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jsw50  
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## Short Assignment 5

To uphold the Duke Community Standard:

- I will not lie, cheat, or steal in my academic endeavors;
- I will conduct myself honorably in all my endeavors; and
- I will act if the Standard is compromised.

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```
(define square-int
  (lambda ((n <integer>))
    (if (= n 0)
        0
        (+ n (- n 1) (square-int (- n 1))))))
```

Proof by induction on variable  $n$

$P(n)$ :  $(\text{square-int } n) = n^2$

### Base Case:

Prove that  $(\text{square-int } \{0\}) = 0^2$

$(\text{square-int } \{0\})$  by the substitution model is:

```
(if (= {0} 0)
    0
```

which is  $\{0\}$  and that is right as  $0 = 0$

### Inductive Hypothesis:

There exists  $k$ , an element of the set of all natural numbers, less than  $n$  such that

$(\text{square-int } k) = k^2$

### Inductive Step:

By the substitution model:

[{proc ((n <integer>)) ...} {k+1}]

As  $k+1 > k$  and  $k$  is a natural number,  $k+1 > 0$  so

[if ({#f})

(+ {k+1} (- {k+1} 1) (square-int (- {k+1} 1)))]

Which simplifies to:

(+ {k+1} {k} (square-int({k})))

By the inductive hypothesis,

(+ {k+1} {k}  $k^2$ )

=  $k^2 + 2k + 1 = (k+1)^2$

Thus by induction, (square-int  $n$ ) =  $n^2$