Стецук Максим 2гр. 1п.4.

Система ур-ий:

$$5X_1 + 35X_2 - 15X_3 + 55X_4 - 25X_5 = 185$$
$$3X_1 + 28X_2 + 12X_3 + 47X_4 - 29X_5 = 188$$
$$4X_1 + 31X_2 + 0 \cdot X_3 + 56X_4 - 29X_5 = 205$$
$$1X_1 + 8X_2 + 4X_3 + 20X_4 - 14X_5 = 40$$
$$2X_1 + 12X_2 - 19X_3 + 9X_4 + 20X_5 = 240$$

$A \cdot X = B$

Перепишем в виде матрицы с присоединённым столбцом свободных членов:

$$\left(\begin{array}{ccccc|c} 5 & 35 & -15 & 55 & -25 & 185 \\ 3 & 28 & 12 & 47 & -29 & 188 \\ 4 & 31 & 0 & 56 & -29 & 205 \\ 1 & 8 & 4 & 20 & -14 & 40 \\ 2 & 12 & -19 & 9 & 20 & 240 \end{array}\right)$$

Решение:

1 способ Приводим матрицу к верхней Δ-й

$$\left(\begin{array}{ccccc|c} 5 & 35 & -15 & 55 & -25 & 185 \\ 3 & 28 & 12 & 47 & -29 & 188 \\ 4 & 31 & 0 & 56 & -29 & 205 \\ 1 & 8 & 4 & 20 & -14 & 40 \\ 2 & 12 & -19 & 9 & 20 & 240 \end{array}\right) \quad \text{I} : 5 \quad \sim$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 7 & -3 & 11 & -5 & 37 \\ 3 & 28 & 12 & 47 & -29 & 188 \\ 4 & 31 & 0 & 56 & -29 & 205 \\ 1 & 8 & 4 & 20 & -14 & 40 \\ 2 & 12 & -19 & 9 & 20 & 240 \end{array}\right) \begin{array}{l} \\ \text{II}-3\text{I} \\ \text{III}-4\text{I} \\ \text{IV}-\text{I} \\ \text{V}-2\text{I} \end{array} \sim$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 7 & -3 & 11 & -5 & 37 \\ 0 & 7 & 21 & 14 & -14 & 77 \\ 0 & 3 & 12 & 12 & -9 & 57 \\ 0 & 1 & 7 & 9 & -9 & 3 \\ 0 & -2 & -13 & -13 & 30 & 166 \end{array}\right) \text{II}:7 \sim$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 7 & -3 & 11 & -5 & 37 \\ 0 & 1 & 3 & 2 & -2 & 11 \\ 0 & 3 & 12 & 12 & -9 & 57 \\ 0 & 1 & 7 & 9 & -9 & 3 \\ 0 & -2 & -13 & -13 & 30 & 166 \end{array}\right) \begin{array}{l} \\ \\ \text{III}-3\text{II} \\ \text{IV}-\text{II} \\ \text{V}+2\text{II} \end{array} \sim$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 7 & -3 & 11 & -5 & 37 \\ 0 & 1 & 3 & 2 & -2 & 11 \\ 0 & 0 & 3 & 6 & -3 & 24 \\ 0 & 0 & 4 & 7 & -7 & -8 \\ 0 & 0 & -7 & -9 & 26 & 188 \end{array}\right) \text{III}:3 \sim$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 7 & -3 & 11 & -5 & 37 \\ 0 & 1 & 3 & 2 & -2 & 11 \\ 0 & 0 & 1 & 2 & -1 & 8 \\ 0 & 0 & 4 & 7 & -7 & -8 \\ 0 & 0 & -7 & -9 & 26 & 188 \end{array}\right) \begin{array}{l} \\ \\ \\ \text{IV}-4\text{III} \\ \text{V}+7\text{III} \end{array} \sim$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 7 & -3 & 11 & -5 & 37 \\ 0 & 1 & 3 & 2 & -2 & 11 \\ 0 & 0 & 1 & 2 & -1 & 8 \\ 0 & 0 & 0 & -1 & -3 & -40 \\ 0 & 0 & 0 & 5 & 19 & 244 \end{array}\right) \text{IV}\cdot(-1) \sim$$

$$\sim\left(\begin{array}{ccccc|c} 1 & 7 & -3 & 11 & +5 & 37 \\ 0 & 1 & 3 & 2 & -2 & 11 \\ 0 & 0 & 1 & 2 & -1 & 8 \\ 0 & 0 & 0 & 1 & 3 & 40 \\ 0 & 0 & 0 & 5 & 19 & 244 \end{array}\right) \begin{array}{l} \\ \\ \\ \\ \text{V}-5\,\text{IV} \end{array} \sim$$

$$\sim\left(\begin{array}{ccccc|c} 1 & 7 & -3 & 11 & -5 & 37 \\ 0 & 1 & 3 & 2 & -2 & 11 \\ 0 & 0 & 1 & 2 & -1 & 8 \\ 0 & 0 & 0 & 1 & 3 & 40 \\ 0 & 0 & 0 & 0 & 4 & 44 \end{array}\right)$$

Находим корни:

1) $4X_5 = 44 \Rightarrow X_5 = \underline{11}$

2) $X_4 + 3X_5 = 40$; $X_4 = 40 - 3X_5 = 40 - 3 \cdot 11 = \underline{7}$

3) $X_3 + 2X_4 - X_5 = 8$; $X_3 = 8 - 2 \cdot 7 + 11 = \underline{5}$

4) $X_2 + 3X_3 + 2X_4 - 2X_5 = 11$; $X_2 = 11 - 3 \cdot 5 - 2 \cdot 7 + 2 \cdot 11 = \underline{4}$

5) $X_1 + 7X_2 - 3X_3 + 11X_4 - 5X_5 = 37$; $X_1 = 37 - 7 \cdot 4 + 3 \cdot 5 - 11 \cdot 7 +$

$+ 5 \cdot 11 = 37 - 28 + 15 - 77 + 55 = 9 + 15 - 22 = \underline{2}$

Ответ: $X_1 = 2$, $X_2 = 4$, $X_3 = 5$, $X_4 = 7$, $X_5 = 11$

2 способ Приводим матрицу к нижней Δ-й

$$\left(\begin{array}{ccccc|c} 5 & 35 & -15 & 55 & -25 & 185 \\ 3 & 28 & 12 & 47 & -29 & 188 \\ 4 & 31 & 0 & 56 & -29 & 205 \\ 1 & 8 & 4 & 20 & -14 & 40 \\ 2 & 12 & -19 & 9 & 20 & 240 \end{array}\right) \quad \text{V} : 20 \quad \sim$$

$$\sim \left(\begin{array}{ccccc|c} 5 & 35 & -15 & 55 & -25 & 185 \\ 3 & 28 & 12 & 47 & -29 & 188 \\ 4 & 31 & 0 & 56 & -29 & 205 \\ 1 & 8 & 4 & 20 & -14 & 40 \\ 0{,}1 & 0{,}6 & -0{,}95 & 0{,}45 & 1 & 12 \end{array}\right) \begin{array}{l} \text{I} + 25\,\text{V} \\ \text{II} + 29\,\text{V} \\ \text{III} + 29\,\text{V} \\ \text{IV} + 14\,\text{V} \\ \\ \end{array}$$

$$\sim \left(\begin{array}{ccccc|c} \frac{15}{2} & 50 & -\frac{155}{4} & \frac{265}{4} & 0 & 485 \\ \frac{59}{10} & \frac{227}{5} & -\frac{311}{20} & \frac{1201}{20} & 0 & 536 \\ \frac{69}{10} & \frac{242}{5} & -\frac{551}{20} & \frac{1381}{20} & 0 & 553 \\ \frac{12}{5} & \frac{82}{5} & -\frac{93}{10} & \frac{263}{10} & 0 & 208 \\ \frac{1}{10} & \frac{3}{5} & -\frac{19}{20} & \frac{9}{20} & 1 & 12 \end{array}\right) \quad \text{IV} : \frac{263}{10} \quad \sim$$

$$\sim \left(\begin{array}{ccccc|c} \frac{15}{2} & 50 & -\frac{155}{4} & \frac{265}{4} & 0 & 485 \\ \frac{59}{10} & \frac{227}{5} & -\frac{311}{20} & \frac{1201}{20} & 0 & 536 \\ \frac{69}{10} & \frac{242}{5} & -\frac{551}{20} & \frac{1381}{20} & 0 & 553 \\ \frac{24}{263} & \frac{164}{263} & -\frac{93}{263} & 1 & 0 & \frac{2080}{263} \\ \frac{1}{10} & \frac{3}{5} & -\frac{19}{20} & \frac{9}{20} & 1 & 12 \end{array}\right) \begin{array}{l} \text{I} - \frac{265}{4} \cdot \text{IV} \\ \text{II} - \frac{1201}{20} \cdot \text{IV} \\ \text{III} - \frac{1381}{20} \cdot \text{IV} \sim \\ \\ \end{array}$$

$$\left(\begin{array}{ccccc|c}
\frac{765}{526} & \frac{2285}{263} & -\frac{4030}{263} & 0 & 0 & -\frac{10245}{263} \\
\frac{221}{526} & \frac{2092}{263} & \frac{1495}{263} & 0 & 0 & \frac{16064}{263} \\
\frac{315}{526} & \frac{1405}{263} & -\frac{824}{263} & 0 & 0 & \frac{1815}{263} \\
\frac{24}{263} & \frac{164}{263} & -\frac{93}{263} & 1 & 0 & \frac{2080}{263} \\
\frac{1}{10} & \frac{3}{5} & -\frac{19}{20} & \frac{9}{20} & 1 & 12
\end{array}\right) \text{III} : \left(-\frac{824}{263}\right) \sim$$

$$\sim \left(\begin{array}{ccccc|c}
\frac{765}{526} & \frac{2285}{263} & -\frac{4030}{263} & 0 & 0 & -\frac{10245}{263} \\
\frac{221}{526} & \frac{2092}{263} & \frac{1495}{263} & 0 & 0 & \frac{16064}{263} \\
-\frac{315}{1648} & -\frac{1405}{824} & 1 & 0 & 0 & -\frac{1815}{824} \\
\frac{24}{263} & \frac{164}{263} & -\frac{93}{263} & 1 & 0 & \frac{2080}{263} \\
\frac{1}{10} & \frac{3}{5} & -\frac{19}{20} & 0 & 1 & 12
\end{array}\right) \begin{array}{l} \text{I} + \frac{4030}{263} \cdot \text{III} \\ \text{II} - \frac{1495}{263} \cdot \text{III} \end{array} \sim$$

$$\sim \left(\begin{array}{ccccc|c}
-\frac{1215}{824} & -\frac{7185}{412} & 0 & 0 & 0 & -\frac{29955}{412} \\
\frac{2483}{1648} & \frac{14541}{824} & 0 & 0 & 0 & \frac{60647}{824} \\
-\frac{315}{1648} & -\frac{1405}{824} & 1 & 0 & 0 & -\frac{1815}{824} \\
\frac{24}{263} & \frac{164}{263} & -\frac{93}{263} & 1 & 0 & \frac{2080}{263} \\
\frac{1}{10} & \frac{3}{5} & -\frac{19}{20} & \frac{9}{20} & 1 & 12
\end{array}\right) \text{II} : \frac{14541}{824} \sim$$

$$\sim \left(\begin{array}{ccccc|c}
-\frac{1215}{824} & -\frac{7185}{412} & 0 & 0 & 0 & -\frac{29955}{412} \\
\frac{2483}{29082} & 1 & 0 & 0 & 0 & \frac{60647}{14541} \\
-\frac{315}{1648} & -\frac{1405}{824} & 1 & 0 & 0 & -\frac{1815}{824} \\
\frac{24}{263} & \frac{164}{263} & -\frac{93}{263} & 1 & 0 & \frac{2080}{263} \\
\frac{1}{10} & \frac{3}{5} & -\frac{19}{20} & \frac{9}{20} & 1 & 12
\end{array}\right) \text{I} + \frac{7185}{412} \text{II} \sim$$

$$\sim \left(\begin{array}{ccccc|c} \frac{70}{4847} & 0 & 0 & 0 & 0 & \frac{140}{4847} \\ \frac{2483}{29082} & 1 & 0 & 0 & 0 & \frac{60647}{14541} \\ -\frac{315}{1648} & -\frac{1405}{824} & 1 & 0 & 0 & -\frac{1815}{824} \\ \frac{84}{263} & \frac{164}{263} & -\frac{93}{263} & 1 & 0 & \frac{2080}{263} \\ \frac{1}{10} & \frac{3}{5} & -\frac{19}{20} & \frac{9}{20} & 1 & 12 \end{array}\right)$$

Находим корни:

1) $\frac{70}{4847} X_1 = \frac{140}{4847} \Rightarrow X_1 = \underline{2}$

2) $X_2 = \frac{60647}{14541} - \frac{2483}{29082} \cdot 2 = \frac{58164}{14541} = \underline{4}$

3) $X_3 = -\frac{1815}{824} + \frac{315}{1648} \cdot 2 + \frac{1405}{824} \cdot 4 = \frac{4120}{824} = \underline{5}$

4) $X_4 = \frac{2080}{263} - \frac{24}{263} \cdot 2 - \frac{164}{263} \cdot 4 + \frac{93}{263} \cdot 5 = \frac{1841}{263} = \underline{7}$

5) $X_5 = 12 - \frac{1}{10} \cdot 2 - \frac{3}{5} \cdot 4 + \frac{19}{20} \cdot 5 - \frac{9}{20} \cdot 7 =$

$= 12 - 0,2 - 2,4 + 4,75 - 3,15 = 12 - 2,6 + 1,6 = \underline{11}$

Ответ: $X_1 = 2, X_2 = 4, X_3 = 5, X_4 = 7, X_5 = 11.$