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Вариант 14

1.1

$$\int\frac{3x\,dx}{(24-17x^2)^9}=\int-\frac{3x\,dx}{(17x^2-24)^9}=-3\int\frac{x\,dx}{(17x^2-24)^9}=$$

$$=\left[t=17x^2-24;\ dt=d(17x^2-24)=(17x^2-24)'_x dx=\right.$$

$$\left.=34x\,dx\ \Rightarrow\ x\,dx=\frac{1}{34}dt\right]=-3\int\frac{\frac{1}{34}dt}{t^9}=$$

$$=-\frac{3}{34}\int\frac{dt}{t^9}=-\frac{3}{34}\int t^{-9}dt=[\text{табличный}]=$$

$$=-\frac{3}{34}\cdot\frac{t^{-9+1}}{-9+1}+C=-\frac{3}{34\cdot(-8)\cdot t^8}+C=\frac{3}{272\cdot t^8}+C=$$

$$=\frac{3}{272(17x^2-24)^8}+C$$

1.2

$$\int-2{,}5\cdot(-4\cdot x^3+2{,}5\cdot x^4)\cdot\ln(x^4)\cdot dx=\int-2{,}5(-4\cdot x^3+$$

$$+2{,}5\cdot x^4)\cdot 4\cdot\ln(x)dx=-10\int(-4x^3+2{,}5x^4)\ln(x)dx=$$

$$=-10\int(-4x^3\ln x+2{,}5x^4\ln x)dx=-10\cdot(-4)\int x^3\ln x\,dx-$$

$$-10\cdot(2{,}5)\int x^4\ln x=40\int x^3\ln x\,dx-25\int x^4\ln x\,dx=$$

$= [1)\ u_1 = \ln x,\ v_1' = x^3 \Rightarrow u_1' = \frac{1}{x},\ v_1 = \frac{x^4}{4};$

$2)\ u_2 = \ln x,\ v_2' = x^4 \Rightarrow u_2' = \frac{1}{x},\ v_2 = \frac{x^5}{5}] =$

$= 40\left(\ln x \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x} dx\right) - 25\left(\ln x \cdot \frac{x^5}{5} -\right.$

$\left. - \int \frac{x^5}{5} \cdot \frac{1}{x} dx\right) = 10x^4 \cdot \ln x - 10\int x^3 dx - 5x^5 \ln x +$

$+ 5\int x^4 dx = 10x^4 \ln x - 10 \cdot \frac{x^4}{4} - 5x^5 \ln x + 5 \cdot \frac{x^5}{5} + C =$

$= x^4\left(10 \ln x - \frac{5}{2} - 5x \ln x + x\right) + C =$

$= x^4\left(\frac{20 \ln x - 5 - 10x \ln x + 2x}{2}\right) + C =$

$= \frac{x^4((20-10x)\ln x + 2x - 5)}{2} + C$

1.3

$\int \frac{x - 17/5}{x^3 - 7x^2 + 4x - 28} dx = \frac{1}{5}\int \frac{5x - 17}{x^2(x-7) + 4(x-7)} =$

$= \frac{1}{5}\int \frac{5x-17}{(x-7)(x^2+4)} = [(x-7)(x^2+4) = 0;\ x_1 = 7;$

$\frac{5x-17}{(x-7)(x^2+4)} = \frac{A}{x-7} + \frac{Bx+C}{x^2+4};$

$5x - 17 = A(x^2+4) + (Bx+C)(x-7);$

$$5x-17=Ax^2+4A+Bx^2-7Bx+Cx-7C;$$

$$5x-17=x^2(A+B)+x(C-7B)-(7C-4A);$$

$$\begin{cases}A+B=0\\C-7B=5\\7C-4A=17\end{cases}\quad\begin{cases}A=-B\\C=5+7B\\35+49B+4B=17\end{cases}\quad\begin{cases}A=-B\\C=5+7B\\53B=-18\end{cases}$$

$$\left.\begin{cases}A=\frac{18}{53}\\C=5+7\cdot\left(-\frac{18}{53}\right)=\frac{265-126}{53}=\frac{139}{53}\\B=-\frac{18}{53}\end{cases}\right]=$$

$$=\frac{1}{5}\int\left(\frac{18}{53(x-7)}+\frac{-18x+139}{53(x^2+4)}\right)dx=\frac{18}{265}\int\frac{dx}{x-7}-\frac{9}{265}\cdot$$

$$\cdot\int\frac{2x\,dx}{(x^2+4)}+\frac{139}{265}\cdot\int\frac{dx}{x^2+4}=\Big[1)\ d(x-7)=(x-7)'_x dx=$$

$$=1\cdot dx=dx;\ 2)\ t=x^2+4;\ dt=d(x^2+4)=2x\,dx;$$

$$3)\int\frac{dx}{x^2+a^2}=\frac{1}{a}\operatorname{arctg}\frac{x}{a}+C\Big]=\frac{18}{265}\ln|x-7|-\frac{9}{265}\cdot\int\frac{dt}{t}+$$

$$+\frac{139}{265}\cdot\frac{1}{2}\operatorname{arctg}\frac{x}{2}+C_1=\frac{18}{265}\ln|x-7|-\frac{9}{265}\ln|t|+$$

$$+\frac{139}{530}\operatorname{arctg}\frac{x}{2}+C=\frac{18\ln|x-7|}{265}-\frac{9\ln|x^2+4|}{265}+$$

$$+\frac{139\operatorname{arctg}\frac{x}{2}}{530}+C=\frac{18\ln|x-7|}{265}-\frac{9\ln(x^2+4)}{265}+\frac{139\operatorname{arctg}\frac{x}{2}}{530}+C$$

(1.4)

$$\int \frac{2}{14+\sqrt{15x+41}}dx = \Big[\sqrt[2]{(15x+41)^1} \Rightarrow 2 \text{ случай} \Rightarrow k=2,$$

$$\text{тогда } 15x+41=t^2, \; dx=\left(\frac{t^2-41}{15}\right)'_t dt = \frac{2}{15}t\,dt\Big] =$$

$$= \int \frac{2\cdot\frac{2}{15}t\,dt}{14+t} = \frac{4}{15}\int\frac{t\,dt}{t+14} = \frac{4}{15}\int\frac{t+14-14}{t+14}dt =$$

$$= \frac{4}{15}\int\frac{t+14}{t+14}dt - \frac{4}{15}\cdot\int\frac{14}{t+14}dt = \Big[2)\; d(t+14) = (t+14)'_t dt =$$

$$= 1\cdot dt = dt\Big] = \frac{4}{15}\int dt - \frac{4\cdot 14}{15}\int\frac{d(t+14)}{t+14} =$$

$$= \frac{4}{15}t - \frac{4\cdot14}{15}\ln|t+14| + C = \frac{4(t-14\ln|t+14|)}{15} +$$

$$+ C = \frac{4(\sqrt{15x+41} - 14\ln|\sqrt{15x+41}+14|)}{15} + C =$$

$$= \frac{4(\sqrt{15x+41} - 14\ln(\sqrt{15x+41}+14))}{15} + C$$

(1.5)

$$\int\frac{2}{14\cdot\cos^2 x + 17\cdot\sin^2 x}dx = \Big[R(\sin x;\cos x) = R(-\sin x;$$

$$-\cos x), \text{ тогда } t=\operatorname{tg}x: \sin x = \frac{t}{\sqrt{1+t^2}}, \; \cos x = \frac{1}{\sqrt{1+t^2}};$$

$$dx = \frac{dt}{1+t^2}\Big] = \int\frac{2}{\frac{14}{1+t^2}+\frac{17t^2}{1+t^2}}\cdot\frac{dt}{1+t^2} = 2\int\frac{1}{\frac{14+17t^2}{1+t^2}}\cdot\frac{dt}{1+t^2} =$$

$$= 2\int\frac{1+t^2}{14+17t^2}\cdot\frac{dt}{1+t^2} = 2\int\frac{dt}{14+17t^2} = 2\cdot\frac{1}{17}\cdot\int\frac{dt}{t^2+\frac{14}{17}} =$$

$$= \left[\text{табл. } \int\frac{dx}{x^2+a^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a}+C\right] = \frac{2}{17}\cdot\frac{1}{\left(\frac{\sqrt{14}}{\sqrt{17}}\right)}\cdot$$

$$\cdot\operatorname{arctg}\frac{t}{\frac{\sqrt{14}}{\sqrt{17}}}+C = \frac{2}{17}\cdot\frac{\sqrt{17}}{\sqrt{14}}\cdot\operatorname{arctg}\frac{\sqrt{17}\,t}{\sqrt{14}}+C =$$

$$= \frac{2\operatorname{arctg}\left(\frac{\sqrt{17}\cdot tgx}{\sqrt{14}}\right)}{\sqrt{14}\cdot\sqrt{17}}+C = \frac{2\operatorname{arctg}\left(\frac{17\cdot tgx}{\sqrt{238}}\right)}{\sqrt{238}}+C$$

1.6

$$\int x\cdot\arcsin(18\cdot x)\,dx = \Big[u=\arcsin(18x),\ v'=x \Rightarrow$$

$$\Rightarrow u' = (\arcsin(18x))'_x = \frac{1}{\sqrt{1-(18x)^2}}\cdot(18x)'_x = \frac{18}{\sqrt{1-324x^2}},$$

$$v=\frac{x^2}{2}\Big] = \underbrace{\arcsin(18x)\cdot\frac{x^2}{2}}_{A=} - \int\frac{x^2}{2}\cdot\frac{18}{\sqrt{1-(18x)^2}}\,dx =$$

$$= A - 9\int\frac{x^2}{\sqrt{1-(18x)^2}}\,dx = \Big[x=\frac{\sin u}{18} \Rightarrow x^2=\frac{\sin^2u}{324};$$

$$dx = \left(\frac{\sin u}{18}\right)'_u du = \frac{\cos u}{18}\,du\Big] = A - 9\int\frac{\frac{\sin^2u}{324}}{\sqrt{1-\left(18\cdot\frac{\sin u}{18}\right)^2}}\cdot$$

$$\cdot\left(-\frac{\cos u}{18}\,du\right) = A - 9\cdot\left(\frac{1}{18}\right)\int\frac{\sin^2u\cdot\cos u}{324\cdot\left(\sqrt{1-\sin^2u}\right)}\cdot du =$$

$$= A - \frac{1}{2}\int\frac{\sin^2u}{324\cos u}\cdot\cos u\,du = A - \frac{1}{648}\int\sin^2u\,du =$$

$$= A - \frac{1}{648}\int\frac{1-\cos2u}{2}\,du = A - \frac{1}{648}\cdot\int\frac{1}{2}\,du + \frac{1}{648}\cdot\int\frac{\cos2u}{2}\,du =$$

$$= A - \frac{1}{1296}\int dU + \frac{1}{1296}\int \cos(2U)\,dU =$$

$$= [\text{1) табл.; 2) 5-я формула}] = A - \frac{U}{1296} +$$

$$+ \frac{1}{1296}\cdot\frac{1}{2}\cdot \sin(2U) + C = \arcsin(18x)\cdot\frac{x^2}{2}$$

$$- \frac{\arcsin 18x}{1296} + \frac{1}{2}\cdot\frac{1}{1296}\cdot \sin(2\arcsin 18x) + C =$$

$$= \frac{648\arcsin(18x)\cdot x^2 - \arcsin 18x}{1296} + \frac{1}{\cancel{2}}\cdot\frac{1}{1296}\cdot\cancel{2}\cdot 18x\cdot$$

$$\cdot\sqrt{1-(18x)^2} + C = \underline{\frac{(648x^2-1)\arcsin 18x + 18x\sqrt{1-324x^2}}{1296} + C}$$

1.7

$$\int_{-4}^{-2}\frac{dx}{\sqrt{60-4x-x^2}} = \int_{-4}^{-2}\frac{dx}{\sqrt{64-4-4x-x^2}} = \int_{-4}^{-2}\frac{dx}{\sqrt{8^2-(x+2)^2}} =$$

$$= [d(x+2) = (x+2)'_x\,dx = 1\cdot dx = dx] = \int_{-4}^{-2}\frac{d(x+2)}{\sqrt{8^2-(x+2)^2}} =$$

$$= \arcsin\frac{x+2}{8}\Bigg|_{-4}^{-2} = \arcsin\frac{-2+2}{8} - \arcsin\frac{-4+2}{8} =$$

$$= \arcsin(0) - \arcsin\left(-\frac{1}{4}\right) = 0 - \left(-\arcsin\left(\frac{1}{4}\right)\right) =$$

$$= \underline{\arcsin\left(\frac{1}{4}\right)}$$

(2.1)

$y' = -2 \cdot y$, $y(0) = 15$

$\frac{dy}{dx} = -2y$, тогда: $\frac{dy}{y} = -2dx$;

$\int \frac{dy}{y} = \int -2dx$; $\ln|y| = -2x + C'$, значит

$|y| = e^{-2x+C'}$; $|y| = e^{C'} \cdot e^{-2x} \Rightarrow |y| = C \cdot e^{-2x}$,

т.к. $e^{C'} > 0$ и $e^{-2x} > 0$, то: $y = C \cdot e^{-2x}$ – общее реш-е

при $x=0$, $y=15$: $15 = C \cdot e^{-2 \cdot 0} = C \cdot e^0 = C \cdot 1 = C$,

значит частное решение: $\underline{y = 15 \cdot e^{-2x}}$

(2.2)

$x \cdot y' = 2 \cdot \sqrt{16x^2 + y^2} + y$;

$\frac{xdy}{dx} = 2 \cdot \sqrt{y^2 + 16x^2} + y \quad | \cdot dx$;

$xdy = (2\sqrt{y^2 + 16x^2} + y)dx$; (вынесем x из ( ))

$xdy = x(2\sqrt{\frac{y^2}{x^2} + 16} + \frac{y}{x})dx$ (*);

$\exists\, y = ux$, $dy = xdu + udx \Rightarrow u = \frac{y}{x}$;

тогда (*) имеет вид:

$$x(x\,du + u\,dx) = (2\sqrt{u^2+16} + u)\,x\,dx$$

$$x^2 du + \cancel{ux\,dx} = 2\sqrt{u^2+16}\,x\,dx + \cancel{ux\,dx}$$

$$x^2 du = 2\sqrt{u^2+16}\,x\,dx$$

$$\frac{du}{\sqrt{u^2+16}} = \frac{2x\,dx}{x^2}; \quad \frac{du}{\sqrt{u^2+16}} = \frac{2dx}{x}$$

$$\int\frac{du}{\sqrt{u^2+16}} = \int\frac{2dx}{x} \Rightarrow \ln|u+\sqrt{u^2+16}| = 2\ln|x| + C_1$$

$$\ln(u+\sqrt{u^2+16}) = \ln|x^2| + \ln(e^{C_1})$$

$$\ln(u+\sqrt{u^2+16}) = \ln(e^{C_1}\cdot x^2)$$

$u+\sqrt{u^2+16} = e^{C_1}\cdot x^2$, заменим $u = \frac{y}{x}$ обратно:

$$\sqrt{\frac{y^2}{x^2}+16} + \frac{y}{x} = Cx^2 \quad (e^{C_1} = C\text{-const})$$

$$\sqrt{y^2+16x^2} + y = Cx^3$$