Практическая работа часть 5

11.4.4

$z = x^2 + y^2 + xy$, $x = a \cdot \sin t$, $y = a \cdot \cos t$; $\frac{dz}{dt} = ?$

$\square$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{\partial z}{\partial x} = (x^2 + y^2 + xy)'_x = 2x + y$$

$$\frac{\partial z}{\partial y} = (x^2 + y^2 + xy)'_y = 2y + x$$

$$\frac{\partial x}{\partial t} = (a \cdot \sin t)'_t = a \cdot \cos t$$

$$\frac{\partial y}{\partial t} = (a \cdot \cos t)'_t = -a \cdot \sin t$$

Тогда:

$$\frac{dz}{dt} = (2x + y) \cdot a \cdot \cos t + (2y + x) \cdot (-a \cdot \sin t) =$$

$$= \underline{a(2x + y) \cdot \cos t - a(2y + x) \sin t}$$

$\blacksquare$

11.4.5

$z = \cos(2t + 4x^2 - y)$; $x = \frac{1}{t}$; $y = \frac{\sqrt{t}}{\ln t}$; $\frac{dz}{dt} = ?$

$t = t$

$\square$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial z}{\partial t} \cdot \frac{dt}{dt};$$

$$\frac{\partial z}{\partial x}=(\cos(2t+4x^2-y))'_x=-\sin(2t+4x^2-y)\cdot 8x$$

$$\frac{\partial z}{\partial y}=(\cos(2t+4x^2-y))'_y=-\sin(2t+4x^2-y)\cdot(-1)$$

$$\frac{dx}{dt}=\left(\frac{1}{t}\right)'_t=-\frac{1}{t^2};\ \frac{dy}{dt}=\left(\frac{\sqrt{t}}{\ln t}\right)'_t=\frac{(\sqrt{t})'_t\cdot\ln t-\sqrt{t}\cdot(\ln t)'_t}{(\ln t)^2}=\frac{\frac{1}{2\sqrt{t}}\cdot\ln t-\sqrt{t}\cdot\frac{1}{t}}{(\ln t)^2}=\frac{\ln t-2}{2\sqrt{t}(\ln t)^2}$$

$$\frac{\partial z}{\partial t}=(\cos(2t+4x^2-y))'_t=-\sin(2t+4x^2-y)\cdot 2$$

Тогда:

$$\frac{dz}{dt}=-\sin(2t+4x^2-y)\cdot\left(2-\frac{8x}{t^2}-\frac{\ln t-2}{2\sqrt{t}(\ln t)^2}\right)=$$

$$=-\sin\left(2t+\frac{4}{t^2}-\frac{\sqrt{t}}{\ln t}\right)\left(2-\frac{8}{t^3}-\frac{\ln t-2}{2\sqrt{t}\cdot\ln^2 t}\right)$$

11.4.6

$z=x^2y^3u;\ x=t;\ y=t^2;\ u=\sin t;\ \frac{dz}{dt}=?$

$\square$

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}+\frac{\partial z}{\partial u}\cdot\frac{du}{dt}$$

$$\frac{\partial z}{\partial x}=(x^2y^3u)'_x=2xy^3u$$

$$\frac{\partial z}{\partial y}=(x^2y^3u)'_y=3x^2y^2u$$

$\frac{\partial z}{\partial u} = (x^2y^3u)'_u = x^2y^3$

$\frac{dx}{dt} = 1$; $\frac{dy}{dt} = 2t$; $\frac{du}{dt} = \cos t$

Тогда:

$$\frac{dz}{dt} = \underline{2xy^3u + 6x^2y^2ut + x^2y^3 \cdot \cos t} \quad \blacksquare$$

<u>11.4.7</u>

$z = e^{xy}\ln(x+y)$; $x = t^3$, $y = 1 - t^3$; $\frac{dz}{dt} = ?$

$\square$

$$z = e^{xy} \cdot \ln(x+y) = e^{t^3 \cdot (1-t^3)} \cdot \ln(t^3 + 1 - t^3) =$$

$$= e^{t^3 \cdot (1-t^3)} \cdot \ln(1) = e^{t^3 \cdot (1-t^3)} \cdot 0 = 0$$

Значит:

$$\frac{dz}{dt} = \underline{0} \quad \blacksquare$$

<u>11.4.8</u>

$z = xy \cdot \operatorname{arctg}(xy)$, $x = t^2 + 1$, $y = t^3$; $\frac{dz}{dt} = ?$

$\square$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{\partial z}{\partial x} = (xy \cdot \operatorname{arctg}(xy))'_x = y \cdot \operatorname{arctg}(xy) + \frac{xy^2}{1 + x^2y^2}$$

$$\frac{\partial z}{\partial y} = (xy \cdot \operatorname{arctg}(xy))'_y = x \cdot \operatorname{arctg}(xy) + \frac{x^2 y}{1+x^2y^2}$$

$$\frac{dx}{dt} = (t^2+1)'_t = 2t \; ; \; \frac{dy}{dt} = (t^3)'_t = 3t^2$$

$$\frac{dz}{dt} = \underline{\left(y \cdot \operatorname{arctg}(xy) + \frac{xy^2}{1+x^2y^2}\right) \cdot 2t} + \underline{+\left(x \cdot \operatorname{arctg}(xy) + \frac{x^2y}{1+x^2y^2}\right) \cdot 3t^2} \quad \blacksquare$$

11.4.9

$z = e^{2x-3y}$; $x = \operatorname{tg} t$; $y = t^2 - 1$; $\frac{dz}{dt} = ?$

$\square$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{\partial z}{\partial x} = (e^{2x-3y})'_x = e^{2x-3y} \cdot 2$$

$$\frac{\partial z}{\partial y} = (e^{2x-3y})'_y = e^{2x-3y} \cdot (-3)$$

$$\frac{dx}{dt} = (\operatorname{tg} t)'_t = \frac{1}{\cos^2 t} \; ; \; \frac{dy}{dt} = (t^2-1)'_t = 2t$$

Тогда:

$$\frac{dz}{dt} = \underline{2e^{2x-3y} \cdot \frac{1}{\cos^2 t} - 6 \cdot e^{2x-3y} \cdot t} \quad \blacksquare$$

11.4.10

$z=x^y, x=\ln t, y=\sin t; \frac{dz}{dt}=?$

□

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$$

$$\frac{\partial z}{\partial x}=(x^y)'_x=y\cdot x^{y-1};\ \frac{\partial z}{\partial y}=(x^y)'_y=x^y\cdot\ln x$$

$$\frac{dx}{dt}=(\ln t)'_t=\frac{1}{t};\ \frac{dy}{dt}=(\sin t)'_t=\cos t$$

Тогда:

$$\frac{dz}{dt}=\underline{y\cdot x^{y-1}\cdot\frac{1}{t}+x^y\cdot\ln x\cdot\cos t}$$

■

11.4.14

$z=x^3+y^3, x=uv, y=\frac{u}{v}; \frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}, dz=?$

□

$$dz=\frac{\partial z}{\partial x}\cdot dx+\frac{\partial z}{\partial y}\cdot dy$$

$$dx=\frac{\partial x}{\partial u}\cdot du+\frac{\partial x}{\partial v}\cdot dv$$

$$dy=\frac{\partial y}{\partial u}\cdot du+\frac{\partial y}{\partial v}\cdot dv$$

$$\frac{\partial z}{\partial u}=\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u}+\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial u}$$

$$\frac{\partial z}{\partial v}=\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial v}+\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial v}$$

$$\frac{\partial z}{\partial x} = (x^3+y^3)'_x = 3x^2$$

$$\frac{\partial z}{\partial y} = (x^3+y^3)'_y = 3y^2$$

$$\frac{\partial x}{\partial u} = (uv)'_u = v; \quad \frac{\partial x}{\partial v} = (uv)'_v = u$$

$$\frac{\partial y}{\partial u} = \left(\frac{u}{v}\right)'_u = \frac{1}{v}; \quad \frac{\partial y}{\partial v} = \left(\frac{u}{v}\right)'_v = -\frac{u}{v^2}$$

Тогда:

$$\frac{\partial z}{\partial u} = 3x^2 \cdot v + 3y^2 \cdot \frac{1}{v}; \quad \frac{\partial z}{\partial v} = 3x^2 \cdot u - 3y^2 \cdot \frac{u}{v^2}$$

$$dx = v \cdot du + u \cdot dv$$

$$dy = \frac{1}{v} \cdot du - \frac{u}{v^2} \cdot dv$$

$$dz = 3x^2 \cdot (v\,du + u\,dv) + 3y^2\left(\frac{1}{v}du - \frac{u}{v^2}dv\right)$$

■

11.4.15

$z = \sqrt{x^2 - y^2}$, $x = u^v$, $y = u \ln v$; $\frac{\partial z}{\partial u}$, $\frac{\partial z}{\partial v}$, dz = ?

□

$$dz = \frac{\partial z}{\partial x} \cdot dx + \frac{\partial z}{\partial y} \cdot dy$$

$$dx = \frac{\partial x}{\partial u} \cdot du + \frac{\partial x}{\partial v} \cdot dv$$

$$dy = \frac{\partial y}{\partial u} \cdot du + \frac{\partial y}{\partial v} \cdot dv$$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u}$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial v}$$

$$\frac{\partial z}{\partial x} = \left(\sqrt{x^2-y^2}\right)'_x = \frac{1}{2\sqrt{x^2-y^2}} \cdot 2x = \frac{x}{\sqrt{x^2-y^2}}$$

$$\frac{\partial z}{\partial y} = \left(\sqrt{x^2-y^2}\right)'_y = \frac{-y}{\sqrt{x^2-y^2}}$$

$$\frac{\partial x}{\partial u} = (u^v)'_u = v \cdot u^{v-1}; \quad \frac{\partial x}{\partial v} = (u^v)'_v = u^v \cdot \ln u$$

$$\frac{\partial y}{\partial u} = (u \ln v)'_u = \ln v; \quad \frac{\partial y}{\partial v} = (u \cdot \ln v)'_v = u \cdot \frac{1}{v} = \frac{u}{v}$$

$$\frac{\partial z}{\partial u} = \frac{x}{\sqrt{x^2-y^2}} \cdot v \cdot u^{v-1} - \frac{y}{\sqrt{x^2-y^2}} \cdot \ln v$$

$$\frac{\partial z}{\partial v} = \frac{x}{\sqrt{x^2-y^2}} \cdot u^v \cdot \ln u - \frac{y}{\sqrt{x^2-y^2}} \cdot \frac{u}{v}$$

$$dx = v \cdot u^{v-1} du + u^v \cdot \ln u \cdot dv$$

$$dy = \ln v \cdot du + \frac{u}{v} \cdot dv$$

$$dz = \left(\frac{x}{\sqrt{x^2-y^2}}\right) \cdot \left(v \cdot u^{v-1} du + u^v \ln u \cdot dv\right) +$$

$$+ \left(\frac{-y}{\sqrt{x^2-y^2}}\right) \cdot \left(\ln v \cdot du + \frac{u}{v} \cdot dv\right) =$$

$$= \left(\frac{x}{\sqrt{x^2-y^2}} \cdot v \cdot u^{v-1} - \frac{y}{\sqrt{x^2-y^2}} \cdot \ln v\right) du +$$

$$+ \left(\frac{x u^v}{\sqrt{x^2-y^2}} \ln u - \frac{y}{\sqrt{x^2-y^2}} \cdot \frac{u}{v}\right) dv$$

11.4.16

$z=\cos xy,\ x=ue^{v},\ y=v\ln u;\ \frac{\partial z}{\partial u},\frac{\partial z}{\partial v},\ dz=?$

$\square$

$\frac{\partial z}{\partial x}=(\cos xy)'_x=(-\sin xy)\cdot y$

$\frac{\partial z}{\partial y}=(\cos xy)'_y=(-\sin xy)\cdot x$

$\frac{\partial x}{\partial u}=(u\cdot e^{v})'_u=e^{v};\ \frac{\partial x}{\partial v}=(u\cdot e^{v})'_v=ue^{v}$

$\frac{\partial y}{\partial u}=(v\ln u)'_u=\frac{v}{u};\ \frac{\partial y}{\partial v}=(v\ln u)'_v=\ln u$

$\frac{\partial z}{\partial u}=\underline{(-\sin xy)\cdot y\cdot e^{v}+(-\sin xy)\cdot x\cdot\frac{v}{u}}$

$\frac{\partial z}{\partial v}=\underline{(-\sin xy)\cdot y\cdot ue^{v}+(-\sin xy)\cdot x\cdot\ln u}$

$dx=e^{v}\cdot du+ue^{v}\cdot dv$

$dy=\frac{v}{u}\cdot du+\ln u\cdot dv$

$dz=(-\sin xy)\cdot y\cdot(e^{v}du+ue^{v}\cdot dv)+$

$+(-\sin xy)\cdot x\cdot(\frac{v}{u}\cdot du+\ln u\cdot dv)=$

$=\underline{-\sin xy\cdot(ye^{v}+x\frac{v}{u})du-\sin xy\cdot(yue^{v}+}$

$\underline{+x\ln u)dv}$

$\blacksquare$

11.4.17

$z = \operatorname{arctg} xy$, $x = \sqrt{u^2+v^2}$, $y = u - v$; $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}, dz = ?$

$\square$

$\frac{\partial z}{\partial x} = (\operatorname{arctg} xy)'_x = \frac{y}{1+x^2y^2}$; $\frac{\partial z}{\partial y} = \frac{x}{1+x^2y^2}$ (ан-но)

$\frac{\partial x}{\partial u} = \left(\sqrt{u^2+v^2}\right)'_u = \frac{u}{\sqrt{u^2+v^2}}$; $\frac{\partial x}{\partial v} = \left(\sqrt{u^2+v^2}\right)'_v = \frac{v}{\sqrt{u^2+v^2}}$

$\frac{\partial y}{\partial u} = (u-v)'_u = 1$; $\frac{\partial y}{\partial v} = (u-v)'_v = -1$

$$\frac{\partial z}{\partial u} = \frac{y}{1+x^2y^2} \cdot \frac{u}{\sqrt{u^2+v^2}} + \frac{x}{1+x^2y^2}$$

$$\frac{\partial z}{\partial v} = \frac{y}{1+x^2y^2} \cdot \frac{v}{\sqrt{u^2+v^2}} - \frac{x}{1+x^2y^2}$$

$$dx = \frac{u}{\sqrt{u^2+v^2}} \cdot du + \frac{v}{\sqrt{u^2+v^2}} \cdot dv$$

$$dy = 1 \cdot du - 1 \cdot dv$$

$$dz = \frac{y}{1+x^2y^2} \cdot \left(\frac{u}{\sqrt{u^2+v^2}} du + \frac{v}{\sqrt{u^2+v^2}} dv\right) + \frac{x}{1+x^2y^2} \cdot (du - dv) = \frac{1}{1+x^2y^2}\left(\left(\frac{uy}{\sqrt{u^2+v^2}} + x\right) du + \left(\frac{vy}{\sqrt{u^2+v^2}} - x\right) dv\right)$$

$\blacksquare$

11.4.18

$z=\sqrt{x+y}$, $x=u\,tg\,v$, $y=u\,ctg\,v$; $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}, dz=?$

$\square$

$$\frac{\partial z}{\partial x}=(\sqrt{x+y})'_x=\frac{1}{2\sqrt{x+y}};\ \frac{\partial z}{\partial y}=(\sqrt{x+y})'_y=\frac{1}{2\sqrt{x+y}}$$

$$\frac{\partial x}{\partial u}=(u\,tg\,v)'_u=tg\,v;\ \frac{\partial x}{\partial v}=(u\,tg\,v)'_v=\frac{u}{\cos^2 v}$$

$$\frac{\partial y}{\partial u}=(u\,ctg\,v)'_u=ctg\,v;\ \frac{\partial y}{\partial v}=(u\,ctg\,v)'_v=\frac{-u}{\sin^2 v}$$

$$\frac{\partial z}{\partial u}=\frac{1}{2\sqrt{x+y}}\cdot tg\,v+\frac{1}{2\sqrt{x+y}}\cdot ctg\,v$$

$$\frac{\partial z}{\partial v}=\frac{1}{2\sqrt{x+y}}\cdot\frac{u}{\cos^2 v}-\frac{1}{2\sqrt{x+y}}\cdot\frac{u}{\sin^2 v}$$

$$dz=\frac{1}{2\sqrt{x+y}}\left((tg\,v+ctg\,v)du+\left(\frac{1}{\cos^2 v}-\frac{1}{\sin^2 v}\right)u\,dv\right)$$

$\blacksquare$

11.4.19

$z=\ln\sqrt[7]{x^2+3y^5}$, $x=u\cos v$, $y=u\sin v$; $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}, dz=?$

$\square$

$$\frac{\partial z}{\partial x}=\left(\ln\sqrt[7]{x^2+3y^5}\right)'_x=\frac{1}{(x^2+3y^5)^{\frac{1}{7}}}\cdot\frac{1}{7}\cdot(x^2+3y^5)^{-\frac{6}{7}}\cdot 2x=$$

$$=\frac{2x}{7\cdot(x^2+3y^5)^{\frac{1}{7}}\cdot(x^2+3y^5)^{6/7}}=\frac{2x}{7x^2+21y^5}$$

$$\frac{\partial z}{\partial y}=\left[\text{ан-но } \frac{\partial z}{\partial x}\right]=\frac{15y^4}{7x^2+21y^5}$$

$$\frac{\partial x}{\partial u}=(u\cos v)'_u=\cos v;\quad \frac{\partial x}{\partial v}=(u\cos v)'_v=-u\sin v$$

$$\frac{\partial y}{\partial u}=(u\sin v)'_u=\sin v;\quad \frac{\partial y}{\partial v}=(u\sin v)'_v=u\cos v$$

$$\frac{\partial z}{\partial u}=\frac{2x}{7x^2+21y^5}\cdot\cos v+\frac{15y^4}{7x^2+21y^5}\cdot\sin v$$

$$\frac{\partial z}{\partial v}=\frac{2x}{7x^2+21y^5}\cdot(-u\sin v)+\frac{15y^4}{7x^2+21y^5}\cdot u\cos v$$

$$dz=\frac{1}{7(x^2+3y^5)}\cdot\Big((2x\cos v+15y^4\cdot\sin v)du+$$

$$+(-2xu\sin v+15y^4u\cos v)dv\Big)$$

■

11.4.22

$$xe^{2y}-y\ln x=8;\quad y'(x)=?$$

□

$$F(x;y)=xe^{2y}-y\ln x-8$$

$$y'_x=-\frac{F'_x(x;y)}{F'_y(x;y)}$$

$$F'_x=(xe^{2y}-y\ln x-8)'_x=e^{2y}-\frac{y}{x}$$

$$F'_y=(xe^{2y}-y\ln x-8)'_y=2xe^{2y}-\ln x$$

$$y'_x=-\frac{e^{2y}-\frac{y}{x}}{2xe^{2y}-\ln x}=\frac{e^{2y}-\frac{y}{x}}{\ln x-2xe^{2y}}$$

■

11.4.23

$e^y+9x^2e^{-y}-26x=0;\ y'(x)=?$

$\square$

$F(x;y)=e^y+9x^2e^{-y}-26x$

$$y'_x=-\frac{F'_x(x;y)}{F'_y(x;y)}$$

$F'_x=(e^y+9x^2e^{-y}-26x)'_x=18xe^{-y}-26$

$F'_y=(e^y+9x^2e^{-y}-26x)'_y=e^y-9x^2e^{-y}$

$$y'_x=-\frac{18xe^{-y}-26}{e^y-9x^2e^{-y}}=\frac{18xe^{-y}-26}{9x^2e^{-y}-e^y}$$

$\blacksquare$

11.4.24

$\ln\frac{\sqrt{x^2+y^2}}{2}=\operatorname{arctg}\frac{y}{x};\ y'(x)=?$

$\square$

$F(x;y)=\ln\frac{\sqrt{x^2+y^2}}{2}-\operatorname{arctg}\frac{y}{x}$

$$F'_x=\left(\ln\frac{\sqrt{x^2+y^2}}{2}-\operatorname{arctg}\frac{y}{x}\right)'_x=$$

$$=\frac{2}{\sqrt{x^2+y^2}}\cdot\frac{1}{2}\cdot\frac{1}{2}\frac{1}{\sqrt{x^2+y^2}}\cdot 2x-\frac{1}{1+\frac{y^2}{x^2}}\cdot y\cdot(-1)\cdot\frac{1}{x^2}=$$

$$=\frac{x}{x^2+y^2}+\frac{y}{x^2+y^2}=\frac{x+y}{x^2+y^2}$$

$F'_y = (\ln \frac{\sqrt{x^2+y^2}}{2} - \operatorname{arctg} \frac{y}{x})'_y =$

$= \frac{y}{x^2+y^2} - \frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{1}{x} \cdot 1 = \frac{y}{x^2+y^2} - \frac{x}{x^2+y^2} = \frac{y-x}{x^2+y^2}$

$y'_x = -\frac{\frac{x+y}{x^2+y^2}}{\frac{y-x}{x^2+y^2}} = -\frac{x+y}{y-x} = \frac{x+y}{x-y}$ ■

11.4.25

$x^2 \ln y - y^2 \ln x = 0$; $y'(x) = ?$

□

$F(x;y) = x^2 \ln y - y^2 \ln x$

$F'_x = (x^2 \ln y - y^2 \ln x)'_x = 2x \ln y - y^2 \frac{1}{x}$

$F'_y = (x^2 \ln y - y^2 \ln x)'_y = x^2 \cdot \frac{1}{y} - 2y \ln x$

$y'_x = -\frac{\frac{2x\ln y \cdot x - y^2}{x}}{\frac{x^2 - 2y \ln x \cdot y}{y}} = \frac{y^3 - 2x^2 y \ln y}{x^3 - 2y^2 x \ln x}$ ■

11.4.26

$1 + xy - \ln(e^{xy} + e^{-xy}) = 0$

□

$F(x;y) = 1 + xy - \ln(e^{xy} + e^{-xy})$

$$F'_x = (1 + xy - \ln(e^{xy} + e^{-xy}))'_x =$$

$$= y - \frac{1}{e^{xy} + e^{-xy}} \cdot (e^{xy} \cdot y + e^{-xy} \cdot (-y)) = y$$

$$F'_y = (1 + xy - \ln(e^{xy} + e^{-xy}))'_y = [\text{ан-но } F'_x] = x$$

$$y'_x = -\frac{y}{x}$$

■

11.4.35

$z^3 - 3xyz = R^2$; $z'_x, z'_y, dz = ?$

□

$$F'_z = (z^3 - 3xyz - R^2)'_z = 3z^2 - 3xy$$

$$F'_x = (z^3 - 3xyz - R^2)'_x = -3yz$$

$$F'_y = (z^3 - 3xyz - R^2)'_y = -3xz$$

$$z'_x = -\frac{-3yz}{3z^2 - 3xy} = \frac{3yz}{3z^2 - 3xy} = \frac{yz}{z^2 - xy}$$

$$z'_y = -\frac{-3xz}{3z^2 - 3xy} = \frac{3xz}{3z^2 - 3xy} = \frac{xz}{z^2 - xy}$$

$$dz = \frac{yz}{z^2 - xy} dx + \frac{xz}{z^2 - xy} dy = \frac{yz\,dx + xz\,dy}{z^2 - xy}$$

■

11.4.36

$x+y+z=e^z$; $z'_x, z'_y, dz=?$

$\square$

$F'_z=(x+y+z-e^z)'_z=1-e^z$

$F'_x=(x+y+z-e^z)'_x=1$

$F'_y=(x+y+z-e^z)'_x=1$

$z'_x=-\frac{1}{1-e^z}=\frac{1}{e^z-1}$; $z'_y=-\frac{1}{1-e^z}=\frac{1}{e^z-1}$

$$dz=\frac{1}{e^z-1}dx+\frac{1}{e^z-1}dy \quad \blacksquare$$

11.4.37

$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$; $z'_x, z'_y, dz=?$

$\square$

$F'_z=\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1\right)'_z=\frac{2z}{c^2}$; $F'_x=\frac{2x}{a^2}$; $F'_y=\frac{2y}{b^2}$

$z'_x=-\frac{\frac{2x}{a^2}}{\frac{2z}{c^2}}=-\frac{xc^2}{za^2}$; $z'_y=-\frac{\frac{2y}{b^2}}{\frac{2z}{c^2}}=-\frac{yc^2}{zb^2}$

$$dz=-\frac{xc^2}{za^2}dx-\frac{yc^2}{zb^2}dy \quad \blacksquare$$