Домашняя работа часть 6

9.1.27

$$\int_1^{e^2} \frac{\ln^3 x}{3x}\,dx = \left[t = \ln x,\ \text{тогда: } dt = d(\ln x) = \right.$$

$$\left. = (\ln x)'_x\,dx = \frac{1}{x}dx;\ \begin{array}{c|c|c} x & 1 & e^2 \\ \hline t=\ln x & 0 & 2 \end{array} \right] =$$

$$= \int_0^2 \frac{t^3 dt}{3} = \frac{1}{3}\int_0^2 t^3 dt = \frac{1}{3}\cdot\frac{t^4}{4}\Big|_0^2 = \frac{1}{12}\cdot 2^4 - \frac{1}{12}\cdot 0^4 =$$

$$= \frac{1}{12}\cdot 16 - 0 = \frac{4}{3}$$

9.1.28

$$\int_{\pi}^{2\pi} \frac{x+\cos x}{x^2+2\sin x}\,dx = \left[t = x^2+2\sin x,\ \text{тогда: } dt = \right.$$

$$= (x^2+2\sin x)'_x\,dx = (2x+2\cos x)\,dx \Rightarrow \frac{1}{2}dt = (x+\cos x)dx$$

$$\left. \begin{array}{c|c|c} x & \pi & 2\pi \\ \hline t=x^2+2\sin x & \pi^2 & 4\pi^2 \end{array} \right] =$$

$$= \int_{\pi^2}^{4\pi^2} \frac{\frac{1}{2}dt}{t} = \frac{1}{2}\int_{\pi^2}^{4\pi^2} \frac{dt}{t} = \frac{1}{2}\ln|t|\Big|_{\pi^2}^{4\pi^2} = \frac{1}{2}\cdot\ln(4\pi^2) -$$

$$- \frac{1}{2}\ln(\pi^2) = \frac{1}{2}\cdot 2\cdot\ln(2\pi) - \frac{1}{2}\cdot 2\cdot \ln(\pi) =$$

$$= \ln(2\pi) - \ln(\pi) = \ln\left(\frac{2\pi}{\pi}\right) = \ln(2)$$

9.1.29

$$\int_0^1 \frac{4\,\mathrm{arctg}x - x}{1+x^2}dx = 4\cdot\int_0^1 \frac{\mathrm{arctg}x}{1+x^2}dx - \int_0^1 \frac{xdx}{1+x^2} =$$

$$= \Big[1)\ t = \mathrm{arctg}x, \text{ тогда: } dt = (\mathrm{arctg}x)'_x dx =$$

$$= \frac{dx}{1+x^2};\quad \begin{array}{c|c|c} x & 0 & 1 \\ \hline t=\mathrm{arctg}x & 0 & \pi/4 \end{array};\quad 2)\ u = 1+x^2 \Rightarrow du =$$

$$= 2x\,dx,\quad \frac{1}{2}du = xdx;\quad \begin{array}{c|c|c} x & 0 & 1 \\ \hline u=1+x^2 & 1 & 2 \end{array} \Big] =$$

$$= 4\int_0^{\pi/4} t\,dt - \frac{1}{2}\int_1^2 \frac{du}{u} = 4\cdot\frac{t^2}{2}\Big|_0^{\pi/4} - \frac{1}{2}\ln|u|\Big|_1^2 =$$

$$= \left(2\cdot\left(\frac{\pi}{4}\right)^2 - 2\cdot 0\right) - \frac{1}{2}\left(\ln|2| - \ln|1|\right) =$$

$$= 2\cdot\frac{\pi^2}{16} - \frac{1}{2}\ln(2) = \underline{\frac{\pi^2}{8} - \frac{\ln(2)}{2}}$$

9.1.30

$$\int_1^e \frac{\sin(\ln x)}{x}dx = \Big[t = \ln x \Rightarrow dt = (\ln x)'_x dx =$$

$$= \frac{1}{x}dx;\quad \begin{array}{c|c|c} x & 1 & e \\ \hline t=\ln x & 0 & 1 \end{array} \Big] = \int_0^1 \sin(t) =$$

$$= -\cos(t)\Big|_0^1 = -\cos(1) - (-1)\cdot\cos(0) = -\cos(1) + 1 =$$

$$= \underline{1 - \cos(1)}$$

9.1.31

$$\int_{-1}^{0}\frac{3^x-2^x}{6^x}dx=\int_{-1}^{0}\frac{3^x}{6^x}dx-\int_{-1}^{0}\frac{2^x}{6^x}dx=\int_{-1}^{0}2^{-x}dx-\int_{-1}^{0}3^{-x}dx=$$

$$=\int_{-1}^{0}\left(\frac{1}{2}\right)^x dx-\int_{-1}^{0}\left(\frac{1}{3}\right)^x dx=\frac{\left(\frac{1}{2}\right)^x}{\ln\left(\frac{1}{2}\right)}\Bigg|_{-1}^{0}-\frac{\left(\frac{1}{3}\right)^x}{\ln\left(\frac{1}{3}\right)}\Bigg|_{-1}^{0}=$$

$$=\frac{1}{3^x\cdot\ln 3}\Bigg|_{-1}^{0}-\frac{1}{2^x\cdot\ln 2}\Bigg|_{-1}^{0}=\left(\frac{1}{3^0\cdot\ln 3}-\frac{1}{3^{-1}\ln 3}\right)-$$

$$-\left(\frac{1}{2^0\cdot\ln 2}-\frac{1}{2^{-1}\ln 2}\right)=\frac{1}{\ln 3}-\frac{3}{\ln 3}-\frac{1}{\ln 2}+\frac{2}{\ln 2}=$$

$$=\frac{1}{\ln 2}-\frac{2}{\ln 3}$$

9.1.32

$$\int_{0}^{1}\frac{x\,dx}{\sqrt{x^4+x^2+1}}=\int_{0}^{1}\frac{x\,dx}{\sqrt{x^4+2\cdot\frac{1}{2}\cdot x^2+\frac{1}{4}+\frac{3}{4}}}=\int_{0}^{1}\frac{x\,dx}{\sqrt{(x^2+\frac{1}{2})^2+\frac{3}{4}}}=$$

$$=\Big[t=x^2+\frac{1}{2}\Rightarrow dt=(x^2+\frac{1}{2})'_x dx=2x\,dx\Rightarrow$$

$$\Rightarrow\frac{1}{2}dt=x\,dx\quad \begin{array}{c|c|c} x & 0 & 1 \\ \hline t=x^2+\frac{1}{2} & \frac{1}{2} & 1\frac{1}{2} \end{array}\Big]=$$

$$=\frac{1}{2}\int_{\frac{1}{2}}^{1\frac{1}{2}}\frac{dt}{\sqrt{t^2+\frac{3}{4}}}=\frac{1}{2}\ln\left|t+\sqrt{t^2+\frac{3}{4}}\right|\Bigg|_{\frac{1}{2}}^{1\frac{1}{2}}=$$

$$=\frac{1}{2}\left(\ln\left(\frac{3}{2}+\sqrt{\frac{9}{4}+\frac{3}{4}}\right)-\ln\left(\frac{1}{2}+\sqrt{\frac{1}{4}+\frac{3}{4}}\right)\right)=$$

$$= \frac{1}{2}\left(\ln\left(\frac{3}{2}+\sqrt{3}\right) - \ln\left(\frac{1}{2}+1\right)\right) = \frac{1}{2}\left(\ln\left(\frac{3+2\sqrt{3}}{2}\right) - \right.$$

$$\left. - \ln\left(\frac{3}{2}\right)\right) = \frac{1}{2}\cdot\ln\left(\frac{\frac{3+2\sqrt{3}}{2}}{\frac{3}{2}}\right) = \underline{\frac{1}{2}\cdot\ln\left(\frac{3+2\sqrt{3}}{3}\right)}$$

9.1.33

$$\int_0^{\pi/4} tg^3x\,dx = \int_0^{\pi/4} tgx\cdot\left(\frac{1}{\cos^2x}-1\right)dx = \int_0^{\pi/4}\frac{tgx}{\cos^2x}dx - \int_0^{\pi/4} tgx\,dx$$

$= \Big[$ 1) $t = tgx \Rightarrow dt = (tgx)'_x\,dx = \frac{dx}{\cos^2x}$, тогда:

x	0	$\pi/4$
$t = tgx$	0	1

$$\Big] = \int_0^1 t\,dt - \int_0^{\pi/4} tgx\,dx =$$

$$= \frac{t^2}{2}\Big|_0^1 + \ln|\cos x|\Big|_0^{\pi/4} = \left(\frac{1^2}{2}-\frac{0^2}{2}\right) + \left(\ln\left|\cos\frac{\pi}{4}\right| - \ln|\cos 0|\right) =$$

$$= \frac{1}{2} + \ln\left|\frac{\cos\frac{\pi}{4}}{\cos 0}\right| = \frac{1}{2} + \ln\frac{\sqrt{2}}{2} = \frac{1}{2} + \ln\frac{1}{\sqrt{2}} =$$

$$= \frac{1}{2} + \ln 2^{-\frac{1}{2}} = \underline{\frac{1}{2} - \frac{1}{2}\ln(2)}$$

9.1.34

$$\int_0^{\pi/3} tgx\cdot\ln(\cos x)\,dx = \Big[t = \ln(\cos x) \Rightarrow dt = -tgx\,dx,$$

$-dt = tgx\,dx$;

x	0	$\pi/3$
$t = \ln(\cos x)$	0	$-\ln 2$

$$\Big] = \int_0^{-\ln 2} -t\,dt =$$

$$= \int_{-\ln 2}^{0} t\,dt = \frac{t^2}{2}\Big|_{-\ln 2}^{0} = \frac{0^2}{2} - \frac{(-\ln 2)^2}{2} = \underline{-\frac{\ln^2(2)}{2}}$$

9.1.35

$$\int_0^2 \frac{x\,dx}{\sqrt{x+2}+\sqrt{3x+2}} = \Big[\text{домножим числ. и знамен. на } \sqrt{3x+2}-\sqrt{x+2}\Big] = \int_0^2 \frac{x(\sqrt{3x+2}-\sqrt{x+2})\,dx}{(\sqrt{3x+2})^2-(\sqrt{x+2})^2} =$$

$$= \int_0^2 \frac{x(\sqrt{3x+2}-\sqrt{x+2})}{2x}dx = \frac{1}{2}\int_0^2 (\sqrt{3x+2}-\sqrt{x+2})\,dx =$$

$$= \frac{1}{2}\int_0^2 (3x+2)^{\frac{1}{2}}dx - \frac{1}{2}\int_0^2 (x+2)^{\frac{1}{2}}dx = \Big[\text{1) 5-я ф-ла; 2) } dx = d(x+2)\Big] = \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{(3x+2)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2}\cdot\frac{(x+2)^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_0^2 =$$

$$= \frac{(3x+2)^{\frac{3}{2}}}{9} - \frac{(x+2)^{\frac{3}{2}}}{3}\Bigg|_0^2 = \frac{(8)^{\frac{3}{2}}}{9} - \frac{(4)^{\frac{3}{2}}}{3} - \frac{(2)^{\frac{3}{2}}}{9} + \frac{(2)^{\frac{3}{2}}}{3} =$$

$$= \frac{16\sqrt{2}}{9} - \frac{8}{3} - \frac{2^{\frac{3}{2}}}{9} + \frac{3\cdot 2^{\frac{3}{2}}}{9} = \frac{8\cdot 2^{\frac{3}{2}} + 2\cdot 2^{\frac{3}{2}}}{9} - \frac{24}{9} =$$

$$= \frac{10\cdot 2^{\frac{3}{2}} - 24}{9} = \frac{20\sqrt{2}-24}{9}$$

9.1.66

$$\int_0^{\pi/2} \frac{5\,dx}{1+\cos x} = \Big[t = \operatorname{tg}\frac{x}{2},\ \text{тогда: } \cos x = \frac{1-t^2}{1+t^2},\ dx = \frac{2dt}{1+t^2};\ \begin{array}{c|c|c} x & 0 & \pi/2 \\ \hline t=\operatorname{tg}x & 0 & 1 \end{array}\Big] = \int_0^1 \frac{5}{1+\frac{1-t^2}{1+t^2}}\cdot\frac{2dt}{1+t^2} =$$

$$= \int_0^1 \frac{5(1+t^2)}{1+t^2+1-t^2}\cdot\frac{2dt}{1+t^2} = \int_0^1 \frac{5\cdot 2dt}{2} = 5\int_0^1 dt = 5t\Big|_0^1 =$$

$= 5\cdot 1 - 5\cdot 0 = 5 - 0 = \underline{5}$

9.1.67

$$\int_0^{\ln 4} \sqrt{e^x - 1}\, dx = \Big[t = e^x - 1,\ dt = (e^x - 1)'_x\, dx = e^x dx,$$

$$e^x = t + 1 \Rightarrow dx = \frac{dt}{t+1};\ \begin{array}{c|c|c} x & 0 & \ln 4 \\ \hline t = e^x - 1 & 0 & 3 \end{array} \Big] =$$

$$= \int_0^3 \frac{\sqrt{t}\, dt}{t+1} = \Big[u = \sqrt{t},\ du = (\sqrt{t})'_t\, dt = \frac{dt}{2\sqrt{t}} \Rightarrow$$

$$\Rightarrow 2u\, du = dt\ \begin{array}{c|c|c} t & 0 & 3 \\ \hline u = \sqrt{t} & 0 & \sqrt{3} \end{array} \Big] = \int_0^{\sqrt{3}} \frac{2u^2 du}{u^2+1} =$$

$$= 2\int_0^{\sqrt{3}} \frac{u^2 + 1 - 1}{u^2+1}\, du = 2\int_0^{\sqrt{3}} du - 2\int_0^{\sqrt{3}} \frac{du}{u^2+1} = 2u \Big|_0^{\sqrt{3}} -$$

$$- 2\,\mathrm{arctg}\, u \Big|_0^{\sqrt{3}} = (2\cdot\sqrt{3} - 2\cdot 0) - (2\cdot \mathrm{arctg}\sqrt{3} -$$

$$- 2\,\mathrm{arctg}\, 0) = 2\sqrt{3} - \left(2\cdot\frac{\pi}{3} - 0\right) = \underline{2\sqrt{3} - \frac{2\pi}{3}}$$

9.1.68

$$\int_0^{\pi/2} \frac{dx}{1 + \cos x + \sin x} = \Big[t = \mathrm{tg}\frac{x}{2},\ \text{тогда: } \sin x = \frac{2t}{1+t^2},\ \cos x =$$

$$= \frac{1-t^2}{1+t^2},\ dx = \frac{2dt}{1+t^2}\ \begin{array}{c|c|c} x & 0 & \pi/2 \\ \hline t = \mathrm{tg}\frac{x}{2} & 0 & 1 \end{array} \Big] = \int_0^1 \frac{1}{1 + \frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2}} \cdot \frac{2dt}{1+t^2} =$$

$$= \int_0^1 \frac{1+t^2}{2+2t} \cdot \frac{2dt}{1+t^2} = \int_0^1 \frac{dt}{1+t} = \big[dt = d(t+1) \big] = \ln|t+1| \Big|_0^1 =$$

$= \ln|1+1| - \ln|0+1| = \ln(2) - 0 = \underline{\ln(2)}$

9.1.69

$$\int_0^{\pi/2} \frac{\cos x\,dx}{6-5\sin x+\sin^2 x} = \Big[R(\sin x; -\cos x) = -R(\sin x; \cos x);$$

$$t=\sin x \Rightarrow \cos x = \sqrt{1-t^2},\ dx = \frac{dt}{\sqrt{1-t^2}},$$

$$\begin{array}{c|c|c} x & 0 & \pi/2 \\ \hline t=\sin x & 0 & 1 \end{array} \Big] = \int_0^1 \frac{\sqrt{1-t^2}}{6-5t+t^2} \cdot \frac{dt}{\sqrt{1-t^2}} =$$

$$= \int_0^1 \frac{dt}{(t-3)(t-2)} = \int_0^1 \left(\frac{1}{t-3} - \frac{1}{t-2}\right) dt = \int_0^1 \frac{dt}{t-3} -$$

$$- \int_0^1 \frac{dt}{t-2} = \big[1)\,dt = d(t-3);\ 2)\,dt = d(t-2)\big] =$$

$$= (\ln|t-3| - \ln|t-2|)\Big|_0^1 = (\ln|-2| -$$

$$- \ln|-1|) - (\ln|-3| - \ln|-2|\big] =$$

$$= \ln 2 - 0 - \ln 3 + \ln 2 = \underline{2\ln(2) - \ln 3}$$

9.1.70

$$\int_1^2 3x(1-x)^{17}dx = \Big[t = 1-x,\ \text{тогда: } x = 1-t,$$

$$dt = (1-x)'_x\,dx = -dx,\ -dt = dx:\ \begin{array}{c|c|c} x & 1 & 2 \\ \hline t=1-x & 0 & -1 \end{array} \Big]$$

$$=\int_{0}^{-1} 3(1-t)t^{17}\cdot(-1)\,dt = 3\int_{-1}^{0}(t^{17}-t^{18})\,dt =$$

$$=3\int_{-1}^{0} t^{17}dt - 3\int_{-1}^{0} t^{18}dt = 3\cdot\frac{t^{18}}{18}\Big|_{-1}^{0} - 3\cdot\frac{t^{19}}{19}\Big|_{-1}^{0} =$$

$$=\left(0-\frac{3}{18}\right)-\left(0-\frac{3\cdot(-1)^{19}}{19}\right) = -\frac{3}{18}-\frac{3}{19} = \underline{-\frac{37}{114}}$$

9.1.100

$$\int_{\pi/6}^{\pi/2}\frac{x\,dx}{\sin^2 x} = \int_{\pi/6}^{\pi/2} x\cdot\frac{1}{\sin^2 x}dx = \Big[\int u v' dx = uv - \int v u' dx;$$

$$u = x;\ v' = 1/\sin^2 x \Rightarrow u' = 1;\ v = -\mathrm{ctg}\,x\Big] =$$

$$= -x\cdot\mathrm{ctg}\,x\Big|_{\pi/6}^{\pi/2} - \int_{\pi/6}^{\pi/2} -\mathrm{ctg}\,x\,dx = -x\cdot\mathrm{ctg}\,x\Big|_{\pi/6}^{\pi/2} +$$

$$+\int \mathrm{ctg}\,x\,dx = -x\cdot\mathrm{ctg}\,x\Big|_{\pi/6}^{\pi/2} + \ln|\sin x|\Big|_{\pi/6}^{\pi/2} =$$

$$=\left(-\frac{\pi}{2}\cdot\mathrm{ctg}\frac{\pi}{2}+\frac{\pi}{6}\cdot\mathrm{ctg}\frac{\pi}{6}\right)+\left(\ln\left|\sin\frac{\pi}{2}\right|-\ln\left|\sin\frac{\pi}{6}\right|\right) =$$

$$=\left(-\frac{\pi}{2}\cdot 0+\frac{\pi}{6}\cdot\sqrt{3}\right)+\left(\ln(1)-\ln\left(\frac{1}{2}\right)\right) =$$

$$=\frac{\pi\sqrt{3}}{6}+\ln\left(\frac{1}{\frac{1}{2}}\right) = \underline{\frac{\pi\sqrt{3}}{6}+\ln(2)}$$

9.1.101

$$\int_{0}^{0{,}2} x\cdot e^{5x} = \Big[u = x,\ v' = e^{5x} \Rightarrow u' = 1,\ v = \frac{e^{5x}}{5}\Big] =$$

$$= x\cdot\frac{e^{5x}}{5}\Big|_0^{0,2} - \int\frac{e^{5x}}{5}dx = x\cdot\frac{e^{5x}}{5}\Big|_0^{0,2} - \frac{1}{5}\int e^{5x}dx =$$

$$= x\cdot\frac{e^{5x}}{5}\Big|_0^{0,2} - \frac{1}{5}\cdot\frac{1}{5}\cdot e^{5x}\Big|_0^{0,2} = \left(\frac{x\cdot e^{5x}}{5} - \frac{e^{5x}}{25}\right)\Big|_0^{0,2} =$$

$$= \frac{0,2\cdot e^{5\cdot 0,2}}{5} - \frac{e^{5\cdot 0,2}}{25} - \left(\frac{0\cdot e^{5\cdot 0}}{5} - \frac{e^{5\cdot 0}}{25}\right) =$$

$$= \frac{e}{25} - \frac{e}{25} - \left(0 - \frac{1}{25}\right) = \underline{\frac{1}{25}}$$

<u>9.1.102</u>

$$\int_{\pi/4}^{\pi/3} 4x\,tg^2x\,dx = 4\int_{\pi/4}^{\pi/3} x\cdot\left(\frac{1}{\cos^2x} - 1\right)dx = 4\int_{\pi/4}^{\pi/3} x\cdot\frac{1}{\cos^2x}\,dx -$$

$$-4\int_{\pi/4}^{\pi/3} x\,dx = \left[1)\ u = x,\ v' = \frac{1}{\cos^2x} \Rightarrow u' = 1,\ v = tg\,x\right] =$$

$$= 4x\cdot tg\,x\Big|_{\pi/4}^{\pi/3} - 4\cdot\int tg\,x\,dx - 2x^2\Big|_{\pi/4}^{\pi/3} =$$

$$= \left(4x\cdot tg\,x - 2x^2 + 4\ln|\cos(x)|\right)\Big|_{\pi/4}^{\pi/3} =$$

$$= \left(4\cdot\frac{\pi}{3}\cdot tg\left(\frac{\pi}{3}\right) - 2\cdot\left(\frac{\pi}{3}\right)^2 + 4\ln\left|\cos\frac{\pi}{3}\right|\right) -$$

$$-\left(4\cdot\frac{\pi}{4}\cdot tg\left(\frac{\pi}{4}\right) - 2\cdot\left(\frac{\pi}{4}\right)^2 + 4\ln\left|\cos\frac{\pi}{4}\right|\right) =$$

$$= \frac{4\pi\sqrt{3}}{3} - \frac{2\pi^2}{9} - 4\ln(2) - \pi + \frac{\pi^2}{8} + 2\ln(2) =$$

$$= \underline{\frac{4\pi\sqrt{3}}{3} - \frac{7\pi^2}{72} - 2\ln(2) - \pi}$$

9.1.103

$$\int_1^{e^2} \ln^2 x\,dx = \int_1^{e^2} 1\cdot \ln^2 x\,dx = [u=\ln^2 x,\ v'=1 \Rightarrow$$

$$\Rightarrow v = x,\ u' = (\ln^2 x)'_x = 2\cdot \ln x \cdot \frac{1}{x}] = x\cdot \ln^2 x\Big|_1^{e^2} -$$

$$-\int_1^{e^2} x\cdot 2\cdot \ln x\cdot \frac{1}{x}dx = x\cdot \ln^2 x\Big|_1^{e^2} - 2\int_1^{e^2} \ln x\,dx =$$

$$= x\cdot \ln^2 x\Big|_1^{e^2} - 2\int_1^{e^2} 1\cdot \ln x\,dx = [u=\ln x,\ v'=1 \Rightarrow$$

$$\Rightarrow v = x,\ u' = \frac{1}{x}] = x\cdot \ln^2 x\Big|_1^{e^2} - 2x\ln x\Big|_1^{e^2} +$$

$$+2\int_1^{e^2} x\cdot \frac{1}{x}dx = x\cdot \ln^2 x\Big|_1^{e^2} - 2x\ln x\Big|_1^{e^2} + 2\int_1^{e^2} dx =$$

$$= (\ln^2(x)\cdot x - 2x\ln(x) + 2x)\Big|_1^{e^2} = (\ln^2(e^2)\cdot e^2 -$$

$$-2\cdot e^2\cdot \ln(e^2) + 2e^2) - (\ln^2(1)\cdot 1 - 2\cdot 1\cdot \ln(1) + 2\cdot 1) =$$

$$= (2^2\cdot e^2 - 2\cdot e^2\cdot 2 + 2e^2) - (0^2 - 2\cdot 0 + 2) = (4e^2 -$$

$$-4e^2 + 2e^2) - (0 - 0 + 2) = \underline{2e^2 - 2}$$

9.1.104

$$\int_{\pi/3}^{\pi/2} \frac{x\cdot\cos x}{\sin^2 x}\,dx = \left[u=x,\ v'=\frac{\cos x}{\sin^2 x} \Rightarrow u'=1,\right.$$

$$v=\int\frac{\cos x}{\sin^2 x}\,dx = \left[t=\sin x \Rightarrow \cos x=\sqrt{1-t^2},\ dx=\frac{dt}{\sqrt{1-t^2}}\right]=$$

$$\left.=\int\frac{dt}{t^2} = -\frac{1}{t} = -\frac{1}{\sin x}\right] = -\frac{x}{\sin x}\Big|_{\pi/3}^{\pi/2} - \int_{\pi/3}^{\pi/2}\frac{-dx}{\sin x} =$$

$$= -\frac{x}{\sin x}\Big|_{\pi/3}^{\pi/2} + \int_{\pi/3}^{\pi/2}\frac{dx}{\sin x} = \left(-\frac{x}{\sin x}+\ln\left|tg\frac{x}{2}\right|\right)\Big|_{\pi/3}^{\pi/2}=$$

$$=\left(-\frac{\frac{\pi}{2}}{\sin(\frac{\pi}{2})}+\ln\left(tg\left(\frac{\frac{\pi}{2}}{2}\right)\right)\right)-\left(-\frac{\frac{\pi}{3}}{\sin(\frac{\pi}{3})}+\ln\left(tg\left(\frac{\frac{\pi}{3}}{2}\right)\right)\right)=$$

$$=\left(-\frac{\pi}{2}+\ln(1)\right)-\left(-\frac{2\pi}{3\sqrt{3}}+\ln\left(\frac{\sqrt{3}}{3}\right)\right)=$$

$$=-\frac{\pi}{2}+\frac{2\pi\sqrt{3}}{9}-\ln\left(3^{-\frac{1}{2}}\right) = \underline{-\frac{\pi}{2}+\frac{2\pi\sqrt{3}}{9}+\frac{1}{2}\ln(3)}$$

9.1.105

$$\int_0^2 \frac{x^3}{\sqrt{1+x^2}}\,dx = \left[t=1+x^2;\ x^2=t-1,\ dt=2x\,dx \Rightarrow\right.$$

$$\left.\Rightarrow \frac{1}{2}dt = x\,dx;\ \begin{array}{c|c|c} x & 0 & 2 \\ \hline t=1+x^2 & 1 & 5 \end{array}\right] =$$

$$=\int_1^5\frac{(t-1)\cdot\frac{1}{2}dt}{\sqrt{t}} = \frac{1}{2}\int_1^5\frac{t-1}{\sqrt{t}}\,dt = \left[u=t-1,\ v'=\frac{1}{\sqrt{t}}\Rightarrow\right.$$

$$\left.\Rightarrow u'=1,\ v=2\sqrt{t}\right] = \frac{1}{2}(t-1)\cdot 2\sqrt{t}\Big|_1^5 - \frac{1}{2}\int_1^5 2\sqrt{t}\,dt=$$

$$= (t-1)\sqrt{t}\,\Big|_1^5 - \int_1^5 \sqrt{t}\,dt = \left((t-1)\sqrt{t} - \frac{2t\sqrt{t}}{3}\right)\Big|_1^5 =$$

$$= (5-1)\cdot\sqrt{5} - \frac{2\cdot 5\cdot\sqrt{5}}{3} - \left((1-1)\sqrt{1} - \frac{2\cdot 1\cdot\sqrt{1}}{3}\right) =$$

$$= \frac{12\sqrt{5}}{3} - \frac{10\sqrt{5}}{3} - 0 + \frac{2}{3} = \frac{2\sqrt{5}+2}{3}$$

9.1.106

$$\int_0^{\sqrt{3}} \frac{x^2}{(1+x^2)^2}\,dx = \left[u = x,\ v' = \frac{x}{(1+x^2)^2} \Rightarrow u' = 1,\ v = \right.$$

$$= \int \frac{x}{(1+x^2)^2}\,dx = \left[t = 1+x^2 \Rightarrow dt = 2x\,dx \Rightarrow \tfrac{1}{2}dt = x\,dx\right] =$$

$$\left. = \frac{1}{2}\int \frac{dt}{t^2} = \frac{1}{2}\cdot(-1)\cdot\frac{1}{t} = -\frac{1}{2t} = -\frac{1}{2+2x^2}\right] =$$

$$= -\frac{x}{2+2x^2}\Big|_0^{\sqrt{3}} - \int_0^{\sqrt{3}} -\frac{dx}{2+2x^2} = -\frac{x}{2+2x^2}\Big|_0^{\sqrt{3}} + \frac{1}{2}\int_0^{\sqrt{3}} \frac{dx}{x^2+1} =$$

$$= \left(-\frac{1}{2}\cdot\frac{x}{1+x^2} + \frac{1}{2}\operatorname{arctg} x\right)\Big|_0^{\sqrt{3}} = \left(\frac{1}{2}\operatorname{arctg}\sqrt{3} - \frac{1}{2}\cdot\frac{\sqrt{3}}{1+(\sqrt{3})^2}\right) -$$

$$-\left(\frac{1}{2}\operatorname{arctg} 0 - \frac{1}{2}\frac{0}{1+0^2}\right) = \left(\frac{1}{2}\cdot\frac{\pi}{3} - \frac{\sqrt{3}}{8}\right) - (0-0) =$$

$$= \frac{\pi}{6} - \frac{\sqrt{3}}{8} = \frac{4\pi - 3\sqrt{3}}{24}$$

9.1.107

$$\int_0^{\pi^2/4} \sin\sqrt{x}\,dx = \left[t = \sqrt{x},\ dt = (\sqrt{x})'_x\,dx = \frac{1}{2\sqrt{x}}\,dx \Rightarrow\right.$$

$$\Rightarrow 2t\,dt = dx; \ \begin{array}{c|c|c} x & 0 & \frac{\pi^2}{4} \\ \hline t=\sqrt{x} & 0 & \pi/2 \end{array} \Big] =$$

$$= \int_0^{\frac{\pi}{2}} 2t\sin(t)\,dt = 2\int_0^{\frac{\pi}{2}} t\cdot\sin(t)\,dt = \Big[u = t,\ v' = \sin t \Rightarrow$$

$$\Rightarrow u' = 1,\ v = -\cos t \Big] = 2t\cdot(-\cos t)\Big|_0^{\frac{\pi}{2}} - 2\int_0^{\frac{\pi}{2}} -\cos t\,dt =$$

$$= -2t\cos t\Big|_0^{\pi/2} + 2\int_0^{\frac{\pi}{2}} \cos t\,dt = (-2t\cos t + 2\sin t)\Big|_0^{\frac{\pi}{2}} =$$

$$= \left(-2\cdot\frac{\pi}{2}\cdot\cos\frac{\pi}{2} + 2\cdot\sin\frac{\pi}{2}\right) - (-2\cdot 0\cdot\cos 0 +$$

$$+ 2\sin 0) = \left(-2\cdot\frac{\pi}{2}\cdot 0 + 2\cdot 1\right) - (0 + 2\cdot 0) =$$

$$= (0+2) - (0+0) = \underline{2}$$

9.1.108

$$\int_0^9 e^{\sqrt{x}}\,dx = \Big[t = \sqrt{x},\ dt = \frac{1}{2\sqrt{x}}\,dx \Rightarrow 2t\,dt = dx;$$

$$\begin{array}{c|c|c} x & 0 & 9 \\ \hline t=\sqrt{x} & 0 & 3 \end{array} \Big] = \int_0^3 2te^t\,dt = 2\int_0^3 te^t\,dt =$$

$$= \Big[u = t,\ v' = e^t \Rightarrow u' = 1,\ v = e^t \Big] =$$

$$= 2te^t\Big|_0^3 - 2\int_0^3 e^t\,dt = (2te^t - 2e^t)\Big|_0^3 =$$

$$= (2\cdot 3\,e^3 - 2\cdot e^3) - (2\cdot 0\cdot e^0 - 2\cdot e^0) = \underline{4e^3 + 2}$$