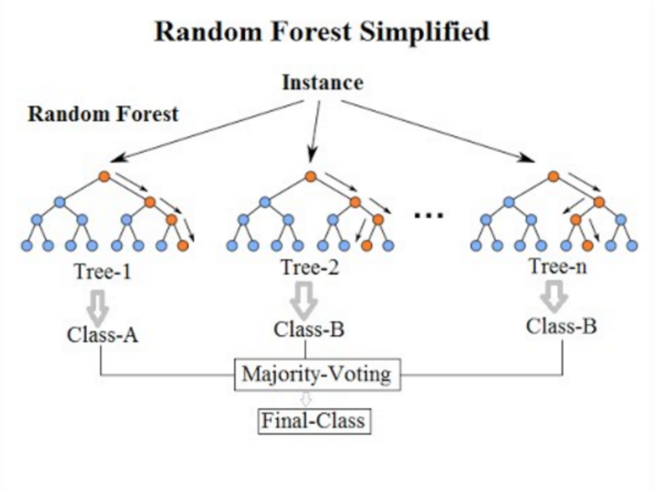

Image from *Venkata Jagannath* (wikimedia.org)

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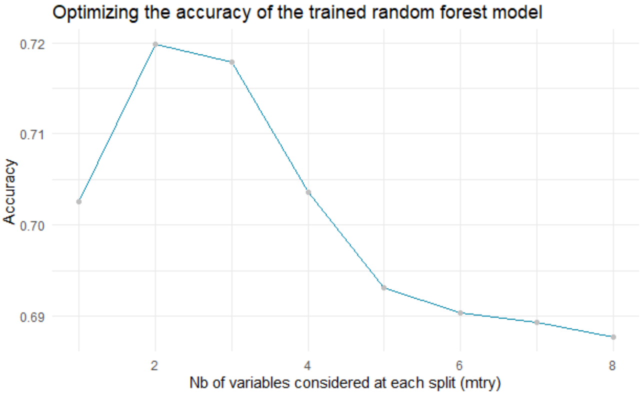
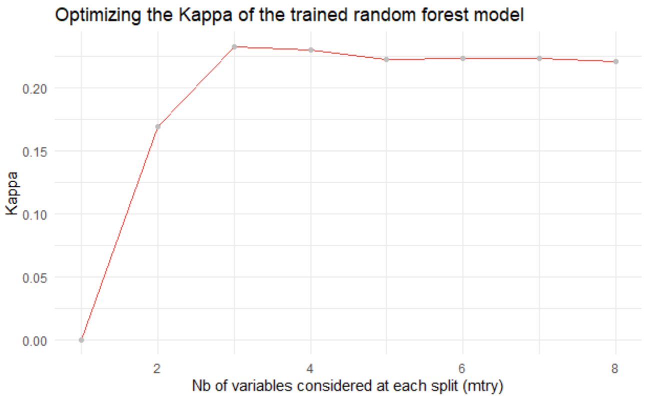


The impact of the number of variables (*m*) on accuracy

[Random Forest on an Example]



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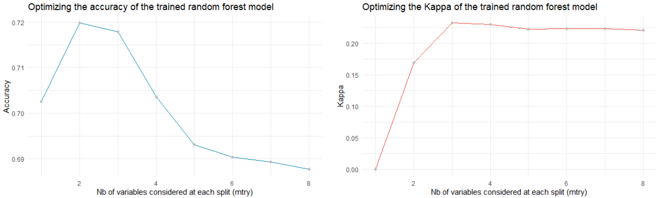

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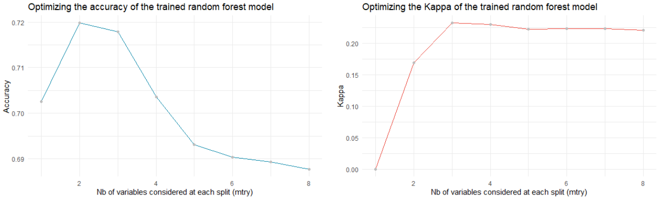

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[RANDOM FOREST ON AN EXAMPLE]

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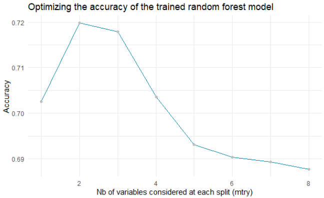

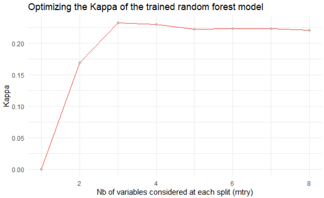

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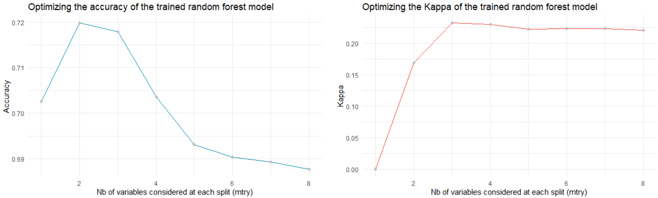

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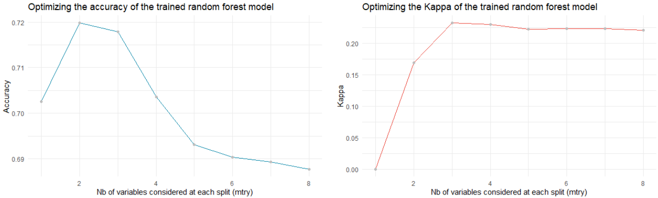

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- Optimum for **3** (out of 7) regressors only at each node

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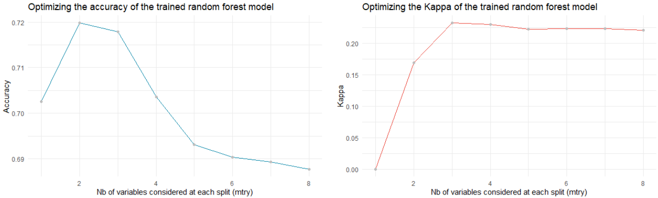

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- Optimum for **3** (out of 7) regressors only at each node

$\hookrightarrow$ Rule of thumb: $m \approx \sqrt{p}$

[Random Forest on an Example]

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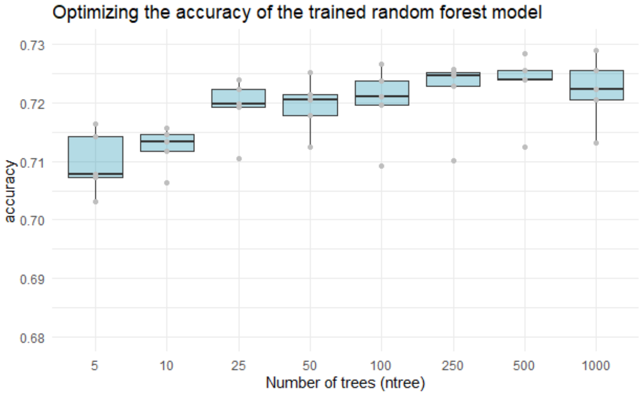

The impact of the number of trees on accuracy

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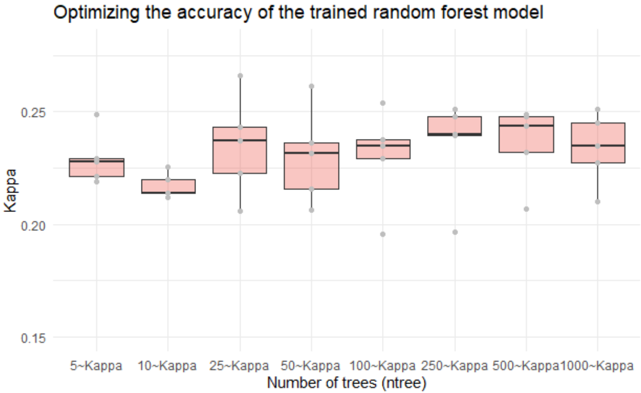

The impact of the number of trees on *kappa*

[Random Forest on an Example]

The impact of the number of trees:

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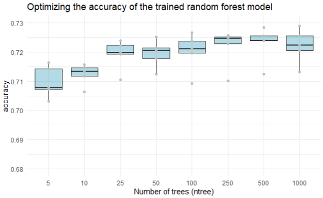

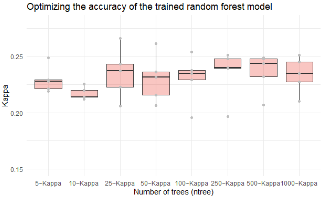

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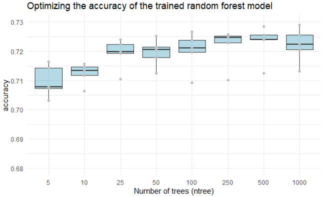

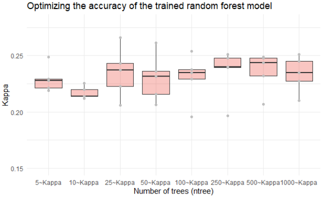

- No clear optimum $\hookrightarrow$ Moderate number of trees is OK

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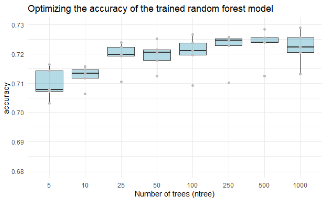

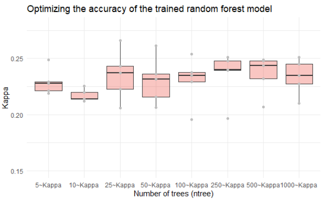

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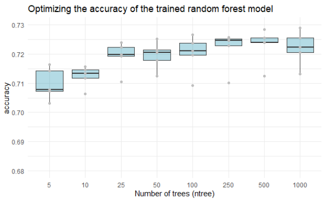

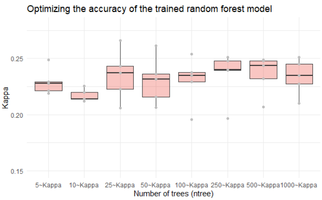

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$\hookrightarrow$ More tree reduces variance (up-to a certain point)

[Improvements]

Trees can also grow differently:

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$\hookrightarrow$ "*Boosting*"

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Boosting helps construct trees sequentially

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- *"Mistakes"* at tree nb t are overweighed for tree nb $t + 1$

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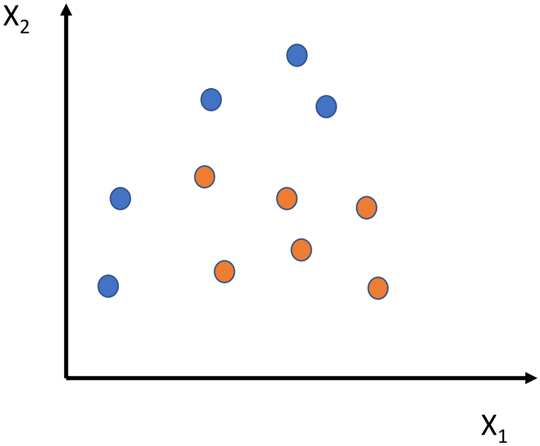
Boosting a tree (iterative process)



Two classes of observations: Orange and blue

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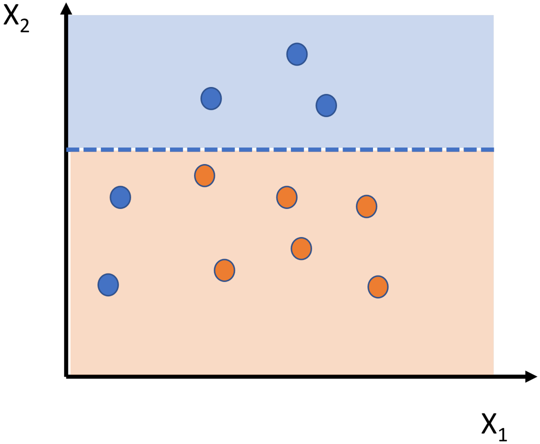

First weak learner

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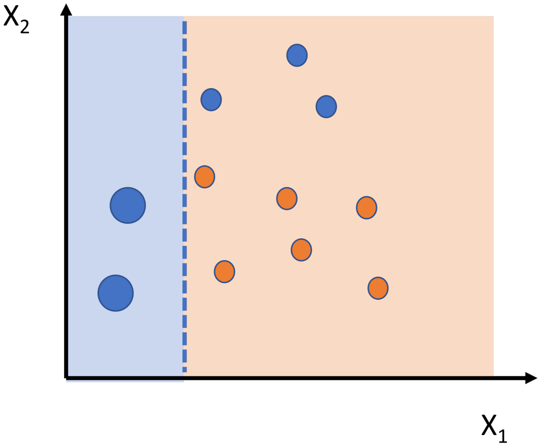

Second weak learner. Misclassification overweighed

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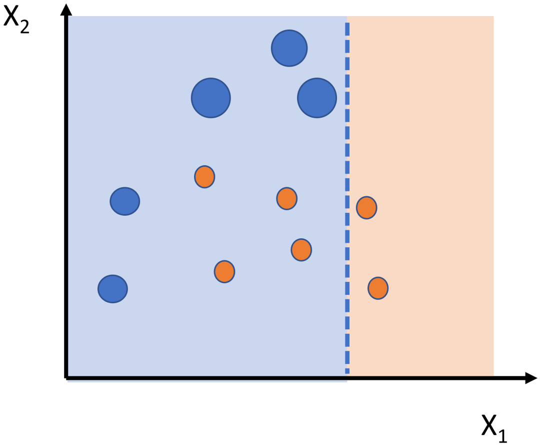

Third weak learner. New misclassification overweighed

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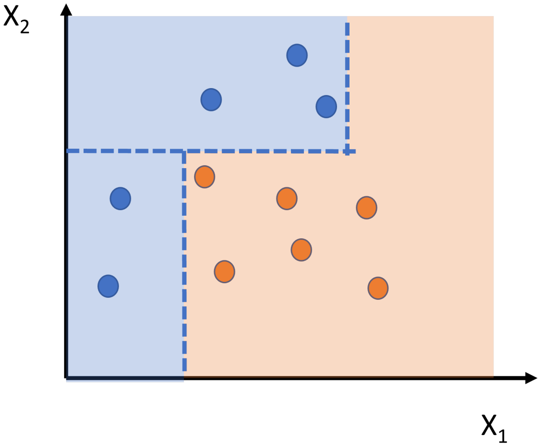

Combining weak learners.

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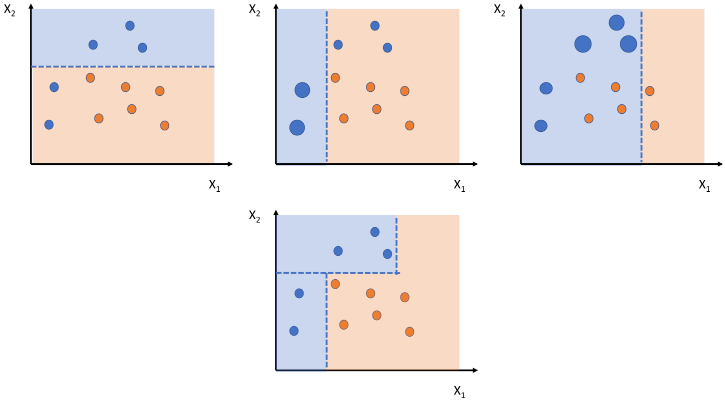

Combining weak learners. Classification tree completed

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- Implemented in many software!