Обратная матрица.

Матричные уравнения

Домашняя работа.

1.4.37

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0,5 & 0 \end{pmatrix}$$

$\square$

1) $\det A = \begin{vmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0,5 & 0 \end{vmatrix} = (-1)\cdot\begin{vmatrix} 0 & 2 \\ 0,5 & 0 \end{vmatrix} - 0 + 0 =$

$= (-1)\cdot(0 - 2\cdot 0,5) = (-1)\cdot(-1) = 1 \neq 0 \Rightarrow \exists A^{-1}$

2) $A_{11} = (-1)^{1+1}\cdot\begin{vmatrix} 0 & 2 \\ 0,5 & 0 \end{vmatrix} = -1$; $A_{12} = (-1)^{1+2}\cdot\begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = 0$;

$A_{13} = (-1)^{1+3}\cdot\begin{vmatrix} 0 & 0 \\ 0 & 0,5 \end{vmatrix} = 0$; $A_{21} = (-1)^{2+1}\cdot\begin{vmatrix} 0 & 0 \\ 0,5 & 0 \end{vmatrix} = 0$;

$A_{22} = (-1)^{2+2}\cdot\begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0$; $A_{23} = (-1)^{2+3}\cdot\begin{vmatrix} -1 & 0 \\ 0 & 0,5 \end{vmatrix} = 0,5$;

$A_{31} = (-1)^{3+1}\cdot\begin{vmatrix} 0 & 0 \\ 0 & 2 \end{vmatrix} = 0$; $A_{32} = (-1)^{3+2}\cdot\begin{vmatrix} -1 & 0 \\ 0 & 2 \end{vmatrix} = 2$;

$A_{33} = (-1)^{3+3}\cdot\begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0$;

3) $\widetilde{A} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0,5 \\ 0 & 2 & 0 \end{pmatrix}^T = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0,5 & 0 \end{pmatrix}$

4) $A^{-1} = \frac{1}{\det A} \cdot \widetilde{A} = \frac{1}{1} \cdot \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0{,}5 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0{,}5 & 0 \end{pmatrix}$

1.4.38

$$\begin{pmatrix} 1 & 1 & -1 \\ 8 & 3 & -6 \\ -4 & -1 & 3 \end{pmatrix}$$

$\square$

1) $\det A = \begin{vmatrix} 1 & 1 & -1 \\ 8 & 3 & -6 \\ -4 & -1 & 3 \end{vmatrix} = 1 \cdot 3 \cdot 3 + (-1) \cdot 8 \cdot (-1) + 1 \cdot (-6) \cdot$

$\cdot (-4) - (-1) \cdot 3 \cdot (-4) - 1 \cdot 8 \cdot 3 - 1 \cdot (-6) \cdot (-1) =$

$= 9 + 8 + 24 - 12 - 24 - 6 = -1 \neq 0 \Rightarrow \exists A^{-1}$

2) $A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 3 & -6 \\ -1 & 3 \end{vmatrix} = 9 - 6 = 3$

$A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 8 & -6 \\ -4 & 3 \end{vmatrix} = (8 \cdot 3 - (-6) \cdot (-4)) = 0$

$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 8 & 3 \\ -4 & -1 \end{vmatrix} = 8 \cdot (-1) - 3 \cdot (-4) = 4$

$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix} = -(3 - 1) = -2$

$A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 1 & -1 \\ -4 & 3 \end{vmatrix} = 3 - (-1) \cdot (-4) = -1$

$A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 1 & 1 \\ -4 & -1 \end{vmatrix} = -(-1 - (-4)) = -3$

$$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 1 & -1 \\ 3 & -6 \end{vmatrix} = -6+3 = -3$$

$$A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 1 & -1 \\ 8 & -6 \end{vmatrix} = -(-6+8) = -2$$

$$A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 1 & 1 \\ 8 & 3 \end{vmatrix} = 3-8 = -5$$

3) $\widetilde{A} = \begin{pmatrix} 3 & 0 & 4 \\ -2 & -1 & -3 \\ -3 & -2 & -5 \end{pmatrix}^T = \begin{pmatrix} 3 & -2 & -3 \\ 0 & -1 & -2 \\ 4 & -3 & -5 \end{pmatrix}$

4) $A^{-1} = \frac{1}{\det A} \cdot \widetilde{A} = \frac{1}{-1} \cdot \begin{pmatrix} 3 & -2 & -3 \\ 0 & -1 & -2 \\ 4 & -3 & -5 \end{pmatrix} = \begin{pmatrix} -3 & 2 & 3 \\ 0 & 1 & 2 \\ -4 & 3 & 5 \end{pmatrix}$ ■

1.4.39

$$\begin{pmatrix} 1 & 1 & 2 \\ 2 & -1 & 2 \\ 4 & 1 & 4 \end{pmatrix}$$

□

1) $\det A = \begin{vmatrix} 1 & 1 & 2 \\ 2 & -1 & 2 \\ 4 & 1 & 4 \end{vmatrix} = 1 \cdot (-1) \cdot 4 + 1 \cdot 2 \cdot 4 + 2 \cdot 2 \cdot 1 -$

$- 2 \cdot (-1) \cdot 4 - 1 \cdot 2 \cdot 4 - 1 \cdot 2 \cdot 1 = -4 + 8 + 4 + 8 - 8 - 2 = 6 \neq 0 \Rightarrow \exists A^{-1}$

2) $A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -1 & 2 \\ 1 & 4 \end{vmatrix} = -4-2 = -6$

$$A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 2 & 2 \\ 4 & 4 \end{vmatrix} = -(8-8) = 0$$

$$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix} = 2-(-4) = 6$$

$$A_{21}=(-1)^{2+1}\cdot\begin{vmatrix}1 & 2\\ 1 & 4\end{vmatrix}=-(4-2)=-2$$

$$A_{22}=(-1)^{2+2}\cdot\begin{vmatrix}1 & 2\\ 4 & 4\end{vmatrix}=4-8=-4$$

$$A_{23}=(-1)^{2+3}\cdot\begin{vmatrix}1 & 1\\ 4 & 1\end{vmatrix}=-(1-4)=3$$

$$A_{31}=(-1)^{3+1}\cdot\begin{vmatrix}1 & 2\\ -1 & 2\end{vmatrix}=2-(-2)=4$$

$$A_{32}=(-1)^{3+2}\cdot\begin{vmatrix}1 & 2\\ 2 & 2\end{vmatrix}=-(2-4)=2$$

$$A_{33}=(-1)^{3+3}\cdot\begin{vmatrix}1 & 1\\ 2 & -1\end{vmatrix}=-1-2=-3$$

3) $\tilde{A}=\begin{pmatrix}-6 & 0 & 6\\ -2 & -4 & 3\\ 4 & 2 & -3\end{pmatrix}^T=\begin{pmatrix}-6 & -2 & 4\\ 0 & -4 & 2\\ 6 & 3 & -3\end{pmatrix}$

4) $A^{-1}=\frac{1}{\det A}\cdot\tilde{A}=\frac{1}{6}\cdot\begin{pmatrix}-6 & -2 & 4\\ 0 & -4 & 2\\ 6 & 3 & -3\end{pmatrix}=\begin{pmatrix}-1 & -1/3 & 2/3\\ 0 & -2/3 & 1/3\\ 1 & 1/2 & -1/2\end{pmatrix}$ ■

1.4.40

$$\begin{pmatrix}3 & 4 & 2\\ 2 & -4 & -3\\ 1 & 5 & 1\end{pmatrix}$$

□

1) $\det A=\begin{vmatrix}3 & 4 & 2\\ 2 & -4 & -3\\ 1 & 5 & 1\end{vmatrix}=3\cdot(-4)\cdot 1+4\cdot(-3)\cdot 1+2\cdot 2\cdot 5-$

$-2\cdot(-4)\cdot 1-4\cdot 2\cdot 1-3\cdot(-3)\cdot 5=-12-12+20+8-8+45=$

$= 41 \neq 0 \Rightarrow \exists A^{-1}$

2) $A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -4 & -3 \\ 5 & 1 \end{vmatrix} = -4-(-15) = 11$

$A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -(2-(-3)) = -5$

$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 2 & -4 \\ 1 & 5 \end{vmatrix} = 10-(-4) = 14$

$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 4 & 2 \\ 5 & 1 \end{vmatrix} = -(4-10) = 6$

$A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3-2 = 1$

$A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 3 & 4 \\ 1 & 5 \end{vmatrix} = -(15-4) = -11$

$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 4 & 2 \\ -4 & -3 \end{vmatrix} = -12-(-8) = -4$

$A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix} = -(-9-4) = 13$

$A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 3 & 4 \\ 2 & -4 \end{vmatrix} = -12-8 = -20$

3) $\widetilde{A} = \begin{pmatrix} 11 & -5 & 14 \\ 6 & 1 & -11 \\ -4 & 13 & -20 \end{pmatrix}^T = \begin{pmatrix} 11 & 6 & -4 \\ -5 & 1 & 13 \\ 14 & -11 & -20 \end{pmatrix}$

4) $A^{-1} = \frac{1}{\det A} \cdot \widetilde{A} = \frac{1}{41} \cdot \begin{pmatrix} 11 & 6 & -4 \\ -5 & 1 & 13 \\ 14 & -11 & -20 \end{pmatrix} = \begin{pmatrix} \frac{11}{41} & \frac{6}{41} & -\frac{4}{41} \\ -\frac{5}{41} & \frac{1}{41} & \frac{13}{41} \\ \frac{14}{41} & -\frac{11}{41} & -\frac{20}{41} \end{pmatrix}$

1.4.41

$$\begin{pmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{pmatrix}$$

□

1) $\det A = \begin{vmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{vmatrix} = 3\cdot(-3)\cdot 0 + (-1)\cdot 3\cdot 1 + 2\cdot 4\cdot 3 -$

$-2\cdot(-3)\cdot 1 - (-1)\cdot 4\cdot 0 - 3\cdot 3\cdot 3 = 0 - 3 + 24 + 6 - 0 - 27 =$

$= 0 \Rightarrow \nexists A^{-1}$

■

1.4.42

$$\begin{pmatrix} 5 & 8 & -1 \\ 2 & -3 & 2 \\ 1 & 2 & 3 \end{pmatrix}$$

□

1) $\det A = \begin{vmatrix} 5 & 8 & -1 \\ 2 & -3 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 5\cdot(-3)\cdot 3 + 8\cdot 2\cdot 1 + (-1)\cdot 2\cdot 2 -$

$-(-1)\cdot(-3)\cdot 1 - 8\cdot 2\cdot 3 - 5\cdot 2\cdot 2 = -45 + 16 - 4 - 3 -$

$-48 - 20 = -36 - 48 - 20 = -104 \neq 0 \Rightarrow \exists A^{-1}$

$A_{11} = (-1)^{1+1}\cdot \begin{vmatrix} -3 & 2 \\ 2 & 3 \end{vmatrix} = -9 - 4 = -13$

$A_{12} = (-1)^{1+2}\cdot \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix} = -(6-2) = -4$

$A_{13} = (-1)^{1+3}\cdot \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} = 4 - (-3) = 7$

$$A_{21}=(-1)^{2+1}\cdot\begin{vmatrix}8 & -1\\ 2 & 3\end{vmatrix}=-(24+2)=-26$$

$$A_{22}=(-1)^{2+2}\cdot\begin{vmatrix}5 & -1\\ 1 & 3\end{vmatrix}=15-(-1)=16$$

$$A_{23}=(-1)^{2+3}\cdot\begin{vmatrix}5 & 8\\ 1 & 2\end{vmatrix}=-(10-8)=-2$$

$$A_{31}=(-1)^{3+1}\cdot\begin{vmatrix}8 & -1\\ -3 & 2\end{vmatrix}=16-3=13$$

$$A_{32}=(-1)^{3+2}\cdot\begin{vmatrix}5 & -1\\ 2 & 2\end{vmatrix}=-(10+2)=-12$$

$$A_{33}=(-1)^{3+3}\cdot\begin{vmatrix}5 & 8\\ 2 & -3\end{vmatrix}=-15-16=-31$$

3) $$\widetilde{A}=\begin{pmatrix}-13 & -4 & 7\\ -26 & 16 & -2\\ 13 & -12 & -31\end{pmatrix}^T=\begin{pmatrix}-13 & -26 & 13\\ -4 & 16 & -12\\ 7 & -2 & -31\end{pmatrix}$$

4) $$A^{-1}=\frac{1}{\det A}\cdot\widetilde{A}=\frac{1}{-104}\cdot\begin{pmatrix}-13 & -26 & 13\\ -4 & 16 & -12\\ 7 & -2 & -31\end{pmatrix}=\begin{pmatrix}\frac{1}{8} & \frac{1}{4} & -\frac{1}{8}\\ \frac{1}{26} & -\frac{2}{13} & \frac{3}{26}\\ -\frac{7}{104} & \frac{1}{52} & \frac{31}{104}\end{pmatrix}$$

1.4.43

$$\begin{pmatrix}1 & -1 & -1\\ -1 & 2 & 1\\ -1 & 1 & 2\end{pmatrix}$$

□

$$\det A=\begin{vmatrix}1 & -1 & -1\\ -1 & 2 & 1\\ -1 & 1 & 2\end{vmatrix}=1\cdot2\cdot2+(-1)\cdot1\cdot(-1)+(-1)\cdot(-1)\cdot1\ -$$

$-(-1)\cdot 2\cdot(-1)-(-1)\cdot(-1)\cdot 2-1\cdot 1\cdot 1=4+1+1-2-2-1=1 \neq$
$\neq 0 \Rightarrow \exists A^{-1}$

$$\Gamma=\left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 & 1 & 0 \\ -1 & 1 & 2 & 0 & 0 & 1 \end{array}\right)\begin{matrix} \\ \underline{II}+I \\ \underline{III}+I \end{matrix} \sim \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right)^{\underline{I}+II+III} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right)=\Gamma_2$$

$$A^{-1}=\begin{pmatrix} 3 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

■

1.4.44

$$\begin{pmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{pmatrix}$$

□

$$\det A=\begin{vmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{vmatrix}=2\cdot 9\cdot 3+7\cdot 4\cdot 1+3\cdot 3\cdot 5-3\cdot 9\cdot 1-7\cdot 3\cdot 3-$$
$$-2\cdot 4\cdot 5=54+28+45-27-63-40=127-130=-3\neq 0 \Rightarrow$$
$\Rightarrow \exists A^{-1}$

$$\Gamma=\left(\begin{array}{ccc|ccc} 2 & 7 & 3 & 1 & 0 & 0 \\ 3 & 9 & 4 & 0 & 1 & 0 \\ 1 & 5 & 3 & 0 & 0 & 1 \end{array}\right)\begin{matrix} \\ 2\cdot\underline{II}-3\cdot I \\ 2\cdot\underline{III}-I \end{matrix} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 2 & 7 & 3 & 1 & 0 & 0 \\ 0 & -3 & -1 & -3 & 2 & 0 \\ 0 & 3 & 3 & -1 & 0 & 2 \end{array}\right)\begin{matrix} \\ \\ \underline{III}+II \end{matrix} \sim$$

$$\sim\left(\begin{array}{ccc|ccc} 2 & 7 & 3 & 1 & 0 & 0 \\ 0 & -3 & -1 & -3 & 2 & 0 \\ 0 & 0 & 2 & -4 & 2 & 2 \end{array}\right) \begin{array}{l} I + \frac{7}{3}\cdot II - \frac{1}{3} III \\ II + \frac{1}{2}\cdot III \end{array} \sim$$

$$\sim\left(\begin{array}{ccc|ccc} 2 & 0 & 0 & -14/3 & 4 & -2/3 \\ 0 & -3 & 0 & -5 & 3 & 1 \\ 0 & 0 & 2 & -4 & 2 & 2 \end{array}\right) \begin{array}{l} I : 2 \\ II : (-3) \\ III : 2 \end{array} \sim$$

$$\sim\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -7/3 & 2 & -1/3 \\ 0 & 1 & 0 & 5/3 & -1 & -1/3 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right) = \Gamma_2$$

$$A^{-1} = \begin{pmatrix} -\frac{7}{3} & 2 & -\frac{1}{3} \\ \frac{5}{3} & -1 & -\frac{1}{3} \\ -2 & 1 & 1 \end{pmatrix}$$

$\blacksquare$

1.4.45

$$\begin{pmatrix} 1 & 2 & -2 & 4 \\ 2 & 6 & 1 & 0 \\ 3 & 0 & 1 & 2 \\ -1 & 4 & 5 & -4 \end{pmatrix}$$

$\square$

$$\det A = \begin{vmatrix} 1 & 2 & -2 & 4 \\ 2 & 6 & 1 & 0 \\ 3 & 0 & 1 & 2 \\ -1 & 4 & 5 & -4 \end{vmatrix} = 1\cdot \begin{vmatrix} 6 & 1 & 0 \\ 0 & 1 & 2 \\ 4 & 5 & -4 \end{vmatrix} - 2\cdot \begin{vmatrix} 2 & 1 & 0 \\ 3 & 1 & 2 \\ -1 & 5 & -4 \end{vmatrix} -$$

$$-2\cdot \begin{vmatrix} 2 & 6 & 0 \\ 3 & 0 & 2 \\ -1 & 4 & -4 \end{vmatrix} - 4\cdot \begin{vmatrix} 2 & 6 & 1 \\ 3 & 0 & 1 \\ -1 & 4 & 5 \end{vmatrix} = (6\cdot 1\cdot(-4) + 1\cdot 2\cdot 4 + 0 - 0 - 0 -$$

$$-6\cdot 2\cdot 5) - 2\cdot(2\cdot 1\cdot(-4) + 0 + 1\cdot 2\cdot(-1) - 0 - 3\cdot 1\cdot(-4) - 2\cdot 2\cdot 5) -$$

$$-2\cdot(0 + 6\cdot 2\cdot(-1) + 0 - 0 - 6\cdot 3\cdot(-4) - 2\cdot 2\cdot 4) - 4\cdot(0 + 6\cdot 1\cdot(-1) +$$

$$+1\cdot3\cdot4-0-6\cdot3\cdot5-2\cdot4\cdot1)=(-24+8-60)-$$
$$-2\cdot(-8-2+12-20)-2\cdot(-12+72-16)-$$
$$-4\cdot(-6+12-90-8)=-76+36-88+368=$$
$$=240\neq0 \Rightarrow \exists A^{-1}$$

$$\Gamma=\left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 2 & 6 & 1 & 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 1 & 2 & 0 & 0 & 1 & 0 \\ -1 & 4 & 5 & -4 & 0 & 0 & 0 & 1 \end{array}\right)\begin{array}{l} \\ \mathrm{II}-2\cdot\mathrm{I} \\ \mathrm{III}-3\mathrm{I} \\ \mathrm{IV}+\mathrm{I} \end{array} \sim$$

$$\sim\left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & 5 & -8 & -2 & 1 & 0 & 0 \\ 0 & -6 & 7 & -10 & -3 & 0 & 1 & 0 \\ 0 & 6 & 3 & 0 & 1 & 0 & 0 & 1 \end{array}\right)\begin{array}{l} \\ \\ \mathrm{III}+3\cdot\mathrm{II} \\ \mathrm{IV}-3\cdot\mathrm{II} \end{array} \sim$$

$$\sim\left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & 5 & -8 & -2 & 1 & 0 & 0 \\ 0 & 0 & 22 & -34 & -9 & 3 & 1 & 0 \\ 0 & 0 & -12 & 24 & 7 & -3 & 0 & 1 \end{array}\right)\begin{array}{l} \\ \\ \\ 11\cdot\mathrm{IV}+6\cdot\mathrm{III} \end{array} \sim$$

$$\sim\left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & 5 & -8 & -2 & 1 & 0 & 0 \\ 0 & 0 & 22 & -34 & -9 & 3 & 1 & 0 \\ 0 & 0 & 0 & 60 & 23 & -15 & 6 & 11 \end{array}\right)\begin{array}{l} \mathrm{I}-\mathrm{II} \\ \\ \\ \\ \end{array} \sim$$

$$\sim\left(\begin{array}{cccc|cccc} 22 & 0 & -7 & 12 & 3 & -1 & 0 & 0 \\ 0 & 44 & 5 & -8 & -2 & 1 & 0 & 0 \\ 0 & 0 & 22 & -34 & -9 & 3 & 1 & 0 \\ 0 & 0 & 0 & 60 & 23 & -15 & 6 & 11 \end{array}\right)\begin{array}{l} 22\cdot\mathrm{I}+7\cdot\mathrm{III} \\ 22\cdot\mathrm{II}-5\cdot\mathrm{III} \\ \\ \\ \end{array} \sim$$

$$\sim\left(\begin{array}{cccc|cccc} 22 & 0 & 0 & 26 & 3 & -1 & 7 & 0 \\ 0 & 44 & 0 & -6 & 1 & 7 & -5 & 0 \\ 0 & 0 & 22 & -34 & -9 & 3 & 1 & 0 \\ 0 & 0 & 0 & 60 & 23 & -15 & 6 & 11 \end{array}\right)\begin{array}{l} 30\cdot\mathrm{I}-13\cdot\mathrm{IV} \\ 10\cdot\mathrm{II}+\mathrm{IV} \\ 30\cdot\mathrm{III}+17\cdot\mathrm{IV} \\ \\ \end{array}$$

$$\sim\left(\begin{array}{cccc|cccc} 660 & 0 & 0 & 0 & -209 & 165 & 132 & -143 \\ 0 & 440 & 0 & 0 & 33 & 55 & -44 & 11 \\ 0 & 0 & 660 & 0 & 121 & -165 & 132 & 187 \\ 0 & 0 & 0 & 60 & 23 & -15 & 6 & 11 \end{array}\right) \begin{array}{l} \mathrm{I}/660 \\ \mathrm{II}/440 \\ \mathrm{III}/660 \\ \mathrm{IV}/60 \end{array} \sim$$

$$\sim\begin{pmatrix} -19/60 & 1/4 & 1/5 & -13/60 \\ 3/40 & 1/8 & -1/10 & 1/40 \\ 11/60 & -1/4 & 1/5 & 17/60 \\ 23/60 & -1/4 & 1/10 & 11/60 \end{pmatrix} = \Gamma_2$$

$$A^{-1} = \begin{pmatrix} -\frac{19}{60} & \frac{1}{4} & \frac{1}{5} & -\frac{13}{60} \\ \frac{3}{40} & \frac{1}{8} & -\frac{1}{10} & \frac{1}{40} \\ \frac{11}{60} & -\frac{1}{4} & \frac{1}{5} & \frac{17}{60} \\ \frac{23}{60} & -\frac{1}{4} & \frac{1}{10} & \frac{11}{60} \end{pmatrix}$$

$\blacksquare$

1.4.50

$$X \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$X \cdot A = B \Rightarrow X = B \cdot A^{-1}$$

$$\det A = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1\cdot 4 - 2\cdot 3 = -2 \neq 0 \Rightarrow \exists A^{-1}$$

$$\Gamma = \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right)_{\mathrm{II}-3\cdot\mathrm{I}} \sim \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{array}\right)^{\mathrm{I}+\mathrm{II}} \sim$$

$$\sim \left(\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & -2 & -3 & 1 \end{array}\right)_{\mathrm{II}/-2} \sim \left(\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & 1{,}5 & -0{,}5 \end{array}\right) \Rightarrow A^{-1} = \begin{pmatrix} -2 & 1 \\ 1{,}5 & -0{,}5 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} -2 & 1 \\ 1{,}5 & -0{,}5 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

1.4.51

$$X \cdot \begin{pmatrix} 4 & 3 \\ -5 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$X \cdot A = B \Rightarrow X = B \cdot A^{-1}$$

$$\det A = \begin{vmatrix} 4 & 3 \\ -5 & -4 \end{vmatrix} = -16-(-15) = -1 \neq 0 \Rightarrow \exists A^{-1}$$

$$\Gamma = \left(\begin{array}{cc|cc} 4 & 3 & 1 & 0 \\ -5 & -4 & 0 & 1 \end{array}\right) 4 \cdot \mathrm{II} + 5 \cdot \mathrm{I} \sim \left(\begin{array}{cc|cc} 4 & 3 & 1 & 0 \\ 0 & -1 & 5 & 4 \end{array}\right) \mathrm{I}+3\mathrm{II} \sim$$

$$\sim \left(\begin{array}{cc|cc} 4 & 0 & 16 & 12 \\ 0 & -1 & 5 & 4 \end{array}\right) \begin{matrix} \mathrm{I}/4 \\ \mathrm{II}/-1 \end{matrix} \sim \left(\begin{array}{cc|cc} 1 & 0 & 4 & 3 \\ 0 & 1 & -5 & -4 \end{array}\right) \Rightarrow A^{-1} = \begin{pmatrix} 4 & 3 \\ -5 & -4 \end{pmatrix}$$

$$X = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 4 & 3 \\ -5 & -4 \end{pmatrix} = \underline{\begin{pmatrix} 4 & 3 \\ -5 & -4 \end{pmatrix}}$$

1.4.52

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$A \cdot X = B \Rightarrow X = A^{-1} \cdot B$$

$$\det A = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1-1 = 0 \Rightarrow \nexists A^{-1}$$

Ответ: решений нет.

1.4.53

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$A \cdot X = B \Rightarrow X = A^{-1} \cdot B$$

$\det A = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1 - 1 = 0 \Rightarrow \nexists A^{-1}$

Ответ: решений нет

1.4.54

$\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \cdot X \cdot \begin{pmatrix} -5 & 6 \\ -4 & 5 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

$A \cdot X \cdot C = B \Rightarrow X = A^{-1} \cdot B \cdot C^{-1}$

$\det A = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3 + 2 = 5 \Rightarrow \exists A^{-1}$

$\det C = \begin{vmatrix} -5 & 6 \\ -4 & 5 \end{vmatrix} = -25 + 24 = -1 \Rightarrow \exists C^{-1}$

$A^{-1} = \frac{1}{\det A} \cdot \tilde{A} = \frac{1}{5} \cdot \begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix}$

$C^{-1} = \frac{1}{\det C} \cdot \tilde{A} = \frac{1}{-1} \cdot \begin{pmatrix} 5 & -6 \\ 4 & -5 \end{pmatrix} = \begin{pmatrix} -5 & 6 \\ -4 & 5 \end{pmatrix}$

$X = \frac{1}{5} \cdot \begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} -5 & 6 \\ -4 & 5 \end{pmatrix} = \frac{1}{5} \cdot \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \cdot \begin{pmatrix} -5 & 6 \\ -4 & 5 \end{pmatrix} =$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -5 & 6 \\ -4 & 5 \end{pmatrix} = \underline{\begin{pmatrix} -5 & 6 \\ -4 & 5 \end{pmatrix}}$

1.4.55

$\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \cdot X \cdot \begin{pmatrix} 2 & -2 \\ -4 & 5 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

$A \cdot X \cdot C = B \Rightarrow X = A^{-1} \cdot B \cdot C^{-1}$

$A^{-1} = \frac{1}{5} \cdot \begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix}$ (см. №1.4.54)

$$\det C = \begin{vmatrix} 2 & -2 \\ -4 & 5 \end{vmatrix} = 10-8=2 \neq 0 \Rightarrow \exists C^{-1}$$

$$C^{-1} = \frac{1}{\det C} \cdot \widetilde{C} = \frac{1}{2} \cdot \begin{pmatrix} 5 & 2 \\ 4 & 2 \end{pmatrix}$$

$$X = \frac{1}{5} \cdot \begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \cdot \frac{1}{2} \cdot \begin{pmatrix} 5 & 2 \\ 4 & 2 \end{pmatrix} =$$

$$= \frac{1}{10} \cdot \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \cdot \begin{pmatrix} 5 & 2 \\ 4 & 2 \end{pmatrix} = \frac{1}{2} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 5 & 2 \\ 4 & 2 \end{pmatrix} =$$

$$= \frac{1}{2} \cdot \begin{pmatrix} 5 & 2 \\ 4 & 2 \end{pmatrix} = \underline{\begin{pmatrix} 2{,}5 & 1 \\ 2 & 1 \end{pmatrix}}$$

1.4.56

$$X \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{pmatrix}$$

$$X \cdot A = B \Rightarrow X = B \cdot A^{-1}$$

$$\det A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix} = 1 \cdot 2 \cdot 3 = 6 \neq 0 \Rightarrow \exists A^{-1}$$

$$A_{11} = +\begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 6;\ A_{12} = -\begin{vmatrix} 0 & 0 \\ 0 & 3 \end{vmatrix} = 0;\ A_{13} = +\begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = 0$$

$$A_{21} = -\begin{vmatrix} 0 & 0 \\ 0 & 3 \end{vmatrix} = 0;\ A_{22} = +\begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} = 3;\ A_{23} = -\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$A_{31} = +\begin{vmatrix} 0 & 0 \\ 2 & 0 \end{vmatrix} = 0;\ A_{32} = -\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0;\ A_{33} = +\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2$$

$$\widetilde{A} = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}^T = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det A} \cdot \tilde{A} = \frac{1}{6} \cdot \begin{pmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{pmatrix} \cdot \frac{1}{6} \cdot \begin{pmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \frac{1}{6} \cdot \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix} =$$

$$= \frac{1}{6} \cdot \begin{pmatrix} 0\cdot6+0\cdot0+1\cdot0 & 0\cdot0+0\cdot3+1\cdot0 & 0\cdot0+0\cdot0+1\cdot2 \\ 0\cdot6+2\cdot0+0\cdot0 & 0\cdot0+2\cdot3+0\cdot0 & 0\cdot0+2\cdot0+0\cdot2 \\ 3\cdot6+0\cdot0+0\cdot0 & 3\cdot0+0\cdot3+0\cdot0 & 3\cdot0+0\cdot0+0\cdot2 \end{pmatrix} = \frac{1}{6} \cdot \begin{pmatrix} 0 & 0 & 2 \\ 0 & 6 & 0 \\ 18 & 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & 0 & 1/3 \\ 0 & 1 & 0 \\ 3 & 0 & 0 \end{pmatrix}$$

1.4.57

$$\begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ 0 & -2 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

$$A \cdot X = B \Rightarrow X = A^{-1} \cdot B$$

$$\det A = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ 0 & -2 & 1 \end{vmatrix} = 1\cdot3\cdot1 + 0 + 3\cdot2\cdot(-2) - 0 - 1\cdot(-2)\cdot(-1) -$$

$$- 2\cdot(-2)\cdot1 = 3 - 12 - 2 + 4 = -7 \neq 0 \Rightarrow \exists A^{-1}$$

$$A_{11} = +\begin{vmatrix} 3 & -1 \\ -2 & 1 \end{vmatrix} = 1; \quad A_{12} = -\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} = -2; \quad A_{13} = +\begin{vmatrix} 2 & 3 \\ 0 & -2 \end{vmatrix} = -4;$$

$$A_{21} = -\begin{vmatrix} -2 & 3 \\ -2 & 1 \end{vmatrix} = -4; \quad A_{22} = +\begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} = 1; \quad A_{23} = -\begin{vmatrix} 1 & -2 \\ 0 & -2 \end{vmatrix} = 2;$$

$$A_{31} = +\begin{vmatrix} -2 & 3 \\ 3 & -1 \end{vmatrix} = -7; \quad A_{32} = -\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = 7; \quad A_{33} = +\begin{vmatrix} 1 & -2 \\ 2 & 3 \end{vmatrix} = 7;$$

$$\tilde{A} = \begin{pmatrix} 1 & -2 & -4 \\ -4 & 1 & 2 \\ -7 & 7 & 7 \end{pmatrix}^T = \begin{pmatrix} 1 & -4 & -7 \\ -2 & 1 & 7 \\ -4 & 2 & 7 \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det A} \cdot \tilde{A} = \frac{1}{-7} \cdot \begin{pmatrix} 1 & -4 & -7 \\ -2 & 1 & 7 \\ -4 & 2 & 7 \end{pmatrix}$$

$$X = \frac{1}{-7} \cdot \begin{pmatrix} 1 & -4 & -7 \\ -2 & 1 & 7 \\ -4 & 2 & 7 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \frac{1}{-7} \cdot \begin{pmatrix} 2+4-21 \\ -4-1+21 \\ -8-2+21 \end{pmatrix} =$$

$$= \frac{1}{-7} \cdot \begin{pmatrix} -15 \\ 16 \\ 11 \end{pmatrix} = \begin{pmatrix} 15/7 \\ -16/7 \\ -11/7 \end{pmatrix}$$

1.4.58

$$\underbrace{\begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ 0 & -2 & 1 \end{pmatrix}}_{A} \cdot X \cdot \underbrace{\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix}}_{C} = \underbrace{\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix}}_{B}$$

$A \cdot X \cdot C = B \Rightarrow X = A^{-1} \cdot B \cdot C^{-1}$

Заметим, что $C = B \Rightarrow B \cdot C^{-1} = \underline{B \cdot B^{-1} = E} \Rightarrow$

$\Rightarrow X = A^{-1} \cdot E = A^{-1}$, если $\exists A^{-1}$ и C^{-1}

$$A^{-1} = \frac{1}{-7} \cdot \begin{pmatrix} 1 & -4 & -7 \\ -2 & 1 & 7 \\ -4 & 2 & 7 \end{pmatrix} \text{ (см. №1.4.57)}$$

$$\det C = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{vmatrix} = 7 \cdot \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} - 8 \cdot \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} =$$

$$= 7 \cdot (12-15) - 8 \cdot (6-12) = -21 + 48 = 27 \neq 0 \Rightarrow \exists C^{-1}$$

Значит $\exists A^{-1}$ и C^{-1} =>

$$\Rightarrow X = A^{-1} \cdot E = A^{-1} = \frac{1}{-7} \cdot \begin{pmatrix} 1 & -4 & -7 \\ -2 & 1 & 7 \\ -4 & 2 & 7 \end{pmatrix} = \begin{pmatrix} -\frac{1}{7} & \frac{4}{7} & 1 \\ \frac{2}{7} & -\frac{1}{7} & -1 \\ \frac{4}{7} & -\frac{2}{7} & -1 \end{pmatrix}$$