Домашняя работа часть 5

8.5.19

$$\int\frac{dx}{\cos x}=[\text{табличный}]=\ln\left|tg\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|+C$$

8.5.20

$$\int\frac{dx}{1-\sin x}=\left[t=tg\frac{x}{2},\ \text{тогда: } \sin x=\frac{2t}{1+t^2},\right.$$

$$\left.dx=\frac{2dt}{1+t^2}\right]=\int\frac{1}{1-\frac{2t}{1+t^2}}\cdot\frac{2dt}{1+t^2}=\int\frac{1}{\frac{1+t^2-2t}{1+t^2}}\cdot\frac{2dt}{1+t^2}=$$

$$=\int\frac{1+t^2}{(t-1)^2}\cdot\frac{2dt}{1+t^2}=2\int\frac{dt}{(t-1)^2}=[dt=d(t-1)]=$$

$$=2\int\frac{d(t-1)}{(t-1)^2}=2\cdot\frac{(t-1)^{-2+1}}{-2+1}+C=\frac{-2}{t-1}+C=-\frac{2}{tg\frac{x}{2}-1}+C$$

8.5.21

$$\int\frac{dx}{5+4\sin x}=\left[t=tg\frac{x}{2},\ \text{тогда: } \sin x=\frac{2t}{1+t^2},\ dx=\right.$$

$$\left.=\frac{2dt}{1+t^2}\right]=\int\frac{1+t^2}{5+5t^2+8t}\cdot\frac{2dt}{1+t^2}=\int\frac{2dt}{5t^2+2\cdot\sqrt{5}\cdot t\cdot\frac{4}{\sqrt{5}}+\frac{16}{5}+\frac{9}{5}}=$$

$$=2\int\frac{dt}{\left(\sqrt{5}t+\frac{4}{\sqrt{5}}\right)^2+\left(\frac{3}{\sqrt{5}}\right)^2}=\left[5\text{-я ф-ла: } f(ax+b)dx=\frac{1}{a}\cdot\right.$$

$$\left.\cdot F(ax+b)+C\right]=\frac{2}{\sqrt{5}}\cdot\frac{1}{\frac{3}{\sqrt{5}}}\cdot arctg\left(\frac{\sqrt{5}}{3}\cdot\left(\sqrt{5}\cdot t+\frac{4}{\sqrt{5}}\right)\right)+C=$$

$$= \frac{2}{3}\cdot arctg\left(\frac{5t+4}{3}\right)+C = \frac{2}{3} arctg\left(\frac{5tg\left(\frac{x}{2}\right)+4}{3}\right)+C$$

8.5.22

$$\int \frac{2-\sin x}{2+\cos x}dx = \left[t = tg\frac{x}{2}, \text{тогда: } \sin x = \frac{2t}{1+t^2},\right.$$

$$\left.\cos x = \frac{1-t^2}{1+t^2}, dx = \frac{2dt}{1+t^2}\right] = \int \frac{2+2t^2-2t}{2+2t^2+1-t^2}\cdot\frac{2dt}{1+t^2} =$$

$$= 4\int \frac{t^2-t+1}{(t^2+3)(t^2+1)} = \left[\frac{t^2-t+1}{(t^2+3)\cdot(t^2+1)} = \frac{At+B}{t^2+3}+\frac{Ct+D}{t^2+1};\right.$$

$$t^2-t+1 = At^3+Bt^2+At+B+Ct^3+Dt^2+3Ct+3D$$

$$t^2-t+1 = t^3(A+C)+t^2(B+D)+t(A+3C)+(B+3D)$$

$$\left.\begin{cases} A+C=0 \\ B+D=1 \\ A+3C=-1 \\ B+3D=1 \end{cases} \begin{cases} A+C=0 \\ B+D=1 \\ 2C=-1 \\ 2D=0 \end{cases} \begin{cases} A=\frac{1}{2} \\ B=1 \\ C=-\frac{1}{2} \\ D=0 \end{cases}\right] =$$

$$= 4\cdot\left(\frac{1}{2}\int\frac{t+2}{t^2+3}dt - \frac{1}{2}\int\frac{t\,dt}{t^2+1}\right) = 4\cdot\left(\frac{1}{2}\int\frac{t}{t^2+3}dt + \int\frac{dt}{t^2+3} -\right.$$

$$\left.-\frac{1}{2}\int\frac{t\,dt}{t^2+1}\right) = \left[1)\ U = t^2+3,\ dU = 2t\,dt,\ \frac{1}{2}dU = t\,dt;\right.$$

$$\left.2)\ \text{табл.};\ 3)\ V = t^2+1,\ dV = 2t\,dt,\ \frac{1}{2}dV = t\,dt\right] =$$

$$= \int\frac{dU}{U} - \int\frac{dV}{V} + 4\int\frac{dt}{t^2+3} = (\ln|U| - \ln|V|) +$$

$$+ 4\cdot\frac{1}{\sqrt{3}}\cdot arctg\frac{t}{\sqrt{3}} + C = \ln\left|\frac{t^2+3}{t^2+1}\right| + \frac{4}{\sqrt{3}}\cdot arctg\frac{t}{\sqrt{3}} + C =$$

$$=\ln\left|1+\frac{2}{(tg\frac{x}{2})^2+1}\right|+\frac{4}{\sqrt{3}}\cdot arctg\left(\frac{tg\frac{x}{2}}{\sqrt{3}}\right)+C=$$

$$=\left[\frac{2}{\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}}{\cos^2\frac{x}{2}}}=\frac{2}{\frac{1}{\cos^2\frac{x}{2}}}=2\cos^2\frac{x}{2}=2\cdot\frac{1+\cos x}{2}=\right.$$

$$\left.=1+\cos x\right]=\ln(2+\cos x)+\frac{4}{\sqrt{3}}\cdot arctg\left(\frac{tg\frac{x}{2}}{\sqrt{3}}\right)+C$$

8.5.23

$$\int\frac{dx}{2\sin x-\cos x+5}=\left[t=tg\frac{x}{2},\text{ тогда: }\sin x=\frac{2t}{1+t^2},\right.$$

$$\left.\cos x=\frac{1-t^2}{1+t^2},\ dx=\frac{2dt}{1+t^2}\right]=\int\frac{1}{\frac{4t}{1+t^2}-\frac{1-t^2}{1+t^2}+5}\cdot\frac{2dt}{1+t^2}=$$

$$=\int\frac{1+t^2}{4t-1+t^2+5+5t^2}\cdot\frac{2dt}{1+t^2}=\int\frac{2dt}{6t^2+4t+4}\doteq\int\frac{dt}{3t^2+2\cdot\sqrt{3}t\cdot\frac{1}{\sqrt{3}}+\frac{1}{3}+\frac{5}{3}}=$$

$$=\int\frac{dt}{(\sqrt{3}t+\frac{1}{\sqrt{3}})^2+(\frac{\sqrt{5}}{\sqrt{3}})^2}=[\text{5-я ф-ла, табличный}]=$$

$$=\frac{1}{\sqrt{3}}\cdot\frac{1}{\frac{\sqrt{5}}{\sqrt{3}}}\cdot arctg\left(\frac{\sqrt{3}}{\sqrt{5}}\cdot\left(\sqrt{3}t+\frac{1}{\sqrt{3}}\right)\right)+C=\frac{1}{\sqrt{5}}arctg\left(\frac{3t+1}{\sqrt{5}}\right)+C=$$

$$=\frac{1}{\sqrt{5}}\cdot arctg\left(\frac{3tg(\frac{x}{2})+1}{\sqrt{5}}\right)+C$$

8.5.24

$$\int\frac{1+\sin x}{(1+\cos x)\sin x}dx=\left[t=tg\frac{x}{2},\text{ тогда: }\sin x=\frac{2t}{1+t^2},\right.$$

$$\left.\cos x=\frac{1-t^2}{1+t^2},\ dx=\frac{2dt}{1+t^2}\right]=$$

$$=\int \frac{1+\frac{2t}{1+t^2}}{\left(1+\frac{1-t^2}{1+t^2}\right)\cdot\frac{2t}{1+t^2}}\cdot\frac{2dt}{1+t^2}=\int\frac{\frac{1+t^2+2t}{1+t^2}}{\frac{(1+t^2+1-t^2)\cdot 2t}{(1+t^2)\cdot(1+t^2)}}\cdot\frac{2dt}{1+t^2}=$$

$$=\int\frac{(t+1)^2\cdot(1+t^2)}{4t}\cdot\frac{2dt}{1+t^2}=\frac{1}{2}\int\frac{t^2+2t+1}{t}dt=$$

$$=\frac{1}{2}\int t\,dt+\int dt+\frac{1}{2}\int\frac{dt}{t}=\frac{1}{2}\cdot\frac{t^2}{2}+t+\frac{1}{2}\ln|t|+C=$$

$$=\frac{1}{4}\mathrm{tg}^2\frac{x}{2}+\mathrm{tg}\frac{x}{2}+\frac{1}{2}\ln\left|\mathrm{tg}\frac{x}{2}\right|+C$$

8.5.25

$$\int\frac{dx}{5\sin^2x-3\cos^2x+4}=\Big[R(\sin x;\cos x)=R(-\sin x;-\cos x),$$

тогда $t=\mathrm{tg}\,x$: $\sin x=\frac{t}{\sqrt{1+t^2}}$, $\cos x=\frac{1}{\sqrt{1+t^2}}$, $dx=$

$$=\frac{dt}{1+t^2}\Big]=\int\frac{1}{\frac{5t^2}{1+t^2}-\frac{3}{1+t^2}+\frac{4+4t^2}{1+t^2}}\cdot\frac{dt}{1+t^2}=\int\frac{dt}{9t^2+1}=$$

$$=\frac{1}{9}\int\frac{dt}{t^2+(\frac{1}{3})^2}=\frac{1}{9}\cdot\frac{1}{\frac{1}{3}}\cdot\mathrm{arctg}\left(\frac{t}{\frac{1}{3}}\right)+C=$$

$$=\frac{1}{3}\mathrm{arctg}(3t)+C=\frac{1}{3}\mathrm{arctg}(3\,\mathrm{tg}\,x)+C$$

8.5.26

$$\int\frac{dx}{4\sin^2x+9\cos^2x}=\Big[R(\sin x;\cos x)=R(-\sin x;-\cos x),$$

тогда $t=\mathrm{tg}\,x$: $\sin x=\frac{t}{\sqrt{1+t^2}}$, $\cos x=\frac{1}{\sqrt{1+t^2}}$,

$$dx=\frac{dt}{1+t^2}\Big] = \int \frac{1}{\frac{4t^2}{1+t^2}+\frac{9}{1+t^2}}\cdot\frac{dt}{1+t^2} = \int\frac{1+t^2}{4t^2+9}\cdot\frac{dt}{1+t^2} =$$

$$=\int\frac{dt}{4t^2+9} = \frac{1}{4}\int\frac{dt}{t^2+\left(\frac{3}{2}\right)^2} = \frac{1}{4}\cdot\frac{1}{\frac{3}{2}}\cdot arctg\left(\frac{t}{\frac{3}{2}}\right)+C=$$

$$=\frac{1}{6}arctg\left(\frac{2\,tgx}{3}\right)+C$$

8.5.27

$$\int\frac{dx}{1+3\cos^2 x} = \Big[R(\sin x;\cos x)=R(-\sin x;-\cos x), \text{тогда}$$

$$t=tgx:\ \cos x=\frac{1}{\sqrt{1+t^2}},\ dx=\frac{dt}{1+t^2}\Big] = \int\frac{1}{1+\frac{3}{1+t^2}}\cdot\frac{dt}{1+t^2}=$$

$$=\int\frac{t^2+1}{t^2+4}\cdot\frac{dt}{t^2+1} = \int\frac{dt}{t^2+2^2} = \frac{1}{2}arctg\frac{t}{2}+C=$$

$$=\frac{1}{2}arctg\left(\frac{tgx}{2}\right)+C$$

8.5.28

$$\int\frac{dx}{\sin^4 x} = \Big[R(\sin x;\cos x)=R(-\sin x;-\cos x), \text{тогда}$$

$$t=tgx:\ \sin x=\frac{t}{\sqrt{1+t^2}},\ dx=\frac{dt}{1+t^2}\Big] = \int\frac{1}{\frac{t^4}{(1+t^2)^2}}\cdot\frac{dt}{1+t^2}=$$

$$=\int\frac{(1+t^2)^2}{t^4}\cdot\frac{dt}{1+t^2} = \int\frac{(1+t^2)dt}{t^4} = \int\frac{dt}{t^4}+\int\frac{dt}{t^2}=$$

$$=\frac{t^{-3}}{-3}+\frac{t^{-1}}{-1}+C = -\frac{1}{3tg^3x}-\frac{1}{tgx}+C = -\frac{1}{3}ctg^3x-ctgx+C$$

8.5.29

$$\int \sin^5 x\, dx = \Big[R(-\sin x; \cos x) = -R(\sin x; \cos x)$$

$$\Rightarrow t = \cos x;\ \sin x = \sqrt{1-t^2},\ dx = -\frac{dt}{\sqrt{1-t^2}} \Big] =$$

$$= \int \left(\sqrt{1-t^2}\right)^5 \cdot \frac{-dt}{\sqrt{1-t^2}} = -\int (1-t^2)^2 dt = -\int (t^4 - 2t^2 + 1) dt =$$

$$= -\int t^4 dt + 2\int t^2 dt - \int dt = -\frac{t^5}{5} + \frac{2t^3}{3} - t + C =$$

$$= -\frac{\cos^5 x}{5} + \frac{2\cos^3 x}{3} - \cos x + C$$

8.5.30

$$\int \sin^4 x \cdot \cos^5 x\, dx = \Big[R(\sin x; -\cos x) = -R(\sin x; \cos x)$$

$$\Rightarrow t = \sin x;\ \cos x = \sqrt{1-t^2},\ dx = \frac{dt}{\sqrt{1-t^2}} \Big] =$$

$$= \int t^4 \cdot \left(\sqrt{1-t^2}\right)^5 \cdot \frac{dt}{\sqrt{1-t^2}} = \int t^4 (1-t^2)^2 dt =$$

$$= \int t^4 dt - 2\int t^6 dt + \int t^8 dt = \frac{t^5}{5} - \frac{2t^7}{7} + \frac{t^9}{9} + C =$$

$$= \frac{\sin^9 x}{9} - \frac{2\sin^7 x}{7} + \frac{\sin^5 x}{5} + C$$

8.5.31

$$\int\frac{\sin 2x\,dx}{\cos^7 x}=\int\frac{2\cos x\sin x}{\cos^7 x}dx=2\int\frac{\sin x}{\cos^6 x}dx=$$

$$=\Big[R(-\sin x;\cos x)=-R(\sin x;\cos x)\Rightarrow t=\cos x;$$

$$\sin x=\sqrt{1-t^2},\ dx=-\frac{dt}{\sqrt{1-t^2}}\Big]=2\int\frac{\sqrt{1-t^2}}{t^6}\cdot\frac{-dt}{\sqrt{1-t^2}}=$$

$$=-2\int\frac{dt}{t^6}=-2\cdot\frac{t^{-5}}{-5}+C=\frac{2}{5t^5}+C=\frac{2}{5\cos^5 x}+C$$

8.5.32

$$\int\frac{\sin^4 x}{\cos x}dx=\Big[R(\sin x;-\cos x)=-R(\sin x;\cos x)\Rightarrow$$

$$\Rightarrow t=\sin x;\ \cos x=\sqrt{1-t^2},\ dx=\frac{dt}{\sqrt{1-t^2}}\Big]=$$

$$=\int\frac{t^4}{\sqrt{1-t^2}}\cdot\frac{dt}{\sqrt{1-t^2}}=\int\frac{t^4dt}{1-t^2}=-\int\frac{t^4dt}{t^2-1}=-\int\frac{t^4-1+1}{t^2-1}dt=$$

$$=-\int\frac{(t^2-1)(t^2+1)+1}{t^2-1}dt=-\int(t^2+1)dt-\int\frac{dt}{t^2-1}=$$

$$=-\int t^2dt-\int dt-\int\frac{dt}{t^2-1}=-\frac{t^3}{3}-t-\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|+C=$$

$$=-\frac{\sin^3 x}{3}-\sin x-\frac{1}{2}\ln\left|\frac{\sin x-1}{\sin x+1}\right|+C$$

8.5.33

$$\int \sin^6 x\,dx = \int \left(\frac{1-\cos 2x}{2}\right)^3 dx = \frac{1}{8}\int (1-\cos 2x)^3$$

$$= \frac{1}{8}\int (-\cos^3 2x + 3\cos^2 2x - 3\cos 2x + 1)\,dx =$$

$$= -\frac{1}{8}\int \frac{1+\cos 4x}{2}\cdot \cos 2x\,dx + \frac{3}{8}\int \frac{1+\cos 4x}{2}dx - \frac{3}{8}\int \cos 2x\,dx +$$

$$+\frac{1}{8}\int dx = -\frac{1}{16}\int \cos 2x\,dx - \frac{1}{16}\int \cos 4x\cdot\cos 2x\,dx +$$

$$+\frac{3}{16}\int dx + \frac{3}{16}\int \cos 4x\,dx - \frac{3}{8}\int \cos 2x\,dx + \frac{1}{8}\int dx =$$

$$= \Big[\cos 4x\cdot \cos 2x = \frac{1}{2}(\cos(4x-2x) + \cos(4x+2x)) =$$

$$= \frac{1}{2}\cos 2x + \frac{1}{2}\cos 6x\Big] = -\frac{1}{16}\int \cos 2x\,dx - \frac{1}{32}\int \cos 2x\,dx -$$

$$-\frac{1}{32}\int \cos 6x\,dx + \frac{3}{16}\int dx + \frac{3}{16}\int \cos 4x\,dx - \frac{3}{8}\int \cos 2x\,dx +$$

$$+\frac{1}{8}\int dx = -\frac{\sin 2x}{32} - \frac{\sin 2x}{64} - \frac{\sin 6x}{192} + \frac{3x}{16} + \frac{3\sin 4x}{64} -$$

$$-\frac{3\sin 2x}{16} + \frac{x}{8} + C = -\frac{15\sin 2x}{64} + \frac{5}{16}x - \frac{\sin 6x}{192} +$$

$$+\frac{3\sin 4x}{64} + C = -\frac{\sin 6x - 9\sin 4x + 45\sin 2x - 60x}{192} + C$$

8.5.34

$$\int \sin^2 x \cdot \cos^4 x\, dx = \int (\sin x \cdot \cos x)^2 \cdot \cos^2 x\, dx =$$

$$= \int \left(\frac{1}{2}\sin 2x\right)^2 \cdot \frac{1+\cos 2x}{2}\, dx = \frac{1}{8}\int \sin^2 2x (1+\cos 2x)\, dx =$$

$$= \frac{1}{8}\int \sin^2 2x\, dx + \frac{1}{8}\int \sin^2 2x \cdot \cos 2x\, dx = \Big[1)\ \sin^2 2x =$$

$$= \frac{1-\cos 4x}{2};\ 2)\ t = \sin 2x \Rightarrow dt = 2\cos 2x\, dx \Rightarrow$$

$$\Rightarrow \cos 2x\, dx = \frac{dt}{2} \Big] = \frac{1}{8}\int \frac{1-\cos 4x}{2}\, dx + \frac{1}{16}\int t^2\, dt =$$

$$= \frac{1}{16}\int dx - \frac{1}{16}\int \cos 4x + \frac{1}{16}\int t^2\, dt = \frac{1}{16}x - \frac{1}{16}\cdot\frac{1}{4}\cdot \sin 4x +$$

$$+ \frac{1}{16}\cdot\frac{t^3}{3} + C = \frac{1}{16}x - \frac{1}{64}\sin 4x + \frac{1}{48}\cdot \sin^3 2x + C =$$

$$= \frac{4x - \sin 4x}{64} + \frac{\sin^3 2x}{48} + C$$

8.5.35

$$\int \sin^4 x \cdot \cos^4 x\, dx = \int (\sin x \cdot \cos x)^4 dx = \int \left(\frac{1}{2}\cdot \sin(2x)\right)^4 dx =$$

$$= \frac{1}{16}\int \sin^4 2x\, dx = \frac{1}{16}\cdot \int \left(\frac{1-\cos 4x}{2}\right)^2 dx = \frac{1}{64}\int (1-\cos 4x)^2 dx =$$

$$= \frac{1}{64}\int (1 - 2\cos 4x + \cos^2 4x)\, dx = \frac{1}{64}\int dx - \frac{1}{32}\int \cos 4x\, dx +$$

$$+\frac{1}{64}\int\cos^2 4x\,dx=\frac{1}{64}\int dx-\frac{1}{32}\int\cos 4x\,dx+$$

$$+\frac{1}{64}\int\frac{1+\cos 8x}{2}dx=\frac{1}{64}\int dx-\frac{1}{32}\int\cos 4x\,dx+$$

$$+\frac{1}{128}\int dx+\frac{1}{128}\int\cos 8x\,dx=\frac{1}{64}x-\frac{1}{32}\cdot\frac{1}{4}\sin 4x+$$

$$+\frac{1}{128}x+\frac{1}{128}\cdot\frac{1}{8}\sin 8x+C=\frac{\sin 8x}{1024}-\frac{\sin 4x}{128}+$$

$$+\frac{3x}{128}+C=\frac{\sin 8x-8\sin 4x+24x}{1024}+C$$

8.5.36

$$\int\sin x\cdot\sin 3x\,dx=\frac{1}{2}\int(\cos(3x-x)-\cos(3x+x))dx=$$

$$=\frac{1}{2}\int\cos 2x\,dx-\frac{1}{2}\int\cos 4x\,dx=\frac{1}{2}\cdot\frac{1}{2}\cdot\sin 2x-$$

$$-\frac{1}{2}\cdot\frac{1}{4}\sin 4x+C=\frac{\sin 2x}{4}-\frac{\sin 4x}{8}+C$$

8.5.37

$$\int\sin\frac{x}{12}\cdot\cos\frac{x}{3}\,dx=\int\cos\frac{4x}{12}\cdot\sin\frac{x}{12}\,dx=$$

$$=\left[\cos\alpha\cdot\sin\beta=\frac{1}{2}(\sin(\alpha+\beta)-\sin(\alpha-\beta))\right]=$$

$$=\frac{1}{2}\int\sin\frac{5x}{12}dx-\frac{1}{2}\int\sin\frac{x}{4}dx=-\frac{1}{2}\cdot\frac{1}{\frac{5}{12}}\cdot\cos\frac{5x}{12}-$$

$-\frac{1}{2}\cdot\frac{1}{\frac{1}{4}}\cdot(-1)\cdot\cos\frac{x}{4}+C = 2\cos\left(\frac{x}{4}\right)-\frac{6}{5}\cos\left(\frac{5x}{12}\right)+C$

8.5.38

$\int\cos x\cdot\cos 3x\,dx = \int\cos 3x\cdot\cos x\,dx = [\cos\alpha\cdot\cos\beta =$

$= \frac{1}{2}(\cos(\alpha+\beta)+\cos(\alpha-\beta))] = \frac{1}{2}\int\cos 4x\,dx + \frac{1}{2}\int\cos 2x\,dx =$

$= \frac{1}{8}\sin 4x + \frac{1}{4}\sin 2x + C$

8.5.39

$\int\cos x\cdot\cos 3x\cdot\cos 5x\,dx = [\cos 5x\cdot\cos 3x\cdot\cos x =$

$= \frac{1}{2}(\cos 8x+\cos 2x)\cdot\cos x = \frac{1}{2}(\cos 8x\cdot\cos x +$

$+\cos 2x\cdot\cos x) = \frac{1}{4}(\cos 9x+\cos 7x) + \frac{1}{4}(\cos 3x+\cos x)] =$

$= \frac{1}{4}\int\cos 9x\,dx + \frac{1}{4}\int\cos 7x\,dx + \frac{1}{4}\int\cos 3x\,dx + \frac{1}{4}\int\cos x\,dx =$

$= \frac{1}{4}\cdot\frac{1}{9}\sin 9x + \frac{1}{4}\cdot\frac{1}{7}\cdot\sin 7x + \frac{1}{4}\cdot\frac{1}{3}\cdot\sin 3x + \frac{1}{4}\cdot\sin x +$

$+C = \frac{\sin 9x}{36} + \frac{\sin 7x}{28} + \frac{\sin 3x}{12} + \frac{\sin x}{4} + C$

8.5.40

$$\int ctg^6x\,dx=\int ctg^4x\cdot\left(\frac{1}{\sin^2x}-1\right)dx=$$

$$=\int\frac{ctg^4x}{\sin^2x}dx-\int ctg^4x\,dx=\int\frac{ctg^4x}{\sin^2x}dx-$$

$$-\int ctg^2x\cdot\left(\frac{1}{\sin^2x}-1\right)dx=\int\frac{ctg^4x}{\sin^2x}dx-\int\frac{ctg^2x}{\sin^2x}dx+$$

$$+\int ctg^2x\,dx=\int\frac{ctg^4x}{\sin^2x}dx-\int\frac{ctg^2x}{\sin^2x}dx+$$

$$+\int\frac{dx}{\sin^2x}-\int dx=\Big[1,2)\ t=ctg x,\ dt=d(ctg x)=$$

$$=-\frac{dx}{\sin^2x}\Rightarrow\frac{dx}{\sin^2x}=-dt;\ 3,4)\text{ табличные}\Big]=$$

$$=-\int t^4dt+\int t^2dt-ctg x-x+C_1=$$

$$=-\frac{t^5}{5}+\frac{t^3}{3}-ctg x-x+C=-\frac{ctg^5x}{5}+\frac{ctg^3x}{3}-$$

$$-ctg x-x+C$$

8.5.41

$$\int tg^4\frac{x}{2}dx=\int tg^2\frac{x}{2}\cdot\left(\frac{1}{\cos^2\frac{x}{2}}-1\right)dx=\int\frac{tg^2\frac{x}{2}}{\cos^2\frac{x}{2}}dx-$$

$$-\int tg^2\frac{x}{2}dx=\int\frac{tg^2\frac{x}{2}}{\cos^2\frac{x}{2}}dx-\int\frac{dx}{\cos^2\frac{x}{2}}+\int dx=$$

$= [1)\ t = tg\frac{x}{2},\ dt = d(tg\frac{x}{2}) = \frac{1}{2\cos^2\frac{x}{2}}dx \Rightarrow$

$\Rightarrow \frac{dx}{\cos^2 x} = 2dt] = 2\int t^2 dt - \int \frac{dx}{\cos^2\frac{x}{2}} + \int dx =$

$= [1,3)$ табличные; $2)$ 5-я ф-ла, табличный$] =$

$= 2\cdot\frac{t^3}{3} - \frac{1}{\frac{1}{2}}\cdot tg\frac{x}{2} + x + C = \frac{2tg^3\frac{x}{2}}{3} - 2tg\frac{x}{2} + x + C$

8.5.42

$\int tg^7 x\,dx = \int tg^5 x(\frac{1}{\cos^2 x} - 1)dx = \int \frac{tg^5 x}{\cos^2 x}dx - \int tg^5 x\,dx =$

$= \int\frac{tg^5 x}{\cos^2 x}dx - \int tg^3 x(\frac{1}{\cos^2 x} - 1)dx = \int\frac{tg^5 x}{\cos^2 x}dx - \int\frac{tg^3 x}{\cos^2 x}dx +$

$+ \int tg^3 x\,dx = \int\frac{tg^5 x}{\cos^2 x}dx - \int\frac{tg^3 x}{\cos^2 x}dx + \int tg\,x(\frac{1}{\cos^2 x} - 1)dx =$

$= \int\frac{tg^5 x}{\cos^2 x}dx - \int\frac{tg^3 x}{\cos^2 x}dx + \int\frac{tg\,x}{\cos^2 x}dx - \int tg\,x\,dx =$

$= [1,2,3)\ t = tg\,x,\ dt = d(tg\,x) = \frac{dx}{\cos^2 x}] =$

$= \int t^5 dt - \int t^3 dt + \int t\,dt - \int tg\,x\,dx = [1,2,3,4)$ табл.$] =$

$= \frac{t^6}{6} - \frac{t^4}{4} + \frac{t^2}{2} + \ln|\cos x| + C = \frac{tg^6 x}{6} - \frac{tg^4 x}{4} +$

$+ \frac{tg^2 x}{2} + \ln|\cos x| + C$