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CS 230
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Short Assignment 3

```
(define multiply
  (lambda ((a <number>) (b <integer>))
    (cond ((zero? b) 0)
          ((odd? b)
           (+ a (multiply (+ a a) (quotient b 2))))
          (else
           (multiply (+ a a) (quotient b 2))))))
```

Proof by induction on variable b, natural number
P(b): (multiply a b) = a*b

Base Case:
Prove that (multiply a {0}) = a*0
(multiply a {0}) by the substitution model is
(cond ((zero? {0}) 0 …)
which is {0}
and that is right as a*0 = 0

Inductive Hypothesis:
For all k, elements of the set of all natural numbers, less than b, assume that (multiply a k) = a*k

Inductive Step:
Show that (multiply a b) = a*b

```
cond ((zero? {b}) 0)
          ((odd? {b})
           (+ {a} (multiply (+ {a} {a}) (quotient {b} 2))))
          (else
           (multiply (+ {a} {a}) (quotient {b} 2))))))
```

{b} can't be 0 as k is a natural number and less than b

if (b) is odd,
(+ {a} (multiply (+ {a} {a}) (quotient {b} 2)))
We know that (+ {a} {a}) is {2a}
Since b is odd, (quotient {b} 2) is {b/2 – 1/2} which is less than k
So we can simplify to (+ {a} (multiply {2a} {b/2 – 1/2}) and because b/2 is less than b, by the inductive hypothesis,
(multiply {2a} {b/2 – 1/2}) = {2a} * {b/2 – 1/2} = a*(b – 1) = a*b – a
Plugging that into the formula,
(+ {a} (multiply (+ {a} {a}) (quotient {b} 2))) = (+ {a} {a*b – a}) = a*b

if (b) is even,
(multiply (+ {a} {a}) (quotient {b} 2)))
We know that (+ {a} {a}) is {2a} and
Because b is even, (quotient {b} 2) is {b/2} which is less than k
So we can simplify to (multiply {2a} {b/2}) and because b/2 is less than b, by the inductive hypothesis,
(multiply (+ {a} {a}) (quotient {b} 2))) = (multiply {2a} {b/2} = {2a} * {b/2}
= a*b

Thus by induction, (multiply a b) = a*b