"Интегралы"

Домашняя работа часть 1.

8.1.29

$$\int \frac{dx}{x^2\sqrt{x}} = \int x^{-\frac{5}{2}} dx = \frac{x^{-\frac{5}{2}+1}}{-\frac{5}{2}+1} + C = x^{-\frac{3}{2}} \cdot \left(-\frac{2}{3}\right) + C =$$

$$= -\frac{2}{3x\sqrt{x}} + C$$

8.1.30

$$\int \frac{dx}{x^2+3} = \int \frac{dx}{x^2+(\sqrt{3})^2} = \left[\int \frac{dx}{x^2+a^2};\ a=\sqrt{3}\right] =$$

$$= \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{x}{\sqrt{3}} + C = \frac{\sqrt{3} \operatorname{arctg}\left(\frac{\sqrt{3}x}{3}\right)}{3} + C$$

8.1.31

$$\int \frac{1}{5^x} dx = \int 5^{-x} dx = \left[\int a^x dx;\ a=5\right] = \frac{5^{-x}}{\ln(\frac{1}{5})} + C =$$

$$= -\frac{1}{\ln(5) \cdot 5^x} + C$$

8.1.32

$$\int \frac{dx}{\sqrt{4-x^2}} = \int \frac{dx}{\sqrt{2^2-x^2}} = \left[\int \frac{dx}{\sqrt{a^2-x^2}};\ a=2\right] = \arcsin \frac{x}{2} + C$$

8.1.33

$$\int \frac{dx}{\sqrt{x^2-1}} = \left[\frac{dx}{\sqrt{x^2+a}};\ a=-1\right] = \ln(|x+\sqrt{x^2-1}|) + C$$

8.1.34

$$\int\frac{dx}{x^2-25}=\int\frac{dx}{x^2-5^2}=\left[\int\frac{dx}{x^2-a^2};\ a=5\right]=$$

$$=\frac{1}{2\cdot5}\cdot\ln\left|\frac{x-5}{x+5}\right|+C=\frac{1}{10}\cdot\ln\left|\frac{x-5}{x+5}\right|+C$$

8.1.35

$$\int\left(x+\frac{2}{x}\right)^2dx=\int\left(x^2+4+\frac{4}{x^2}\right)dx=\int x^2dx+$$

$$+\int4dx+\int\frac{4}{x^2}dx=\frac{x^3}{3}+4x-\frac{4}{x}+C$$

8.1.36

$$\int\frac{dx}{4x^2+1}=\int\frac{1}{4}\cdot\frac{dx}{x^2+\frac{1}{4}}=\frac{1}{4}\int\frac{dx}{x^2+(\frac{1}{2})^2}=\frac{1}{4}\cdot2\operatorname{arctg}(2x)+C=$$

$$=\frac{\operatorname{arctg}2x}{2}+C$$

8.1.37

$$\int\left(7^x-\frac{8}{x}+4\cos x\right)dx=\int7^xdx-8\int\frac{1}{x}dx+4\int\cos x\,dx=$$

$$=\frac{7^x}{\ln7}-8\cdot\ln|x|+4\cdot\sin x+C$$

8.1.38

$$\int\left(\frac{\sqrt{3}}{\cos^2x}-\sqrt[3]{x}-\frac{2}{x^4}\right)dx=\sqrt{3}\cdot\int\frac{dx}{\cos^2x}-\int x^{\frac{1}{3}}dx-$$

$$-2\int x^{-4}dx=\sqrt{3}\operatorname{tg}x-\frac{3}{4}\cdot x^{\frac{4}{3}}-2\cdot\frac{x^{-3}}{-3}+C=$$

$$=\sqrt{3}\,tg x-\frac{3x\sqrt[3]{x}}{4}+\frac{2}{3x^3}+C$$

8.1.39

$$\int\frac{\sqrt{x}-3\sqrt[5]{x^2}+1}{\sqrt[4]{x}}dx=\int\frac{x^{\frac{1}{2}}}{x^{\frac{1}{4}}}dx-3\int\frac{x^{\frac{2}{5}}}{x^{\frac{1}{4}}}dx+\int x^{-\frac{1}{4}}dx=$$

$$=\int x^{\frac{1}{4}}dx-3\int x^{\frac{3}{20}}dx+\int x^{-\frac{1}{4}}dx=\frac{x^{\frac{5}{4}}}{\frac{5}{4}}-3\cdot\frac{x^{\frac{23}{20}}}{\frac{23}{20}}+$$

$$+\frac{x^{\frac{3}{4}}}{\frac{3}{4}}+C=\frac{4x\sqrt[4]{x}}{5}-\frac{60x\sqrt[20]{x^3}}{23}+\frac{4\sqrt[4]{x^3}}{3}+C$$

8.1.40

$$\int(0{,}7\cdot x^{-0{,}1}+0{,}2\cdot(0{,}5)^x)dx=0{,}7\cdot\int x^{-0{,}1}dx+$$

$$+0{,}2\int(0{,}5)^x dx=\frac{7}{10}\cdot\frac{x^{\frac{9}{10}}}{\frac{9}{10}}+\frac{1}{5}\cdot\frac{0{,}5^x}{\ln 0{,}5}+C=$$

$$=\frac{7\sqrt[10]{x^9}}{9}-\frac{1}{5\ln(2)\cdot 2^x}+C$$

8.1.41

$$\int(5\,sh x-7\,ch x+1)dx=5\int sh x\,dx-7\int ch x\,dx+$$

$$+\int 1dx=5\,ch x-7\,sh x+x+C$$

8.1.42

$$\int(x^2-1)(\sqrt{x}+4)dx=\int(x^2\sqrt{x}-\sqrt{x}+4x^2-4)dx=$$

$$=\int x^{\frac{5}{2}}dx-\int x^{\frac{1}{2}}dx+4\int x^2dx-4\int 1dx=$$

$$=\frac{2x^3\sqrt{x}}{7}-\frac{2x\sqrt{x}}{3}+\frac{4x^3}{3}-4x+C$$

8.1.43

$$\int\frac{7-\sqrt{x^2+\pi}}{\sqrt{x^2+\pi}}dx=\int\left(\frac{7}{\sqrt{x^2+\pi}}-1\right)dx=$$

$$=7\cdot\int\frac{dx}{\sqrt{x^2+\pi}}-\int 1dx=7\cdot\ln(|x+\sqrt{x^2+\pi}|)-$$

$$-x+C$$

8.1.44

$$\int\left(\frac{\sqrt{x}-5}{x}\right)^3dx=\int\left(\frac{x\sqrt{x}}{x^3}-\frac{15x}{x^3}+\frac{75\sqrt{x}}{x^3}-\frac{125}{x^3}\right)dx=$$

$$=\int x^{-\frac{3}{2}}dx-15\int x^{-2}dx+75\int x^{-\frac{5}{2}}dx-125\int\frac{dx}{x^3}=$$

$$=\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}-15\cdot\frac{x^{-1}}{-1}+75\frac{x^{-\frac{3}{2}}}{-\frac{3}{2}}-125\frac{x^{-2}}{-2}+C=$$

$$=-\frac{2}{\sqrt{x}}+\frac{15}{x}-\frac{50}{x\sqrt{x}}+\frac{125}{2x^2}+C$$

8.1.45

$$\int \sin(7x)dx = \left[ax+b=7x+0;\ \int \sin x\,dx = -\cos x + C\right] =$$

$$= \frac{1}{7}\cdot(-\cos(7x)) + C = -\frac{\cos 7x}{7} + C$$

8.1.46

$$\int \sqrt[5]{2x-8}\,dx = \int (2x-8)^{\frac{1}{5}}dx = \left[ax+b=2x+(-8);\ \int x^{\alpha}dx\right] =$$

$$= \frac{1}{2}\cdot\frac{(2x-8)^{1+\frac{1}{5}}}{1+\frac{1}{5}} + C = \frac{1}{2}\cdot\frac{5}{6}\cdot(2x-8)^{1\frac{1}{5}} + C = \frac{5(2x-8)\sqrt[5]{2x-8}}{12} +$$

$$+ C = \frac{5(x-4)\sqrt[5]{2x-8}}{6} + C$$

8.1.47

$$\int (1-4x)^{2001}dx = \left[ax+b=(-4)\cdot x+1;\ \int x^{\alpha}dx\right] =$$

$$= \frac{1}{(-4)}\cdot\frac{(1-4x)^{2001+1}}{2001+1} + C = -\frac{1}{4}\cdot\frac{(1-4x)^{2002}}{2002} + C = -\frac{(1-4x)^{2002}}{8008} + C$$

8.1.48

$$\int \frac{dx}{9x+7} = \left[ax+b=9x+7;\ \int \frac{dx}{x} = \ln|x| + C\right] =$$

$$= \frac{1}{9}\cdot\ln|9x+7| + C$$

8.1.49

$$\int\frac{dx}{(6x+11)^4}=\int(6x+11)^{-4}dx=\left[ax+b=6x+11;\ \int x^{\alpha}dx\right]=$$

$$=\frac{1}{6}\cdot\frac{(6x+11)^{-3}}{-3}+C=-\frac{1}{18(6x+11)^3}+C$$

8.1.50

$$\int\frac{dx}{25x^2+1}=\int\frac{dx}{(5x)^2+1}=\left[ax+b=5x+0;\ \int\frac{dx}{x^2+1}\right]=$$

$$=\frac{1}{5}\cdot\operatorname{arctg}(5x)+C=\frac{\operatorname{arctg}(5x)}{5}+C$$

8.1.51

$$\int 3^{2-11x}dx=\left[ax+b=(-11)\cdot x+2;\ \int a^x dx\right]=$$

$$=\frac{1}{-11}\cdot\frac{3^{2-11x}}{\ln 3}+C=-\frac{3^{2-11x}}{11\cdot\ln 3}+C$$

8.1.52

$$\int\frac{dx}{\sqrt{4x^2-1}}=\int\frac{dx}{\sqrt{(2x)^2-1}}=\int\frac{dx}{\sqrt{4}\cdot\sqrt{x^2-\frac{1}{4}}}=\frac{1}{2}\int\frac{dx}{\sqrt{x^2-\frac{1}{4}}}=$$

$$=\left[\int\frac{dx}{\sqrt{x^2+a}}\right]=\frac{1}{2}\cdot\ln\left|x+\sqrt{x^2-\frac{1}{4}}\right|+C=$$

$$=\frac{1}{2}\ln\left|x+\frac{\sqrt{4x^2-1}}{2}\right|+C$$

8.1.53

$$\int \sin^2 3x\,dx = \int \frac{1-\cos(6x)}{2}dx = \int \frac{1}{2}dx -$$

$$-\frac{1}{2}\int \cos(6x)\,dx = \frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{6}\cdot\sin(6x) + C =$$

$$= \frac{1}{2}x - \frac{\sin(6x)}{12} + C$$

8.1.54

$$\int \cos^2 8x\,dx = \int \frac{1+\cos(16x)}{2}dx = \int \frac{1}{2}dx +$$

$$+\frac{1}{2}\int \cos(16x) = \frac{1}{2}x + \frac{1}{2}\cdot\frac{1}{16}\cdot\sin 16x + C =$$

$$= \frac{1}{2}x + \frac{\sin(16x)}{32} + C$$

8.1.55

$$\int tg^2 x\,dx = \int \frac{\sin^2 x}{\cos^2 x}dx = \int \frac{1-\cos^2 x}{\cos^2 x}dx = \int \frac{1}{\cos^2 x}dx -$$

$$-\int 1\,dx = tg x - x + C$$

8.1.56

$$\int \frac{4x+1}{x-5}dx = \int \frac{4x-20+21}{x-5}dx = \int 4dx + 21\int \frac{1}{x-5}dx =$$

$$= 4x + 21\ln(|x-5|) + C$$

8.1.57

$$\int(3tgx-2ctgx)^2dx=\int(9tg^2x-12tgx\cdot ctgx+$$

$$+4ctg^2x)dx=\int 9tg^2x\,dx-\int 12dx+\int 4ctg^2x\,dx=$$

$$=9\int\frac{1-\cos^2x}{\cos^2x}dx+4\int\frac{1-\sin^2x}{\sin^2x}dx-\int 12dx=$$

$$=9\left(\int\frac{1}{\cos^2x}dx-\int 1dx\right)+4\left(\int\frac{1}{\sin^2x}dx-\int 1dx\right)-\int 12dx=$$

$$=9tgx-9x+4\cdot(-ctgx)-4x-12x+C=$$

$$=\underline{9tgx-4ctgx-25x+C}$$

8.1.58

$$\int\frac{4\sqrt{1-x^2}+3x^2}{x^2-1}dx=\int\frac{4\sqrt{1-x^2}}{x^2-1}dx+\int\frac{3x^2-3+3}{x^2-1}dx=$$

$$=-4\int\frac{1}{\sqrt{1-x^2}}dx+\int 3dx+3\int\frac{1}{x^2-1}dx=$$

$$=-4\arcsin x+3x+3\cdot\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|+C=$$

$$=\underline{\frac{3}{2}\ln\left|\frac{x-1}{x+1}\right|+3x-4\arcsin x+C}$$

8.1.59

$$\int \frac{\cos 2x\,dx}{\sin^2 x \cos^2 x} = \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x} dx = \int \frac{1}{\sin^2 x} dx - \int \frac{1}{\cos^2 x} dx = -ctg x - tg x + C$$

8.1.60

$$\int \frac{\sin 2x}{\cos x} dx = \int \frac{2\sin x \cdot \cos x}{\cos x} dx = 2\int \sin x\,dx = -2\cos x + C$$