Домашняя работа часть 4.

8.4.12

$$\int \frac{dx}{x+\sqrt[3]{x^2}} = [k = \text{НОК}(1;3) = 3 \Rightarrow x = t^k = t^3 \Rightarrow$$

$$\Rightarrow \sqrt[3]{x^2} = \sqrt[3]{t^6} = t^2;\ dx = d(t^3) = 3t^2dt] =$$

$$= \int \frac{3t^2dt}{t^3+t^2} = 3\int \frac{t^2dt}{t^2(t+1)} = 3\int \frac{dt}{t+1} = [dt = d(t+1)] =$$

$$= 3\ln|t+1| + C = 3\ln|\sqrt[3]{x}+1| + C$$

8.4.13

$$\int \frac{\sqrt{x}}{1+\sqrt[4]{x^3}}dx = [k = \text{НОК}(2;4) = 4, \Rightarrow x = t^4 \Rightarrow \sqrt{x} =$$

$$= t^2, \sqrt[4]{x^3} = t^3; dx = d(t^4) = 4t^3dt] =$$

$$= \int \frac{t^2 \cdot 4t^3dt}{1+t^3} = \int \frac{4t^5}{1+t^3}dt = 4\int \frac{t^5}{1+t^3}dt =$$

$$= [u = 1+t^3,\ du = (1+t^3)'_t dt = 3t^2dt \Rightarrow \frac{1}{3}du =$$

$$= t^2dt,\ t^3 = u-1] = 4 \cdot \int \frac{1}{3} \cdot \frac{(u-1)}{u}du =$$

$$= \frac{4}{3}\int du - \frac{4}{3}\int \frac{1}{u}du = \frac{4}{3}u - \frac{4}{3}\ln|u| + C =$$

$$= \frac{4}{3}(1+t^3) - \frac{4}{3}\ln|1+t^3| + C = \frac{4}{3} \cdot \sqrt[4]{x^3} - \frac{4}{3}\ln|1+\sqrt[4]{x^3}| + C$$

8.4.14

$$\int \frac{x+\sqrt[3]{x^2}+\sqrt[6]{x}}{x(1-\sqrt[3]{x})}\,dx = \Big[k = НОК(3,6) = 6 \Rightarrow x = t^6 \Rightarrow$$

$$\Rightarrow \sqrt[3]{x^2} = t^4,\ \sqrt[6]{x} = t,\ \sqrt[3]{x} = t^2;\ dx = (t^6)'_t\,dt =$$

$$= 6t^5dt\Big] = \int \frac{t^6+t^4+t}{t^6-t^8}\cdot 6t^5dt =$$

$$= 6\int \frac{t(t^5+t^3+1)}{t(1-t^2)}dt = 6\int \frac{t^5+t^3+1}{1-t^2}dt =$$

$$= -6\int \frac{t^5+t^3+1}{t^2-1}dt = \Bigg[\begin{array}{l|l} t^5+0\cdot t^4+t^3+0\cdot t^2+0\cdot t+1 & t^2-1 \\ \underline{-\ t^5 \qquad\qquad -t^3} & t^3+2t \\ \qquad\qquad -\,2t^3 & \\ \qquad\qquad \underline{\ \ 2t^3 \qquad\qquad -2t} & \\ \qquad\qquad\qquad\qquad\quad 2t+1 & \end{array}$$

$$\Bigg] = -6\int \frac{(t^2-1)(t^3+2t)+2t+1}{t^2-1} = -6\int \frac{(2t+1)dt}{t^2-1} - 6\int t^3dt -$$

$$-12\int t\,dt = \Big[1)\int\frac{2t+1}{t^2-1} = 2\int\frac{t\,dt}{t^2-1} + \int\frac{dt}{t^2-1}\Big] =$$

$$= -12\int\frac{t\,dt}{t^2-1} - 6\int\frac{dt}{t^2-1} - 6\int t^3dt - 12\int t\,dt =$$

$$= \Big[1)\ u = t^2-1 \Rightarrow du = 2t\,dt\Big] = -6\int\frac{du}{u} -$$

$$-6\int\frac{dt}{t^2-1} - 6\int t^3dt - 12\int t\,dt = -6\ln|u| -$$

$$-6\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| - 6\cdot\frac{t^4}{4} - 12\frac{t^2}{2} + C =$$

$$= -6\ln|\sqrt[3]{x}-1| - 3\ln\left|\frac{\sqrt[6]{x}-1}{\sqrt[6]{x}+1}\right| - \frac{3}{2}\sqrt[3]{x^2} - 6\sqrt[3]{x} + C =$$

$$= -6\ln|\sqrt[3]{x}-1| - 3\ln|\sqrt[6]{x}-1| + 3\ln|\sqrt[6]{x}+1| -$$

$$-\frac{3}{2}\sqrt[3]{x^2} - 6\sqrt[3]{x} + C = \underline{-9\ln|\sqrt[6]{x}-1| + 3\ln|\sqrt[6]{x}+1| -}$$

$$\underline{-\frac{3}{2}\sqrt[3]{x^2} - 6\sqrt[3]{x} + C}$$

<u>8.4.15</u>

$$\int \frac{\sqrt{x}\,dx}{x-\sqrt[3]{x^2}} = \Big[k = \text{НОК}(2;3) = 6 \Rightarrow x = t^6;\ \sqrt{x} = t^3;$$

$$\sqrt[3]{x^2} = t^4;\ dx = 6t^5dt\Big] = 6\int \frac{t^3\cdot t^5}{t^4(t^2-1)}\,dt = 6\int \frac{t^4}{t^2-1}\,dt =$$

$$= \left[\begin{array}{l|l} t^4+0\cdot t^3+0\cdot t^2+0\cdot t+0 & t^2-1 \\ \cline{2-2} -\ t^4+0\cdot t^3-t^2 & t^2+1 \\ \cline{1-1} \qquad\qquad -\ t^2 & \\ \qquad\qquad\ \ t^2 \qquad -1 & \\ \cline{1-1} \qquad\qquad\qquad\quad 1 & \end{array}\right] =$$

$$= 6\int \frac{(t^2-1)(t^2+1)+1}{t^2-1}\,dt = 6\int\frac{dt}{t^2-1} + 6\int t^2dt + 6\int dt =$$

$$= 6\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + 6\frac{t^3}{3} + 6t + C = \underline{3\ln\left|\frac{\sqrt[6]{x}-1}{\sqrt[6]{x}+1}\right| +}$$

$$\underline{+2\sqrt{x} + 6\sqrt[6]{x} + C}$$

8.4.16

$$\int\frac{\sqrt{x}\,dx}{1+\sqrt{x}}=\left[k=2 \Rightarrow x=t^2;\ \sqrt{x}=t;\ dx=2t\,dt\right]=$$

$$=2\int\frac{t\cdot t\,dt}{1+t}=2\int\frac{t^2-1+1}{t+1}dt=2\int\frac{(t-1)(t+1)}{t+1}dt+$$

$$+2\int\frac{dt}{t+1}=2\int t\,dt-2\int dt+2\int\frac{dt}{t+1}=$$

$$=2\cdot\frac{t^2}{2}-2t+2\ln|t+1|+C=\underline{x-2\sqrt{x}+2\ln(\sqrt{x}+1)+C}$$

8.4.17

$$\int\frac{\sqrt{x}\,dx}{1-\sqrt[3]{x}}=\left[k=\text{НОК}(2;3)=6 \Rightarrow x=t^6;\ \sqrt{x}=t^3;\right.$$

$$\left.\sqrt[3]{x}=t^2;\ dx=6t^5dt\right]=6\int\frac{t^8}{1-t^2}dt=-6\int\frac{t^8}{t^2-1}dt=$$

$$=\left[t^8=t^8-t^6+t^6-t^4+t^4-t^2+t^2-1+1=\right.$$

$$\left.=(t^2-1)(t^6+t^4+t^2+1)+1\right]=-6\int\frac{(t^2-1)(t^6+t^4+t^2+1)+1}{t^2-1}dt$$

$$=-6\int\frac{dt}{t^2-1}-6\int t^6dt-6\int t^4dt-6\int t^2dt-6\int dt=$$

$$=-3\ln\left|\frac{t-1}{t+1}\right|-6\cdot\frac{t^7}{7}-6\cdot\frac{t^5}{5}-6\cdot\frac{t^3}{3}-6t+C=$$

$$=\underline{-3\ln\left|\frac{x^{1/6}-1}{x^{1/6}+1}\right|-\frac{6}{7}\cdot x^{\frac{7}{6}}-\frac{6}{5}\cdot x^{\frac{5}{6}}-2\cdot x^{1/2}-6x^{1/6}+C}$$

8.4.18

$$\int \frac{\sqrt{x+2}}{x}dx = \Big[k=2 \Rightarrow x+2=t^2 \Rightarrow x=t^2-2;$$

$$dx = d(t^2-2) = 2t\,dt\Big] = \int \frac{t\cdot 2t}{t^2-2}dt =$$

$$= 2\int \frac{t^2}{t^2-2}dt = 2\int \frac{t^2-2}{t^2-2}dt + 2\int \frac{2}{t^2-2}dt =$$

$$= 2\int dt + 4\int \frac{dt}{t^2-2} = 2t + 4\cdot\frac{1}{2\sqrt{2}}\ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| + C =$$

$$= 2\sqrt{x+2} + \sqrt{2}\cdot\ln\left|\frac{\sqrt{x+2}-\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}\right| + C$$

8.4.19

$$\int \frac{x\,dx}{\sqrt{x+1}+\sqrt[3]{x+1}} = \Big[k = \text{НОК}(2;3) = 6 \Rightarrow x+1 = t^6;$$

$$\sqrt{x+1} = t^3, \sqrt[3]{x+1} = t^2, x = t^6-1; dx = d(t^6-1) = 6t^5dt\Big] =$$

$$= \int \frac{(t^6-1)\cdot 6t^5 dt}{t^3+t^2} = 6\int \frac{t^9-t^3}{t+1}dt = \Big[t^9 - t^3 = t^3\cdot(t^6-1) =$$

$$= t^9 + t^8 - t^8 - t^7 + t^7 + t^6 - t^6 - t^5 + t^5 + t^4 - t^4 - t^3 =$$

$$= (t+1)(t^8 - t^7 + t^6 - t^5 + t^4 - t^3)\Big]\Big] = 6\int(t^8 - t^7 + t^6 -$$

$$- t^5 + t^4 - t^3)dt = 6\int t^8dt - 6\int t^7dt + 6\int t^6dt -$$

$$-6\int t^5 dt+6\int t^4 dt-6\int t^3 dt=6\cdot\frac{t^9}{9}-$$

$$-6\cdot\frac{t^8}{8}+6\cdot\frac{t^7}{7}-6\cdot\frac{t^6}{6}+6\cdot\frac{t^5}{5}-6\cdot\frac{t^4}{4}+C=$$

$$=6\cdot\frac{(\sqrt[6]{x+1})^9}{9}-6\cdot\frac{(\sqrt[6]{x+1})^8}{8}+6\cdot\frac{(\sqrt[6]{x+1})^7}{7}-6\cdot\frac{(\sqrt[6]{x+1})^6}{6}+$$

$$+6\cdot\frac{(\sqrt[6]{x+1})^5}{5}-6\cdot\frac{(\sqrt[6]{x+1})^4}{4}+C=\frac{2}{3}\cdot(x+1)^{\frac{3}{2}}-$$

$$-\frac{3}{4}(x+1)^{\frac{4}{3}}+\frac{6}{7}\cdot(x+1)^{\frac{7}{6}}-x+\frac{6}{5}(x+1)^{\frac{5}{6}}-$$

$$-\frac{3}{2}\cdot(x+1)^{\frac{2}{3}}+C$$

8.4.20

$$\int\frac{dx}{(x+1)^{3/2}+(x+1)^{1/2}}=[k=2\Rightarrow x+1=t^2;\ (x+1)^{3/2}=t^3;$$

$$(x+1)^{1/2}=t;\ dx=d(t^2-1)=2t\,dt]=$$

$$=2\int\frac{t\,dt}{t^3+t}=2\int\frac{dt}{t^2+1}=2\cdot\frac{1}{1}\operatorname{arctg}\frac{t}{1}+C=$$

$$=2\operatorname{arctg}\sqrt{x+1}+C$$

8.4.21

$$\int\frac{\sqrt{1+x}+1}{\sqrt{1+x}-1}dx=[k=2\Rightarrow x+1=t^2;\ \sqrt{1+x}=t;$$

$dx=(t^2-1)'_t dt = 2t\,dt] = 2\int \frac{t^2+t}{t-1}dt =$

$=[t^2+t = t^2-t+2t-2+2 = (t-1)(t+2)+2] =$

$=2\int \frac{(t-1)(t+2)+2}{t-1}dt = 2\int t\,dt + 4\int dt + 4\int \frac{dt}{t-1} =$

$= t^2+4t+4\cdot \ln|t-1| = x+1+\sqrt{x+1}\cdot 4 +$

$+4\cdot \ln|\sqrt{x+1}-1| + C = \underline{x+4\sqrt{x+1}+4\ln|\sqrt{x+1}-1|+C}$

<u>8.4.22</u>

$\int \frac{x-1}{\sqrt{2x-1}}dx = [k=2 \Rightarrow 2x-1=t^2;\ \sqrt{2x-1}=t,\ x=\frac{t^2+1}{2};$

$dx = d\left(\frac{t^2+1}{2}\right) = \frac{1}{2}\cdot 2t\,dt = t\,dt] = \int \frac{t^2-1}{2t}\cdot t\,dt =$

$=\frac{1}{2}\int t^2 dt - \frac{1}{2}\int dt = \frac{1}{2}\cdot\frac{t^3}{3} - \frac{1}{2}t + C = \frac{\sqrt{(2x-1)^3}}{6} -$

$-\frac{\sqrt{2x-1}}{2} + C = \frac{(2x-1)\sqrt{2x-1}}{6} - \frac{3\sqrt{2x-1}}{6} + C =$

$=\frac{(2x-4)\sqrt{2x-1}}{6} + C = \underline{\frac{(x-2)\sqrt{2x-1}}{3} + C}$

<u>8.4.23</u>

$\int \frac{dx}{\sqrt{1-2x}-\sqrt[4]{1-2x}} = [k = \text{НОК}(2;4) = 4 \Rightarrow 1-2x = t^4;$

$$\sqrt{1-2x}=t^2;\ \sqrt[4]{1-2x}=t,\ x=\frac{1-t^4}{2};\ dx=$$

$$=-\frac{1}{2}\cdot 4\cdot t^3dt=-2t^3dt=-2\int\frac{t^3dt}{t^2-t}=$$

$$=-2\int\frac{t^2dt}{t-1}=[t^2=t^2-1+1=(t-1)(t+1)+1]=$$

$$=-2\int(t+1)dt-2\int\frac{dt}{t-1}=-2\int t\,dt-2\int dt-$$

$$-2\int\frac{dt}{t-1}=-2\cdot\frac{t^2}{2}-2t-2\ln|t-1|+C=$$

$$=-t^2-2t-2\ln|t-1|+C=\underline{-\sqrt{1-2x}-2\sqrt[4]{1-2x}-}$$

$$\underline{-2\ln|\sqrt[4]{1-2x}-1|+C}$$

8.4.24

$$\int\frac{1}{(2-x)^2}\cdot\sqrt{\frac{2-x}{2+x}}dx=[\text{3 случай; } k=2\Rightarrow$$

$$\Rightarrow\frac{2-x}{2+x}=t^2;\ \sqrt{\frac{2-x}{2+x}}=t;\ 2-x=2t^2+t^2x\Rightarrow$$

$$\Rightarrow 2-2t^2=t^2x+x\Rightarrow x=\frac{2-2t^2}{t^2+1};\ dx=d\left(\frac{2-2t^2}{t^2+1}\right)=$$

$$=\frac{-8t}{(t^2+1)^2}dt]=\int\frac{1}{\left(2-\frac{2-2t^2}{t^2+1}\right)^2}\cdot t\cdot\frac{(-8t)}{(t^2+1)^2}dt=$$

$$=\int\frac{(t^2+1)^2}{16t^4}\cdot t\cdot\frac{(-8t)}{(t^2+1)^2}dt=\int-\frac{dt}{2t^2}=-\frac{1}{2}\int t^{-2}dt=$$

$$= -\frac{1}{2}\cdot\frac{t^{-2+1}}{-2+1} + C = +\frac{1}{2}\cdot t^{-1} + C = \frac{1}{2t} + C =$$

$$= \frac{\sqrt{2+x}}{2\sqrt{2-x}} + C$$

8.4.25

$$\int\frac{dx}{\sqrt{(x-1)^3(x-2)}} = \int\frac{\sqrt{x-1}\,dx}{\sqrt{(x-1)^4}\cdot\sqrt{x-2}} = \int\frac{dx}{(x-1)^2\cdot\sqrt{\frac{x-2}{x-1}}} =$$

$$= \Big[k=2 \Rightarrow \frac{x-2}{x-1} = t^2;\ \sqrt{\frac{x-2}{x-1}} = t;\ x-2 = t^2\cdot x - t^2;$$

$$x - t^2x = 2 - t^2,\ x = \frac{2-t^2}{1-t^2} = \frac{t^2-2}{t^2-1};\ dx =$$

$$= d\left(\frac{t^2-2}{t^2-1}\right) = \frac{2t}{(t^2-1)^2}\Big] = \int\frac{1}{\left(\frac{t^2-2}{t^2-1}-1\right)^2\cdot t}\cdot\frac{2t}{(t^2-1)^2}dt =$$

$$= \Big[\frac{t^2-2}{t^2-1} - 1 = \frac{t^2-2-t^2+1}{t^2-1} = \frac{-1}{t^2-1}\Big] = \int\frac{(t^2-1)^2}{t}\cdot\frac{2t}{(t^2-1)^2}dt =$$

$$= \int 2dt = 2\int dt = 2t + C = 2\sqrt{\frac{x-2}{x-1}} + C$$

8.4.26

$$\int\frac{dx}{\sqrt[3]{(x-1)^2(x+1)}} = \int\frac{dx}{(x-1)\sqrt[3]{\frac{x+1}{x-1}}} = \Big[k=3 \Rightarrow \frac{x+1}{x-1} = t^3;\ \sqrt[3]{\frac{x+1}{x-1}} = t,$$

$$x+1 = xt^3 - t^3,\ xt^3 - x = 1 + t^3,\ x = \frac{t^3+1}{t^3-1},\ dx = d\left(\frac{t^3+1}{t^3-1}\right) =$$

$$= -\frac{6t^2}{(t^3-1)^2}dt\Big] = \int\frac{1}{\left(\frac{t^3+1}{t^3-1}-1\right)\cdot t}\cdot\frac{-6t^2}{(t^3-1)^2}dt = -3\int\frac{t}{t^3-1}dt =$$

$$= -3\int \frac{t}{(t-1)(t^2+t+1)}\,dt = \Big[\frac{t}{(t-1)(t^2+t+1)} =$$

$$= \frac{A}{t-1} + \frac{Bt+C}{t^2+t+1}; \quad t = At^2 + At + A + Bt^2 + Ct - Bt - C$$

$$t = (A+B)t^2 + (A+C-B)t + (A-C)$$

$$\begin{cases} 0 = A - C \\ 1 = A + C - B \\ 0 = A + B \end{cases} \quad \begin{cases} A = C \\ 2A - B = 1 \\ A = -B \end{cases} \quad \begin{cases} A = C \\ 3A = 1 \\ A = -B \end{cases} \quad \begin{cases} C = \frac{1}{3} \\ A = \frac{1}{3} \\ B = -\frac{1}{3} \end{cases} \Big] =$$

$$= -3\int \frac{1}{3(t-1)}\,dt - 3\int \frac{-t+1}{3(t^2+t+1)}\,dt =$$

$$= -\int \frac{1}{t-1}\,dt + \int \frac{t-1}{t^2+t+1}\,dt = \Big[2)\ A = 1,\ B = -1,$$

$$p = 1,\ q = 1; = -\int \frac{dt}{t-1} + \frac{1}{2}\int \frac{(2t+1)\,dt}{t^2+t+1} +$$

$$+ \left(-1 - \frac{1}{2}\right)\int \frac{dt}{t^2+t+1} = \Big[2)\ u = t^2 + t + 1;\ du = (2t+1)\,dt;$$

$$3)\ y = t + \frac{1}{2} \Rightarrow dy = dt \Big] = -\int \frac{dt}{t-1} + \frac{1}{2}\int \frac{du}{u} -$$

$$- \frac{3}{2}\int \frac{dy}{y^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = -\ln|t-1| + \frac{1}{2}\ln|u| -$$

$$- \frac{3}{2} \cdot \frac{1}{\frac{\sqrt{3}}{2}} \cdot \operatorname{arctg} \frac{2y}{\sqrt{3}} + C = -\ln\left|\sqrt[3]{\frac{x+1}{x-1}} - 1\right| +$$

$$+ \frac{1}{2}\ln\left|\sqrt[3]{\left(\frac{x+1}{x-1}\right)^2} + \sqrt[3]{\frac{x+1}{x-1}} + 1\right| - \sqrt{3} \cdot \operatorname{arctg}\left(\frac{2\sqrt[3]{\frac{x+1}{x-1}} + 1}{\sqrt{3}}\right) + C$$

8.4.27

$$\int\frac{dx}{(1-x)\sqrt{1-x^2}} = \int\frac{dx}{(1-x)\sqrt{(1-x)(1+x)}} = \int\frac{dx}{(1-x)^2\cdot\sqrt{\frac{1+x}{1-x}}} =$$

$$= \Big[k=2 \Rightarrow \frac{1+x}{1-x}=t^2;\ \sqrt{\frac{1+x}{1-x}}=t;\ 1+x=t^2-t^2x,\ x+t^2x=t^2-1,$$

$$x=\frac{t^2-1}{t^2+1};\ dx=d\left(\frac{t^2-1}{t^2+1}\right)=\frac{4t}{(t^2+1)^2}\Big] = \int\frac{1}{\left(1-\frac{t^2-1}{t^2+1}\right)^2\cdot t}\cdot\frac{4t}{(t^2+1)^2}dt=$$

$$= \int\frac{(t^2+1)^2}{4t}\cdot\frac{4t}{(t^2+1)^2}dt = \int dt = t+C = \underline{\sqrt{\frac{1+x}{1-x}}+C}$$

8.4.28

$$\int\frac{dx}{x(1+\sqrt[3]{x})^3} = \int x^{-1}(1+x^{1/3})^{-3}dx = \Big[m=-1,\ a=1,\ b=1,$$

$$n=\frac{1}{3},\ p=-3;\ 1)\ p\in\mathbb{Z} \Rightarrow k=3 \Rightarrow x=t^3;$$

$$dx=(t^3)'_t dt = 3t^2dt\Big] = \int t^{-3}(1+t)^{-3}\cdot 3t^2dt =$$

$$= 3\int t^{-1}\cdot(t^3+3t^2+3t+1)^{-3}dt = 3\int(t^4+3t^3+3t^2+t)^{-1}dt=$$

$$= 3\int\frac{dt}{t(t+1)^3} = \Big[t(t+1)^3=0;\ t=0,\ t=-1;\ \frac{1}{t(t+1)^3} =$$

$$= \frac{A}{t}+\frac{B}{t+1}+\frac{C}{(t+1)^2}+\frac{D}{(t+1)^3};\ 1=A(t+1)^3+B\cdot t\cdot(t+1)^2+$$

$$+C\cdot(t+1)\cdot t + Dt;$$

$$1 = At^3 + A\cdot 3t^2 + A\cdot 3t + A + Bt^3 + B\cdot 2t^2 + Bt +$$
$$+ Ct^2 + Ct + Dt$$
$$1 = t^3(A+B) + t^2(3A+2B+C) + t(3A+B+C+D) + A$$

$$\begin{cases} A+B=0 \\ 3A+2B+C=0 \\ 3A+B+C+D=0 \\ A=1 \end{cases} \quad \begin{cases} B=-1 \\ 3-2+C=0 \\ 3-1+C+D=0 \\ A=1 \end{cases} \quad \begin{cases} B=-1 \\ C=-1 \\ D=-1 \\ A=1 \end{cases} \Big] =$$

$$= 3\int\frac{dt}{t} - 3\int\frac{dt}{t+1} - 3\int\frac{dt}{(t+1)^2} - 3\int\frac{dt}{(t+1)^3} =$$

$$= [1) \text{ табл.}; \ 2,3,4)\ dt = d(t+1)] =$$

$$= 3\ln|t| - 3\ln|t+1| - 3\cdot\frac{(t+1)^{-1}}{-1} -$$

$$- 3\cdot\frac{(t+1)^{-2}}{-2} + C = 3\ln|x^{1/3}| - 3\ln|x^{1/3}+1| +$$

$$+ 3(x^{1/3}+1)^{-1} + \frac{3}{2}(x^{1/3}+1)^{-2} + C = \ln|x| - 3\ln|\sqrt[3]{x}+1| +$$

$$+ \frac{3}{\sqrt[3]{x}+1} + \frac{3}{2(\sqrt[3]{x}+1)^2} + C$$

8.4.29

$$\int x^3\cdot\sqrt{1+x^2}\,dx = \int x^3\cdot(1+x^2)^{\frac{1}{2}}dx = [m=3,\ n=2,\ p=\frac{1}{2},$$

$$1)\ p=\frac{1}{2}\notin\mathbb{Z};\ 2)\ \frac{m+1}{n} = \frac{3+1}{2} = 2\in\mathbb{Z} \Rightarrow 1+x^2=t^2;$$

$$x=\sqrt{t^2-1};\ dx=\frac{t}{\sqrt{t^2-1}}dt\Big]=\int\sqrt{(t^2-1)^3}\cdot t\cdot\frac{t}{\sqrt{t^2-1}}dt=$$

$$=\int(t^2-1)\cdot t^2dt=\int t^4dt-\int t^2dt=\frac{t^5}{5}-\frac{t^3}{3}+C=$$

$$=\frac{1}{5}\cdot\left(\sqrt{1+x^2}\right)^5-\frac{1}{3}\left(\sqrt{1+x^2}\right)^3+C=\frac{(1+2x^2+x^4)\sqrt{1+x^2}}{5}-$$

$$-\frac{(1+x^2)\sqrt{1+x^2}}{3}+C$$

8.4.30

$$\int\frac{dx}{x^{11}\sqrt{x^4+1}}=\int x^{-11}\cdot(x^4+1)^{-\frac{1}{2}}dx=\Big[m=-11,\ n=4,\ p=-\frac{1}{2};$$

$$1)\ p=-\frac{1}{2}\notin\mathbb{Z};\ 2)\ \frac{m+1}{n}=\frac{-11+1}{4}=-\frac{10}{4}\notin\mathbb{Z};\ 3)\ \frac{m+1}{n}+p=$$

$$=\frac{-11+1}{4}-\frac{1}{2}=2,5-0,5=2\in\mathbb{Z}\Rightarrow x^{-4}+1=t^2,$$

$$\frac{1}{x^4}+1=t^2,\ \frac{x^4+1}{x^4}=t^2,\ x^4+1=t^2x^4,\ x^4=\frac{1}{t^2-1},\ x=\sqrt[4]{\frac{1}{t^2-1}};$$

$$dx=-\frac{t\,dt}{2\sqrt[4]{t^2-1}\cdot|t^2-1|}\Big]=\int\sqrt[4]{(t^2-1)^{11}}\cdot\sqrt{\frac{t^2-1}{t^2}}\cdot(-1)\cdot\frac{t\,dt}{2\sqrt[4]{t^2-1}(t^2-1)}=$$

$$=-\frac{1}{2}\int(t^2-1)^2\cdot\sqrt[4]{(t^2-1)^3}\cdot\frac{\sqrt[4]{(t^2-1)^2}}{t}\cdot\frac{t\,dt}{\sqrt[4]{(t^2-1)}\cdot(t^2-1)}=$$

$$=-\frac{1}{2}\int(t^2-1)^2dt=-\frac{1}{2}\int t^4dt+\frac{1}{2}\int2t^2dt-\frac{1}{2}\int dt=$$

$$=-\frac{1}{2}\cdot\frac{t^5}{5}+\frac{t^3}{3}-\frac{1}{2}t+C=-\frac{(x^4+1)^2\cdot\sqrt{x^4+1}}{10x^{10}}+\frac{(x^4+1)\sqrt{x^4+1}}{3x^6}-$$

$$-\frac{\sqrt{x^4+1}}{2x^2}+C=-\frac{(3x^8+6x^4+3)\sqrt{x^4+1}}{30x^{10}}+\frac{(10x^8+10x^4)\sqrt{x^4+1}}{30x^{10}}$$

$$-\frac{15x^8\sqrt{x^4+1}}{30x^{10}}+C=-\frac{(8x^8-4x^4+3)\sqrt{x^4+1}}{30x^{10}}+C$$

8.4.31

$$\int\frac{dx}{\sqrt{x}(1-\sqrt{x})^2}=\int x^{-\frac{1}{2}}(1-x^{\frac{1}{2}})^{-2}dx=[m=-\frac{1}{2},n=\frac{1}{2},$$

$p=-2$; 1) $p=-2\in\mathbb{Z}\Rightarrow k=\text{НОК}(2;2)=2$, $x=t^2$,

$x^{\frac{1}{2}}=t$, $x^{-\frac{1}{2}}=t^{-1}$, $dx=2t\,dt]=$

$$=\int\frac{2t\,dt}{t(1-t)^2}=2\int\frac{dt}{(t-1)^2}=[dt=d(t-1)]=$$

$$=2\int(t-1)^{-2}d(t-1)=2\cdot\frac{(t-1)^{-1}}{-1}+C=-\frac{2}{t-1}+C=$$

$$=\frac{2}{1-\sqrt{x}}+C$$

8.4.32

$$\int x^5\sqrt[3]{(1+x^3)^2}dx=\int x^5(1+x^3)^{\frac{2}{3}}dx=[m=5,n=3,$$

$p=\frac{2}{3}$; 1) $p=\frac{2}{3}\notin\mathbb{Z}$; 2) $\frac{m+1}{n}=\frac{5+1}{3}=2\in\mathbb{Z}$;

$k=3\Rightarrow 1+x^3=t^3$, $x=\sqrt[3]{t^3-1}$; $dx=\frac{t^2}{\sqrt[3]{(t^3-1)^2}}dt]$

$$= \int \sqrt[3]{(t^3-1)^5} \cdot t^2 \cdot \frac{t^2}{\sqrt[3]{(t^3-1)^2}} dt = \int (t^3-1) \cdot t^4 dt =$$

$$= \int t^7 dt - \int t^4 dt = \frac{t^8}{8} - \frac{t^5}{5} + C = \frac{\sqrt[3]{(1+x^3)^8}}{8} - \frac{\sqrt[3]{(1+x^3)^5}}{5} + C =$$

$$= \frac{(1+x^3) \cdot (1+x^3)^{\frac{5}{3}}}{8} - \frac{(1+x^3)^{5/3}}{5} + C = \underline{\frac{(5x^3-3)(x^3+1)^{\frac{5}{3}}}{40} + C}$$

<u>8.4.33</u>

$$\int \frac{dx}{x^3 \sqrt[3]{2-x^3}} = \int x^{-3} \cdot (2-x^3)^{-\frac{1}{3}} dx = [m=-3, n=3, p=-\frac{1}{3};$$

$$1)\ p = -\frac{1}{3} \notin \mathbb{Z};\ 2)\ \frac{m+1}{n} = \frac{-3+1}{3} \notin \mathbb{Z};\ 3)\ \frac{m+1}{n} + p =$$

$$= \frac{-3+1}{3} - \frac{1}{3} = -\frac{2}{3} - \frac{1}{3} = -1 \in \mathbb{Z} => k=3,\ 2x^{-3} - 1 = t^3;$$

$$x^{-3} = \frac{t^3+1}{2},\ x^3 = \frac{2}{t^3+1},\ x = \sqrt[3]{\frac{2}{t^3+1}};\ dx = -\frac{\sqrt[3]{2} \cdot t^2 dt}{(t^3+1)\sqrt[3]{t^3+1}}] =$$

$$= \int \frac{\cancel{t^3+1}}{2} \cdot \left(2 - \frac{2}{t^3+1}\right)^{-\frac{1}{3}} \cdot (-1) \frac{\sqrt[3]{2} \cdot t^2}{\cancel{(t^3+1)}\sqrt[3]{t^3+1}} dt =$$

$$= -\frac{1}{2} \int \left(\frac{2t^3}{t^3+1}\right)^{-\frac{1}{3}} \cdot \frac{\sqrt[3]{2} t^2}{\sqrt[3]{t^3+1}} dt = -\frac{1}{2} \int \frac{\sqrt[3]{t^3+1}}{\sqrt[3]{2} \cdot t} \cdot \frac{\sqrt[3]{2} \cdot t^2}{\sqrt[3]{t^3+1}} dt =$$

$$= -\frac{1}{2} \int t\, dt = -\frac{1}{2} \cdot \frac{t^2}{2} + C = -\frac{t^2}{4} + C =$$

$$= -\frac{\sqrt[3]{(2x^{-3}-1)^2}}{4} + C = -\frac{1}{4} \cdot \sqrt[3]{\left(\frac{2-x^3}{x^3}\right)^2} + C = -\frac{1}{4} \cdot \frac{\sqrt[3]{(x^3-2)^2}}{x^2} + C =$$

$$= \underline{-\frac{(x^3-2)^{\frac{2}{3}}}{4x^2} + C}$$

8.4.34

$$\int \sqrt{x}(1+\sqrt{x})^3 dx = \int \sqrt{x}(1+3\sqrt{x}+3x+x\sqrt{x})dx =$$

$$= \int (x^{\frac{1}{2}}+3x+3x^{\frac{3}{2}}+x^2)dx = \int x^{\frac{1}{2}}dx + 3\int x dx +$$

$$+ 3\int x^{\frac{3}{2}}dx + \int x^2 dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 3\cdot\frac{x^2}{2} + 3\cdot\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^3}{3} + C =$$

$$= \frac{2x\sqrt{x}}{3} + \frac{3x^2}{2} + \frac{6x^2\sqrt{x}}{5} + \frac{x^3}{3} + C$$

8.4.35

$$\int \sqrt[3]{x^3-4}\cdot x^2 dx = \int x^2\cdot(x^3-4)^{\frac{1}{3}}dx = [m=2, n=3, p=\frac{1}{3};$$

$$1)\ p=\frac{1}{3}\notin \mathbb{Z};\ 2)\ \frac{m+1}{n} = \frac{2+1}{3} = 1 \in \mathbb{Z} \Rightarrow k=3,$$

$$x^3-4=t^3;\ x^2=\sqrt[3]{(t^3+4)^2},\ x=\sqrt[3]{t^3+4};\ dx =$$

$$= \frac{t^2}{\sqrt[3]{(t^3+4)^2}}dt] = \int \sqrt[3]{(t^3+4)^2}\cdot t\cdot\frac{t^2}{\sqrt[3]{(t^3+4)^2}}dt =$$

$$= \int t^3 dt = \frac{t^4}{4} + C = \frac{(x^3-4)\sqrt[3]{x^3-4}}{4} + C$$

8.4.36

$$\int \frac{dx}{\sqrt{1-2x-x^2}} = \int \frac{dx}{\sqrt{2-(x+1)^2}} = [p=x+1;\ dp=dx] =$$

$$=\int\frac{dp}{\sqrt{(\sqrt{2})^2-p^2}}=\arcsin\frac{p}{\sqrt{2}}+C=\arcsin\left(\frac{x+1}{\sqrt{2}}\right)+C$$

8.4.37

$$\int\frac{(x-2)dx}{\sqrt{x^2-10x+29}}=\int\frac{(x-5)+3}{\sqrt{x^2-10x+25+4}}dx=\int\frac{(x-5)dx}{\sqrt{x^2-10x+29}}+$$

$$+3\int\frac{dx}{\sqrt{(x-5)^2+4}}=[1)\ t=x^2-10x+29\Rightarrow dt=(2x-10)dx,$$

$$\frac{1}{2}dt=(x-5)dx;\ 2)\ (x-5)=y,\ dx=dy]=$$

$$=\frac{1}{2}\int\frac{dt}{\sqrt{t}}+3\int\frac{dy}{\sqrt{y^2+4}}=\frac{1}{2}\cdot\frac{t^{\frac{1}{2}}}{\frac{1}{2}}+3\cdot\ln|y+\sqrt{y^2+4}+C=$$

$$=\sqrt{x^2-10x+29}+3\ln\left|x-5+\sqrt{x^2-10x+29}\right|+C$$

8.4.38

$$\int\frac{3x-5}{\sqrt{x^2-4x+5}}dx=\int\frac{3(x-2)+1}{\sqrt{x^2-4x+5}}dx=3\int\frac{(x-2)dx}{\sqrt{x^2-4x+5}}+$$

$$+\int\frac{dx}{\sqrt{x^2-4x+5}}=[1)\ x^2-4x+5=t,\ dt=(2x-4)dx\Rightarrow$$

$$\Rightarrow\frac{1}{2}dt=(x-2)dx;\ 2)\ y=x-2,\ dx=dy]=$$

$$=3\cdot\frac{1}{2}\cdot\int\frac{dt}{\sqrt{t}}+\int\frac{dy}{\sqrt{y^2+1}}=\frac{3}{2}\cdot\frac{t^{\frac{1}{2}}}{\frac{1}{2}}+\ln|y+\sqrt{y^2+1}|+C=$$

$$=3\sqrt{x^2-4x+5}+\ln\left|x-2+\sqrt{x^2-4x+5}\right|+C$$

8.4.39

$$\int \frac{x+1}{\sqrt{2x-x^2}}dx = \int \frac{(x-1)+2}{\sqrt{2x-x^2}}dx = \int \frac{(x-1)dx}{\sqrt{2x-x^2}} +$$

$$+2\int \frac{dx}{\sqrt{2x-x^2}} = [1)\ 2x-x^2=t,\ dt=(2-2x)dx,$$

$$\frac{1}{2}dt=(1-x)dx,\ -\frac{1}{2}dt=(x-1)dx;\ 2)\ y=x-1;$$

$$2x-x^2 = 1-1+2x-x^2 = 1-(x-1)^2 = 1-y^2;\ dx=dy] =$$

$$= -\frac{1}{2}\int \frac{dt}{\sqrt{t}} + 2\cdot\int \frac{dy}{\sqrt{1-y^2}} = -\frac{1}{2}\cdot\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + 2\cdot \arcsin\frac{y}{1} +$$

$$+C = 2\arcsin(x-1) - \sqrt{2x-x^2} + C$$

8.4.40

$$\int \frac{\sqrt{1-x^2}}{x}dx = \int x^{-1}\cdot(1-x^2)^{\frac{1}{2}}dx = [m=-1,\ n=2,$$

$$p=\frac{1}{2};\ 1)\ p=\frac{1}{2}\notin \mathbb{Z}\ \ 2)\ \frac{m+1}{n} = \frac{-1+1}{2} = 0 \in \mathbb{Z} \Rightarrow$$

$$\Rightarrow k=2,\ 1-x^2=t^2;\ x^2=1-t^2,\ x=\sqrt{1-t^2},$$

$$dx = d(\sqrt{1-t^2}) = -\frac{t\,dt}{\sqrt{1-t^2}}] = \int \frac{t}{\sqrt{1-t^2}}\cdot(-1)\cdot\frac{t\,dt}{\sqrt{1-t^2}} =$$

$$= -\int \frac{t^2dt}{1-t^2} = \int \frac{t^2dt}{t^2-1} = [t^2=(t^2-1)+1] =$$

$$= \int \frac{(t^2-1)+1}{t^2-1}\,dt = \int dt + \int \frac{dt}{t^2-1} = t + \frac{1}{2\cdot 1}\cdot \ln\left|\frac{t-1}{t+1}\right| +$$

$$+ C = \sqrt{1-x^2} + \frac{1}{2}\cdot \ln\left|\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right| + C$$

8.4.41

$$\int \sqrt{4-x^2}\,dx\ [x = 2\sin t,\ dx = 2\cos t\,dt] = \int \sqrt{4-(2\sin t)^2}\,\cdot$$

$$\cdot\, 2\cos(t)\,dt = 2\int \sqrt{4-4\sin^2 t}\cdot \cos t\,dt = 4\int \sqrt{1-\sin^2 t}\,\cdot$$

$$\cdot \cos t\,dt = 4\int \cos t \cdot \cos t\,dt = 4\int \cos^2 t\,dt =$$

$$= 4\int \frac{1+\cos 2t}{2}\,dt = 2\int (1+\cos 2t)\,dt = 2\int dt +$$

$+ 2\int \cos 2t\,dt = $ [1) табл.; 2) 5-я ф-ла] $=$

$$= 2t + \sin 2t + C = 2\arcsin\frac{x}{2} + \sin\left(2\arcsin\frac{x}{2}\right) +$$

$$+ C = 2\arcsin\frac{x}{2} + \frac{x\sqrt{4-x^2}}{2} + C$$

8.4.42

$$\int x\cdot \sqrt[5]{x-2}\,dx = \int x^1\cdot (x^1-2)^{\frac{1}{5}}\,dx = [m=1,\ n=1,\ p=\frac{1}{5};$$

$$1)\ p = \frac{1}{5} \notin \mathbb{Z};\ 2)\ \frac{m+1}{n} = \frac{1+1}{1} = 2 \in \mathbb{Z} \Rightarrow k=5,\ x-2=t^5;$$

$x=t^5+2;\ dx=5t^4dt\ ]=$

$=\int (t^5+2)\cdot t\cdot 5t^4dt = 5\int t^{10}dt+10\int t^5dt=$

$=5\cdot\frac{t^{11}}{11}+10\cdot\frac{t^6}{6}+C=\frac{5\sqrt[5]{(x-2)^{11}}}{11}+\frac{5\cdot\sqrt[5]{(x-2)^6}}{3}+C=$

$=\frac{15\cdot(x-2)\cdot(x-2)^{\frac{6}{5}}}{33}+\frac{55\cdot(x-2)^{\frac{6}{5}}}{33}+C=$

$=\frac{5\cdot(x-2)^{\frac{6}{5}}\cdot(3x+5)}{33}+C$

8.4.43

$\int\frac{dx}{x^2\sqrt{x^2+1}}=\int x^{-2}(x^2+1)^{-\frac{1}{2}}dx=[m=-2,\ n=2,\ p=-\frac{1}{2};$

1) $p=-\frac{1}{2}\notin\mathbb{Z}$; 2) $\frac{m+1}{n}=\frac{-2+1}{2}=-\frac{1}{2}\notin\mathbb{Z}$;

3) $\frac{m+1}{n}+p=\frac{-2+1}{2}-\frac{1}{2}=-1\in\mathbb{Z}\Rightarrow k=2\Rightarrow x^{-2}+1=t^2;$

$x^{-2}=t^2-1;\ x^2=\frac{1}{t^2-1},\ x=\frac{1}{\sqrt{t^2-1}},\ dx=\frac{-tdt}{\sqrt{t^2-1}(t^2-1)}]=$

$=\int\frac{t^2-1}{\sqrt{\frac{1}{t^2-1}+1}}\cdot(-1)\cdot\frac{tdt}{\sqrt{t^2-1}\cdot(t^2-1)}=-\int\frac{(t^2-1)\sqrt{t^2-1}}{t}\cdot\frac{tdt}{\sqrt{t^2-1}(t^2-1)}=$

$=-\int dt=-t+C=-\sqrt{x^{-2}+1}+C=-\sqrt{\frac{1}{x^2}+1}+C=$

$=-\frac{\sqrt{x^2+1}}{x}+C$