„Производные"

Практическая работа часть 1.

7.1.4

$y=\sqrt{x}$

$\square$

$\Delta y = y(x+\Delta x) - y(x) = \sqrt{x+\Delta x} - \sqrt{x}$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\frac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x} = \lim_{\Delta x\to 0}\frac{(\sqrt{x+\Delta x}-\sqrt{x})\cdot(\sqrt{x+\Delta x}+\sqrt{x})}{\Delta x\cdot(\sqrt{x+\Delta x}+\sqrt{x})} =$$

$$= \lim_{\Delta x\to 0}\frac{x+\Delta x - x}{\Delta x\cdot(\sqrt{x+\Delta x}+\sqrt{x})} = \lim_{\Delta x\to 0}\frac{\Delta x}{\Delta x(\sqrt{x+\Delta x}+\sqrt{x})} =$$

$$= \lim_{\Delta x\to 0}\frac{1}{\sqrt{x+\Delta x}+\sqrt{x}} = \frac{1}{2\sqrt{x}} \quad \blacksquare$$

7.1.5

$y=\frac{1}{x}$

$\square$

$$\Delta y = y(x+\Delta x) - y(x) = \frac{1}{x+\Delta x} - \frac{1}{x} = \frac{-\Delta x}{x(x+\Delta x)}$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\frac{\frac{-\Delta x}{(x+\Delta x)\cdot x}}{\Delta x} = \lim_{\Delta x\to 0}\frac{-\Delta x}{\Delta x\cdot x\cdot(x+\Delta x)} =$$

$$= \lim_{\Delta x\to 0} -\frac{1}{x(x+\Delta x)} = -\frac{1}{x(x+0)} = -\frac{1}{x^2} \quad \blacksquare$$

7.1.8

$y'_x=(ax^2+bx+c)'_x=(ax^2)'_x+(bx)'_x+c'_x=$

$=2ax+b$

7.1.9

$y'_x=(6x^7+4x^3-\frac{1}{8}x)'_x=(6x^7)'_x+(4x^3)'_x-$

$-(\frac{1}{8}x)'_x=42x^6+12x^2-\frac{1}{8}$

7.1.12

$y'_x=\left(\frac{1}{\sqrt[3]{x^2}}-\frac{2}{x^3}+\sqrt{7}\cdot x\right)'_x=(x^{-\frac{2}{3}})'_x-(2\cdot x^{-3})'_x+$

$+(\sqrt{7}\cdot x)'_x=-\frac{2}{3}\cdot x^{-\frac{5}{3}}+6\cdot x^{-4}+\sqrt{7}=-\frac{2}{3x\sqrt[3]{x^2}}+\frac{6}{x^4}+\sqrt{7}$

7.1.13

$y'_x=(x\sqrt[4]{x}+3\sin 1)'_x=(x^{\frac{5}{4}})'_x+(3\sin 1)'_x=$

$=\frac{5}{4}\cdot x^{\frac{1}{4}}+0=\frac{5\sqrt[4]{x}}{4}$

7.1.15

$y'_x=(\operatorname{tg}x-\operatorname{ctg}x)'_x=(\operatorname{tg}x)'_x-(\operatorname{ctg}x)'_x=$

$=\frac{1}{\cos^2x}-\left(-\frac{1}{\sin^2x}\right)=\frac{1}{\cos^2x}+\frac{1}{\sin^2x}=$

$=\frac{\sin^2x+\cos^2x}{\cos^2x\cdot\sin^2x}=\frac{1}{(\sin x\cdot\cos x)^2}=\frac{1\cdot 2\cdot 2}{(\frac{1}{2}\cdot\sin x\cdot\cos x)^2}=$

$$= \frac{4}{(\sin 2x)^2} = \frac{4}{\sin^2(2x)}$$

7.1.16

$$y'_x = (-10\,\mathrm{arctg}\,x + 7\cdot e^x)'_x = (-10\,\mathrm{arctg}\,x)'_x +$$
$$+ (7\cdot e^x)'_x = -10\cdot\frac{1}{1+x^2} + 7\cdot e^x = \frac{-10}{1+x^2} + 7\cdot e^x$$

7.1.20

$$z'_y = ((\sqrt{y}+1)\arcsin y)'_y = (\sqrt{y}+1)'_y\cdot\arcsin y +$$
$$+ (\arcsin y)'_y\cdot(\sqrt{y}+1) = \frac{1}{2\sqrt{y}}\cdot\arcsin y +$$
$$+ \frac{1}{\sqrt{1-y^2}}\cdot(\sqrt{y}+1) = \frac{\arcsin y}{2\sqrt{y}} + \frac{\sqrt{y}+1}{\sqrt{1-y^2}}$$

7.1.21

$$u'_v = \left(\frac{21^v}{21^v+1}\right)'_v = \frac{(21^v)'_v\cdot(21^v+1) - 21^v\cdot(21^v+1)'_v}{(21^v+1)^2} =$$

$$= \frac{21^v\cdot\ln 21\cdot 21^v + 21^v\cdot\ln 21\cdot 1 - 21^v\cdot 21^v\cdot\ln 21}{(21^v+1)^2} = \frac{21^v\cdot\ln 21}{(21^v+1)^2}$$

7.1.24 $y'(x_0) - ?$

$y = x^4 + x^3 - 17^5,\ x_0 = 1$

□

$$y'_x = (x^4 + x^3 - 17^5)'_x = (x^4)'_x + (x^3)'_x - (17^5)'_x = 4x^3 + 3x^2$$

$y'_x(x_0) = y'_x(1) = 4\cdot 1^3 + 3\cdot 1^2 = 4+3 = \underline{7}$ ■

<u>7.1.25</u> $y'(x_0) - ?$

$y = \frac{\ln x}{x}$, $x_0 = e$

□

$$y'_x = \left(\frac{\ln x}{x}\right)'_x = \frac{(\ln x)'_x \cdot x - \ln x \cdot (x)'_x}{x^2} =$$

$$= \frac{\frac{1}{x}\cdot x - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2}$$

$$y'_x(x_0) = y'_x(e) = \frac{1 - \ln(e)}{e^2} = \frac{1-1}{e^2} = \underline{0}$$ ■

<u>7.1.26</u> $y'(x_0) - ?$

$y = \frac{\sqrt{x}}{\sqrt{x}+1}$, $x_0 = 9$

□

$$y'_x = \left(\frac{\sqrt{x}}{\sqrt{x}+1}\right)'_x = \frac{(\sqrt{x})'_x \cdot (\sqrt{x}+1) - (\sqrt{x})\cdot(\sqrt{x}+1)'_x}{(\sqrt{x}+1)^2} =$$

$$= \frac{\frac{1}{2\sqrt{x}}\cdot\sqrt{x} + \frac{1}{2\sqrt{x}} - \sqrt{x}\cdot\frac{1}{2\sqrt{x}}}{(\sqrt{x}+1)^2} = \frac{1}{2\sqrt{x}\cdot(\sqrt{x}+1)^2}$$

$$y'_x(x_0) = y'_x(9) = \frac{1}{2\cdot\sqrt{9}\cdot(\sqrt{9}+1)^2} = \frac{1}{2\cdot 3\cdot 4^2} = \underline{\frac{1}{96}}$$ ■

7.1.35

$$y'_x = (\ln\sin x)'_x = (\ln\sin x)'_{\sin x} \cdot (\sin x)'_x =$$

$$= \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \underline{\operatorname{ctg} x}$$

7.1.36

$$y'_x = (e^{\operatorname{ctg} x})'_x = (e^{\operatorname{ctg} x})'_{\operatorname{ctg} x} \cdot (\operatorname{ctg} x)'_x =$$

$$= e^{\operatorname{ctg} x} \cdot \left(-\frac{1}{\sin^2 x}\right) = \underline{-\frac{e^{\operatorname{ctg} x}}{\sin^2 x}}$$

7.1.37

$$y'_x = (\arccos(e^x))'_x = (\arccos(e^x))'_{e^x} \cdot (e^x)'_x =$$

$$= -\frac{1}{\sqrt{1-e^{2x}}} \cdot e^x = \underline{-\frac{e^x}{\sqrt{1-e^{2x}}}}$$

7.1.38

$$y'_x = (\operatorname{arctg}^2 \tfrac{1}{x})'_x = (\operatorname{arctg}^2(\tfrac{1}{x}))'_{\operatorname{arctg}\frac{1}{x}} \cdot (\operatorname{arctg}\tfrac{1}{x})'_{\frac{1}{x}} \cdot (\tfrac{1}{x})'_x =$$

$$= 2 \cdot \operatorname{arctg}\frac{1}{x} \cdot \frac{1}{1+x^2} \cdot \left(-\frac{1}{x^2}\right) = \frac{-2\operatorname{arctg}\frac{1}{x} \cdot x^2}{(1+x^2)\cdot x^2} = \underline{-\frac{2\operatorname{arctg}\frac{1}{x}}{1+x^2}}$$

7.1.42

$$y'_x = \left(\ln\sqrt{\frac{1+\operatorname{tg} x}{1-\operatorname{tg} x}}\right)'_x = \left(\frac{1}{2}(\ln(1+\operatorname{tg} x) - \ln(1-\operatorname{tg} x))\right)'_x =$$

$$= \frac{1}{2}\cdot(\ln(1+tgx))'_{1+tgx}\cdot(1+tgx)'_x - \frac{1}{2}\cdot(\ln(1-tgx))'_{1-tgx}\cdot$$

$$\cdot(1-tgx)'_x = \frac{1}{2}\cdot\frac{1}{1+tgx}\cdot\frac{1}{\cos^2 x} + \frac{1}{2}\cdot\frac{1}{1-tgx}\cdot\frac{1}{\cos^2 x} =$$

$$= \frac{1}{2\cdot\cos^2 x}\cdot\left(\frac{2}{1-tg^2 x}\right) = \frac{1}{\cos^2 x\cdot(1-tg^2 x)} =$$

$$= \frac{1}{\cos^2 x - \sin^2 x} = \frac{1}{\cos 2x}$$

7.1.43

$$y'_x = \left((1+tg^2 3x)\cdot e^{-\frac{x}{2}}\right)'_x = (1+tg^2 3x)'_x\cdot e^{-\frac{x}{2}} +$$

$$+ (1+tg^2 3x)\cdot\left(e^{-\frac{x}{2}}\right)'_x = (*)$$

$$1)\ (1+tg^2 3x)'_x = (tg^2 3x)'_x = (tg^2 3x)'_{tg3x}\cdot$$

$$\cdot(tg3x)'_{3x}\cdot(3x)'_x = 2\cdot tg3x\cdot\frac{1}{\cos^2 3x}\cdot 3 = \frac{6tg3x}{\cos^2 3x}$$

$$2)\ \left(e^{-\frac{x}{2}}\right)'_x = \left(e^{-\frac{x}{2}}\right)'_{-\frac{x}{2}}\cdot\left(-\frac{x}{2}\right)'_x = -\frac{1}{2}\cdot e^{-\frac{x}{2}}$$

$$(*) = e^{-\frac{x}{2}}\cdot\frac{6tg3x}{\cos^2 3x} - \frac{1}{2}\cdot e^{-\frac{x}{2}}\cdot\frac{1}{\cos^2 3x} = \frac{e^{-\frac{x}{2}}(12tg3x-1)}{2\cdot\cos^2 3x} =$$

$$= \frac{e^{-\frac{x}{2}}\left(12\frac{\sin 3x}{\cos 3x} - \frac{\cos 3x}{\cos 3x}\right)}{2\cdot\cos^2 3x} = \frac{e^{-\frac{x}{2}}(12\sin 3x - \cos 3x)}{2\cos^3 3x}$$

7.1.44

$$y'_x = (\ln(x+\sqrt{x^2-1}))'_x = \frac{1}{x+\sqrt{x^2-1}} \cdot (x+\sqrt{x^2-1})'_x =$$

$$= \frac{1}{x+\sqrt{x^2-1}} \cdot (1 + (\sqrt{x^2-1})'_x) = \frac{1}{x+\sqrt{x^2-1}} (1+\frac{1}{2\sqrt{x^2-1}} \cdot 2x) =$$

$$= \frac{1}{x+\sqrt{x^2-1}} \cdot \frac{x+\sqrt{x^2-1}}{\sqrt{x^2-1}} = \underline{\frac{1}{\sqrt{x^2-1}}}$$

7.1.45

$$y'_x = (tg\,4x + \frac{2}{3} tg^3 4x + \frac{1}{5} tg^5 4x)'_x = (tg\,4x)'_x +$$

$$+ (\frac{2}{3} tg^3 4x)'_x + (\frac{1}{5} tg^5 4x)'_x = \frac{1}{\cos^2 4x} \cdot 4 + \frac{2}{3} \cdot 3 \cdot tg^2 4x \cdot$$

$$\cdot \frac{1}{\cos^2 4x} \cdot 4 + \frac{1}{5} \cdot 5 \cdot tg^4 4x \cdot \frac{1}{\cos^2 4x} \cdot 4 =$$

$$= \frac{4}{\cos^2 4x} (1 + 2 tg^2 4x + tg^4 4x) = \frac{4}{\cos^2 4x} \cdot (1 + tg^2 4x)^2 =$$

$$= \frac{4}{\cos^2 4x} \cdot \left(\frac{1}{\cos^2 4x}\right)^2 = \underline{\frac{4}{\cos^6 4x}}$$

7.1.46

$$y'_x = (x^3 \cdot \sin(\cos x))'_x = (x^3)'_x \cdot \sin(\cos x) + x^3 \cdot (\sin(\cos x))'_x =$$

$$= 3x^2 \cdot \sin(\cos x) + x^3 \cdot \cos(\cos x) \cdot (-\sin x) =$$

$$= \underline{x^2 (3 \sin(\cos x) - x \cdot \cos(\cos x) \cdot \sin x)}$$

7.1.47

$$y'_x = \left(3^{x^2}\cdot\sqrt{x^3-5x}\right)'_x = \left(3^{x^2}\right)'_x\cdot\sqrt{x^3-5x}\ +$$

$$+\,3^{x^2}\cdot\left(\sqrt{x^3-5x}\right)'_x = 3^{x^2}\cdot\ln 3\cdot 2x\cdot\sqrt{x^3-5x}\ +$$

$$+\,3^{x^2}\cdot\frac{1}{\sqrt{x^3-5x}\cdot 2}\cdot(3x^2-5) =$$

$$= \frac{3^{x^2}}{2\sqrt{x^3-5x}}\cdot\left(\ln 3\cdot 2x\cdot\sqrt{x^3-5x}\cdot 2\sqrt{x^3-5x} + (3x^2-5)\right) =$$

$$= \frac{3^{x^2}}{2\sqrt{x^3-5x}}\cdot\left(4x^4\cdot\ln 3 + x^2\cdot(3-20\ln 3)-5\right)$$

7.1.48

$$y'_x = \left(\log_6(\sin 4x)\right)'_x = \left(\log_6(\sin 4x)\right)'_{\sin 4x}\cdot$$

$$\cdot(\sin 4x)'_{4x}\cdot(4x)'_x = \frac{1}{\sin 4x\cdot\ln 6}\cdot\cos 4x\cdot 4 =$$

$$= \frac{4\cdot\cos 4x}{\sin 4x\cdot\ln 6} = \frac{4\cdot\operatorname{ctg} 4x}{\ln 6}$$

7.1.49

$$y'_x = \left(\cos\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)'_x = -\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\cdot\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)'_x =$$

$$= -\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\cdot\frac{(1-\sqrt{x})'\cdot(1+\sqrt{x})-(1-\sqrt{x})(1+\sqrt{x})'}{(1+\sqrt{x})^2}$$

$$= -\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\cdot\frac{-\frac{1}{2\sqrt{x}}-\frac{1}{2}-\frac{1}{2\sqrt{x}}+\frac{1}{2}}{(1+\sqrt{x})^2}=$$

$$= -\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\cdot\frac{-\frac{2}{2\sqrt{x}}}{(1+\sqrt{x})^2} = \sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\cdot\frac{1}{\sqrt{x}\cdot(1+\sqrt{x})^2}$$

7.1.50

$$y'_x = \left(\ln\frac{(x+1)(x+3)^3}{(x+2)^3(x+4)}\right)'_x = \frac{(x+2)^3(x+4)}{(x+1)(x+3)^3}\cdot\overset{①}{\left(\frac{(x+1)(x+3)^3}{(x+2)^3(x+4)}\right)'_x} = (**)$$

$$① \left(\frac{(x+1)(x+3)^3}{(x+2)^3(x+4)}\right)'_x = \frac{((x+1)(x+3)^3)'_x\cdot(x+2)^3(x+4) -}{(x+2)^6(x+4)^2}$$

$$\underline{-(x+1)(x+3)^3\cdot((x+2)^3(x+4))'_x} = (*)$$

$$\cdot\ ((x+1)(x+3)^3)'_x = (x+1)'_x(x+3)^3 + (x+1)((x+3)^3)'_x =$$

$$= (x+3)^3 + (x+1)\cdot 3(x+3)^2 = (x+3)^2(4x+6)$$

$$\cdot\ ((x+2)^3(x+4))'_x = ((x+2)^3)'_x(x+4) + (x+2)^3(x+4)'_x =$$

$$= 3\cdot(x+2)^2\cdot(x+4) + (x+2)^3 = (x+2)^2(4x+14)$$

$$(*) = \frac{(x+3)^2(4x+6)(x+2)^3(x+4) - (x+1)(x+3)^3\cdot(x+2)^2(4x+14)}{(x+2)^6(x+4)^2} =$$

$$= \frac{(x+3)^2(x+2)^2\cdot((4x+6)(x+2)(x+4) - (x+1)(x+3)\cdot(4x+14))}{(x+2)^6(x+4)^2} =$$

$$= \frac{(x+3)^2\cdot(4x^3+16x^2+14x^2+56x+12x+48-4x^3-30x^2-68x-42)}{(x+2)^4(x+4)^2} =$$

$$= \frac{(x+3)^2 \cdot 6}{(x+2)^4 \cdot (x+4)^2}$$

$$(**) = \frac{(x+2)^3(x+4)}{(x+1)(x+3)^3} \cdot \frac{(x+3)^2 \cdot 6}{(x+2)^4(x+4)^2} = \frac{6}{(x+1)(x+2)(x+3)(x+4)}$$

2сп

$$y'_x = \left(\ln\frac{(x+1)(x+3)^3}{(x+2)^3(x+4)}\right)'_x = \left(\ln((x+1)(x+3)^3) -\right.$$

$$\left.-\ln((x+2)^3(x+4))\right)'_x = (\ln(x+1) + 3\ln(x+3) - 3\ln(x+2) -$$

$$-\ln(x+4))'_x = \frac{1}{x+1} + \frac{3}{x+3} - \frac{3}{x+2} - \frac{1}{x+4} =$$

$$= \frac{(x+3)(x+2)(x+4) + 3\cdot(x+1)(x+2)(x+4) - 3\cdot(x+1)\cdot(x+3)\cdot(x+4) - (x+1)(x+3)(x+2)}{(x+1)(x+2)(x+3)(x+4)} =$$

$$= \frac{x^3+9x^2+26x+24+3x^3+21x^2+42x+24-3x^3-24x^2-57x-36-x^3-6x^2-11x-6}{(x+1)(x+2)(x+3)(x+4)} = \frac{6}{(x+1)(x+2)(x+3)(x+4)}$$

7.1.51

$$y'_x = \left(\operatorname{arctg}(x-2) + \frac{x-3}{x^2-4x+5}\right)'_x = (\operatorname{arctg}(x-2))'_x +$$

$$+\left(\frac{x-3}{x^2-4x+5}\right)'_x = \frac{1}{1+(x-2)^2} + \frac{(x-3)'_x\cdot(x^2-4x+5)-(x-3)\cdot(x^2-4x+5)'_x}{(x^2-4x+5)^2} =$$

$$= \frac{1}{1+(x-2)^2} + \frac{x^2-4x+5-(x-3)(2x-4)}{(x^2-4x+5)^2} =$$

$$= \frac{1}{x^2-4x+5} + \frac{x^2-4x+5-2x^2+10x-12}{(x^2-4x+5)^2} =$$

$$= \frac{x^2-4x+5+x^2-4x+5-2x^2+10x-12}{(x^2-4x+5)^2} =$$

$$= \frac{2x-2}{(x^2-4x+5)^2}$$

7.1.52

$$y'_x = \left(\sin^4\frac{x}{2} + \cos^4\frac{x}{2}\right)'_x = \left[\sin^4 a + \cos^4 a = 1 - \frac{1}{2}\cdot\sin^2 2a\right] =$$

$$= \left(1 - \frac{1}{2}\sin^2 x\right)'_x = (1)'_x - \left(\frac{1}{2}\sin^2 x\right)'_x =$$

$$= -\frac{1}{2}\cdot(\sin^2 x)'_{\sin x}\cdot(\sin x)'_x = -\frac{1}{2}\cdot 2\cdot\sin x\cdot\cos x =$$

$$= -\frac{1}{2}\sin 2x$$

7.1.53

$$y'_x = \left(e^{sh^2 5x}\right)'_x = \left(e^{sh^2 5x}\right)'_{sh^2 5x}\cdot(sh^2 5x)'_{sh 5x}\cdot(sh 5x)'_{5x}\cdot$$

$$\cdot(5x)'_x = e^{sh^2 5x}\cdot 2\cdot sh 5x\cdot ch 5x\cdot 5 = 5\cdot e^{sh^2 5x}\cdot sh 10x$$

7.1.54

$$y'_x = \left(\frac{x+e^{3x}}{x-e^{3x}}\right)'_x = \frac{(x+e^{3x})'_x(x-e^{3x})-(x+e^{3x})(x-e^{3x})'_x}{(x-e^{3x})^2} =$$

$$= \frac{(1+3e^{3x})(x-e^{3x})-(x+e^{3x})(1-3\cdot e^{3x})}{(x-e^{3x})^2} =$$

$$= \frac{x-e^{3x}+3xe^{3x}-3e^{6x}-x-e^{3x}+3xe^{3x}+3e^{6x}}{(x-e^{3x})^2} =$$

$$= \frac{6xe^{3x}-2e^{3x}}{(x-e^{3x})^2} = \frac{2e^{3x}(3x-1)}{(x-e^{3x})^2}$$

7.1.55

$$y'_x = (\arccos\sqrt{x}+\sqrt{x-x^2})'_x = (\arccos\sqrt{x})'_x +$$

$$+(\sqrt{x-x^2})'_x = -\frac{1}{\sqrt{1-x}}\cdot\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x-x^2}}\cdot(1-2x) =$$

$$= \frac{1-2x}{2\sqrt{x}\cdot\sqrt{1-x}} - \frac{1}{2\sqrt{x}\cdot\sqrt{1-x}} = \frac{-2x}{2\sqrt{x}\sqrt{1-x}} = -\frac{x}{\sqrt{x}\cdot\sqrt{1-x}} =$$

$$= -\sqrt{\frac{x}{1-x}}$$

7.1.56

$$y'_x = \left(\operatorname{arctg}\frac{x+1}{x-1}\right)'_x = \frac{1}{1+\left(\frac{x+1}{x-1}\right)^2}\cdot\left(\frac{x+1}{x-1}\right)'_x =$$

$$= \frac{1}{\frac{(x-1)^2+(x+1)^2}{(x-1)^2}} \cdot \frac{(x+1)'_x(x-1)-(x+1)\cdot(x-1)'_x}{(x-1)^2} =$$

$$= \frac{(x-1)^2}{x^2-2x+1+x^2+2x+1} \cdot \frac{x-1-x-1}{(x-1)^2} = \frac{-2}{2x^2+2} = \frac{-1}{x^2+1}$$

7.1.57

$$y'_x = \left(\frac{\sin^2 x}{ctg x+1} + \frac{\cos^2 x}{tg x+1}\right)'_x = \left(\frac{\sin^2 x}{\frac{\cos x+\sin x}{\sin x}} + \frac{\cos^2 x}{\frac{\sin x+\cos x}{\cos x}}\right)'_x =$$

$$= \left(\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}\right)'_x = \left(\frac{(\sin x+\cos x)\cdot(\sin^2 x-\sin x\cdot\cos x+\cos^2 x)}{\sin x+\cos x}\right)'_x =$$

$$= (\sin^2 x+\cos^2 x-\sin x\cdot\cos x)'_x = (1-\sin x\cdot\cos x)'_x =$$

$$= (1)'_x - (\sin x\cdot\cos x)'_x = -((\sin x)'_x\cdot\cos x+\sin x\cdot(\cos x)'_x) =$$

$$= -(\cos^2 x-\sin^2 x) = \sin^2 x-\cos^2 x = -\cos 2x$$

7.1.62

$$y = \frac{(x^3-2)\cdot\sqrt[3]{(x-1)}}{(x+5)^4}$$

$$\ln y = \ln\left(\frac{(x^3-2)\cdot\sqrt[3]{x-1}}{(x+5)^4}\right)$$

$$\ln y = \ln(x^3-2) + \frac{1}{3}\ln(x-1) - 4\cdot\ln(x+5)$$

$$(\ln y)'_x = (\ln(x^3-2)+\frac{1}{3}\ln(x-1)-4\cdot\ln(x+5))'_x$$

$$\frac{1}{y}\cdot y'_x = \frac{3x^2}{x^3-2} + \frac{1}{3x-3} - \frac{4}{x+5}$$

$$y'_x = y\cdot\left(\frac{3x^2}{x^3-2} + \frac{1}{3x-3} - \frac{4}{x+5}\right)$$

$$y'_x = \frac{(x^3-2)\sqrt[3]{x-1}}{(x+5)^4}\cdot\left(\frac{3x^2}{x^3-2} + \frac{1}{3(x-1)} - \frac{4}{x+5}\right)$$

7.1.63

$$y = (tgx)^{\cos x}$$

$$\ln y = \ln\left((tgx)^{\cos x}\right)$$

$$\ln y = \cos(x)\cdot\ln(tgx)$$

$$(\ln y)'_x = (\cos(x)\cdot\ln(tgx))'_x$$

$$\frac{1}{y}\cdot y'_x = (\cos(x))'_x\cdot\ln(tgx) + \cos(x)\cdot(\ln(tgx))'_x$$

$$\frac{1}{y}\cdot y'_x = -\sin(x)\cdot\ln(tgx) + \cos(x)\cdot\frac{1}{tgx}\cdot\frac{1}{\cos^2 x}$$

$$y'_x = y\cdot\left(\cos x\cdot\frac{\cos x}{\sin x}\cdot\frac{1}{\cos^2 x} - \sin(x)\cdot\ln(tgx)\right)$$

$$y'_x = (tgx)^{\cos x}\cdot\left(\frac{1}{\sin x} - \sin(x)\cdot\ln(tgx)\right)$$

7.1.64

$$y=\frac{(1-x^2)\cdot\cos^6 x}{\sqrt[7]{x^5}}$$

$$\ln y=\ln\left(\frac{(1-x^2)\cdot\cos^6 x}{\sqrt[7]{x^5}}\right)$$

$$\ln y=\ln(1-x^2)+6\ln(\cos x)-\frac{5}{7}\cdot\ln(x)$$

$$(\ln y)'_x=(\ln(1-x^2)+6\ln(\cos x)-\frac{5}{7}\ln(x))'_x$$

$$\frac{1}{y}\cdot y'_x=\frac{1}{1-x^2}\cdot(-2x)+6\cdot\frac{1}{\cos x}\cdot(-\sin x)-\frac{5}{7}\cdot\frac{1}{x}$$

$$y'_x=y\cdot(-1)\cdot\left(\frac{2x}{1-x^2}+6\cdot tgx+\frac{5}{7x}\right)$$

$$y'_x=\frac{-(1-x^2)\cdot\cos^6 x}{\sqrt[7]{x^5}}\cdot\left(\frac{2x}{1-x^2}+6tgx+\frac{5}{7x}\right)$$

$$y'_x=\frac{(x^2-1)\cdot\cos^6 x}{\sqrt[7]{x^5}}\cdot\left(\frac{2x}{1-x^2}+6tgx+\frac{5}{7x}\right)$$

7.1.66

$$e^{xy}-\cos(x^2+y^2)=0 \qquad y'_x=?$$

$$(e^{xy})'_x=(\cos(x^2+y^2))'_x$$

$$Л.Ч.=(e^{xy})'_x=e^{xy}\cdot(y+x\cdot y'_x)$$

$$П.Ч.=(\cos(x^2+y^2))'_x=-\sin(x^2+y^2)(2x+2y\cdot y'_x)$$

$$e^{xy}\cdot(y + x\cdot y'_x) = -\sin(x^2+y^2)\cdot(2x+2y\cdot y'_x)$$

$$e^{xy}\cdot y + y'_x\cdot x\cdot e^{xy} = -\sin(x^2+y^2)\cdot 2x - \sin(x^2+y^2)\cdot 2y\cdot y'_x$$

$$y'_x\left(x\cdot e^{xy} + 2y\cdot\sin(x^2+y^2)\right) = -\left(\sin(x^2+y^2)\cdot 2x + e^{xy}\cdot y\right)$$

$$y'_x = -\frac{2x\cdot\sin(x^2+y^2) + y\cdot e^{xy}}{2y\cdot\sin(x^2+y^2) + x\cdot e^{xy}}$$

7.1.67

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad y'_x = ?$$

$$\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)'_x = (1)'_x$$

$$\left(\frac{x^2}{a^2}\right)'_x + \left(\frac{y^2}{b^2}\right)'_x = 0$$

$$\frac{(x^2)'_x\cdot a^2 - x^2\cdot(a^2)'_x}{(a^2)^2} + \frac{(y^2)'_x\cdot b^2 - y^2\cdot(b^2)'_x}{(b^2)^2} = 0$$

$$\frac{2x\cdot a^2 - x^2\cdot 0}{(a^2)^2} + \frac{2y\cdot y'_x\cdot b^2 - y^2\cdot 0}{(b^2)^2} = 0$$

$$\frac{2x}{a^2} = -\frac{2y\cdot y'_x}{b^2}; \quad y'_x = \frac{2x}{a^2}\cdot\left(-\frac{b^2}{2y}\right)$$

$$y'_x = -\frac{b^2\cdot x}{a^2\cdot y}$$

7.1.68

$x^2+y^2=\ln\frac{y}{x}+7\ ,\ y'_x=?$

$(x^2+y^2)'_x=(\ln\frac{y}{x}+7)'_x$
① ②

① $(x^2+y^2)'_x=(x^2)'_x+(y^2)'_x=2x+2y\cdot y'_x$

② $(\ln\frac{y}{x}+7)'_x=(\ln\frac{y}{x})'_x+(7)'_x=\frac{x}{y}\cdot(\frac{y}{x})'_x+0=$

$=\frac{x}{y}\cdot\frac{(y)'_x\cdot x-y\cdot(x)'_x}{x^2}=\frac{y'_x\cdot x-y}{xy}$

$2x+2y\cdot y'_x=\frac{y'_x\cdot x}{xy}-\frac{y}{xy}$

$2y\cdot y'_x-\frac{y'_x}{y}=-\frac{1}{x}-2x$

$\frac{y'_x\cdot(2y^2-1)}{y}=-\left(\frac{1+2x^2}{x}\right)$

$y'_x=\frac{y\cdot(1+2x^2)}{x\cdot(1-2y^2)}$

7.1.69

$x\cdot\sin y+y\cdot\sin x=0\quad,\ y'_x=?$

$(x\cdot\sin y)'_x+(y\cdot\sin x)'_x=(0)'_x$

$(1\cdot\sin y+x\cdot\cos y\cdot y'_x)+(y'_x\cdot\sin x+y\cdot\cos x)=0$

$x\cdot\cos y\cdot y'_x+y'_x\cdot\sin x=-(y\cdot\cos x+\sin y)$

$y'_x(x\cdot\cos y+\sin x)=-(y\cdot\cos x+\sin y)$; $y'_x=-\frac{y\cdot\cos x+\sin y}{x\cdot\cos y+\sin x}$

7.1.70

$x^4 - y^4 = x^2 y^2$, $y'_x = ?$

$(x^4 - y^4)'_x = (x^2 y^2)'_x$

$(x^4)'_x - (y^4)'_x = (x^2)'_x \cdot y^2 + x^2 \cdot (y^2)'_x$

$4x^3 - 4y^3 \cdot y'_x = 2x \cdot y^2 + 2y \cdot x^2 \cdot y'_x$

$2yx^2 \cdot y'_x + 4y^3 \cdot y'_x = 4x^3 - 2xy^2$

$y'_x \cdot 2y \cdot (2y^2 + x^2) = 2x(2x^2 - y^2)$

$y'_x \cdot y \cdot (2y^2 + x^2) = x \cdot (2x^2 - y^2)$

$$y'_x = \frac{x \cdot (2x^2 - y^2)}{y \cdot (2y^2 + x^2)}$$

7.1.71

$e^y = e - xy$, найти y' в точке $(0;1)$

$(e^y)'_x = (e - xy)'_x$

$(e^y)'_x = (e)'_x - (xy)'_x$

$e^y \cdot y'_x = 0 - y - x \cdot y'_x$

$e^y \cdot y'_x + x \cdot y'_x = -y$

$y'_x(e^y + x) = -y$

$$y'_x = -\frac{y}{e^y + x}$$

$\exists x=0;\ y=1$

Тогда

$$y'_x = -\frac{y}{e^y+x} = -\frac{1}{e^1+0} = -\frac{1}{e}$$

7.1.73

$x=t^3+t,\ y=t^2+t+1 \qquad y'(x)-?$

$$y'(x) = \frac{y'(t)}{x'(t)} = \frac{(t^2+t+1)'_t}{(t^3+t)'_t} = \frac{2t+1}{3t^2+1}$$

7.1.74

$x=t-\sin t,\ y=1-\cos t \qquad y'(x)-?$

$$y'(x) = \frac{y'(t)}{x'(t)} = \frac{(1-\cos t)'_t}{(t-\sin t)'_t} = \frac{0-(-\sin t)}{1-\cos t} = \frac{\sin t}{1-\cos t}$$

7.1.75

$x=e^t\cdot\sin t,\ y=e^t\cdot\cos t \qquad y'(x)-?$

$$x'(t) = (e^t\cdot\sin t)'_t = (e^t)'_t\cdot\sin t + e^t\cdot(\sin t)'_t =$$
$$= e^t\cdot\sin t + e^t\cdot\cos t = e^t(\cos t+\sin t)$$
$$y'(t) = (e^t\cdot\cos t)'_t = (e^t)'_t\cdot\cos t + e^t\cdot(\cos t)'_t =$$
$$= e^t\cdot\cos t - e^t\cdot\sin t = e^t(\cos t-\sin t)$$
$$y'(x) = \frac{y'(t)}{x'(t)} = \frac{e^t\cdot(\cos t-\sin t)}{e^t\cdot(\cos t+\sin t)} = \frac{\cos t-\sin t}{\cos t+\sin t}$$

7.1.76

$x = \sin^2 t,\ y = \cos^2 t \qquad y'(x) - ?$

$$y'(x) = \frac{y'(t)}{x'(t)} = \frac{(\cos^2 t)'_t}{(\sin^2 t)'_t} = \frac{2\cos t \cdot (-\sin t)}{2\sin t \cdot \cos t} = \underline{-1}$$

7.1.77

$x = 5\operatorname{ch} t,\ y = 4\operatorname{sh} t \qquad y'(x) - ?$

$$y'(x) = \frac{y'(t)}{x'(t)} = \frac{(4\operatorname{sh} t)'_t}{(5\operatorname{ch} t)'_t} = \frac{4 \cdot \operatorname{ch} t}{5 \cdot \operatorname{sh} t} = \frac{4}{5}\operatorname{cth} t = \underline{0{,}8\operatorname{cth}(t)}$$

7.1.84

$y = \operatorname{tg} 3x,\ y'' - ?$

$$y'_x = (\operatorname{tg} 3x)'_x = \frac{1}{\cos^2 3x} \cdot 3 = \frac{3}{\cos^2 3x}$$

$$y''_{xx} = (y'_x)'_x = \left(\frac{3}{\cos^2 3x}\right)'_x = \frac{(3)'_x \cdot \cos^2 3x - 3 \cdot (\cos^2 3x)'_x}{(\cos^2 3x)^2} =$$

$$= \frac{-3 \cdot (2 \cdot \cos 3x) \cdot (-\sin 3x) \cdot 3}{\cos^4 3x} = \underline{\frac{18\sin 3x}{\cos^3 3x}}$$

7.1.85

$y = -x \cdot \cos x,\ y'' - ?$

$$y'_x = (-x \cdot \cos x)'_x = (-x)'_x \cdot \cos x + (-x)(\cos x)'_x =$$

$$= -\cos x + x \cdot \sin x$$

$y''_{xx} = (-\cos x + x\cdot\sin x)'_x = \sin x + (1\cdot\sin x +$
$+ x\cdot\cos x) = \underline{2\sin x + x\cdot\cos x}$

7.1.86

$y = \ln^2 x$, $y'' - ?$

$y'_x = (\ln^2 x)'_x = 2\ln x \cdot \frac{1}{x} = \frac{2\ln x}{x}$

$y''_{xx} = \left(\frac{2\ln x}{x}\right)'_x = \frac{(2\ln x)'_x\cdot x - 2\ln x\cdot(x)'_x}{x^2} =$

$= \frac{\frac{2}{x}\cdot x - 2\ln x}{x^2} = \frac{2 - 2\ln x}{x^2} = \underline{\frac{2(1-\ln x)}{x^2}}$

7.1.87

$y = x\cdot\ln x$, $y''' - ?$

$y'_x = (x\cdot\ln x)'_x = 1\cdot\ln x + x\cdot\frac{1}{x} = \ln x + 1$

$y''_{xx} = (1+\ln x)'_x = 0 + \frac{1}{x} = \frac{1}{x}$

$y'''_{xxx} = \left(\frac{1}{x}\right)'_x = (x^{-1})'_x = -1\cdot x^{-2} = \underline{-\frac{1}{x^2}}$

7.1.88

$y = e^{2x}$, $y^{(V)} - ?$

$y'(x) = (e^{2x})'_x = e^{2x}\cdot 2$

$$y''(x)=(e^{2x}\cdot 2)'_x=e^{2x}\cdot 2\cdot 2=e^{2x}\cdot 4$$
$$y'''(x)=(e^{2x}\cdot 4)'_x=e^{2x}\cdot 2\cdot 4=e^{2x}\cdot 8$$
$$y^{(IV)}(x)=(e^{2x}\cdot 8)'_x=e^{2x}\cdot 2\cdot 8=e^{2x}\cdot 16$$
$$y^{(V)}(x)=(e^{2x}\cdot 16)'_x=e^{2x}\cdot 2\cdot 16=\underline{32\cdot e^{2x}}$$

7.1.89

$y=\ln(1+x)$, $y^{(n)}$ - ?

$$y'(x)=(\ln(1+x))'_x=\frac{1}{1+x}\cdot(1+x)'_x=\frac{1}{1+x}$$

$$y''(x)=\left(\frac{1}{1+x}\right)'_x=\frac{0\cdot(1+x)-1\cdot(1+x)'_x}{(1+x)^2}=\frac{-1}{(1+x)^2}$$

$$y'''(x)=\left(\frac{-1}{(1+x)^2}\right)'_x=\frac{(-1)'_x\cdot(1+x)^2-(-1)((1+x)^2)'_x}{(1+x)^4}=\frac{2(1+x)}{(1+x)^4}=\frac{2}{(1+x)^3}$$

$$y^{(IV)}(x)=\left(\frac{2}{(1+x)^3}\right)'_x=\frac{0\cdot(1+x)^3-2\cdot 3\cdot(1+x)^2}{(1+x)^6}=\frac{-6(1+x)^2}{(1+x)^6}=\frac{-6}{(1+x)^4}$$

$$y^{(V)}(x)=\left(\frac{-6}{(1+x)^4}\right)'_x=\frac{0\cdot(1+x)^4-(-6)\cdot 4(1+x)^3}{(1+x)^8}=\frac{24}{(1+x)^5}$$

$$\vdots$$

Заметим, что данные производные имеют общий вид, зависящий от степени производной: $\dfrac{(-1)^{n-1}(n-1)!}{(1+x)^n}$

Докажем, что это верно для $\forall n \in \mathbb{N}$:

Индукция

База $n=1$: $y'(x) = \frac{(-1)^{1-1} \cdot (1-1)!}{(1+x)^1} = \frac{1 \cdot 1}{1+x} = \frac{1}{1+x}$ $\oplus$

Переход: $n \to n+1$

$$y^{(n)}(x) = \frac{(-1)^{n-1}(n-1)!}{(1+x)^n}$$

$$y^{(n+1)}(x) = \left(\frac{(-1)^{n-1} \cdot (n-1)!}{(1+x)^n}\right)'_x = \frac{0 \cdot (1+x)^n - (-1)^{n-1} \cdot (n-1)! \cdot n \cdot (1+x)^{n-1}}{(1+x)^{2n}} =$$

$$= \frac{(-1)^n \cdot n!}{(1+x)^{n+1}} = \frac{(-1)^{(n+1)-1} \cdot ((n+1)-1)!}{(1+x)^{n+1}}$$

ч.т.д.

Значит: $y^{(n)}(x) = \frac{(-1)^{n-1} \cdot (n-1)!}{(1+x)^n}$

7.1.90

$x = t^3$, $y = t^2$, $y''_{xx} = ?$

$\square$

$y'_t = (t^2)'_t = 2t$; $y''_{tt} = (2t)'_t = 2$

$x'_t = (t^3)'_t = 3t^2$; $y''_{tt} = (3t^2)'_t = 6t$

$$y''_{xx} = \frac{y''_t \cdot x'_t - x''_t \cdot y'_t}{(x'_t)^3} = \frac{2 \cdot 3t^2 - 6t \cdot 2t}{27t^6} = \frac{-6t^2}{27t^6} = -\frac{2}{9t^4}$$

$\blacksquare$

7.1.91

$x=\cos t,\ y=\sin t,\ y''_{xx}-?$

$\square$

$y'_t=(\sin t)'_t=\cos t,\ y''_{tt}=(\cos t)'_t=-\sin t$

$x'_t=(\cos t)'_t=-\sin t,\ x''_{tt}=(-\sin t)'_t=-\cos t$

$$y''_{xx}=\frac{y''_t\cdot x'_t-x''_t\cdot y'_t}{(x'_t)^3}=\frac{(-\sin t)\cdot(-\sin t)-(-\cos t)\cdot\cos t}{(-\sin t)^3}=$$

$$=\frac{\sin^2 t+\cos^2 t}{-\sin^3 t}=-\frac{1}{\sin^3 t}$$

$\blacksquare$