Практическая работа часть 4

11.3.3

$$z=\frac{x^2y^2}{x^2y^2-(x-y)^2};\ M_0(2;2);\ \Delta x=-0,2;\ \Delta y=0,1$$

$\square$

$x_0=2 \Rightarrow x_0+\Delta x=2-0,2=1,8$

$y_0=2 \Rightarrow y_0+\Delta y=2+0,1=2,1$

$$z(x_0;y_0)=\frac{2^2\cdot 2^2}{2^2\cdot 2^2-(2-2)^2}=1$$

$$z(x_0+\Delta x;y_0)=\frac{1,8^2\cdot 2^2}{1,8^2\cdot 2^2-(1,8-2)^2}=\frac{\left(\frac{18}{5}\right)^2}{\left(\frac{18}{5}\right)^2-\left(\frac{1}{5}\right)^2}=\frac{18^2}{18^2-1}=\frac{324}{323}$$

$$z(x_0;y_0+\Delta y)=\frac{2^2\cdot 2,1^2}{2^2\cdot 2,1^2-(2-2,1)^2}=\frac{\left(\frac{21}{5}\right)^2}{\left(\frac{21}{5}\right)^2-\frac{1}{4}\cdot\left(\frac{1}{5}\right)^2}=$$

$$=\frac{21^2}{21^2-\frac{1}{4}}=\frac{4\cdot 21^2}{4\cdot 21^2-1}=\frac{1764}{1763}$$

$$z(x_0+\Delta x;y_0+\Delta y)=\frac{1,8^2\cdot 2,1^2}{1,8^2\cdot 2,1^2-(1,8-2,1)^2}=\frac{\left(\frac{378}{100}\right)^2}{\left(\frac{378}{100}\right)^2-\frac{9}{100}}=$$

$$=\frac{378^2}{378^2-900}=\frac{142884}{141984}=\frac{3969}{3944}$$

$$\Delta_x z=\frac{324}{323}-1=\frac{1}{323}\approx 0,0031$$

$$\Delta_y z=\frac{1764}{1763}-1=\frac{1}{1763}\approx 0,0006$$

$$\Delta z=\frac{3969}{3944}-1=\frac{25}{3944}\approx 0,0063$$

$\blacksquare$

11.3.4

$Z=\left(\frac{x^2+y^2}{xy}\right)^2$; $M_0(1;1)$; $\Delta x=-0,1$; $\Delta y=-0,1$

□

$x_0=1 \Rightarrow x_0+\Delta x=0,9$

$y_0=1 \Rightarrow y_0+\Delta y=0,9$

$Z(x_0;y_0)=\left(\frac{1^2+1^2}{1\cdot1}\right)^2=4$

$Z(x_0+\Delta x;y_0)=\left(\frac{0,9^2+1^2}{0,9\cdot1}\right)^2=\left(\frac{1,81}{0,9}\right)^2=\frac{181^2}{90^2}=$
$=\frac{32761}{8100}\approx4,04$

$Z(x_0;y_0+\Delta y)=$ [ан-но, т.к. Z симметрична относительно X и Y] $\approx4,04$

$Z(x_0+\Delta x;y_0+\Delta y)=\left(\frac{0,9^2+0,9^2}{0,9^2}\right)^2=2^2=4$

$\Delta_x Z=4,04-4=\underline{0,04}$

$\Delta_y Z=4,04-4=\underline{0,04}$

$\Delta Z=4-4=\underline{0}$

■

11.3.5

$Z = 3x^2 + xy - y^2 + 1$; $M_0(2;1)$; $\Delta x = 0,1$; $\Delta y = 0,2$

$\square$

$x_0 = 2 \Rightarrow x_0 + \Delta x = 2,1$

$y_0 = 1 \Rightarrow y_0 + \Delta y = 1,2$

$Z(x_0; y_0) = 3 \cdot 2^2 + 2 \cdot 1 - 1^2 + 1 = 14$

$Z(x_0 + \Delta x; y_0 + \Delta y) = 3 \cdot 2,1^2 + 2,1 \cdot 1,2 - 1,2^2 + 1 = 15,31$

$\Delta Z = 15,31 - 14 = \underline{1,31}$ $\blacksquare$

11.3.6

$Z = 3x^2 + xy - y^2 + 1$; $M_0(2;1)$; $\Delta x = 0,01$; $\Delta y = 0,02$

$\square$

$x_0 = 2 \Rightarrow x_0 + \Delta x = 2,01$

$y_0 = 1 \Rightarrow y_0 + \Delta y = 1,02$

$Z(x_0; y_0) = 3 \cdot 2^2 + 2 \cdot 1 - 1^2 + 1 = 14$

$Z(x_0 + \Delta x; y_0 + \Delta y) = 3 \cdot 2,01^2 + 2,01 \cdot 1,02 - 1,02^2 + 1 =$

$= 12,1203 + 2,0502 - 1,0404 + 1 = 14,1301$

$\Delta Z = 14,1301 - 14 = \underline{0,1301}$ $\blacksquare$

11.3.7

$z = x^2 - xy + y^2$; $M_0(2;1)$; $M_1(2{,}1;1{,}2)$

$\square$

$z(x_0;y_0) = 2^2 - 2\cdot1 + 1^2 = 3$

$z(x_0+\Delta x; y_0+\Delta y) = 2{,}1^2 - 2{,}1\cdot1{,}2 + 1{,}2^2 = 3{,}33$

$\Delta z = 3{,}33 - 3 = 0{,}33$

$\blacksquare$

11.3.8

$z = \lg(x^2+y^2)$; $M_0(2;1)$; $M_1(2{,}1;0{,}9)$

$\square$

$z(x_0;y_0) = \lg(2^2+1^2) = \lg(5)$

$z(x_0+\Delta x; y_0+\Delta y) = \lg(2{,}1^2+0{,}9^2) = \lg(5{,}22)$

$\Delta z = \lg(5{,}22) - \lg(5) = \lg\left(\frac{5{,}22}{5}\right) =$
$= \lg(1{,}044) \approx 0{,}0187$

$\blacksquare$

11.3.13

$v = x^4\cdot\cos^2 y - y^4\cdot\sin^3 x^5$; $v'_x, v'_y = ?$

$\square$

$v'_x = (x^4\cdot\cos^2 y - y^4\cdot\sin^3 x^5)'_x =$
$= (x^4)'_x\cdot\cos^2 y + x^4\cdot(\cos^2 y)'_x - (y^4)'_x\cdot\sin^3 x^5 -$

$-y^4\cdot(\sin^3 x^5)'_x = 4x^3\cdot\cos^2 y - y^4\cdot 3\cdot\sin^2 x^5\cdot\cos x^5\cdot$
$\cdot 5x^4 = \underline{4x^3\cos^2 y - 15x^4y^4\sin^2 x^5\cdot\cos x^5}$
$v'_y = (x^4\cdot\cos^2 y - y^4\cdot\sin^3 x^5)'_y =$
$= (x^4)'_y\cdot\cos^2 y + x^4\cdot(\cos^2 y)'_y - (y^4)'_y\cdot\sin^3 x^5 -$
$-y^4\cdot(\sin^3 x^5)'_y = x^4\cdot 2\cdot\cos y\cdot(-\sin y) -$
$-4y^3\cdot\sin^3 x^5 = \underline{-x^4\sin 2y - 4y^3\sin^3 x^5}$

$\blacksquare$

11.3.14

$z = x^2\cdot\cos 2xy - y^2\cdot\sin(x+y)$; $z'_x, z'_y = ?$

$\square$

$z'_x = (x^2\cdot\cos 2xy - y^2\cdot\sin(x+y))'_x =$
$= (x^2)'_x\cdot\cos 2xy + x^2(\cos 2xy)'_x - (y^2)'_x\cdot\sin(x+y) -$
$-y^2\cdot(\sin(x+y))'_x = 2x\cdot\cos 2xy + x^2\cdot(-\sin(2xy)\cdot 2y -$
$-y^2\cdot\cos(x+y)\cdot(1+0) = \underline{2x\cos 2xy - 2yx^2\cdot\sin 2xy -}$
$\underline{-y^2\cdot\cos(x+y)}$
$z'_y = (x^2\cdot\cos 2xy - y^2\cdot\sin(x+y))'_y = (x^2)'_y\cdot\cos 2xy +$
$+x^2\cdot(\cos 2xy)'_y - (y^2)'_y\cdot\sin(x+y) - y^2\cdot(\sin(x+y))'_y =$
$= x^2\cdot(-\sin 2xy)\cdot 2x - 2y\cdot\sin(x+y) - y^2\cdot\cos(x+y)\cdot 1 =$
$= \underline{-2x^3\sin 2xy - 2y\cdot\sin(x+y) - y^2\cos(x+y)}$

$\blacksquare$

11.3.15

$u = x^y + (xy)^z + z^{xy}$; $u'_x, u'_y, u'_z = ?$

$\square$

$u'_x = (x^y + (xy)^z + z^{xy})'_x = y \cdot x^{y-1} + y^z \cdot z \cdot x^{z-1} +$
$+ z^{xy} \cdot \ln(z^y) = \underline{yx^{y-1} + y^z z x^{z-1} + y \cdot z^{xy} \cdot \ln(z)}$

$u'_y = (x^y + (xy)^z + z^{xy})'_y = x^y \cdot \ln x + x^z \cdot z \cdot y^{z-1} +$
$+ z^{xy} \cdot \ln(z^x) = \underline{x^y \ln x + x^z \cdot z \cdot y^{z-1} + x \cdot z^{xy} \cdot \ln(z)}$

$u'_z = (x^y + (xy)^z + z^{xy})'_z = 0 + (xy)^z \cdot \ln(xy) +$
$+ xy \cdot z^{xy-1} = \underline{(xy)^z \cdot \ln(xy) + xyz^{xy-1}}$ $\blacksquare$

11.3.19

Найти $1{,}04^{2{,}03}$

$\square$

$z = f(x; y) = x^y$

$x = 1{,}04 \quad y = 2{,}03$

$x = 1{,}04 = x_0 + \Delta x = 1 + 0{,}04$

$y = 2{,}03 = y_0 + \Delta y = 2 + 0{,}03$

$f(x_0; y_0) = f(1; 2) = 1^2 = 1$

$f'_x = (x^y)'_x = y \cdot x^{y-1} \Rightarrow f'_x(x_0; y_0) = f'_x(1;2) = 2 \cdot 1^{2-1} = 2$

$f'_y = (x^y)'_y = x^y \cdot \ln x \Rightarrow f'_y(x_0, y_0) = f'_y(1;2) = 1^2 \cdot \ln(1) = 0$

$\Downarrow$

$1{,}04^{2{,}03} \approx 1 + 2 \cdot 0{,}04 + 0 \cdot 0{,}03 = \underline{1{,}08}$ ■

11.3.20

Найти $\sqrt{(1{,}04)^2 + (3{,}01)^2}$

□

$z = f(x; y) = \sqrt{x^2 + y^2}$

$x = 1{,}04 = x_0 + \Delta x = 1 + 0{,}04$

$y = 3{,}01 = y_0 + \Delta y = 3 + 0{,}01$

$f(x_0; y_0) = f(1;3) = \sqrt{1^2 + 3^2} = \sqrt{10}$

$f'_x = (\sqrt{x^2+y^2})'_x = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}} \Rightarrow$

$\Rightarrow f'_x(x_0; y_0) = f'_x(1;3) = \frac{1}{\sqrt{10}}$

Ан-но получаем:

$f'_y = \frac{y}{\sqrt{x^2+y^2}} \Rightarrow f'_y(x_0, y_0) = f'_y(1;3) = \frac{3}{\sqrt{10}}$

$\Downarrow$

$\sqrt{(1{,}04)^2 + (3{,}01)^2} \approx \sqrt{10} + \frac{1}{\sqrt{10}} \cdot 0{,}04 + \frac{3}{\sqrt{10}} \cdot 0{,}01 =$

$= \frac{10{,}07}{\sqrt{10}} = 1{,}007 \cdot \sqrt{10} \approx \underline{3{,}185}$ ■

11.3.21

$\sin 28° \cdot \cos 61° \approx ?$

$\square$

$z = f(x;y) = \sin x \cdot \cos y$

$x = 28° = x_0 + \Delta x = 30 + (-2)$

$y = 61° = y_0 + \Delta y = 60 + 1$

$f(x_0;y_0) = \sin 30° \cdot \cos 60° = \frac{1}{4} = 0,25$

$f'_x = (\sin x \cdot \cos y)'_x = \cos x \cdot \cos y \Rightarrow$

$\Rightarrow f'_x(x_0;y_0) = f'_x(30°;60°) = \cos 30° \cdot \cos 60° = \frac{\sqrt{3}}{4}$

$f'_y = (\sin x \cdot \cos y)'_y = -\sin x \cdot \sin y \Rightarrow$

$\Rightarrow f'_y(x_0;y_0) = f'_y(30°;60°) = -\sin 30° \cdot \sin 60° = -\frac{\sqrt{3}}{4}$

$\Downarrow$

$\sin 28° \cdot \cos 61° \approx 0,25 + \frac{\sqrt{3}}{4} \cdot \left(\frac{-\pi}{90}\right) + \left(-\frac{\sqrt{3}}{4}\right) \cdot \frac{\pi}{180} =$

$= 0,25 - \frac{\sqrt{3}}{4}\left(\frac{3\pi}{180}\right) = 0,25 - \frac{\sqrt{3}}{4} \cdot \frac{\pi}{60} \approx \underline{0,227}$ $\blacksquare$

11.3.23

$\operatorname{arctg} \frac{1,02}{0,95} \approx ?$

$\square$

$x = 1,02 = x_0 + \Delta x = 1 + 0,02$

$y = 0,95 = y_0 + \Delta y = 1 + (-0,05)$

$f(x_0;y_0) = arctg\left(\frac{1}{1}\right) = \frac{\pi}{4} = 0,785$

$f'_x = \left(arctg\,\frac{x}{y}\right)'_x = \frac{y}{y^2+x^2} \Rightarrow f'_x(x_0;y_0) = \frac{1}{1+1} = \frac{1}{2}$

$f'_y = \left(arctg\,\frac{x}{y}\right)'_y = -\frac{x}{y^2+x^2} \Rightarrow f'_y(x_0;y_0) = -\frac{1}{1+1} = -\frac{1}{2}$

$\Downarrow$

$arctg\,\frac{1,02}{0,95} \approx 0,785 + \frac{1}{2}\cdot 0,02 - \frac{1}{2}\cdot(-0,05) =$

$= 0,785 + 0,01 + 0,025 = \underline{0,82}$ $\blacksquare$

11.3.24

$\sqrt{5e^{0,02} + 2,03^2} \approx ?$

$\square$

$f(x;y) = \sqrt{5e^x + y^2}$

$x = 0,02 = x_0 + \Delta x = 0 + 0,02$

$y = 2,03 = y_0 + \Delta y = 2 + 0,03$

$f(x_0;y_0) = \sqrt{5\cdot e^0 + 2^2} = \sqrt{5+4} = 3$

$f'_x = \left(\sqrt{5e^x+y^2}\right)'_x = \frac{1}{2\sqrt{5e^x+y^2}}\cdot 5e^x = \frac{5e^x}{2\sqrt{5e^x+y^2}} \Rightarrow$

$\Rightarrow f'_x(x_0;y_0) = \frac{5\cdot e^0}{2\sqrt{5\cdot e^0+4}} = \frac{5}{2\cdot 3} = \frac{5}{6}$

$f'_y = \left(\sqrt{5e^x+y^2}\right)'_y = \frac{2y}{2\cdot\sqrt{5e^x+y^2}} = \frac{y}{\sqrt{5e^x+y^2}} \Rightarrow$

$\Rightarrow f'_y(x_0;y_0)=\frac{2}{3}$

$\Downarrow$

$\sqrt{5e^{0,02}+2,03^2}\approx 3+\frac{5}{6}\cdot 0,02+\frac{2}{3}\cdot 0,03=$

$=3+\frac{1}{60}+\frac{1}{50}\approx \underline{3,037}$ ■

11.3.25

$\ln(0,09^3+0,99^3)\approx ?$

□

$f(x;y)=\ln(x^3+y^3)$

$x=0,09=x+\Delta x=0+0,09$

$y=0,99=y+\Delta y=1+(-0,01)$

$f(x_0;y_0)=\ln(0^3+1^3)=\ln(1)=0$

$f'_x=(\ln(x^3+y^3))'_x=\frac{1}{x^3+y^3}\cdot 3x^2=\frac{3x^2}{x^3+y^3}\Rightarrow$

$\Rightarrow f'_x(x_0;y_0)=\frac{3\cdot 0^2}{0^3+1^3}=0$

Ан-но: $f'_y=\frac{3y^2}{x^3+y^3}\Rightarrow f'_y(x_0;y_0)=\frac{3\cdot 1^2}{0^3+1^3}=3$

$\Downarrow$

$\ln(0,09^3+0,99^3)\approx 0+0\cdot 0,09+3\cdot(-0,01)=\underline{-0,03}$

■

11.3.27

$1{,}002 \cdot 2{,}003^2 \cdot 3{,}004^3 \approx ?$

$\square$

$f(x;y;z) = x \cdot y^2 \cdot z^3$

$x = 1{,}002 = x_0 + \Delta x = 1 + 0{,}002$

$y = 2{,}003 = y_0 + \Delta y = 2 + 0{,}003$

$z = 3{,}004 = z_0 + \Delta z = 3 + 0{,}004$

$f(x_0;y_0;z_0) = 1 \cdot 2^2 \cdot 3^3 = 1 \cdot 4 \cdot 27 = 108$

$f'_x = (x \cdot y^2 \cdot z^3)'_x = y^2 z^3 \Rightarrow f'_x(x_0;y_0;z_0) = 2^2 \cdot 3^3 = 108$

$f'_y = (x \cdot y^2 \cdot z^3)'_y = 2y \cdot x \cdot z^3 \Rightarrow f'_y(x_0;y_0;z_0) = 2 \cdot 2 \cdot 1 \cdot 3^3 = 108$

$f'_z = (x \cdot y^2 \cdot z^3)'_z = x \cdot y^2 \cdot 3 \cdot z^2 \Rightarrow f'_z(x_0;y_0;z_0) = 1 \cdot 2^2 \cdot 3 \cdot 3^2 = 108$

$\Downarrow$

$1{,}002 \cdot 2{,}003^2 \cdot 3{,}004^3 \approx 108 + 108 \cdot 0{,}002 + 108 \cdot 0{,}003 + 108 \cdot 0{,}004 =$

$= 108 \cdot 1{,}009 = \underline{108{,}972}$

$\blacksquare$

11.3.28

$$\frac{1{,}03^2}{\sqrt[3]{0{,}98 \cdot \sqrt[4]{1{,}05^3}}} \approx ?$$

$\square$

$$f(x;y;z) = \frac{x^2}{y^{\frac{1}{3}} \cdot z^{\frac{1}{4}}}$$

$$X = 1{,}03 = X_0 + \Delta X = 1 + 0{,}03$$

$$Y = 0{,}98 = Y_0 + \Delta y = 1 + (-0{,}02)$$

$$Z = 1{,}05 = Z_0 + \Delta Z = 1 + (0{,}05)$$

$$f(x_0; y_0; z_0) = \frac{1^2}{1^{\frac{1}{3}} \cdot 1^{\frac{1}{4}}} = \frac{1}{2} = 1$$

$$f'_x = \left(\frac{x^2}{y^{\frac{1}{3}} \cdot z^{\frac{1}{4}}}\right)'_x = \frac{2x}{y^{\frac{1}{3}} \cdot z^{\frac{1}{4}}} \Rightarrow f'_x(x_0, y_0, z_0) = \frac{2 \cdot 1}{1 \cdot 1} = 2$$

$$f'_y = \left(\frac{x^2}{y^{1/3} \cdot z^{1/4}}\right)'_y = \frac{x^2}{z^{1/4}} \cdot \left(y^{-\frac{1}{3}}\right)'_y = \frac{x^2}{z^{1/4}} \cdot \left(-\frac{1}{3}\right) \cdot y^{-\frac{4}{3}} \Rightarrow$$

$$\Rightarrow f'_y(x_0; y_0; z_0) = -\frac{1}{3} \cdot \frac{1}{1 \cdot 1} = -\frac{1}{3}$$

$$f'_z = \left(\frac{x^2}{y^{1/3} \cdot z^{1/4}}\right)'_z = \frac{x^2}{y^{1/3}} \cdot \left(-\frac{1}{4}\right) \cdot z^{-\frac{5}{4}} \Rightarrow$$

$$\Rightarrow f'_z(x_0; y_0; z_0) = -\frac{1}{4} \cdot \frac{1}{1 \cdot 1} = -\frac{1}{4}$$

$$\Downarrow$$

$$\frac{1{,}03^2}{\sqrt[3]{0{,}98 \cdot \sqrt[4]{1{,}05^3}}} \approx 1 + 2 \cdot 0{,}03 - \frac{1}{3} \cdot (-0{,}02) - \frac{1}{4} \cdot 0{,}05 \approx$$

$$\approx \underline{1{,}054}$$