MATHEMATICAL MODELING AND SIMULATION OF STEPPING MOTOR SYSTEMS

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Abstract. A mathematical model is developed for a variable reluctance stepping motor and then a numerical simulation is provided in view of studying the behaviour of the motor chiefly in dynamic start regime. The voltage equations are written considering the magnetic circuit unsaturated and the torque equation contains inertia, viscous friction and load torque components. The six first order differential equations of the model are solved using Basic language programs for a Wang 2200 computer. The motor is tested by numerical simulation in dynamic start regime with constant frequency of input pulses and also with different laws for varying this frequency determining the optimal law for motor acceleration. The method presented here for this aim is generally valid for a wide range of VR stepping motors.

Keywords. Stepping motors; computer programming; frequency control; optimal acceleration.

INTRODUCTION

The advantages of using stepping motors in open-loop positioning control systems don't exclude certain limits of frequency at motor acceleration and deceleration. The improvement of stepping motor dynamic performances is achieved by its control based on variable frequency upon a given law.

Such a study has been made before by experimental tests (Page, 1975).

In this work the method of numerical simulation is used to establish the optimal law for processing the pulse frequency at start and also for more detailed study of the motor behaviour at start and stop.

The aim of the work is to find a mathematical model and a computer program to test the dynamic behaviour of all the single-stack variable reluctance stepping motors by numerical simulation.

MATHEMATICAL MODEL

A single-stack variable reluctance stepping motor is taken to deduce the mathematical model (fig. 1).

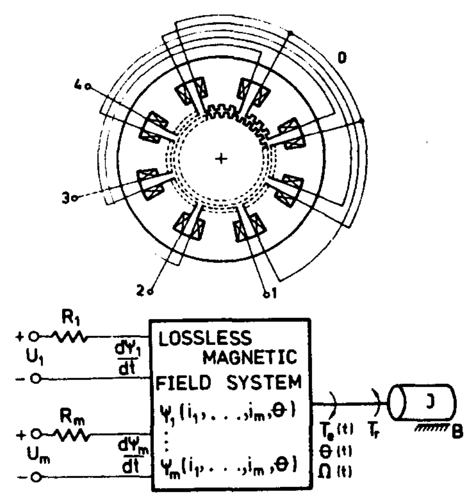

Fig. 1. Motor and system structure

The stator has 2 m salient poles which contain m phase windings. Two diametrally-placed windings are series connected and form one motor phase. Each stator pole has z_s teeth, each one identical with the z_r teeth of the rotor.

The mathematical model of an open-loop stepping motor positioning control system consists of a system of differential equations, which contains the equations of electrical equilibrium of the motor phases and the torque equation (fig. 1). This system of equations forms a polyphase model in real coordinates of the motor and has the advantage of a direct physical interpretation versus the mathematical model written in transformed coordinates (Kuo, 1981).

The voltage equations

The voltage of the phase k is:

$$u_k=R_k i_k+\frac{d\Psi_k}{dt}, \quad k=1,2,\ldots,m \qquad (1)$$

where Ψ_k is the total flux that flows through the phase k and is a function of the rotor position and phase currents:

$$\Psi_k=\Psi_k(i_1,i_2,\ldots,i_m,\theta) \qquad (2)$$

where θ defines position of the rotor relative to a reference position.

Neglecting the saturation, the flux Ψ_k can be expressed using phase inductivities and currents as:

$$\Psi_k=\sum_{j=1}^{m} L_{kj} i_j \qquad (3)$$

where L_{kj} is static self inductivity of phase k, when k=j and static mutual inductivity between phases k and j, when $k \neq j$, all being functions of rotor angle θ.

Using expression (3), the voltage equation (1) becomes:

$$u_k=R_k i_k+\sum_{j=1}^{m} L_{kj}\frac{di_j}{dt}+\frac{dL_{kj}}{d\theta}\cdot\frac{d\theta}{dt} i_j \qquad (4)$$

The dependence of the inductivities on the rotor angle is well enough approximated by cosinusoidal laws (Kuo, 1981), as:

$$L_{kk}=\frac{m-1}{m}L_0+\frac{m-2}{m}L_1\cos\left[z_r\theta-\frac{2\pi}{m}(k-1)\right]$$

$$\pm L_{jk}=\frac{1}{m}L_0+ \qquad (5)$$

$$+\frac{2}{m}\cos\frac{\pi}{m}(j-k)L_1\cos\left[z_r\theta-\frac{\pi}{m}(j+k-2)\right]$$

where L_0 and L_1 are calculated as functions of the number of wires per a stator pole and of the average value, respectively the amplitude of the first harmonic of magnetic permeance.

Torque equation

The differential equation of the rotor equilibrium is:

$$J\frac{d^2\theta}{dt^2}+B\frac{d\theta}{dt}+T_r=T_e \qquad (6)$$

where:

- J - total moment of inertia;
- B - coefficient of viscous friction;
- T_r - load torque;
- T_e - electromagnetic torque.

The expression of electromagnetic torque is deduced from the derivative of the energy of magnetic field with respect to angle θ :

$$T_e=\frac{1}{2}\sum_{k=j}^{m}\sum_{k=j}^{m}\frac{dL_{kj}}{d\theta} i_k i_j=\sum_{k=1}^{m}\sum_{j=1}^{m}T_{kj} \qquad (7)$$

where torque components T_{kj} are calculated using the expressions of the inductivities (5).

$$T_{kk}=-\frac{m-2}{2m}L_1 z_r i_k^2\sin\left[z_r\theta-\frac{2\pi}{m}(k-1)\right], \quad k=1,2,\ldots,m \qquad (8)$$

$$T_{kj} = -\frac{1}{m}\cos\frac{\pi}{m}(j-k)\times$$

$$\times L_1 z_r i_k i_j \sin\left[z_r\theta - \frac{\pi}{m}(j+k-2)\right],$$

$$k,j = 1,2,\dots,m;\ k \neq j$$

The torque equation is written as two first order differential equations using the rotor speed Ω as derivative of the angle:

$$\frac{d\theta}{dt} = \Omega$$

$$\frac{d\Omega}{dt} = \frac{1}{J}(T_e - B\Omega - T_r) \qquad (9)$$

Equations (4) together with (9) consist in the mathematical model which will be used for numerical simulation of the motor.

The motor specifications are: $m=4$, $z_s=4$, $z_r=34$, step angle $\theta_p=2{,}65^0$ (136 steps per revolution) phase resistance $R_f=10\Omega$, d.c supply voltage $U=56$ V.

PROGRAMMING ALGORITHMS

Basic language programs were written out for numerical simulation of the stepping motor behaviour in dynamic regime at start, using a WANG 2200 computer. The library of the computer was used in order to solve the system of equations that belong to mathematical model, based on the integrating method of fourth order Runge Kutta.

Figure 2 shows the algorithm in PASCAL designation for testing the system, when the motor is started with a constant frequency of pulses and then operates at this frequency for a certain number of steps.

The following symbols are used in the algorithm:

x(20) - matrix of variable values $x_1,\dots,x_6$;

f(10) - matrix of variable derivatives $dx_1/dt,\dots,dx_6/dt$;

k(4,10)-matrix that saves the derivatives in the first integrating point;

w, w_1, w_2, w_3 - matrices for graphical representation of stored values for: time, rotor angle, speed, and torque;

s_1 - index of matrices w, w_1, w_2, w_3;

n - number of differential eqs. of the system;

t - initial value of time;

d - value of integrating step;

a - initial value of integrating interval;

b - final value of integrating interval;

j - number of integrating steps.

```
var  x: array [1 ... 20]
     f: array [1 ... 10]
     k: array [1 ... 4, 1...10]
     w, w1, w2, w3: array [1...800]
begin
  sl: = 1;
  read (n,t,d,b);
  for  i: = 1 to n do read(x(i));
  write ("title");
  a: = t;
  j: = a - d;
  repeat
     j: = j + d;
     write (t);
     for i: = 1 to n do write (x(i));
     w [sl]: = t;
     w1 [sl]: = x [5];
     w2 [sl]: = x [6];
     w3 [sl]: = h5 ;
     sl: = sl + 1;
     "Runge-Kutta integration procedure";
  until  j≥b;
  "axes drawing procedure";
  "procedure fordrawing graphics";
end
```

Fig. 2. Main algorithm for numerical simulation.

Figure 3 shows the algorithm for defining the inductivities, the system of diff. eqs., and also for selecting the two phase-on energization of the motor.

```
procedure systdif;
   var tt:integer;
begin
   read (a5,b5,c5,d5,j1,b1,m1,r,t1);
   tt: = t/t1;
   case  tt of
     0,4 : begin  u1:=0;u2:=u;u3:=u;u4:=0  end;
     1,5 : begin  u1:=0;u2:=0;u3:=u;u4:=u  end;
     2,6 : begin  u1:=u;u2:=0;u3:=u;u4:=u  end;
     3,7 : begin  u1:=u;u2:=u;u3:=0;u4:=0  end;
   end;
   {definition inductances}
    a:=a5 + b5 * cos(34 * x);
    :
   {system of differential equations}
     f[1]: = .... ;
     :
     f[6]: = .... ;
end
```

Fig. 3. Algorithm for defining the system.

The following symbols are used:

a_5,b_5,c_5,d_5 - coefficients of the inductivities;

j_1 - total moment of inertia;

m_1 - load torque;

r - total resistance per phase;

t_1 - period of input pulse train;

$u_1,\ldots,u_4$- supplying voltages.

The second problem studied by numerical simulation is the start with variable frequency. The motor is first started with a minimal constant frequency and then this frequency is increased by a given law to a maximal constant value,which corresponds to the steady-state regime. Also, the law for processing the frequency is valid for a given number of motor steps.

The time interval between the two input pulses is variable and therefore the integrating steps have to be modified from an input pulse to another.

For the cases studied the variation of the frequency is made into an interval of 25 pulses, from 100 pps to 500 pps then the motor operates at this frequency for a 5 pulses.

NUMERICAL SIMULATION RESULTS

The simulation results for starting the motor with constant frequency input pulses are plotted in fig.4,5.

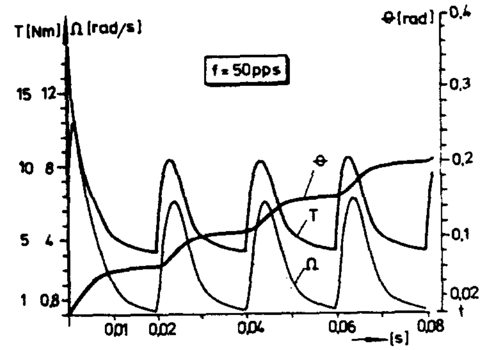

Fig. 4. The variation of torque, speed and rotor angle at 50 pps.

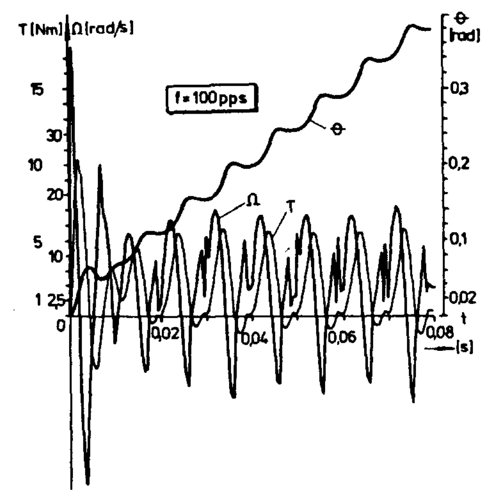

Fig. 5. The variation of torque, speed and rotor angle at 100 pps.

Certain laws were tested for motor start with variable frequency input pulses. The variation of frequency (frequency profile), of its slope and also of the torque, speed and angle are plotted for each case, as indicated in fig. 6, 7..., 11.

Figures 12 and 13 show the start with linear frequency profile for two extreme cases: small slope (fig.12) and high slope (fig.13). The loss of synchronism is observed for the second case.

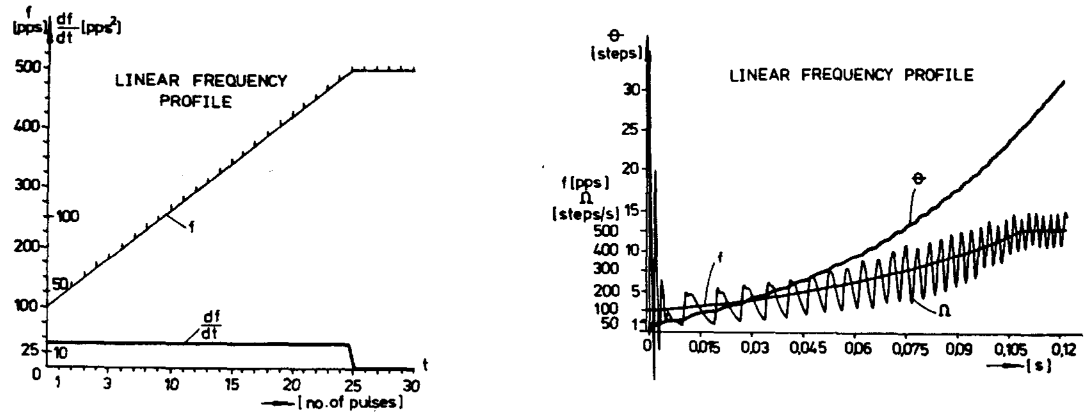

Fig. 6 - 7. Linear frequency profile

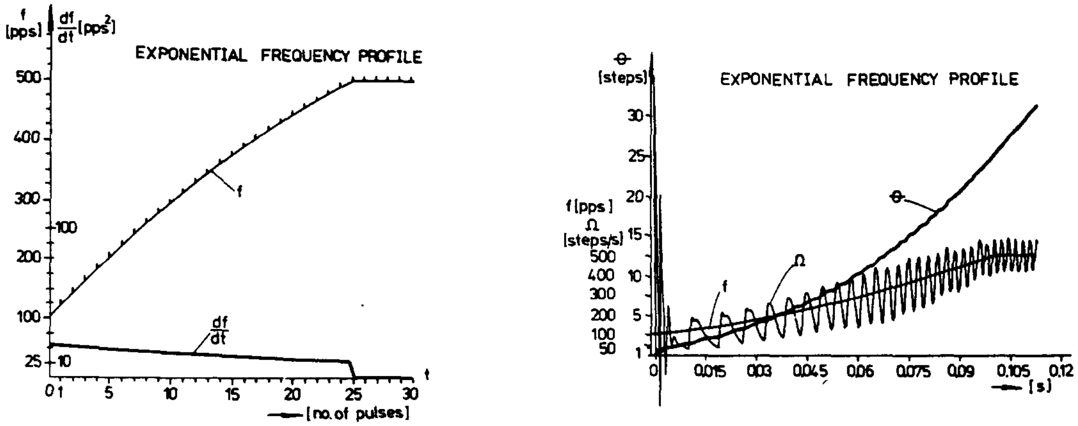

Fig. 8 - 9. Exponential frequency profile

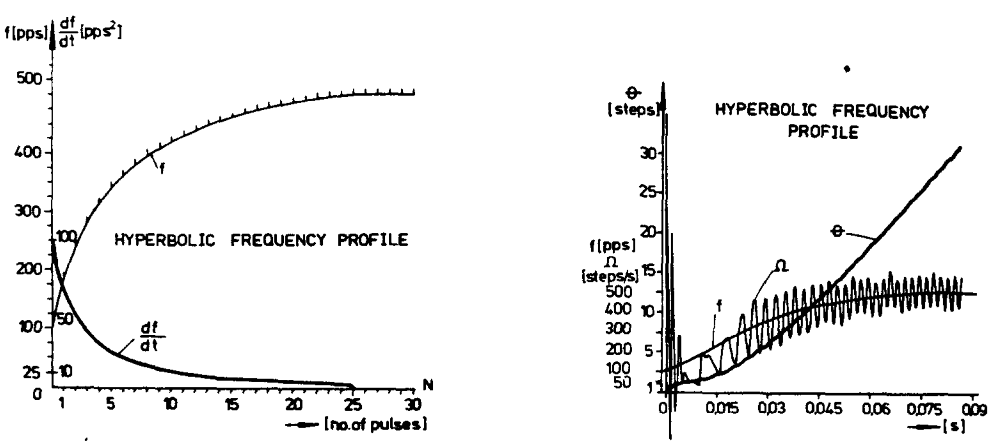

Fig. 10 - 11. Hyberbolic frequency profile

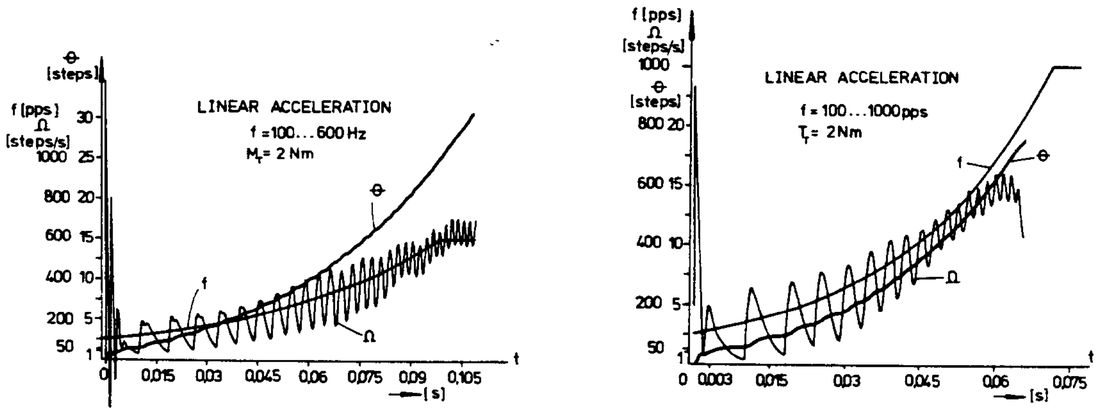

Fig. 12 - 13. Linear acceleration

CONCLUSIONS

The mathematical model and programs presented allow to test any VR stepping motor if its specifications as well as those of the load are given.

Examining the plotted results the following conclusions can be drawn:

- hyperbolic frequency profile provides the minimal starting time without losing steps;
- the slope of frequency profile must be high at the beginning of the start and then it must decrease;
- in the case of linear acceleration the maximal slope is limited by load parameters.

REFERENCES

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