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Short Assignment 4 Problem 2

The nth line causes the plane to separate into an additional n regions versus if there were n-1 lines. Therefore, when there are n lines, the number of regions is [n*(n+1)/2]+1.

Theorem:
When n lines are drawn in a plane, the plane is separated into P(n) = [n*(n+1)/2]+1 regions

Proof by induction on variable n

Base Case:
Prove that when n = 0, P(n) = 1
When there are no lines on an empty plane, the plane is not separated at all so there is only 1 large region.

Inductive Hypothesis:
There exists k that is an element of the set of all natural numbers such that P(k) = [k*(k+1)/2]+1

Inductive Step:
P(k) + k+1 = [k*(k+1)/2]+1 + k+1
= [k(k+1)/2]+ 1+ k + 1
= [k(k+1)/2]+ 2/2 + k/2 + 1
= [(k^2 + k)/2] + 2/2 + k/2 + 1
= [(k^2 + 2k + 2)/2] + 1
= [(k+1)(k+2)/2] + 1

Thus by induction, P(n) = [n*(n+1)/2] + 1