New simulations of the paper to be submitted to JPA (2014)

Christophe Bontemps, Celine Nauges, Vincent Requillart, & Michel Simioni

May 28, 2014

1 Simulation framework

We present a single output - single input simulation exercice allowing us to compare DEA frontiers and Bias-corrected frontiers efficiencies.

1.1 Parameters

The parameters used in this simulation are :

- Sample size (initial): 200
- Size of the evaluation sample[1] : 100
- Number of bootstraps for Bias correction 99
- Level for the bias correction $\alpha = 0.05$

1.2 Recalling simulation case (1: Regress)

We start by generating a dataset of N =200 single-input single-output firms over three years from the following equation:

$$y_t = x_t^{0.5} \times \exp\{-0.25 \times (t-1)\} / (1+u_t) \tag{1}$$

with $x_t \sim U[0,1]$ and $u_t \sim \mathcal{N}^+(0.2, 0.25)$. This procedure generates input-output pairs for year 1, year 2, and year 3, and incorporates an assumption of technical regress.

First we simulate a technical **regress** from year 1 to year 2 and from year 2 to year 3. We simulate the true frontier each year by $y = F(x)$:

- For year 1 : $y = x^{1/2}$
- For year 2 : $y = x^{1/2} \cdot e^{-0.25}$
- For year 3 : $y = x^{1/2} \cdot e^{-0.5}$

[1] Randomly drawn from the complete sample of all firms all years

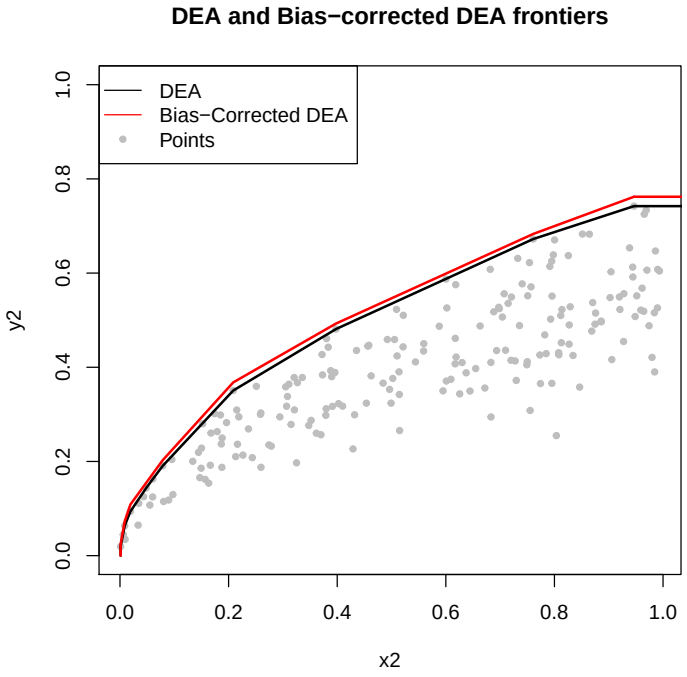
2 Bias-corrected frontiers

2.1 Comparing the Biais-Corrected DEA Frontier with the DEA-frontier

We use here the sample of **200** firms on year 2 (x_2), and compute the DEA frontier (VRS) as well as the bias-corrected frontier.

Variable	**n**	**Min**	$\bar{\mathbf{x}}$	$\widetilde{\mathbf{x}}$	**Max**
DEA.efficiency..vrs.	200	0.370	0.750	0.742	1.000
Bias.corrected.efficiency	200	0.366	0.730	0.729	0.989

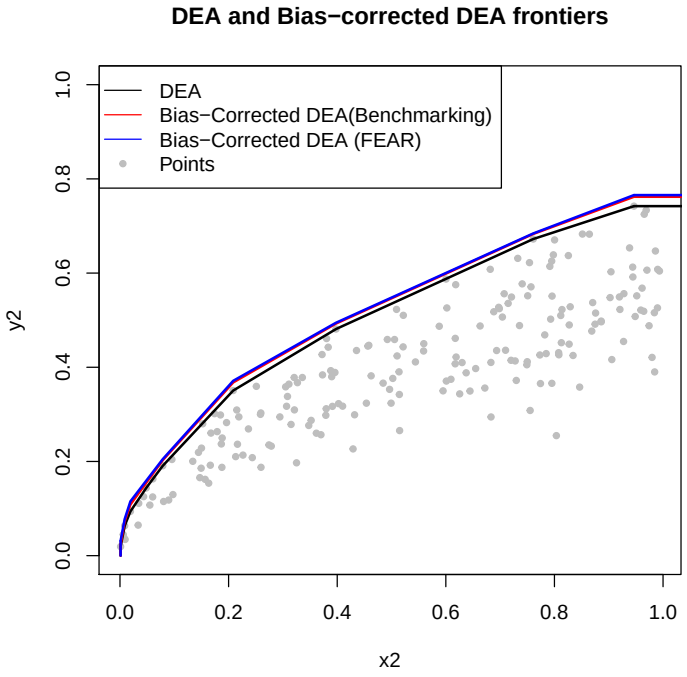
Table 1: Efficiencies and bias-corrected efficiencies (ns= 200)



2.2 Computing Bias-Frontier efficiencies with FEAR

Variable	n	Min	$\bar{x}$	$\widetilde{x}$	Max
DEA.efficiency..vrs.	200	0.370	0.750	0.742	1.000
Bias.corrected.efficiency	200	0.366	0.730	0.729	0.989
Bias.corrected.efficiency..verif.	200	0.366	0.730	0.729	0.988
Bias.corrected.efficiency..FEAR.	200	0.365	0.726	0.727	0.987

Table 2: Efficiencies and bias-corrected efficiencies (ns= 200)



2.3 Is the Biais changing with sample size ?

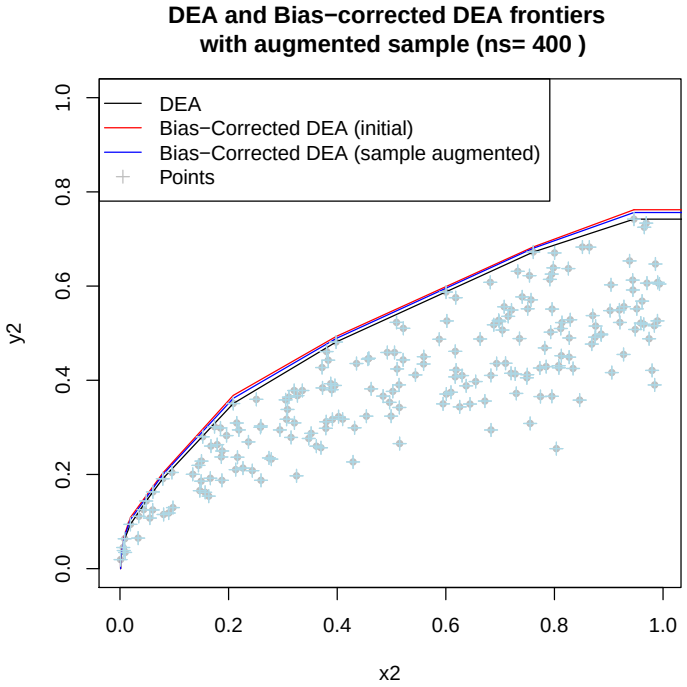
2.3.1 In the case where we increase the number of points by replicating the sample

We first use the same sample of points (x_2, y_2) **replicated** so that the total sample size is doubled (n = 400), with absolute no changes in the points and thus in the initial DEA frontier estimated.[2]

Variable	**n**	**Min**	$\bar{\mathbf{x}}$	$\widetilde{\mathbf{x}}$	**Max**
DEA.efficiency..vrs.	400	0.370	0.750	0.742	1.000
Bias.corrected.efficiency	400	0.367	0.737	0.734	0.993

Table 3: Efficiencies and bias-corrected efficiencies (n= 400)

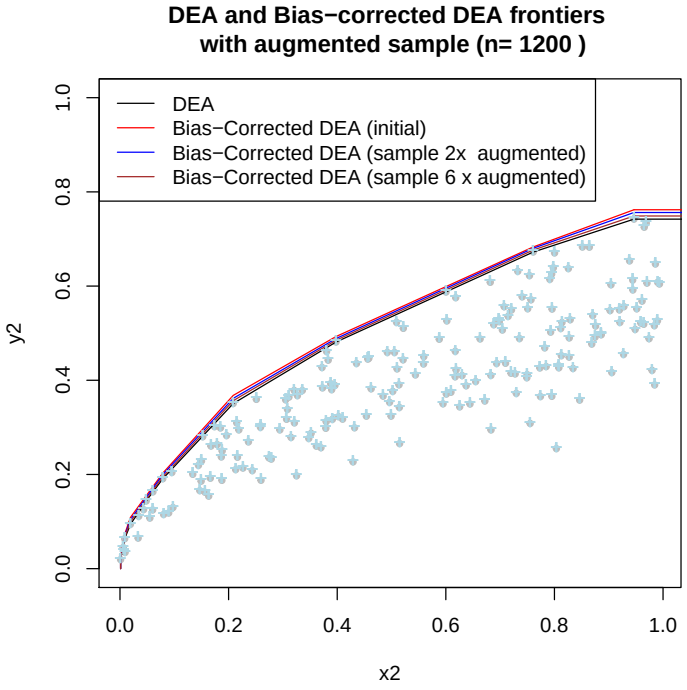
[2]Using a new draw from the distribution of (x_2, y_2) induces changes in the estimated frontier, and thus on the bias corrected frontier. I found that it was more accurate to see the "pure" effect of sample size using a replication of the initial sample.



We now increase 6 time the size of the original sample (still by replicating the points) so that n= 1200

Variable	**n**	**Min**	$\bar{x}$	$\widetilde{x}$	**Max**
DEA.efficiency..vrs.	1200	0.370	0.750	0.742	1.000
Bias.corrected.efficiency	1200	0.369	0.745	0.738	0.997

Table 4: Efficiencies and bias-corrected efficiencies (ns= 1200)

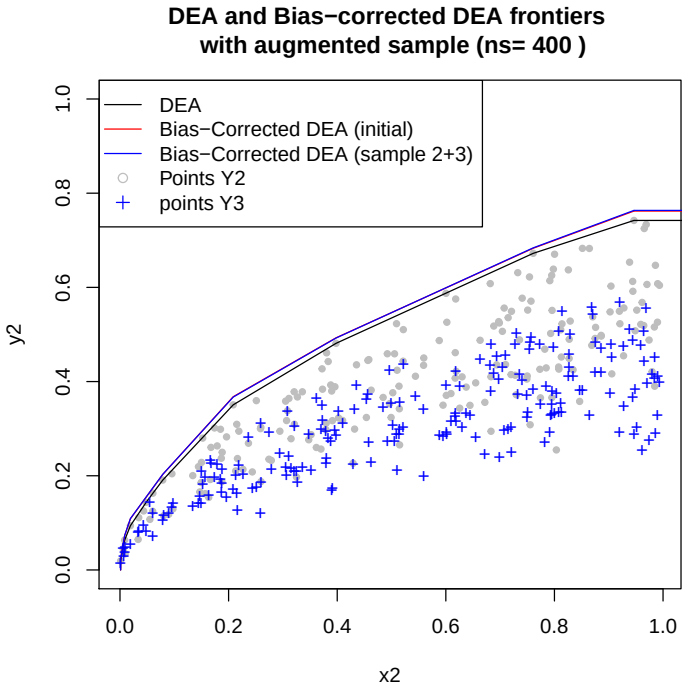

DEA and Bias−corrected DEA frontiers
with augmented sample (n= 1200)
DEA
Bias−Corrected DEA (initial)
Bias−Corrected DEA (sample 2x augmented)
Bias−Corrected DEA (sample 6 x augmented)
y2
x2
0.0
0.2
0.4
0.6
0.8
1.0

2.3.2 In the case where we increase the number of points by adding points from another year

We now add the points from year 3 (x_3, y_3, in blue) to our sample (n= 400)

Variable	n	Min	$\bar{x}$	$\widetilde{x}$	Max
DEA.efficiency..vrs.	400	0.304	0.663	0.649	1.000
Bias.corrected.efficiency	400	0.297	0.646	0.637	0.989

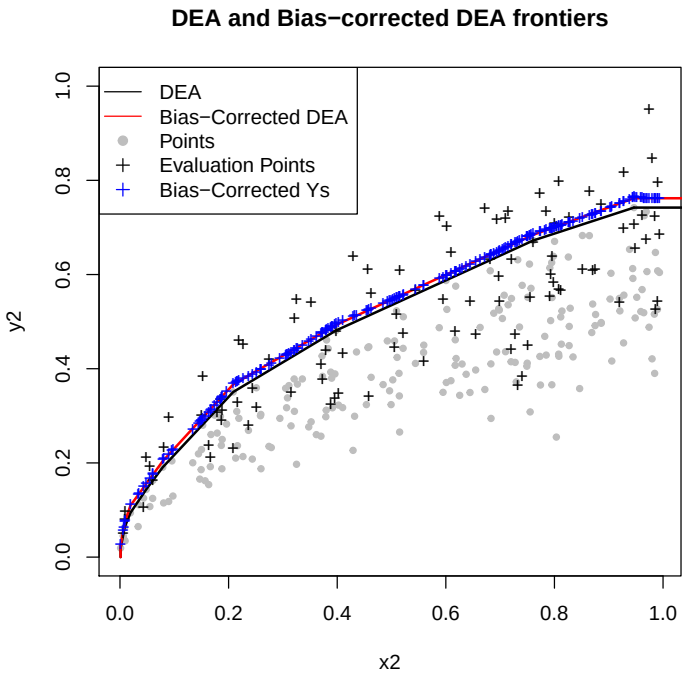
Table 5: Efficiencies and bias-corrected efficiencies when adding year 3 to year 2 (ns= 400)



3 Computing Bias-Frontrier efficiencies for points not in the sample used for frontier construction

Variable	n	$\bar{x}$	Min	$\widetilde{x}$	Max	$\bar{N}b_{Eff}$	$\bar{N}b_{Super}$	$\bar{X}_{not}$	#NA
Efficiency.to.DEA.Frontier	100	0.965	0.552	0.948	1.456	0	39	0.836	0
Efficiency.to.BC.DEA.Frontier	100	0.932	0.542	0.914	1.350	0	36	0.819	0

Table 6: Efficiencies and bias-corrected efficiencies (ns= 100)



NB :This program runned for 227.73 seconds.

4 FIPS and BIPS with Bias -corrected Frontiers

TO DO (In another program) !!