$$\nabla f(x,y) = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}.$$

$$\nabla f(x_1, x_2, \cdots, x_n) = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}.$$

$$f_x(x,y) = 2Ax \pm 2Ax_0.$$

$$f_y(x,y) = 2By - 2By_0.$$

$$x^T = (3\ 1\ 4) \quad y^T = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} \quad x \cdot x = 26$$

$$x \cdot y^T = 15 \quad B^T = \begin{pmatrix} 3 & 5 & 1 \\ 5 & 2 & 4 \end{pmatrix} \quad x \times y = \begin{pmatrix} 6 & 15 & 3 \\ 2 & 5 & 1 \\ 8 & 20 & 4 \end{pmatrix}$$

$$y \times x = 15 \quad A \times x = \begin{pmatrix} 25 \\ 30 \\ 34 \end{pmatrix} \quad A \times B = \begin{pmatrix} 39 & 38 \\ 19 & 37 \\ 41 & 50 \end{pmatrix}$$

B.reshape(1, 6) = (3 5 5 2 1 4)

LLS: $L(p) = \sum_{i=1}^{N} (y_i - (mx_i + b))^2 = \sum y_i^2 + m^2 \sum x_i^2 + nb^2 - 2m \sum x_i y_i$

$- 2b \sum y_i + 2mb \sum x_i$

the (m_0, b_0) that minimizes the function $L(p)$ ~~that~~ should satisfies

$$\begin{cases} \frac{\partial L(p)}{\partial m} = 0 \\ \frac{\partial L(p)}{\partial b} = 0 \end{cases} \Rightarrow \begin{cases} 2\sum x_i^2 m - 2\sum x_i y_i + 2b\sum x_i = 0. \\ 2nb - 2\sum y_i + 2m\sum x_i = 0 \end{cases}$$

$$\Rightarrow \begin{cases} m = \frac{cov(x,y)}{Var(x)} \\ b = \bar{y} - \frac{cov(x,y)}{Var(x)} \bar{x} \end{cases}$$ minimize $L(p)$

$\Rightarrow L(p)$ is convace function for m, b

For any integer n ($n \geq 2$) variable linear regression

denote $y = M(X|p) = m_1 x_1 + m_2 x_2 + \cdots + m_n x_n + b$

let $X = (1, x_1, \cdots, x_n)$ $m = (b, m_1, \cdots, m_n)^T$

so $y = Xm$

for k sets of data points $X_i = (1, x_{i1}, \cdots, x_{in})$

let $X' = (X_1^T, X_2^T, \cdots, X_k^T)^T$

the loss function $L(p) = (X'm - Y)^T (X'm - Y)$

the m that minimize $L(p)$ satisfies

$$\frac{\partial L(p)}{\partial m} = 2X'^T X'm - 2X'^T Y$$

$$\therefore m = (X'^T X')^{-1} X'^T Y.$$