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Stochastic network design for disaster preparedness

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This article introduces a risk-averse stochastic modeling approach for a pre-disaster relief network design problem under uncertain demand and transportation capacities. The sizes and locations of the response facilities and the inventory levels of relief supplies at each facility are determined while guaranteeing a certain level of network reliability. A probabilistic constraint on the existence of a feasible flow is introduced to ensure that the demand for relief supplies across the network is satisfied with a specified high probability. Responsiveness is also accounted for by defining multiple regions in the network and introducing local probabilistic constraints on satisfying demand within each region. These local constraints ensure that each region is self-sufficient in terms of providing for its own needs with a large probability. In particular, the Gale–Hoffman inequalities are used to represent the conditions on the existence of a feasible network flow. The solution method rests on two pillars. A preprocessing algorithm is used to eliminate redundant Gale–Hoffman inequalities and then proposed models are formulated as computationally efficient mixed-integer linear programs by utilizing a method based on combinatorial patterns. Computational results for a case study and randomly generated problem instances demonstrate the effectiveness of the models and the solution method.

Keywords: Stochastic programming, probabilistic constraint, feasibility of second-stage, feasible flow, network reliability, facility location, disaster preparedness, humanitarian logistics

1. Introduction

Natural disasters lead to increasingly higher death tolls and material losses due to a multitude of factors, such as unplanned urbanization and increase in population and poverty. The recent massive 2010 Haiti earthquake resulted in death and injuries to hundreds of thousands of people (Walton and Ivers, 2011). The frequent occurrence of severe natural disasters has captured the attention of governments, humanitarian relief organizations, and researchers all over the world and highlighted the need to enhance the effectiveness of humanitarian relief management. Humanitarian logistics plays an important role in this field (Van Wassenhove, 2006) and differs greatly from business logistics (Van Wassenhove, 2006). Humanitarian logistics involves a high level of uncertainty and raises specific issues, such as the quickness and the fairness of the response to the affected areas (Huang *et al.*, 2011; Van Wassenhove and Pedraza Martinez, 2012). Considering the complex structure of humanitarian relief systems and the need to allocate scarce resources in a way that improves the effectiveness of the relief operations, humanitarian logistics could greatly benefit from operations research methods (Altay and Green, 2006; Van Wassenhove, 2006; Van Wassenhove and Pedraza Martinez, 2012). It is thus not surprising to see a growing body of literature devoted to the development of optimization models for humanitarian relief logistics (see, e.g., Caunhye *et al.* (2012)).

Our study focuses on the preparedness phase of disaster management for long-term planning purposes and considers the stochastic Pre-disaster Relief Network Design Problem (PRNDP) to respond to sudden-onset natural disasters. This problem determines the location of the response facilities (distribution centers) and the positioning of the relief supplies in order to improve the effectiveness of the post-disaster relief operations. The importance of long-term pre-disaster planning has been emphasized in the recent literature (see e.g., Balcık and Beamon (2008) and Salmerón and Apte (2010)). In particular, Balcık and Beamon (2008, p. 106) state that "the facility location and stock pre-positioning decisions in the relief chain are critical components of disaster preparedness and hence, require long-term planning to achieve a high-performance disaster response." The number of people affected and thus, the demand for relief supplies varies with the location, severity, and time of the disaster. Moreover, roads may be damaged and therefore, transportation capacities fluctuate according to the location and severity of the disaster. It is crucial to

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develop models that incorporate the inherent uncertainty in order to make sound relief network design decisions. To this end, we use stochastic programming to model the uncertainty in the demand and transportation capacities. The uncertain parameters are represented by a finite set of scenarios as in most applied stochastic programming models. We use the "network reliability" as the performance metric and define it as the probability that a feasible flow of relief supplies exists in the immediate post-disaster response phase. A flow is said to be feasible if it satisfies the demand for relief supplies across the network. If there is no such feasible distribution when a disaster occurs, then there can be a significant delay in providing an adequate amount of relief supplies to the people in need. To avoid such situations, we obtain a reliable relief network design by introducing a global probabilistic constraint to guarantee the existence of a distribution plan under which the network demand is satisfied with a specified high probability.

In a humanitarian relief system, positioning the resources reasonably close to the potential demand locations is essential to decreasing the response times and consequently, to alleviate the suffering of the people in need. We incorporate the issue of responsiveness by defining multiple regions (e.g., states, counties, district, etc.) in the network and locating adequate capacity and resources in each region. The goal is to guarantee that the regions are self-sufficient in terms of providing for their relief supply needs with a high probability. This is achieved by introducing local probabilistic constraints on satisfying the demand within each region. By enforcing the same reliability level for every region, we ensure an equitable response and service distribution among regions.

The PRNDP can be modeled as a two-stage stochastic programming problem. The first-stage decisions typically concern the size and the location of the response facilities as well as the inventory level at each facility. The second-stage (recourse) decisions pertain to the distribution of the relief supplies and depend on the predetermined first-stage decisions and the observed realization of the random parameters. Developing such two-stage stochastic models for the PRNDPs with inventory pre-positioning and relief distribution has recently received particular attention (Balcık and Beamon, 2008; Mete and Zabinsky, 2010; Rawls and Turnquist, 2010; Salmerón and Apte, 2010; Döyen *et al.*, 2012; Noyan, 2012). Most of these existing studies propose risk-neutral two-stage stochastic programming models; i.e., they are based on expected values. However, soley considering expected values may not be good enough for rarely occurring disaster events. Incorporating the risk concept is crucial in order to model the random variability inherent in disaster relief systems. The importance of managing uncertainty and risk is also discussed in Van Wassenhove (2006) and Noyan (2012). In this spirit, we propose new risk-averse models for the PRNDP.

The standard two-stage stochastic programming formulation enforces the feasibility of the second-stage problem for each joint realization of the random parameters and may consequently lead to over-conservative solutions. A well-known approach to avoid such conservative solutions is to relax some constraints in the second-stage problem and penalize the amount by which the relaxed constraints are violated. For example, in a disaster management context, Rawls and Turnquist (2010) relax the demand satisfaction constraints and include the expected penalty cost for the unmet demand in the cost function of the second-stage problem. However, in practice it may be challenging to estimate the shortage penalty costs. As an alternative, a qualitative approach can be used in order to control the violation of the relaxed constraints. In line with this approach, we introduce a probabilistic constraint to control the infeasibility of the second-stage problem.

In general, a two-stage stochastic programming problem is formulated as a large-scale Mixed-Integer Programming (MIP) model by introducing a potentially huge number of second-stage decision variables and constraints. Noyan (2012) and Rawls and Turnquist (2010) emphasize the computational difficulty of solving such an MIP formulation for the PRNDP, in particular when the number of scenarios is large. As in Noyan (2012) and Rawls and Turnquist (2010), one could use a scenario decomposition method, but the problem remains hard to solve when many scenarios are considered. In this study, we also focus on the two-stage programming framework. We propose a novel modeling approach that ensures the feasibility of the second-stage problem without requiring the introduction of recourse decisions. Indeed, the feasibility of the second-stage problem is solely defined in terms of the first-stage decisions. This is the key feature of our study and it is accomplished by using the Gale–Hoffman theorem (Gale, 1957; Hoffman, 1960). This theorem represents the conditions on the existence of a feasible network flow with a set of linear inequalities expressed in terms of demand and arc capacities. In particular, we introduce a single-stage stochastic programming model with a joint probabilistic constraint on the feasibility of the second-stage problem. Then, we reformulate the joint probabilistic constraint in terms of the first-stage decision variables only by using the Gale–Hoffman inequalities. This approach allows us to incorporate the effectiveness of the post-disaster relief operations into the strategic level pre-disaster decision making without modeling the operational-level relief distribution explicitly.

In addition to the presence of integer variables pertaining to the location of facilities, two computational challenges linked to the probabilistic constraints need careful attention. First, the number of Gale–Hoffman inequalities for which we impose a joint probabilistic constraint is extremely large (see Section 4.1). As a remedy, we eliminate the redundant inequalities by adopting the procedures proposed in Prékopa and Boros (1991) and Wallace and Wets (1995). However, these preprocessing procedures become a computational bottleneck as the network size increases. To alleviate this issue, following a common practice (see, e.g., Klibi and Martel (2010) and Galindo and Batta (2013)), we propose to use a node aggregation (clustering) technique and apply the preprocessing procedure on an aggregated version of an originally large network. Second, we use a method based on the concept of combinatorial patterns (Lejeune, 2012a) in order to solve the proposed models. This method reformulates a chance-constrained model as an MIP problem in which the number of binary variables does not depend on the number of scenarios. Being able to solve a stochastic network design model for a large number of scenarios is a significant contribution, since it may be crucial to use many scenarios to represent the uncertainty in humanitarian relief environments (Snyder, 2006; Campbell and Jones, 2011). We also propose a scenario generation method allowing for the representation of dependency structures specific to disaster relief systems.

We have several significant contributions in this study. We design a novel approach to model the reliability of a disaster relief network and introduce new risk-averse stochastic programming models for the pre-disaster relief network design problem. Moreover, we develop computationally efficient methods to solve the probabilistically constrained models for moderate-size networks and a large number of scenarios.

The article is organized as follows. In Section 2, we present the related literature. In Section 3, we propose alternate optimization models for the stochastic PRNDP. Section 4 first discusses how to derive the set of non-redundant Gale–Hoffman inequalities. It continues with the presentation of the combinatorial pattern-based method used to reformulate the probabilistically constrained models as MIP problems. Section 5 is devoted to the computational results for a case study and a set of randomly generated problem instances based on a real disaster network. Section 6 provides managerial implications and concluding remarks.

2. Literature review

We first review the relevant literature on the design of pre-disaster relief networks in the presence of uncertainty. Then, we briefly review some of the studies related to the methodological aspects of our work.

There is an increasing number of studies that develop mathematical models for designing relief distribution networks. The high level of uncertainty characterizing humanitarian relief environments and the need to allocate scarce relief resources efficiently led to the development of stochastic optimization models for the PRNDP in the recent literature. Within this body of literature, there are few studies that address only the facility location (e.g., Jia *et al.* (2007) and Ukkusuri and Yushimito (2008)) or only the inventory problem (e.g., Beamon and Kotleba (2006)). Among the studies about preparedness, we focus on the ones that propose stochastic optimization models involving both facility location and inventory decisions. In general, such studies propose two-stage stochastic programming models, where the first-stage decisions are for locating the response facilities and pre-positioning the relief supplies and the second-stage decisions are related to the distribution of the relief supplies. Basically, such models determine the pre-disaster decisions considering the potential post-disaster decisions.

Balcık and Beamon (2008) develop a maximal covering-type model to determine the location of the distribution centers and the inventory levels of multiple relief items at each facility under demand and transportation time uncertainty. The recourse decisions represent the proportion of demand for a particular type of item satisfied by each distribution center. Thus, they assume that there is only one demand location under each scenario. They also assume that the demand is fully satisfied under each scenario and hence, the model may provide conservative solutions. Rawls and Turnquist (2010) allow multiple locations to be demand points under each disaster scenario and allow for the infeasibility of the demand satisfaction constraints. They consider the PRNDP with uncertainty in the pre-positioned supplies, demand, and transportation capacities. Different from a coverage-type model as the one proposed in Balcık and Beamon (2008), the second-stage problem in Rawls and Turnquist (2010) involves detailed distribution decisions representing the flow of relief supplies on each arc of the network and penalizes the demand shortages. Other recent and related studies are by Mete and Zabinsky (2010), Salmerón and Apte (2010), and Döyen *et al.* (2012). In contrast with the other studies, Döyen *et al.* (2012) consider facility location decisions in both the first- and second-stage problems.

Almost all of the above studies propose risk-neutral models; i.e., they are based on expected values. More recently, the literature addresses the importance of incorporating the concept of risk into disaster management decision making (Van Wassenhove, 2006; Noyan, 2012). Beraldi and Bruni (2009) introduce a risk-averse model for an emergency service vehicle location problem under demand uncertainty. They consider a coverage-type model and formulate it as a two-stage stochastic program with a probabilistic constraint. The second-stage decisions represent the assignment of supply nodes to demand points under each scenario. The probabilistic constraint enforces a lower bound on the probability that each demand node is covered and the demand at each node is fully served. The idea of developing such a stochastic programming model with a probabilistic constraint on the feasibility of the second-stage problem was first proposed by Prékopa (1980). Noyan (2012) incorporates a risk measure to develop a risk-averse two-stage stochastic model for the PRNDP. Noyan (2012) extends the model proposed in Rawls and Turnquist (2010) by incorporating the Conditional Value-at-Risk (CVaR) as the risk measure on the total cost in addition to the expectation criterion. We refer to Noyan (2012) for the definition of the CVaR and further details. Alternatively, Rawls and Turnquist (2011) extend their earlier model (Rawls and Turnquist, 2010) by introducing a lower bound on the probability that the network demand is satisfied and the resulting average total shipment distance is less than a specified threshold. Different from the existing studies, our article proposes reliability-based models with a joint probabilistic constraint on the feasibility of the second-stage problem without introducing any recourse decision. This approach essentially provides an alternative way of modeling "network reliability."

There is a huge literature on modeling network reliability and designing reliable networks. We shall briefly review the studies relevant to a disaster context. Peeta *et al.* (2010) propose a pre-disaster investment problem that identifies a subset of links to be strengthened. The goals are to maximize the post-disaster connectivity and minimize the expected transportation cost between various origin–destination pairs. They assume that there are only two possible states for each link—i.e., a link either survives or fails after the disaster—and the link failure probabilities are known *a priori*. Under the assumption of independent link failures, they propose a two-stage stochastic programming model where the investment and flow decisions are represented by first- and second-stage decisions, respectively. They characterize the network reliability in terms of its connectivity through non-failed links. Thus, their only focus is to provide access in case of a disaster and they do not take demand for relief supplies and capacity into consideration. Note that the common and simplifying assumption that link failures are independent can be criticized in a disaster context where it is crucial to model the dependency between link failures. Gunnec and Salman (2011) focus on assessing network reliability considering dependency among link failures. According to their dependency model, the links are partitioned into sets. Links in different sets are assumed to fail independently of each other, while links within the same set fail according to a specified dependency model. As in Peeta *et al.* (2010), Gunnec and Salman (2011) view network reliability as the probability of connectedness and define the performance of the network as the expected shortest path distance. Gunnec and Salman (2011) propose methods to calculate these measures in order to evaluate the post-disaster performance of a network. Differing from these two studies, ours takes demand and capacity into consideration. Moreover, each link is allowed to be partially damaged and the dependencies among link failures are accounted for by using a scenario-based approach. For a comprehensive review of network reliability measures and reliable network design problems, we refer the reader to Gunnec and Salman (2011) and the references therein. A recent study by Nolz *et al.* (2011) also considers several risk measures to design a post-disaster delivery system under the assumption that link failures are independent.

For other types of mathematical models used to develop disaster preparedness policies, we refer to the review papers by Brandeau *et al.* (2009) and Caunhye *et al.* (2012) and the references therein. Another major stream of literature in humanitarian logistics is related to the distribution of emergency supplies and vehicle routing in the post-disaster response phase (Haghani and Oh, 1996; Barbarosoğlu and Arda, 2004; Ozdamar *et al.*, 2004). For a recent and comprehensive review of the humanitarian logistics literature, we also refer to the special issues of *International Journal of Production Economics* edited by Boin *et al.* (2010) and *OR Spectrum* edited by Doerner *et al.* (2011).

As mentioned above, we propose probabilistically constrained optimization problems. Stochastic programming problems with joint probabilistic (chance) constraints in which the uncertainty is represented by a set of scenarios are typically NP-hard and extremely difficult to solve. Developing solution methods for this class of problems has been receiving sustained attention in the last decade. In general, the proposed solution methods utilize the p-efficiency concept (Dentcheva *et al.*, 2001; Lejeune and Ruszczyński, 2007; Lejeune and Noyan, 2010; Dentcheva and Martinez, 2012), valid inequalities to derive strengthened MIP reformulations (Ruszczyński, 2002; Luedtke *et al.*, 2010), and conservative convex approximations. In order to be able to consider a large number of scenarios for characterizing the uncertainty, we use the method proposed by Lejeune (2012a) to reformulate and solve the chance-constrained optimization problems.

3. Stochastic optimization models

In this study, we consider the problem of long-term pre-disaster planning and develop models that support response facility location and stock pre-positioning decisions. These decisions are to be implemented well before a disaster strikes. Such long-term considerations are markedly different from short-term relief planning, where the goal is to position the supplies near a potentially affected area in anticipation of an imminent disaster. For example, Galindo and Batta (2013) propose a pre-disaster management model, where the pre-positioning decisions are made two days before a foreseeable hurricane, based on the forecasts available about five days prior to the landfall. However, such a short time period might not be sufficient to ensure a desired level of service, and response operations can be difficult to carry out effectively without appropriate long-term pre-disaster planning.

In short-term planning situations, forecasts about the location, severity, and time of a disaster are often available. In such cases, replacing an uncertain parameter (such as the random demand for relief commodities) by its expected value (as in Galindo and Batta (2013)) may be justified due to the lower inherent uncertainty. However, as our focus is on long-term planning in the absence of accurate predictions, it is crucial to consider a more detailed way of modeling uncertainty. To this end, we use a finite set of scenarios to represent potential future outcomes; in particular, each scenario consists of a joint realization of the arc capacities and the demand for relief supplies at each node. This scenario-based approach allows us to incorporate the dependency between uncertain parameters into our models.

In the remainder of this section, we present our optimization models for the PRNDP and their MIP reformulations. In Section 3.1, we present a two-stage stochastic programming model. Based on this two-stage model, we develop in Section 3.2 an optimization model enforcing a network-wide (global) probabilistic constraint. This constraint ensures the existence of a feasible flow of relief supplies to satisfy the demand across the network with a specified high probability. Next, the model is extended in Section 3.3 to account for the responsiveness and equity criteria that play a crucial role in disaster management. This is achieved by introducing local probabilistic constraints that ensure that each region provides for its own commodity needs with a certain reliability. In Section 3.2, we utilize the Gale–Hoffman inequalities to define the probabilistic existence of a feasible network flow. Additionally, we employ a preprocessing algorithm to eliminate the redundant Gale–Hoffman inequalities and obtain compact chance-constrained models for the PRNDP. In the last step of the modeling approach, we employ a combinatorial method to derive equivalent MIP reformulations for the chance-constrained models proposed in Sections 3.2 and 3.3.

3.1. *Two-stage model*

We present now a two-stage stochastic programming model for the PRNDP with stochastic demand and arc capacities. This model constitutes a basis for the development of our new chance-constrained optimization models. We assume that each node represents a candidate facility location. This is not a restrictive assumption, since the corresponding formulation can be easily modified to allow the set of candidate facility locations to differ from the set of demand points. We consider a single type of commodity that can be a bundle of critical relief supplies, such as prepackaged food, medical kits, blankets, and water.

Here we introduce the notation:

I: set of nodes (locations);
A: set of arcs;
L: set of facility types differentiated by their size;
Ω: set of scenarios;
ξ_i^d: random demand for the bundled commodity at location i;
ξ_{ij}^v: random capacity of arc linking locations i and j;
d_i^s: realization of demand for the bundled commodity at location i under scenario s;
v_{ij}^s: realization of the capacity of arc (i, j) expressed in units of the bundled commodity under scenario s;
p: prespecified probability (reliability) level;
M_l: total capacity of a facility of type l;
F_{il}: fixed cost of opening a type l facility at location i;
q: unit acquisition cost for the bundled commodity.

The $|I|$-dimensional random vector associated with the demand at each location and the $|A|$-dimensional random vector associated with the arc capacities are denoted by $\boldsymbol{\xi}^d$ and $\boldsymbol{\xi}^v$, respectively. Using these notations, we represent the inherent uncertainty in the network by the $(|I| + |A|)$-dimensional random vector $\boldsymbol{\xi} = (\boldsymbol{\xi}^d, \boldsymbol{\xi}^v)$ and denote its realization under scenario s by $\boldsymbol{\omega}^s$. Thus, $\boldsymbol{\omega}^s$ is the vector whose components correspond to the elements of $\{d_i^s,\ i \in I,\ v_{ij}^s,\ (i, j) \in A\}$ and denote the joint realizations of the random variables $\xi_i^d,\ i \in I,\ \xi_{ij}^v,\ (i, j) \in A$, under scenario s.

The first-stage decision variables define the size and location of the facilities and the amount of commodity pre-positioned at each facility. Let the binary variable $y_{il},\ i \in I,\ l \in L$, equal one if a facility of type l is located at node i, while $r_i,\ i \in I$, designates the inventory level of the facility located at node i. The main second-stage decision variables define the distribution of the relief supplies, while the auxiliary second-stage decision variables measure the amount of shortages and surplus of commodity. Let x_{ij}^s designate the amount of the bundled commodity shipped through arc (i, j) under scenario s, while w_i^s and a_i^s respectively denote the amount of shortage and surplus at location i under scenario s. The first-stage problem takes the form

$$\min \quad \sum_{i\in I}\sum_{l\in L} F_{il} y_{il} + \sum_{i\in I} q r_i + E[Q(\mathbf{r}, \mathbf{y}, \xi)] \tag{1}$$

$$\text{s.t.} \quad r_i \le \sum_{l\in L} M_l y_{il}, \quad i \in I \tag{2}$$

$$\sum_{l\in L} y_{il} \le 1, \quad i \in I \tag{3}$$

$$y_{il} \in \{0, 1\}, \quad i \in I, l \in L \tag{4}$$

$$r_i \ge 0, \quad i \in I. \tag{5}$$

Here $Q(\mathbf{r}, \mathbf{y}, \xi)$ denotes the random objective function value of the second-stage problem and its realization $Q(\mathbf{r}, \mathbf{y}, \boldsymbol{\omega}^s)$ under scenario $s \in \Omega$ is obtained by solving the following problem:

$$Q(\mathbf{r}, \mathbf{y}, \boldsymbol{\omega}^s) = \min \ f(\mathbf{x}^s, \mathbf{w}^s, \mathbf{a}^s) \tag{6}$$

$$\text{s.t.} \ r_i + \sum_{(j,i)\in A} x_{ji}^s - \sum_{(i,j)\in A} x_{ij}^s = d_i^s - w_i^s + a_i^s, \quad i \in I \tag{7}$$

$$w_i^s,\ a_i^s \ge 0, \quad i \in I \tag{8}$$

$$0 \le x_{ij}^s \le v_{ij}^s, \quad (i, j) \in A \tag{9}$$

The objective function (1) of the first-stage problem minimizes the sum of the cost of opening facilities and purchasing the relief supplies and the expected total cost of the second-stage. Constraint (2) ensures that each facility is sufficiently large to store the amount of the commodity pre-positioned at that facility. Constraint (3) guarantees that at most one facility is located at each node. Constraints (4) and (5) are the binary and non-negativity restrictions. In the formulation of the second-stage problem, constraint (7) represents the flow conservation constraints. For instance, the shortfall variable w_i^s is positive if the amount of the commodity available at node i is not sufficient to cover the associated demand d_i^s under scenario s. Constraint (9) prevents the flow on arc (i, j) from exceeding the capacity of the arc under scenario s.

The second-stage objective function (6) includes the demand shortage costs and in general, the surplus and the transportation costs (see, e.g., Rawls and Turnquist (2010)). In contrast, we mainly focus on satisfying demand for relief supplies and do not explicitly consider such costs. Surplus costs are implicitly accounted for due to the first-stage objective of minimizing the acquisition costs, and transportation costs are relatively small compared with the cost of building the pre-disaster relief network (e.g., Rawls and Turnquist (2010)). Nevertheless, considering the transportation costs may be helpful in reducing response times. We partially address this issue by proposing a model that incorporates a responsiveness criterion. As the model presented in Rawls and Turnquist (2010), the model (1)–(9) penalizes the expected violation amounts associated with the relaxed demand satisfaction constraints and thus, requires estimating the shortfall cost parameter. Although such penalty parameters are often used in the disaster management literature (see, e.g., Barbarosoğlu and Arda (2004), Salmerón and Apte (2010), and Rottkemper *et al.* (2011)), it may be challenging to quantify the cost of failing to satisfy the demand (see, e.g., Prékopa (1995) and Rottkemper *et al.* (2011)). As an alternative, we introduce a probabilistic constraint to control the existence of a feasible flow of relief supplies to satisfy the network demand. The proposed probabilistically constrained optimization model is described in the next section.

3.2. *Chance-constrained models*

We shall now present an alternative modeling approach to the ones that inflict penalty costs for the unsatisfied demand (see the previous subsection and, e.g., Rawls and Turnquist (2010) and Noyan (2012)). Our approach focuses on the probability of failing to satisfy the demand in the network and avoid inflicting a penalty per unit of shortfall as in constraint (6). Hence, it does not require the estimation of the shortfall cost parameters. The proposed chance-constrained model, which we refer to as **SP1**, probabilistically prevents the demand shortage in case of a disaster:

$$\textbf{SP1} \quad \min \quad \sum_{i\in I}\sum_{l\in L} F_{il}y_{il} + \sum_{i\in I} qr_i \tag{10}$$

$$\text{s.t. } (2), (3), (4), (5),$$

$$\mathbb{P}\begin{pmatrix} r_i + \sum\limits_{(j,i)\in A} x_{ji} - \sum\limits_{(i,j)\in A} x_{ij} \geq \xi_i^d, \quad i \in I \\ 0 \leq x_{ij} \leq \xi_{ij}^v, \quad (i,j) \in A \end{pmatrix} \geq p. \tag{11}$$

Here we elaborate only on the new probabilistic constraint (11). First, recall that the corresponding two-stage problem (6)–(9) involves scenario-dependent second-stage variables x_{ij}^s, $(i, j) \in A$. Basically, the recourse decisions depend on the realizations of the random vector $\boldsymbol{\xi}$ and are thus random variables. In the above formulation, by abuse of notation, we drop the scenario index s from x_{ij}^s to denote the random recourse decisions. Constraint (11) is a joint chance constraint enforcing the probability of satisfying the second-stage constraints (7) and (9) with $w_i^s = 0$ to be at least p. In fact, the set of inequalities featured in the probabilistic constraint (11) represents the conditions on *the existence of a flow such that the flow along each edge is no greater than its capacity, and the net flow into each node is greater than or equal to the demand at that node*. Thus, any location–allocation policy defined by a feasible solution $(\mathbf{y}, \mathbf{r})$ of **SP1** guarantees *the existence of a feasible network flow* with probability at least p. We refer to the risk parameter p as the "network-wide reliability level." Introducing binary variables β^s, $s \in \Omega$, to identify whether there is such a feasible distribution of relief supplies under each scenario, we formulate **SP1** as the large-scale MIP problem **SP1a**:

$$\textbf{SP1a} \quad \min \quad \sum_{i\in I}\sum_{l\in L} F_{il}y_{il} + \sum_{i\in I} qr_i$$

$$\text{s.t. } (2), (3), (4), (5),$$

$$r_i + \sum_{(j,i)\in A} x_{ji}^s - \sum_{(i,j)\in A} x_{ij}^s \geq d_i^s(1-\beta^s), \quad i \in I,\ s \in \Omega \tag{12}$$

$$0 \leq x_{ij}^s \leq v_{ij}^s + Z\beta^s, \quad (i,j) \in A,\ s \in \Omega \tag{13}$$

$$\sum_{s\in\Omega} \pi^s \beta^s \leq 1 - p. \tag{14}$$

Here π^s denotes the probability of scenario s and Z is a sufficiently large positive number. Constraints (12) and (13) force β^s to take a value of one if the network demand cannot be satisfied without exceeding any arc capacity under scenario s. Thus, constraint (14) implies that the probability of the flow of relief supplies to be feasible is lower bounded by p. Note that MIP formulations stronger than **SP1a** can be obtained. However, any such deterministic equivalent formulation introduces recourse decisions associated with each scenario and hence, involves a potentially large number of second-stage decision variables. To obtain a computationally efficient reformulation of **SP1**, we propose to avoid adding the scenario-dependent second-stage variables. In particular, we use the Gale–Hoffman theorem (Gale, 1957; Hoffman, 1960) to represent the conditions on the existence of a feasible network flow for the second-stage problem. The feasibility of the network flow is modeled with a set of linear inequalities defined in terms of demand and arc capacities. This, in turn, leads to the reformulation of the probabilistic constraint (11) in terms of first-stage decision variables only. We shall now discuss the concept of a demand function defined on the set of nodes and present the Gale–Hoffman inequalities.

Consider a directed network defined by a set of nodes I and a set of arcs $A \subseteq I \times I$. Each arc $(i, j) \in A$ and each node $i \in I$ has an associated non-negative arc capacity v_{ij} and resulting net demand $d(i)$, respectively. For any $H \subseteq I$, we define $d(H) = \sum_{i \in H} d(i)$. Let $\bar{H}$ refer to the complement of a set H and $v(\bar{H}, H)$ be the sum of the capacities of the arcs connecting the nodes in $\bar{H}$ to those in H. Using these notations, we next present the Gale–Hoffman theorem providing the necessary and sufficient conditions on the existence of a feasible network flow (equivalently, conditions on the feasibility of a net demand function).

Theorem 1. *(Gale and Hoffman) There exists a feasible network flow if and only if the inequality*

$$d(H) \leq v(\bar{H}, H) \tag{15}$$

holds for every $H \subseteq I$.

The set of constraints (15) will be thereafter referred to as the Gale–Hoffman inequalities. If the stocked inventory is not sufficient to satisfy the demand at a node, the resulting net demand is positive; i.e., the node is in need of relief supplies. Since the total capacity of the incoming arcs to a node is essentially an upper bound on the amount of supplies that can be delivered to that node, the net demand at each node must be smaller than or equal to the total capacity of the incoming arcs in order to guarantee the satisfaction of the network demand. For our stochastic network design problem, the net demand associated with the subset H under scenario s is obtained as $d(H) = \sum_{i \in H}(d_i^s - r_i)$, and a feasible network flow exists if and only if the Gale–Hoffman inequality

$$\sum_{i \in H} \left(d_i^s - r_i\right) \leq \sum_{i \in \bar{H}, j \in H} v_{ij}^s \tag{16}$$

holds for every $H \subseteq I$. If this is the case, the demand at all of the nodes of the network can be satisfied under scenario s. We use the convention that $v_{ij}^s = 0,\ s \in \Omega$, for all $(i, j) \notin A$ and therefore, the equality in (16) holds true.

The next lemma, which is a direct consequence of Theorem 1, utilizes a stochastic version of the Gale–Hoffman inequalities (15) and provides us with an equivalent formulation of the chance constraint (11).

Lemma 1. *The chance constraint* (11) *can be equivalently rewritten as*

$$\mathbb{P}\left(\sum_{i \in H} \left(\xi_i^d - r_i\right) \leq \sum_{i \in \bar{H}, j \in H} \xi_{ij}^v\ , \ \ \textit{for every } H \subseteq I\right) \geq p. \tag{17}$$

Proof. The net demand at each location and the capacity of each arc are random. This implies that $d(H)$ and $v(\bar{H}, H)$ are random for a subset $H \subseteq I$. Hence, we utilize a stochastic version of the Gale–Hoffman inequalities to represent the stochastic conditions on the existence of the feasible network flow. Requiring that the Gale–Hoffman inequalities hold jointly with probability at least p gives

$$\mathbb{P}\left(d(H) \leq v(\bar{H}, H), \ \ \text{for every } H \subseteq I\right) \geq p. \tag{18}$$

Substituting $d(H) = \sum_{i \in H}(\xi_i^d - r_i)$ and $v(\bar{H}, H) = \sum_{i \in \bar{H}, j \in H} \xi_{ij}^v$ into (18) provides the desired probabilistic constraint (17). Then, the assertion follows from Theorem 1 and the definition of a feasible network flow. ■

The Gale–Hoffman inequalities in (17) define the sufficient and necessary conditions for the existence of a flow to satisfy the network demand and are written in terms of the first-stage decision variables r_i, $i \in I$, only. This is a marked difference from model **SP1**, where the probabilistic feasibility of the second-stage problem (11) is defined in terms of the second-stage distribution variables x_{ij}. Replacing (11) by (17), we obtain an equivalent formulation of **SP1**, referred to as **SP2**:

$$\textbf{SP2} \quad \min \left\{ \sum_{i \in I} \sum_{l \in L} F_{il} y_{il} + \sum_{i \in I} q r_i : (2), (3), (4), (5), (17), \right.$$
$$\left. \mathbf{r} \in \mathbb{R}^{|I|}, \mathbf{y} \in \mathbb{R}^{|I| \times |L|} \right\}$$

Typically, scenario-based approaches use a limited number of scenarios to represent the uncertain future outcomes in order to avoid introducing a large number of decision variables and obtain computationally tractable models (see e.g., Snyder (2006) and Campbell and Jones (2011)). A significant contribution of our approach is that it does not require the introduction of the scenario-dependent second-stage decision variables. Thus, it permits the efficient handling of a large number of scenarios, which is crucial to obtain a fine characterization of the uncertainty in humanitarian relief environments (Lempert *et al.*, 2003; Comfort, 2005; Snyder, 2006).

3.3. *Responsiveness and equity*

As for other types of emergency service systems (e.g., Brotcorne *et al.* (2003)), it is also crucial to take the responsiveness and fairness criteria into consideration while designing a humanitarian relief network (Tzeng *et al.*, 2007; Huang *et al.*, 2011; Van Wassenhove and Pedraza Martinez, 2012).

Responsiveness refers to the quickness of the response and is related to the positioning of the resources reasonably close to the potential demand locations. We ensure responsiveness by defining multiple regions (e.g., states, counties, districts) in the network and by requiring the allocation of enough resources to make the regions self-sufficient. This means that each region will be able to provide for its own needs with a large probability, which we call the local (region-wide) reliability. Regions can be viewed as clusters of locations and can be constructed based on threshold distances or response times. Chang *et al.* (2007) also focus on dividing the disaster area into several regions and assume that a facility gives a higher priority to the demand points in its own region before extending help to others. As in Chang *et al.* (2007), a mathematical programming model can also be used to cluster the nodes in the network. For each cluster of nodes representing a certain region, we introduce a so-called local probabilistic constraint on satisfying the demand within each region. Moreover, we ensure an equitable service distribution at the region level by enforcing the same reliability level for every region. Noyan (2010) also models equity by setting the risk parameter to be equal for different regions but considers a different type of risk measure for an emergency service system design problem. Thus, we propose an alternative way of modeling equity which is based on the reliability level of satisfying the demand within each region.

In this section, we introduce additional stochastic constraints in order to incorporate the responsiveness and equity into the proposed model **SP2**. These constraints are referred to as the local probabilistic constraints. Let K denote the number of regions in the network and I^k be the set of nodes within region k. Then, we introduce the following K local (region-wide) chance constraints:

$$\mathbb{P}\left(\sum_{i\in H}(\xi_i^d - r_i) \leq \sum_{i\in \bar{H}^k,\, j\in H} \xi_{ij}^v, \text{ for every } H \subseteq I^k\right) \geq p'_k \qquad k \in \{1, \dots, K\} \tag{19}$$

where $\bar{H}^k = I^k \setminus H$ and p'_k is the specified minimal acceptable probability level of satisfying the demand within region k. We assign the same value to each parameter p'_k: $p'_k = p',\ k = 1, \dots, K$. We refer to the risk parameter p' as the "region-wide reliability level." Incorporating constraint (19) into **SP2** we obtain the model with global and local probabilistic constraints, which we refer to as **SP3**:

$$\textbf{SP3} \quad \min\left\{\sum_{i\in I}\sum_{l\in L} F_{il} y_{il} + \sum_{i\in I} q r_i : (2), (3), (4), (5), (17), (19), \mathbf{r} \in \mathbb{R}^{|I|}, \mathbf{y} \in \mathbb{R}^{|I|\times|L|}\right\}$$

Constraint (19) ensures that the inventories pre-positioned within a region are sufficient to satisfy the demand in that region with a probability of at least p'. This would ensure that the demand for relief commodities can be satisfied from nearby facilities with probability at least p' and thus, the introduction of the local reliability constraints can improve the effectiveness of the immediate post-disaster operations in terms of the response times. This novel model allows the simultaneous control of the reliability-based performances of individual regions and the entire network.

In our computational study we consider local reliability levels p' that are smaller than the network-wide reliability level p. This choice is supported by the observation that individual region-wide Gale–Hoffman inequalities $\sum_{i\in H}(\xi_i^d - r_i) \leq \sum_{i\in \bar{H}^k, j\in H} \xi_{ij}^v$ imply their global counterparts $\sum_{i\in H}(\xi_i^d - r_i) \leq \sum_{i\in \bar{H}, j\in H} \xi_{ij}^v$. However, we note that local probabilistic constraints in general do not imply the global one, since the latter is enforced in a joint fashion. Thus, it could also be meaningful to specify a p' value that is higher than p.

We remark that while we incorporate equity by setting the local reliability levels to be equal, relief organizations might prefer to follow some other alternative approach that gives different priorities to different areas. Such a prioritizing approach can be accommodated by assigning different local reliability levels. We also note that it is possible to incorporate existing facilities into our model. For example, if there already exists a facility of type l located at node i with an inventory level of ρ, we set $y_{il} = 1$. Furthermore, if the relocation of the inventory at this facility is not allowed, we add the constraint $r_i \geq \rho$ to our models.

4. Solution methods

As mentioned in the Literature Review section, probabilistically constrained optimization problems are typically NP-hard and difficult to solve. It is therefore important to develop computationally tractable formulations and solution methods. In this section, we first present computationally more efficient formulations of **SP2** and **SP3**. They are obtained by eliminating the redundant Gale–Hoffman inequalities with the procedures proposed by Prékopa and Boros (1991) and Wallace and Wets (1995). Then, we use a recently proposed method based on combinatorial patterns (Lejeune, 2012a) to reformulate the proposed probabilistically constrained models as MIP problems including a limited number of integer variables.

4.1. *Elimination of redundancy*

Each Gale–Hoffman inequality corresponds to a subset of I. This implies that the number $(2^{|I|} - 1)$ of inequalities required to hold jointly by the global probabilistic constraint (17) is very large even for small networks. Thus, solving **SP2** and **SP3** would be extremely challenging, if not hopeless. Fortunately, as mentioned in Prékopa and Boros (1991), the number of Gale–Hoffman inequalities can be drastically reduced by eliminating the redundant ones. For example, Prékopa and Boros (1991) study a 15-node network with a total of 32 767 Gale–Hoffman inequalities and show that only 13 of those remain after the elimination procedure. Motivated by these discussions, we use the methods proposed by Prékopa and Boros (1991) and Wallace and Wets (1995) to eliminate the redundant Gale–Hoffman inequalities. The elimination procedure uses lower and upper bounds on the random demand and arc capacities, which are assumed to be known. We present the details of the elimination procedure in Appendix A for completeness. However, as will be shown in Section 5.2.1, the time needed to eliminate the redundant Gale–Hoffman inequalities also rapidly increases with the size of the network. Therefore, our modeling approach will be most beneficial for networks of small and moderate sizes. In the case of a large network, we propose to construct an aggregated version of the original network. This is a commonly used practice for the design of humanitarian logistics networks (Klibi and Martel, 2010; Galindo and Batta, 2013) that pools some of the individual demand nodes to form aggregated nodes. For example, Klibi and Martel (2010) suggest merging the demand points (based on characteristics such as geographical and political) into a geographical demand zone with a computable centroid. In this study, we shall pool some of the adjacent individual nodes. The resulting aggregated nodes will form the basis to analyze the relief commodity needs and design the relief network. If a disaster happens, more detailed post-disaster decisions can be made on the basis of the original network.

Suppose that we perform the elimination procedure for the Gale–Hoffman inequalities associated with all of the subsets of I. Let $V_t \subseteq I, t \in T \subseteq \{1, \ldots, 2^{|I|} - 1\}$, be the subsets of I that remain after performing the elimination procedure. Thus, these are the subsets of I for which the associated Gale–Hoffman inequalities are non-redundant. Then, the model **SP2** is equivalently reformulated as

$$\textbf{SP2a} \quad \min \sum_{i\in I}\sum_{l\in L} F_{il}y_{il} + \sum_{i\in I} qr_i$$

$$\text{s.t.} \quad (2), (3), (4), (5),$$

$$\mathbb{P}\left(\sum_{i\in V_t}\left(\xi_i^d - r_i\right) \leq \sum_{i\in \bar{V}_t, j\in V_t} \xi_{ij}^v,\ t\in T\right) \geq p\ . \tag{20}$$

The number of inequalities subjected to the probabilistic condition (20) is $|T|$ and is typically much smaller than the number $(2^{|I|} - 1)$ of the inequalities involved in constraint (17). The elimination procedure provides a much more compact formulation of the probabilistic constraint (17), which, from a computational point of view, is much easier to deal with than constraint (17).

Obviously, we can use the same procedure to obtain a more compact reformulation of **SP3**. Recall that we have defined K regions in the network, where region k comprises the nodes included in the set I^k. Let $V_t \subseteq I^k, t \in T^k \subseteq \{1, \ldots, 2^{|I|} - 1\}$ be the subsets of I^k that remain after performing the elimination procedure for the network defined by the set of nodes I^k, $k = 1, \ldots, K$. Then, the compact reformulation of **SP3**, referred to as **SP3a**, is obtained by adding the following local chance constraints to **SP2a**:

$$\mathbb{P}\left(\sum_{i\in V_t}\left(\xi_i^d - r_i\right) \leq \sum_{i\in \bar{V}_t^k, j\in V_t} \xi_{ij}^v,\ t\in T^k\right) \geq p'_k,$$
$$k = 1, \ldots, K,$$

where $\bar{V}_t^k = I^k \setminus V_t$ for all $k = 1, \ldots, K,\ t \in T^k$, and $p'_k = p'$ for all $k = 1, \ldots, K$.

The remaining computational challenge is to solve the models **SP2a** and **SP3a** when the number of scenarios characterizing the uncertainty in the network is large, as it may be desirable in a disaster context.

4.2. *Combinatorial pattern-based formulations*

In this section, we use the combinatorial pattern-based framework proposed by Lejeune (2012a) to reformulate the models **SP2a** and **SP3a**. It has been successfully used to solve probabilistically constrained stochastic programming problems in which the uncertainty is described by a huge (up to 50 000) number of scenarios. For self-containment, purpose we succinctly describe the main components of the method and refer the reader to Lejeune (2012a) and Kogan and Lejeune (2013) for a more thorough description and an illustration of the method.

The following notation is used. Recall that $|T|$ is the number of inequalities in constraint (20). To ease the notation, we introduce the $|T|$-dimensional random vector $\boldsymbol{\zeta}$ whose components, denoted by $\zeta_t,\ t \in T$, are linear combinations of random demand and arc capacities:

$$\zeta_t = \sum_{i\in V_t} \xi_i^d - \sum_{i\in \bar{V}_t, j\in V_t} \xi_{ij}^v,\ t\in T.$$

Moreover, the marginal cumulative probability distribution of ζ_t is designated by F_t. Then, we rewrite the chance constraint (20) in its canonical form with the random variables in the right-hand sides of the inequalities:

$$\mathbb{P}\left(\sum_{i\in V_t} r_i \geq \zeta_t\ ,\ t\in T\right) \geq p\ . \tag{21}$$

Let us also introduce the $|T|$-dimensional deterministic vector $\boldsymbol{\varpi}^s$, $s \in \Omega$, denoting the realization of the random vector $\boldsymbol{\zeta}$ under scenario s:

$$\varpi_t^s = \sum_{i\in V_t} d_i^s - \sum_{i\in \bar{V}_t, j\in V_t} v_{ij}^s,\ \ t\in T, s\in \Omega.$$

The overall objective for the use of the combinatorial method is to come up with an equivalent reformulation of the joint probabilistic constraint (20) that is amenable to an efficient numerical solution. The method involves two main steps. The first one (Section 4.2.1) consists of the derivation of the set of relevant points. In order to derive an equivalent reformulation of the probabilistic constraint, we must identify the minimal conditions required for its satisfaction. The set of relevant points includes all of the points (thus including the minimal ones) that can possibly define sufficient conditions for the probabilistic constraint to hold. The second step (Section 4.2.2) involves the binarization of the probability distribution using a selected set of cut points. The motivation behind the binarization is that it will allow the derivation of two reformulations of the probabilistic constraint. First, it will enable the reformulation of the probabilistic constraint as a partially defined Boolean function (pdBf), which will subsequently lead to a mixed-integer reformulation in which the number of binary variables is typically very low (much smaller than the number of scenarios) and equal to the number of cut points. A third sub-section describes the implementation of the reformulation method for the local probabilistic constraints associated with each region. The different steps of the method are illustrated in Appendix B with an example (A18).

4.2.1. *Construction of a set of relevant points*

We first partition the set Ω of scenarios into two disjoints subsets $\Omega^+ = \{s \in \Omega : P(\boldsymbol{\zeta} \leq \boldsymbol{\varpi}^s) \geq p\}$ and $\Omega^- = \{s \in \Omega : P(\boldsymbol{\zeta} \leq \boldsymbol{\varpi}^s) < p\}$. We refer to the scenarios in Ω^+ and Ω^- as the p-sufficient and p-insufficient scenarios, respectively.

In order to derive sufficient conditions for the chance constraint (20) to be satisfied, we shall consider all the scenarios that can be p-sufficient; i.e., satisfy the basic necessary conditions:

$$F_t\big(\varpi_t^s\big) \geq p,\ t \in T\,. \tag{22}$$

The second step of the pattern-based modeling approach involves the generation of the exhaustive set of points satisfying (22). This is accomplished by constructing the sets

$$C_t = \left\{\varpi_t^s : F_t\big(\varpi_t^s\big) \geq p,\ s \in \Omega\right\},\ \ t \in T,$$

whose Cartesian product $\bar{\Omega} = \prod_{t \in T} C_t$ provides the set $\bar{\Omega}$ of *relevant points* including all of the points that can possibly be p-sufficient. A relevant point $s \in \bar{\Omega}$ is represented by a vector $\boldsymbol{\varpi}^s$ satisfying the $|T|$ conditions defined by (22). The disjoint sets $\bar{\Omega}^+ = \{s \in \bar{\Omega} : F(\boldsymbol{\varpi}^s) \geq p\}$ and $\bar{\Omega}^- = \{s \in \bar{\Omega} : F(\boldsymbol{\varpi}^s) < p\}$ are called the sets of p-sufficient and p-insufficient relevant points (see Table A3 in the Appendix).

4.2.2. *Binarization and reformulation process*

We shall now binarize the probability distribution F of $\boldsymbol{\zeta}$ and use in that regard the concept of cut point (Boros *et al.*, 1997). The binarization process is based on the set of cut points and provides a pdBf that represents the feasibility of the joint chance constraint. The notation $\{0, 1\}^n$ refers to the n-dimensional unit cube.

Definition 1. The binarization process is the mapping $\mathbb{R}^{|T|} \to \{0, 1\}^n$ of a real-valued vector $\boldsymbol{\varpi}^s$ into a binary one β^s in such a way that the value of each component β^s_{jt} is defined with respect to a cut point c_{jt} as follows:

$$\beta^s_{jt} = \begin{cases} 1 & \text{if } \varpi_t^s \geq c_{jt} \\ 0 & \text{otherwise} \end{cases},$$

where c_{jt} denotes the jth cut point associated with component ξ_t,

$$j' < j \ \Rightarrow\ c_{j't} < c_{jt}\ ,\ \ t \in T,\ \ j = 1, \ldots, n_t, \tag{23}$$

and $n = \sum_{t \in T} n_t$ is the sum of the number n_t of cut points for each component ξ_t.

The binarization process defines the binary projection $\bar{\Omega}_B \subseteq \{0, 1\}^n$ of $\bar{\Omega}$. The set of relevant Boolean points is denoted by $\bar{\Omega}_B = \bar{\Omega}^+_B \bigcup \bar{\Omega}^-_B$, and $\bar{\Omega}^+_B$ (resp., $\bar{\Omega}^-_B$) is the set of p-sufficient (resp., insufficient) relevant Boolean points. Evidently, the cut points are parameters whose values must not be defined arbitrarily. In order to identify the conditions that are necessary for (21) to hold, we need to use a consistent set of cut points (Boros *et al.*, 1997) such that the resulting binarization process (23) preserves the disjointedness between the binary projections $\bar{\Omega}^+_B$ and $\bar{\Omega}^-_B$ of $\bar{\Omega}^+$ and $\bar{\Omega}^-$, respectively. This is achieved by carrying out the binarization process by using the sufficient-equivalent set of cut points (Lejeune 2012a; 2012b; Kogan and Lejeune 2013):

$$C^e = \bigcup_{t \in T} C_t\,.$$

The binarization process of the probability distribution with a consistent set of cut points permits the representation of the combination (F, p) of the probability distribution F of $\boldsymbol{\zeta}$ and of the prescribed probability level p as a pdBf defined by the pair of disjoint sets $\Omega^+_B, \Omega^-_B \subseteq \{0, 1\}^n$ (see Table A3 in the Appendix). The pdBf is a mapping $g : (\Omega^+_B \bigcup \Omega^-_B) \to \{0, 1\}$ such that $g(s) = 1$ if $s \in \Omega^+_B$ and $g(s) = 0$ if $s \in \Omega^-_B$. The domain of the mapping g is a cube $\{0, 1\}^n$ of dimension n equal to the number of cut points. As demonstrated by Lejeune (2012a), this in turn allows the derivation of a deterministic MIP reformulation **SP2_IP** equivalent to **SP2a** and containing $n = \sum_{t \in T} n_t$ binary variables.

Theorem 2. *Let ξ be a $|T|$-variate random variable and c_{jt}, $t \in T$, $j = 1, \ldots, n_t$, be the cut points in the sufficient-equivalent set. The MIP problem* **SP2_IP**

$$\textbf{SP2\_IP} \quad \min \sum_{i\in I}\sum_{l\in L} F_{il}\, y_{il} + \sum_{i\in I} q r_i$$

$$\text{s.t. } (2), (3), (4), (5),$$

$$\sum_{t\in T}\sum_{j=1}^{n_t} \beta^s_{jt}\, u_{jt} \le |T| - 1, \qquad s \in \bar{\Omega}^-_B \tag{24}$$

$$\sum_{i\in V_t} r_i \ge \sum_{j=1}^{n_t} c_{jt}\, u_{jt}, \qquad t \in T \tag{25}$$

$$\sum_{j=1}^{n_t} u_{jt} = 1, \qquad t \in T \tag{26}$$

$$u_{jt} \in \{0,1\}, \qquad t \in T, j = 1,\ldots,n_t \tag{27}$$

is a deterministic equivalent formulation of the probabilistically constrained model **SP2a**.

Constraints (24), together with constraints (26), ensure that the binary vector **u** does not coincide with the binary image β^s of any of the p-insufficient relevant points. By construction of the set of relevant points, (24) and (26) also guarantee **u** to be such that $\sum_{j=1}^{n_t} c_{jt}\ u_{jt} = c_{jt'}\ \beta^s_{jt'}, j \in J$, for at least one of the p-sufficient relevant points $s \in \bar{\Omega}^+_B$ with $t' = \max\{t\ : \beta^s_{jt} = 1\}$. It follows that the terms $\sum_{j=1}^{n_t} c_{jt}\ u_{jt}, j \in J$, in the right-hand side of constraint (25) define the sufficient conditions for the satisfiability of the probabilistic constraint (21). Then, constraints (25) guarantee that the pre-positioned commodities $\sum_{i\in V_t} r_i$ are sufficiently large to satisfy constraints (21). Constraints (26) ensure that exactly one variable u_{jt} taking value one is associated with each component ζ_t of the random vector. Thus, each component of the random vector is accounted for in the inequalities defining the conditions required for (21) to hold. Constraints (27) define the binary character of the variables u_{jt} (see Table A3 in the Appendix).

4.2.3. *Reformulation of Local Chance Constraints*

We shall now present the MIP formulation of the model **SP3a** including a local probabilistic constraint for each region k, $k = 1,\ldots,K$. Let $|T^k|$ be the number of non-redundant Gale–Hoffman inequalities associated with region k. The components of the $|T^k|$-dimensional random vector $\boldsymbol{\zeta}^k$, $k = 1,\ldots,K$, are

$$\zeta^k_t = \sum_{i\in V_t} \xi^d_i - \sum_{i\in \bar{V}^k_t, j\in V_t} \xi^v_{ij}, t \in T^k.$$

Thus, the local chance constraint associated with region k reads:

$$\mathbb{P}\left(\sum_{i\in V_t} r_i \ge \zeta^k_t\ ,\ t \in T^k\right) \ge p'. \tag{28}$$

As done for model **SP2a**, we also define the $|T^k|$-dimensional deterministic vector $\boldsymbol{\varpi}^{s,k}, s \in \Omega,\ k = 1,\ldots,K$, denoting the realization of the random variable $\boldsymbol{\zeta}^k$ under scenario s:

$$\varpi^{s,k}_t = \sum_{i\in V_t} d^s_i - \sum_{i\in \bar{V}^k_t, j\in V_t} v^s_{ij},\ t \in T^k, s \in \Omega. \tag{29}$$

Let $c^k_{jt}, t \in T^k,\ j = 1,\ldots,n^k_t$, be the sufficient-equivalent cut points associated with region k, the number of which is equal to $n^k = \sum_{t\in T^k} n^k_t$. Additionally, $\beta^{s,k}_{jt}$ is the binary mapping of $\varpi^{s,k}_t$ using the cut points $c^k_{jt},\ t \in T^k,\ j = 1,\ldots,n^k_t$, and $u^k_{jt}, t \in T^k,\ j = 1,\ldots,n^k_t$, are the binary variables corresponding to the cut points c^k_{jt}. The notation $\bar{\Omega}^-_{B^k}$ refers to the set of relevant p'-insufficient scenarios for region k. The deterministic formulation of **SP3a** with local probabilistic constraints, referred to as **SP3_IP**, is given by

$$\textbf{SP3\_IP} \quad \min \sum_{i\in I}\sum_{l\in L} F_{il}\, y_{il} + \sum_{i\in I} q r_i$$

$$\text{s.t.} \quad (2), (3), (4), (5), (24), (25), (26), (27),$$

$$\sum_{t\in T^k}\sum_{j=1}^{n^k_t} \beta^{s,k}_{jt}\, u^k_{jt} \le |T^k| - 1, \qquad k = 1,\ldots,K,\ s \in \bar{\Omega}^-_{B^k}$$

$$\sum_{i\in V_t} r_i \ge \sum_{j=1}^{n^k_t} c^k_{jt}\, u^k_{jt}, \qquad k = 1,\ldots,K,\ t \in T^k$$

$$\sum_{j=1}^{n^k_t} u^k_{jt} = 1, \qquad k = 1,\ldots,K,\ t \in T^k$$

$$u^k_{jt} \in \{0,1\}, \qquad k = 1,\ldots,K,\ t \in T^k, j = 1,\ldots,n^k_t.$$

The optimal solution of **SP2_IP** (resp., **SP3_IP**) is equivalent to that of the probabilistic programming model **SP2** (resp., **SP3**).

A critical feature of the combinatorial pattern method is that the number of binary variables in the MIP reformulations **SP2_IP** and **SP3_IP** is not an increasing function of the number of scenarios. It contains a significantly lower number of binary variables, which is equal to the number of cut points (Definition 1) used for the binarization process (n for **SP2_IP** and $n + \sum_{k=1}^{K} n^k$ for **SP3_IP**). This is what makes it applicable to problems in which uncertainty is described with a large number of scenarios.

5. Computational study and modeling insights

This section is decomposed into two main parts. In the first subsection, we conduct a case study and sensitivity analysis that provide insights about the impact of the main features of the proposed models. The case study focuses on designing a pre-disaster relief network for responding to hurricane disasters in the Southeast part of the United

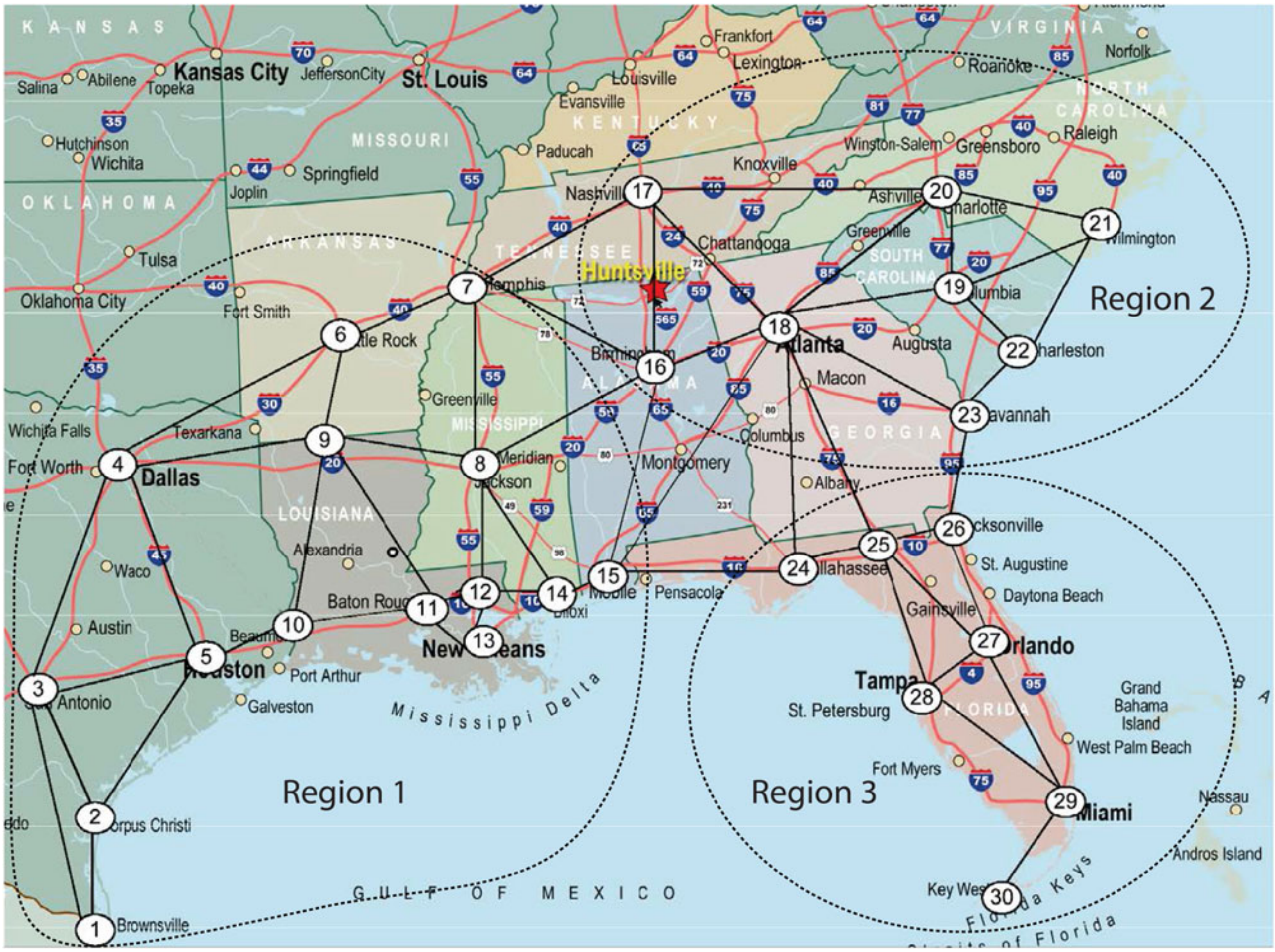

Fig. 1. Southeastern U.S. region facing hurricane risk.

States. The description of the structure of the network was first presented in Rawls and Turnquist (2010). The second subsection assesses the computational effectiveness of the proposed solution method. We show that it allows to find the optimal location-allocation policies when the uncertainty is characterized with a large number of scenarios.

The preprocessing algorithm used for eliminating the redundant Gale–Hoffman inequalities and the binarization process employed for deriving the proposed MIP formulations are implemented in MATLAB. AMPL and CPLEX 12.3 are used to formulate and solve the mathematical programming problems. Each problem instance is solved on a 64-bit Dell Precision T5400 Workstation with Quad Core Xeon Processor X5460 3.16 GHz CPU and 4X2GB of RAM.

5.1. *Case study—hurricane threat in the U.S. Southeast region and sensitivity analysis*

The case study concerns the PRNDP for the threat of hurricanes in the Southeastern U.S. region, which is represented by a graph including 30 nodes and 55 links (see Fig. 1). The case study focuses on satisfying the demand for water. Each node represents a demand point and facilities of three different sizes can be located at any node in the network. The uncertainty is described using 51 scenarios (and the corresponding probabilities) constructed by Rawls and Turnquist (2010). Each scenario represents a joint realization of random demand and arc capacities. We use the parameter values as described by Rawls and Turnquist (2010). The demand for water is expressed in units of 1000 gallons, and the unit acquisition price is \$647.7. Up to 252 (resp., 2823 and 5394) units of water can be stored in a small (resp., medium and large) facility. The setup cost for opening a facility of small (resp., medium and large) size is \$19 600 (resp., \$188 400 and \$300 000). The unit shortage penalty cost and the unit holding cost for unused material are assumed to be 10 times the purchasing cost and 25% of the purchase price, respectively. We refer the reader to Rawls and Turnquist (2010) for a more detailed description of the structure of the network.

As discussed in Section 3.3, we introduce one local probabilistic constraint for each region. We have decomposed the U.S. Southeast area into three disjoint regions as illustrated in Fig. 1:

- Region 1: Brownsville, Corpus Christi, San Antonio, Dallas, Houston, Little Rock, Memphis, Biloxi, Jackson, Monroe, Lake Charles, Baton Rouge, Hammond, New Orleans, Mobile;
- Region 2: Birmingham, Nashville, Atlanta, Savannah, Columbia,Charlotte,Wilmington,Charleston;
- Region 3: Tallahassee, Lake City, Jacksonville, Orlando, Tampa, Miami, Key West.

5.1.1. *Benchmarking*

We benchmark our results with those obtained by solving the model proposed in Rawls and Turnquist (2010) under the assumption that all the stocked supplies remain usable after a disaster occurs. The benchmark model is thereafter referred to as **RTM**. It is in the form of the two-stage model (1)–(9), where the objective function (6) includes the shortage, transportation, and holding costs as follows:

$$\sum_{i\in I} h w_i^s + \sum_{(i,j)\in A} c_{ij} x_{ij}^s + \sum_{i\in I} \tau a_i^s.$$

Here, h, c_{ij}, and τ denote the unit shortage cost, the unit transportation cost on arc (i, j), and the unit holding cost, respectively. In the base problem instance of **RTM**, referred to as the Base Problem, the parameters take the values described in the previous subsection, and $c_{ij} = 0.3$ dollars per unit-mile. Solving the large-scale MIP formulation of the Base Problem provides an optimal solution at which a total of 8973 units of water is pre-positioned in two small facilities located at Charlotte and Charleston and three medium-sized facilities located at Wilmington, Tallahassee, and Miami. The total facility setup and acquisition costs amount to \$6416 210. Moreover, the optimal solution of **RTM** leads to the occurrence of demand shortage with a probability of 14.49%. Table A7 (column 6 with $h = \$6477$) provides an in-depth description of the optimal solution of the Base Problem. The probability of demand shortage associated with a location-allocation policy is basically computed as

$$\sum_{s\in\Omega} \left\{ \pi^s : \max_{i\in N} \; w_i^{s*} > 0 \right\} \stackrel{\text{def}}{=} 1-\alpha, \qquad (30)$$

where w_i^{s*} is the amount of demand shortage for node i under scenario s at the specified solution. Clearly, $\max_{i\in N} w_i^{s*}$ taking a strictly positive value implies that a demand shortage occurs for at least one of the nodes at the network under scenario s, and α denotes the achieved region-wide reliability level. Similarly, the achieved region-wide reliability for region k associated with an optimal location-allocation policy represented by $(\mathbf{r}^*, \mathbf{y}^*)$ is given by

$$\alpha_k' \stackrel{\text{def}}{=} 1 - \sum_{s\in\Omega} \left\{ \pi^s : \max_{t\in T^k} \; \varpi_t^{s,k} - \sum_{i\in V_t} r_i^* \; > 0 \right\}. \qquad (31)$$

Observe that by (28) and (29), $\varpi_t^{s,k} - \sum_{i\in V_t} r_i^*$ indicates by how much the Gale–Hoffman inequality corresponding to the set of nodes V_t, $t \in T^k$, is violated at the optimal solution (r^*, y^*) under scenario s. Thus, while calculating α_k' we consider the scenarios under which at least one of the Gale–Hoffman inequalities associated with region k is violated; i.e., a demand shortage occurs within the region.

Different from the benchmark model **RTM**, our models do not explicitly take into consideration the operational costs of the second-stage problem while determining the location-allocation policies. We here describe how we calculate the operational costs associated with our solutions in order to compare different models. In practice, when a disaster happens, we can make the post-disaster decisions on distributing the relief supplies by solving the second-stage problem under the realized uncertain parameters and the location-allocation decisions determined *a priori* by our models. The objective function of the second-stage problem could then include the operational costs. Following this approach, we evaluate our optimal policies in terms of the operational costs. This requires the solution of the second-stage problem of **RTM** under each scenario, where the first-stage decisions are set equal to those at an optimal solution of our models. The solutions of the second-stage problems and the resulting expected operational costs can be obtained all together by solving a large-scale model which we call **RTM2**. Basically, **RTM2** is the large-scale linear programming formulation of **RTM** with fixed first-stage decisions (obtained by solving **SP2_IP** or **SP3_IP**).

5.1.2. *Model **SP2_IP***

As mentioned in Section 4.1, the elimination of the redundant Gale–Hoffman inequalities is computationally challenging. Therefore, we consider an aggregated version of the network representing the Southeastern U.S. region. Recall that the original network includes 30 nodes and 55 links. As in Klibi and Martel (2010), we use the proximity criterion to construct an aggregated network comprising 16 nodes. The aggregated network is only used to eliminate the redundant Gale–Hoffman inequalities, and to formulate the network-wide probabilistic constraint. We mention that while expressing the Gale–Hoffman inequalities for the aggregated network, we sum the capacities of the original arcs and the net demands of the original nodes for the groups of the nodes that are aggregated. Thus, we directly incorporate the randomness in the capacities of the original arcs connecting the nodes in the aggregated network. However, we do not consider the arcs within a group of aggregated original nodes and consequently, ignore the capacity issues within a group. Therefore, one should pay attention to the distances within the group while constructing the aggregated network. On the other hand, the region-wide probabilistic constraints are formulated based on the original network. Thus, we solve the PRNDP for the original 30-node network while the specified

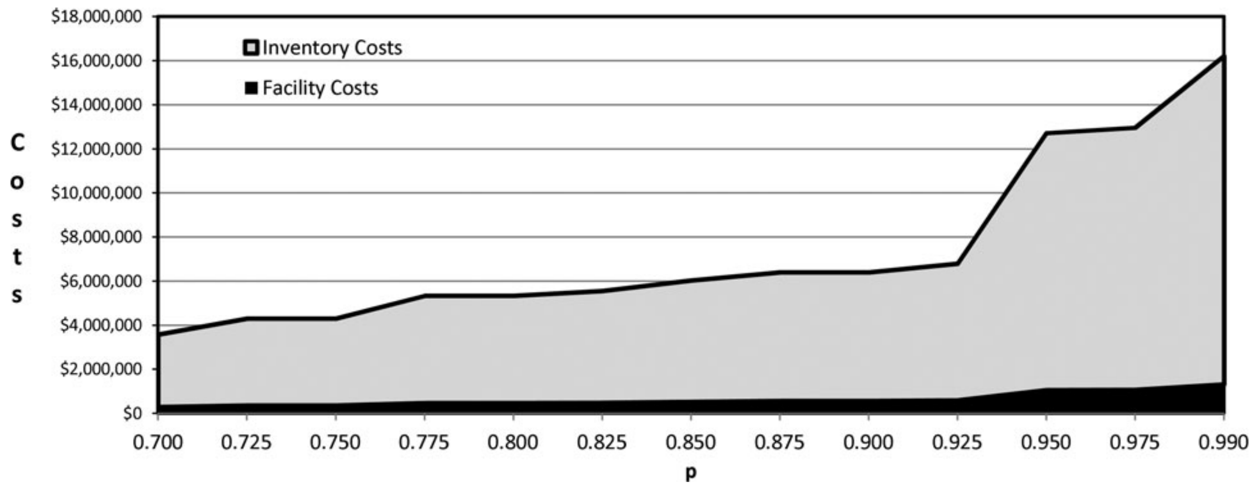

Fig. 2. Total cost, acquisition, and facility setup costs versus network-wide reliability level p.

network-wide reliability is ensured at the aggregated network level.

We solve 13 instances of **SP2_IP**, which are constructed by successively setting the parameter p to values between 0.7 and 0.975 in increments of 0.025, in addition to $p = 0.99$. We then conduct a sensitivity analysis to investigate how the location-allocation decisions, the associated costs, and the network and region-wide reliability levels vary with the risk parameter p. In what follows, we use excerpts from Table A4 to present a detailed analysis of the optimal solutions to the 13 instances of **SP2_IP**.

First, we focus on how the total cost, defined as the sum of the acquisition (light-grey shaded area in Fig. 2) and the facility setup costs (black area in Fig. 2), evolves with the parameter p. The total facility setup cost accounts for roughly 8% of the total cost for any considered level of reliability. Figure 2 shows that each type of cost is non-decreasing with respect to the enforced reliability level, and the rate at which the total cost increases with the network-wide reliability level is relatively smooth for p ranging between 0.7 and 0.925. In contrast, the total cost increases by 87% and 25% as we change the value of p from 0.925 to 0.95 and from 0.975 to 0.99, respectively. Moreover, Fig. 3 displays the total inventory level for different network-wide reliability levels. It is clear that the total inventory level and the cost associated with a more reliable network design (more risk-averse policy) are in general higher. The network-wide inventory level increases at a quasilinear rhythm for $p \leq 0.925$ and increases sharply for larger reliability levels. We note here that above a sufficiently large reliability level the corresponding probabilistic constraint becomes too demanding and leads to significantly more conservative solutions. However, the exact threshold above which the reliability level leads to such significant differences is instance-dependent.

We shall now compare the optimal location-allocation policies provided by **SP2_IP** for different values of p with the optimal policy obtained by solving the base problem instance of **RTM** (Base Problem). The optimal solution of the Base Problem does not involve opening any large facility, whereas the optimal solution of **SP2_IP** requires the opening of at least one large facility for all the considered values of p. At the optimal solution of the Base Problem, the facility setup and acquisition costs amount to \$6416 210, and the probability of satisfying the demand across the network is 0.8551. The optimal solution of **SP2_IP** with $p = 0.8551$ shows that it is possible to design a network with a reliability reaching 0.8682 at a lower total cost of \$6 321 510. This network design policy suggests to open one medium-sized facility in Corpus Christi, three small facilities in Nashville, Atlanta, and Wilmington, and one large facility in Tampa. The decrease in the total cost is achieved by savings in both the facility setup and acquisition costs; the decrease in the facility setup cost by 9.46% is more noticeable. However, since **SP2_IP** minimizes the facility setup and acquisition costs and does not explicitly factor in the second-stage operational costs, these latter costs might be higher than those of **RTM**. The following results provide some indication about the relative increase in the operational costs. At the optimal solution of **RTM**, the transportation, holding, and shortage costs amount to \$412 275, \$808 372, and \$4 641 900, respectively. On the other hand, these costs at the optimal solution of **SP2_IP** are equal to \$501 702, \$801 723, and \$4750 953, respectively, when the enforced global reliability level p is set to 0.8551. If we add the facility setup and acquisition costs (\$6 321 510) accounted for by **SP2_IP** to the second-stage operational costs the total costs reach \$12 375 888, while the total costs for **RTM** are equal to \$12 278 757. Thus, **SP2_IP** results in a 1.48% relative decrease in the facility setup and acquisition costs, whereas the relative increase

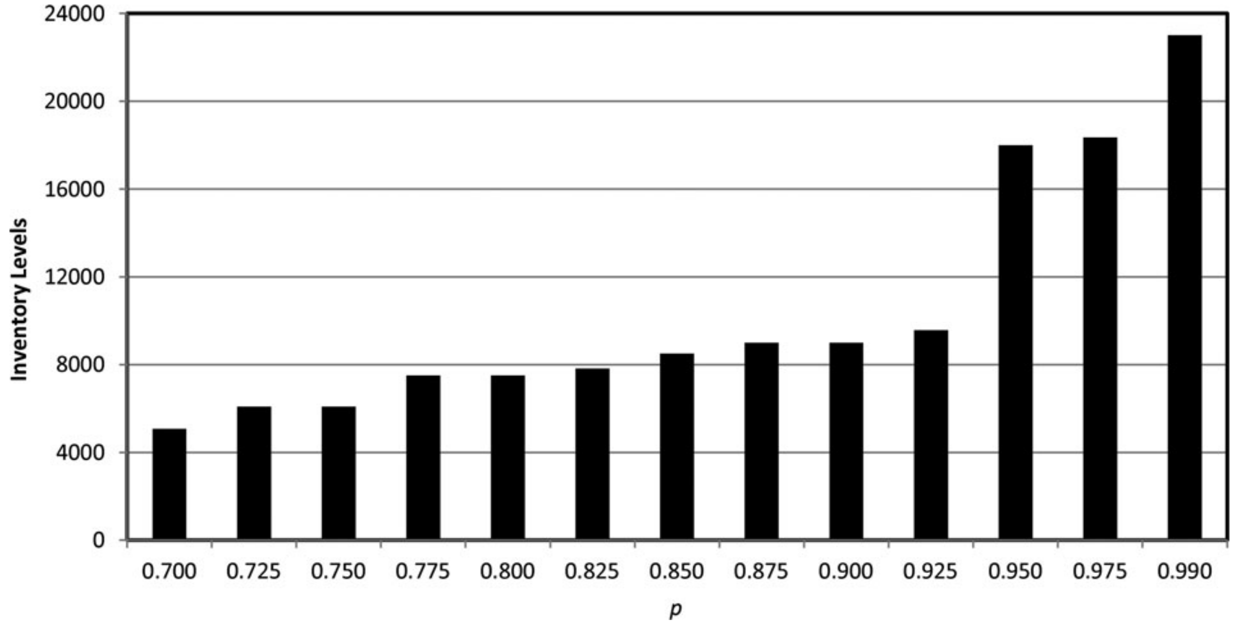

Fig. 3. Inventory level versus network-wide reliability level p.

in the expected total second-stage cost and the overall total cost with respect to **RTM** is only 3.27% and 0.79%, respectively.

We finally utilize the model **RTM2** to evaluate our optimal policies in terms of the operational costs. This approach basically requires the solution of the second-stage problem of **RTM** under each scenario, where the first-stage decisions are set equal to those at the optimal solution ($\mathbf{r}^*$, $\mathbf{y}^*$) of **SP2_IP**. The expected value of the resulting second-stage transportation, shortage, and holding costs are displayed in Table A4. The holding costs increase as the network-wide reliability level increases. Such an increase in the holding costs is expected, since the proposed models focus only on satisfying the demand. However, the transportation and, most interesting, the shortage costs do not change monotonically with respect to p. For example, the expected shortage costs decrease as p increases in the range between 0.7 and 0.95 but counterintuitively they increase beyond that range.

5.1.3. *Model **SP3_IP***

We shall now analyze the effect of adding a local reliability constraint for each region. Based on (31) we calculate the region-wide reliability level for each region achieved by an optimal solution of **SP2_IP**. Table A4 reports these region-wide reliability levels for various values of the network-wide reliability level p. It appears clearly that satisfying the network-wide reliability constraint does not guarantee that the local probabilistic constraints are also satisfied. Thus, it is not guaranteed that the demand at a node is satisfied by the relief supplies pre-positioned within the same region with a specified high probability. For example, at the optimal solution of **SP2_IP**, the region-wide reliability level for region 2 (α_2') is only 0.32 (resp., 0.47) when the network-wide reliability parameter p is 0.75 (resp., 0.99). Whereas, when we solve **SP3_IP** enforcing the local probabilistic constraints for $p' = 0.60$, the optimal policy leads to a region-wide reliability level of 0.909 (resp., 0.915) for region 2 when p is 0.75 (resp., 0.99). These observations emphasize the contribution of incorporating the local probabilistic constraints into **SP2_IP**. Table A5 (resp. Table A6) provides the detailed solutions of **SP3_IP**, where the region-wide reliability is enforced to be at least 0.60 (resp., 0.7) at each region while p ranges from 0.7 to 0.99.

Let z_p^* be the optimal objective function value of **SP2_IP** enforcing the network-wide reliability level of p. Further, let $z_{p,p'}^*$ denote the optimal objective value of **SP3_IP** enforcing the network-wide reliability of p and the region-wide reliability level of p' for each region. The values of the ratio $z_{p,p'}^*/z_p^*$ showing the relative cost increases due to the incorporation of the local probabilistic constraints are reported in Table 1.

As shown by Table 1, **SP3_IP** provides network design policies under which the regions are self-sufficient (i.e., in terms of fulfilling their own commodity needs with probability of at least 0.6 or 0.7) without incurring almost any additional costs when the network-wide reliability is reasonably large ($p \geq 0.725$). For instance, the total cost at the optimal solution of **SP3_IP** with $p = 0.975$ and $p' = 0.70$ is the same as the one at the optimal solution of **SP2_IP** with $p = 0.975$, for which the achieved region-wide reliability is 0.47 for region 2. The impact of the local

Table 1. Impact of incorporating local probabilistic constraints on total cost

	p												
	0.7	*0.725*	*0.75*	*0.775*	*0.8*	*0.825*	*0.85*	*0.875*	*0.9*	*0.925*	*0.95*	*0.975*	*0.99*
$\frac{z^*_{p,0.6}}{z^*_p}$	1.011	1	1	1	1	1.001	1	1	1	1	1	1	1
$\frac{z^*_{p,0.7}}{z^*_p}$	1.207	1.005	1.005	1	1	1.001	1.032	1	1	1	1	1	1

constraints on the total cost diminishes as p increases; i.e., the global probabilistic constraint becomes more restrictive. Figure 4 displays the optimal inventory level at each region determined by solving **SP3_IP** when the region-wide reliability level is 0.7.

Here we briefly discuss some comparative results for **SP3_IP** and **RTM**. At the optimal solution of the Base Problem, the achieved network-wide reliability is 0.8551 and the achieved region-wide reliability levels for regions 1, 2, and 3 are 0.5722, 0.8098, and 0.7848, respectively. However, the solution of **SP3_IP** with the required reliability levels $p = 0.8551$, $p'_1 = 0.5722$, $p'_2 = 0.8098$, and $p'_3 = 0.7848$ indicates that a network with reliability levels exceeding the required ones can be constructed at a lower total cost of \$6321 510. We recall that the facility setup and acquisition costs associated with the optimal solution of the Base Problem reach \$6416 210. However, **RTM** achieves a smaller expected total second-stage cost of \$5862 547 (transportation cost = \$412 275, holding cost = \$808 372, and shortage cost = \$4641 900). The expected total second-stage cost and the overall total cost amount to \$6077 637 (transportation cost = \$524 961, holding cost = \$801 723, and shortage cost = \$4750 953) and \$12 399 147 for **SP3_IP** with the above specified reliability levels. Thus, **SP3_IP** results in a 1.48% relative decrease in the facility setup and acquisition costs, while the relative increase in the expected total second-stage cost and the overall total cost with respect to **RTM** is only 3.67% and 0.98%, respectively.

As above for **SP2_IP**, we fix the first-stage decisions of **RTM** equal to those at the optimal solution $(\boldsymbol{r}^*, \boldsymbol{y}^*)$ of **SP3_IP** and solve the model **RTM2**. The expected values of the resulting second-stage transportation, shortage, and holding costs are displayed in Tables A5 and A6 for $p' = 0.6$ and $p' = 0.7$, respectively. Similar to **SP2_IP**, it can be seen that the resulting holding costs in general increase and the expected second-stage transportation and shortage costs do not change monotonically with respect to the risk parameter p. When the required network-wide reliability level p is larger than or equal to 0.875, the solution of **SP3_IP** is the same regardless of whether the required local reliability levels are equal to 0.6 or 0.7. The same comment applies when p is equal to 0.775, 0.8, and 0.825. A larger local reliability level $p' = 0.7$ (instead of 0.6) triggers a cost increase of 0.3% (resp., 0.4, 0.4, and 19%) when the network-wide reliability level is equal to 0.85 (resp., 0.75, 0.725, and 0.70). Clearly, the effect of enforcing a higher local reliability level becomes more visible as p decreases.

Considering **RTM** as the benchmark, not explicitly accounting for the operational costs does not lead to a significant increase in the total expected second-stage costs. Moreover, **SP3_IP** allows us to incorporate the equity and responsiveness criteria in addition to the network reliability.

5.1.4. *Alternate approach based on penalizing the amount of demand shortage*

One can argue that penalizing the demand shortage amounts and enforcing an upper bound on the probability of a demand shortage are alternate ways of controlling the existence of a feasible flow to satisfy the demand. Thus, **RTM** and our proposed models involve alternate stockout measures. We analyze the policies obtained with **RTM** and our proposed models by varying the shortage penalty cost in **RTM** and the reliability levels in our models. We solve **RTM** for various values of the unit shortage cost, which is defined as $h = \lambda \times 6477$ with $\lambda = 1$ in the Base Problem. We construct 11 problem instances by setting the coefficient λ to values between 0.5 and 1.5 in increments of 0.1. Table A7 presents the detailed results for the optimal solutions of **RTM** and Fig. 5 displays the inventory level in each region for different values of the unit shortage cost. It is significant to observe that **RTM** provides policies in which no relief supplies are pre-positioned within region 1 for $h \geq \$6477$. This is due to the fact that, in the constructed set of scenarios, the demand in region 1 is in general smaller than that in the other regions. However, not pre-positioning relief supplies within region 1 can be an issue to respond quickly, in particular if a disaster affects the western part of region 1 (see Fig. 1). In that respect, our policies, which require the pre-positioning of commodities in region 1, may perform better in terms of responsiveness.

Recall that **RTM** does not impose any explicit constraint on the occurrence of a demand shortage. At the optimal solution of **RTM**, the achieved network-wide reliability ranges from 0.7685 (for $h = \$3886$) to 0.9148 (for h between \$7125 and \$9716). Next, given the optimal first-stage decisions of **RTM**, which define the number, location, and size of the facilities, and the allocation of commodities, we compute the largest network-wide reliability level that can

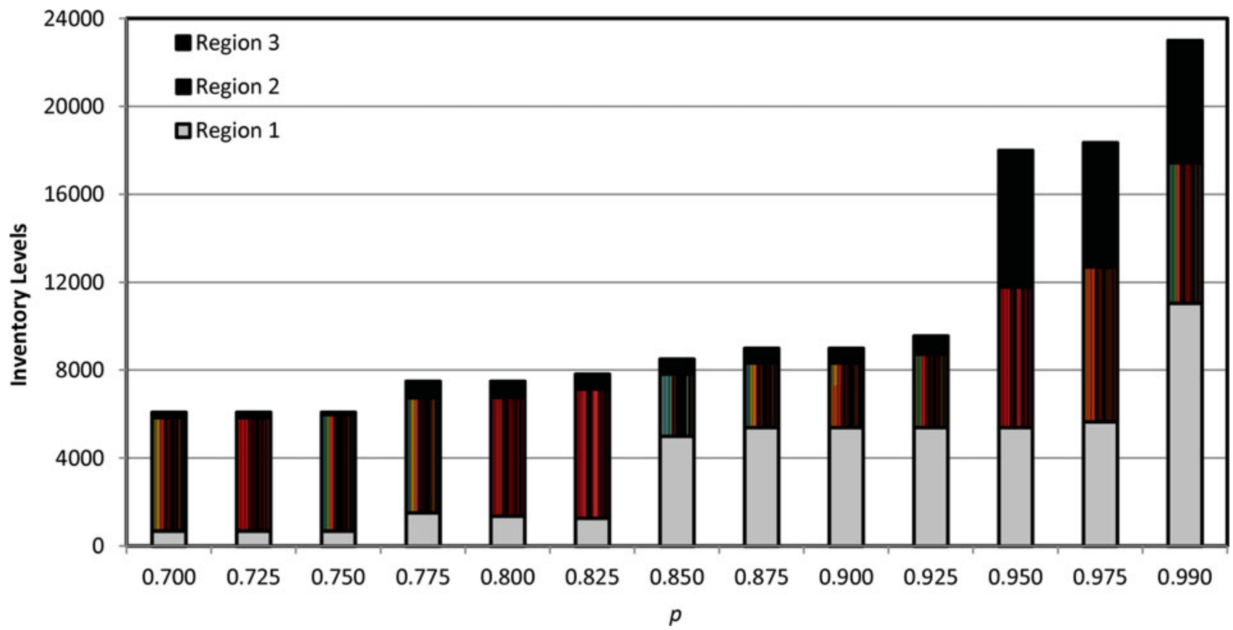

Fig. 4. Inventory level in each region versus the network-wide reliability level p when $p' = 0.7$.

be achieved. This is done by solving an MIP problem in which the network-wide reliability is maximized. The decision variables are the transportation, shortage, and surplus variables, while the inventory pre-positioning and facility location variables are fixed. The largest (resp., smallest) value of the highest possible network-wide reliability level is 0.9148 (resp., 0.7741) and obtained when h takes the values \$7125, \$7772, \$8420, \$9068, and \$9716 (resp., \$3886). These results show that a very high network-wide reliability level is not achieved with **RTM** even if the unit shortage cost is increased by 50% (from \$6477 to \$9716). Thus, it can be challenging to implement a (very) high-reliability policy with the **RTM** model. When the reliability levels less than 0.95 (e.g., 0.9148 as mentioned above) are targeted, **RTM** and our models **SP2_IP** and **SP3_IP** produce policies that perform closely in terms of the network-wide

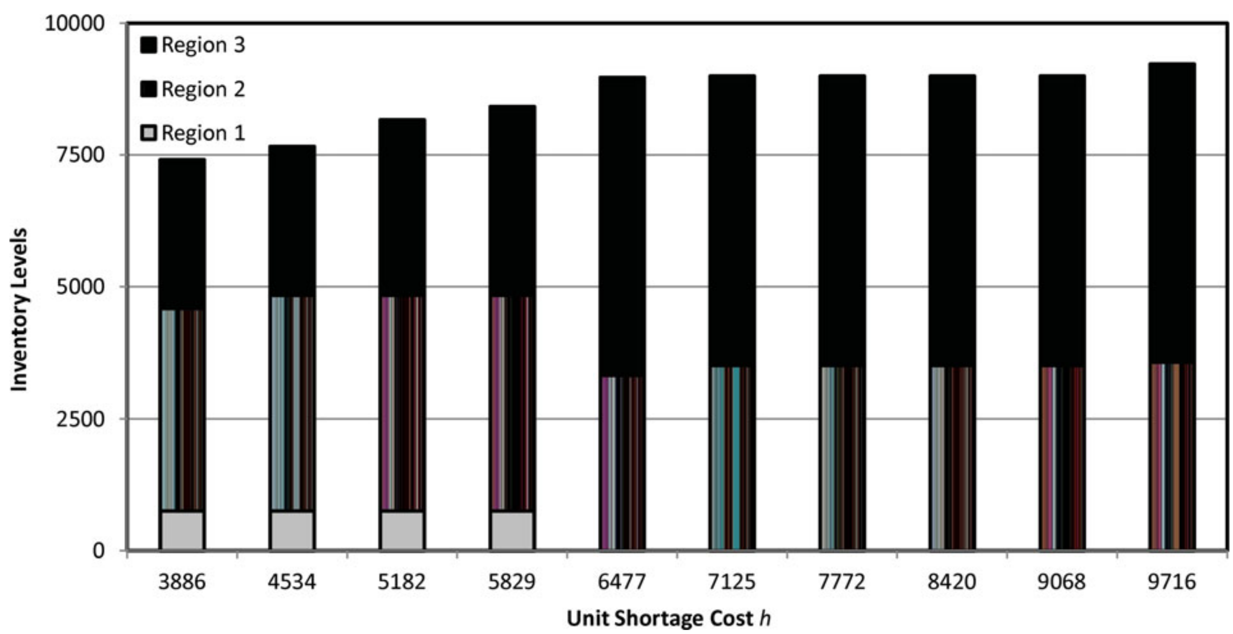

Fig. 5. Model **RTM**: inventory level in each region versus unit shortage cost parameter h.

reliability. However, our modeling approach might be preferable for practical purposes, as it avoids the difficult problem of estimating the cost of unsatisfied demand (required by **RTM**). Moreover, **SP3_IP** allows the enforcement of high levels of region-wide reliability.

5.2. *Scalability of the solution method*

In this section, we study the scalability of the modeling approach. First, we assess the reduction in the number of Gale–Hoffman inequalities achieved by using the elimination process. Second, we analyze the computational efficiency of the proposed combinatorial reformulation method used to solve the PRNDP models. The scalability study is based on the computational time needed to solve the MIP formulations **SP2_IP** and **SP3_IP** to optimality and on the striking contrast between the (large) number of considered scenarios and the (small) number of binary variables in the MIP formulations.

The computational study is based on the hurricane network discussed in Section 5.1. Several network configurations detailed in the next subsections are generated for the PRNDP problem and used to conduct the computational analysis.

5.2.1. *Elimination of Gale–Hoffman inequalities*

We consider four configuration variants of the hurricane network in the Southeastern U.S. region. The configurations include 16 aggregated nodes and 29 links and differ in terms of the upper (v^u_{ij}) and lower (v^l_{ij}) bounds of the capacity of each arc (i, j). For each network configuration, we derive the non-redundant Gale–Hoffman inequalities using the elimination process described in Section 4.1. The first two columns of Table 2 define the network configuration. Column 4 displays the total number ($2^{|I|} - 1$) of Gale–Hoffman inequalities, whereas columns 5 to 8 report the number S_i of Gale–Hoffman inequalities eliminated at each stage i of the elimination process. The notation $|T|$ (column 9) denotes the number of non-redundant Gale–Hoffman inequalities; i.e., those that have not been eliminated through the elimination process. The last column indicates the CPU time in seconds to carry out the elimination process. For each network configuration, we report the results for the entire network and for each region. Note that the number of Gale–Hoffman inequalities is invariant to the set of scenarios.

Table 2 shows that the first stage in the elimination process removes more than 85% of the 65 535 Gale–Hoffman inequalities describing the conditions for the existence of a feasible flow at the entire network. At the end of the elimination process, more than 99.97% of the Gale–Hoffman inequalities have been discarded. Note that the computational time rapidly increases with the number of nodes in the network. Although the elimination process is completed in less than two minutes for region 1 that includes 15 nodes (see Fig. 1), the process takes almost 105 minutes for the aggregated network that comprises 16 nodes.

5.2.2. *Scalability and computational efficiency of combinatorial reformulation method*

To evaluate the scalability of our modeling approach, we consider a set of randomly generated problem instances with a large number of scenarios. We propose a scenario generation method that takes into account the dependency structures inherent in disaster relief networks and utilize it to construct 80 problem instances involving up to 20 000 scenarios. This scenario generation method, presented in Appendix C, has minimal data requirements, relying for the most part on publicly available geographical and population data. For each network configuration of the hurricane network, we consider 20 problem instances that differ in terms of (i) the network-wide reliability level p (0.85, 0.9, 0.95, 0.975, 0.99); (ii) the number $|\Omega|$ of scenarios (5000, 20 000), and (iii) the inclusion of local reliability constraints. For each instance, all the scenarios are assumed to be equally likely.

Table 3 reports the solution times and the number of binary variables n and $n' = n + \sum_{k=1}^{3} n^k$ included in **SP2_IP** and **SP3_IP**, respectively. It can be seen that the number of integer variables increases as the enforced reliability level decreases. Even for low values taken by p (i.e., 0.85) and p' (i.e., 0.70), the number of integer variables remains low. We also observe that the number of binary variables is significantly smaller than the number of scenarios used to represent uncertainty. Indeed, when 5000 (resp., 20 000) scenarios are considered, the maximum number of binary variables is 42 (resp., 46) in **SP2_IP** and 150 (resp., 164) in **SP3_IP**. This also indicates that the number of binary variables is about the same regardless of the number (5000, 20 000) of scenarios. This comment applies to both **SP2_IP** and **SP3_IP** and highlights the scalability of the method, as well as its applicability to problems in which uncertainty is finely represented. This is confirmed by the computational times needed to solve the problem instances to optimality. Reformulating each model with the Boolean framework takes less than 1.5 seconds and the solution time for the reformulated model is less than two seconds for any of the 80 problem instances.

As a benchmark, we attempted to solve the MIP reformulation **SP1a**, equivalent to **SP2_IP**, for the problem instance with $v^u_{ij} = 12\,000$, $v^l_{ij} = 6000$, $p = 0.9$, and $|\Omega| = 20\,000$. While **SP2_IP** includes 135 binary variables (90 variables y_{il} indicating whether a facility of size l is located at node i and 45 variables u_{jt} used for the binarization process), **SP1a** contains 20 090 binary variables. Moreover, it includes a very large number of continuous decision variables and constraints that are not needed in **SP2_IP**. It is thus not surprising that **SP1a** for the above-described instance could not be solved within two hours of computing time. This is a striking contrast with the formulation **SP2_IP** that can

Table 2. Gale–Hoffman inequalities for the hurricane network in the U.S. southeastern region

Network configurations			*Gale–Hoffman inequalities: elimination process*						
v^u_{ij}	v^l_{ij}	*Reliability for*	$2^{\|I\|}-1$	S_1	S_2	S_3	S_4	$\|T\|$	*Time (sec)*
12 000	6000	Entire network	65 535	55 772	7605	2114	34	10	6394
		Region 1	32 767	28 100	2309	2325	18	15	86
		Region 2	255	102	148	4	0	1	1
		Region 3	127	67	34	24	1	1	1
10 000	5000	Entire network	65 535	55 772	9445	292	21	5	6393
		Region 1	32 767	28 100	4141	489	26	11	86
		Region 2	255	102	152	0	0	1	1
		Region 3	127	67	51	7	0	2	1
8000	4000	Entire network	65 535	55 772	9002	695	58	8	6396
		Region 1	32 767	28 100	3549	985	107	26	93
		Region 2	255	102	151	1	0	1	1
		Region 3	127	67	42	10	4	4	1
7000	3500	Entire network	65 535	55 772	8491	1111	143	18	6392
		Region 1	32 767	28 100	3028	1372	220	47	112
		Region 2	255	102	150	2	0	1	1
		Region 3	127	67	38	10	8	4	1

be solved to optimality in less than 0.1 seconds. The overall reformulation process takes for that instance 6394.05 seconds: 6394 seconds are needed for the elimination of the redundant inequalities (Table 2). To provide additional information on the computational performance of CPLEX, we solved the formulation **SP1a** for the instances with 5000 scenarios. CPLEX took at least 162 minutes and on average more than three hours to solve the relaxation at the root node, and it could not find a feasible solution within four hours of computing time for any of these instances.

The computational study shows that our solution method enables the use of a large number of scenarios, and can be efficiently used to design highly reliable, fair and reasonably responsive pre-disaster relief networks.

6. Conclusions, implications, and future research

The principal goal of this study is to support relief planners in making long-term facility location and stock pre-positioning decisions that enable an efficient immediate response to sudden-onset natural disasters. To this end, we propose a new scenario-based modeling approach for the design of a reliable relief network. The stochastic nature of our models allows us to take into account the uncertainty in the severity of the disasters, their impact on the transportation network, and the demand for relief commodities. Our models feature probabilistic constraints that ensure that the demand for relief supplies across the network is satisfied with a high probability. In addition, we consider local probabilistic constraints that can be incorporated in order to guarantee a certain level of fairness and enhance the responsiveness of the system. The local constraints ensure that each region can provide for its needs with a certain probability.

Our modeling and solution methods rest on three fundamental ideas:

- We use a stochastic version of the Gale–Hoffman inequalities to characterize the probabilistic existence of a feasible flow of relief commodities in the network.
- We implement a preprocessing algorithm (Prékopa and Boros, 1991; Wallace and Wets, 1995) to eliminate the redundant Gale–Hoffman inequalities.
- We use a method based on combinatorial patterns (Lejeune, 2012a) to reformulate the stochastic programming models.

The reformulated models take the form of MIP problems that can be solved extremely quickly; more than 97% of the MIPs are solved within one second. The reason for this is that the number of binary variables is typically several orders of magnitude smaller than the number of scenarios. For example, we have reformulated one of the stochastic models for an instance featuring 20 000 scenarios as an MIP with 45 binary variables. The ability to account for a very large number of scenarios is a key contribution of our approach, allowing for a fine characterization of the underlying uncertainties, including the dependencies between various random parameters. In the current implementation, the computational bottleneck is the elimination of the redundant Gale–Hoffman inequalities; reducing the necessary computational burden for larger networks is a subject of ongoing research.

Table 3. Scalability of approach—hurricane network without local reliability constraints ($p' = 0.7$)

Problem Instances		***SP2_IP***		***SP3_IP***		*Problem Instances*		***SP2_IP***		***SP3_IP***	
$v^u_{ij} = 12\,000$, p	$v^l_{ij} = 6000$ $\lvert\Omega\rvert$	n	*Time*	n'	*Time*	$v^u_{ij} = 10\,000$, p	$v^l_{ij} = 5000$ $\lvert\Omega\rvert$	n	*Time*	n'	*Time*
0.85	5000	42	0.05	98	0.06	0.85	5000	25	0.02	59	0.03
	20 000	46	0.08	107	0.11		20 000	26	0.02	60	0.04
0.9	5000	41	0.02	97	0.03	0.9	5000	23	0.03	57	0.04
	20 000	45	0.03	106	0.05		20 000	26	0.02	60	0.04
0.95	5000	41	0.03	97	0.03	0.95	5000	20	0.06	54	0.06
	20 000	45	0.05	106	0.06		20 000	26	0.05	60	0.06
0.975	5000	40	1.73	96	1.73	0.975	5000	20	0.03	54	0.04
	20 000	44	0.36	105	0.38		20 000	25	0.02	59	0.03
0.99	5000	33	0.06	89	0.85	0.99	5000	13	0.05	47	0.05
	20 000	39	0.03	100	0.03		20 000	20	0.03	54	0.04
$v^u_{ij} = 8000$, p	$v^l_{ij} = 4000$ $\lvert\Omega\rvert$	n	*Time*	n'	*Time*	$v^u_{ij} = 7000$, p	$v^l_{ij} = 3500$ $\lvert\Omega\rvert$	n	*Time*	n'	*Time*
0.85	5000	37	0.01	99	0.02	0.85	5000	41	0.05	150	0.07
	20 000	37	0.02	104	0.04		20 000	46	0.02	164	0.06
0.9	5000	35	0.28	97	0.31	0.9	5000	40	0.02	149	0.08
	20 000	35	0.02	103	0.05		20 000	45	0.05	163	0.08
0.95	5000	33	0.03	95	0.04	0.95	5000	40	0.05	149	0.03
	20 000	35	0.08	102	0.11		20 000	45	0.03	163	0.09
0.975	5000	29	0.03	91	0.10	0.975	5000	40	0.05	149	0.06
	20 000	29	0.02	96	0.05		20 000	45	0.03	163	0.06
0.99	5000	18	0.11	80	0.15	0.99	5000	35	0.06	144	0.14
	20 000	22	0.06	89	0.07		20 000	39	0.08	157	0.12

A sensitivity analysis of our models can provide valuable managerial insights on the role of various input parameters. Our numerical results show that the facility setup and acquisition costs are mainly driven by the global reliability level. In particular, even small additional increases to high reliability levels can lead to substantial increases in cost. This highlights the need for decision-makers to perform a careful sensitivity analysis before committing to a particular global reliability level. On the other hand, while the local reliability level significantly affects the pre-positioning policies, it appears to have little impact on the total cost. This shows the possibility of significantly enhancing the responsiveness of the relief system without a substantial increase in the budget, which is a key insight for decision-makers. Our results underline the importance of pre-disaster planning, which is frequently underestimated compared with the post-disaster relief management.

A key feature of our models is that they do not require estimating the cost of unsatisfied demand. Obtaining these cost estimates may be challenging in the context of disaster management when a shortfall in relief commodities could lead to the death of human beings. We have performed a computational experiment to highlight the differences between our reliability-based models and models in which infeasibility is not constrained but is instead penalized. This experiment concerns a case study that focuses on the risk of hurricanes in the Southeastern U.S. region. The results reveal that the penalty-based benchmark model does not guarantee a highly reliable network, even with significantly increased unit shortage costs. Moreover, we have observed that our models can provide policies with higher reliability levels and lower costs than benchmark policies.

The proposed models can support relief organizations in making both strategic (facility location) and tactical (inventory level) decisions. Although facility location decisions typically imply long-term commitments, inventory level decisions are easier to adjust if more accurate information becomes available. Short-term tactical concerns are particularly relevant for certain types of natural disasters, such as hurricanes. For example, the National Hurricane Center provides predictions about the path of a hurricane about five days in advance (Galindo and Batta, 2013). Such predictions allow us to identify facilities that could be destroyed by the disaster and to update our estimates of the potential damage sustained by the transportation network. Based on this updated information, it is possible to make recourse decisions about modifying the existing relief network by repositioning supplies, either to existing facilities or to temporary locations. Studying the joint use and interaction of short-term and long-term planning approaches is an exciting direction for future research.

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Appendices

Appendix A: Elimination process

We briefly discuss the preprocessing method to eliminate the redundant Gale–Hoffman inequalities. The elimination procedure uses lower and upper bounds on the demand and capacity functions, which are assumed to be known. We denote the lower and upper bounds of the random capacity of arc (i, j) by v_{ij}^l and v_{ij}^u, respectively: $P(v_{ij}^l \le \xi_{ij}^v \le v_{ij}^u) = 1, \forall (i, j) \in A$. Recall that the random net demand at location i is obtained as $\xi_i^d - r_i$ and M_l is the capacity of a facility of type l. Denoting the largest and the smallest possible values of the demand at node i by u_i^d and l_i^d, respectively, the random net demand is bounded from below by $l_i = -\max_{l \in L} M_l$ and bounded from above by $u_i = u_i^d$. The lower and upper bounds of the random total net demand associated with a subset $B \subseteq I$ of nodes are

$$l(B) = \textstyle\sum_{i \in B} l_i, \qquad u(B) = \sum_{i \in B} u_i,$$
$$v^l(\bar{B}, B) = \sum_{i \in \bar{B}, j \in B} v_{ij}^l \quad v^u(\bar{B}, B) = \sum_{i \in \bar{B}, j \in B} v_{ij}^u.$$

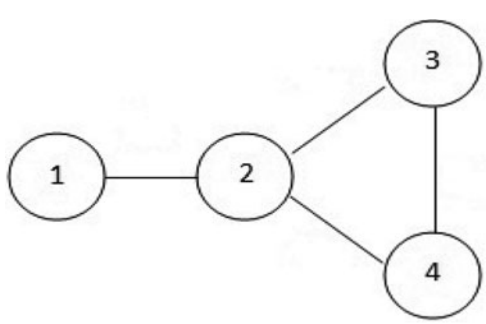

Fig. A1. Four-node network.

Table A1. Upper and lower bounds of the random demand and arc capacities for the four-node network

	Lower bound (l_i^d)	*Upper bound* (u_i^d)		*Lower bound* (v_{ij}^l)	*Upper bound* (v_{ij}^u)
ξ_1^d	−10	20	ξ_{12}^v	10	20
ξ_2^d	−25	10	ξ_{23}^v	5	10
ξ_3^d	−15	18	ξ_{24}^v	5	10
ξ_4^d	−25	10	ξ_{34}^v	5	10

Example 1. Consider the four-node network presented in Fig. A1. The lower and upper bounds of the random demand at each node and the random arc capacities are given in Table A1.

Assuming $\xi_{ij}^v = \xi_{ji}^v$ for all $i < j$ and denoting the prepositioned inventory levels by $r_i,\ i = 1, \ldots, 4$, the corresponding stochastic Gale–Hoffman inequalities are given by

$$\xi_1^d - r_1 + \xi_2^d - r_2 + \xi_3^d - r_3 + \xi_4^d - r_4 \le 0 \tag{A1}$$
$$\xi_1^d - r_1 \le \xi_{12}^v \tag{A2}$$
$$\xi_2^d - r_2 \le \xi_{12}^v + \xi_{23}^v + \xi_{24}^v \tag{A3}$$
$$\xi_3^d - r_3 \le \xi_{23}^v + \xi_{34}^v \tag{A4}$$
$$\xi_4^d - r_4 \le \xi_{24}^v + \xi_{34}^v \tag{A5}$$
$$\xi_1^d - r_1 + \xi_2^d - r_2 \le \xi_{23}^v + \xi_{24}^v \tag{A6}$$
$$\xi_1^d - r_1 + \xi_3^d - r_3 \le \xi_{12}^v + \xi_{23}^v + \xi_{34}^v \tag{A7}$$
$$\xi_1^d - r_1 + \xi_4^d - r_4 \le \xi_{12}^v + \xi_{24}^v + \xi_{34}^v \tag{A8}$$
$$\xi_2^d - r_2 + \xi_3^d - r_3 \le \xi_{12}^v + \xi_{24}^v + \xi_{34}^v \tag{A9}$$

$$\begin{array}{lclr}
\xi_2^d - r_2 + \qquad\qquad \xi_4^d - r_4 & \le & \xi_{12}^v + \xi_{23}^v + \qquad \xi_{34}^v & \text{(A10)} \\
\xi_3^d - r_3 + \xi_4^d - r_4 & \le & \xi_{23}^v + \xi_{24}^v & \text{(A11)} \\
\xi_1^d - r_1 + \xi_2^d - r_2 + \xi_3^d - r_3 & \le & \xi_{24}^v + \qquad \xi_{34}^v & \text{(A12)} \\
\xi_1^d - r_1 + \xi_2^d - r_2 + \qquad \xi_4^d - r_4 & \le & \xi_{23}^v + \qquad \xi_{34}^v & \text{(A13)} \\
\xi_1^d - r_1 + \qquad \xi_3^d - r_3 + \xi_4^d - r_4 & \le & \xi_{12}^v + \xi_{23}^v + \xi_{24}^v & \text{(A14)} \\
\xi_2^d - r_2 + \xi_3^d - r_3 + \xi_4^d - r_4 & \le & \xi_{12}^v & \text{(A15)}
\end{array}$$

◀

For this example, we can show that eight of 15 Gale–Hoffman inequalities are redundant.

The elimination procedure presented here consists of four stages; generating facets, elimination by upper bounds, elimination by lower bounds, and elimination by linear programming.

Stage 1. Generation of all the facets: We generate the facet inequalities using the recursive algorithm proposed by Wallace and Wets (1995). The method involves the introduction of a slack node with uncapacitated arcs connecting it to all other nodes. This allows us to formulate the so-called facet generating problem as a balanced supply and demand problem that is used to generate the facets of the feasible set of a network flow problem. In Stages 2, 3, and 4, we then withdraw the facets that become superfluous when we take into consideration the additional information about the lower and upper bounds of the demand and arc capacities. For the four-node network example, 12 inequalities ((A1)–(A6), (A9)–(A13), and (A15)) are generated.

Stage 2. Elimination by upper bounds: (Prékopa and Boros, 1991): In contrast to Prékopa and Boros (1991), we model the arc capacities as random variables. We refer to the Gale–Hoffman inequality corresponding to the subset of nodes $F \subseteq I$ as "inequality (F)." We eliminate an inequality (B) if $u(B) \le v^l(\bar{B}, B)$. Indeed, since

$$v^{\mathrm{l}}(\bar{B}, B) = \sum_{i\in\bar{B}, j\in B} v_{ij}^{\mathrm{l}} \le \sum_{i\in\bar{B}, j\in B} \xi_{ij}^v = v(\bar{B}, B) \quad \text{and}$$
$$u(B) \le v^l(\bar{B}, B),$$

we have

$$u(B) \le v(\bar{B}, B).$$

The three inequalities (A3), (A5), and (A10) are eliminated after executing Stage 2.

Stage 3. Elimination by lower bounds: If $l(F)$ is finite, the inequality (F) can be rewritten as

$$d(F) - l(F) \le v(\bar{F}, F) - l(F). \tag{A16}$$

If $G \subset F$, then $l(G)$ is also finite. Furthermore, if we have the inequality

$$v^u(\bar{F}, F) - l(F) \le v^l(\bar{G}, G) - l(G) \tag{A17}$$

then (A16) implies that

$$d(G) - l(G) \le v(\bar{G}, G) - l(G)$$

holds true. Indeed, the sum in $d(F) - l(F)$ has non-negative terms and thus,

$$d(G) - l(G) \le d(F) - l(F) \le v(\bar{F}, F) - l(F) \le v^u(\bar{F}, F) - l(F) \le v^l(\bar{G}, G) - l(G) \le v(\bar{G}, G) - l(G).$$

The above result is the basis to eliminate the inequalities using the lower bounds on the net demand. Let H be the collection of the subsets of I, which have not been eliminated so far. For each $F \in H$, we identify all the subsets $G \subseteq F$ for which (A17) holds. Such subsets $G \in H$ are eliminated and the set H representing the remaining inequalities are updated accordingly. The inequality (A15) is eliminated after executing Stage 3.

Stage 4. Elimination by linear programming: Suppose that the set H is composed of the subsets of I which have not been eliminated so far. To check whether a subset $F_0 \in H$ can be eliminated, we solve a linear programming problem. Let us introduce the auxiliary decision variables $z_i, i \in I$. Considering the random arc capacities, it is easy to see that the inequality (F_0) is a consequence of the other remaining inequalities (all $F \in H,\ F \ne F_0$) only when the optimum value of the following linear problem

$$\begin{aligned}
\max\ & \sum_{i\in F_0} z_i \\
\text{s.t.}\ & \sum_{i\in F} z_i \le v^l(\bar{F}, F), \quad \forall F \in H,\ F \ne F_0 \\
& l_i \le z_i \le u_i, \quad i \in I
\end{aligned}$$

is smaller than or equal to $v^l(\bar{F}_0, F_0)$. In our example, the linear programming stage eliminates the inequality (A13). Seven of the 15 inequalities remain after the elimination procedure.

Appendix B: Illustrative example for the combinatorial pattern-based reformulation method

Consider the following probabilistically constrained problem (see Lejeune, 2012a):

$$\begin{aligned}
\min\ & x_1 + 2x_2 \\
\text{s. t. } & \mathbb{P}\left\{ \begin{aligned} 8 - x_1 - 2x_2 &\ge \xi_1 \\ 8x_1 + 6x_2 &\ge \xi_2 \end{aligned} \right\} \ge 0.7 \\
& x_1, x_2 \ge 0,
\end{aligned} \tag{A18}$$

where the random vector $\boldsymbol{\xi} = [\xi_1, \xi_2]$ has 10 equally likely realizations represented by $\boldsymbol{\omega}^s = [\omega_1^s, \omega_2^s],\ s = 1, \ldots, 10$. The corresponding bivariate probability distribution is presented in Table A2.

Table A2. Probability distribution

s	ω_1^s	ω_2^s	$F_1(\omega_1^s)$	$F_2(\omega_2^s)$	$F(\omega^s)$
1	6	3	1	0.2	0.2
2	2	3	0.3	0.2	0.1
3	1	4	0.2	0.3	0.1
4	4	5	0.7	0.4	0.3
5	3	6	0.4	0.5	0.3
6	4	8	0.7	0.7	0.5
7	6	8	1	0.7	0.7
8	1	9	0.2	0.9	0.2
9	4	9	0.7	0.9	0.7
10	5	10	0.8	1	0.8

The sufficient-equivalent set of the cut points is obtained as

$$C^e = \{c_{11} = 4; c_{21} = 5; c_{31} = 6; c_{12} = 8; \\ c_{22} = 9; c_{32} = 10\}, \tag{A19}$$

which includes three cut points associated with each component ($n_1 = n_2 = 3$ and $n = 6$). The central part of Table A3 displays the binarization of the probability distribution of $\boldsymbol{\xi}$ with the set C^e. The right-hand side of Table A3 displays the truth table of the pdBf obtained with the set C^e for $p = 0.7$.

The MIP reformulation (see formulation **SP2_IP**) of the probabilistically constrained optimization problem (A18) obtained with the combinatorial pattern reformulation method is as follows:

$$\begin{aligned} z = \min\ & x_1 + 2x_2 \\ \text{s.t. } & u_{11} + u_{12} \le 1 \\ & u_{11} + u_{21} + u_{12} \le 1 \\ & 8 - x_1 - 2x_2 \ge 4u_{11} + 5u_{21} + 6u_{31} \\ & 8x_1 + 6x_2 \ge 8u_{12} + 9u_{22} + 10u_{32} \\ & u_{11} + u_{21} + u_{31} = 1 \\ & u_{12} + u_{22} + u_{32} = 1 \\ & u_{ij} \in \{0, 1\},\ i = 1, 2, 3,\ j = 1, 2 \\ & x_1, x_2 \ge 0 \end{aligned} \tag{A20}$$

Table A3. Realizations, binary images, and truth table of pdBf

	Numerical representations		Truth table of pdBf: Binarized images							
k	ω_1^k	ω_2^k	β_{11}^k	β_{21}^k	β_{31}^k	β_{12}^k	β_{22}^k	β_{32}^k	$g(k)$	
1	6	3	1	1	1	0	0	0	0	Set Ω_B^- of p-insufficient realizations
2	2	3	0	0	0	0	0	0	0	
3	1	4	0	0	0	0	0	0	0	
4	4	5	1	0	0	0	0	0	0	
5	3	6	0	0	0	0	0	0	0	
6	4	8	1	0	0	1	0	0	0	
8	1	9	0	0	0	1	1	0	0	
7	6	8	1	1	1	1	0	0	1	Set Ω_B^+ of p-sufficient realizations
9	4	9	1	0	0	1	1	0	1	
10	5	10	1	1	0	1	1	1	1	

The optimal solution is obtained as $(\mathbf{u}^*, \mathbf{x}^*) = (0, 0, 1, 1, 0, 0, 1, 0)$ with the objective value of one.

Appendix C: Scenario generation

In this section, we present a scenario generation method that takes into account the dependency structures inherent in disaster relief networks. As in Rawls and Turnquist (2010), we consider five severity levels for hurricanes. We refer to levels 1 and 5 as the least and the most severe levels, respectively. We postulate that the magnitude of the commodity demand at a node and the damage inflicted to the capacity of an arc depend on the proximity to the epicenter of the disaster. Furthermore, we assume that the epicenter is at a coastal location.

To generate a scenario, the first step is to randomly select a coastal node as the epicenter and determine the severity level of the hurricane. In the second step, we define the nodes and arcs that are affected by the disaster and its impact on them. To that end, we determine three areas A_1, A_2, and A_3 that are defined as concentric areas with radius R_1, R_2, and R_3 centered at the epicenter A of the disaster (see Fig. A2). Next, an intensity level is assigned to each concentric area. The innermost area is most severely impacted by the disaster. Nodes within R_1 (resp., within R_2 but not within R_1; within R_3 but not within R_2) are impacted with the intensity assigned to A_1 (resp., A_2; A_3). Nodes that are not within distance R_3 of the epicenter are

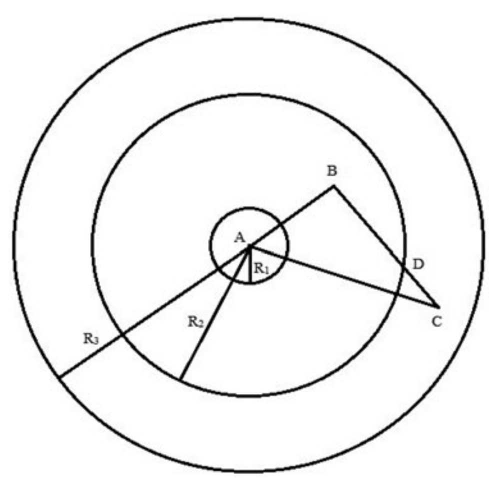

Fig. A2. Illustration of the region for the scenario generation method.

Appendix D: Detailed numerical results

Table A4. Location-allocation decisions with model **SP2_IP**.

Enforced network-wide reliability level p		0.7	0.725	0.75	0.775	0.8	0.825	0.85	0.875	0.9	0.925	0.95	0.975	0.99
Total facility setup cost		300 000	358 800	358 800	476 400	476 400	488 400	527 600	566 800	566 800	600 000	1056 800	1076 400	1317 600
Total acquisition cost		3282 540	3944 040	3944 040	4857 170	4857 170	5063 070	5504 220	5828 650	5828 650	6191 880	11658 600	11885 000	14896 500
Total cost		3582 540	4302 840	4302 840	5333 570	5333 570	5551 470	6031 820	6395 450	6395 450	6791 880	12 715 400	12961 400	16214 100
Total inventory level		5068	6089	6089	7499	7499	7817	8498	8999	8999	9560	18 000	18 350	22 999
	Number of													
Region 1	Small facilities	0	0	0	2	2	0	1	3	3	0	1	0	4
	Medium facilities	0	0	0	0	0	0	0	0	0	0	0	0	0
	Large facilities	0	0	0	0	0	0	0	0	0	0	1	2	1
Region 2	Small facilities	0	3	3	3	3	0	1	1	1	0	4	7	1
	Medium facilities	0	0	0	0	0	1	0	1	1	0	0	0	0
	Large facilities	0	1	1	1	1	0	1	0	0	1	1	0	2
Region 3	Small facilities	0	0	0	4	4	0	0	0	0	0	3	2	1
	Medium facilities	0	0	0	0	0	0	1	0	0	0	0	0	0
	Large facilities	1	0	0	0	0	1	0	1	1	1	1	1	1
Achieved region-wide reliability level (see (31))	α'_1	57.22%	57.22%	57.22%	60.66%	74.586%	57.21%	57.21%	89.64%	89.64%	57.22%	89.64%	89.64%	89.64%
	α'_2	80.97%	32.29%	32.29%	38.84%	38.84%	66.17%	80.98%	46.60%	46.60%	25.27%	88.28%	47.21%	47.21%
	α'_3	69.82%	85.29%	85.29%	85.29%	85.29%	78.48%	78.48%	78.48%	78.48%	69.82%	85.29%	93.11%	93.11%
Achieved network-wide reliability level		70.03%	75.60%	75.60%	81.66%	81.66%	83.16%	85.51%	90.73%	90.73%	92.55%	97.27%	97.79%	99.39%
Solve **RTM** where the first-stage decisions are fixed to be the ones presented above (Solve **RTM2**)														
Total first-stage cost (=Total cost of **SP2_IP**)		3582 540	4302 840	4302 840	5333 570	5333 570	5551 470	6031 820	6395 450	6395 450	6791 880	12 715 400	12 961 400	16 214 100
Expected second-stage costs	Transportation	540 794	569 931	563 112	506 961	506 961	519 050	564 073	506 215	506 215	569 772	496 427	505 266	344 756
	holding	320 937	439 436	439 436	614 628	614 628	651 304	743 109	813 291	813 291	899 855	2206 820	2325 370	3051 770
	shortage	10 437 200	8580 640	8562 180	6438 610	6438 610	5846 610	5107 330	4670 280	4670 280	4500 510	2112 030	4590 140	3531 290
Expected total second-stage cost		11 298 931	9590 007	9564 728	7560 199	7560 199	7016 964	6414 512	5989 786	5989 786	5970 137	4815 277	7420 776	6927 816
Total cost of model **RTM2**		14 881 471	13 892 847	13 867 568	12 893 769	12 893 769	12 568 434	12 446 332	12 385 236	12 3852 36	12 762 017	17 530 677	20 382 176	23 141 916
Achieved network-wide reliability level α (see (30))		69.48%	72.18%	72.18%	81.85%	81.85%	81.85%	85.51%	90.92%	90.92%	91.99%	92.55%	86.67%	93.11%

In Tables A4 to A7, the costs reported are expressed in U.S. dollars and the unit holding cost is \$647.7/4. In Tables A4 to A6, the unit shortage cost is \$6477.

Table A5. Location-allocation decisions with model **SP3_IP** - $p' = 0.60$.

Enforced network-wide reliability level p		0.7	0.725	0.75	0.775	0.8	0.825	0.85	0.875	0.9	0.925	0.95	0.975	0.99
Total facility setup cost		339 200	358 800	358 800	476 400	476 400	496 000	527 600	566 800	566 800	606 000	1056 800	1076 400	1317 600
Total acquisition cost		3282 540	3944 040	3944 040	4857 170	4857 170	5063 070	5504 220	5828 650	5828 650	6191 880	11 658 600	11 885 000	14 896 500
Total cost		3621 740	4302 840	4302 840	5333 570	5333 570	5559 070	6031 820	6395 450	6395 450	6797 880	12 715 400	12 961 400	16 214 100
Total inventory level		5068	6089	6089	7499	7499	7817	8498	8999	8999	9560	18 000	18 350	22 999
	Number of													
Region 1	Small facilities	2	2	2	5	5	6	2	3	3	3	5	2	3
	Medium facilities	0	0	0	0	0	0	0	0	0	0	0	0	0
	Large facilities	0	0	0	0	0	0	0	0	0	0	1	1	2
Region 2	Small facilities	0	0	0	1	1	1	0	1	1	0	0	6	1
	Medium facilities	0	0	0	0	0	0	1	1	1	1	0	0	0
	Large facilities	1	1	1	1	1	1	0	0	0	1	1	1	1
Region 3	Small facilities	0	1	1	3	3	3	0	0	0	3	3	1	2
	Medium facilities	0	0	0	0	0	0	0	0	0	0	0	0	0
	Large facilities	0	0	0	0	0	0	1	1	1	0	1	1	1
Achieved region-wide reliability level (see (31))	α'_1	60.66%	60.66%	60.66%	81.26%	81.26%	82.34%	60.66%	81.26%	81.26%	66.91%	89.64%	93.67%	93.67%
	α'_2	83.32%	90.89%	90.89%	84.33%	84.33%	84.33%	80.41%	66.17%	80.41%	92.96%	83.59%	91.46%	91.46%
	α'_3	69.82%	74.59%	74.59%	74.59%	74.59%	74.59%	78.48%	78.48%	78.48%	74.59%	85.29%	85.29%	85.29%
Achieved network-wide reliability level		70.03%	75.60%	75.60%	82.41%	82.41%	83.16%	85.51%	90.73%	90.73%	92.55%	97.27%	97.79%	99.39%
Solve **RTM** where the first-stage decisions are fixed to be the ones presented above (Solve **RTM2**)														
Total first-stage cost (=Total cost of **SP3_IP**)		3621 740	4302 840	4302 840	5333 570	5333 570	5559 070	6031 820	6395 450	6395 450	6797 880	12 715 400	12 961 400	16 214 100
Expected second-stage costs	Transportation	507 846	538 621	572 713	597 545	601 373	593 853	532 370	587 094	577 105	607 162	400 438	371 881	344 491
	holding	319 873	435 218	435 218	620 833	620 833	666 279	743 659	814 147	813 403	898 315	2245 180	2299 270	3009 820
	shortage	10 394 600	8393 470	8393 470	6686 790	6686 790	6445 620	5129 330	4704 490	4674 750	4438 920	3646 170	3546 310	1853 400
Expected total second-stage cost		11 222 319	9367 309	9401 401	7905 168	7908 996	7705 752	6405 359	6105 731	6065 258	5944 397	6291 788	6217 461	5207 711
Total cost of model **RTM2**		14 844 059	13 670 149	13 704 241	13 238 738	13 242 566	13 264 822	12 437 179	12 501 181	12 460 708	12 742 277	19 007 188	19 178 861	21 421 811
Achieved network-wide reliability level α (see (30))		69.47%	75.04%	75.04%	81.85%	81.85%	81.85%	84.95%	84.80%	90.17%	91.99%	93.11%	93.11%	93.11%

Table A6. Location-allocation decisions with model **SP3_IP** - $p' = 0.70$.

Enforced network-wide reliability level p		0.7	0.725	0.75	0.775	0.8	0.825	0.85	0.875	0.9	0.925	0.95	0.975	0.99
Total facility setup cost		378 400	378 400	378 400	476 400	476 400	496 000	547 200	566 800	566 800	606 000	1056 800	1076 400	1317 600
Total acquisition cost		3944 040	3944 040	3944 040	4857 170	4857 170	5063 070	5504 220	5828 650	5828 650	6191 880	11 658 600	11 885 000	14 896 500
Total cost		4322 440	4322 440	4322 440	5333 570	5333 570	5559 070	6051 420	6395 450	6395 450	6797 880	12 715 400	12 961 400	16 214 100
Total inventory level		6089	6089	6089	7499	7499	7817	8498	8999	8999	9560	18 000	18 350	22 999
	Number of													
Region 1	Small facilities	3	3	3	6	6	5	0	0	0	0	0	1	1
	Medium facilities	0	0	0	0	0	0	0	0	0	0	0	0	0
	Large facilities	0	0	0	0	0	0	1	1	1	1	1	1	2
	Number of													
Region 2	Small facilities	0	0	0	0	0	2	0	1	1	2	4	7	4
	Medium facilities	0	0	0	0	0	0	1	1	1	1	0	0	0
	Large facilities	1	1	1	1	1	1	0	0	0	0	1	1	1
Region 3	Small facilities	1	1	1	3	3	3	3	3	3	4	4	1	1
	Medium facilities	0	0	0	0	0	0	0	0	0	0	0	0	0
	Large facilities	0	0	0	0	0	0	0	0	0	0	1	1	1
Achieved region-wide reliability level (see (31))	α'_1	71.22%	71.22%	71.22%	80.70%	80.70%	81.26%	89.64%	89.64%	89.64%	89.64%	89.64%	93.67%	93.67%
	α'_2	90.89%	90.89%	90.89%	90.89%	90.89%	84.33%	80.41%	80.41%	80.41%	80.98%	91.46%	91.46%	88.28%
	α'_3	74.59%	74.59%	74.59%	74.59%	74.59%	74.59%	74.59%	74.59%	74.59%	74.59%	85.29%	85.29%	85.29%
Achieved network-wide reliability level		75.60%	75.60%	75.60%	82.41%	82.41%	83.16%	85.51%	90.73%	90.73%	92.55%	97.73%	97.79%	99.94%
Solve **RTM** where the first-stage decisions are fixed to be the ones presented above (Solve **RTM2**)														
Total first-stage cost (=Total Cost of **SP3_IP**)		4322 440	4322 440	4322 440	5333 570	5333 570	5559 070	6051 420	6395 450	6395 450	6797 880	12 715 400	12 961 400	16 214 100
Expected second-stage costs	Transportation	535 715	535 715	546 214	739 471	746 064	669 506	671 388	662 664	662 664	913 548	501 835	458 648	472 113
	holding	435 218	435 218	435 218	620 258	620 258	660 229	745 309	840 875	840 875	897 830	2266 550	2320 390	3076 550
	shortage	8393 466	8393 466	8393 466	6663 818	6663 818	6203 603	5195 322	5773 628	5773 628	4419 518	4501 230	4390 910	4522 800
Expected total second-stage costs		9364 399	9364 399	9374 898	8023 548	8030 141	7533 338	6612 019	7277 167	7277 167	6230 896	7269 615	7169 948	8071 463
Total cost of model **RTM2**		13 686 838	13 686 838	13 697 337	13 357 118	13 363 711	13 092 408	12 663 439	13 672 617	13 672 617	13 028 776	19 985 015	20 131 348	24 285 563
Achieved network-wide reliability level α (see (30))		75.04%	75.04%	75.04%	81.85%	81.85%	81.85%	84.95%	84.80%	84.80%	91.99%	86.67%	86.67%	85.92%

Table A7. Location-allocation policies obtained by model **RTM** for various values of the unit shortage Cost.

Unit shortage cost h		3886	4534	5182	5829	6477	7125	7772	8420	9068	9716
Total cost		10 088 691	10 704 270	11 271 522	11 793 649	12 278 757	12 741 014	13 202 764	13 664 514	14 126 274	14 578 082
Total inventory level		7410	7662	8166	8418	8973	8999	8999	8999	8999	9225
	Number of										
Region 1	Small facilities	3	3	3	3	0	0	0	0	0	0
	Medium facilities	0	0	0	0	0	0	0	0	0	0
	Large facilities	0	0	0	0	0	0	0	0	0	0
Region 2	Small facilities	4	5	5	5	2	3	3	3	3	3
	Medium facilities	1	1	1	1	1	1	1	1	1	1
	Large facilities	0	0	0	0	0	0	0	0	0	0
Region 3	Small facilities	0	0	2	3	0	0	0	0	0	0
	Medium facilities	1	1	1	1	2	2	2	2	2	2
	Large facilities	0	0	0	0	0	0	0	0	0	0
First-stage costs	Facility setup	514 000	533 600	572 800	592 400	604 400	624 000	624 000	624 000	624 000	624 000
	acquisition	4799 460	4962 680	5289 120	5452 340	5811 810	5828 650	5828 650	5828 650	5828 650	5975 030
Total first-stage cost		5313 460	5496 280	5861 920	6044 740	6416 210	6452 650	6452 650	6452 650	6452 650	6599 030
Expected second-stage costs	Transportation	380 860	375 650	367 191	366 228	412 275	397 122	397 122	397 122	397 122	394 464
	holding	597 441	630 120	697 431	731 651	808 372	811 972	811 972	811 972	811 972	845 448
	shortage	3796 930	4202 220	4344 980	4651 030	4641 900	5079 270	5541 020	6002 770	6464 530	6739 140
Expected total second-stage cost		4775 231	5207 990	5409 602	5748 909	5862 547	6288 364	6750 114	7211 864	7673 624	7979 052
Achieved region-wide reliability level (see (31))	α'_1	77.18%	77.18%	77.18%	77.18%	57.22%	57.22%	57.22%	57.22%	57.22%	57.22%
	α'_2	80.98%	80.98%	80.98%	80.98%	80.98%	80.98%	80.98%	80.98%	80.98%	80.98%
	α'_3	78.48%	78.48%	78.48%	78.48%	78.48%	78.48%	78.48%	78.48%	78.48%	78.48%
Achieved network-wide reliability level α (see (30))		76.85%	81.85%	83.10%	84.91%	85.51%	91.48%	91.48%	91.48%	91.48%	91.48%
Highest possible network-wide reliability level		77.41%	82.41%	83.66%	84.91%	86.11%	91.48%	91.48%	91.48%	91.48%	91.48%

not affected. Considering the dependency structure based on the proximity to the epicenter, the intensity levels of three areas under five severity levels of hurricanes are specified. The demand at a node is a function of the population size associated with the node, the severity of the disaster, and a random coefficient based on the intensity level of the area to which the node belongs. This random coefficient is generated from a uniform distribution with lower and upper limits defined according to the corresponding intensity level. To determine how the disaster affects the capacity of an arc (i, j), we verify whether (i, j) crosses the areas A_1, A_2, and A_3. If the link (i, j) does not cross A_1, A_2, and A_3, then its capacity is not affected. Otherwise, we calculate the segment c^1_{ij} (resp. c^2_{ij}, c^3_{ij}) of link (i, j) that spans over A_1 (resp., A_2, A_3) and define $\kappa^{ij} = \arg\max_l c^l_{ij}$. Thus, $A_{\kappa^{ij}}$ is the disaster-stricken area that is "most crossed" by link (i, j). We use the intensity level of $A_{\kappa^{ij}}$ to define the proportion by which the nominal capacity of link (i, j) is reduced. For example, for the capacity of the link BC (i.e., connecting B and C) in Fig. A2, the segment BD is longer than DC. Thus, the largest proportion of link BC is in area A_2 and BC is affected by the intensity level of A_2. We calculate the length of segments using the cosine formula for triangles.

Biographies

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