Практическая работа часть 3

7.3.20

$$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right)=\lim_{x\to 0}\left(\frac{\sin x - x}{x\cdot\sin x}\right)=\left[\frac{0}{0}\right]=$$

$$=\lim_{x\to 0}\frac{(\sin x - x)'_x}{(x\cdot\sin x)'_x}=\lim_{x\to 0}\left(\frac{\cos x - 1}{\sin x + x\cdot\cos x}\right)=$$

$$=\lim_{x\to 0}\frac{\cos x - 1}{\sin x + x\cdot\cos x}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{(\cos x - 1)'_x}{(\sin x + x\cdot\cos x)'_x}=$$

$$=\lim_{x\to 0}\frac{-\sin x}{\cos x + 1\cdot\cos x + x\cdot(-\sin x)}=\lim_{x\to 0}\frac{-\sin x}{2\cos x - x\cdot\sin x}=$$

$$=\frac{-0}{2\cdot 1 - 0\cdot 0}=\frac{0}{2}=\underline{0}$$

7.3.21

$$\lim_{x\to\infty} x\cdot\left(e^{\frac{1}{x}}-1\right)=[\infty\cdot 0]=\lim_{x\to\infty}\frac{\left(e^{\frac{1}{x}}-1\right)}{\left(\frac{1}{x}\right)}=$$

$$=\left[\frac{0}{0}\right]=\lim_{x\to\infty}\frac{\left(e^{\frac{1}{x}}-1\right)'_x}{\left(\frac{1}{x}\right)'_x}=\lim_{x\to\infty}\frac{e^{\frac{1}{x}}\cdot\left(\frac{1}{x}\right)'_x}{\left(\frac{1}{x}\right)'_x}=$$

$$=\lim_{x\to\infty}\frac{e^{\frac{1}{x}}\cdot\left(-\frac{1}{x^2}\right)}{\left(-\frac{1}{x^2}\right)}=\lim_{x\to\infty}\frac{e^{\frac{1}{x}}\cdot(-x^2)}{(-x^2)}=$$

$$=\lim_{x\to\infty} e^{\frac{1}{x}}=e^{\frac{1}{\infty}}=\underline{e^0=1}$$

7.3.22

$$\lim_{x\to 1}\left(\frac{1}{1-x^3}-\frac{1}{1-x^2}\right)=[\infty-\infty]=$$

$$=\lim_{x\to 1}\left(\frac{1-x^2-1+x^3}{(1-x^3)(1-x^2)}\right)=\lim_{x\to 1}\frac{x^3-x^2}{(1-x^2)(1-x^3)}=\left[\frac{0}{0}\right]=$$

$$=\lim_{x\to 1}\frac{(x^3-x^2)'_x}{((1-x^2)(1-x^3))'_x}=\lim_{x\to 1}\frac{3x^2-2x}{-2x(1-x^3)+(-3x^2)(1-x^2)}=$$

$$=\lim_{x\to 1}\frac{3x^2-2x}{-2x+2x^4-3x^2+3x^4}=\lim_{x\to 1}\frac{3x^2-2x}{5x^4-3x^2-2x}=$$

$$=\frac{3\cdot 1^2-2\cdot 1}{5\cdot 1^4-3\cdot 1^2-2\cdot 1}=\frac{3-2}{5-3-2}=\frac{1}{0}=\underline{\infty}$$

7.3.24

$$\lim_{x\to 0} x^{tgx}=?$$

$\square$

$$\lim_{x\to 0} x^{tgx}=[0^0]$$

$$\lim_{x\to 0}\ln y=\lim_{x\to 0}\ln x^{tgx}=\lim_{x\to 0} tgx\cdot\ln x=$$

$$=\lim_{x\to 0}\frac{\ln x}{ctgx}=\left[\frac{\infty}{\infty}\right]=\lim_{x\to 0}\frac{(\ln x)'_x}{(ctgx)'_x}=$$

$$=\lim_{x\to 0}\frac{-\sin^2 x}{x}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{(-\sin^2 x)'_x}{(x)'_x}=$$

$$=\lim_{x\to 0}\frac{-2\sin x\cdot\cos x}{1}=\frac{-2\cdot 0\cdot 1}{1}=0$$

$$\ln\lim_{x\to 0} y=\lim_{x\to 0}\ln y=0$$

$$\Downarrow$$

$$\lim_{x\to 0} y=\lim_{x\to 0}(x^{tgx})=\underline{1}$$

■

7.3.25

$$\lim_{x\to 0}(\cos 2x)^{\frac{1}{x^2}}=?$$

□

$$\lim_{x\to 0}(\cos 2x)^{\frac{1}{x^2}}=[1^\infty]$$

$$\lim_{x\to 0}\ln y=\lim_{x\to 0}\ln\left((\cos 2x)^{\frac{1}{x^2}}\right)=\lim_{x\to 0}\frac{\ln(\cos 2x)}{x^2}=$$

$$=\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{(\ln\cos 2x)'_x}{(x^2)'_x}=\lim_{x\to 0}\frac{\frac{1}{\cos 2x}\cdot(-\sin 2x)\cdot 2}{2x}=$$

$$=\lim_{x\to 0}\frac{-2\,tg2x}{2x}=\lim_{x\to 0}\frac{-tg2x}{x}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{(-tg2x)'_x}{(x)'_x}=$$

$$=\lim_{x\to 0}\frac{\frac{-1}{\cos^2 2x}\cdot 2}{1}=\lim_{x\to 0}\frac{-2}{\cos^2 2x}=\frac{-2}{1^2}=-2$$

$$\ln \lim_{x\to 0} y = \lim_{x\to 0} \ln y = -2$$

$$\Downarrow$$

$$\lim_{x\to 0} y = \lim_{x\to 0} (\cos 2x)^{\frac{1}{x^2}} = e^{-2}$$

$\blacksquare$

7.3.26

$$\lim_{x\to 0} \left(\frac{1}{x}\right)^{x^2} = ?$$

$\square$

$$\lim_{x\to 0} \left(\frac{1}{x}\right)^{x^2} = [\infty^0]$$

$$\lim_{x\to 0} \ln y = \lim_{x\to 0} \ln\left(\frac{1}{x}\right)^{x^2} = \lim_{x\to 0} x^2 \cdot \ln\left(\frac{1}{x}\right) =$$

$$= \lim_{x\to 0} \frac{\ln\left(\frac{1}{x}\right)}{\frac{1}{x^2}} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to 0} \frac{\left(\ln\left(\frac{1}{x}\right)\right)'_x}{\left(\frac{1}{x^2}\right)'_x} =$$

$$= \lim_{x\to 0} \frac{x \cdot (-1) \cdot x^{-2}}{-2 \cdot x^{-3}} = \lim_{x\to 0} \frac{x^3}{x} = \left[\frac{0}{0}\right] =$$

$$= \lim_{x\to 0} \frac{(x^3)'_x}{(x)'_x} = \lim_{x\to 0} \frac{3x^2}{1} = \frac{3\cdot 0}{1} = 0$$

$$\ln \lim_{x\to 0} y = \lim_{x\to 0} \ln y = 0$$

$\Downarrow$

$\lim\limits_{x\to 0} y = \lim\limits_{x\to 0} \left(\frac{1}{x}\right)^{x^2} = \underline{1}$ ■

<u>7.3.27</u>

$\lim\limits_{x\to 0} x^{\frac{1}{1+\ln x}} = ?$

□

$\lim\limits_{x\to 0} x^{\frac{1}{1+\ln x}} = [0^0]$

$\lim\limits_{x\to 0} \ln x^{\frac{1}{1+\ln x}} = \lim\limits_{x\to 0} \frac{\ln x}{1+\ln x} = \left[\frac{\infty}{\infty}\right] =$

$= \lim\limits_{x\to 0} \frac{(\ln x)'_x}{(1+\ln x)'_x} = \lim\limits_{x\to 0} \frac{\frac{1}{x}}{0+\frac{1}{x}} = \lim\limits_{x\to 0} 1 = 1$

$\ln \lim\limits_{x\to 0} y = \lim\limits_{x\to 0} \ln y = 1$

$\Downarrow$

$\lim\limits_{x\to 0} y = \lim\limits_{x\to 0} \left(x^{\frac{1}{1+\ln x}}\right) = \underline{e}$ ■

7.3.30

$P(x) = x^5 - 3x^4 + 7x + 2$

разложить по степеням, при $x_0 = 2$

по ф. Тейлора

$\square$

$P(2) = 2^5 - 3\cdot 2^4 + 7\cdot 2 + 2 = 32 - 48 + 14 + 2 = 0$

$P'_x = (x^5 - 3x^4 + 7x + 2)'_x = 5x^4 - 12x^3 + 7$

$\Rightarrow P'(2) = 5\cdot 2^4 - 12\cdot 2^3 + 7 = -9$

$P''_x = (5x^4 - 12x^3 + 7)'_x = 20x^3 - 36x^2$

$\Rightarrow P''(2) = 20\cdot 2^3 - 36\cdot 4 = 16$

$P'''_x = (20x^3 - 36x^2)'_x = 60x^2 - 72x$

$\Rightarrow P'''(2) = 60\cdot 2^2 - 72\cdot 2 = 96$

$P^{(4)}_x = (60x^2 - 72x)'_x = 120x - 72$

$\Rightarrow P^{(4)}(2) = 120\cdot 2 - 72 = 168$

$P^{(5)}_x = (120x - 72)'_x = 120$; $P^{(6)}_x = (120)'_x = 0 \Rightarrow P^{(6)}(2) = 0 \Rightarrow$

$\Rightarrow P^{(6)} = P^{(7)} = \ldots = 0$

$P(x) = 0 + \frac{-9}{1!}(x-2) + \frac{16}{2!}(x-2)^2 + \frac{96}{3!}(x-2)^3 +$

$+ \frac{168}{4!}(x-2)^4 + \frac{120}{5!}(x-2)^5 =$

$= -9(x-2) + 8(x-2)^2 + 16(x-2)^3 + 7(x-2)^4 + (x-2)^5$

7.3.32

$f(x) = 2^x$, $x_0 = \log_2 3$, ф. Тейлора

$\square$

$$f(\log_2 3) = 2^{\log_2 3} = 3$$

$$f'_x = (2^x)'_x = 2^x \cdot \ln 2 \Rightarrow f'(\log_2 3) = 2^{\log_2 3} \ln 2 = 3\ln 2$$

$$f''_x = (2^x \ln 2)' = 2^x \cdot \ln^2 2 \Rightarrow f''(\log_2 3) = 3 \ln^2 2$$

$$f'''_x = (2^x \ln^2 2)' = 2^x \cdot \ln^3 2 \Rightarrow f'''(\log_2 3) = 3 \ln^3 2$$

Тогда

$$f^{(n)}(\log_2 3) = 3 \cdot \ln^n 2$$

$$f(x) = 3 + \frac{3\ln 2}{1!} \cdot (x - \log_2 3) + \frac{3\ln^2 2}{2!} \cdot (x - \log_2 3)^2 + \frac{3\ln^3 2}{3!}(x - \log_2 3)^3 + \ldots + \frac{3\ln^n 2}{n!} \cdot (x - \log_2 3)^n + o((x - \log_2 3)^n) = 3 + 3\ln 2 (x - \log_2 3) + \frac{3\ln^2 2 (x - \log_2 3)^2}{2!} + \frac{3\ln^3 2 \cdot (x - \log_2 3)^3}{3!} + \ldots + \frac{3\ln^n 2 (x - \log_2 3)^n}{n!} + o((x - \log_2 3)^n), \; x \to x_0$$

$\blacksquare$

7.3.33

$f(x) = \frac{x^2 \ln x}{2}$, $x_0 = 1$, ф. Тейлора

$\square$

$f(1) = \frac{1^2 \cdot \ln 1}{2} = \frac{1 \cdot 0}{2} = 0$

$f'(x) = \left(\frac{x^2 \ln x}{2}\right)'_x = \frac{1}{2} \cdot (x^2 \cdot \ln x)'_x =$

$= \frac{1}{2}(2x \cdot \ln x + x^2 \cdot \frac{1}{x}) = \frac{1}{2} x (2\ln x + 1) \Rightarrow$

$\Rightarrow f'(1) = \frac{1}{2} \cdot 1 (\ln 1 \cdot 2 + 1) = \frac{1}{2}$

$f''(x) = (\frac{1}{2} x (2\ln x + 1))'_x = \frac{1}{2}(x'_x (2\ln x + 1) +$

$+ x \cdot (2\ln x + 1)'_x) = \frac{1}{2}(2\ln x + 1 + x \cdot \frac{2}{x}) =$

$= \ln x + \frac{3}{2} \Rightarrow f''(1) = \ln 1 + \frac{3}{2} = \frac{3}{2}$

$f'''(x) = (\ln x + \frac{3}{2})'_x = \frac{1}{x} \Rightarrow f'''(1) = 1$

$f^{(4)}(x) = (\frac{1}{x})'_x = -\frac{1}{x^2} \Rightarrow f^{(4)}(1) = -1$

$f^{(5)}(x) = (-\frac{1}{x^2})'_x = \frac{2}{x^3} \Rightarrow f^{(5)}(1) = 2$

$f^{(6)}(x) = (\frac{2}{x^3})'_x = -\frac{6}{x^4} \Rightarrow f^{(6)}(1) = -6 = -(2 \cdot 3)$

$f^{(7)}(x) = (-\frac{6}{x^4})'_x = \frac{24}{x^5} \Rightarrow f^{(7)}(1) = 24 = 2 \cdot 3 \cdot 4$

Тогда $(n \geqslant 4)$

$$f^{(n)}(1) = (-1)^{n-1} \cdot (n-3)!$$

$$f(x) = 0 + \frac{\frac{1}{2}}{1!} \cdot (x-1) + \frac{\frac{3}{2}}{2!}(x-1)^2 + \frac{1}{3!}(x-1)^3 - \frac{(4-3)!}{4!} \cdot (x-1)^4 + \frac{(5-3)!}{5!}(x-1)^5 + \ldots + \frac{(-1)^{n-1}(n-3)!}{n!} \cdot (x-1)^n + o((x-1)^n) = \frac{1}{2}(x-1) + \frac{3(x-1)^2}{2 \cdot 2!} + \frac{(x-1)^3}{3!} - \frac{(x-1)^4}{2 \cdot 3 \cdot 4} + \frac{(x-1)^5}{3 \cdot 4 \cdot 5} + \ldots + \frac{(-1)^{n-1}(x-1)^n}{(n-2)(n-1) \cdot n} + o((x-1)^n), \ x \to 1$$

■

7.3.34

$$f(x) = e^{2-x}, \ k = 4$$

□

$$f(0) = e^{2-0} = e^2$$

$$f'_x = (e^{2-x})'_x = e^{2-x} \cdot (2-x)'_x = -e^{2-x} \Rightarrow f'(0) = -e^{2-0} = -e^2$$

$$f''_x = (-e^{2-x})'_x = -(e^{2-x})'_x = e^{2-x} \Rightarrow f''(0) = e^2$$

$$f'''_x = (e^{2-x})'_x = -e^{2-x} \Rightarrow f'''(0) = -e^2$$

$$f^{(4)}_x = (-e^{2-x})'_x = e^{2-x} \Rightarrow f^{(4)}(0) = e^2$$

$$f(x) = e^2 - e^2 \cdot x + \frac{e^2 \cdot x^2}{2!} - \frac{e^2 \cdot x^3}{3!} + \frac{e^2 \cdot x^4}{4!} + o(x^4)$$

■

7.3.35

$f(x) = \arcsin x$, $k = 3$

$\square$

$f(0) = \arcsin 0 = 0$

$f'_x = (\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \Rightarrow f'(0) = 1$

$f''_x = \left(\frac{1}{\sqrt{1-x^2}}\right)'_x = \frac{-1}{(\sqrt{1-x^2})^2} \cdot \frac{1}{2} \cdot (\sqrt{1-x^2})^{-\frac{1}{2}} \cdot (-2x) =$

$= \frac{x}{\sqrt{1-x^2} \cdot (1-x^2)} \Rightarrow f''(0) = 0$

$f'''_x = \left(\frac{x}{\sqrt{1-x^2}(1-x^2)}\right)'_x = \frac{x'_x \cdot (\sqrt{1-x^2}(1-x^2)) - x \cdot (\sqrt{1-x^2}(1-x^2))'_x}{(\sqrt{1-x^2}(1-x^2))^2} =$

$= \frac{1 \cdot \sqrt{1-x^2}(1-x^2) - x \cdot \left(\frac{-2x}{2\sqrt{1-x^2}} \cdot (1-x^2) + \sqrt{1-x^2} \cdot (-2x)\right)}{(\sqrt{1-x^2}(1-x^2))^2} =$

$= \frac{\sqrt{1-x^2}(1-x^2) - x \cdot (-x \cdot \sqrt{1-x^2} + \sqrt{1-x^2}(-2x))}{(\sqrt{1-x^2}(1-x^2))^2} =$

$= \frac{\sqrt{1-x^2} - x^2 \cdot \sqrt{1-x^2} + x^2 \cdot \sqrt{1-x^2} + 2x^2\sqrt{1-x^2}}{(\sqrt{1-x^2})^2 \cdot (1-x^2)^2} =$

$= \frac{1+2x^2}{\sqrt{1-x^2} \cdot (1-x^2)^2} \Rightarrow f'''(0) = \frac{1+2 \cdot 0}{\sqrt{1-0} \cdot (1-0)^2} = 1$

$f(x) = x + \frac{x^3}{3!} + o(x^3)$ $\blacksquare$